Worksheet 18

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Topics

- Linear Regression

Optional Challenge

Every day my alarm goes off at seemingly random times… I've recorded the times at which it goes off for the past year of so (1 - 355 days). Today is day 356. Can you predict when my alarm will ring using `data.csv`?

Please fill out the piazza poll if you think you found the answer.

Linear Regression

Where does randomness come from?

```
1  pip install ipympl
```

```
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  Downloading ipympl-0.9.3-py2.py3-none-any.whl (511 kB)
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```
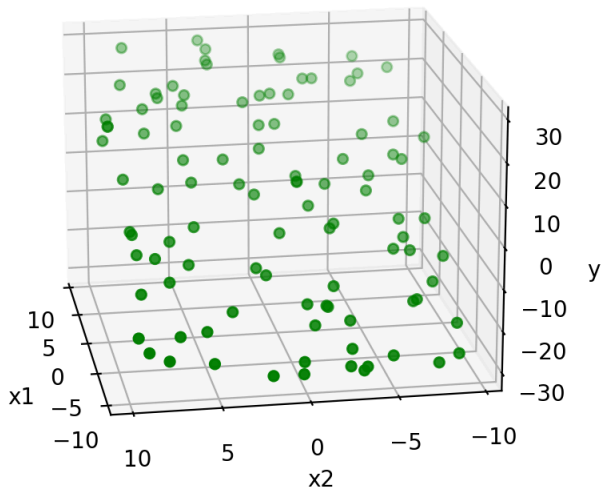

```
from google.colab import output
output.enable_custom_widget_manager()
```

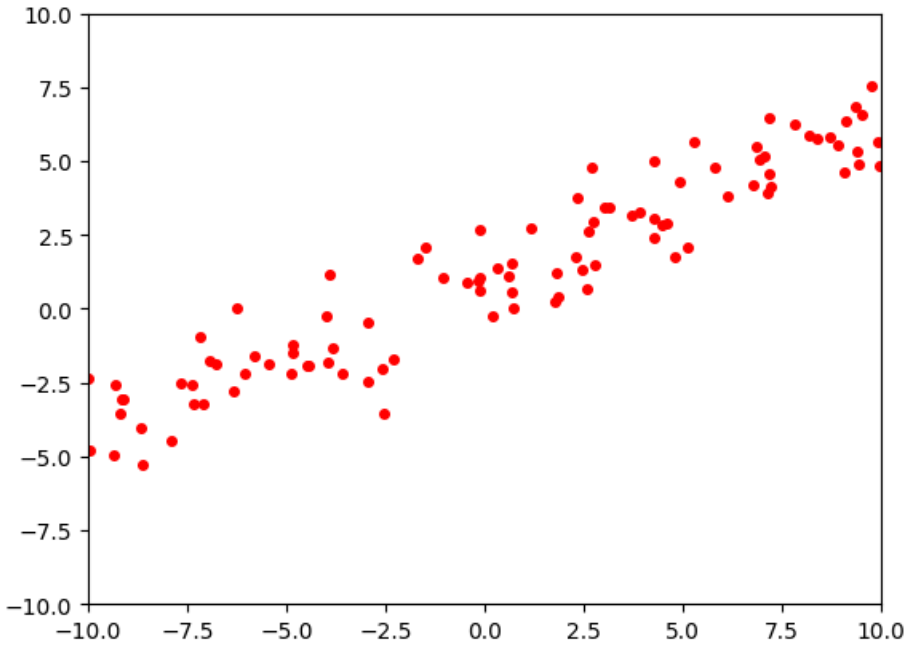
```
%matplotlib widget
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

SAMPLE_SIZE = 100

# it's possible for y = 3 * x_1 + (1/4) * x_2
# but we don't know to look for x_2 and we only have x_1 as a feature
x1 = -10.0 + 20.0 * np.random.random(SAMPLE_SIZE)
x2 = -10.0 + 20.0 * np.random.random(SAMPLE_SIZE)
y = 3 * x1 + (1/4) * x2

# Create the figure
fig = plt.figure()

# Add an axes
ax = fig.add_subplot(111,projection='3d')

# and plot the point
ax.scatter(x1 , x2 , y,  color='green')
ax.set_xlabel("x1")
ax.set_ylabel("x2")
ax.set_zlabel("y")
plt.show()
```

Figure 1



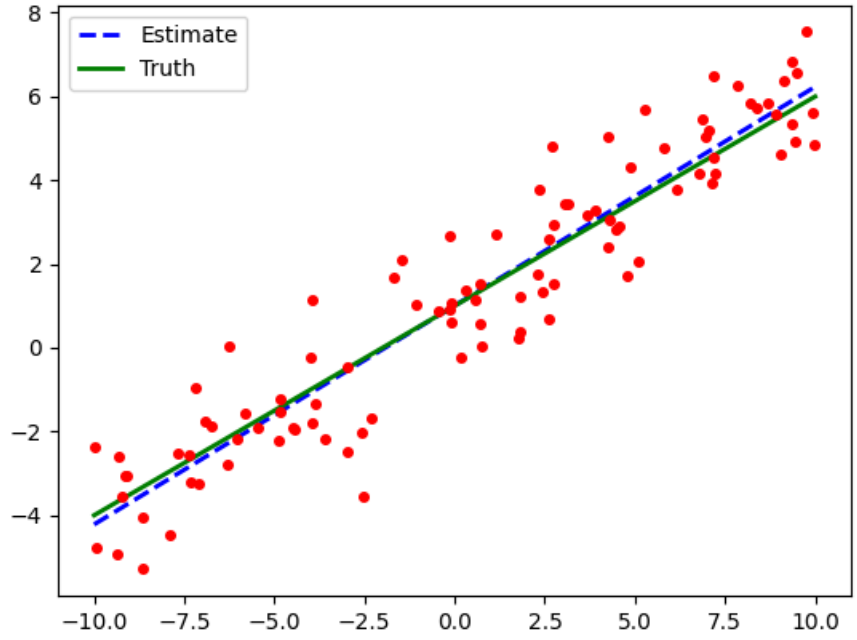
a) Create and plot a dataset of 100 (Y, X) points where `Y = 1 + 0.5 * X + eps`.

```
import numpy as np
import matplotlib.pyplot as plt

SAMPLE_SIZE = 100

fig = plt.figure()
ax = fig.subplots()

true_beta = np.array([ 1 , 0.5 ])
x = -10.0 + 20.0 * np.random.random(SAMPLE_SIZE)
y = true_beta[0] + true_beta[1] * x + np.random.randn(SAMPLE_SIZE)
"""
x_plot = np.linspace(-10,10,50)
plt.plot(x_plot, true_beta[0] + true_beta[1] * x_plot, lw=2, c='g', label='Truth')
"""

plt.plot(x,y,'ro',markersize=4)
ax.set_xlim(-10, 10)
ax.set_ylim(-10,10)
plt.show()
```



b) Plot the least squares estimate line through the scatter plot.

```
intercept = np.ones(np.shape(x)[0])
X = np.array([intercept, x]).T
beta_hat = np.linalg.inv(X.T @ X) @ X.T @ y

x_plot = np.linspace(-10,10,50)
y_est = beta_hat[0] + beta_hat[1] * x_plot
plt.plot(x_plot, y_est,'b--', lw=2, label='Estimate')
plt.plot(x_plot, true_beta[0] + true_beta[1] * x_plot, lw=2, c='g', label='Truth')
plt.plot(x, y,'ro',markersize=4)
plt.legend()

plt.show()
```



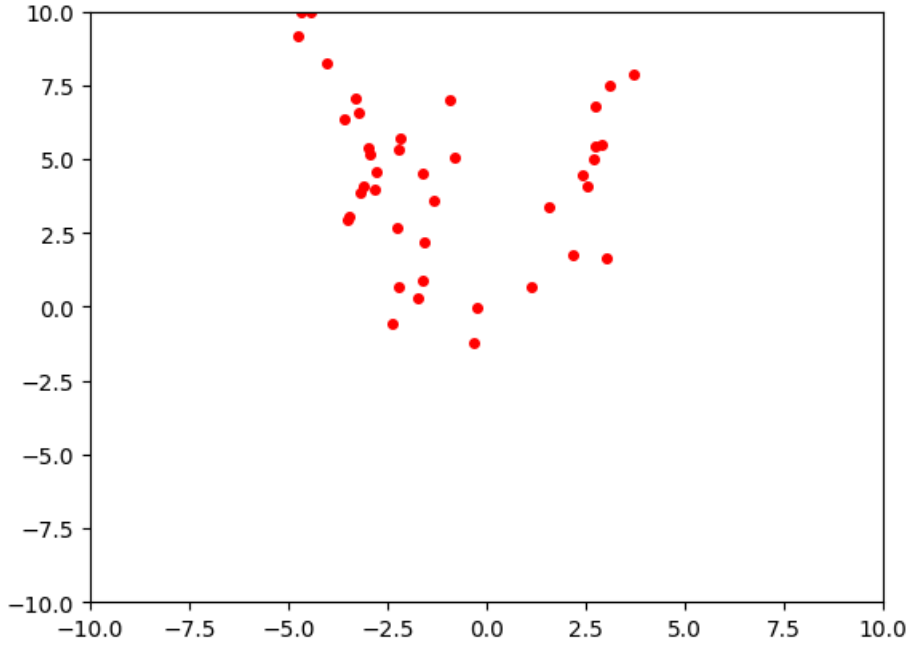
c) How does the estimate of `beta` compare to the parameters we used to generate the data?

```
print(f"The true beta is {true_beta}")
print(f"The estimated beta is {beta_hat}")
```

```
The true beta is [1.  0.5]
The estimated beta is [1.00651849 0.52254326]
```

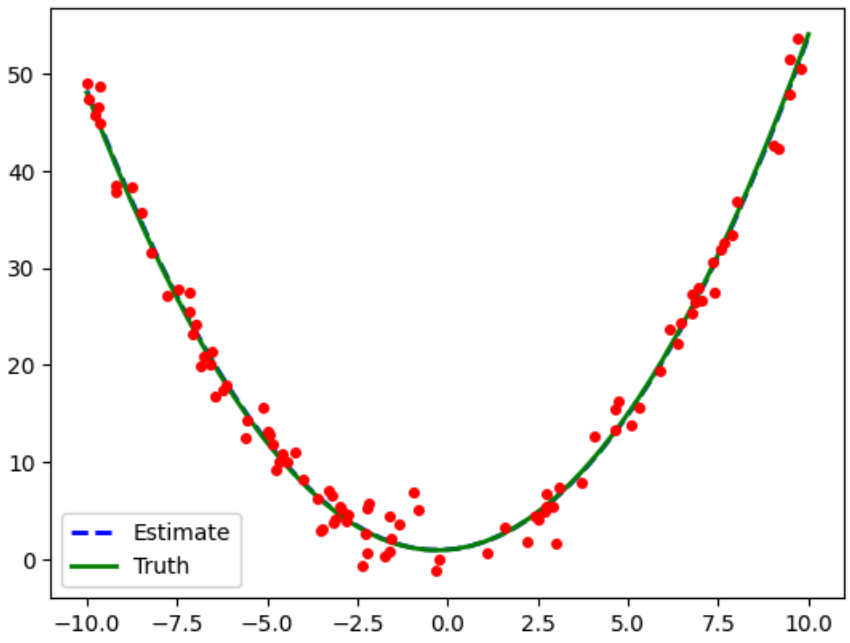
d) Create and plot a dataset of 100 (Y, X) points where `Y = 1 + 3 * X + .5 * X^2 + eps`.

```
import numpy as np
import matplotlib.pyplot as plt

SAMPLE_SIZE = 100

fig = plt.figure()
ax = fig.subplots()

true_beta = np.array([ 1 , 0.3, 0.5 ])
x = -10.0 + 20.0 * np.random.random(SAMPLE_SIZE)
y = true_beta[0] + true_beta[1] * x + true_beta[2] * (x**2) + np.random.randn(SAMPLE_SIZE) * 2
"""
x_plot = np.linspace(-10,10,50)
plt.plot(x_plot, true_beta[0] + true_beta[1] * x_plot, lw=2, c='g', label='Truth')
"""

plt.plot(x,y,'ro',markersize=4)
ax.set_xlim(-10, 10)
ax.set_ylim(-10,10)
plt.show()
```



e) Plot the least squares estimate line through the scatter plot.

```
intercept = np.ones(np.shape(x)[0])
X = np.array([intercept, x, x**2]).T
beta_hat = np.linalg.inv(X.T @ X) @ X.T @ y

x_plot = np.linspace(-10,10,50)
y_est = beta_hat[0] + beta_hat[1] * x_plot + beta_hat[2] * (x_plot ** 2)
plt.plot(x_plot, y_est,'b--', lw=2, label='Estimate')
plt.plot(x_plot, true_beta[0] + true_beta[1] * x_plot + true_beta[2] * (x_plot ** 2), lw=2, c='g', label='Truth')
plt.plot(x, y,'ro',markersize=4)
plt.legend()

plt.show()
```



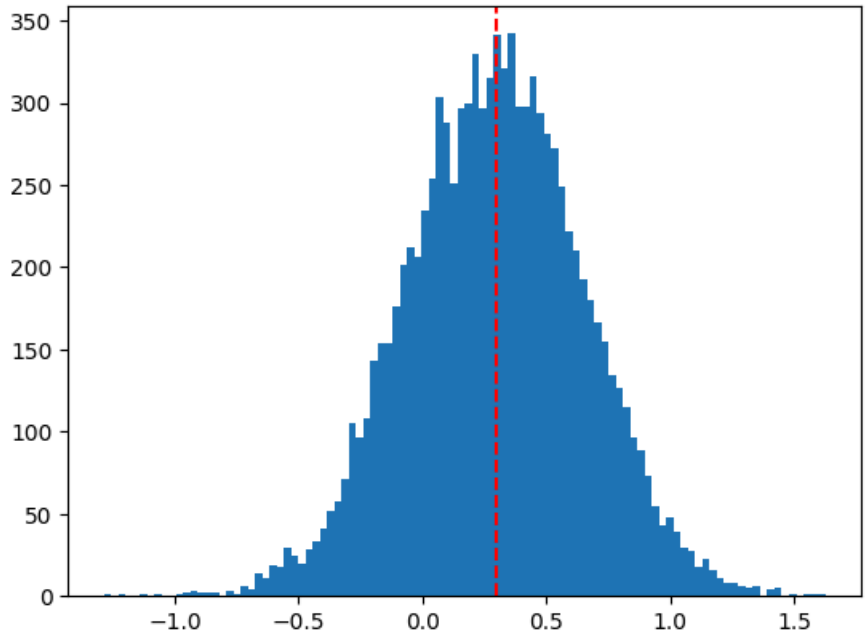
f) How does the estimate of `beta` compare to the parameters we used to generate the data?

```
print(f"The true beta is {true_beta}")
print(f"The estimated beta is {beta_hat}")
```

```
The true beta is [1.  0.3 0.5]
The estimated beta is [1.01106703 0.27819109 0.49990981]
```

g) Let's repeat d) and f) a large number of times to see how close our estimates are on average and what that distribution looks like.

```
import numpy as np
import matplotlib.pyplot as plt

betas = []
true_beta = np.array([ 1, 0.3, 0.5 ])
for _ in range(10000):
    x = -1 + 2 * np.random.random(SAMPLE_SIZE)
    y = true_beta[0] + true_beta[1] * x + true_beta[2] * (x**2) + np.random.randn(SAMPLE_SIZE) * 2
    intercept = np.ones(np.shape(x)[0])
    X = np.array([intercept, x, x ** 2]).T
    betas.append(np.linalg.inv(X.T @ X) @ X.T @ y)

plt.hist(np.array(betas)[:, 1], bins=100)
plt.axvline(x=true_beta[1], c='r', linestyle='dashed')
plt.show()
```




```
코딩을 시작하거나 AI로 코드를 생성하세요.
```