EQUATION 1.

$$P(y_{tn} \mid X_t = K) = \mathrm{Bin}(M_{tn} \mid M_{tn} + U_{tn}, E_{Kn}) \quad \underline{1.1}$$

$$= E_{Kn}^{M_{tn}} (1 - E_{Kn})^{U_{tn}} \quad \underline{1.1.2}$$

$X_t = ?$ > REGULAR DISCRETE LIKELIHOOD

$$\hookrightarrow P(Y_n \mid X_t) = \prod_K P(Y_{tn} \mid X_t = K)^{\mathbb{1}[X_t = K]} \quad \underline{1.2}$$

$$\mathbb{E}_Q(\mathbb{1}[X_t = K]) = \gamma_{Kt} \quad \therefore \gamma \text{ COMES FROM FORWARD/BACK} \quad \underline{1.2.1}$$

COMPLETE LIKELIHOOD

$$P(Y \mid X) = \prod_{tn} P(Y_{tn} \mid X_t) \quad \underline{1.3}$$

GIVEN $X_t = K$ THE LIKELIHOOD FOR AN INDIVIDUAL POSITION & CELL

ON 1.3 log

$$\log(P(Y \mid X)) = \sum_{tn} \log(\underbrace{P(Y_{tn} \mid X_t)}_{}^{1.2})$$

$$= \prod_K \underbrace{P(Y_{tn} \mid X_t = K)}_{1.1.2}^{\mathbb{1}[X_t = K]}$$

$$= E_{Kn}^{M_{tn}} (1 - E_{Kn})^{U_{tn}}$$

$\underline{1.4}$

$$\hookrightarrow \log(P(Y \mid X)) = \sum_{tn} \sum_K \underbrace{\mathbb{1}[X_t = K]}_{1.2.1 = \gamma_{Kt}} \; M_{tn} \log E_{Kn} + U_{tn}(1 - E_{Kn})$$

$\underline{1.5}$

$$\text{EXPECT} \rightarrow \mathbb{E}_Q \log(P(Y \mid X)) = \sum_{tn} \sum_K \gamma_{Kt} \, M_{tn} \log E_{Kn} + U_{tn}(1 - E_{Kn})$$

MAXIMIZATION WRT E_{Kn}

$$\frac{\partial}{\partial E_{Kn}} \mathbb{E}_Q \underbrace{(\log(P(Y \mid X))}_{1.5} = 0$$

$$\frac{\partial}{\partial E_{kn}} \sum_{ktn} \gamma_{kt} M_{tn} \log E_{kn} + U_{tn} (1-E_{kn}) \qquad 1.6$$

$$= \sum_t \gamma_{kt} \left(\frac{M_{tn}}{E_{kn}} - \frac{U_{tn}}{1-E_{kn}} \right) \qquad 1.7$$

SOLVE FOR E_{kn} IN 1.7

$$\sum_t \gamma_{kt} \left(\frac{M_{tn}}{E_{kn}} \right) = \sum_t \gamma_{kt} \left(\frac{U_{tn}}{1-E_{kn}} \right)$$

$$\sum_t \gamma_{kt} \left(\frac{M_{tn}}{\cancel{E_{kn}}} \right) \cancel{E_{kn}} = \sum_t \gamma_{kt} \left(\frac{U_{tn}}{1-E_{kn}} \right) E_{kn}$$

$$\sum_t \gamma_{kt} \left(M_{tn} \right) = \sum_t \gamma_{kt} \left(\frac{U_{tn}}{\cancel{1-E_{kn}}} \right) (\cancel{1-E_{kn}}) (E_{kn})$$

$$(1-E_{kn}) \sum_t \gamma_{kt} \left(M_{tn} \right) = E_{kn} \sum_t \gamma_{kt} \left(U_{tn} \right) \qquad 1.8$$

IN TERMS OF E_{kn} IN 1.8

$$E_{kn} \sum_t \gamma_{kt} (U_{tn}) + E_{kn} \sum_t \gamma_{kt} (M_{tn}) = \sum_t \gamma_{kt} (M_{tn})$$

$$E_{kn} \left(\sum_t \gamma_{kt} U_{tn} + \sum_t \gamma_{kt} M_{tn} \right) = \sum_t \gamma_{kt} M_{tn}$$

$$E_{kn} = \frac{\sum_t \gamma_{kt} M_{tn}}{\left(\sum_t \gamma_{kt} U_{tn} + \sum_t \gamma_{kt} M_{tn} \right)} \qquad 1.9$$

INCORPORATING $\beta(2,2)$ PRIOR IN 1.9

$\alpha = 2 \quad \beta = 2$

$$\log \text{Beta}(E \mid \alpha, \beta) = (\alpha - 1)\log E + (\beta - 1)\log(1-E) + C$$

$$\log \text{Beta}(E \mid 2,2) = \log E + \log(1-E) + C \qquad \underline{2.0}$$

$\underline{1.5}$ $\quad \mathbb{E}_Q[\log P(Y \mid X)] + \underline{\log \text{Beta}[E \mid 2,2]} =$

$$\sum_{tkn} \gamma_{kt} \left(M_{tn} \log E_{kn} + U_{tn} \log(1 - E_{kn}) \right) +$$

$\underline{2.0} \longrightarrow \log E_{kn} + \log(1 - E_{kn})$

$$\boxed{E_{kn} = \frac{\sum_t \gamma_{kt} M_{tn} + 1}{\left(\sum_t \gamma_{kt} U_{tn} + \sum_t \gamma_{kt} M_{tn}\right) + 2}}$$

$\beta(2,2)$ SUCCESS

$\beta(2,2)$ SUCCESS + FAILURE.

EQUATION 2.

DERIVATE

$z_{tj} = P(Y_t^{MISS} = j \mid Y_{OBS}, E, T)$ PREDICTOR VAR

IN TERMS OF $E_{ij} = P(Y_{tj} = 1 \mid X_t = i)$ PROB MATRIX

STATE i

AND $\gamma_{ti} = P(X_t = i \mid Y_{OBS})$

Y_t^{MISS} AT TIME t TAKES j VAL

Y_{tj} IS 1 IF $X_t = i$

time

Y_{OBS} INFLUENCES THE PRED VAR ON γ_{ti}

↳ SO IT HELPS TO INFORM THE PROBS OF x

$$z_{ti} = \sum_i P(\underbrace{Y_t^{miss} = j, X_t = i}_{} \mid Y_{OBS})$$

ALL OVER STATES

BOTH DEPEND ON Y_{OBS} THROUGH γ_{ti} AND OUR MATRIX

CONDITIONAL PROB

$$z_{ti} = \sum_i \underbrace{P(Y_t^{miss} = j \mid X_t = i}_{E_{ij}}, Y_{OBS}) \cdot \underbrace{P(X_t = i \mid Y_{OBS})}_{\gamma_{ti}}$$

Y_t^{miss} IS COND IND? OF Y_{OBS} GIVEN X_i?

$$\boxed{z_{ti} = \sum_i E_{ij} \gamma_{ti}}$$

$$P(Y^M, X_t \mid Y^O) = P(Y^M \mid Y^O)\, P(X_t \mid Y^O)$$
$$= P(Y^M \mid X_t, Y^O)\, P(X_t \mid Y^O)$$

$$P(Y^M \mid X_t, Y^O) = P(Y^M \mid Y^O)$$

CONDITIONAL INDEPENDENCE.

$P(A \mid B, C) = P(A \mid C)$

$P(A, B \mid C) = P(A \mid C) P(B \mid C)$

$= P(A \mid B, C) P(B \mid C)$

IF I KNOW C, THEN B TELLS ME NOTHING ABOUT A

$A \perp\!\!\!\perp B \mid C$

EQUATION 3.

ADDING LOSSES IN ALL CELLS

$$L = -\sum_{j=1}^{J} \sum_{t=1}^{T-1} \log Bin(M_{t+1,j} \mid M_{t+1,j} + U_{t+1,j}, p_{tj})$$

LOSSES PER POSITION IN CELL j

where

p_{tj} = RNN OUTPUT

t = POSITION

j = CELL

'LOSS IS FOR SCENARIOS WHERE WE PREDICT SEQ DATA'

?

$$Bin(x \mid n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

APPLYING LOG

$$= \log(p^x) \log((1-p)^{n-x})$$

$$= (x \log p) + (n - x \log(1-p))$$

$$= M_{t+1,j} \log p_{tj} + (\cancel{M_{t+1,j}} + U_{t+1,j} - \cancel{M_{t+1,j}} \log(1 - p_{tj}))$$

$$= M_{t+1,j} \log p_{tj} + U_{t+1,j} \log(1 - p_{tj})$$

$$\boxed{= -\sum_{j=1}^{J} \sum_{t=1}^{T-1} M_{t+1,j} \log p_{tj} + U_{t+1,j} \log(1 - p_{tj})}$$

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