

Given a set of 2D points  $X$  on a line that makes 45 degree to the x-axis:

$$X = \{[1, 1]^T, [2, 2]^T, [3, 3]^T, [4, 4]^T, [5, 5]^T\}$$

We compute the covariance matrix, and its eigen values and eigen vectors. Then:

- (A) **[Ans]**  $\lambda_2 = 0$
- (B)  $\lambda_1 = \lambda_2$
- (C)  $\lambda_1 = -1$
- (D) **[Ans]**  $\Sigma$  is singular
- (E) none of the above

Given a set of 2D points  $X$  on a line that makes 45 degree to the x-axis:

$$X = \{[-2, 2]^T, [-3, 3]^T, [-4, 4]^T, [-5, 5]^T, [-6, 6]^T\}$$

We compute the covariance matrix, and its eigen values and eigen vectors. Then:

- (A) **[Ans]**  $\lambda_2 = 0$
- (B)  $\lambda_1 = \lambda_2$
- (C)  $\lambda_1 = -1$
- (D) **[Ans]**  $\Sigma$  is singular
- (E) none of the above

Given a set of 2D points  $X$  on the vertical line  $x_1 = 5$ ,

$$X = \{[5, 1]^T, [5, 2]^T, [5, 3]^T, [5, 4]^T, [5, 5]^T\}$$

We now add an additional point  $[4, 3]^T$  to  $X$ . We compute the covariance matrix, and its eigen values and eigen vectors. Then:

- (A) **[Ans]**  $\lambda_1 \geq \lambda_2$
- (B)  $\mathbf{u}_1$  and  $\mathbf{u}_2$  are nearly orthogonal, but not perfectly orthogonal.
- (C)  $\Sigma$  is singular
- (D) **[Ans]**  $\Sigma$  is diagonal
- (E) None of the above.

Given a set of 2D points  $X$  on the vertical line  $x_2 = 5$ ,

$$X = \{[1, 5]^T, [2, 5]^T, [3, 5]^T, [4, 5]^T, [5, 5]^T\}$$

We compute the covariance matrix, and its eigen values and eigen vectors. Then:

- (A) **[Ans]**  $\lambda_1 \geq \lambda_2$
- (B) **[Ans]**  $\mu$  is on the same line.
- (C) **[Ans]**  $\Sigma$  is singular
- (D) **[Ans]**  $\Sigma$  is diagonal
- (E) None of the above

Set  $X$  has 10 points. 5 of them are on a line that makes 45 degrees with the  $x_1$  axis and another 5 from on a line that makes 135 degrees with the  $x_1$  axis. We compute the covariance matrix, and its eigen values and eigen vectors. Then:

- (A)  $\lambda_1 = \lambda_2 \neq 0$
- (B)  $\Sigma$  is singular
- (C)  $\Sigma$  is diagonal
- (D)  $\mu$  is on either of these lines.
- (E) **[Ans]** None of the above