Konvexe Optimierung

5. Übungsserie

Aufgabe 33

$$h_{\delta}(x,y) = \sum_{i}^{n} \delta^{2} \left(\sqrt{1 + \left(\frac{\xi_{i}x_{0} + x_{1} - \eta_{i}}{\delta} \right)^{2}} - 1 \right)$$

$$= \delta^{2} \left(\sum_{i}^{n} \left(1 + \left(\frac{\xi_{i}x_{0} + x_{1} - \eta_{i}}{\delta} \right)^{2} \right)^{\frac{1}{2}} - \sum_{i}^{n} 1 \right)$$

$$= \delta^{2} \left(\sum_{i}^{n} \left(1 + \left(\frac{\xi_{i}x_{0} + x_{1} - \eta_{i}}{\delta} \right)^{2} \right)^{\frac{1}{2}} - n \right)$$

$$= \delta^{2} \sum_{i}^{n} \left(1 + \left(\frac{\xi_{i}x_{0} + x_{1} - \eta_{i}}{\delta} \right)^{2} \right)^{\frac{1}{2}} - \delta^{2}n$$

$$\frac{\partial}{\partial x_0} h_{\delta}(x, y) = \frac{\partial}{\partial x_0} \sum_{i=1}^{n} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{\frac{1}{2}} - \delta^2 n \tag{1}$$

$$= \sum_{i}^{n} \frac{\partial}{\partial x_{0}} \left(1 + \left(\frac{\xi_{i} x_{0} + x_{1} - \eta_{i}}{\delta} \right)^{2} \right)^{\frac{1}{2}} - \frac{\partial}{\partial x_{0}} \delta^{2} n$$
 (2)

$$= \sum_{i}^{n} \frac{\partial}{\partial x_0} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{\frac{1}{2}} \tag{3}$$

$$= \sum_{i=1}^{n} \frac{1}{2} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{-\frac{1}{2}} \cdot \frac{\partial}{\partial x_0} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)$$
(4)

$$= \sum_{i}^{n} \frac{1}{2} \left(1 + \left(\frac{\xi_{i} x_{0} + x_{1} - \eta_{i}}{\delta} \right)^{2} \right)^{-\frac{1}{2}} \cdot 2 \left(\frac{\xi_{i} x_{0} + x_{1} - \eta_{i}}{\delta} \right) \cdot \frac{\partial}{\partial x_{0}} \left(\frac{\xi_{i} x_{0} + x_{1} - \eta_{i}}{\delta} \right)$$
 (5)

$$= \sum_{i}^{n} \frac{1}{2} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{-\frac{1}{2}} \cdot 2 \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right) \cdot \frac{\xi_i \cdot \delta}{\delta^2}$$
 (6)

$$= \sum_{i}^{n} \frac{\left(\frac{\xi_{i}x_{0} + x_{1} - \eta_{i}}{\delta}\right) \cdot \frac{\xi_{i} \cdot \delta}{\delta^{2}}}{\sqrt{1 + \left(\frac{\xi_{i}x_{0} + x_{1} - \eta_{i}}{\delta}\right)^{2}}}$$
 (7)

$$= \sum_{i}^{n} \frac{\left(\frac{\xi_{i}x_{0} + x_{1} - \eta_{i}}{\delta}\right) \cdot \frac{\xi_{i}}{\delta}}{\sqrt{1 + \left(\frac{\xi_{i}x_{0} + x_{1} - \eta_{i}}{\delta}\right)^{2}}}$$
(8)

$$= \sum_{i}^{n} \frac{\xi_{i}^{2} x_{0} + \xi_{i} x_{1} - \xi_{i} \eta_{i}}{\delta^{2} \sqrt{1 + \left(\frac{\xi_{i} x_{0} + x_{1} - \eta_{i}}{\delta}\right)^{2}}}$$
(9)

(10)

$$\frac{\partial}{\partial x_1} h_{\delta}(x, y) = \sum_{i}^{n} \frac{\left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right) \cdot \frac{\delta}{\delta^2}}{\sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2}}$$
(11)

$$= \sum_{i}^{n} \frac{\frac{\xi_{i}x_{0} + x_{1} - \eta_{i}}{\delta^{2}}}{\sqrt{1 + \left(\frac{\xi_{i}x_{0} + x_{1} - \eta_{i}}{\delta}\right)^{2}}}$$
(12)

$$= \sum_{i}^{n} \frac{\xi_{i} x_{0} + x_{1} - \eta_{i}}{\delta^{2} \sqrt{1 + \left(\frac{\xi_{i} x_{0} + x_{1} - \eta_{i}}{\delta}\right)^{2}}}$$
(13)

(14)

$$\frac{\partial}{\partial x_1} h_{\delta}(x, y) = \sum_{i}^{n} \frac{\left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right) \cdot \frac{\delta}{\delta^2}}{\sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2}}$$

$$= \sum_{i}^{n} \frac{\frac{\xi_i x_0 + x_1 - \eta_i}{\delta^2}}{\sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2}}$$

$$= \sum_{i}^{n} \frac{\xi_i x_0 + x_1 - \eta_i}{\delta^2 \sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2}}$$