The Cycle-7 Complex: Relations of Diatonic Set Theory to the Evolution of Ancient Tonal Systems

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The recent application of set theory[1] and the concepts of pattern matching and position finding[2] to the diatonic set 7–35 has served to refocus attention on the manner in which pitch and interval classes operate within a tonal hierarchy. To quote Richmond Browne, "The notions of *interval content* and *intervallic context* will serve as vehicles for . . . position finding and pattern matching operations in tonal usages of the diatonic set. Content and context, like position finding and pattern matching, are in some senses dialectically related ways of looking at the same data."[3] Little attempt, however, has been made toward the possible employment of these principles in a historical perspective—in particular, the evolution of musical systems in early civilizations. It is conceivable that the resultant wedding of old and new could shed illumination on the question of common tonal elements in diverse cultures of the ancient world. It is this topic to which the present paper is addressed.

Its basic purpose is threefold: (1) to survey the existent research dealing with the origins of certain tonal systems, concentrating primarily on those documented with theoretical writings, (2) to suggest several reasons why specific pitch collections reappear in a number of apparently widely divergent cultures, and (3) to propose a hypothesis concerning the relative frequency/infrequency of particular modes. Obviously such an undertaking is fraught with dangers. Even assuming correct notational deciphering, very little of the music survives. Nor is it certain whether any reconstruction of musical style, based on extant theoretical treatises, closely approximates actual performance practices. Finally, some of the research by other scholars cited here is itself highly speculative.

The codification of religious and court duties in ancient civilizations necessitated a corps of trained musicians whose function it was to create an art form suitable for the aesthetic appreciation of the aristocracy. This in turn led logically to the establishment of tonal systems, which attempted to organize musical space in accord with specific theoretical principles. Sachs notes that such systems tended to crystallize in "one or

[1]See, for example, the recent articles of John Clough, although he employs a mod 7 system.

[2]The entire issue of *In Theory Only* 5, nos. 6-7 (July-August 1981) is devoted to this area of research. Included are Richmond Browne, "Tonal Implications of the Diatonic Set," pp. 3–21; Edwin Hantz, "Recognizing Recognition: A Problem of Musical Empiricism," pp. 22–38; Helen Brown and David Butler, "Diatonic Trichords as Minimal Cue-Cells," pp.39–55.

[3]Browne, "Tonal Implications," p. 3.

more of the three consonant intervals innate in man: the fourth, the fifth, and the octave."[4] If this statement is valid, the significant role of perfect intervals in early music should be evident.[5]

Although ancient instrumental music was doubtless patterned originally after vocal models, the need for consistent tuning procedures led to more concrete theoretical foundations. Investigations concerning the overblowing of early wind instruments are somewhat disappointing in this regard.[6] With strings, however, we are on safer ground. The feasibility of sounding two strings simultaneously allowed precise tuning by eliminating beats between harmonic components of the octave, fourth, and fifth.[7] It is therefore no surprise that a number of ancient plucked instruments incorporated a fourth and fifth above the lowest sounding string as basic pivotal tones, thereby partitioning the octave into two empty disjunct tetrachords (↑ E A B e).[8] The old four-stringed Greek lyre is a classic case

Figure 1

in point.[9]. These pitches could be derived either through *cyclic* tuning, using a series of alternating ascending/descending fifths and fourths, or by the *divisive* principle of dividing a string into arithmetic lengths of 1/2, 1/3, and 1/4 (see Figure 1). Mathematical proportion confirms this arrangement, with pitch classes A and B functioning as the harmonic and arithmetic means of the octave respectively. The resultant tones served conveniently as framing references for tetrachordal melodic construction, noted in many early vocal melodies.[10] It is also possible that the fourth and/or the fifth occasionally functioned as harmonic entities[11] or drones.[12]

The asymmetrical partitioning of the octave by the fourth or fifth is not trivial.[13] Sachs mentions that there has always been

[4]Curt Sachs, *The Rise of Music in the Ancient World* (New York: W.W. Norton, 1943), p. 64.

[5]The frequent use of the octave as a framing boundary in "tumbling strains" may be noted in primitive vocal melodies; see Curt Sachs, *The Wellsprings of Music*, edited by Jaap Kunst (New York: McGraw-Hill, 1965), pp. 54–55. It may have also occurred as a fleeting vertical entity in music that incorporated an underlying drone. Its employment as a doubling interval in vocal ensembles was probably restricted, however, since most prehistoric cultures rarely featured mixed male and female voices.

[6]There is some debate concerning the dependability of overblown fifths, as in the closed bamboo pipes of ancient China. Bukofzer has pointed out the crucial role of length to diameter, producing twelfths of varying intonation; see his "Präzionmessungen an primitiven Musikinstrumenten," *Zeitschrift für Physik* 99 (1936), pp. 643ff. On the other hand, see Kathleen Schlesinger, *The Greek Aulos* (London: Methuen, 1939), pp. 313–37.

[7]By matching aurally the unisons occurring between the third and second harmonics of a perfect fifth (C to G) or the fourth and third harmonics of a perfect fourth (C to F).

[8]In the following examples the signs ↑ or ↓ will denote whether the pitch collection is rising or falling from the initial pitch class. The upper octave will be represented by a lowercase letter.

[9]Ludwig Deubner, "Die viersaitige Leier," *Athenische Mitteilungen* 54 (1929), pp. 194–200. See also Otto Gombosi, "Key, Mode, Species," *Journal of the American Musicological Society* 4 (1951), pp. 24–25, regarding the significance of these pitch classes within the Greek octave species.

[10]It should be pointed out, however, that many vocal melodies were based on conjunct tetrachords (↑ E A D) in which the octave does not play a significant role. In this respect some stringed instruments, such as the ancient Persian *'ud*, were tuned in consecutive fourths. For a further reference in relation to Gregorian chant, see below.

[11]See the section dealing with the transcription of an ancient Babylonian hymn on p. 48 and n. 61.

[12]The *tambura* drones in Indian music, which normally feature a fifth or fourth, probably originated not earlier than the fifteenth century; see B. Chaitanya Deva, *The Psychoacoustics of Music and Speech* (Madras: The Music Academy, 1967), p. 71.

[13]One may also mention certain isotonic systems, such as those found in Bali and Java. Even here, the distance between tones is not exactly equal but,

an "inhibition of artists against squares for the benefit of rectangles."[14] The division of the octave into a conjunct pentachord/tetrachord (♮ E B e or E A e) may have formed the basis for authentic/plagal classifications in many modal systems.

This resultant pitch collection constitutes the first three tones of a projection of perfect fifths: ♮A E b. Expressed in set theoretic terms, the trichord 0, 2, 7 (3–9) is generated by successive applications of the multiplier ic 7. For purposes of comparison, future references to this superimposition will employ the pitch class F as the generating tone whenever feasible. Further extension of this process results in a group of sets which may be conveniently labelled the *Cycle-7 Complex*.[15] This study will concentrate on the pair of complementary sets 5–35 and 7–35, the familiar pentatonic and diatonic collections.

Pentatonic Systems

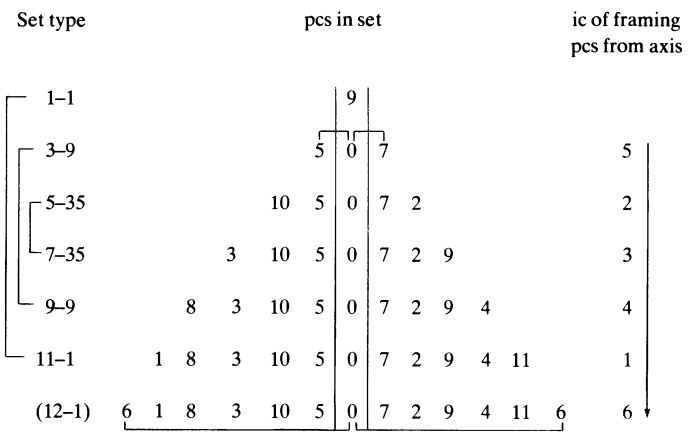
Numerous authors have commented on the prevalence of pentatonic formations in the ancient music of widely divergent geographical areas.[16] Even isotonic tuning procedures have of-

in the case of five notes, may vary as much as 60 cents, depending upon octave placement. Nor is the octave itself always in a 2:1 relation. Isotonic tunings also occur in the music of the Uganda and Baganda tribes of Africa.

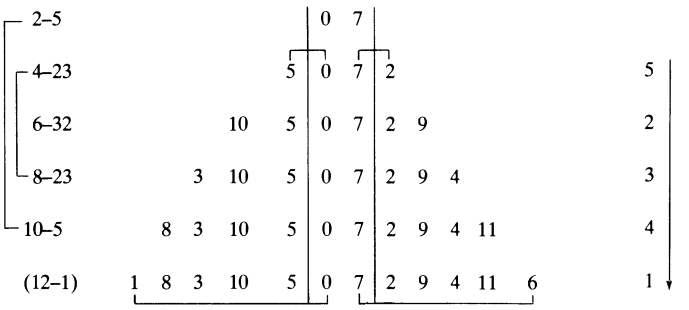
[14]Curt Sachs, *The Wellsprings of Music*, p. 153.

[15]For reference, the set theoretic properties of the cycle-7 complex are cited below. It consists of symmetrical formations around either a pitch-class or interval-class axis.

PC AXIS SYSTEM



IC AXIS SYSTEM



Properties of PC Axis System:

1. Bracketed pairs of set types are complementary (3–9 and 9–9, etc.); the set of smaller cardinality is literally included within the larger.
2. Pairs of framing pcs are inversely related.
3. The union of all framing pcs (with the axis tone) produces the aggregate.
4. The ic distance of framings pcs from the axis produces all ics in order of their respective frequency in the vector.
5. The ic difference between pairs of framing pcs is always even—2, 4, or 6 (true of any pc axis system).

Properties of IC Axis System:

1. Properties 1 and 3 hold.
2. With the exception of the tritone, property 4 holds.
3. With property 5 the ic difference is always odd—1, 3, or 5 (true of any ic axis system).

The ic vectors for the above sets are given in n. 36.

[16]One of the first references occurs in Carl Engel, *The Music of the Most Ancient Nations* (London: J. Murray, 1864), pp. 141–62. See also Constantin Brailiou, "Sur une mélodie russe," *La musique russe 2* (1953), pp. 331ff, and

ten featured an underlying five-tone basis.[17] Is this occurrence mere coincidence, the result of cyclic string tuning, or are there indeed certain inherent features of pentatonic collections which not only permitted the construction of a practical and coherent theoretical system but, more importantly, provided an interplay between tones that was both aesthetically pleasing and meaningful?

One method of generating pentatonic pitch sets consisted of the filling-in of the two empty tetrachords with an additive note or *infix*, which partitioned the fourths asymmetrically.[18] Apparently two possibilities were exploited: M2 and m3 (ics 2 and 3) or M3 and m2 (ics 4 and 1), producing ♮ d c A G F D and ♮ d B♭ A G E♭ D respectively. Both forms are encountered in ancient music. Several occurrences of the latter hemitonic collection may be cited. Josephus observed that the Egyptian temple harps were tuned in this manner (*órganon trígnon enarmónion*).[19] The 4 + 1 tetrachordal construction may have represented the prototype of the chromatic genus in Greek music, forming the possible tonal basis of the two extant Delphic Hymns.[20] In addition, other instances can be excerpted from early melodic constructs of various cultures.[21] From a theoretical standpoint this set is 5–20 (0,1,3,7,8) with an interval vector of 211231.

The 2 + 3 anhemitonic pentatonic set is more central to this study. Although examples may be drawn from innumerable world cultures, attention will focus on those which developed documented theoretical systems. This tonal collection, as existent in ancient China (probably up to the Chou dynasty of about 1100 B.C.), was produced from a cycle of pure fifths (*lü*), in which each tone "gave birth" to its succeeding fifth.[22] The first five notes from the generating pitch *huang-chung* (Yellow Bell), arranged in ascending order within the octave, resulted in the *kung tiao* (♮ F G A C D), which has been cited as the basic form.[23] The existence of a modal system is well documented, allowing tones other than *kung* (F) to function as the

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Tran van Khé, "Is the Pentatonic Universal? A Few Reflections on Pentatonicism," *The World of Music* 19 (1977), pp. 76–84. This entire issue of *The World of Music* is devoted to the question of universals in world music. Obviously, pentatonic practices continue in recent folk music. For instance, see Zoltan Kodaly, "Pentatonicism in Hungarian Music," *Ethnomusicology* 14 (1970), pp. 228–42, where he concentrates on the ♮ D F G A C mode.

[17]This is true of the *slendro* system of Bali and Java. However, the unearthing of ancient metallophones has revealed two large intervals approximating 300 cents, which suggests a prototype of 2 + 3 pentatonic origin; see J. Kunst, C.J.A. Kunst, and V. Waly, *Toonkunst van Bali* II (Weltevreden: Druk G. Kolff, 1925), pp. 476–77.

[18]This does not imply that vocal music began with fourths and later inserted successive infixes. Indeed, the process was probably the reverse, with the tetrachord marking the boundaries of secundal accumulation. This certainly appears to be the case with archaic Vedic and possibly Hebraic chant.

[19]This may have resulted from the extension of divisive tuning, in which the next ratio (1/5 of the string length) produces a M3 above the open string and a m2 below the 1/4 division. Apparently Sachs had trouble interpreting this phrase; contrast p. 71 in his *Rise of Music in the Ancient World* with p. 95 of his earlier *The History of Musical Instruments* (New York: W.W. Norton, 1940).

[20]For a transcription see Theodore Reinach, *La musique grecque* (Paris: Payot, 1926), pp. 177–91. A more recent version of the first hymn may be found in Isobel Henderson, "Ancient Greek Music," in the *New Oxford History of Music*, vol. 1 (London: Oxford University Press, 1957), pp. 364–67. Also consult the comments in Sachs, *The Rise of Music in the Ancient World*, pp. 240–42.

[21]See Sachs, *The Wellsprings of Music*, p. 66.

[22]As is customarily the case where practice precedes theory, the underlying pentatonic basis of ancient Chinese music was probably already in existence before the more abstract concept of the *lü* was established. The pitch class C is most often cited for the *huang-chung*. According to *Kuan-tzu* tuning, the oldest surviving procedure from about 300 B.C., the projection of fifths was produced by successively adding and subtracting 1/3 of each previous string length. For more information on this topic see Ernest McClain, "Chinese Cyclic Tuning in Late Antiquity," *Ethnomusicology* 23 (1979), pp. 205–24.

[23]In addition to musical tones, the number five was also related symbolically to the planets, elements, directions, and colors. Kazu Nakaseko, "Symbolism in Ancient Chinese Music Theory," *Journal of Music Theory* 1 (1957), pp. 147–80, deals in detail with this topic, including an excellent list of early sources.

basic reference note or *finalis*.[24] Similar modal procedures may also be noted in the early pentatonic practices of the Far East, including Japan[25] and Korea.[26] Sachs doubts whether the *kung tiao* actually represented the "original, standard form," since it is not tetrachordally oriented but tends to gravitate around the pentachord F–C.[27] Unfortunately, it is impossible to ascertain early modal tendencies, since almost no purely pentatonic melodies survive from earlier than the Song dynasty (960-1279 A.D.).[28] However, the preference for heptatonic modes on G (*shang*), D (*yü*), and, to some degree, F (*kung*) during this later period suggests that these may also have been the most common pentatonic modes.[29] This is substantiated by the pentatonic modal practices in Japan, Korea, and even the isotonic *slendro* modes of Java.[30]

Other cultures which employed cyclic tuning, such as Babylonia and Greece, may also have originally been based on a 2+3 pentatonic model similar to that of the Far East. We do know that the most common methods of tuning the open strings of the Greek *kithara* produced pentatonic sequences not unlike the oriental *ch'in*.[31] Even the further extension of divisive tuning employing arithmetic and harmonic means, as found in the system of Archytas, yielded a basic pentatonic foundation from which the remaining diatonic tones were derived.[32] The role of pentatonicism in archaic Hebraic and Persian music is less certain, although some scholars have attempted to make a case for the former.[33] Later discussion of heptatonic collections would lead one to believe that the existence of a pentatonic foundation was rather widespread throughout the ancient world.

Although the principle of cyclic string tuning was no doubt crucial in the derivation of pentatonic pitch collections,[34] this alone would not account for the retention of such a system had not it proved workable from both a practical and aesthetic standpoint. Before examining those theoretical features of the anhemitonic pentatonic set, it is necessary to consider first some properties which a given pitch set should exhibit in order to function ideally as the basis of a viable hierarchical tonal system. At least four conditions may be cited:

1. A relatively large number of distinct subset interval classes as compared to the cardinality of the set, thereby assur-

[24]Apparently the *kung* (or "tonic") changed periodically according to the calendar.

[25]The underlying structural tones of the *ryō* and *ritsu* systems were pentatonic in nature: ♮ F G A C D and ♮ C D F G A respectively. While the *ryō* was most certainly the result of Chinese influence, the *ritsu* may have developed from folk origins.

[26]The two basic modes of the *Pyongjo* pentatonic collection were ♮ C D F G A and ♮ D F G A C.

[27]Curt Sachs, *The Rise of Music in the Ancient World*, p. 124.

[28]See Walter Kaufmann, *Musical References in the Chinese Classics* (Detroit: Information Coordinators, 1976), pp. 9–11.

[29]See n. 45.

[30]The Japanese *ryō* and *ritsu* systems cited in n. 25 correspond to the Chinese *kung* and *chih* modes, while the two Korean collections of n. 26 are equivalent to the Chinese *chih* and *yü* modes. The modes in *slendro patēt* are *nem* (G = *shang*), *sanga* (C = *chih*), and *manyura* (D = *yü*); see p. 438 of Harold Power's article on "Mode" in *The New Grove's Dictionary of Music and Musicians* (London: Macmillan, 1980).

[31]The tuning of the archaic Japanese *wagon* may also be cited: ♮ C D F G A. In addition, see Eta Harich-Schneider, *A History of Japanese Music* (London: Oxford University Press, 1973), pp. 123–24.

[32]Scott Makeig, "Means, Meaning, and Music: Pythagoras, Archytas, and Plato," *ex tempore* 1 (1981), p. 49.

[33]The writings of A. Z. Idelsohn and H. G. Farmer are among the most authoritative in these respective fields. However, for those scholars who stress an underlying pentatonicism, see David Maggid, "On Ancient Hebrew Music and Psalmody," *De Musica* 3 (1927); Bence Szabolcsi, "About Five-Tone Scales in the Early Hebrew Melodies," *Ignaz Goldziher Memorial Volume*, pt. 1 (Budapest, 1948), pp. 310–12; and Alfred Sendrey, *Music in Ancient Israel* (New York: Philosophical Library, 1969), pp. 212–14.

[34]It should be noted that Pythagorean projections of ic 7 up to seven notes produce an eminently practical system for melodic purposes. Indeed, studies have shown that even modern string players tend to emulate the Pythagorean diatonic set in monodic passages. The closure of the octave following twelve successive fifths preoccupied early Chinese theorists for some time.

ing the composer of a variety of different intervallic relations as fundamental melodic building blocks.

2. A distribution of the number of occurrences of these distinct interval classes within the *total* intervallic content of the set (or vector) in such a manner that there exists a gradation from common dyads to those of comparative rarity, thereby providing a built-in system of more-or-less normative/deviant relations. This condition is particularly crucial in the resultant aesthetic play of notes, since the more frequent interval classes are correspondingly more ambiguous as to their function or position within the system (pattern matching), whereas the rarest interval class(es) are relatively "fixed" as to their function (position finding).

3. A high frequency of stable acoustical intervals, in particular perfect fourths or fifths, which are not only useful for cyclic tuning, but even more importantly provide for the tonal establishment of a specific reference or key note and the possible shifting of that key note for tonal contrast.

4. A modal system which not only allows a choice of different key notes within the confines of the set but also a realignment of functional intervallic relations in regard to the various reference tones.

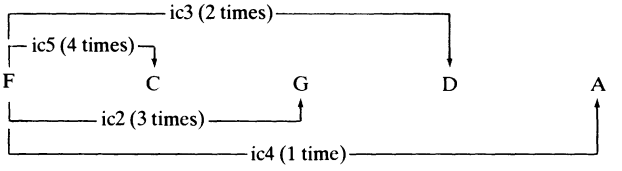
Generally speaking, sets based on an equal division of the octave are less satisfactory in all four respects, since they tend to feature a relatively small number of uniformly recurring interval classes in the vector and a symmetrical arrangement that discourages the formation of a modal system.[35] On the other hand, the partitioning of the octave by means of a five-tone projection of fifths (or fourths), resulting in the 2 + 3 pentatonic set 5–35 (0,2,4,7,9), provides an almost ideal environment for our hypothetical tonal hierarchy. In terms of the above conditions:

[35]The *Yang/Yin* (masculine/feminine) *lü* of ancient China, although they were never used practically, form such a system, the familiar wholetone hexachord 6–35. Its vector is 060603, omitting any perfect intervals; no modal system of distinctive rotations is possible.

1. It contains four different interval classes (ics 2, 3, 4, and 5) as compared to a cardinality of five. Although ics 1 and 6 are absent, there are, as we shall see, compensating factors.

2. Each existing interval class occurs a unique number of times in the vector (032140).[36] In addition, the number of interval class occurrences in the vector for each successive pitch class in the fifth projection is always one less than the preceding (see Figure 2).

Figure 2



3. The perfect fifth/fourth is the most common interval class (ic5), providing four such stable intervals (FC, CG, GD, and DA) with the potential for establishing a key note.

4. The structure of the set allows a complete modal system with no duplication of successive intervallic relations. The classification in Figure 3 is based on that of Gilchrist in her work

[36]The interval vectors for the cycle-7 complex are given below. Up to 8–32 the different ics increase by successive increments of one.

Set Type	Vector	
2–5	000010	
3–9	010020	
4–23	021030	
5–35	032140	
6–32	143250	(unique multiplicity)
7–35	254361	
8–23	465472	
9–9	676683	
10–5	888884	

with Gaelic folksongs.[37] Note that modes I, II, and IV are tetrachordally oriented; this system is symmetrical around the axis mode IV, with inversional relations occurring between I and II, III and V.

Figure 3

Mode		Intervallic sequence (in ics)
I	C D F G A c	(2 3 2 2 3)
II	D F G A c d	(3 2 2 3 2)
III	F G A c d f	(2 2 3 2 3)
IV	G A c d f g	(2 3 2 3 2)
V	A c d f g a	(3 2 3 2 2)

The only other five-tone set which operates in an analogous fashion is the chromatic collection 5–1, by projection of ic 1. Although it duplicates conditions 1 (omitting ics 5 and 6) and 2, it contains no perfect intervals, and its "lopsided" partitioning of the octave would prove problematic from a modal standpoint.

Condition 2 requires additional comment. Another method of demonstrating the successive gradation of interval class subsets in the pentatonic system is diagrammed in the left column of Figure 4.[38] In addition it will be noted that the trichord subsets of the collection operate in exactly the same manner (see the right column of Figure 4). Again 5–1 is the only other five-tone set with this particular property. There thus exists an ordered hierarchy of decreasing subsets, not only for two-note but also for three-note melodic fragments.[39] This would prove especially useful in the construction of musical phrases from a defined repertory of standardized motivic gestures, as in the case of the Greek *nomoi* or *echoi*.[40] This procedure of compos-

[37]Annie Gilchrist, "Note on the Modal System in Gaelic Tunes," *Journal of the Folk Song Society* 4 (1911), pp. 150–53.

[38]This hierarchy of dyads is mentioned by Donald Sur in his discussion of Korean pentatonic practice, when he states that "this represents a fair approximation of the frequency with which the intervals occur." See "Korea" in the *Harvard Dictionary of Music*, 2d ed. (Cambridge: Harvard University Press, 1970), pp. 457–58.

[39]The subsets with cardinalities of 2, 3, and 4 occurring in those tonal collections most central to this study (5–35, 6–32, and 7–35) are listed below. Note that their frequency increases by successive increments of one or two.

Subsets (cardinality 2 to 4) in 5–35, 6–32, and 7–35

Two-note subsets

2–	1	2	3	4	5	6
5–35	0	3	2	1	4	0
6–32	1	4	3	2	5	0
7–35	2	5	4	3	6	1

Three-note subsets

3–	2	4	5	6	7	8	9	10	11
5–35	0	0	0	1	4	0	3	0	2
6–32	2	2	0	2	6	0	4	0	4
7–35	4	4	2	3	8	2	5	1	6

Four-note subsets

4–	8	10	11	13	14	16	20	21	22	23	26	27	30
5–35	0	0	0	0	0	0	0	0	2	2	1	0	0
6–32	0	1	2	0	2	0	1	0	4	3	2	0	0
7–35	1	2	4	2	4	2	2	1	6	4	3	2	2

The pentatonic set may be constructed by overlapping the most common trichord subset 3–7 (CDF DFG GAC ACD). This subset appears frequently in the melodic fragments of young children. For an earlier study along this line see Heinz Werner, "Die melodische Erfindung in frühen Kindalter," *Akademie der Wissenschaften* 4 (1917). One may also cite the pentatonic basis of the Orff and Kodaly "methods" for children; Orff begins with subset 3–7.

[40]See pp. 11–12 and 71–75 in Gustave Reese, *Music in the Middle Ages* (New York: W. W. Norton, 1940). Other melody types include the Syrian *risqolo*, the Hindu *rāga*, the Arabian *maqam*, and the Javanese *patět*.

ing "new" pieces by rearranging the components of a collection of existing "motives" (or subsets) was a widespread technique in ancient music and may well have represented the first evolutionary step toward modal designation.[41]

Figure 4

	Two-note subsets					Three-note subsets			
2–4	FA				3–6	F(G)A			
2–3	FD	CA			3–11	FD(A)	CA(F)		
2–2	FG	CD	GA		3–9	FG(C)	CD(G)	GA(D)	
2–5	FC	CG	GD	DA	3–7	FC(D)	CG(A)	GD(F)	DA(C)

This attribute has enormous compositional and affective implications. The relative frequency of 2–5 and 3–7 provides the basis for more normative tonal relations, while the rarest subsets 2–4 and 3–6 tend to function as deviants within the system. This again evokes the principles of pattern matching for the more ambiguous common subsets and that of position finding for those subsets which are rarer or more "fixed" as to their position. There is little doubt that over centuries of actual musical practice, the musicians of antiquity would have become aurally sensitive to these features of the collection and thereby exploited them in a meaningful aesthetic manner.

As regards condition 4, it would seem likely that modal systems did actually exist in ancient pentatonic practice, since the limited number of available pitch classes would have necessitated a choice of different key or reference note in order to vary the functional relations within the set. This appears to be substantiated in the Orient, although it is less certain in Babylonian, Greek or other cultures. Unfortunately, there are few extant pieces from early times on which to determine whether this may have indeed been the case. Even those surviving works that have been handed down aurally may have taken on the characteristics of later periods.

Nevertheless, based on an observation of certain properties of the set, several hypotheses may be advanced. It seems doubtful that the mode on A was often employed, since it lacks the structural upper fifth with which to stabilize its sense as a key note. Also, since the rarest pitch relation (F to A) probably operated as a deviant gesture, it is unlikely that those modes in which this interval class lay in *direct relation* to the key note (those on A or F) would have been particularly frequent. In this respect it is analogous to the single tritone of the heptatonic set (B to F), thereby restricting the use of modes on B or F; this point will be discussed later under heptatonic sets. As Yasser notes in his discussion of Chinese harmonic practice, the setting of the pentatonic tones with consecutive perfect fourths proves problematic in the case of pitch class A ($\binom{C\ D\ F\ G\ A}{G\ A\ C\ D(F)}$);[42] the identical situation recurs centuries later with the harmonization of B (and the tritone F) in medieval organum. Neither mode on F or A is tetrachordally oriented, as are the other modes on C, D, and G.

In summary, one may quote Walter Kaufmann in his essay on "The Five Notes" when he states that the "familiarity, the 'remembering' of the anhemitonic pentatonic character of a melody appears to be deeply rooted in the depths of the human mind and its realization into actual sound provides a more or less distinct sensation of recognition, of pleasure, to know again something that had been forgotten."[43]

Heptatonic Systems

It seems probable that the tonal evolution of some ancient musical systems witnessed an expansion from pentatonic toward the heptatonic collection 7–35, with structural gaps of the

[41]See Idelsohn's definition of "mode" on pp. 24–25 of his *Jewish Music in its Historical Development* (New York: Henry Holt, 1929).

[42]Joseph Yasser, *A Theory of Evolving Tonality* (New York: American Library of Musicology, 1932), pp. 63–78.

[43]Walter Kaufmann, *Musical References in the Chinese Classics*, p. 113.

five-tone set bridged by secundal motion. In the case of 5–35, the m3 was cleft with a whole- and half-step, thereby continuing the asymmetrical process noted earlier in the division of the fourth: 𝄞 d C (B) A G F (E) D.

The term *infra-diatonic*, coined by Yasser, denotes a seven-tone diatonic set of which five notes act as basic structural notes, while the remaining two function as secondary or "filling-in" tones, resulting in a 5 + 2 complex.[44] This relation between structural versus auxiliary tones is most clearly illustrated in the development of the Chinese heptatonic system. The two subsidiary notes shown in parentheses [𝄞 F G A (B) C D (E) f] were designated as *pien*-tones (literally "becoming" tones) and took their name from the half-step neighbor: B = *pien chih* (to C) and E = *pien kung* (to F). Although this procedure created a potential seven-note modal complex, the *pien*-tones B or E never functioned as a *finalis*. Analysis of the earliest extant monodic pieces (from the *Song* dynasty) has revealed that the most common modes employed were those on G(*shang*), D (*yü*), and to some degree F (*kung*).[45] In each case the fifth above the *finalis* is a structural pentatonic note. This influence may be seen in the early Japanese *ryō* system, based on the above collection, in which the two prevailing modes were also on G (*Ichikotsu-chō* = *shang*) and D (*Ōshiki-chō* = *yü*). In the later Togaku court music the *pien*-tones were modified with the following result: *ryō* = 𝄞 G A B (C) D E (F) g (from the older *shang*) and *ritsu* = 𝄞 D E (F) G A B (C) d (from the older *yü*), thus switching the *pien*-tones from E-B to F-C and producing both a "major and minor" prototype.[46]

The hypothesis that the diatonic Greek *harmoniai* were derived from pentatonic/*pien*-tone origins is largely speculative. Although Plutarch and Nichomachus both mention incomplete heptatonic collections,[47] it seems doubtful, as Riemann and Yasser have deduced, that these were purely pentatonic in nature.[48] Certainly the open string tunings of the classical *kithara* might suggest an underlying pentatonic basis.[49] Several historians, including Shirlaw,[50] appear to take this approach, although no specific mention is made of the *pien*-tone principle. Perhaps the most comprehensive theory advanced to account for the origins of the Greek heptatonic system is that of Sachs; however, his reference to seventeenth-century Japanese Endo tonal practice is inappropriate.[51] In brief, he argues that the pentatonic basis of the Dorian *harmonia* lies in the hemitonic collection with later auxiliary filler (or *pien*) tones added (5+2): 𝄞 e (D) C B A (G) F E. The two other "original modes,"[52] the Lydian (later Hypolydian) and the Phrygian, evolved from the 2 + 3 pentatonic set. In ascent the latter two would approximate the Chinese modes *kung* (transposed to the E Dorian octave for comparison) [𝄞 E F♯ G♯ (A♯) B C♯ (D♯) e] and *yü* [𝄞 E (F♯) G A B (C♯) D e] or again the " major-minor"

[44]Yasser, *A Theory of Evolving Tonality*, pp. 40–61.

[45]See the analyses in Rulan Chao Pian, *Sonq Dynasty Musical Sources and their Interpretation* (Cambridge: Harvard University Press, 1967), pp. 59–67, and John Hazeldal Lewis, *Foundations of Chinese Musical Art*, 2nd ed. (New York: Paragon, 1963), pp. 97–178.

[46]One of the most comprehensive discussions of Japanese modal theory may be found in Robert Garfias, *Music of a Thousand Autumns: The Togaku Style of Japanese Court Music* (Berkeley: University of California Press, 1975).

[47]Plutarch, *De Musica*, chap. 18; Nichomachus, *Encheiridion*, chap. 9.

[48]Yasser, *A Theory of Evolving Tonality*, pp. 151–52, and Hugo Riemann, *Handbuch der Musikgeschichte* I, 1 (Leipzig: Breitkopf und Härtel, 1904), p. 162.

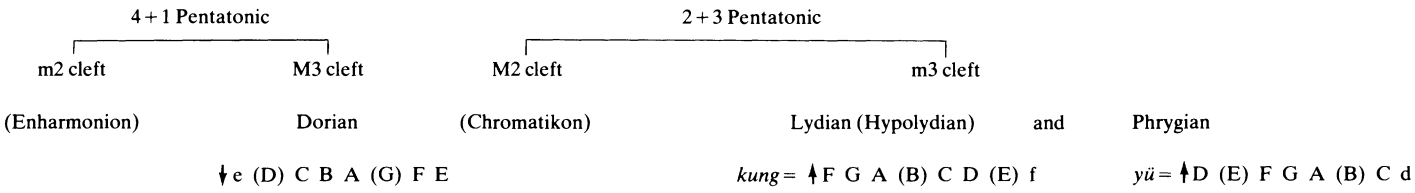
[49]Curt Sachs, "Die griechische Instrumentalnotenschrift," *Zeitschrift für Musikwissenschaft* 6 (1924), pp. 289–301. This is confirmed by Gombosi in his *Tonarten und Stimmungen der Antiken Musik* (Copenhagen: Ejnar Munksgaard, 1939), pp. 37–77.

[50]Matthew Shirlaw, "The Music and Tone Systems of Ancient Greece," *Music Review* 4 (1943), pp. 14–27. See also Theo Reiser, *Das Geheimnis der Pythagoreischen Tetraktys* (Heidelberg: Verlag Lambert Schneider, 1967), pp. 37–45.

[51]Sachs, *The Rise of Music in the Ancient World*, pp. 216–21.

[52]According to Aristides Quintilianus. Note the tetrachordal construction: Dorian = 𝄞 TTS, Phrygian = 𝄞 TST, and (Hypo)Lydian = 𝄞 STT. See also Gombosi, *Tonarten*, pp. 83–85.

Figure 5



prototypes. An amplification of his illustrative diagram, using a generating note of F, is given in Figure 5.[53]

Despite the intriguing nature of this hypothesis, it is not easy to make a convincing case for it on the basis of extant music literature, most of which dates from later periods. Indeed, the overall question of early Greek tonal practice is still far from resolved; in particular, see Gombosi's distinction between key (*tonos*) and octave species (*eide diapason*).[54]

Although the diatonic set 7–35 is firmly established in ancient Babylonian and Indian practice, there are few if any clues with which to postulate a preexistent pentatonic foundation. The recent deciphering of Assyrian-Babylonian cuneiform tablets reveals that the open strings of plucked instruments correspond to the diatonic set.[55] The tuning systems, based on cycles of pure fifths/fourths,[56] yield seven octave species which coincide with those of the Greeks,[57] although the sequence of steps is reversed, reading low to high. One mathematical treatise even mentions consonant harmonic dyads,[58] thus confirming Sach's earlier speculation.[59] Considering the early date of about 1800-1400 B.C., these characteristics would appear to indicate a highly evolved musical language, further strengthening the argument that this locale represented the cradle of civilization. On the basis of only one extant composition, it is impossible to ascertain any modal tendencies, if a modal system existed at all.[60] Kilmer's transcription of a Hurrian cult song shows a decided preference for the fifth "scale degree" (the piece is in the *nid gabli* tuning: ♭ E F♯ G♯ A *B* C♯ D♯ e).[61] Since B is the only pitch

[53]The original diagram may be found on p. 221 of Sachs, *The Rise of Music in the Ancient World.*

[54]Otto Gombosi, "Key, Mode, Species," pp. 20–26. His views are based in part on Francis Haskins Eyles Stiles, "Explanation of the Modes or Tones in the Ancient Graecian Music," *Philosophical Transactions* 51 (1870), pt. 2, pp. 695–773, and D. B. Monro, *The Modes of Ancient Greek Music* (Oxford: Clarendon Press, 1894). An excellent bibliography may be found on the first page of Gombosi's article.

[55]This research is summarized in the record and booklet entitled *Sounds from Silence* (Berkeley: Bit Enki Publications, 1976) by Anne Kilmer, Richard Crocker, and Robert Brown, although much of it is based on the work of O.R. Gurney, M. Duchesne-Guillemin, D. Wulstan, and K. Kümmel.

[56]*Sounds from Silence,* p. 11.

[57]This may, of course, be mere coincidence. However, it does further substantiate the claim for Babylonian influence on such early Greek theorists as Pythagoras.

[58]As noted in the so-called "mathematical text" coded CBS 10996. Kilmer's song transcription illustrates their use.

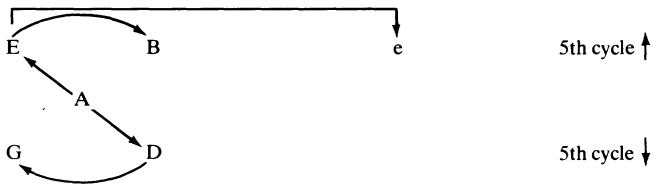
[59]Curt Sachs, "Zweiklänge in Altertum," *Festschrift für Johannes Wolf* (Berlin: M. Breslauer, 1929), pp. 168–70, where he interprets the harpists in Assyrian and Egyptian bas reliefs as sounding simultaneous unisons, octaves, and fifths.

[60]One may conveniently transfer Gombosi's suspicion of Greek modes to this case as well, noting the similarity of octave species.

[61]*Sounds from Silence,* pp. 12–17. Francis Galpin has offered a transcription of the famous Sumerian *Hymn on the Creation of Man,* but his pitch choice and octave placement in the instrumental part are highly suspect; see pp. 99–104 of his *The Music of the Sumerians and their Immediate Successors the Babylonians and Assyrians* (Cambridge: The University Press, 1937). For a further

class that remains constant throughout the tuning cycles, it tended perhaps to function as a basic referential key note. The designation of the fourth string (A in the basic tuning of *isartu*: ↟ E F G *A* B C D e) as the God-Creator raises the question of even earlier pentatonic origins. It may have operated as the generator note to tune other ascending/descending fifths/fourths in the same manner as for the open strings of the *kithara*, employing the *mese* A (see Figure 6).[62]

Figure 6



Legend has it that Tumburu, the first singer of Indian antiquity, expanded the *Samaveda* chant from five to six or seven tones, but knowledge of that melodic style suggests that the "five tones" probably consisted of a filled-in pentachord rather than a pentatonic collection.[63] Archaeological excavations in the Indus valley have uncovered lyre-types which seem to have had seven strings;[64] this corresponds to most descriptions of the archaic *vīṇā* as a seven-stringed instrument. Ludwig Riemann, in his study of extant flutes, noted that most were based either on the fourth or a disjunct fourth/fifth with sharpened octave.[65] The historian William Hunter hypothesizes that the pitch names (*svaras*) of the diatonic set (Sa Ri Ga Ma Pa Dha Ni) were already in existence about the time of Pāṇini (fourth century B.C.), the great Sanskrit grammarian.[66] The first concrete theoretical evidence, contained in the fabeled *Nāṭyaśāstra*, is of comparatively late date (ca. A.D. 100–500), although certain passages may refer to more ancient practice.[67] The basic heptatonic pitch collections (*ṣaḍja-grāma* and *madhyama-grāma*)[68] are derived from arrangements of the microtonal *śrutis*.[69] From

discussion of this issue see pp. 248–50 of Henry G. Farmer, "The Music of Mesopotamia" in the *New Oxford History of Music*, vol. 1 (London: Oxford University Press, 1957).

[62]See Gustave Reese, *Music in the Middle Ages*, p. 46 and the reference to Henry Macran, *The Harmonics of Aristoxenus* (Oxford: Clarendon Press, 1902), p. *206*, not 236 as cited in Reese.

[63]A. H. Fox Strangways, *The Music of Hindostan* (Oxford: Clarendon Press, 1904), p. 75.

[64]Stuart Piggott, *Pre-historic India to 1000 B.C.* (Harmondsworth: Penguin Books, 1950), p. 270.

[65]Ludwig Riemann, *Über eigentümliche bei Natur-und orientalischen Kulturvölkern vorkommende Tonreihen* (Essen: Baedeker, 1899), pp. 3, 27–30.

[66]William W. Hunter, *The Indian Empire: Its People, Its History and Products* (1892), 3d ed. (New York: AMS Press, 1966), p. 152.

[67]For an English translation see Manomohan Ghosh, *The Nāṭyaśāstra*, 2 vols. (Calcutta: Royal Asiatic Society, 1950 and 1961) and the translation/commentary of chap. 28 in Ernest Clements, *Introduction to the Study of Indian Music* (London: Longmans, Green and Co., 1913), pp. 49–53. An excellent guide to English writings on Indian music is contained in Harold Powers, "Indian Music and the English Language," *Ethnomusicology* 9 (1965), pp. 1–12.

[68]Sachs believes that these tone collections represent plagal (*ṣaḍja-grāma* = ↟ $\overline{\text{D G}}$ d) and authentic (*madhyama-grāma* = ↟ $\overline{\text{G D}}$ g) forms. See p. 168 in his *Rise of Music in the Ancient World*.

[69]Scholars are still divided as to the function of the twenty-two *śrutis* in the archaic Hindu tuning systems. For example, see P. R. Bhandarkar, "Contribution to the Study of Ancient Hindu Music," *The Indian Antiquary* (1912), pp. 192–94 and 260–62; Nazir Jairazbhoy, "An Interpretation of the 22 *śrutis*," *Asian Music* 6 (1975), pp. 52ff. Harold Powers is of the opinion that these microtonal increments were of equal size, stating that fifths and fourths (of 13 and 9 *śrutis* respectively) produced two cycles of "a — D — G — c — F" and "B — E," with a distance of 7 *śrutis* between G and B; see pp. 59–60 of his "An Historical and Comparative Approach to the Classification of *Ragas*," in *Selected Reports*, vol. 1, no. 3 (U.C.L.A. Institute of Ethnomusicology, 1970). While it is true that the original cycle is an F pentatonic collection, it is not derived by *pure* fifths, as the final "f" is about a syntonic comma off from the *śrutis* tuning. Sachs, on the other hand, reasons that there were three sizes of *śrutis* and thus postulates a divisive just tuning for the *Sa-grāma* (see pp. 166–71 of his *Rise of Music in the Ancient World*).

these, various modes (*mūrcchanās*) and melodic prototypes (*jātis*) were evolved.[70] However, references to five- or six-tone sets (*tānas*) were apparently considered as "deficient" heptatonic modes.[71]

The origins of Western chant continue to be shrouded in mystery. Apel is probably correct in stating that "the truly Gregorian, and, even more, any pre-Gregorian repertory must have been of a more elementary character";[72] however, as in the case of Roman chant, one must guard against the generalization that music invariably evolved from simpler toward more complex formats. Although historians have cited Hebraic, Syrian, Byzantine, Gallic, and even European folk influence,[73] the earliest roots may well lie in ancient Jewish cantillation, as proposed by Idelsohn in his study of isolated Yemenite communities.[74] Clement of Alexandria (ca. A.D. 200) tantalizes us when he states that the *tropos spondeiakos*, of possible pentatonic origins, reminds him of Jewish psalmody and recommends it for Christian singers.[75]

[70]An excellent commentary on this subject, based on the *Nāṭyaśāstra*, is found in Lewis Rowell, "Early Indian Musical Speculation and the Theory of Melody," *Journal of Music Theory* 25 (1981), pp. 217–44. Several comparisons are made between Greek and Indian melodic theory, noting, for instance, that the Indians have no equivalent term for interval (GK.:*diastema*). His analyses of several *jātis* (the *ṣāḍjī* and *Nandayantī*) are particularly revealing.

[71]Sachs relates the seven "fixed" *jātis* to heptatonic modes with possible pentatonic origin; see pp. 176–77 of his *Rise of Music in the Ancient World*.

[72]Willi Apel, *Gregorian Chant* (Bloomington: Indiana University Press, 1958), p. 76.

[73]Earlier surveys of the origins of Gregorian chant include Peter Wagner, *Einführung in der gregorianischen Melodien*, vol. 1 (Leipzig: Breitkopf und Härtel, 1911–21) and Amédée Gastoué, *Les Origins du chant romain* (Paris: A. Picard et fils, 1907). Bruno Stäblein's theory of a "pre-Gregorian" repertory should also be mentioned (see his "Zur Entstehung der gregorianischen Melodien," *Kirchenmusikisches Jahrbuch* 35 [1951], pp. 5–10).

[74]A. Z. Idelsohn, *Thesaurus of Hebrew Oriental Melodies*, vol. 1 (Berlin: Benjamin Harz, 1922-28).

[75]See Clement of Alexandria, *Paidogōgos* 2.4 and the discussion on pp. 213–14 of Alfred Sendrey, *Music in Ancient Israel*.

There are several problems associated with this line of investigation. The earliest decipherable form of chant notation did not appear until relatively late (with the treatises of ca. A.D. 900[76] and the neumatic manuscripts of approximately two centuries later). Another complicating factor is the imposition of modal classification upon the preexistent repertory, already mentioned ca. 800.[77] And finally, geographical influences (such as the Frankish-Roman conflict) have tended to obscure tonal origins.

Several scholars, among them the redoubtable Riemann, have suggested an underlying pentatonic basis in chant,[78] and an extensive argument along this line has been proposed by Joseph Yasser.[79] Although his thesis of an infra-diatonic system, in which the pentatonic foundation is extended through the use of two *pien*-tones (5 + 2), parallels this discussion, his presentation lacks sufficient analytical data and early sources upon which to confirm his conclusions. In particular, his almost sole dependence upon the *quilisma* as an auxiliary note to bridge the m3 gaps (D-F and A-C) of the pentatonic collection is questionable. However, his basic hypothesis has resurfaced in the re-

[76]These include the *Musica enchiriadis*, *Commemoratio brevis*, and Odo's *Dialogus de musica*.

[77]See Alcuin's *Musica*. By the time of Guido the modal classification was firmly established. Nevertheless, such anti-modal theorists as Theinred of Dover (ca. A.D. 1150?) employed the Greek pentachord/tetrachord species to explain the presence of such "note mobile" as F♯ und E♭ (see his *De legitimis ordinibus pentachordorum et tetrachordorum*).

[78]Hugo Riemann, *Handbuch der Musikgeschichte*, vol. 1 (Leipzig: Breitkopf und Härtel, 1904–1922), pp. 62–74. Other authors reasoning along this same line include Jacques Chailley (*Une nuovelle méthod d'approache pour l'analyse modale du chant gregorien*), Hubert Kupper (*Statistische Untersuchungen zur Modusstruktur der Gregorianik*), Knud Jeppesen (*Counterpoint*), and Max Schneider ("Der Wechsel der Modalität im Lichte der Tonalitätkreislehre," in *Kirchenmusikalisches Jahrbuch* 30).

[79]Joseph Yasser, *Medieval Quartal Harmony* (New York: American Library of Musicology, 1938), pp. 18–26. Reese's discussion of this topic draws heavily on Yasser's work (see pp. 159–61 of his *Music in the Middle Ages*).

search of Egeland Hansen and his *Grammar of Gregorian Tonality*. This controversial work is based on a computer-assisted analysis of the Montpellier Codex H 159.[80] Although it is impossible to summarize adequately his methodology, results, and conclusions within the narrow confines of the present study, his basic premise is that pre-Gregorian chant finds its roots in the pentatonic collection F G A C D, to which microtonal inflections were first appended below F and C (*tonal alienation*).[81] These expressive embellishments eventually emerged as secondary *pien*-tones (E and B).[82] The extension of the gamut through transposition of the F collection (*quintal/quartal identity*) produced other *pien*-tones: [♮ C D (E) F G A (B♭/B) c d e (f) g a].[83] The final step was the crystallization of a *bifurcate system*, producing primary hexachordal collections.[84] Of particular interest is his analysis of cadence and *finalis* tones in the oldest Tracts and Graduals,[85] which show that the preferred pitches were G (= *shang*), D (= *yü*), and A.[86] His reasoning for the avoidance of F and C was that these tones were precisely those most affected by microtonal inflection.[87] In conclusion, to quote Hansen, "Even though the idea of the pentatonic character of chant is not new, the development of the *pien*-tone models and the description of the various systems have contributed substantially to the understanding of this tonal principle."[88]

Additional research into the origins and tonal structure of chant is currently being carried on by Hendrick VanderWerf, based on the comparison of Antiphon Introits in early manuscripts and the employment of certain procedures codified in his previous volume on early secular monody.[89] Although he does not emphasize the pentatonic/*pien*-tone approach, VanderWerf has indicated to the author (in conversation) that many of his underlying "structural tones" do correspond to the F G A C D pentatonic set.

If we can assume that the modal system has some valid theoretical relation to the chant repertory, several observations are in order. With the sole exception of Phrygian,[90] the collection of *finalis/tenor* pitch classes comprises the pentatonic ♮ D F G A C d.[91] There is also a tendency of many of the melodic lines to fall within framing conjunct tetrachords, the prevailing series being ♮ D G C f.[92] The final pitch class of the seven-tone ic 7 projection (B) was handled with particular care. Since it lacks an upper structural perfect fifth to support it, the mode on that tone (Locrian) was forbidden. The fact that the finalis of Phrygian is itself a *pien*-tone may account for certain particular properties of that modal complex. Indeed the *tenor* in Phrygian often preferred C to that of B.

If our hypothesis is correct that the last pitch class in an ic 7

[80]Hansen's choice of this tonary was based on five characteristics: (1) a wide range of historical period, (2) the oldest form of transcribable notation, (3) early distinction between B♮/B♭, (4) possible use of microtones, and (5) various chant types grouped and assigned specific modal classification.

[81]Finn Egeland Hansen, *The Grammar of Gregorian Tonality* (Copenhagen: Dan Fog Musikforlag, 1976), pp. 180–83. This might also account for the extra tones of F♯ and E♭ mentioned in several early treatises.

[82]Hansen, *Grammar*, pp. 30–126. He refers to the basic pentatonic set (F C G D A) as the *fa* system, with other possibilities of *ce* and *gb* as well. Here the fifth is the generator with octave identity.

[83]This collection is produced by an octave generator with quintal/quartal identity, eventually resulting in the Daseian scale of *Musica enchiriadis*.

[84]See pp. 150–52 of Hansen, *Grammar*. This is a system of conjunct/disjunct primary tone tetrachords, which produces a *fe* hexachord (F C G D A E).

[85]In particular, the Tracts in *protus* and *tetrardus*, as well as the Graduals of the "Justus ut palma" type.

[86]Hansen, *Grammar*, pp. 183–87.

[87]Hansen, *Grammar*, pp. 185–86.

[88]Hansen, *Grammar*, p. 279.

[89]Hendrick VanderWerf, *The Chansons of the Troubadours and Trouvères* (Utrecht: A. Oosthoek, 1972).

[90]In keeping with the broader scope of this study, the geographical terminology for the Gregorian modes will be employed in preference to the Greek numerical designations (*protus*, etc.).

[91]The earliest list of *tenors* occurs in *Commemoratio brevis*; a transcription may be found in Wagner, *Einführung*, vol. 3, p. 89.

[92]This does not result in a closed octave system.

projection (B in this case) should not fall in direct relation to the key note (or *finalis*), then the Lydian mode requires closer examination. It would be gratifying to report that the most common modes were those on D and G, corresponding to *yü* and *shang* in Oriental practice, and that the least frequent was that of Lydian. According to Apel's calculations, the most common chant type, the Antiphons, along with Alleluias and hymns, do confirm this distribution.[93] In Ambrosian chant, which does not employ the normal modal classification, Jesson notes a low percentage of *finalis* on F and B (9 and 1% respectively), with the most frequent being on D and G (22 and 41%).[94] On the other hand, the Graduals are heavily weighted toward Lydian.

A more accurate indicator may be the presence of B♭ or B♮, a sticky problem at best, since there are often contradictions between equally reliable sources. *If* one assumes, as does Hansen,[95] that the Montpellier Codex H 159 is not ambiguous in this respect, and *if* one applies a simple statistical analysis, again an oft-criticized procedure, the results are revealing (see Figure 7).

Figure 7

	B♮	B♭
Dorian	1.9%	1.4%
Phrygian	6.0%	1.5%
Lydian	3.6%	2.7%
Mixolydian	8.7%	1.3%

In Lydian, B♭ occurs almost twice as often as in the other modal complexes, whereas in Phrygian/Mixolydian there is more than a 4:1 preference for B♮ over B♭. Because F is an important structural tone in both Dorian and Lydian, its relation to B♮, producing the rarest interval class (the tritone), results in a lower overall percentage of both B♮/B♭ but with a more equal balance between their occurrences. Using the same reasoning one might speculate that in the case of the two remaining modes the decided preference for B♮ would also produce a corresponding decline in the frequency of F. This indeed does prove to be the case.[96]

The final stop in our journey westward concerns the modal

[93]Apel, *Gregorian Chant,* p. 27ff.

[94]Apel, *Gregorian Chant,* p. 480.

[95]Hansen, *Grammar*, p. 15.

[96]It is interesting to apply set-theoretical principles to practical problems of a specific historical era. A case in point is the medieval practice of hexachordal solmization. As mentioned previously, the pitch classes employed in chant consisted for the most part of the diatonic set plus B♭, which may be represented as a projection of fifths upward from B♭ (4 B♭ F C G D A E B♮) or set 8-23. Nevertheless, the tonal gamut was still probably viewed as a basic heptatonic system. The problem from a didactic standpoint was to select a smaller, more manageable collection of pitches, preferably in stepwise order, from the eight-tone set. This subset (A) could then act as a template, in that its subsequent transposition (at some T level) would allow maximal overlapping in such a way as to facilitate mutation, which was in turn dependent upon the range of the chant melodies. In order to satisfy this situation, two conditions must be met:

1. The total pitch content of subset A and its T's must be literally included in the pitch content of the original set S (8–23); or expressed in another way: $S = A \cup T_x(A) \cup T_y(A)$ etc.
2. Maximal overlapping or invariance must occur between A and its T's, in order to facilitate the mutational process.

Since the most common interval class in the vector of 8–23 is ic 5, transposition by perfect fourth or fifth (T^5 or T^7) will produce the highest occurrence of pitch class invariance. Thus the only subset collection which will satisfy both conditions is the familiar hexachord [C D E F G A] or 6–32, which is itself a subprojection of fifths. Note that the pentachord [C D E F G] satisfies the first condition, but the E (or *Mi*) of the "naturale" form is not found in either transposition. Four pitch classes are held invariant (C G D A), while *Fa* (T^0) = *Ut* (T^5) and *Mi* (T^0) = *La* (T^7). See the diagram below:

Molle	(T^5)	B♭	F	C	G	D	A		
Naturale	(T^0)		F	C	G	D	A	E	
Durum	(T^7)			C	G	D	A	E	B♮

evolution of early English folksong traditions. It was Sharp's contention that the original basis for this literature was the anhemitonic pentatonic collection, with the "missing" notes filled in by auxiliaries (or *pien*-tones) of E-B, E-B♭, and E♭-B♭.[97] Further research along this line by Gilchrist[98] and Bronson[99] has substantiated this thesis, demonstrating the manner in which the five-tone sets were gradually expanded through hexachordal collections to an ultimate seven-tone diatonic system. I will spare the reader the deciphering of Bronson's seven-pointed "lode-star," and instead arrange his relationships in the manner of Figure 8.[100] Here the additional *pien*-tones are consistently F and B, transposed to a fundamental scale of C (see bottom diagram). The respective collections under each column are 5–35, 6–32, and 7–35. Each grouping shows an overall symmetrical configuration within the octave, with the original pentatonic modes numbered in parentheses (after Gilchrist). It is a simple matter to trace the growth of the *pien*-tones, observing the retention of invariant pitch classes from the original pentatonic modes. Note the close relation of Lydian/Ionian and Dorian/Aeolian, perhaps accounting for the "borrowing" of B♭ (as denoted by the dotted line) in the Lydian and Dorian modes. It should be added that the Locrian and Lydian modes, with their direct relation of the final projected pitch class to the key note, are almost non-existent in this repertory, while the problematic Phrygian is quite rare.[101]

Based on the above survey, a summary of *pien*-tone procedure is given in Figure 9. It will be observed that these are the *only* three arrangements that will preserve a normative pentatonic background, which must be generated by *successive* fifth relations.

As noted at the outset of this study, the theoretical properties of the diatonic set have been previously explored.[102] Suffice it to say that the seven-tone projection of fifths (F C G D A E B) constitutes set 7–35 (0,1,3,5,7,9,10). As with its hexachordal subset 6–32, its vector (254361) displays the property of unique multiplicity, with ic 1 and 6 appearing twice and once respectively. Sets 5–35 and 7–35 are not only complementary, but the five-note collection, as our studies have demonstrated from a practical standpoint, is literally included in the heptatonic set. The most commonly employed *pien*-tones, placed either at the head or end of the fifth projection, serve to produce the rarest and therefore most deviant interval classes in the set. We have noted those modes (Locrian and Lydian) in which a direct relation between the key note and final pitch projection (either F or B) occurs. The satisfaction of those conditions conducive to the construction of an idealized hierarchical tonal system, which were found applicable in the pentatonic collection, is even further realized with 7–35. The qualification of condition 1 is now removed, since all interval classes are present. Thus the diatonic set emerges as the *sine qua non* upon which a viable tonal hierarchy may be built. I would not be the least surprised to find it alive, well, and living on some distant planet.

We have observed the extension of the cycle-7 complex, producing in turn sets of three (3–9) to five (5–35) to seven tones (7–35), through the successive filling-in of structural gaps in each collection. The final logical step is the bridging of the whole-steps with semitones, thereby completing the aggregate. May we assume that the chromatic complex was arrived at in a manner analogous to that of the preceding sets, a 7 + 5 collection in which the chromatics now function as auxiliary *pien*-

[97]Cecil Sharp, *English Folk Song: Some Conclusions,* 4th ed. prepared by Maud Karpeles (Belmont: Wadsworth, 1965), pp. 65–67.

[98]Gilchrist, "Note on the Modal System," pp. 150–53.

[99]Bertrand Bronson, "Folksong and the Modes," *Musical Quarterly* 32 (1946), pp. 37–49.

[100]The arabic numerals refer to missing scale degrees in terms of the complete diatonic modes. The "lode-star" may be found on p. 44 of Bronson's "Folksong and the Modes."

[101]Sharp, *English Folk Song,* pp. 68–69.

[102]For a complete analysis of the subsets in 7–35 see Browne, "Tonal Implications."

Figure 8

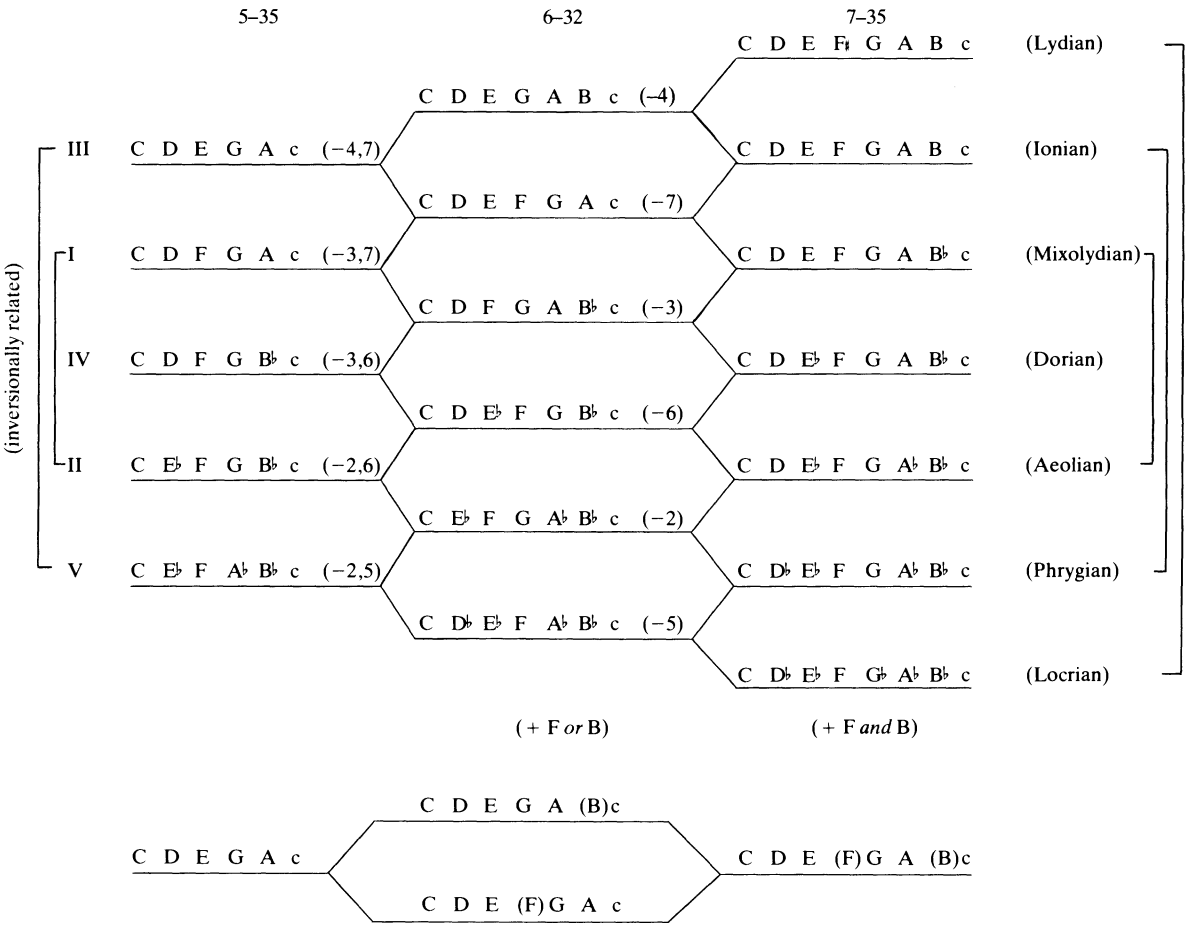

5–35
6–32
7–35
C D E F♯ G A B c (Lydian)
C D E G A B c (–4)
III C D E G A c (–4,7)
C D E F G A B c (Ionian)
C D E F G A c (–7)
I C D F G A c (–3,7)
C D E F G A B♭ c (Mixolydian)
C D F G A B♭ c (–3)
IV C D F G B♭ c (–3,6)
C D E♭ F G A B♭ c (Dorian)
C D E♭ F G B♭ c (–6)
II C E♭ F G B♭ c (–2,6)
C D E♭ F G A♭ B♭ c (Aeolian)
C E♭ F G A♭ B♭ c (–2)
V C E♭ F A♭ B♭ c (–2,5)
C D♭ E♭ F G A♭ B♭ c (Phrygian)
C D♭ E♭ F A♭ B♭ c (–5)
C D♭ E♭ F G♭ A♭ B♭ c (Locrian)
(inversionally related)
(+ F *or* B)
(+ F *and* B)
C D E G A (B)c
C D E G A c
C D E (F) G A (B)c
C D E (F) G A c

Figure 9

1. Chinese, early Japanese *Ryō*, Greek Lydian/Phrygian (after Sachs), Gregorian chant (after Hansen)	F C G D A (E B)
2. Japanese Togaku, Greek *kithara* tuning, Babylonian (?)	(F C) G D A E B
3. Gaelic	(F) C G D A E (B)

tones to the structural diatonic tones? This ultimate process is diagrammed in Figure 10.[103] The chromatics listed in 12–1 correspond to those employed during the early Renaissance; their enharmonic equivalents were added later. In this respect the aggregate may be considered as a complete cycle of ic 7 generated from Eb.

If the general outline of this presentation seems vaguely familiar, I can assure you that it is, being based on a theory proposed by Joseph Yasser some years ago in his *Theory of Evolving Tonality*. Indeed, I consider it a further vindication of his remarkable hypothesis, and it is to his memory that the present study is dedicated.

[103]This represents a modification of the diagram appearing on p. 141 of Yasser's *Theory of Evolving Tonality*. Yasser refers to the aggregate as the *diatonic system* (7 + 5).

Figure 10

3–9	D				G		A						d
5–35 (3+2)	D			(F)	G		A				(C)		d
7–35 (5+2)	D		(E)	F	G		A			(B)	C		d
12–1 (7+5)	D	(E♭)	E	F	(F♯)	G	(G♯)	A	(B♭)	B	C	(C♯)	d
	‖		‖		‖		‖		‖	‖		‖	
	D♯		F♭		G♭		A♭		A♯	C♭		D♭	

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