CAS CS 365
Lec 6

Probability

1. General overview
    a. If Pr(B >0), then Pr(A|B) = Pr(A n B)/Pr(B)
        i. Ex) What are the chances that blood pressure is high given the patient is greater than 50 years old?
            1. Let A = set of patients with high blood pressure
            2. Let B = set of patients who's age is greater than 50 years old
            3. Let C = set of all patients
            4. Pr(patient that is greater than 50 years old) = B/C
            5. Pr(pressure is high | patient is more than 50 years old)
               = A n B
               —---------
               B
        ii. Ex) A family has two kids. One of them is a boy. What are the chances that they have two boys?
            1. Sample Size = {BB, BG, GB, GG}
            2. Pr(BB | BB or GB or BG) = Pr(BB n (BG or GB or BB))
               —-----------------------------
               Pr(BG or GB or BB)
               = 1/4
               —-------
               3/4
               = ⅓

        iii. Pr(A) = Pr(A|B1)*Pr(B1) + Pr(A|B2)*Pr(B2) → holds when B1 and B2 are partitions of B
            Pr(A) = Pr(A n B1) + Pr(A n B2)
            A = (A n B1) or (A n not B1)
            Pr(A) = Pr(A n B) + Pr(A n not B1) - Pr((A n B1) n (A n not B1))

            Since Pr((A n B1) n (A n not B1)) = 0 if B1 and B2 are partitions of B,
            Pr(A) = Pr(A n B) + Pr(A n not B1)
            1. ex) factory 1: makes mistake 20%, twice as productive as factory 2
            2. Factory 2: makes mistake 5%
                a. Let W = random watch is Ok. What is Pr(W)?
                b. Pr(random watch is OK)

= Pr(W n factory1) + Pr(W n factory2)
= Pr(W| factory1) * Pr(factory1) + Pr(W| factory2) * Pr(factory2)
= (0.8) * (⅔) + (0.95) * (⅓)

3. ex) If I get a watch that is not ok, what is the probability that it came from factory 1?
    a. Pr(factory1 | not W)
       = Pr(not W | factory 1) * Pr(factory 1)
       —--
       Pr(not W)
       = Pr(not W | factory 1) * Pr(factory 1)
       —--
       Pr(not W | factory 1) * Pr(factory 1) + Pr(not W | factory 2) * Pr(factory 2)

       = (0.2) * (⅔)
       —--------------------------
       (0.2) * (⅔) + (0.05) * (⅓)

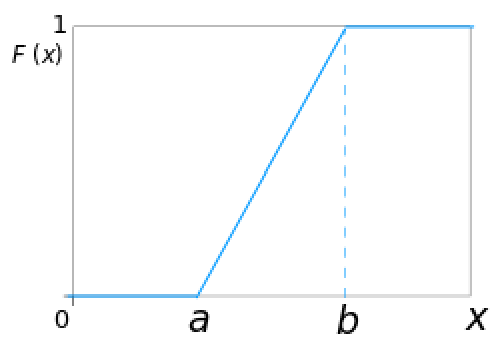
iv. Ex) Suppose there is a disease 1 out of 10^5 humans. Test is correct with probability 99%. What is the probability that somebody is sick given that the test is positive? Pr(sick | correct)
    1. Pr(sick | positive) = Pr(correct | positive) * Pr(positive)
       —-----------------------
       Pr(correct)
       = Pr(correct | sick) * Pr(sick)
       —-----------------—--
       Pr(correct | sick) * Pr(sick) + Pr(correct | not sick) * Pr(not sick)

       = (0.99) * (10^-5)
       —--------------------------------------
       (0.99) * (10^-5) + (0.01) * (1-10^-5)

v. Pr(water flows from s to t)
   = Pr(water flows from s to B and water flows from B to t)
   = Pr(water flows from s to B) * Pr(water flows from B to t)
   = (1-p^2) * (1-p^2)

vi. Basic concepts:
   1. Discrete → can count (# of cats)
   2. Continuous → can measure (temperature degree)
   3. PDF → the probability that x equals a value (probability that coin toss gives H=3)
   4. CDF → the probability that x is less than a value (probability that coin toss gives H <3)
   5. CDF = Fx(X)
      a. Example: Distribution function Fx of Random variable X
         i. FX: R → [0,1] (domain is uniform distribution from 0 to 1)
         ii. FX(x) = Pr(X <= x)
         iii. X is Bernoulli distribution (0 with probability of 1-p and 1 with probability of p)
         iv. FX(x) = 0 if x < 0
             = 1-p if 0 <= x < 1
             = p if x >= 1
   6. Density function
   7. CDF = integral of density function from (lower bound, x)
      a. In other words, FX(x) = integral of fX from (lower bound, x)

vii. Example of uniform distribution
   1. X ~ U(a,b)
   2. FX(x) = 0 if x < a
      = (x-a)/(b-a) if a <= x <= b
      = 1 if x > b



   3. Let X, Y ~ U(0,1)
   4. U = min(X, Y)
      V = max(X, Y)
   5. Find E(U), E(V), Cov(U,V)
      a. Cov(U,V) = E(UV) - E(U)*E(V)
         E(UV) = E(XY) = E(X)E(Y) = ½ ^2 = ¼

E(V) = integral of v * fV(v) dv from (0,1)
FV(v) = Pr(V <= v)
= Pr (max(X,Y) <= V)
= Pr(X <= V and Y <= V)
= Pr(X <= V) * Pr(Y <= V)
= V^2

fV(v) = derivative of V^2 = 2V

E(V) = integral of v * fV(v) dv from (0,1)
= integral of V * 2V
= integral of 2V^2 from (0,1)
= 2 * (⅓ * V^3) from (0,1)
= 2 * ⅓
= ⅔

E(U) = integral of u * fU(u) du from (0,1)
FU(u) = 1 - Pr(min(X,Y) >= U)
= Pr(X >= U and Y>= U)
= Pr(X >= U) * Pr(Y >=U)
= 1 - (1-U)^2

fU(u) = derivative of 1 - (1-U)^2 = 2(1-U)

E(U) = integral of u *fU(u) from (0,1)
= integral of 2(u-u^2) from (0,1)
= 2(½ - ⅓) = 2(⅙)
= ⅓