1 (a) 1 $\neg r \to \neg s$ premise

2 $p \to u$ premise

3 $\neg t \to \neg r$ premise

4 $u \to s$ premise

5 $t \to q$ premise

6 $\neg t \to \neg s$ hypothetical syllogism 3, 1

7 $p \to s$ hypothetical syllogism 2, 4

8 $\neg\neg s \to \neg\neg t$ Contrapositive Rule 6

9 $s \to t$ Double negation Laws 8

10 $p \to t$ hypothetical syllogism 7, 9

$\therefore p \to q$ hypothetical syllogism 10, 5

(b) 1 $p \to (q \land r)$ premise

2 $s \to r$ premise

3 $r \to p$ premise

4 $s \to p$ hypothetical syllogism 2, 3

5 $p \to q$ Conditional simplification 1

$\therefore s \to q$ hypothetical syllogism 4, 5

② ① $P(A\cup B) \neq P(A)\cup P(B)$

Assume $A=\{1,3,5\}$ and $B=\{1,4,\}$

$A\cup B=\{1,3,4,5\}$

$P(A\cup B)=\{\emptyset, \{1\}, \{3\}, \{4\}, \{5\}, \{1,3\} \{1,4\} \{1,5\} \{3,4\}, \{3,5\},$
$\{4,5\} \{1,3,4\} \{1,3,5\} \{1,4,5\} \{3,4,5\}, \{1,3,4,5\}\}$

$|P(A\cup B)| = 2^{|(A\cup B)|} = 2^4 = 16$

$P(A)=\{\emptyset, \{1\}, \{3\} \{5\}, \{1,3\}, \{1,5\} \{3,5\}, \{1,3,5\}\}$

$P(B)=\{\emptyset, \{1\}, \{4\}, \{1,4\}\}$

$P(A)\cup P(B)=\{\emptyset, \{1\}, \{3\}, \{5\}, \{1,3\}, \{1,5\}, \{3,5\}, \{1,3,5\}, \{4\}, \{1,4\}\}$

$|P(A)\cup P(B)| = 10$

$P(A\cup B)$ includes the set $\{3,4\}, \{4,5\}, \{1,3,4\}, \{1,4,5\}, \{3,4,5\},$
$\{1,3,4,5\}$, and all the sets in $P(A)\cup P(B)$.

Since $P(A)\cup P(B)$ not only contains the same amount of sets as $P(A\cup B)$ but $P(A\cup B)$ also has sets that are not in $P(A)\cup P(B)$, $P(A)\cup P(B) \neq P(A\cup B)$

↳ Moreover, $P(A)\cup P(B)$ is a subset of $P(A\cup B)$

$P(A)\cup P(B) \subseteq P(A\cup B)$

Therefore, $P(A)\cup P(B)$ does not always equal $P(A\cup B)$

② ② $P(A)\cap P(B) = P(A\cap B)$

claim 1 : $P(A)\cap P(B) \subseteq P(A\cap B)$

claim 2 : $P(A\cap B) \subseteq P(A)\cap P(B)$

Since both $S\in P(A)\cap P(B) \to S\in P(A\cap B)$ and $S\in P(A\cap B) \to S\in P(A)\cap P(B)$ are true, the two equations must be equal

$\to P(A)\cap P(B) = P(A\cap B)$

Claim 1: $P(A)\cap P(B) \subseteq P(A\cap B)$

1. $S\in P(A)\cap P(B)$ Premise
2. $S\in P(A) \wedge S\in P(B)$ Definition of $\wedge$ 2
3. $S\in P(A)$ Simplification 2
4. $S\in P(B)$ simplification 2
5. $S\subseteq A$ definition of Powerset 3
6. $S\subseteq B$ definition of Powerset 4
7. $x\in S\to x\in A$ definition of $\subseteq$ 5
8. $x\in S\to x\in B$ definition of $\subseteq$ 6
9. $x\in S\to x\in A \wedge x\in B$ conditional Conjunction 7, 8
10. $x\in S\to x\in (A\cap B)$ definition of $\cap$ 9
11. $S\subseteq (A\cap B)$ definition of $\subseteq$ 10
12. $S\in P(A\cap B)$ definition of Power set 11
13. $S\in P(A)\cap P(B) \to S\in P(A\cap B)$ hypothesis elimination 1, 13

Claim 2: $P(A\cap B) \subseteq P(A)\cap P(B)$

1. $S\in P(A\cap B)$ Premise
2. $S\subseteq (A\cap B)$ definition of Powerset 1
3. $x\in S\to x\in (A\cap B)$ definition of $\subseteq$ 2
4. $x\in S\to((x\in A)\wedge(x\in B))$ definition of $\cap$ 3
5. $(x\in S\to(x\in A))\wedge(x\in S\to(x\in B))$ conditional Conjunction 4
6. $x\in S\to(x\in A)$ Simplification 5
7. $x\in S\to(x\in B)$ Simplification 5
8. $S\subseteq A$ definition of $\subseteq$ 6
9. $S\subseteq B$ definition of $\subseteq$ 7
10. $S\in P(A)$ definition of Powerset 8
11. $S\in P(B)$ definition of Powerset 9
12. $S\in P(A)\cap P(B)$ Conjunction 10, 11
13. $S\in P(A\cap B) \to S\in P(A)\cap P(B)$ hypothesis elimination 1, 12