SIMULATING MUSICAL SKILLS BY DIGITAL COMPUTER

John Rothgeb
State University of New York
Binghamton, New York

I. Introduction. It was perhaps inevitable that the digital computer, an insrument of such enormous potential and accomplishment in the study of both natural and social sciences and humanities, should have been applied to the investigation of problems in the fine arts as well. Among the latter, music was predictably the art form most directly accessible to computational study. Painting and the plastic arts do not readily lend themselves to direct computer applications; their analysis and elucidation with the aid of computers will necessarily await the development of highly sophisticated optical scanning devices at the very least. Music, on the other hand, is normally set down and transmitted graphically in the form of a notated score that can be almost completely represented in a code consisting of discrete symbols. It must be said at once, however, that the notated musical score, even of the simplest traditional western art music, is incomplete: The musical work, as conceived by the composer and as produced by the sensitive musical performer, deviates from the precision of the printed or written score in countless subtle ways -- most obviously with respect to temporal organization, but also, even, with respect to pitch and timbre.[1] Thus the score is at best an approximation to what can properly be designated as "the" composition. To appreciate fully what this means, it is instructive to imagine, or actually to experience, if possible, a literal realization by synthesizer of, say, a Bach sonata for unaccompanied violin (to choose a particularly extreme example). The hearer of such a realization finds it almost impossible to "follow" the music, to "make sense" of it; and this has far less to do with the timbral inadequacies of the synthesizer's violin tone than with missing temporal and dynamic nuances. (The same music speaks eloquently on ancient phonorecords, despite their obvious timbral deficiencies.) But the printed score and its representation in an appropriate linear code, incompleteness notwithstanding, present a sufficiently detailed picture of the music's structure to render it accessible to analytical probing in a depth not approachable in many other art-forms.

II. Tonal music. Music theorists customarily group musical artworks into classes defined on the basis of shared structural characteristics. Tonal music, or the class of tonal compositions, is one such class; twelve-tone music (indigenous to the twentieth century) is another. It is convenient, moreover, to distinguish between the compositions belonging to a given class and the language underlying those compositions; this distinction is often compared to that made in linguistics between the corpus of sentences in a language and the grammar according to which those sentences are produced and interpreted. (The analogy, although useful, is far from perfect; I shall not develop it further here.) Tonal music, comprising most of the music written between about 1650 and 1900, is especially suitable as an object for rigorous (and therefore computational) study for several reasons. First, its significance as a pinnacle in the history of art music in general is indisputable. Secondly, it has a highly constrained syntax embodying many known regularities: for example, the class of intervals characterized as "dissonant," and the structural obligations incumbent upon such intervals, were firmly established long before 1650 and remained constant throughout the period of tonal music.[2] Finally, tonal compositions are structurally organized in terms of a series of strata (each stratum specifiable in its own musical notation) that relate to one another as simple to (more) complex.[3] This means that one can explain a tonal composition according to the venerable explanatory model wherein complex phenomena or configurations are related through known operations to simple and familiar ones. (This notion elucidates, among many other things, the basis of the art of musical variation as applied not only in works entitled "Theme and Variations" but also from phrase to phrase and bar to bar in the classical masterworks: variations which contrast markedly in notation and -- therefore -- in sonic qualities may share a common underlying stratum.) For convenience, I shall speak here occasionally of the "simple-to-complex array" or simply "array" of strata associated with a given composition.

III. Computer-accessible problems in music theory. In view of the characteristics of music representation and the nature of tonal music as outlined above, and the capabilities of the digital computer, a wide range of possible computer applications in music theory comes to mind. Well-trained musicians have a thorough but inexplicit understanding of the elements and operations that make up the language we call classical tonality. It is part of the essence of theoretical studies, however, to render explicit that which has been inexplicit. The familiar result of such work is that new areas of inexplicitness are exposed, and those areas are then subject to study and explication in their turn. By symbolically representing the elements of musical notation and expressing in formal terms those operations believed to form the basis of inter-stratal relations, the music theorist obtains something comparable to a generative grammar with all of its advantages for the refinement and verification of theories.

There are several avenues through which one may approach such research; they may be characterized in terms of the requirements one might impose on the theory to be tested. In the first place, a theory might be required, in its computational implementation, to compose credible tonal music and only such music, starting from minimal axiomatic bases of some kind and applying the known operations to derive further steps in the simple-to-complex array. In principle, one could require the output of such a machine to include the Goldberg Variations and the Hammerklavier Sonata while excluding such conservative departures from classical tonal syntax as the milder pieces from Hindemith's Ludus Tonalis. (The exclusion clause is obviously necessary: trivial theories could be constructed that would generate authentic tonal pieces among countless arbitrary concatenations of notes.) A weaker requirement would be that the theory be capable, given an authentic tonal composition, of constructing the simple-to-complex array associated with it. Or, a theory could be designed to operate as a decision procedure: given an arbitrary musical score, does the composition specified belong to the class of tonal compositions, or not?

Investigations of each of the above types have been proposed, but to my knowledge none has been brought to a satisfactorily conclusion. Nor is this surprising, in view of the obvious comlexity of the tonal language and of the compositional technique displayed by its greatest "native speakers." One might, therefore, want to consider possibly useful initial limitations of the foregoing experimental designs. For example, work is currently being done[4] toward solution of the following problem: given level $\underline{n}$ in the simple-to-complex array (or possibly levels 0 through $\underline{n}$), compute level $\underline{n} + 1$ (or $\underline{n} + \underline{m}$, for some relatively small value of $\underline{m}$). Alternatively, one could start with an extremely restricted corpus of music -- initially even a single work -- and, proceeding from an apparently correct rational reconstruction thereof, supplement the rules of procedure for moving from stratum $\underline{n}$ to stratum $\underline{n} + 1$ with rule-constraints appropriate to the particular style of composition under investigation.[5] In either case, the advantages afforded by the computational implementation for verification of the theory and in suggesting new lines of inquiry are both obvious and familiar.

IV. The unfigured-bass problem. I was invited to address this gathering as a "pioneer" in the field of computational studies in music. That characterization can be considered accurate only if it is interpreted as indicating that the project that I undertook several years ago, and that I will now describe to you with merciful brevity, belongs properly to the pre-history of such endeavors.[6]

The unfigured-bass problem can be fitted in a loose way into the schema of simple-to-complex arrays outlined above, in that it involves a progression from a less fully specified pitch structure to a more fully specified one. From the locution "unfigured bass," you may correctly infer that there is such a thing as a "figured bass"; the latter entity is, in an important and interesting sense, more advanced in the array terminating in a tonal composition than the former. A figured bass, that is, a bass line with certain combinations of arabic numbers and musical accidental symbols adjoined either above or below it, specifies not only the bass voice of the composition with which it is associated but also the chords that accompany those bass notes, and, to a considerable extent, the voice leading (horizontal progression) of the (as yet unwritten) voices above that bass.

During the seventeenth and eighteenth centuries, the figures bass (or thoroughbass) was applied in practice as a kind of musical shorthand in notating all music accompanied by a basso continuo, normally played by cello, gamba, or bassoon together with a keyboard instrument. The keyboard player was expected to infer and supply chords and voice leading on the basis of the figures given; this was known as realizing the figured bass.[7]

Since a figured-bass realization as performed by a trained accompanist can also be written down in standard musical notation, the art of realizing such a bass is itself a skill that admits of simulation by computer. If the continuo bass was unfigured, as occasionally happened in the thoroughbass period, the accompanist was still required to provide a competent realization. The necessary musical skill in this case involves two distinct steps: (1) infer appropriate figured-bass figures from the structure of the bass itself, and (2) realize the resulting figured bass in accordance with the conventional rules. It was the first step of this process that I endeavored to simulate computationally.

To supply appropriate figures for an unfigured bass is a non-trivial problem; although some latitude exists, an arbitrary assignment of figures would, in general, result in violations of tonal syntax. For example, the penultimate note of a "cadential" configuration must be set with a figure which determines a major triad or a "dominant-seventh" type chord; other combinations of figures such as 6, ${}^{6}_{5}$, ${}^{4}_{3}$, etc., although in themselves possible in figured-bass realization, would be incorrect in such a context.

The problem of the unfigured bass was recognized by a number of eighteenth-century thoroughbass theorists and treated in some detail by a few. Among the more extended informal algorithms for solving the unfigured-bass problem -- that is, for determining appropriate figures from the organization of the bass itself -- were those by Heinichen[8] and Saint-Lambert.[9] One of the main computational aspects of my study, then, was the explication and testing, by computer, of the theories set down by those eighteenth-century authors.

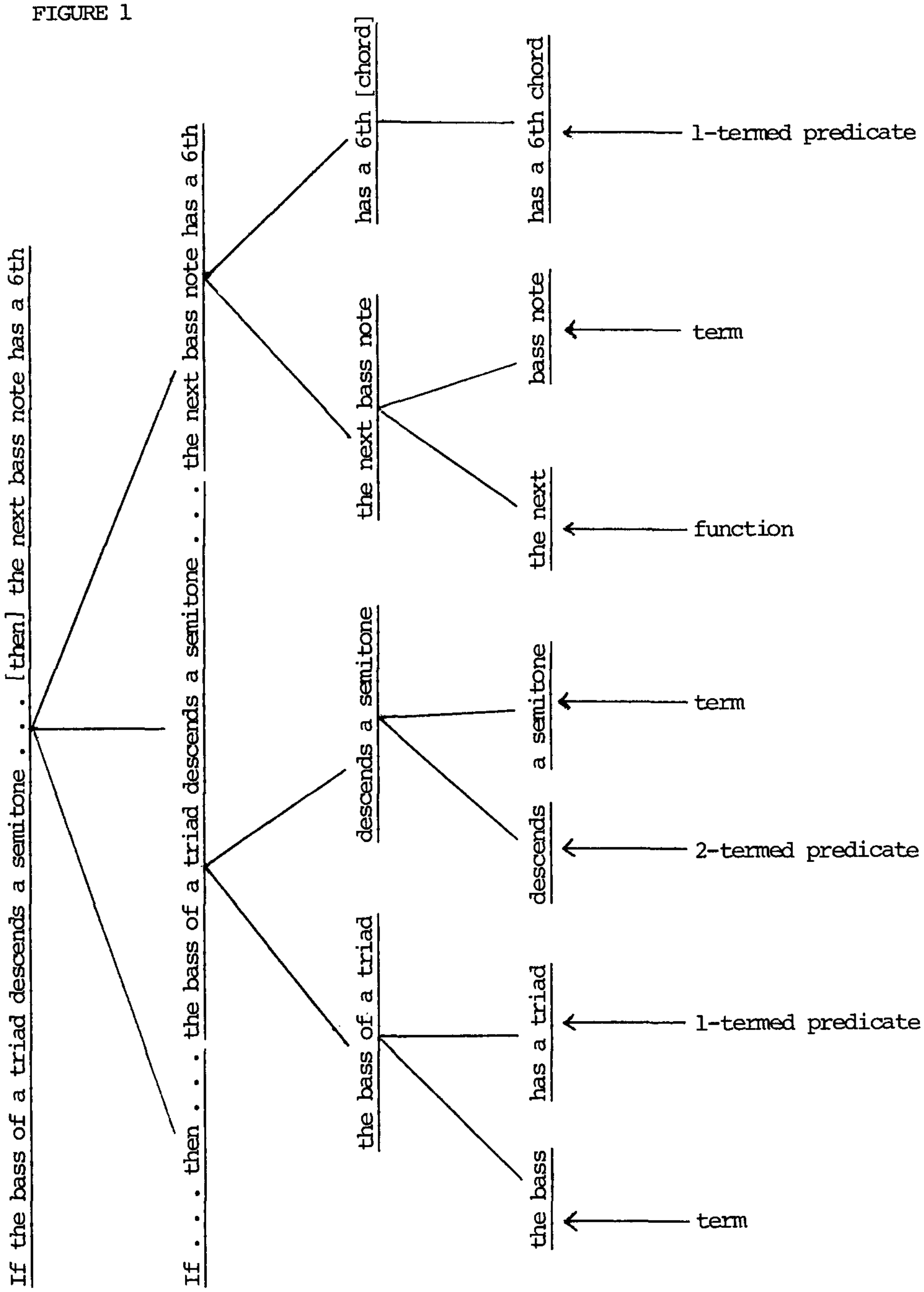
It would far exceed time and space limitations to undertake an exhaustive description of the Heinichen and Saint-Lambert procedures for bass harmonization. For our purposes it is sufficient to state briefly the necessary components of a computational implementation of them. First, the computer must have access to an unfigured bass in a symbolic code readable by it. Secondly, computer programs must be defined in a suitable programming language; these programs must have the capability to examine the encoded bass, derive from it information of various kinds, and generate new symbolic code incorporating the computed figured-bass specifications.

The music-representation system adopted for this purpose was the code known as DARMS (Digital Alternate Representation of Musical Scores), developed by Stefan Bauer-Mengelberg and his associates. DARMS is capable of representing all aspects of even very complex conventional musical scores, including stem-direction, beamings, dynamics, and expression marks. The unfigured-bass programs accept as input data relatively simple monophonic musical utterances, and it was not necessary to retain all of the information included in the musical notation; for example, stem-direction is irrelevant to the harmonization procedures, and therefore was not specified in the code.

The programs were written in the SNOBOL3 programming language, then one of the more sophisticated languages available for problems involving operations on data in the form of free character strings. They comprised a network of variable and constant terms (used for designating individual bass notes and musical-interval constants, respectively), one- and two-termed predicates, and one- and two-termed functions. The relation of these entities to bass-harmonization rules as formulated by the eighteenth-century theorists under consideration can best be shown with reference to a specific example. The first of Heinichen's "General Rules" (his theory also includes "Special Rules," which will not concern us here) for assigning figures to an unfigured bass is stated as follows: "If the bass of a triad descends a semitone . . . (then) the next bass note has a 6th." Figure 1 illustrates the analysis of this rule into its component parts. The predicates (some of which were regarded as "primitive" and others as "defined") accept as arguments individual bass notes as represented in DARMS and/or musical interval names (with or without specification of interval quality and direction) and return truth values. The functions accept bass notes as arguments and return other bass notes. In general the functions enable the program to look forward and backward in the bass line, while the predicates answer questions about the mode of progression of the bass and about the figures already assigned to specific bass notes. The values returned by predicates determine the applicability or inapplicability of a given rule. The operation of the program, then, follows closely the logical structure of the individual rules.

The bass-harmonization procedures set

FIGURE 1


If the bass of a triad descends a semitone . . . [then] the next bass note has a 6th
If . . . then . . .
the bass of a triad descends a semitone . . .
the next bass note has a 6th
the bass of a triad
descends a semitone
the next bass note
has a 6th [chord]
the bass
has a triad
descends
a semitone
the next
bass note
has a 6th chord
1-termed predicate
term
function
term
2-termed predicate
1-termed predicate
term

forth by Heinichen and Saint-Lambert were completely formulated as computer programs and were tested for completeness and adequacy on input data derived from several standard figured-bass treatises of the eighteenth century. The programs were allowed to admit failure and print the message "undefined" when necessary; this permitted discrimination between a program's occasional inability to compute a figure (signalling incompleteness of the harmonization procedure under investigation) and the absence of a figure for reasons consistent with figured-bass practice, wherein the assignment of the figure $\substack{5\\3}$ was regarded as a "default" situation not requiring any explicit figure specification. The adequacy of a given solution was determined by fiat emanating from the investigator's intuitive grasp of figured-bass lore.

As expected, both of the procedures tested were shown to be partially incomplete and to a certain extent inadequate. The deficiencies revealed in both procedures suggested certain refinements which were incorporated in subsequent versions of the programs and which led to improved results. Nevertheless, it gradually became clear that general solutions to the unfigured-bass problem were probably inaccessible to procedures of the type represented by those of Heinichen and Saint-Lambert. The principal source of inadequacy seemed to be that such procedures did not allow for a sufficiently sophisticated analysis of the structure of the bass line; in particular, they failed to take into account the hierarchic, stratified character of the bass. As a result, they were unable to identify cadential configurations and to cope with the implications of compositional elaborations of a single harmony within the bass line.

The musical skill involved in assigning figures to an unfigured bass has not yet, to my knowledge, been fully explicated and computationally simulated. The study that I have briefly reported constituted an initial step toward such a solution. The computer made a significant and well-defined contribution to the study by exposing deficiencies in the theories under investigation and in suggesting further lines of inquiry. Although I have not elected to follow them up systematically, there is no doubt that the experience of attempting to simulate a musical skill with the digital computer has made its mark on my way of thinking about musical problems of all kinds.

REFERENCES

1. During the past three hundred years or so, the general trend in notating music has been to make the written score more and more explicit. At the same time, the interpretative responsibilities of the performer have increased (at least up until the first decades of the twentieth century), so that the written score of a Mahler symphony, however fully specified it may appear, is not an appreciably closer approximation to an orchestral performance than is the score of a Brandenburg Concerto by Bach.

2. Indeed, the structural significance of the various intervals is a defining property of tonal music; the departure from established norms of intervallic behavior, thus, became _the_ salient feature of pitch organization _in_ the "atonal" music of the early twentieth century.

3. The concept of structural strata was most fully and convincingly expounded by the Austrian music theorist Heinrich Schenker from about 1906-1935.

4. By Stephan Haflich, for a Ph.D. disseratation in progress at Yale University.

5. This is the strategy employed by James L. Snell in his _Design for a Formal System for Deriving Tonal Music_ (M.A. thesis, State University of New York, Binghamton, N.Y., 1979).

6. Rothgeb, John, _Harmonizing the Unfigured Bass: A Computational Study_ (Ph.D. Dissertation, Yale University, 1968).

7. With the demise of the _basso continuo_ the figured bass disappeared from finished musical scores; but masters such as Mozart, Beethoven, Mendelssohn, and even Brahms continued to use it in sketching their compositions. It provided them with a record, for their own reference, of the basic harmonic and voice-leading content of compositions in progress. Figured bass remains today an indispensable pedagogical aid in the study of tonal harmony and voice leading.

8. Heinichen, Johann David, _Der Generalbass in der Composition_ (Dresden, 1728).

9. Saint-Lambert, Michel de, _Nouveau traite de l'accompagnement du clavecin_ (Paris, 1707).