

We know $\kappa(\mathbf{p}, \mathbf{q})$ as $\phi(\mathbf{p})^T \phi(\mathbf{q})$.

How we express the squared euclidean distance

$$d(\phi(\mathbf{p}), \phi(\mathbf{q})) = [\phi(\mathbf{p}) - \phi(\mathbf{q})]^T [\phi(\mathbf{p}) - \phi(\mathbf{q})]$$

in terms of the kernels

- (A) $\kappa(\mathbf{p}, \mathbf{q})$
- (B) $\kappa(\mathbf{p}, \mathbf{p}) + \kappa(\mathbf{q}, \mathbf{q}) + 2\kappa(\mathbf{p}, \mathbf{q})$
- (C) **[Ans]** $\kappa(\mathbf{p}, \mathbf{p}) + \kappa(\mathbf{q}, \mathbf{q}) - 2\kappa(\mathbf{p}, \mathbf{q})$
- (D) $(\kappa(\mathbf{p}, \mathbf{p}) - \kappa(\mathbf{q}, \mathbf{q}))^2$
- (E) None of the above.

We know $\kappa(\mathbf{p}, \mathbf{q})$ as $\phi(\mathbf{p})^T \phi(\mathbf{q})$.

Consider a data matrix with a feature map i.e.,

$$\mathbf{X} = [\phi(\mathbf{x}_1), \phi(\mathbf{x}_2), \dots, \phi(\mathbf{x}_N)]$$

Then $\mathbf{X}^T \mathbf{X}$ is

- (A) **[Ans]** $N \times N$
- (B) Can not be computed since $\phi()$ can map to infinite dimension
- (C) **[Ans]** The kernel matrix \mathbf{K} with $K_{ij} = \kappa(\mathbf{x}_i, \mathbf{x}_j)$
- (D) **[Ans]** Symmetric
- (E) None of the above

We know $\kappa(\mathbf{p}, \mathbf{q})$ as $\phi(\mathbf{p})^T \phi(\mathbf{q})$.

Consider two vectors $\phi(\mathbf{p})$ and $\phi(\mathbf{q})$ and their L2 normalized version as $\phi(\mathbf{p})'$ and $\phi(\mathbf{q})'$. i.e.,

$$\phi(\mathbf{p})' = \frac{\phi(\mathbf{p})}{\|\phi(\mathbf{p})\|}$$

How do we compute $\kappa'(\mathbf{p}, \mathbf{q}) = (\phi(\mathbf{p})')^T (\phi(\mathbf{q})')$ in terms of $\kappa(\mathbf{p}, \mathbf{q}) = (\phi(\mathbf{p}))^T (\phi(\mathbf{q}))$.

i.e., $\kappa'(\mathbf{p}, \mathbf{q}) =$

(A) $\frac{\kappa(\mathbf{p}, \mathbf{q})}{\kappa(\mathbf{p}, \mathbf{q}) \kappa(\mathbf{p}, \mathbf{q})}$

(B) $\frac{\kappa(\mathbf{p}, \mathbf{q})}{\kappa(\mathbf{p}, \mathbf{p}) \kappa(\mathbf{q}, \mathbf{q})}$

(C) $\frac{\kappa(\mathbf{p}, \mathbf{q})}{\sqrt{\kappa(\mathbf{p}, \mathbf{q}) \kappa(\mathbf{p}, \mathbf{q})}}$

(D) **[Ans]** $\frac{\kappa(\mathbf{p}, \mathbf{q})}{\sqrt{\kappa(\mathbf{p}, \mathbf{p}) \kappa(\mathbf{q}, \mathbf{q})}}$

(E) None of the above

We know $\kappa(\mathbf{p}, \mathbf{q})$ as $\phi(\mathbf{p})^T \phi(\mathbf{q})$.

Kernel matrix \mathbf{K} is

- (A) **[Ans]** Symmetric
- (B) $d \times d$
- (C) **[Ans]** $N \times N$
- (D) **[Ans]** Depends on $\phi()$
- (E) None of the above

Let μ be the mean of samples $\mathbf{x}_1, \dots, \mathbf{x}_N$.

Also τ be the mean of samples $\phi(\mathbf{x}_1), \dots, \phi(\mathbf{x}_N)$.

Let $\phi()$ be the feature map corresponding to RBF Kernel (or your more familiar quadratic kernel).

(A) μ and τ are identical.

(B) $\tau = \phi(\mu)$

(C) **[Ans]** $\tau \neq \phi(\mu)$

(D) μ and τ are different; but of same dimension.

(E) Two of the above are true.