Sensor Fusion Lab
DEIS Course

This laboratory is about Kalman Filtering and Sensor Fusion. By completing this laboratory, you should be able to

- get familiar with system modeling
- understand and implement Kalman and Extended Kalman Filtering
- understand the concept of sensor fusion

IMPORTANT

- You should make sure that you learn MATLAB programming before you start the laboratory.
- You should also read the lecture slides.

Laboratory Report Submission

- You must submit your laboratory report in 3 weeks after the laboratory.
- Your report must contain
    - Brief description of the procedure
    - MATLAB source code
    - Results plots
- You may choose to work in a group of two people even though I strongly suggest doing it alone.
- Please, send your report to
    - abbas.orand@hh.se

Grading Criteria

You will get a score of

- Pass: if you complete all the tasks with appropriate results
- Fail: if you do not submit a report OR if you submit an incomplete report.

Problem #1

The distance of an object is measured at a frequency of 1 Hz by using a proximity sensor. Assume the measurement just contains the measurement and noise. Therefore,

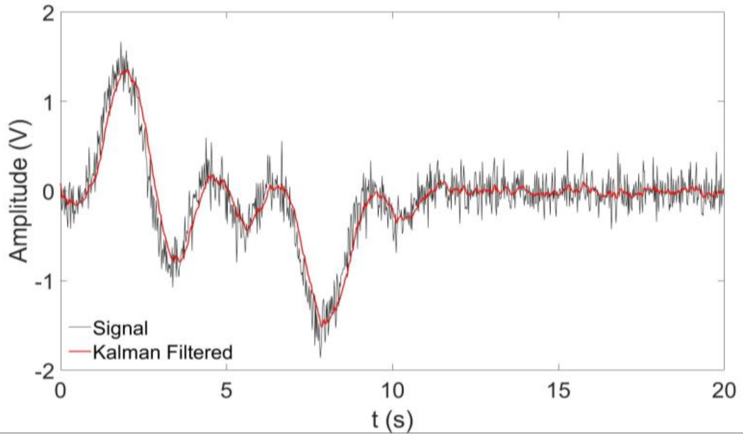
$$x_k = A x_{k-1}$$

$$A = 1,\ H = 1,\ Q = 0,\ R = 1$$

The data is saved as '*ProblemNo1.mat*'. Plot the original raw data and the Kalman Filtered signal.

Problem #2

A collected signal of a system that has a similar pattern to a Sinc function is shown below.



The system's signal pattern

Related data is saved as 'ProblemNo2.mat'. Implement the Kalman Filter to filter the signal.

Hint: The signal includes a noise which can not be predicted but estimated. Therefore, we use statistics to express noise. Since the noise is assumed to have a normal distribution with a mean of zero, we need to know its variance. The noise is expressed in covariance matrices of Q and R.

$$x_{k+1} = Ax_k + w_k,\ z_k = Hx_k + v_k$$

$$x_k: state\ variable, \qquad n \times 1\ column\ vector$$

$$z_k: measurement, \qquad m \times 1\ column\ vector$$

$$A: state\ transition\ matrix, \qquad n \times n\ matrix$$

$$H: state - to - measurement\ matrix, \qquad m \times n\ matrix$$

$$w_k: state\ transition\ noise, \qquad n \times 1\ column\ vector$$

$$v_k: measurement\ noise, \qquad m \times 1\ column\ vector$$

$$\boldsymbol{Q}: \boldsymbol{covariance\ matirx\ of\ w_k}, \qquad \boldsymbol{n \times n\ diagonal\ matrix}$$

$$\boldsymbol{R}: \boldsymbol{covariance\ matrix\ of\ v_k}, \qquad \boldsymbol{m \times m\ diagonal\ matrix}$$

Covariance matrices consist of the variance of the variable. For example, Q matrix is written in terms of the variances of variables of w as follows.

$$\boldsymbol{Q} = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \sigma_n^2 \end{bmatrix}$$

Problem #3:

This problem is regarding the fusion of data from an accelerometer and gyro of the collected from an IMU9250. Further information of the sensor can be found at the following links:

- https://invensense.tdk.com/wp-content/uploads/2015/02/PS-MPU-9250A-01-v1.1.pdf
- https://learn.sparkfun.com/tutorials/mpu-9250-hookup-guide/all

The IMU includes an accelerometer, gyro and a magnetometer. Gyro produces the angular rates of p, q, and r. This rate of changes should be transformed into their respective Euler Angles (https://mathworld.wolfram.com/EulerAngles.html) followed by integration. Euler angles and angular velocities relationship can be expressed as

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}1 & sin\phi\, tan\theta & cos\phi\, tan\theta\\ 0 & cos\phi & -sin\phi\\ 0 & sin\phi/cos\theta & cos\phi/cos\theta\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix} = \begin{bmatrix}p + qsin\phi\, tan\theta + rcos\phi\, tan\theta\\ qcos\phi - rsin\phi\\ qsin\phi sec\theta + rcos\phi sec\theta\end{bmatrix}$$

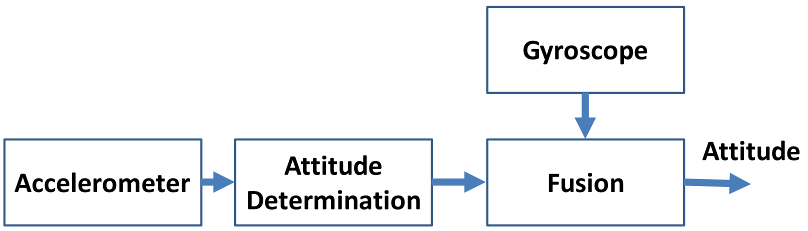
The integration of the Euler angles $(\dot{\emptyset},\ \dot{\theta},\ \dot{\psi})$ over time results in the accumulation of error and the drift of the result.

For the accelerometer, the accelerations along the 3 axes are measured as f_x, f_y, f_z. Considering a system with constant velocity, we would obtain the following expressions for the accelerations

$$\begin{bmatrix}f_x\\ f_y\\ f_z\end{bmatrix} = g\begin{bmatrix}sin\theta\\ -cos\theta sin\phi\\ -cos\theta cos\phi\end{bmatrix} \rightarrow \phi = \sin^{-1}\left(\frac{-f_y}{gcos\theta}\right), \theta = \sin^{-1}\left(\frac{f_x}{g}\right)$$

where g is the gravitational acceleration.

The fusion of the two sensors data is done according to the following diagram.



Gyro and accelerometer data fusion

Since Euler angles suffer from Gimble Lock (http://www.math.umd.edu/~immortal/MATH431/lecturenotes/ch_gimballock.pdf), we choose to use Quaternion for the state variables.

$$x = [q_1;\ q_2;\ q_3;\ q_4]$$

The relationship between angular velocities and the rate of changes in quaternion in a discrete form can be stated as

$$\begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix}_{k+1} = \left[I + \Delta t.\, {}^{1}\!/_{2} \begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix} \right] \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}_k$$

where

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{Bmatrix} \cos\frac{\phi}{2}\cos\frac{\theta}{2}\cos\frac{\psi}{2} + \sin\frac{\phi}{2}\sin\frac{\theta}{2}\sin\frac{\psi}{2} \\ \sin\frac{\phi}{2}\cos\frac{\theta}{2}\cos\frac{\psi}{2} - \cos\frac{\phi}{2}\sin\frac{\theta}{2}\sin\frac{\psi}{2} \\ \cos\frac{\phi}{2}\sin\frac{\theta}{2}\cos\frac{\psi}{2} + \sin\frac{\phi}{2}\cos\frac{\theta}{2}\sin\frac{\psi}{2} \\ \cos\frac{\phi}{2}\cos\frac{\theta}{2}\sin\frac{\psi}{2} - \sin\frac{\phi}{2}\sin\frac{\theta}{2}\cos\frac{\psi}{2} \end{Bmatrix}$$

State-to-measurement matrix should be taken as

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Choose appropriate values for the covariance matrices of Q and R. Open the MATLAB program ‘ProblemNo3_Studetns’. Include the values for the two covariance matrices in lines 93 and 94.

In your report, you should discuss the fusion of the two sensors and provide the screen shot of the appropriate parts that show the fusion of the two sensors.

Problem #4

IN this problem, you are going to try the to implement Extended Kalman Filter for the Problem No 3. Open the MATLAB Function 'ProblemNo4_Students'. Use the functions 'Acc_PhiTheta' and 'Gyro_pqr' to run the first part of the ProblemNo4_Students.

For the second part, you need to implement EKF (lines 61 to 63). Keep the structure of the function as it is

```
[phi theta psi] = Extended_Kalman_Filter([phi theta],[p q r],dt);
```

You just need to implement the function. The same procedure as of Problem No 3 can be used. Except that you need to write the Jacobian Function of State Transition Matrix (A) and function of the estimate ($\hat{x}_{k-1}$). You may check the lecture slides and use the followings.

$$x_k = f_x([acceleration\ variables], [gyro\ variables\ ], dt)$$

$$A = Jacobian([acceleration\ variables], [gyro\ variables\ ], dt)$$

Set H, Q, R as follows,

$$\mathrm{H} = [\,1\ 0\ 0;\ 0\ 1\ 0\,]$$
$$\mathrm{Q} = [\,0.0001\ \ 0\ \ 0;\ 0\ \ 0.0001\ \ 0; 0\ \ 0\ \ 0.1\,];$$
$$\mathrm{R} = [\ 6\ \ 0; 0\ \ 6\,];$$