

✓ Worksheet 16

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Topics

- Support Vector Machines (Non-linear case)

Support Vector Machines

Follow along in class to implement the perceptron algorithm and create an animation of the algorithm.

a) As we saw in class, the form

$$w^T x + b = 0$$

while simple, does not expose the inner product $\langle x_i, x_j \rangle$ which we know w depends on, having done the math. This is critical to applying the "kernel trick" which allows for learning non-linear decision boundaries. Let's modify the above algorithm to use the form

$$\sum_i \alpha_i \langle x_i, x \rangle + b = 0$$

```

1 import numpy as np
2 from PIL import Image as im
3 import matplotlib.pyplot as plt
4 import sklearn.datasets as datasets
5
6 TEMPFILE = "temp.png"
7 CENTERS = [[0, 1], [1, 0]]
8
9 epochs = 100
10 learning_rate = .05
11 expanding_rate = .99
12 retracting_rate = 1.1
13
14 X, labels = datasets.make_blobs(n_samples=10, centers=CENTERS, cluster_std=0.2, random_state=0)
15 Y = np.array(list(map(lambda x : -1 if x == 0 else 1, labels.tolist())))
16
17 alpha_i = np.zeros((len(X),))
18 b = 0
19
20 def snap(x, alpha_i, b, error):
21     # create a mesh to plot in
22     h = .01 # step size in the mesh
23     x_min, x_max = X[:, 0].min() - .5, X[:, 0].max() + .5
24     y_min, y_max = X[:, 1].min() - .5, X[:, 1].max() + .5
25     xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
26                           np.arange(y_min, y_max, h))
27
28     meshData = np.c_[xx.ravel(), yy.ravel()]
29     cs = np.array([x for x in 'gb'])
30     fig, ax = plt.subplots()
31     ax.scatter(X[:,0],X[:,1],color=cs[labels].tolist(), s=50, alpha=0.8)
32
33     if error:
34         ax.add_patch(plt.Circle((x[0], x[1]), .12, color='r',fill=False))
35     else:
36         ax.add_patch(plt.Circle((x[0], x[1]), .12, color='y',fill=False))
37
38     Z = predict_many(alpha_i, b, meshData)
39     Z = np.array([0 if z <=0 else 1 for z in Z]).reshape(xx.shape)
40     ax.contourf(xx, yy, Z, alpha=.5, cmap=plt.cm.Paired)
41     fig.savefig(TEMPFILE)
42     plt.close()
43     return im.fromarray(np.asarray(im.open(TEMPFILE)))
44
45 def predict_many(alpha_i, b, Z):
46     res = []
47     for i in range(len(Z)):
48         res.append(predict(alpha_i, b, Z[i]))
49     return np.array(res)
50
51 def predict(alpha_i, b, x):
52     wx = 0
53     for a in range(len(x)):
54         wx += alpha_i[a] * np.dot(x[a], x[0])
55     break
56     return wx + b
57
58 images = []
59 for _ in range(epochs):
60     # pick a point from X at random
61     i = np.random.randint(0, len(X))
62     error = False
63     x, y = X[i], Y[i]
64     ypred = predict(alpha_i, b, x)
65
66     if (ypred.all() > 0 and y > 0) or (ypred.all() < 0 and y < 0):
67         error = True
68         print(True)
69         if ypred.all() < 1 and ypred.all() > -1:
70             alpha_i[i] += y * learning_rate
71             alpha_i = alpha_i * retracting_rate
72             b += y * learning_rate * retracting_rate

```

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73     else:
74         alpha_i = alpha_i * expanding_rate
75         b += expanding_rate
76
77
78     images.append(snap(x, alpha_i, b, error))
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81
82
83 images[0].save(
84     'svm_dual.gif',
85     optimize=False,
86     save_all=True,
87     append_images=images[1:],
88     loop=0,
89     duration=1000
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55     wx += alpha_i[a] * np.dot(x[a], x[0])
56     break
57     return wx + b
58
59
60 images = []
61 for _ in range(epochs):
62     # pick a point from X at random
63     i = np.random.randint(0, len(X))
64     error = False
65     x, y = X[i], Y[i]
66     ypred = predict(alpha_i, b, x)
67
68     if (ypred.all() > 0 and y > 0) or (ypred.all() < 0 and y < 0):
69         error = True
70         print(True)
71         if ypred.all() < 1 and ypred.all() > -1:
72             alpha_i[i] += y * learning_rate
73             alpha_i = alpha_i * retracting_rate
74             b += y * learning_rate * retracting_rate
75         else:
76             alpha_i = alpha_i * expanding_rate
77             b += expanding_rate
78
79
80     images.append(snap(x, alpha_i, b, error))
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84
85 images[0].save(
86     'svm_dual2.gif',
87     optimize=False,
88     save_all=True,
89     append_images=images[1:],
90     loop=0,
91     duration=100
92 )

```

Write a configurable kernel function to apply in lieu of the dot product. Try it out on a dataset that is not linearly separable.

```

1 def polynomial(x_i, x_j, c, n):
2     return (np.dot(x_i, x_j) + c) ** n
3
4 def predict(alpha_i, b, x):
5     wx = 0
6     for a in range(len(x)):
7         wx += alpha_i[a] * polynomial(X[a], x, C, N)
8     break
9     return wx + b

```

b) Assume we fit an SVM using a polynomial Kernel function and it seems to overfit the data. How would you adjust the tuning parameter n of the kernel function?

We can reduce the degree, the value of n . As the value of n increases, the decision boundaries become more complex and therefore, the model can overfit the dataset. If we decrease the value of n , it might not cause overfitting of the dataset.

c) Assume we fit an SVM using a RBF Kernel function and it seems to underfit the data. How would you adjust the tuning parameter σ of the kernel function?

We can decrease the value of σ since it can allow the model to become more complex and therefore, the model would be able to avoid underfitting.

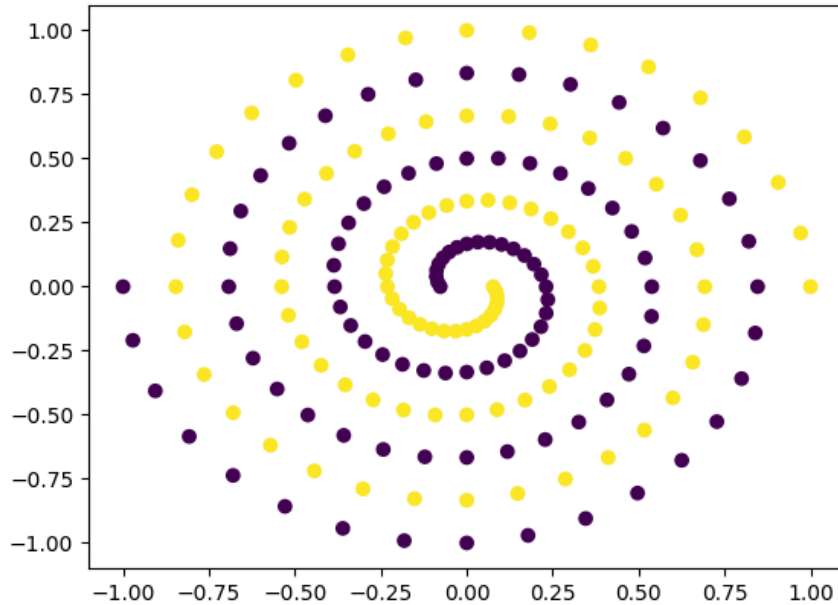
d) Tune the parameter of a specific Kernel function, to fit an SVM (using your code above) to the following dataset:

```

1 import numpy as np
2 import matplotlib.pyplot as plt
3
4 data = np.loadtxt("spiral.data")
5 X, Y = data[:, :2], data[:, 2]
6
7 plt.scatter(X[:,0], X[:,1], c=Y)

```

<matplotlib.collections.PathCollection at 0x7d088472d420>



```

1 C = 1
2 N = 3
3
4 epochs = 100
5 learning_rate = .05
6 expanding_rate = .99
7 retracting_rate = 1.1
8
9 TEMPFILE = "temp3.png"
10 alpha_i = np.zeros((len(X),))
11 b = 0
12
13 def snap(x, alpha_i, b, error):
14     # create a mesh to plot in
15     h = .01 # step size in the mesh
16     x_min, x_max = X[:, 0].min() - .5, X[:, 0].max() + .5
17     y_min, y_max = X[:, 1].min() - .5, X[:, 1].max() + .5
18     xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
19                          np.arange(y_min, y_max, h))
20
21     meshData = np.c_[xx.ravel(), yy.ravel()]
22     cs = np.array([x for x in 'gb'])
23     fig, ax = plt.subplots()
24     ax.scatter(X[:,0],X[:,1],color=cs[labels].tolist(), s=50, alpha=0.8)
25
26     if error:
27         ax.add_patch(plt.Circle((x[0], x[1]), .12, color='r',fill=False))
28     else:
29         ax.add_patch(plt.Circle((x[0], x[1]), .12, color='y',fill=False))
30
31     Z = predict_many(alpha_i, b, meshData)
32     Z = np.array([0 if z <=0 else 1 for z in Z]).reshape(xx.shape)
33     ax.contourf(xx, yy, Z, alpha=.5, cmap=plt.cm.Paired)
34     fig.savefig(TEMPFILE)
35     plt.close()
36     return im.fromarray(np.asarray(im.open(TEMPFILE)))
37
38 def predict_many(alpha_i, b, Z):
39     res = []
40     for i in range(len(Z)):

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```

40     for i in range(len(Z)):
41         res.append(predict(alpha_i, b, Z[i]))
42     return np.array(res)
43
44 def polynomial(x_i, x_j, c, n):
45     return (np.dot(x_i, x_j) + c) ** n
46
47 def predict(alpha_i, b, x):
48     wx = 0
49     for a in range(len(x)):
50         wx += alpha_i[a] * polynomial(X[a], x, C, N)
51     break
52     return wx + b
53

```

```

1
2 images = []
3 for _ in range(epochs):
4     # pick a point from X at random
5     i = np.random.randint(0, len(X))
6     error = False
7     x, y = X[i], Y[i]
8     ypred = predict(alpha_i, b, x)
9

```