Konvexe Optimierung

5. Übungsserie

Aufgabe 33

$$\begin{aligned}
h_\delta(x,y) &= \sum_i^n \delta^2 \left(\sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2} - 1 \right) \\
&= \delta^2 \left(\sum_i^n \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2\right)^{\frac{1}{2}} - \sum_i^n 1 \right) \\
&= \delta^2 \left(\sum_i^n \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2\right)^{\frac{1}{2}} - n \right) \\
&= \delta^2 \sum_i^n \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2\right)^{\frac{1}{2}} - \delta^2 n
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial x_0} h_\delta(x,y) &= \frac{\partial}{\partial x_0} \sum_i^n \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2\right)^{\frac{1}{2}} - \delta^2 n && (1) \\
&= \sum_i^n \frac{\partial}{\partial x_0} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2\right)^{\frac{1}{2}} - \frac{\partial}{\partial x_0} \delta^2 n && (2) \\
&= \sum_i^n \frac{\partial}{\partial x_0} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2\right)^{\frac{1}{2}} && (3) \\
&= \sum_i^n \frac{1}{2} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2\right)^{-\frac{1}{2}} \cdot \frac{\partial}{\partial x_0} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2\right) && (4) \\
&= \sum_i^n \frac{1}{2} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2\right)^{-\frac{1}{2}} \cdot 2 \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right) \cdot \frac{\partial}{\partial x_0} \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right) && (5) \\
&= \sum_i^n \frac{1}{2} \left(1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2\right)^{-\frac{1}{2}} \cdot 2 \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right) \cdot \frac{\xi_i \cdot \delta}{\delta^2} && (6) \\
&= \sum_i^n \frac{\left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right) \cdot \frac{\xi_i \cdot \delta}{\delta^2}}{\sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2}} && (7) \\
&= \sum_i^n \frac{\left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right) \cdot \frac{\xi_i}{\delta}}{\sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2}} && (8) \\
&= \sum_i^n \frac{\xi_i^2 x_0 + \xi_i x_1 - \xi_i \eta_i}{\delta^2 \sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2}} && (9) \\
& && (10)
\end{aligned}$$

$$\frac{\partial}{\partial x_1} h_\delta(x,y) = \sum_i^n \frac{\left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right) \cdot \frac{\delta}{\delta^2}}{\sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2}} \tag{11}$$

$$= \sum_i^n \frac{\frac{\xi_i x_0 + x_1 - \eta_i}{\delta^2}}{\sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2}} \tag{12}$$

$$= \sum_i^n \frac{\xi_i x_0 + x_1 - \eta_i}{\delta^2 \sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2}} \tag{13}$$

$$\tag{14}$$

$$\frac{\partial}{\partial x_1} h_\delta(x,y) = \sum_i^n \frac{\left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right) \cdot \frac{\delta}{\delta^2}}{\sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2}}$$

$$= \sum_i^n \frac{\frac{\xi_i x_0 + x_1 - \eta_i}{\delta^2}}{\sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2}}$$

$$= \sum_i^n \frac{\xi_i x_0 + x_1 - \eta_i}{\delta^2 \sqrt{1 + \left(\frac{\xi_i x_0 + x_1 - \eta_i}{\delta}\right)^2}}$$