(2)

(a) $\neg(\neg A \wedge \neg B \wedge \neg C)$

$\equiv (A \vee B \vee C)$

(b) Since $A \vee B \vee C$ is the case, it is true as long as at least one of them have a muddy case.

ex)

A

B C

Let's assume that only 1 of A, B, or C has a muddy face. Again, let's assume A has the muddy face. If there is only one muddy face, B and C will see that A has the muddy face, which is why they do not go home. However, because A does not go home also, it shows that A also sees one muddy face from B or C. This means that there exists at least two muddy faces from A, B, and C.

$$(A \wedge \neg B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (\neg A \wedge B \wedge C) \vee (A \wedge B \wedge C)$$

(c) Since $(A \wedge \neg B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (\neg A \wedge B \wedge C) \vee (A \wedge B \wedge C)$ is true, there exists at least two muddy faces from A, B, and C.

ex)

A

B C

Let's assume that there exists exactly two muddy faces among A, B, and C. Again, let's assume that A and B are the two people with muddy faces. C does not go home because C sees that there exists two muddy faces A and B. In this case, A should have gone home noticing that there is only one muddy face B (since there exists two muddy faces and A only sees one muddy face, A can conclude that A's face is muddy). Similarly, B should have went back after only seeing one muddy face from A. However, both A and B do not go back, which means that they each saw two muddy faces. This means that C's face is also muddy. Therefore,

$$(A \wedge B \wedge C)$$

(d) A and B and C will all walk home because $(A \wedge B \wedge C)$ is True only if there exists three muddy faces. If there are 3 muddy faces for three people, they all have a muddy face, meaning they all go back.

(e) They would not have walked home without the initial statement, because they would not have known any information about how many muddy faces there are in total and make no assumptions out of it.

① a) If she did not finish her homework, she did not go to the party.
↳ Inverse
Converse: If she went to the party, she finished her homework.
Contrapositive: If she did not go to the party, she did not finish her homework.

b) Inverse: If he did not train for the race, he did not finish the race.
Converse: If he finished the race, he trained for the race.
Contrapositive: If he did not finish the race, he did not train for the race.