Konvexe Optimierung

Projekt

L_1 Loss

$$\min_{x\in\mathbb{R}^n} f_1(x) = \frac{1}{2}w^\intercal w + c \cdot \sum_{i=1}^{k} \left[\max\left\{0, 1 - y_i \cdot \left(w^\intercal x_i + b\right)\right\}\right] \tag{1}$$

L_2 Loss

$$\min_{x\in\mathbb{R}^n} f_2(x) = \frac{1}{2}w^\intercal w + c \cdot \sum_{i=1}^{k} \left[\max\left\{0, 1 - y_i \cdot \left(w^\intercal x_i + b\right)\right\}^2\right] \tag{2}$$

Logistic Loss

$$\min_{x\in\mathbb{R}^n} f_3(x) = \frac{1}{2}w^\intercal w + c \cdot \sum_{i=1}^{k} \left[\ln(1 + \exp(-y_i(w^\intercal x_i + b)))\right] \tag{3}$$

$$\begin{aligned}
\frac{\partial}{\partial w} f_3(x) &= \frac{\partial}{\partial w}\left[\frac{1}{2}w^\intercal w\right] + c \cdot \sum_{i=1}^{k} \frac{\partial}{\partial w}\left[\ln(1 + \exp(-y_i(w^\intercal x_i + b)))\right] \\
&= w + c \cdot \sum_{i=1}^{k} \frac{\partial}{\partial w}\left[\ln(1 + \exp(-y_i(w^\intercal x_i + b)))\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial x} f_3(x) &= \frac{\partial}{\partial x}\frac{1}{2}w^\intercal w + c \cdot \sum_{i=1}^{k} \frac{\partial}{\partial x}\left[\ln(1 + \exp(-y_i(w^\intercal x_i + b)))\right] \\
&= c \cdot \sum_{i=1}^{k} \frac{\partial}{\partial x}\left[\ln(1 + \exp(-y_i\left(w^\intercal x_i + b\right)))\right] \\
&= c \cdot \sum_{i=1}^{k} \frac{\frac{\partial}{\partial x}\left[1 + \exp(-y_i(w^\intercal x_i + b))\right]}{1 + \exp(-y_i(w^\intercal x_i + b))} \\
&= c \cdot \sum_{i=1}^{k} \frac{\frac{\partial}{\partial x}\left[\exp(-y_i(w^\intercal x_i + b))\right]}{1 + \exp(-y_i(w^\intercal x_i + b))} \\
&= c \cdot \sum_{i=1}^{k} \frac{\exp(-y_i(w^\intercal x_i + b))\frac{\partial}{\partial x}\left[-y_i(w^\intercal x_i + b)\right]}{1 + \exp(-y_i(w^\intercal x_i + b))} \\
&= c \cdot \sum_{i=1}^{k} \frac{\exp(-y_i(w^\intercal x_i + b))\frac{\partial}{\partial x}\left[-y_i w^\intercal x_i - y_i b\right]}{1 + \exp(-y_i(w^\intercal x_i + b))} \\
&= c \cdot \sum_{i=1}^{k} \frac{-y_i w \exp(-y_i(w^\intercal x_i + b))}{1 + \exp(-y_i(w^\intercal x_i + b))}
\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial x \partial x} f_3(x) &= \frac{\partial}{\partial x}\left[c \cdot \sum_{i=1}^{k} \frac{-y_i w \exp(-y_i(w^\intercal x_i + b))}{1+\exp(-y_i(w^\intercal x_i + b))}\right] \\ &= c \cdot \sum_{i=1}^{k} \frac{\partial}{\partial x}\left[\frac{-y_i w \exp(-y_i(w^\intercal x_i + b))}{1+\exp(-y_i(w^\intercal x_i + b))}\right] = c \cdot \sum_{i=1}^{k} \frac{\frac{\partial}{\partial x} u(x_i) \cdot v(x_i) - u(x_i) \cdot \frac{\partial}{\partial x_i} v(x_i)}{v(x_i)^2}\end{aligned}$$

$$\begin{aligned}u(x) &= -y_i w \exp(-y_i(w^\intercal x_i + b)) \\ v(x) &= 1 + \exp(-y_i(w^\intercal x_i + b)) \\ \frac{\partial}{\partial x} u(x) &= \frac{\partial}{\partial x}\left[-y_i w \exp(-y_i(w^\intercal x_i + b))\right] \\ &= -y_i w \exp(-y_i(w^\intercal x_i + b)) \frac{\partial}{\partial x}\left[-y_i(w^\intercal x_i + b)\right] \\ &= y_i^2 w^2 \exp(-y_i(w^\intercal x_i + b)) \\ \frac{\partial}{\partial x} v(x) &= \frac{\partial}{\partial x}\left[1 + \exp(-y_i(w^\intercal x_i + b))\right] \\ &= \frac{\partial}{\partial x}\left[\exp(-y_i\left(w^\intercal x_i + b\right))\right] \\ &= u(x)\end{aligned}$$

$$\frac{\partial}{\partial x \partial x} f_3(x) = \frac{y_i^2 w^2 \exp(-y_i(w^\intercal x_i + b)) \cdot \big(1 + \exp(-y_i(w^\intercal x_i + b))\big) - \big(-y_i w \exp(-y_i(w^\intercal x_i + b))\big)^2}{\big(1 + \exp(-y_i(w^\intercal x_i + b))\big)^2}$$