

# Konvexe Optimierung

## 5. Übungsserie

### Aufgabe 33

$$\begin{aligned}
 h_\delta(x, y) &= \sum_i^n \delta^2 \left( \sqrt{1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2} - 1 \right) \\
 &= \delta^2 \left( \sum_i^n \left( 1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{\frac{1}{2}} - \sum_i^n 1 \right) \\
 &= \delta^2 \left( \sum_i^n \left( 1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{\frac{1}{2}} - n \right) \\
 &= \delta^2 \sum_i^n \left( 1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{\frac{1}{2}} - \delta^2 n
 \end{aligned}$$

$$\frac{\partial}{\partial x_0} h_\delta(x, y) = \frac{\partial}{\partial x_0} \sum_i^n \left( 1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{\frac{1}{2}} - \delta^2 n \quad (1)$$

$$= \sum_i^n \frac{\partial}{\partial x_0} \left( 1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{\frac{1}{2}} - \frac{\partial}{\partial x_0} \delta^2 n \quad (2)$$

$$= \sum_i^n \frac{\partial}{\partial x_0} \left( 1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{\frac{1}{2}} \quad (3)$$

$$= \sum_i^n \frac{1}{2} \left( 1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{-\frac{1}{2}} \cdot \frac{\partial}{\partial x_0} \left( 1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right) \quad (4)$$

$$= \sum_i^n \frac{1}{2} \left( 1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{-\frac{1}{2}} \cdot 2 \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right) \cdot \frac{\partial}{\partial x_0} \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right) \quad (5)$$

$$= \sum_i^n \frac{1}{2} \left( 1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2 \right)^{-\frac{1}{2}} \cdot 2 \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right) \cdot \frac{\xi_i \cdot \delta}{\delta^2} \quad (6)$$

$$= \sum_i^n \frac{\left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right) \cdot \frac{\xi_i \cdot \delta}{\delta^2}}{\sqrt{1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2}} \quad (7)$$

$$= \sum_i^n \frac{\left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right) \cdot \frac{\xi_i}{\delta}}{\sqrt{1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2}} \quad (8)$$

$$= \sum_i^n \frac{\xi_i^2 x_0 + \xi_i x_1 - \xi_i \eta_i}{\delta^2 \sqrt{1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2}} \quad (9)$$

$$(10)$$

$$\frac{\partial}{\partial x_1} h_\delta(x, y) = \sum_i^n \frac{\left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right) \cdot \frac{\delta}{\delta^2}}{\sqrt{1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2}} \quad (11)$$

$$= \sum_i^n \frac{\frac{\xi_i x_0 + x_1 - \eta_i}{\delta^2}}{\sqrt{1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2}} \quad (12)$$

$$= \sum_i^n \frac{\xi_i x_0 + x_1 - \eta_i}{\delta^2 \sqrt{1 + \left( \frac{\xi_i x_0 + x_1 - \eta_i}{\delta} \right)^2}} \quad (13)$$

$$(14)$$