Game of Sevens Structural Analysis

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Abstract

This paper examines the strategic complexity of the game of Sevens, with particular emphasis on how this complexity changes as the number of suits and cards is increased. Through statistical analysis and logical modeling, the range of strategic options available to players is characterized. The results indicate that, as the deck size and the number of suits increase, the number of choices available to each player grows linearly. As a consequence, the opportunity for meaningful strategic advantage decreases, and outcomes in large-scale variants of the game become predominantly determined by chance.

***K*eywords** Sevens · Options · Game Theory · Strategic Analysis

1 Introduction

Sevens is notable for its straightforward rules and minimal dependence on previous moves, which makes it an ideal subject for mathematical analysis. The simplicity of its structure allows for systematic investigation into how the game's characteristics change when key parameters—such as the number of suits or the number of cards—are adjusted.

This study aims to address the following questions: How does the difficulty of Sevens evolve as additional suits or cards are introduced? Are there critical thresholds beyond which the strategic nature of the game changes significantly?

The primary finding is that increasing the number of cards and suits causes strategy to play a meaningful role primarily during a specific phase—when both ends of a suit's sequence remain playable, referred to as the "LR phase." Beyond this stage, the progression of the game is largely determined by chance rather than strategic decision-making.

This work was conducted independently and is based entirely on original simulation and modeling. While no formal literature review was performed, this paper builds on foundational ideas from combinatorial game analysis and computational strategy evaluation. The game of Sevens, due to its constrained but expandable ruleset, provides an approachable platform for studying how strategic depth evolves with system complexity. This analysis aims to contribute to that broader conversation, particularly in contexts where chance and constrained choice interact in scaled environments. Future iterations may explore connections with established game-theoretic frameworks or related work in card game modeling.

Code Repository: `https://github.com/chrolo3/sevens`

2 Methodology

2.1 Game Simulations

Simulations were developed in Rust to analyze two variations of the game of Sevens. These simulations incorporated several factors, including the statistical impact of different strategies, card distribution methods, selection procedures, and random assignment of the starting player.

2.1.1 Game Rules

1. The entire deck is dealt evenly among all players, continuing until no cards remain.
2. Players take turns in a fixed order. If a player cannot play, their turn is skipped and play passes to the next participant.
3. The player holding the 7 of Spades begins the game by playing this card.
4. A 7 of any suit may be played to initiate a new row on the table.
5. On each turn, a player may add to an existing row by playing the next sequential card (for example, playing the 3 or 8 of Hearts if the row contains the 4 or 7 of Hearts).
6. **Spades Variation Only:** A player may play a card in a non-spade suit only if its corresponding card already exists in the spades row.

2.2 Data Modeling

To facilitate simulation and analysis, structured representations for both suits and cards were developed.

2.2.1 Suit Representation

Suits are represented using integer indices, with index 0 reserved for spades (i.e., Spades = 0, Hearts = 1, etc.). Apart from distinguishing spades, the ordering of the remaining suits is arbitrary, so representing suits as integers is sufficient for the modeling purposes.

2.2.2 Card Representation

Each card is identified by a pair consisting of its suit index and a numerical value ranging from 1 to the maximum value allowed for a suit (typically 1–13 in a standard deck). Since face cards and aces function identically to numbered cards in this context, they are represented by their respective values without special treatment.

2.2.3 Game Board Structure

The game board is modeled as a collection of suits, each associated with the range of cards currently in play for that suit. For example, if the 4 through 8 of spades have been played, the state is represented as `gameBoard[Spades] = 4..8`. During simulation, the available cards in a player's hand are compared to these ranges to determine which cards can be legally played. The total options for a player are given by the sum of possible plays across all suits.

The set of game rules evaluates the current board state and generates the list of playable options for each player. These rules vary by version of Sevens. A key distinction is that, in certain variations, if a suit's sequence aligns with that of spades, options for that suit may be limited. For instance, if the spades row contains cards 4 through 8 and hearts contains 5 through 8, only the 4 of hearts remains playable in the hearts suit.

2.2.4 Player Strategies

Player strategies are implemented as distinct structures. Each strategy determines which card to play by selecting from the set of playable cards—those found both in the player's hand and among the legal moves on the game board.

Baseline Strategy A fully random strategy is used as the baseline for comparison. In this approach, the available plays are identified by intersecting the player's hand with the set of valid moves. A card is then selected at random from this subset. This strategy provides a reference point against which the performance of other, more deliberate strategies can be measured.

Additional Strategies Several other strategies are examined, including:

- **(Sevens)** Selecting the lowest available card first
- **(Sevens)** Selecting the highest available card first
- **(Spades variant)** Preferring to play a spades card, if available
- **(Spades variant)** Prioritizing spades cards to be played last

- **(Spades variant)** Avoiding spades as long as possible, then playing the highest available card in other suits

This collection of strategies is not exhaustive and does not necessarily represent optimal play for a given hand. However, it offers a useful range of approaches for evaluating overall strategy performance.

2.3 Simulation Parameters

At the start of each simulation, the entire deck is shuffled and cards are distributed equally among all players. This process ensures that the selection of the starting player and initial strategy is random, as the player dealt the 7 of spades is determined by chance.

Each simulation instance is characterized by:

- A predefined set of strategies
- Four players
- Specific suit counts and suit sizes under consideration

A diverse collection of strategies was selected to capture a wide range of strategic behaviors. Each strategy combination was evaluated across all combinations of suit counts and suit sizes, defined as follows:

- **Suit counts:** 4, 8, 16, 32, 48, 64
- **Suit sizes:** 13, 27, 55, 83, 111

These parameters were chosen to maximize observable effects within simulations that could be completed in a reasonable time frame.

For each game variant, strategy combinations were constructed to reflect a broad sampling of interactions between players. These included:

- **Homogeneous strategies:** All four players using the same strategy (e.g., all random).
- **Pairwise matchups:** Games featuring two strategies in varying proportions (e.g., 3 vs. 1, 2 vs. 2, etc.).
- **Mixed strategies:** Combinations with three or all four strategies present, representing more realistic and diverse play environments.

The Game of Sevens simulations used three core strategies, while the Spades variant included four. This yielded 14 unique configurations in the base game and 36 in the Spades variation. These sets were not exhaustive but were sufficient to capture distinct interaction dynamics, including cases where strategic diversity influenced game flow or outcomes.

For every unique parameter combination, 100,000 games were simulated, producing the following totals:

- 35 million games of Sevens
- 90 million games of the Spades variation
- >200 billion cards played

Although this number of simulations does not encompass the entire set of possible games, it provides a sufficiently broad sample to support the conclusions presented below.

2.4 Terminology

N_7 The central card of a suit, typically the 7 in traditional Sevens.

L The lowest card in the range of a suit (e.g., if Hearts includes cards 4 through 10, L represents 4).

R The highest card in the range of a suit (e.g., in the previous example, R represents 10).

2.4.1 Game States

0 Initial state of a suit, with no cards played.

N_7 State in which only the central card N_7 has been played.

LR State where both the lowest (L) and highest (R) cards in the suit remain playable.

LC State where only the left (low) side of the suit is open for play, while the right is closed.

CR State where only the right (high) side is open for play, while the left is closed.

C State indicating that all cards in the suit have been played.

3 Constraints for Basic Games of Sevens

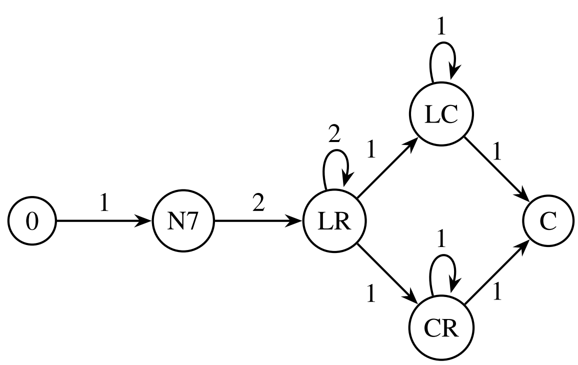
3.1 Analysis from a Player's Perspective

In the game of Sevens, the board can be viewed as a set of playable options available to each participant. Specifically, these options consist both of cards that could potentially be played (if held by the player) and cards that can actually be played (because the player has them in hand). Distinguishing between these two cases provides a straightforward framework for analysis. Each card is dealt randomly and each player has an equal probability of receiving any given card. Therefore, if there are five playable options on the board, the probability that a player holds at least one of these cards depends on their initial hand, which is determined entirely by the random deal.

3.2 Modeling the Game State

In the absence of house rules, Sevens can be modeled as a collection of S distinct suits, each containing N cards. The total number of cards in the deck is then given by $D = N \times S$. Play typically begins with the midpoint card of each suit—traditionally a seven—denoted as N_7.

The progression of the game can be represented using a state diagram, where each transition between states corresponds to the number of cards that can be used to advance from one state to another. In other words, these edges indicate the number of available plays a participant can make for a given suit at a particular moment. The rules of Sevens ensure that each suit evolves independently, which permits modeling the state of each suit separately.



This diagram quantifies the number of possible plays for each state transition, but does not specify the precise conditions under which these transitions occur. As the number of suits and the deck size increase, the LR state (where both the leftmost and rightmost cards can be played) becomes increasingly dominant, since a variety of initial suit configurations can lead to this state.

The table below corresponds to these states but focuses on the probability that a player possesses the relevant card for each transition. Note that in this tabulation, N_L and N_R—the next available cards on the left and right sides—are treated as distinct playable options for the participant.

$$P = \frac{\binom{D-1}{C}}{\binom{D}{C}}$$

This represents the probability that a player does **not** have a specific desired card in their hand, given that their hand contains C cards drawn from a deck of D distinct cards.

Here, P is the probability that none of the C cards in the player's hand is the particular card of interest (for example, a specific suit and rank). The numerator, $\binom{D-1}{C}$, counts the ways to choose C cards that exclude the desired card. The denominator, $\binom{D}{C}$, counts all possible hands of C cards from D.

To find the probability that the player **does** have the specific card, subtract P from 1 (i.e., $1 - P$). Since every card in the deck is unique, and you are checking for precisely one special card, this calculation gives the correct likelihood of that card appearing in the player's hand.

Table 1: State Transition Table with Available Plays

State	N_7 played (N_7)	N_L or N_R played (LR)	N_L played (CR)	N_R played (LC)	N_L or N_R played (C)
N_7	$1-P$	0	0	0	0
N_L	0	$1-P$	0	0	0
N_R	0	$1-P$	0	0	0
N_{CL}	0	0	$1-P$	0	0
N_{CR}	0	0	0	$1-P$	0
N_{CC}	0	0	0	0	$1-P$

Further Analysis To better understand how complex the game can get, it is helpful to look at the number of choices a player faces at different stages. Each state in the game is defined by the cards that remain and where the player is in the sequence of play.

The total number of possible state transitions can be expressed as $O(S_i)$ and, in this approach, is bounded above by $(N_7)^2$, where N_7 is the number of starting midpoint cards in the system.

(N_7)	Only one possibility—the game starts with just the N_7 card available.
(LR)	Two possibilities—transitions happen when either the left or right path closes.
(LC) or (RC)	One option—occurs when there is a single lane left open.
(C)	One possibility—this is the final state when play is complete.

Breaking down the distribution of all game states by transition type gives a clearer view:

$O(S_i)$	There are $(N_7)^2$ possible configurations in total.
(N_7)	Just one initial scenario—where play starts with N_7.
(LR)	$(N_7-1)^2$ configurations—this represents repeated situations with one fewer card on the left and right, each time.
(LC) or (CR)	$2N_7-2$ configurations—derived from considering how the sides close.
(C)	One ending configuration.

Taking the analysis further, we can examine what happens as N_7 becomes very large:

$$
\begin{array}{ll}
(N_7, N_7) & \lim_{N_7\to\infty} \dfrac{1}{(N_7)^2} = 0 \\
(N_L, N_R) & \lim_{N_7\to\infty} \dfrac{(N_7-1)^2}{(N_7)^2} = 1 \\
(N_L, N_C) \text{ or } (N_C, N_R) & \lim_{N_7\to\infty} \dfrac{2N_7-2}{(N_7)^2} = 0 \\
(N_C, N_C) & \lim_{N_7\to\infty} \dfrac{1}{(N_7)^2} = 0
\end{array}
$$

These results indicate that, in sufficiently large games, the board is overwhelmingly likely to be in a state where both the left and right sides of a suit are playable. This aligns with intuition: as the number of suits and card values increases, the most prevalent game state becomes the one that allows the greatest number of playable options.

3.3 Game State Analysis

As previously discussed, when the number of cards in each suit, denoted N_7, increases without bound, the number of available options per suit in a typical game state, represented as $O(S_i) = (LR)$, converges to 1. In practical terms, as

N_7 becomes large, a player's decisions increasingly depend on the limited possible moves available for each suit, which are constrained to two options per suit.

Consequently, as N_7 grows, the likelihood that the game is in a state (LR) at any given moment approaches certainty. In this context, the (LR) state refers to a situation where only the lowest and highest remaining cards of a suit can be played.

Generally, strategic depth increases with the number of available options, particularly when these options are independent of prior moves or when a player can influence the options accessible to themselves or others. In the game of Sevens, however, the number of actionable moves per player per suit approaches two as N_7 increases, and these moves are determined by the initial card distribution. As a result, there is minimal opportunity for meaningful strategy in most cases.

Exceptions may occur when N_7 is small. In these cases, players occasionally have opportunities to withhold or play cards in ways that slightly affect the options available to themselves or to other players. Nonetheless, any such influence is limited: a player can only affect the number of available options by at most one. Therefore, even under these conditions, the overall strategic complexity of the game remains low.

Importantly, the total number of options for all suits, denoted $O(S)$, is given by $2|S|$, where $|S|$ is the number of suits. Thus, as N_7 approaches infinity, the total options available to each player at any time approach twice the number of suits.

3.4 Statistical Analysis

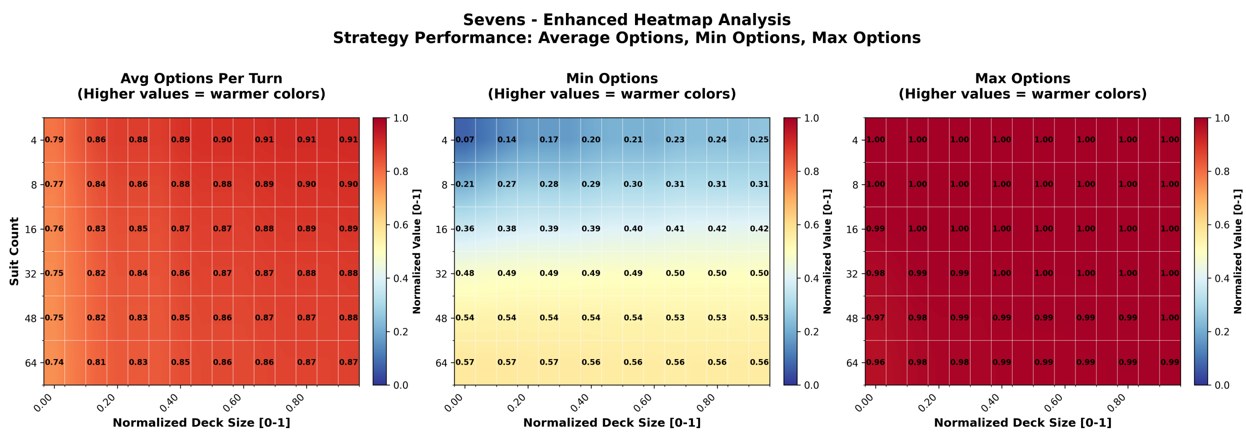


Figure 1: Dependence of Number of Suits

3.4.1 Heatmap Analysis

Figure 1 presents several normalized graphs, each scaled between 0 and the maximum observed value for the corresponding metric, to facilitate direct comparison as suit and deck sizes increase. In these graphs, the y-axis indicates the number of suits, while the x-axis shows increasing deck sizes. Each curve represents the average metric value calculated across all simulations for each suit and deck size combination.

For example, an average minimum options value close to 0 indicates that, on average, players had very few options available per turn, whereas a value near 1 suggests that the average minimum number of options per player equaled the number of suits (e.g., 16 for an 8-suit deck).

The results show that the average number of options per turn increases steadily with deck size and is generally independent of the number of suits. This observation is expected, as smaller suit sizes reduce the number of possible game states, and the overall structure becomes more uniform.

In contrast, the average minimum options per turn demonstrate a stronger correlation with both suit count and deck size, though deck size has a more pronounced effect. As the number of suits increases, so does the average minimum number of options available, and this trend is also present, though less strongly, as deck size increases. These patterns align with earlier arguments that, with larger values for suit count and deck size, the (LR) state becomes dominant.

Finally, the average maximum options per turn remain close to 1, indicating that, on average, the maximum number of options available to a player typically corresponds to the (LR) state, for the reasons previously discussed.

4 Constraints for the Spades Variation in the Game of Sevens

The introduction of the final rule for the Spades variation adds significant complexity to the game's dynamics. When a player is dealt spades, it becomes possible to manipulate the board with greater effectiveness. This increased potential for strategy, however, is primarily determined by the initial distribution of cards, rather than choices made during the game.

4.1 Strategic Considerations in the Spades Variation

Consider a scenario where the spades row on the board covers the range $4, 9$, and the corresponding rows for the other suits are within this interval. If a player holds the 3 of spades and the 10 of spades, a decision must be made regarding which card to play, or whether to play a card from another suit if available. Playing a spade in this context may unlock additional moves for opponents by increasing the number of possible plays on the board. Conversely, choosing not to play a spade can restrict opponents from making certain plays in other suits.

4.2 Modeling the Game State

Under the Spades house rules, the game can still be represented as M distinct suits, each containing N cards. However, in this variation, suit transitions become conditional rather than independent. Specifically, a card at position N_7 in any suit can only be played after the N_7 of spades has been played. Similarly, the states denoted as LR, LC, and RC are accessible only after the corresponding spade has been played. This interdependency between suits increases the complexity of state modeling compared to the standard rules. Nevertheless, several simplifications can still be made for the purpose of analysis, and the general structure of the state diagram and likelihood of each move remain similar to the standard game.

Several key properties can be deduced for the Spades variation:

- The minimum number of available options is achieved by delaying the play of spades. For example, after the N_7 of spades is played, each remaining N_7 is unlocked and $S - 1$ plays—one become available.
- If all players only play spades when no other moves are possible, the resulting pattern in available options resembles a sawtooth graph, peaking at $2S - 1$ options and reaching a minimum at 1. The average across a game is approximately S options per turn.
- In practice, players do not always follow identical strategies, so there will be observable variation around this average, with the average number of options sometimes being exceeded.

As previously established, as N_7 approaches infinity, the number of options per suit in state (LR), $O(S_i) = (LR)$, approaches 1. Thus, as the number of cards in each suit increases, the effectiveness of any strategy becomes increasingly constrained, since each player faces only 2 possible moves per suit. In the limit, as N_7 grows, the probability that a suit is in state (LR) at any point approaches 1.

For small values of N_7, the number of decisions where a player can withhold or play cards to influence the available options is greater. However, because a player can alter the set of options by at most one, the overall strategic depth introduced by these choices remains limited.

4.3 Statistical Analysis

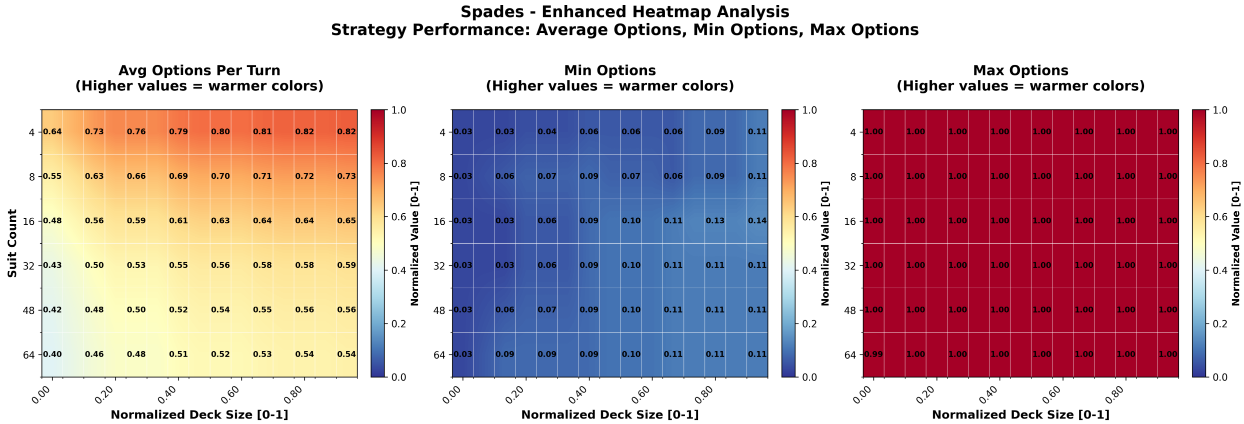


Figure 2: Dependence of Number of Suits

4.3.1 Spades Variation Analysis

Figure 2 shows normalized results for the Spades version of Sevens. These results show different trends from those seen in the basic game.

The average number of options per turn depends on both the number of suits and the size of the deck. When the number of suits increases, the average number of options goes down. This happens because more suits rely on spades being played first. On the other hand, as the deck gets larger, the average number of options increases. This is because many simulated strategies choose to play spades early, which makes other suits available sooner. The combined effect of these two factors tends to produce an average close to 0.5, meaning that players usually have about one playable option per suit.

The average minimum number of options follows a simpler pattern. It increases as both the number of suits and the deck size grow. This happens because it becomes less likely that a suit is blocked by a missing spade, especially when players are using a mix of strategies. These results also suggest that, when players are not always making the best moves, randomness can play a big role, and strong strategies might not always be enough to control the outcome.

The maximum options per turn behave much like they do in the standard version of the game. As expected, especially in larger games, there are turns when all suits reach the (LR) state, which allows for the maximum number of plays. This supports the earlier finding that the (LR) state shows up often in more complex games.

5 Conclusion

Analysis of these variations in the game of Sevens demonstrates that, although some strategies may offer occasional advantages, outcomes are primarily determined by the initial card distribution. This investigation extends existing analyses by considering the behavior of the game as both deck size and suit count increase, highlighting the diminishing influence of strategy and the increasing predominance of chance in large-scale scenarios.