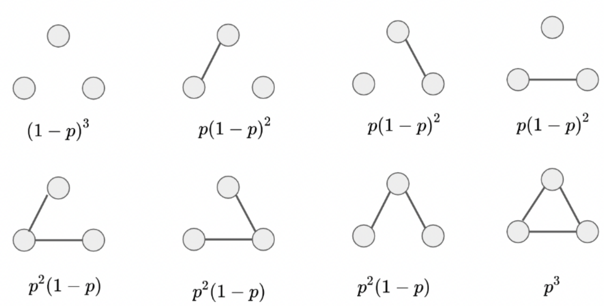
CAS CS 365
Lab 6

1. Asymptotic analysis
    a. In mathematical analysis, asymptotic analysis is a method of describing limiting behavior
    b. Formally, given functions f(x) and g(x), we define a binary relation f(x)~g(x) if and only if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1$ or $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x)(1+o(1))$
    c. For example, x^2 - 2x ~ (x+1)(x)
2. Exercise
    a. In the limit as n goes to infinity, how does (1-1/n)^(nlnn) behave?
        i. Take log and use L'Hopital's rule
        ii. Take log and use taylor series of ln(1+x)
        iii. The answer is 0 as n goes to infinity
    b. What is $\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n$
        i. This is equivalent to (1+1/n)^n, which is just e
3. G(n,p) model
    a. A labeled graph is constructed by connecting labeled nodes randomly. Each edge is included in the graph with probability p, independently from every other edge
    b. 

4. Existence of triangles
    a. Let X be the number of triangles in G(n,p)
    b. Expected number of triangles $E[X] = \binom{n}{3} p^3$
    c. To bound the probability of triangle existence

$$\Pr[X=0] \le \Pr[|X - E[X]| \ge E[X]] \le \frac{Var[X]}{E[X]^2}$$

5. Existence of triangles

   a. To get the variance, define $\triangle_{i,j,k}$ be the indicator random variable that equals to 1 if a triangle exists with vertices i, j and k. Then

$$E[X^2] = E\left[\left(\sum_{i,j,k\in[n]} \triangle_{i,j,k}\right)\left(\sum_{x,y,z\in[n]} \triangle_{x,y,z}\right)\right] = \sum_{i,j,k\in[n]} \sum_{x,y,z\in[n]} E[\triangle_{i,j,k}\triangle_{x,y,z}]$$

   b. Case 1: i, j, k share at most one vertex with x, y, z for such combinations
      i. I, j, k are independent from x, y, and z
      ii.
$$\sum_{case\ 1} E[\triangle_{i,j,k}\triangle_{x,y,z}] = \sum_{case\ 1} E[\triangle_{i,j,k}]E[\triangle_{x,y,z}]$$
$$\leq \sum_{i,j,k\in[n]} E[\triangle_{i,j,k}] \sum_{x,y,z\in[n]} E[\triangle_{x,y,z}] = E[X]^2$$

   c. Case 2: i, j, k and x, y, z share 2 nodes, for these combinations
      i. I, j, k are not independent from x, y, and z
      ii.
$$\sum_{case\ 2} E[\triangle_{i,j,k}\triangle_{x,y,z}] = \binom{n}{4}p^5$$

   d. Case 3: i, j, k and x, y, z are the same, for these combinations
      i.
$$\sum_{case\ 3} E[\triangle_{i,j,k}\triangle_{x,y,z}] = \sum_{i,j,k\in[n]} E[\triangle_{i,j,k}] = E[X]$$

   e. Combine all three cases, Var[X] = E[X^2] - E[X]^2 <= E[X] + O(1)
   f. Finally,

$$\Pr[X = 0] \leq \frac{Var[X]}{E[X]^2} \leq \frac{6}{n^3p^3}$$