

Let  $X = UDV^T$ . Then

- (A) Columns of  $U$  are eigenvectors of  $X^T X$
- (B) **[Ans]** Columns of  $V$  are eigenvectors of  $X^T X$
- (C) Rows of  $U$  are eigenvectors of  $X^T X$
- (D) Rows of  $V$  are eigenvectors of  $X^T X$
- (E) None of these

Let  $X = UDV^T$ . Then

- (A) **[Ans]** Columns of  $U$  are eigenvectors of  $XX^T$
- (B) Columns of  $V$  are eigenvectors of  $XX^T$
- (C) Rows of  $U$  are eigenvectors of  $XX^T$
- (D) Rows of  $V$  are eigenvectors of  $XX^T$
- (E) None of these

Consider  $X$  to be a square matrix of size  $n \times n$  and  $X = UDV^T$ .

- (A) **[Ans]** Both  $X^T X$  and  $XX^T$  have the same eigenvalues
- (B) Both  $X^T X$  and  $XX^T$  have the same eigenvectors
- (C)  $X$ ,  $XX^T X$  and  $XX^T$  have the same eigenvalues
- (D) **[Ans]**  $D^2$  contains the eigenvalues of  $X^T X$  on its diagonal
- (E)  $D$  contains the eigenvalues of  $X^T X$  on its diagonal
- (F) None of these

Consider  $X$  to be a square matrix of size  $n \times n$  and  $X = UDV^T$ .

- (A) **[Ans]** If  $\text{rank}(X) = n$ ,  $D$  has all non-zero entries in diagonal.
- (B) If  $\text{rank}(X) = k$ ,  $D$  has  $k$  zeros in diagonal
- (C) **[Ans]** If  $\text{rank}(X) = k$ ,  $D$  has  $n - k$  zeros in diagonal
- (D) **[Ans]** if  $\text{rank}(X) = n$  but  $|A|$  is a very small number then,  $D$  takes the form  $D = \text{diag}(d_1, d_2, \dots, \epsilon)$  where  $\epsilon$  is a very small number
- (E) None of these

Suppose you want to apply PCA to your data  $X$  which is in 2D and you decompose  $X$  as  $UDV^T$ . Then,

- (A) PCA can be useful if all elements of  $D$  are equal
- (B) **[Ans]** PCA can be useful if all elements of  $D$  are not equal
- (C) **[Ans]**  $D$  is not full-rank if all points in  $X$  lie on a straight line
- (D)  $V$  is not full-rank if all points in  $X$  lie on a straight line
- (E)  $D$  is not full-rank if all points in  $X$  lie on a circle
- (F) None of these