CAS CS 131 Midterm

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TOTAL POINTS

95 / 100

QUESTION 1

Problem 1 20 pts

1.1 part 1 **10 / 10**

- **- 1 pts** 000 incorrect
- **- 1 pts** 001 incorrect
- **- 1 pts** 010 incorrect
- **- 1 pts** 011 incorrect
- **- 1 pts** 100 incorrect
- **- 1 pts** 101 incorrect
- **- 1 pts** 110 incorrect
- **- 1 pts** 111 incorrect
- ✓ **- 0 pts** Correct

1.2 part 2 **10 / 10**

- ✓ **- 0 pts Matches truth table**
- **- 10 pts** does not match truth table
- **- 10 pts** not CNF
- **- 4 pts** a --> not a
- **- 4 pts** terms for 1's are written

QUESTION 2

Problem 2 15 pts

2.1 part 1 **5 / 5**

- ✓ **- 0 pts** Correct
- **- 5 pts** Incorrect
- **- 2 pts** not well justified
- **- 2 pts** wrong property

2.2 part 2 **5 / 5**

- ✓ **- 0 pts** Correct
- **- 5 pts** Incorrect
- **- 2 pts** not well justified
- **- 2 pts** wrong property

2.3 part 3 **5 / 5**

- ✓ **- 0 pts** Correct
- **- 5 pts** Incorrect
- **- 2 pts** not well justified
- **- 2 pts** wrong property / misses property

QUESTION 3

Problem 3 20 pts

3.1 part 1 **9 / 9**

- **- 0 pts** Regrading
- ✓ **- 0 pts** Correct
- **- 3 pts** 1 formula is missing
- **- 6 pts** 2 formulas are missing
- **- 9 pts** 3 formulas are missing
- **- 1 pts** One mistake in the first formula
- **- 2 pts** Two mistakes in the first formula
- **- 3 pts** More than 2 mistakes in the first formula
- **- 1 pts** One mistake in the second formula
- **- 2 pts** Two mistakes in the second formula
- **- 3 pts** More than 2 mistakes in the second formula
- **- 1 pts** One mistake in the third formula
- **- 2 pts** Two mistakes in the third formula
- **- 3 pts** More than 2 mistakes in the third formula

3.2 part 2 **9 / 11**

- **- 0 pts** Correct
- **- 3 pts** No indentation for existential instantiation or missing/wrong application of existential instantiation/generalization
- **- 3 pts** No indentation for universal instantiation or missing/wrong application of universal instantiation
- **- 3 pts** No indentation for Hypothesis or no need for hypothesis or wrong application of hypothesis (or hypothesis elimination) in the proof
- **- 2 pts** One mistake in the last line (the domain of x

is anyone, not the students in the class)

✓ **- 2 pts** **Missing the domain of the variables**

- 2 pts It's not clear (or wrong) what kind of domain restriction is being done for E.I. and U.I.

- 3 pts No Premises or wrong premises

- 10 pts The student only wrote the premises or a premise as correct steps

- 2 pts A lot of missing\wrong explanation\steps

- 3 pts No/wrong conclusion

- 2 pts The student used DeMorgan once

- 11 pts Empty or the whole solution is wrong

- 3 pts The student didn't use conjunction or any rule that gives (not R(x)) and T(x)

- 1 pts One wrong explanation/symbol

QUESTION 4

Problem 4 25 pts

4.1 part 1 9 / 12

+ 0 pts Incorrect / missing

+ 2 pts Correct use of subset definition

+ 2 pts Correct use of set minus definition

+ 2 pts Correct use of conditional identity

+ 7 pts An important step is missing, but the rest is there

✓ **+ 9 pts** **All of the major correct steps are present, but also some mistake or wrong steps**

+ 11 pts All correct except a minor mistake

+ 12 pts Correct

1 not a boolean

4.2 part 2 13 / 13

+ 0 pts Incorrect

+ 2 pts Correct use of empty set definition

+ 2 pts Correct use of conditional identity (assuming the same points weren't already earned in part 4.1)

+ 2 pts Correct use of set minus (assuming the same points weren't already earned in part 4.1)

+ 7 pts Correct proof of the wrong direction; or much correct with one serious mistake

+ 9 pts All of the major correct steps are present, but also some mistakes and/or wrong steps

+ 12 pts All correct except a minor mistake

✓ **+ 13 pts** **Correct**

QUESTION 5

5 Problem 5 20 / 20

✓ **- 0 pts** **Correct**

- 20 pts no answer

- 5 pts incorrect negation of statement. You need to assume that both b and b+1 are divisible by a.

- 20 pts an example is not a proof. You cannot prove by picking specific values.

- 2 pts negation of "for all" is "exists"

- 1 pts need to set up initial claim more precisely

- 2 pts small detail to be fixed. See individual comment

- 1 pts you cannot assume b or b/a is even

- 10 pts you cannot assume "a" or "b" are a specific value

- 5 pts incorrect conclusion

- 1 pts small mistake in negation of statement: assume there exists integers a AND b

- 10 pts the integers b/a and (b+1)/a are not equal

- 18 pts you cannot prove an "exists" statement by showing that it doesn't hold for some example.

- 10 pts unfinished argument - why must a be equal to 1?

- 20 pts incorrect

- 20 pts you need to assume b and b+1 are both divisible by a

- 5 pts how do you know (b+1)/a has a remainder?

- 5 pts why is it that two consecutive integers (other than 1) don't share any divisors?

- 20 pts you cannot assume that b or b+1 are not divisible by a

- 10 pts incorrect def of divisibility

- 10 pts integers are rational numbers too. The fact that a can be expressed as the quotient of two integers doesn't mean it's not an integer itself

- 5 pts how do you know that no multiple of an odd number can divide both b and b+1?

- **20 pts** You cannot assume that b and a can be expressed by the same k
- **18 pts** for proof by contradiction, there is not one specific statement that you have to contradict and you may not be able to.
- **3 pts** why is 1/a not an integer?
- **10 pts** Why is your remainder argument true?
- **18 pts** even numbers can have odd divisors. e.g. 6 is divisible by 3
- **15 pts** you are mixing up the use of contrapoisitve
- **18 pts** Your argument did not lead to contradiction
- **10 pts** even numbers can have odd divisors. e.g. 6 is divisible by 3. You did cover all cases.
- **18 pts** arguments of odd and even are (partially) incorrect and don't cover all cases

BU CAS CS 131 Midterm

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There is a total of 100 points to be earned in 75 minutes. You can write on the front and back of each page. If you cross out some parts of your solution, make sure to clearly mark which solution we should grade. If anything is unclear, please raise your hand and ask. Please do not hand in your cheat sheet.

Problem 1. (20 pts)

1. (10 pts) Convert the following Boolean formula to a truth table $\overline{y}\,(\overline{x}+z)+x\,(y+z)$.

x	y	z	$\neg y \wedge (\neg x \vee z)$	$x \wedge (y \vee z)$	$((\neg y \wedge (\neg x \vee z)) \vee (x \wedge (y \vee z)))$
0	0	0	1	0	1
0	0	1	1	0	1
0	1	0	0	0	0
1	0	0	0	0	0
0	1	1	0	0	0
1	0	1	1	1	1
1	1	0	0	1	1
1	1	1	0	1	1

2. (10 pts) Convert the above formula to CNF. No need to explain the process; simply write down the CNF.

$\neg((\neg x \wedge y \wedge \neg z) \vee (x \wedge \neg y \wedge \neg z) \vee (\neg x \wedge y \wedge z))$

$\neg(\neg x \wedge y \wedge \neg z) \wedge \neg(x \wedge \neg y \wedge \neg z) \wedge \neg(\neg x \wedge y \wedge z)$

$(x \vee \neg y \vee z) \wedge (\neg x \vee y \vee z) \wedge (x \vee \neg y \vee \neg z)$

$\boxed{(x+\overline{y}+z)\cdot(\overline{x}+y+z)\cdot(x+\overline{y}+\overline{z})}$

Problem 2. (15 pts) A relation aRb is called an *equivalence relation* if R is reflexive, symmetric and transitive. Show that the following relations are **not** equivalence relations. *Hint: you can show something is not an equivalence relation by showing a specific example where one of the properties is violated. For each example, explain which property is violated and how.*

1. (5 pts) R is the "less or equal to" relation on real numbers. That is, for real numbers a and b, aRb if and only if $a \leq b$.

Symmetric relation is violated

example) $3R5$ is $3 \leq 5$ which is true, however $5R3$ is $5 \leq 3$ which is false.

↳ $a \leq b$ doesn't always imply $b \leq a$

2. (5 pts) R is the "subset" relation. That is, for sets X and Y, XRY if and only if $X \subseteq Y$.

Symmetric relation is violated

example) let $A = \{1,2\}$, $B = \{1,2,3\}$. ARB = True since all elements of A are in B. However, BRA is false since 3 doesn't exist inside the set A.

↳ $X \subseteq Y$ doesn't always imply $Y \subseteq X$.

3. (5 pts) R is the "xor" operation. That is, for Boolean variables p and q, pRq if and only if $p \oplus q = \text{T}$

$T \oplus F = F \oplus F = T$

Transitive relation is violated

example) let a = True, b = False, c = False. aRb gives False since $T \oplus F \equiv F$. bRc gives true since $F \oplus F \equiv T$. However, aRc gives False because $T \oplus F$ is False.

↳ aRb and bRc doesn't always imply aRc.

Problem 3. (20 pts) Take the two premises *"A student in this class has not read the textbook"* and *"Everyone in this class passed the first exam"* and the conclusion *"Someone who passed the first exam has not read the textbook."*

1. (9 pts – 3 each) Write out the three formulas expressing the statements in the above sentences using first order logic.

Let $P(x)$ = a student in this class
$M(x)$ = Read textbook
$G(x)$ = Passed the first exam

$\exists x\,(P(x) \land \neg M(x))$, $\forall x\,(P(x) \rightarrow G(x))$, $\exists x\,(G(x) \land \neg M(x))$

student in the class has not read the textbook

Everyone in this class passed the first exam

Someone who passed the first exam has not read the textbook

2. (11 pts) Give a logical proof that the conclusion follows from the premises using the inference rules.

(1) $\exists x\,(P(x) \land \neg M(x))$ Premise
(2) $\forall x\,(P(x) \rightarrow G(x))$ Premise
(3) $P(z) \rightarrow G(z)$ Universal instantiation, (2)
(4) $P(y) \land \neg M(y)$ Existential instantiation, (1)
(5) $P(y)$ Simplification, (4)
(6) $\neg M(y)$ Simplification, (4)
(7) $G(y)$ Modus Ponens (5)(3)
(8) $G(y) \land \neg M(y)$ Conjunction (6), (7)
(9) $\exists x\,(G(x) \land \neg M(x))$ Existential Generalization (8)

Problem 4. (25 pts) Suppose A, B and C are sets over the same universal set. Use the inference rules and rules of propositional logic to prove the following two statements. Provide your proofs in column format.

1. (12 pts) Prove that if $A \subseteq B$ then $A - C \subseteq B - C$.

① $A \subseteq B$ Premise
② $\forall x((x\in A)\rightarrow(x\in B))$ def. of $\subseteq$ ①
③ $(z\in A)\rightarrow(z\in B)$ Universal instantiation ②
④ $A - C$ hypothesis
⑤ $\cancel{\forall x}((x\in A)\wedge\neg(x\in C))$ def. of $-$, ④
⑥ $(y\in A)\wedge\neg(y\in C)$ universal instantiation, ⑤
⑦ $y\in A$ Simplification, ⑥
⑧ $\neg(y\in C)$ Simplification, ⑥
⑨ $y\in B$ Modus Ponens ⑦, ③
⑩ $y\in B\wedge\neg(y\in C)$ Conjunction ⑧, ⑨
⑪ $\cancel{\forall x}((x\in B)\wedge\neg(x\in C))$ Universal Generalization ⑩
⑫ $\forall x(((x\in A)\wedge\neg(x\in C))\rightarrow((x\in B)\wedge\neg(x\in C)))$ Hypothesis elimination ⑤, ⑩

unindented
↓
⑬ $\forall x(((x\in A)\wedge\neg(x\in C))\subseteq((x\in B)\wedge\neg(x\in C)))$ def. of $\subseteq$, ⑫
⑭ $A - C \subseteq B - C$ def. of $-$, ⑬

2. (13 pts) Prove that if $A - B = \emptyset$ then $A \subseteq B$.

① $A - B = \emptyset$ Premise
② $\forall x\neg(x\in(A-B))$ Def. of $=\emptyset$, ①
③ $\forall x\neg((x\in A)\wedge\neg(x\in B))$ Def. of $-$, ②
④ $\forall x(\neg(x\in A)\vee\neg\neg(x\in B))$ De Morgan's Law, ③
⑤ $\forall x(\neg(x\in A)\vee(x\in B))$ Double Negation Law, ④
⑥ $\forall x((x\in A)\rightarrow(x\in B))$ Conditional Identity, ⑤
⑦ $A\subseteq B$ Def. of $\subseteq$, ⑥

Problem 5. (20 pts) Let a and b be two integers and $a \geq 2$. Prove that b is **not** divisible by a or $(b+1)$ is **not** divisible by a. You should write your proof in paragraph form.

$\cancel{\frac{b+1}{a} = k}$ $\frac{b}{a} = k$ $\frac{b+1}{a} = n$

$b = ka$ $\frac{ka+1}{a} = n$ $k - n = \frac{1}{a}$

$ka + 1 = na$

$\cancel{(k-n)}\ 1 = (k-n)a$

proof by contradiction.

Assume that b is divisible by a and (b+1) is divisible by a. If b is divisible by a, $\frac{b}{a}$ equals to a number without a remainder. Let's say $\frac{b}{a} = k$, where k is an integer. Similarly, if b+1 is divisible by a, $\frac{b+1}{a}$ is equal to a number without remainder. Let's say $\frac{b+1}{a} = n$, where n is an integer. Since $\frac{b}{a} = k$, $b = ka$ and since $\frac{b+1}{a} = n$, $b+1 = na$. If we plot $b = ka$ to the equation $b+1 = na$, it becomes $ka + 1 = na$. Move ka to the other side by subtracting ka to both sides of equation, $1 = na - ka$. factor a out. $1 = (n-k)a$. Move a to the other side by dividing both sides by a. $\frac{1}{a} = n - k$. Since n and k are both integers, n-k is also an integer. Let's say $n - k = x$, $x = \frac{1}{a}$. In order for x to be an integer, a cannot be greater than 1 or less than -1. However, it says that $a >= 2$, meaning that a is greater than or equal to 2. Therefore, x cannot be an integer, proving that k and n cannot be integer. This shows that b is not divisible by a or (b+1) is not divisible by a, by proof using contradiction.