b) $$y(x[n]) = \sigma(x[n]) = \frac{1}{1+e^{-x[n]}} = (1+e^{-x})^{-1}$$

$$y^{1}(x[n]) = -1\cdot(1+e^{-x[n]})^{-2}\cdot(-e^{-x[n]}) = \frac{e^{-x[n]}}{(1+e^{-x[n]})^2} = \frac{e^{x[n]}}{(1+e^{x[n]})^2} = \underline{\underline{e^{x[n]}\cdot(1+e^{x[n]})^{-2}}}$$

$$y^{2}(x[n]) = e^{x[n]}(1+e^{x[n]})^{-2} - e^{x[n]}(1+e^{x[n]})^{-3}\,2\cdot e^{x[n]} = \frac{e^{x[n]}}{(1+e^{x[n]})^2} - \frac{2e^{2x[n]}}{(1+e^{x[n]})^3}$$

$$= \frac{e^{x[n]} + e^{2x[n]} - 2e^{2x[n]}}{(1+e^{x[n]})^3} = \underline{\underline{\frac{e^{x[n]}(1-e^{x[n]})}{(1+e^{x[n]})^3}}}$$

$$\cancel{y^{3}(x[n]) = \frac{e^{x[n]}}{(1+e^{x[n]})^2} - \frac{2e^{2x[n]}}{(1+e^{x[n]})^3} - \left(4e^{2x[n]}(1+e^{x[n]})^{-3} - 2e^{2x[n]}\cdot 3(1+e^{x[n]})^{-4}\cdot e^{x[n]}\right)}$$

$$y^{3}(x[n]) = (e^{x[n]} - 2e^{2x[n]})(1+e^{x[n]})^{-3} + (e^{x[n]} - e^{2x[n]})\cdot(-3)(1+e^{x[n]})^{-4}\cdot(e^{x[n]})$$

$$= \frac{e^{x[n]} - 2e^{2x[n]}}{(1+e^{x[n]})^3} + \frac{-3e^{2x[n]} + 3e^{3x[n]}}{(1+e^{x[n]})^4} = \frac{(e^{x[n]} - 2e^{2x[n]})(1+e^{x[n]}) - 3e^{2x[n]} + 3e^{3x[n]}}{(1+e^{x[n]})^4}$$

$$= \frac{e^{x[n]} - 2e^{2x[n]} + e^{2x[n]} - 2e^{3x[n]} - 3e^{2x[n]} + 3e^{3x[n]}}{(1+e^{x[n]})^4} = \underline{\underline{\frac{e^{x[n]}\cdot(1-4e^{x[n]} + e^{2x[n]})}{(1+e^{x[n]})^4}}}$$

$c = 0$:

$$y(0) = \frac{1}{2} \qquad \alpha_0 = \frac{1}{2}$$

$$y^{1}(0) = \frac{1}{4} \qquad \alpha_1 = \frac{A}{4} - \frac{A^3}{64}$$

$$y^{2}(0) = 0 \qquad \alpha_2 = 0$$

$$y^{3}(0) = -\frac{1}{8} \qquad \alpha_3 = -\frac{A^3}{192}$$

$c = \ln 2$:

$$y = \frac{2}{3}$$

$$y^{1} = \frac{2}{9}$$

$$y^{2} = -\frac{2}{27}$$

$$y^{3} = -\frac{2}{27}$$

$$\alpha_0 = \frac{2}{3} - \frac{2}{9}\ln 2 + \frac{1}{4}\left(-\frac{2}{27}\right)A^2 + \frac{1}{2}\left(-\frac{2}{27}\right)\ln(2)^2 - \frac{1}{4}\left(-\frac{2}{27}\right)A^2\ln 2 - \frac{1}{6}\left(-\frac{2}{27}\right)\ln(2)^3$$

$$\alpha_1 = \frac{2}{9}A - \left(-\frac{2}{27}\right)A\ln 2 + \frac{1}{8}\left(-\frac{2}{27}\right)A^3 + \frac{1}{2}\left(-\frac{2}{27}\right)A\ln(2)^2$$

$$\alpha_2 = \frac{1}{4}\left(-\frac{2}{27}\right)A^2 - \frac{1}{4}\left(-\frac{2}{27}\right)A^2\ln 2$$

$$\alpha_3 = \frac{1}{24}\left(-\frac{2}{27}\right)A^3$$