

We know that solution to the problem of Maximize $\mathbf{w}^T \mathbf{A} \mathbf{w}$ subject to $\|\mathbf{w}\| = 1$ is the eigen vector corresponding to the largest eigen value.

Note that we assume that a typical eigen value computation assumes to be returning (i) eigen values arranged in non-increasing order (ii) eigen vectors have unit L2 norm.

What is the solution to the problem of Maximize $\mathbf{w}^T \mathbf{A} \mathbf{w}$ subject to $\|\mathbf{w}\|_2^2 = 2$

- (A) Eigen vector correspond to the second eigen value.
- (B) Eigen vector correspond to the first eigen value.
- (C) $2 * \mathbf{u}$ where \mathbf{u} is the eigen vector correspond to the first eigen value.
- (D) **[Ans]** $\sqrt{2} * \mathbf{u}$ where \mathbf{u} is the eigen vector correspond to the first eigen value.
- (E) None of the above.

We are working with N samples each of d dimension. Consider $N < d$

- (A) Solution to the problem of linear regression as a closed form can not be computed because the matrices are no longer compatible for multiplication.
- (B) **[Ans]** Solution to the problem of linear regression as a closed form can not be computed because the matrix can not be inverted.
- (C) Solution to the problem of ridge regularized linear regression as a closed form can not be computed because the matrix can not be inverted.
- (D) None of the above

We are working with N samples each of d dimension. Consider $N \leq d$

- (A) PCA can not be computed
- (B) While computing Eigen values, we will see d zero eigen values.
- (C) **[Ans]** While computing Eigen values, we will see at least $d - N$ zero eigen values.
- (D) **[Ans]** While computing Eigen values, we will see at max $d - N$ zero eigen values.
- (E) **[Ans]** We can not use eigen value/vector computation. We need to use SVD.
- (F) None of the above.

We know the weighted Euclidean distance

$$\tau = [\mathbf{x} - \mathbf{y}]^T [\mathbf{A}] [\mathbf{x} - \mathbf{y}]$$

Where \mathbf{x} is a vector in d dimension \mathbf{A} is a square matrix

- (A) **[Ans]** If \mathbf{A} is a non-identity diagonal matrix. Then “dist” is a scaled version of Euclidean distance or “ $\tau = \alpha$ Euclidean distance”
- (B) If $k < d$ and $A = \sum_{i=1}^k \mathbf{u}_i \mathbf{u}_i^T$, with \mathbf{u}_i s as orthonormal. Then A is rank deficient and Euclidean τ can not be computed.
- (C) **[Ans]** If $k < d$ and $A = \sum_{i=1}^k \mathbf{u}_i \mathbf{u}_i^T$, with \mathbf{u}_i s as orthonormal. Then τ is equivalent to dimensionality reduction $\mathbf{x}' = \mathbf{W}\mathbf{x}$ with \mathbf{W} as $k \times d$ matrix with \mathbf{u}_i^T as the i th row.
- (D) When \mathbf{A} is non-diagonal matrix, τ can not be a metric.
- (E) None of the above.

Consider the covariance matrix Σ

- (A) **[Ans]** Σ is symmetric
- (B) **[Ans]** Σ is PSD
- (C) Σ is Diagonal if the distribution is Normal.
- (D) Σ can not be Diagonal if the distribution is Normal.
- (E) None of the above