

X2 & Z2

1. Higher order function Review

- a. `int1_forall(n, fn i =>`
 `if i < 2 then true`
 `else (n mod i <> 0))`
 `(* int1_forall to test if the value n is prime or not *)`

2. “Cross product”

- a. Ex) `xs = [1,2]`
 `ys = [a,b,c]`
 `cross(xs, ys) = [(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)]`

3. X2 and X3

- a. `fun list_cross2(xs: 'a list, ys: 'b list): ('a * 'b) list =`
 `list_map(xs, fn x => list_map(ys, fn y => (x,y)))`
 `(* for each y, encounter the pair (x,y) *)`
 `(* when we see x, we traverse the map and when we see y, get pair (x,y) *)`
 `(* the output we get is ('a * 'b) list list so we need to flatten it *)`
 `(* [(1,a), (1,b), (1,c)], [(2,a), (2,b), (2,c)] → need to delete inner list *)`
- b. Therefore, need to do
 `fun list_cross2(xs: 'a list, ys: 'b list): ('a * 'b) list =`
 `list_concat(list_map(xs, fn x => list_map(ys, fn y => (x,y))))`
- c. `fun list_enumerate(xs: 'a list): (int * 'a) list =`
 `list_reverse(`
 `#2(list_foldl(xs, (0, nil), fn(i,r), x) => (i + 1, (i,x) :: r)))`
 `)`
- d. `fun list_concat(xss: 'a list list): 'a list =`
 `list_foldr(xss, [], fn(xs, res) => list_append(xs,res))`
 OR
 `case xss of`
 `nil => nil`
 `| xss1: xss => list_append(xss1, list_concat(xss))`
- e. `fun list_zip2(xs: 'a list, ys: 'b list, fopr: 'a * 'b -> 'c): 'c list =`
 `list_map(list_zip2(xs,ys), fopr)`
- f. `fun list_x2map(xs: 'a list, ys: 'b list, fopr: 'a * 'b -> 'c): 'c list =`
 `list_map(list_cross2(xs,ys), fopr)`
- g. `fun list_filter(xs: 'a list, test: 'a -> 'bool): 'a list =`
 `list_foldr(xs,[],fn(x1, res) => if test(x1) then x1 :: res else res)`

- h. `fun list_z2forall(xs: 'a list, ys: 'b list, test: 'a * 'b -> bool): bool =
list_forall(list_zip2(xs,ys), test)`
 - i. `fun list_x2forall(xs: 'a list, ys: 'b list, test: 'a * 'b -> bool): bool =
list_forall(list_cross2(xs,ys), test)`
 - j. `fun list_x2exists(xs: 'a list, ys: 'b list, test : 'a * 'b -> bool): bool =
list_exists(list_cross2(xs,ys),test)`
4. Python library
- a. `def int1_forall(n0, test_func):
i0 = 0
while(i0 < n0):
if not test_func(i0):
return False
i0 = (i0 + 1)
return True`
 - b. `def int1_foreach(n0, work_func):
i0 = 0
while(i0 < n0):
work_func(i0)
i0 = i0 + 1
return None`
 - c. `def int1_rforeach(n0, work_func):
i0 = 0
while(i0 < n0):
work_func(n0-1-i0)
i0 = i0 + 1
return None`
 - d. `def int1_map_fnlist(xs, fopr_func):
return foreach_to_map_fnlist(int1_foreach)(xs, fopr_func)`
 - e. `def int1_map_pylist(xs, fopr_func):
return foreach_to_map_pylist(int1_foreach)(xs, fopr_func)`
 - f. `def foreach_to_map_pylist(foreach):
def map_pylist(xs, fopr_func):
res = []
def work_func(x0):
nonlocal res # want to access variable that is outside
res.append(fopr_func(x0))
return None
foreach(xs, work_func)
return res
return map_pylist`

- g. `def int1_foldleft(xs,r0, fopr_func):`
 `return foreach_to_foldleft(int1_foreach)(xs, r0, fopr_func)`
- h. `def in1_foldright(xs, r0, fopr_func):`
 `return rforeach_to_foldright(int1_foreach)(xs, r0, fopr_func)`
- i.