CAS CS 131 HW10

Jeong Yong Yang

TOTAL POINTS

100 / 100

QUESTION 1

1 9 / 9

- ✓ **- 0 pts Correct**
- **- 3 pts** Did not count two ways for the first 4 digits
- **- 1 pts** Counted additional 3 digits
- **- 2 pts** Used P(9, 4) instead of P(10, 4)
- **- 3 pts** Used C(10, 4) instead of P(10, 4)
- **- 3 pts** Did not account for repeats in last 4 digits (i.e. used 10^4)
- **- 5 pts** Did not adequately explain usage of P(10, 4)
- **- 1 pts** Incorrect notation

QUESTION 2

2 9 / 9

- ✓ **- 0 pts Correct**
- **- 8 pts** Used P instead of C
- **- 1 pts** Added instead of multiplying

QUESTION 3

3 9 / 9

- ✓ **- 0 pts Correct**
- **- 5 pts** Incorrect explanation of answer
- **- 4 pts** Did not write C(37, 2)

QUESTION 4

4 9 / 9

- ✓ **- 0 pts Correct**
- **- 4 pts** Did not use complement rule
- **- 2 pts** All strings = 36^6
- **- 2 pts** Letter strings = 26^6
- **- 1 pts** Digit strings = 10^6

QUESTION 5

5 9 / 9

- ✓ **- 0 pts Correct**
- **- 9 pts** Incorrect
- **- 2 pts** Student didn't correctly calculate the total number of 8-bit strings
- **- 3 pts** Student didn't make observation that no two consecutive bits means alternating bits
- **- 2 pts** Student didn't observe that the number of possible strings with alternating bits is 2
- **- 2 pts** Student didn't apply complement rule

QUESTION 6

6 9 / 9

- ✓ **- 0 pts Correct**
- **- 9 pts** No submission
- **- 9 pts** Incorrect
- **- 9 pts** Correct answer but no explanation at all
- **- 3 pts** Correct work but incorrect final answer
- **- 2 pts** Error in explanation

QUESTION 7

7 9 / 9

- ✓ **- 0 pts Correct**
- **- 6 pts** Partially correct
- **- 9 pts** Incorrect answer

QUESTION 8

8 9 / 9

- ✓ **- 0 pts Correct**
- **- 4 pts** Missing counter example for 7
- **- 5 pts** Missing/Wrong explanation for 8

QUESTION 9

9 9 / 9

- ✓ **- 0 pts Correct**
- **- 3 pts** k != 12, incorrect number of holes
- **- 3 pts** b != 20, incorrect number of pigeons
- **- 3 pts** Incorrect application of PP

QUESTION 10

10 9 / 9

- ✓ **- 0 pts** Correct
- **- 3 pts** k != 7, incorrect number of holes
- **- 3 pts** b != 8, incorrect number of pigeons
- **- 3 pts** Incorrect use of PP

QUESTION 11

11 10 / 10

- ✓ **- 0 pts** correct
- **- 5 pts** incorrect application of formula (n=25; m=6)
- **- 5 pts** incorrect calculation for n+m-1 choose m-1
- **- 5 pts** incorrect number for 1's and 0's
- **- 2 pts** incorrect total length of string
- **- 3 pts** incorrect number of strings
- **- 10 pts** missing

① 11.4.2(b)

Since you can start with 824 or 825, there are two choices on how it starts. There are 4-digits that are remaining after 3 digits have been chosen. Those 4-digits can be any number from 0-9, having total of 10 choices. However, they all have to be different and order in which it goes matters since 9786 and 9768 are considered different. Therefore, we need to apply permutation. Because we are choosing 4 out of 10 possibilities, it is $P(10, 4)$. Applying the product rule, we need to multiply 2 and $P(10,4)$.

$\boxed{\text{Answer: } 2 \cdot P(10,4)}$

② 11.5.5 (a)

There are 30 boys and 35 girls. Since we are selecting 10 out of 35 girls, and 10 out of 30 boys, we need to consider boys and girls differently and multiply them by applying the product rule. When selecting 10 out of 35 girls, the order does not matter, and same applies for selecting 10 out of 30 boys. For girls, it is $C(35,10)$ and boys, it is $C(30,10)$. It is combination since the order in which you pick the 10 doesn't matter. By applying product rule, it is $C(35,10) \cdot C(30,10)$.

$\boxed{\text{Answer: } C(35,10) \cdot C(30,10)}$.

1 9 / 9

- ✓ **- 0 pts** Correct
- **- 3 pts** Did not count two ways for the first 4 digits
- **- 1 pts** Counted additional 3 digits
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✓

① 11.4.2(b)

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$\boxed{\text{Answer: } C(35,10) \cdot C(30,10).}$

2 9 / 9

- ✓ **- 0 pts** Correct
- **- 8 pts** Used P instead of C
- **- 1 pts** Added instead of multiplying

③ 11.5.6 (b)

Since 3 of the 40 computers in the network have a copy of a particular file, there are $40-3=37$ computers that do not have a copy. Out of 5-subsets, if 3 of them have a copy of a particular file, then there are $5-3=2$ of them that do not have a copy. Since we are choosing 2 out of 37 spots that are left for the computers that do not have a copy in which the order in how we choose the subset doesn't matter, we use combination. Therefore, it is $C(37, 2)$.

Answer: $C(37,2)$

④ 11.7.1(b)

There are total of 10 digits and 26 lowercase letters. Therefore, each place can have a total of 36 candidates. Since the length of the password is six, there are total of 36^6 possibilities of password. However, there is a restriction that each password must have at least one digit or at least one character, we can get the total number by subtracting the cases where all places of length 6 password is consisted entirely with digits (no characters) and entirely with characters (no digits). If there are only digits in the password, there are total of 10 candidates. Therefore, it has 10^6 possibilities while if there are no digits, there are total of 26 candidates. Therefore, it has 26^6 possibilities, which means that length 6 password with no digits or no characters has 26^6+10^6 choices. We must subtract these complements from the total number of choices, which is 36^6. Therefore, the answer is $36^6-(26^6+10^6)$

$$\boxed{\text{Answer: } 36^6-(26^6+10^6) \text{ or } 36^6-26^6-10^6}$$

3 **9 / 9**

✓ **- 0 pts** Correct

- 5 pts Incorrect explanation of answer

- 4 pts Did not write C(37, 2)

③ 11.5.6 (b)

Since 3 of the 40 computers in the network have a copy of a particular file, there are $40-3=37$ computers that do not have a copy. Out of 5-subsets, if 3 of them have a copy of a particular file, then there are $5-3=2$ of them that do not have a copy. Since we are choosing 2 out of 37 spots that are left for the computers that do not have a copy in which the order in how we choose the subset doesn't matter, we use combination. Therefore, it is $C(37, 2)$.

Answer: $C(37,2)$

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$\boxed{\text{Answer: } 36^6-(26^6+10^6) \text{ or } 36^6-26^6-10^6}$

4 9 / 9

- ✓ **- 0 pts** Correct
- **- 4 pts** Did not use complement rule
- **- 2 pts** All strings = 36^6
- **- 2 pts** Letter strings = 26^6
- **- 1 pts** Digit strings = 10^6

(5) 11.7.3 (a)

If there are bit strings of length 8, there are total of 2^8 possibilities since bit strings only have two possibilities: 1 or 0. If we want to count bit strings of length 8 that have at least two consecutive 1's or two consecutive 0's, we can subtract the cases where there is not two consecutive 0's and 1's from the total number of choices. Bit strings that don't have two consecutive 0's or 1's must always alternate from 0 to 1, which means it has to be 1010101o or 01010101, which is 2 choices. Therefore, from the total of 2^8 choices, if we subtract 2, it is 2^8-2.

Answer: 2^8-2.

(6) 11.8.1. (c)

Since SUBSETS have the length 7, there are total of $7!$ possibilities or $P(7,7)$, we use permutation here because the order does matter → SUBSETS is different from SUBSEST. Since the letter 'S' repeats 3 times, if we apply the permutation with repetition, we need to divide $7!$ by $3!$, which is $7!/3!$

$\boxed{\text{Answer: } \frac{7!}{3!}}$

(7) 11.8.7 (b)

If she wants to cook the meal same amount of times, each meal is made $20/10=2$ times. Therefore, out of 20 meals, each 10 meals are given twice each, which means there are 10 repetitions of 2. Since there are 20 meals provided total, it is total of $20!$ possible meals that can be given. By applying permutation with repetition, there are $20!/(2!)^{10}$, which is equivalent to $20!/(2^{10})$.

Answer: $\left(\frac{20!}{2^{10}}\right)$

5 9 / 9

✓ **- 0 pts** Correct

- 9 pts Incorrect

- 2 pts Student didn't correctly calculate the total number of 8-bit strings

- 3 pts Student didn't make observation that no two consecutive bits means alternating bits

- 2 pts Student didn't observe that the number of possible strings with alternating bits is 2

- 2 pts Student didn't apply complement rule

⑤ 11.7.3 (a)

If there are bit strings of length 8, there are total of 2^8 possibilities since bit strings only have two possibilities: 1 or 0. If we want to count bit strings of length 8 that have at least two consecutive 1's or two consecutive 0's, we can subtract the cases where there is not two consecutive 0's and 1's from the total number of choices. Bit strings that don't have two consecutive 0's or 1's must always alternate from 0 to 1, which means it has to be 1010l010 or 0101010l, which is 2 choices. Therefore, from the total of 2^8 choices, if we subtract 2, it is 2^8-2.

Answer: 2^8-2.

⑥ 11.8.1. (c)

Since SUBSETS have the length 7, there are total of 7! possibilities or P(7,7), we use permutation here because the order does matter → SUBSETS is different from SUBSEST. Since the letter 'S' repeats 3 times, if we apply the permutation with repetition, we need to divide 7! by 3!, which is $7!/3!$

$\boxed{\text{Answer: } \frac{7!}{3!}}$

⑦ 11.8.7 (b)

If she wants to cook the meal same amount of times, each meal is made $20/10=2$ times. Therefore, out of 20 meals, each 10 meals are given twice each, which means there are 10 repetitions of 2. Since there are 20 meals provided total, it is total of 20! possible meals that can be given. By applying permutation with repetition, there are $20!/(2!)^{10}$, which is equivalent to $20!/(2^{10})$.

Answer: $\left(\frac{20!}{2^{10}}\right)$

6 9 / 9

✓ **- 0 pts** **Correct**

- 9 pts No submission

- 9 pts Incorrect

- 9 pts Correct answer but no explanation at all

- 3 pts Correct work but incorrect final answer

- 2 pts Error in explanation

⑤ 11.7.3 (a)

If there are bit strings of length 8, there are total of 2^8 possibilities since bit strings only have two possibilities: 1 or 0. If we want to count bit strings of length 8 that have at least two consecutive 1's or two consecutive 0's, we can subtract the cases where there is not two consecutive 0's and 1's from the total number of choices. Bit strings that don't have two consecutive 0's or 1's must always alternate from 0 to 1, which means it has to be 1010 1010 or 01010101, which is 2 choices. Therefore, from the total of 2^8 choices, if we subtract 2, it is 2^8-2.

Answer: 2^8-2.

⑥ 11.8.1. (c)

Since SUBSETS have the length 7, there are total of 7! possibilities or $P(7,7)$, we use permutation here because the order does matter → SUBSETS is different from SUBSEST. Since the letter 'S' repeats 3 times, if we apply the permutation with repetition, we need to divide 7! by 3!, which is $7!/3!$

$\boxed{\text{Answer: } \frac{7!}{3!}}$

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If she wants to cook the meal same amount of times, each meal is made $20/10=2$ times. Therefore, out of 20 meals, each 10 meals are given twice each, which means there are 10 repetitions of 2. Since there are 20 meals provided total, it is total of 20! possible meals that can be given. By applying permutation with repetition, there are $20!/(2!)^{10}$, which is equivalent to $20!/(2^{10})$.

Answer: $\left(\frac{20!}{2^{10}}\right)$

7 9 / 9

- ✓ **- 0 pts** Correct
- **- 6 pts** Partially correct
- **- 9 pts** Incorrect answer

⑧ 11.9.1. (b)

If all the 7 or 8 members are faster than 7 minutes and slower than 6 minutes, they all fall in between 6 to 7 minutes. Therefore, each runner cannot be 60 seconds apart from each other since 60 seconds equals to a minute. This is because if it exceeds a minute, at least one of the members are faster than 6 minutes, which is given to be false. In order to have two runners that are 9 seconds apart, we need $(7-1) = 6 \cdot 9$ seconds, which is equivalent to 54 seconds maximum between the fastest and slowest runner out of 7 runners. For example, if the slowest runner completes the race in 6min 59seconds, the fastest runner completes it on $59-54 =$ 6min 5 seconds if there are 9 seconds apart from each runner. According to the pigeonhole principle, there are target of size is 58 seconds (6min 59s - 6min 1s = 58s) with domain of size being 54, it doesn't necessarily have a map that has two elements in the same target.

If there 8 runners, the answer changes. In this case, in order to have two runners that are 9 seconds apart, we need $(8-1) \cdot 9 = 63$ seconds, which is a minute and 3 seconds. If the slowest runner completes in 6min 59 seconds, the fastest runner completes the race in at least 6min 59s - 1min 3s = 5min 56seconds, which is faster than 6 minutes, which is false. According to the pigeonhole principle, there are target size of 58 seconds with domain of size being 63, which means that there must be two elements in the domain that map to the same element in the target, or two runners that do not have a 9 seconds gap in between them.

8 **9 / 9**

- ✓ **- 0 pts** Correct
- **- 4 pts** Missing counter example for 7
- **- 5 pts** Missing/Wrong explanation for 8

(9) 11.9.3 (b)

When you apply the pigeonhole principle for 12 months, if there are 13 people in total, you are guaranteed to have two people that share the same birthday month. When you want at least 20 people that share the same birthday month, you can apply the contrapositive of the generalized pigeonhole principle, there are 12 elements, 20 items, which is $12(20-1)+1 = 229$

$\boxed{\text{Answer: 229 ways}}$

(10) 11.9.4 (a)

Out of the set $\{1,2,3,4,\ldots,13,14\}$, there are 7 sets that add up to 15, which are $\{1,14\}, \{2,13\}, \{3,12\}, \{4,11\}, \{5,10\}, \{6,9\}, \{7,8\}$. We are choosing 8 numbers from the 7 sets, and therefore, according to the pigeonhole principle, one set of the 7 sets is guaranteed to occur. Therefore, two of the eight numbers chosen must sum up to 15.

(11) 12.4.2 (a)

We can express this question into a binary string. We can express 25 as having 25 0s and $(6-1)=5$ 1s that divide up the 25 0s. Therefore, the binary string has length of 30, consisted of 25 0s and 5 1s. Since each x_1, x_2, x_3, x_4, x_5, and x_6 can be 0, there doesn't exist any restrictions, on how you place the 1s. By applying permutation with repetition since order matters, (0011... is different from 1100....) it is $30!/(25! * 5!)$, where 25! comes from 25 0s and 5! comes from 5 1s.

$\boxed{\text{Answer: } \frac{30!}{25!5!}}$

9 9 / 9

- ✓ **- 0 pts** Correct
- **- 3 pts** k != 12, incorrect number of holes
- **- 3 pts** b != 20, incorrect number of pigeons
- **- 3 pts** Incorrect application of PP

⑨ 11.9.3 (b)

When you apply the pigeonhole principle for 12 months, if there are 13 people in total, you are guaranteed to have two people that share the same birthday month. When you want at least 20 people that share the same birthday month, you can apply the contrapositive of the generalized pigeonhole principle, there are 12 elements, 20 items, which is $12(20-1)+1 = 229$

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$$\boxed{\text{Answer: } \frac{30!}{25!\,5!}}$$

10 9 / 9

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- **- 3 pts** b != 8, incorrect number of pigeons
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11 10 / 10

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