Supervised Learning: Regression

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Classification

IRIS Classification, Ronald Fisher (1936)

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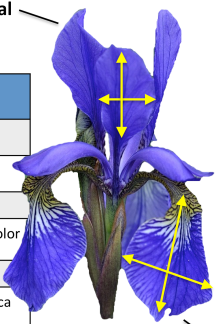
Samples
(instances, observations)

	Sepal length	Sepal width	Petal length	Petal width	Class label
1	5.1	3.5	1.4	0.2	Setosa
2	4.9	3.0	1.4	0.2	Setosa
...					
50	6.4	3.5	4.5	1.2	Versicolor
...					
150	5.9	3.0	5.0	1.8	Virginica

Features
(attributes, measurements, dimensions)

Class labels
(targets)



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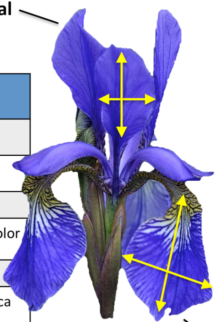

- discrete valued output

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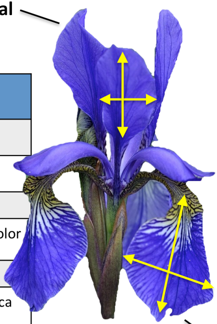
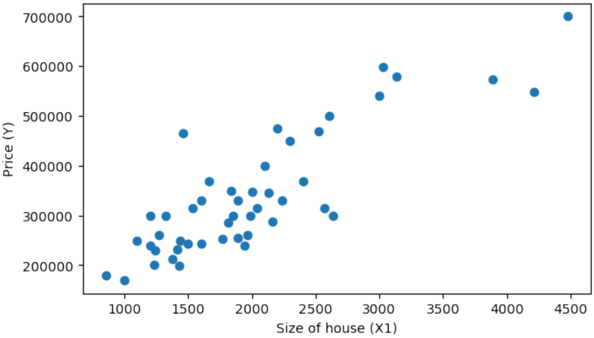


- discrete valued output
- what if output is continuous?

Regression



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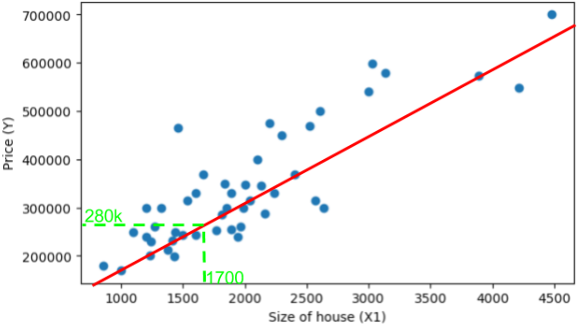
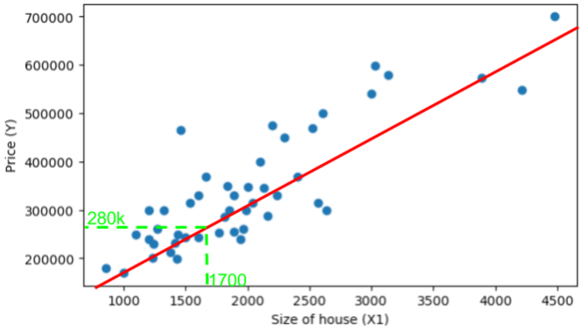
Regression



700000
600000
500000
400000
300000
200000
Price (Y)
280k
1700
1000
1500
2000
2500
3000
3500
4000
4500
Size of house (X1)

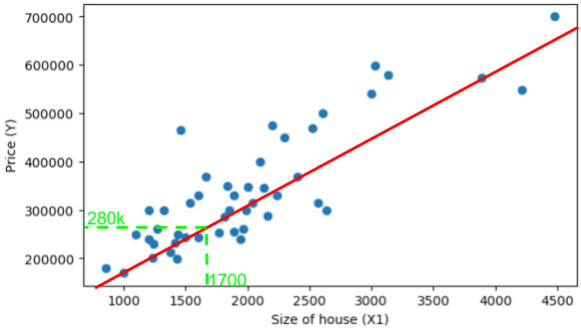
Regression



Supervised Learning

Right answers are given

Regression



Supervised Learning

Right answers are given

Regression Problem

Predict real valued output

Training Set

Size of house in feet^2 (x)	Price in 1000$s (y)
2104	460
1416	232
1534	315
⋮	⋮

m → number of examples
x → input variable/features
y → output variable/target
(x^i, y^i) → i^{th} training example

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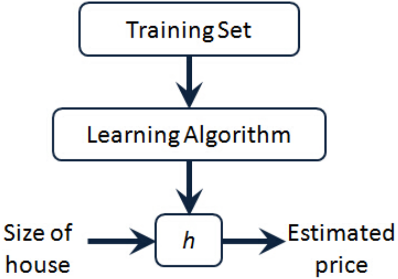

Hypothesis $h : x \rightarrow y$

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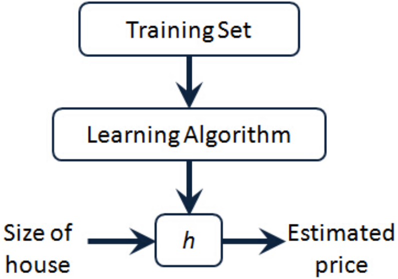

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$$y = h_\theta(x) = \theta_0 + \theta_1 x$$

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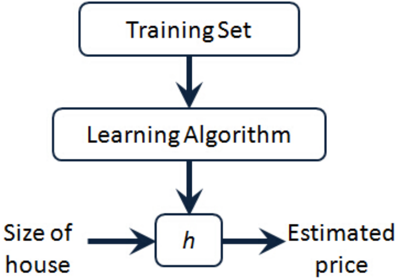

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Univariate Linear Regression

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How to choose the parameters?

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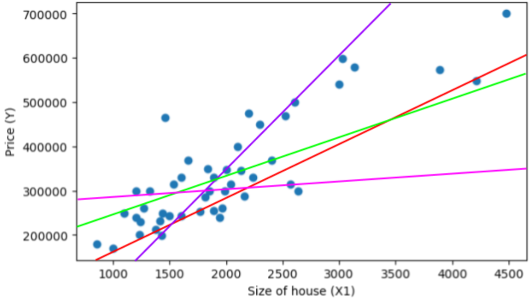
- Different θ_0 and θ_1 will generate different lines



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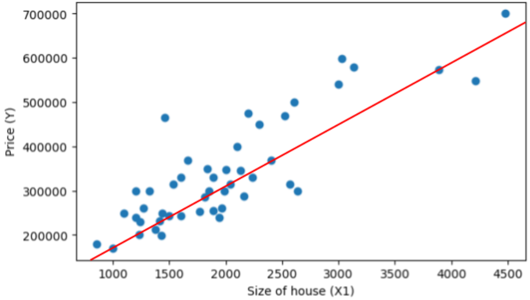

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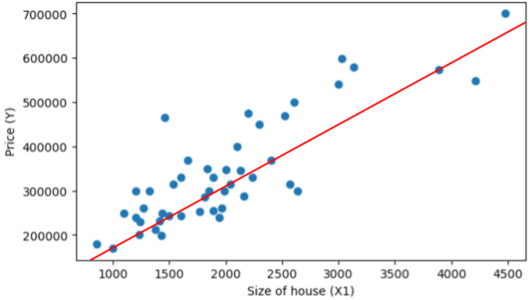

- Need to choose best θ_0 and θ_1

Idea: Choose θ_0 and θ_1 so that $h_\theta(x)$ is close to given outputs y in the training examples (x, y)

Defining a Cost

$$y = h_\theta(x) = \theta_0 + \theta_1 x$$

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Cost Function: Mean Squared Error

$$min_{\theta_0,\theta_1} \ (h_\theta(x) - y)^2$$

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$$min_{\theta_0,\theta_1} \ \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^i) - y^i)^2$$

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$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^i) - y^i)^2$$

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$J(\theta_0, \theta_1)$ is a function of the parameters θ_0 and θ_1

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More about the Cost

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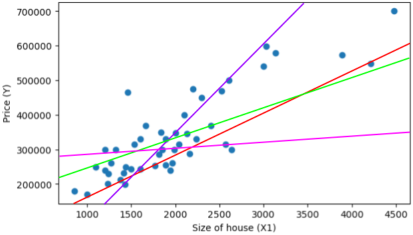

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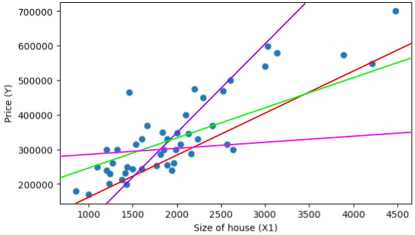
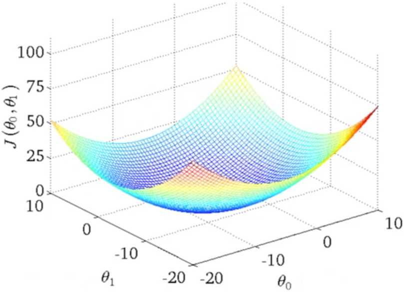


$J(\theta_0, \theta_1)$ is a function of the parameters θ_0 and θ_1



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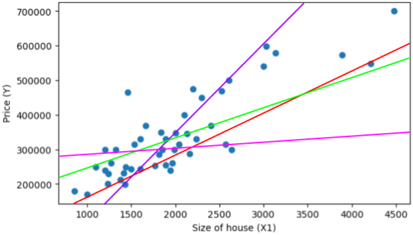

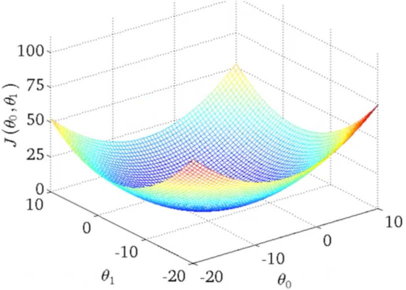

- Goal: to find a $\hat{\theta}_0$ and $\hat{\theta}_1$ providing minimum $J(\theta_0, \theta_1)$
- Corresponding hypothesis will be the trained model: $h_{\hat{\theta}}(x) = \hat{\theta}_0 + \hat{\theta}_1 x$
- $J(\theta_0, \theta_1)$ is convex: can be minimized using Gradient Descent

Multiple Features

Size of house in feet^2 (x)	Price in 1000$s (y)
2104	460
1416	232
1534	315
⋮	⋮

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Multiple Features

Size of house in feet^2 (x)	Number of bedrooms	Number of Floors	Price in 1000$s (y)
2104	5	1	460
1416	3	2	232
1534	3	2	315
$\vdots$	$\vdots$	$\vdots$	$\vdots$

$n \rightarrow$ number of features
$\vec{x}^{i} \rightarrow$ i^{th} input is now a n-dimensional vector
$x_j^i \rightarrow$ j^{th} feature of the i^{th} input

Multivariate Linear Regression

Hypothesis

$$y = h_{\vec{\theta}}(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \ldots + \theta_n x_n$$

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Cost Function

$$J(\theta_0, \theta_1, \theta_2, \ldots, \theta_n) = \frac{1}{2m} \sum_{i=1}^{m} (h_{\vec{\theta}}(\vec{x^i}) - y^i)^2$$

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Multivariate Linear Regression

Hypothesis

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Cost Function

$$J(\theta_0, \theta_1, \theta_2, \ldots, \theta_n) = \frac{1}{2m} \sum_{i=1}^{m} (h_{\vec{\theta}}(\vec{x^i}) - y^i)^2$$

- Goal: to find $\hat{\theta}_0, \ldots, \hat{\theta}_n$ providing minimum $J(\theta_0, \ldots, \theta_n)$
- Here also, $J(\theta_0, \ldots, \theta_n)$ is convex: thus can be minimized using Gradient Descent

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Time Series

A time series is a sequence of observations $s_t \in \mathbb{R}$, usually ordered in time.

Examples of time series can be found in every scientific and applied domain:

- Meteorology: weather variables, like temperature, pressure, wind.
- Economy and finance: economic factors (GNP), financial indexes, exchange rate, spread.
- Marketing: activity of business, sales.
- Industry: electric load, power consumption, voltage, sensors.
- Bio-medicine: physiological signals (EEG), heart-rate, patient temperature.
- Web: clicks, logs.

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Time Series Forecasting

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To do that:

1. Understand or model the stochastic mechanisms generating the data
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Examples:

- Weather forecasting.
- Sales prediction.
- Stock market forecasting.

Formalizing Time Series Forecasting

- Assume we have a time series:

$$x_1, x_2, \dots, x_N$$

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- Assume we have a time series:

$$x_1, x_2, \ldots, x_N$$

- T observed values: also known as history size
- Need to forecast T' future values: also known as forecast horizon
- We want to model the relation:

$$(x_{T+1}, x_{T+2}, \ldots, x_{T+T'}) = r(x_1, x_2, \ldots, x_T)$$

Constructing Regression Matrix

- To learn a regression model: multiple examples of feature value pairs are required

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- Moving window method:

$$x_1, x_2, \ldots, x_T, x_{T+1}, x_{T+2}, \ldots x_{T+T'}, x_{T+T'+1}, \ldots$$

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$$\underbrace{x_1, x_2, \ldots, x_T}_{\text{history}}, \underbrace{x_{T+1}, x_{T+2}, \ldots, x_{T+T'}}_{\text{horizon}}, x_{T+T'+1}, \ldots$$

$$X = \begin{bmatrix} x_1 & x_2 & \ldots & x_T \end{bmatrix} \quad Y = \begin{bmatrix} x_{T+1} & x_{T+2} & \ldots & x_{T+T'} \end{bmatrix}$$

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- Regression models are learned on input X and output y

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A Good ML Course

- Introduction to Machine Learning. Course by Andrew Ng. (Youtube)

Hands On!

- 1_MachineLearning_Regression
- 2_TimeSeriesForecasting