Konvexe Optimierung

5. Übungsserie

Aufgabe 33

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\begin{aligned}
h_\delta(x,y) &= \sum_i^n \delta^2 \left(\sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2} - 1 \right) \\
&= \delta^2 \left(\sum_i^n \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{\frac{1}{2}} - \sum_i^n 1 \right) \\
&= \delta^2 \left(\sum_i^n \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{\frac{1}{2}} - n \right) \\
&= \delta^2 \sum_i^n \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{\frac{1}{2}} - \delta^2 n
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial x_0} h_\delta(x,y) &= \frac{\partial}{\partial x_0} \sum_i^n \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{\frac{1}{2}} - \delta^2 n \\
&= \sum_i^n \frac{\partial}{\partial x_0} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{\frac{1}{2}} - \frac{\partial}{\partial x_0} \delta^2 n \\
&= \sum_i^n \frac{\partial}{\partial x_0} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{\frac{1}{2}} \\
&= \sum_i^n \frac{1}{2} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{-\frac{1}{2}} \cdot \frac{\partial}{\partial x_0} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right) \\
&= \sum_i^n \frac{1}{2} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{-\frac{1}{2}} \cdot 2 \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right) \cdot \frac{\partial}{\partial x_0} \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right) \\
&= \sum_i^n \frac{1}{2} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{-\frac{1}{2}} \cdot 2 \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right) \cdot \frac{\xi_i \cdot \delta}{\delta^2} \\
&= \sum_i^n \frac{\left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right) \cdot \frac{\xi_i \cdot \delta}{\delta^2}}{\sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2}} \\
&= \sum_i^n \frac{\left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right) \cdot \frac{\xi_i}{\delta}}{\sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2}} \\
&= \sum_i^n \frac{\xi_i^2 x_0 + \xi_i x_1 - \xi_i \eta_i}{\delta^2 \sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2}}
\end{aligned}
$$

$$\begin{aligned}
\frac{\partial}{\partial x_1} h_\delta(x,y) &= \sum_i^n \frac{\left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right) \cdot \frac{\delta}{\delta^2}}{\sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2}} \\
&= \sum_i^n \frac{\frac{\xi_i x_0 + x_1 - \eta_i}{\delta^2}}{\sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2}} \\
&= \sum_i^n \frac{\xi_i x_0 + x_1 - \eta_i}{\delta^2 \sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2}}
\end{aligned}$$