

Space for innovative ideas.

4) a) $Z = X + Y$ $p_{X,Y} = p_X \cdot p_Y$

show that $p_Z = p_Y * p_X$

$$p_Z(z) = \int_x p_X(x) p_Y(z-x) dx$$

$$= \int_y p_Y(y) p_X(z-y) dy$$

substitution : $x = z - y$

$$\frac{dx}{dy} = -1 \rightarrow dx = -1 dy$$

$$p_Z(z) = \int_{x_1}^{x_2} p_X(x) p_Y(z-x) dx = - \int_{y_1}^{y_2} p_X(z-y) p_Y(y) dy = \int_{y_2}^{y_1} p_X(z-y) p_Y(y) dy$$

$$\int_x p_X(x) p_Y(z-x) dx = \int_y p_Y(y) p_X(z-y) dy$$

$$p_Z = p_Y * p_X = p_X * p_Y$$

b)