DOI: 10.1111/j.1468-2249.2008.00267.x

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BIRD'S-EYE VIEWS OF THE MUSICAL SURFACE: METHODS FOR SYSTEMATIC PITCH-CLASS SET ANALYSIS

I

Surface Overviews and Detectors

Descriptions of musical style may concentrate on issues of varying magnitude. For instance, musical style is often thought of as depending on deep-level structural properties which call for a global view of the musical work. Alternatively, style may also be understood as the accumulation of minute and musically superficial details. It is notable that levels of description are typically coupled with structural levels: features assigned to deep structural levels are dealt with using large-scale descriptions, whereas foreground features are approached through detailed local descriptions. In this article, we will attempt a different coupling between structural levels and levels of description, presenting global accounts of surface-level stylistic features in musical compositions. Although our concern is with pitch-organisational features within complete musical works or classes of such works, we do not seek to advance an approach to harmonic syntax in the sense of using pitch-reduction techniques. Rather, our purpose is to stay as close as possible to the surface level of music while still making meaningful observations about musical style on a global level. Metaphorically speaking, our intention is to approach a musical landscape not by drawing a map, which necessarily confines itself to a limited set of structurally important features, but by presenting a bird's-eye view of the musical surface – an aerial photograph, as it were, which details the position of every pitched component. Whereas any pitch-reduction graph tends to describe broader musical processes by excluding 'less important' events, our purpose is to produce large-scale descriptions which still convey precise information in respect of local details. This represents a strictly non-hierarchical view of pitch organisation.

The bird's-eye views which we have in mind will reveal only the average characteristics of the pitch material found on the musical surface. Rhythmic considerations of the music as well as many other stylistically important factors having to do with the temporal order of notes will be ignored. To some readers our approach may appear problematic in so far as it concerns only the basic properties of pitch-class sets with no regard for their functional roles. As Joel Lester (1992, pp. 227–8) has observed, however, such an approach does have theoretical precedents (not least in the case of Joseph Riepel, for instance) and may even have surprising relevance for some styles of tonal music. Lester himself has sought to illustrate this thesis with reference to the famous opening of Mozart's A major Piano Sonata, K. 331, in which almost every simultaneity represents either set class 3–11B or one of the inverted forms of 3–7.[1] Although it is possible that some composers may have applied such techniques consciously, we believe that pitch-class set analysis can provide more general insights concerning the surface differences between compositional styles as well as revealing significant tendencies within the surface materials of particular compositions. However, a true bird's-eye view of such features requires a quantitative approach, which understandably remains vulnerable to criticism on account of its potential lack of musical pertinence. Indeed it is difficult to disagree with Jan LaRue's remark that we do not 'gain much insight from the comment "The piece contains many augmented fourths and major sevenths"'. Rather the analyst should, as he goes on to advocate, 'take a further step toward understanding by attempting to show musical meaning' (LaRue 1970, p. 83). Even so, one does not have to concur with his additional suggestion that musical meaning can be elicited merely by supplementing the process of enumeration with expressive terminology evoking the 'function' of the intervals in question. A much better way of rendering a quantitative result meaningful is to study it in relation to other, similar findings. Thus, a quantitative measurement with respect to a given segment of a composition acquires meaning only in the context of the pattern formed by similar measurements throughout the whole composition. In the same way, a quantitative characterisation of certain aspects of a complete musical work is made meaningful first and foremost in relation to other such measurements drawn from a range of musical styles. This need for contextualisation motivates the relatively broad stylistic scope of the present study.

It is obvious that the project of generating surface overviews of musical works by contextualising the quantitative features of pitch organisation with respect to other measurements cannot hope to succeed unless it is highly systematised. In the present context, systematicity has two aspects, both of which can be illustrated by another metaphor – that of a detector, a mechanical sensing device. First of all, it is crucial that the whole field of objects under examination be processed using the same detector. In the following enquiry, detectors will be defined using pitch-class set theory, which is here defined as a systematic and stylistically neutral theory of the combinatory possibilities of a tone system involving twelve pitch classes. A simple example of a detector is provided by Lester's Mozart example. A suitable discovery procedure capable of generating the same analytical results may be construed in terms of a set of nineteen detectors, each reporting the presence of one of the T_n-type trichords. Scanning the beginning of the same sonata movement with these detectors, it may be noted that three of them – those which are attuned to 3–7A, 3–7B and 3–11B – are triggered more frequently than the others, indicating the presence of the appropriate set classes. We can therefore reformulate Lester's assertion by concluding that a surprisingly small number of these simple detectors are required to cover the surface of Mozart's well-known theme. More complex detectors will be similarly sensitive to more complex properties of the musical surface.

The other, and equally important, aspect of systematicity pertains to the way in which the detector is deployed throughout the relevant field of objects. Just as a minesweeper has to move systematically across an entire minefield in order to ensure that every explosive device has been found, so the musical surface has to be scanned in a similarly exhaustive fashion if the quantitative results are to be of any value. In the Mozart example, such systematic scanning requires only that the successive simultaneities be checked for features of interest; however, larger compositions and less transparent musical textures tend to render the issue much more involved. Now, it is well known that the systematic core of pitch-class set theory has often been supplemented by less than systematic analytical procedures, especially when it comes to the question of segmentation and the further identification of key structural features (Apajalahti 1994; Pople 2004b). Merely pointing the detector towards some intuitively salient aspect may perhaps engender pertinent results, yet may just as easily lead to more sweeping implications that are insupportable.

The second section of this study will concentrate on this latter aspect of systematicity, namely the means by which the detector is applied. This is a task which entails the use of computer algorithms.[2] At the same time, we will attempt to establish a framework for developing various kinds of detectors, each sensitive to the differing properties of pitch-class sets. Although our contribution can be seen as a continuation of the tradition of statistical music analysis, the pitch-class set-theoretical approach widens the perspective from the primitive characteristics of individual notes (pitch, duration and so on) to more complex relational features. This change in scope carries a clear advantage: put simply, its findings are more open to qualitative interpretation. For example, whereas early information-theoretic approaches to the analysis of music were concerned, in the main, with the amount of musical information generated (for instance, Winckel 1964; Fucks and Lauter 1965), it now seems that greater attention should be paid to the musical character of the pitch materials being analysed. Even if the differences between musical styles may only be approached fruitfully using information regarding the frequency distributions of various musical elements (Beran 2004; Manaris et al 2005), we prefer a form of statistical style analysis that attaches some musical meaning to these elements from the outset.

For example, the third section of this survey will seek to explore the use of detectors which are sensitive to the information encoded within interval-class vectors. This results in *stretched interval-class vectors* – graphic representations of the changes in local interval-class content within single compositions – as well as in *mean interval-class bias vectors* (that is, statistical summaries of interval-class content within larger sections of music). The following section introduces our main analytical method, *comparison set analysis*, which incorporates similarity measures for set classes within the detector model of systematic pitch-class set analysis. Instead of measuring the similarities, distances or mutual affinities between some individual pitch-class sets A and B within a piece of music, we apply similarity measures as a means of assessing the relationships of both A and B to a constant *comparison set*. Thus similarity measures are not used to address questions of local coherence within particular musical events, but are instead transformed into a powerful computational tool which yields intuitively accessible descriptions of the dynamic processes with respect to selected, qualitatively comprehensible features of the musical material. The next section presents some applications of the comparison-set approach. At this point it will be possible to appraise the relative diatonicism of the musical surface – an application which leads to the facilitation of rough comparisons among the average surface features characteristic of various compositional styles. Furthermore, we will aim to apply our comparison-set analysis in a quantitative approach to set-class referentiality, demonstrating some of the ways in which these methods might be deemed useful in connection with more orthodox reductive techniques. The final section then proceeds to outline a general evaluation of our working methods, concentrating on matters of contextualisation.

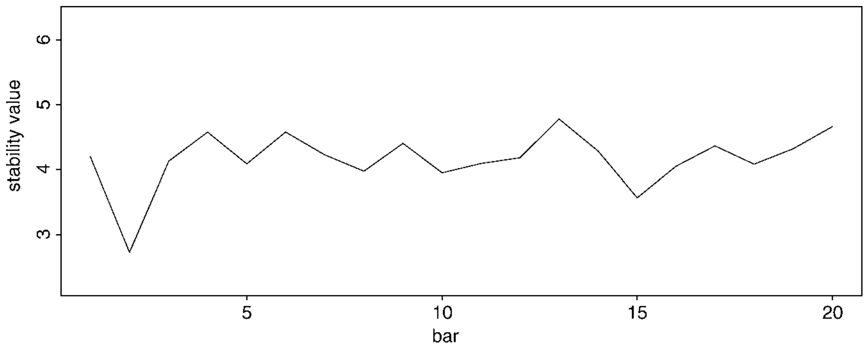
Throughout this article, our focus will be on the development of a particular music-analytical approach rather than on specific interpretative results. If some of our analyses seem only to reveal what was already understood about the structure of the musical works in question, we urge the reader not simply to dismiss these interpretations, but rather to see them as indicating that such methods are capable of generating results which may be accepted intuitively. Even for the most seemingly trivial instances, the computational processes involved will greatly exceed what can be realised through more informal procedures. In this respect, it is important to give due credence to the preliminary illustrations which serve to establish the precise nature of our preferred methodology.[3]

II

Continuous Measurement of the Relational Properties of Notes

Perhaps the most elementary of approaches to the quantitative description of music involves assigning numerical values to individual notes. Each individual note has properties such as pitch level or temporal duration which can be measured and recorded. This information may be represented either as a continuous curve depicting local changes in the values through time, or by computing an average value to characterise a particular facet of the musical material on a more global level. The former approach may be conceptualised in terms of a scanning procedure which tracks across the musical surface, measuring one note after another. The only prerequisite is that the notes be

Fig. 1 J. S. Bach, *Es ist genug*, BWV 60: tonal stability calculated using Krumhansl and Kessler's A major key profile



capable of being treated as successive in some sense, a condition which is of course problematic in the case of pitch simultaneities. In the following instances, we will be working with MIDI files of Western music, and thus each composition is already arranged as a successive event list. In order to avoid arbitrariness in the compilation of these lists, we have arranged each chord as an ordered set, counting upwards from the lowest note. The lists thus consist of event onsets; note durations will be ignored. With this tabulation as a starting point, one can already begin to represent graphically those aspects of the music for which the measurement of individual notes will prove sufficient. As an elementary example, one might assign each pitch class a numerical value according to its supposed stability in the tonal hierarchy of the main key of the piece, and subsequently record the stability values for each successive note in the composition. As stability values, one might use the well-known key profiles advanced by Carol Krumhansl and Edward Kessler (1982; Krumhansl 1990, p. 30). Fig. 1 shows the results of applying an A major key profile to the Bach chorale setting of *Es ist genug*. The stability values assigned to individual notes have been averaged over each bar for the purpose of legibility. Features such as the departure from the main key to the G♯ dominant seventh chord in bar 2 and the prolonged A major harmony in bar 13 are clearly reflected in this representation.

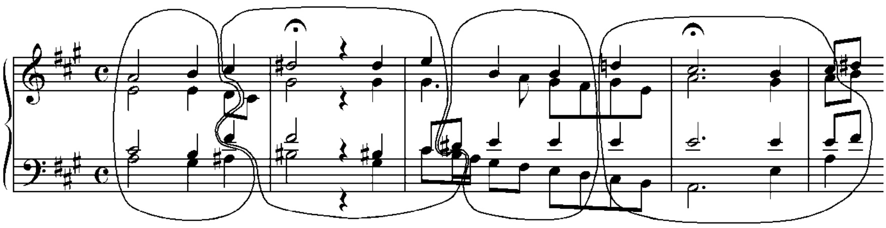
Such an approach may be extended to correlate the music's localised tonal characteristics with the full gamut of major and minor keys, thereby tracing mobile progressions of 'activation' within a spatial model of key relationships (Toiviainen and Krumhansl 2003). However, the one-value-from-one-note approach is rather limited in its applicability. This is because most of the musically interesting properties are relational in kind and cannot be measured for each individual note in isolation. Some of these relational properties may be rather unproblematic, such as the sizes of melodic intervals between successive notes, transitional probabilities and so forth (as variously summarised in Beran 2004; Eerola and Toiviainen 2004). Others are rather more complex and cannot be measured without taking into account larger sets of notes around the one which is in focus. For this purpose, the appropriate sets of notes have to be identified, which requires a pertinent segmentation procedure.

As noted by David Temperley (2001), the question of segmentation is one of the most urgent difficulties to be addressed by any computerised method of music analysis. Since we shall be discussing computer-aided pitch-class set analysis of tonal as well as non-tonal music, it is also appropriate to recognise at this point that Anthony Pople's *Tonalities* project in fact leaves this issue unresolved. In Pople's approach, the 'analyst supplies a segmentation of the passage into harmonic areas based on his or her best intuitions' (Pople 2004a, p. 164). For reasons which will subsequently become clear, we will adopt an alternative course, presenting instead a series of analytical techniques that are centrally dependent on an automated method of segmentation – even though this will require stretching the meaning of the term beyond that which is normally accepted by music analysts.

As Pople's remark implies, segmentation in computer-aided harmonic analysis usually equates with 'identifying those places in the music where the harmony changes' (Pardo and Birmingham 2002, p. 27), a view which reflects the attitudes encapsulated in standard music-analytical practice. In other words, segmentation is understood as the discovery of some natural boundaries that are either inherent in the music itself or characteristic of its perception. In Fred Lerdahl's (1988) terms, segmentation is thus directed either towards an appropriate 'compositional grammar' or towards a composition's 'listening grammar'. In the first approach, the analyst is interested in revealing a segmentation which in some sense respects the composer's decisions or, perhaps, presents the composition as structurally homogeneous by disclosing similarities between various segments. Hence as Christopher Hasty suggests, 'segmentation can be understood not as something imposed upon the work, but rather as something inherent – something to be discovered' (Hasty 1981, p. 59). In the second approach, the analyst (for example, Deliège 1989) aims to segment the composition from the point of view of a listener, whether an ideal one or an average one. In computer-aided studies, the two dominant approaches to listener-oriented segmentation use either Gestalt rules (for instance Tenney and Polansky 1980; Cambouropoulos 1997) or statistical data (Bod 2002) as their starting points.

Both of the above-mentioned approaches are nevertheless somewhat problematic from a statistical viewpoint, which aims at as objective a description of the musical material as possible. Let us assume that we have an algorithm with the capability of producing segmentations either from the point of view of a given compositional grammar, or from that of some listening grammar. According to the rules of the grammar – that is, depending on our previous knowledge of relevant musical categories – we would arrive at different forms of segmentation. Assuming that our aim is a quantitative analytical method which assigns numerical values to musical segments in accordance with their

Ex. 1 Non-overlapping heptachordal segmentation of J. S. Bach, *Es ist genug*, BWV 60, bars 1–6



1 | 2

A3 C#4 E4 A4 G#3 B3 E4 B4 A#3 D4 F#4 C#5 C#4 | B#3 F#4 G#4 D#5 G#3 B#3 G#4 D#5 |

3

C#4 C#4 G#4 E5 B3 D#4 A3 G#3 E4 B4 F#3 A4 E3 E4 G#4 B4 D3 F#4 C#3 E4 G#4 D5 B2 E4 |

4 | 5

A2 E4 A4 C#5 E3 E4 G#4 B4 | A3 E4 A4 C#5 F#4 B4 D#5

structural properties, we would have to choose between different sets of values assigned to different sets of musical segments, each of these analyses being highly dependent on complex networks of segmentation rules. From a statistical viewpoint, it simply does not seem impartial enough to begin by assigning a determinate segmentational interpretation to the composition. However, without any segmentation, we cannot identify pitch-class sets to serve as the basis of our measurements. In a nutshell, the problem is how to segment without interpretation, or – more realistically – with as little interpretative intervention as possible.

Tail-Segment Arrays

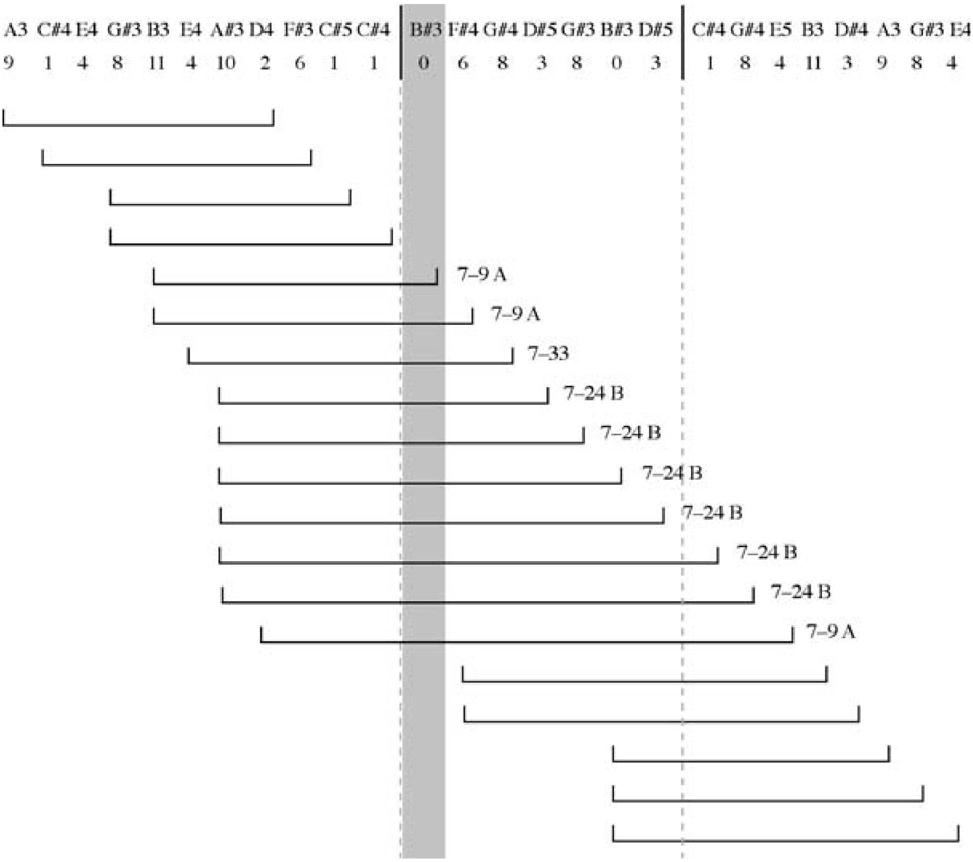
In our analytical method, the systematic movement of the detector through the musical work is constituted by an automatic division of the musical surface into pitch-class sets. Now, given a list of 'successive' notes, one way of minimising the amount of segmentational interpretation might simply be to segment the event list of pitches into consecutive pitch-class sets of a given cardinality. As an example of such an elementary approach, a heptachordal segmentation of Bach's *Es ist genug* is shown as Ex. 1. Here one simply begins with the first note on the list, subsequently grouping each successive note into the initial segment until seven different pitch classes have been accumulated. The next note would initiate a new segment which is again completed with the seventh distinct pitch class, and so on. Having thus decided how to apply the detector, we would then define the detector itself by choosing the properties to be recorded within the consecutive segments.

For a computationally simple but perhaps somewhat artificial example, we might use the standard deviation of the consecutive pitch-class intervals within each pitch-class set to measure their deviation from maximal evenness (as represented in Clough and Douthett 1991). While this example is chosen as one possible instance, any function capable of returning numerical values for set classes would serve just as well for demonstration purposes. For the first four segments of *Es ist genug*, representing set classes 7–Z36A, 7–27A, 7–29A and 7–35, the standard deviations are 1.11, 0.76, 0.76 and 0.49, reflecting an increasing evenness ('scale-likeness', or regular dispersal on the pitch-class circle) which culminates in the maximally even diatonic scale around the second cadence.[4] The problem of determinate segmentational interpretation still remains, however: an arbitrary segmentation is a segmentation nonetheless and will tend to condition our view of the composition in exactly the same way as if the segmentation were advocated for seemingly informed musical reasons. If a musical analysis 'stand[s] or fall[s] on the appropriateness of the segmentation' (Isaacson 1992, p. 195), an arbitrarily determinate segmentation will most probably render it unsatisfactory.

Marcus Castrén has suggested one possible solution: in short, the introduction of an automated segmentation in which 'a pitch succession representing a passage [is] . . . segmented into overlapping segments of several different lengths', thereby allowing the identification of a wide variety of pitch-class sets even in short passages of music (Castrén 1994, p. 164). Such a procedure is reminiscent of what Allen Forte termed 'systematic imbrication' (Forte 1973, pp. 83–4). However, Castrén himself has not gone on to advance a detailed methodological description, in fact relying on automated segmentation only for a preliminary search through his chosen repertoire. In any case, such a procedure ignores the stipulation that the segments be non-overlapping. The requirement for non-overlapping groupings may perhaps be perceptually motivated (as emphasised by Lerdahl and Jackendoff 1983, p. 38), but there is no *a priori* reason to think that a detailed music-analytical system should seek to imitate human perceptual processing. Insisting that segments be non-overlapping makes it possible for a single event list of notes to be segmented in mutually contradictory ways. Following Castrén, we will discard this condition and instead suggest a systematic method of automatically dividing a piece of music into a large number of overlapping segments.

The first step in our procedure is to define a *tail segment* for each note of the composition. A tail segment is a succession of notes with pitch-class content of a given cardinality such that it ends with the note in question. In Fig. 2, the specification of tail segments as well as the further steps in our procedure are demonstrated with reference to the beginning of *Es ist genug*. The figure shows the list of notes corresponding to the first two phrases of the Bach chorale, accompanied by appropriate pitch-class numbers and separated by bar lines. Each simultaneity is listed from the bass note up, and pitch doublings at the unison or one or more octaves are omitted, for reasons which will be explained below.

Fig. 2 Tail segments in J. S. Bach, *Es ist genug*, BWV 60, bars 1–3



As with the simpler segmentation procedure shown in Ex. 1, we will first demonstrate our method with reference to pitch-class sets of cardinality 7. Once again, the first tail segment will consist of the first notes on the event list up to the point where a heptad is completed. With doublings omitted, eight notes are sufficient to complete the first tail segment. The difference here is that we conceive of this segment as a 'tail' for its last note. In other words, the third note of the alto voice – D4, representing pc [2] – is assigned a tail segment which extends back to the very first note on the list. Thereafter, a similar tail segment is defined for each successive note on the list following the D4, beginning with the tenor F♯4 in the third chord (note the voice-crossing which takes place at this point). For this note as well as for each note following it, heptachordal tail segments are found by counting backwards on the list to the seventh distinct pitch class. Each note thus stands at the end of its respective tail segment, which extends at least six notes backwards. In Fig. 2, the heptachordal tail segments for the beginning of *Es ist genug* are indicated by square brackets.

Notice how the repetition of pitch classes [6], [1], [0], [8] and [3] affects the length of the tail segments in and around bar 2. Even though each new tail segment begins from a new note farther ahead on the list, several successive tails have to reach back to the A♯3 in bar 1 in order to obtain a complete heptad. It becomes clear that in less chromatic music such tail segments might easily extend backwards over several bars. In some sense, the tail segment represents a simple kind of 'pitch-class memory': at each point on the list, the analytical system is able to 'recall' a tail segment whose length in time increases in relation to the decreasing complexity of the piece's pitch-class structure. Because we wish to describe the pitch-class structure of compositions and not their orchestration, however, we do not want the length of the tail segment to increase with the number of doublings. For this reason, the list of pitch classes in Fig. 2 includes only a reference to the lowest note of a given pitch class at any temporal location in the piece. Considering the first bar of the Bach example, the doublings in the first two chords are thus omitted, but the two C♯s of the third chord are retained since the appoggiatura makes them appear temporally distinct. Although such pruning of the event list may appear somewhat arbitrary in connection with a Bach chorale, it does succeed in alleviating some rather counter-intuitive consequences when analysing larger orchestral scores.[5]

Let us now focus on the first note of bar 2. This B♯3, representing pitch class 0, has a tail segment of length 8 (including the note itself). In a sense, this note is 'associated' with the earlier notes of its tail segment by virtue of the fact that they all belong to a common heptad. However, as one can see by following the grey shading in Fig. 2, the same note also belongs to several other tail segments, namely those covering the nine subsequent notes. The collection of tail segments which include a given note *n* is hereafter called its *tail-segment array*; each member of the array is a tail segment whose notes are associated with the note *n* in the sense that they belong to a common conjunct pitch-class set on the event list associated with the note *n*. In our example, the B♯3 which initiates bar 2 has an array of ten tail segments, each of them representing a set class of cardinality 7. In this particular tail-segment array, the tail segments represent only three different set classes. In a 'temporal' sense, the ten tail segments form an *array range* with respect to the central note, B♯3. Given a heptachordal tail segmentation, the array range extends seven notes backwards and nine notes forwards from the B♯3.

The point of the exercise should now be clear. Whereas the division into non-overlapping segments in Ex. 1 imposed a unique interpretation on *Es ist genug*, placing each note unequivocally into a single segment, the new procedure evades such decisions, instead recording each and every tail segment of which this note is a member. Whether one should call this 'segmentation' or not, we have no intention of trying to cut nature at its joints. For a tail segmentation of cardinality 7, the method presents each note as belonging to a minimum of seven tail segments, the majority of which would probably not qualify as 'good' segments – according to neither a compositional nor a listening grammar. As

Forte observes, a systematic segmentation procedure 'may often produce units that are of no consequence with respect to structure' (Forte 1973, p. 90). But as the reader will now understand, our purpose in this preliminary phase has not been to propose a new method of segmentation in any traditional sense. To borrow William Benjamin's terminology, tail segments need not correspond to 'structural sets' that 'pervade the composition and are useful in accounting for its progress at all levels'; they are, indeed, more akin to mere 'instrumental segments' (Benjamin 1979, p. 28).

This said, it is time to return to our initial question: how can we measure pitch-class sets in a composition independently of segmentational interpretation? Put briefly, a given note *n* on the list can now be assigned the value of a function *g* by evaluating each tail segment in *n*'s tail-segment array separately by some other function *f*, representing the detector, and letting the arithmetic mean of the resulting values stand for the value of *g* for note *n*. More formally, when A_n is the tail-segment array for a note *n* and $|A_n|$ is its cardinality (that is, the number of tail segments in A_n), the value of function *g* for the note *n* is given by

$$(1) \qquad g(n) = \frac{\sum_{i=1}^{|A_n|} f(X_i)}{|A_n|},$$

where $X_i \in A_n$. Notice that function *f* can take tail segments (or, in our application of the method, pitch-class sets corresponding to the tail segments) as its arguments, whereas the arguments for function *g* are individual notes. The choice of function *f* is understood as the specification of a detector which measures any given properties of the tail segment pitch-class sets. On the left-hand side of the equation, function *g* is just a construct allowing us to assign values of *f* to individual notes, so to speak. Each value of $g(n)$ represents an average of a number of local detector measurements, each of which associates the note *n* with a different set of nearby notes. By calculating $g(n)$ for each successive note on the list, we may now carry out continuous measurements of the relational properties of notes.

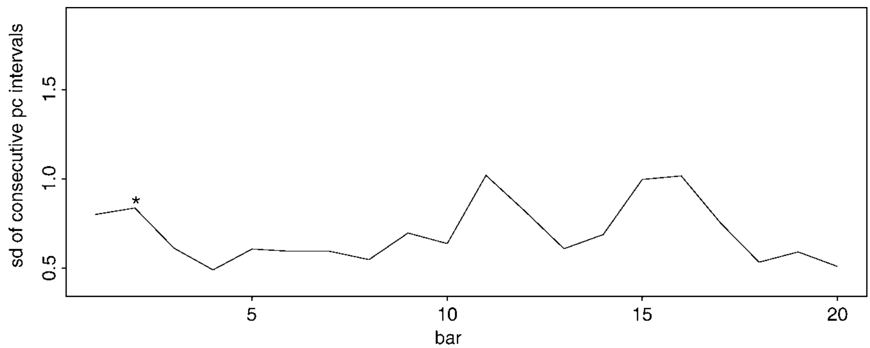
A Preliminary Example: Measuring Evenness

In our demonstration of the simpler segmentation method, we used the standard deviation of the set of consecutive pitch-class intervals (on the pitch-class circle) to measure the relative distance of pitch-class sets from maximal evenness. As an example of the tail-segment array method, let us now apply this tentative measure of non-evenness to Bach's *Es ist genug*. According to equation (1), the B♯3 in the beginning of bar 2, for example, acquires the numerical value 0.836, which is the average of the standard deviations calculated for the pitch-class sets of the ten tail segments. We are thus able to assimilate a set of complex relational properties of the pitch-class sets around the note B♯3 as indexed to the note itself – that is, as if dealing with some non-relational property of this individual note. Carrying out similar computations for all of the notes in *Es ist genug* consequently offers a way of characterising the changes in relative evenness without imposing a determinate segmentation on the piece.

Although the tail segments noted above were conceptualised as a form of 'memory' within the music-analytical system, it is worth emphasising that the methodological objective is not to approximate human musical cognition. Indeed, if our purpose were something like this, we would be better off characterising each note n directly with the value of f as it is applied to n's tail segment. Instead of equation (1), which relies on averaging the values of numerous tail segments, we would simply accept $g(n) = f$ (the tail segment of n). At each new note n on the list, the system would then evaluate the state of its 'short-term memory' by applying the function f to the tail segment of n. Even though such a procedure might work in the context of monophonic melodies, it would clearly be out of place in more complex compositional textures such as the Bach example, given the way in which the event list has been compiled. Indeed, the 'short-term memory' interpretation of such an alternative procedure is immediately lost with a MIDI list, in which simultaneous notes are represented as successive entries. This is a good reason for preferring a procedure in which the set-class associations of a given note are examined forwards as well as backwards, averaging all of the consecutive tail segments (of a given pitch-class set cardinality) of which the note is a member. However, since all of the associated tail segments are taken equally into account, our detector measurements would be highly unrealistic as models of human musical cognition. Notice that in this preliminary example the detector itself is also a rather abstract function, chosen not because such a function operates in listener cognition, but because it is able to highlight some interesting overall characteristics of the musical material at hand.

With this caveat in mind, we are in a position to finalise the preliminary example of non-evenness in *Es ist genug*. A detector graph can now be drawn for the changes in non-evenness by plotting the successive measurements for each note. Because our present focus is trained to observe the overall features of and changes in pitch-class content, however, we will mostly forgo the discussion of small-scale musical events. The detector graphs will therefore be simplified by averaging the values of g over each notated bar, which yields one value representing the entire bar. This has two advantages: it makes for legible analytical graphs while at the same time simplifying the interpretation of further overall means which will subsequently be calculated for entire compositions. Remember, however, that the individual values of g which are averaged in each measure do not force strict segmentational divisions on the musical work. Each value $g(n)$ pertains to the whole array range of note n, and thus the mean g value for each bar also reflects pitch-class associations across the bar lines (see again Fig. 2). Our decision not to enforce a mechanical segmentation into separate bars but rather to average the values within each bar aims merely at lowering the resolution in the interest of easier reference with respect to the score.

Fig. 3 J. S. Bach, *Es ist genug*, BWV 60: heptachordal tail segmentation conjoined with function *f* measuring the standard deviation of consecutive intervals on the pitch-class circle



The basic representational form of our analyses can be seen in the completed detector graph of Bach's *Es ist genug* shown in Fig. 3. The continuous measurements of standard deviation of the consecutive pitch-class intervals on the pitch-class circle produce a bar-by-bar curve in which each point corresponds to a local mean value of *g*. The reader may compare the completed measurement for bar 2, indicated by an asterisk, with the earlier calculation, which yielded the value 0.836 for the first note of the bar. Generally, the 'non-evenness' curve of Fig. 3 appears to correspond well to basic music-analytical intuition. For instance, in bars 2–4, the initial movement away from acute chromaticism towards full diatonicism is borne out by the quick lowering of the curve in the direction of maximal evenness with respect to the diatonic heptad. Similarly, note the increasing evenness brought about by the diatonic material in bars 12–14 (involving only a single cadential accidental) in comparison with the following phrase in bars 15–16 which is built on a chromatically descending bass. Finally, the distinctive opening of the chorale is balanced by the closing cadence, which verges on maximal evenness. As in our later examples, the length of the y-axis is set to indicate the range of possible values (given the tail-segmentation cardinality).[6]

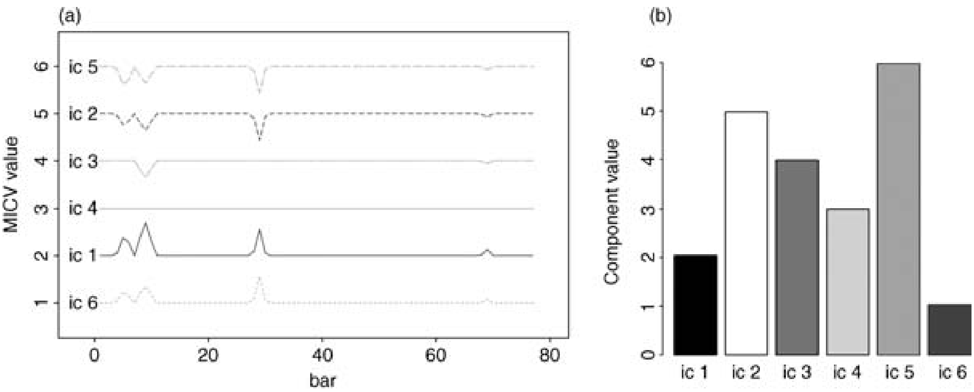
There are some obvious similarities between the key profile values for *Es ist genug* in Fig. 1 and the reading presented in Fig. 3. For example, the two troughs of tonal stability in bars 2 and 15 both correspond to deviations from maximal evenness, which are seen as relatively high peaks in Fig. 3. The two curves are not exact mirror images, however. Indeed, there are three principal differences between them, none of which is dependent on the exact details of the detectors (for example, the set of stability values employed for the first curve). First of all, the tonal-stability curve refers only to the absolute pitch classes in the music, whereas the non-evenness curve deals only with abstract pitch-class sets. Second, the stability curve reflects some values of the notes which have been assigned to them on the basis of a supposed main key – that is, with reference to a scheme which is external to the list of notes – whereas the non-evenness curve is not dependent on similar interpretative assumptions. Third, and most important, the stability curve is a straightforward reflection of certain (apparently) monadic properties assigned directly to individual notes, whereas the non-evenness curve substitutes for each monadic value a computational result which reflects the overall presence of a given relational property in the close environment of the note in question. The three assumptions relating to the stability curve – absolute pitch classes, prior tonal interpretation and non-relationality – are often useful in the sphere of cognitive musicology. In the remainder of this article, however, we will concentrate exclusively on detectors which are sensitive to abstract, relational properties of the pitch-class material that can be discussed on the level of set classes. Thus, even if something like the earlier stability curve might still, with some imagination, be regarded as 'modelling human cognition', no such claims could reasonably be made for the computationally more complex and conceptually more abstract detector measurements. Nonetheless, we hope to show that our methods may successfully furnish intuitively accessible results: music analyses which, in one way or another, correspond to meaningful features of our musical experience.

III

Stretching and Averaging the Interval-Class Vector

The standard way to characterise the intervallic content or, more appropriately, the interval-class content of a pitch-class set is the interval-class vector (ICV). The ICV may be thought of in terms of a set of six functions. Each of the functions assigns to every set class an integer which records the number of intervals belonging to a given interval class that appear within any pitch-class set representing the set class. For example, in case of the diatonic scale (set class 7–35), the six functions thus produce the familiar values [254361], indicating that the scale includes two instances of intervals belonging to interval class 1 (ic 1), five members of ic 2 and so on. A natural development of the tail-segment-array method explored in the previous section of this study is made by substituting these interval-class-counting functions for function f in equation (1). First, ICVs are determined for each tail segment. Second, each note in the composition is assigned six numerical values, corresponding to the arithmetic means of the appropriate entries in the ICVs of the tail-segment array. In other words, each note n is assigned the values of a *mean interval-class vector* (MICV) by averaging the ICVs of those tail segments which contain the note n. For each note in a given piece of music, the MICV values are thus used to represent the average interval-class contents of its associated tail segments. Finally, the third, optional step taken here is applied in order to realise the average for the six MICV values relating to every note within each bar.

Fig. 4 The interval-class contents of the Kyrie from Palestrina's *Missa Papae Marcelli* represented as (a) a stretched ICV and (b) an overall MICV (the tail-segmentation cardinality is 7)



Despite this somewhat forbidding description, the resulting construct should be immediately comprehensible. What is produced is in effect a *stretched ICV* (or, more precisely, a stretched MICV) involving a set of six curves which represents the trajectories for the six ICV entries within any composition. The usual punctual enumeration of instances for each interval class is thereby replaced by a more dynamic view of the temporal changes apparent within the interval-class hierarchy of a musical work. In a stretched ICV, each interval class has its own curve reflecting the temporal development of its particular share of local intervallic content within the music. For a simple example, one may take a look at Fig. 4a, representing the stretched ICV of the Kyrie from Palestrina's *Missa Papae Marcelli*. Given that this is predominantly diatonic music, we have used heptachordal tail segments for this demonstration.[7] (In order to produce local mean values for the graph, we have adopted the bar lines from the familiar Eulenburg edition.) From a music-analytical point of view, there is nothing particularly surprising in such a temporal representation of the Kyrie's interval-class contents. As any music student might surmise, strict diatonicism is maintained throughout the composition – a situation reflected in the flatness of the curves. As a matter of fact, one could easily identify each of the six curves just by recalling that the diatonic scale possesses six instances of ic 5, five of ic 2, and so on.

The slight bulges which might be noted are due to the few F♯s that are grouped into common tail segments with the neighbouring non-accidental pitches, thus producing non-diatonic pitch-class sets on a local level. For example, the last tail segment of which the first F♯ is a member turns out to be {2, 4, 5, 6, 7, 9, 11}, including a 'cluster' of four neighbouring pitch classes. Of course, our belief is not that such pitch collections have any independent status in

Palestrina's compositional style. Rather, the aim of the present method is to remain sensitive to the various juxtapositions of pitch classes and thus, among other things, to accommodate the effects of modulation (or, in Palestrina's case, *musica ficta*). A more traditional segmentation would assign F♮ and F♯ to consecutive diatonic segments, resulting in completely flat curves in which the modulatory effect of such *musica ficta* would be masked. The reader should also note the symmetrical appearance of the F♯ deviations, a reflection of the fact that the tail-segment arrays reach both forwards and backwards from their central notes.

For music whose pitch-class content remains so stable, a stretched ICV provides only limited information over and above what could be gleaned from an overall MICV representing the whole work. In Fig. 4b, such an MICV has been calculated as an arithmetic mean of the separate MICVs for each bar division within the Eulenburg edition. It is immediately evident that the MICV for the complete Kyrie faithfully mirrors the familiar ICV of the diatonic heptachord 7–35, that is [254361]. In this case, the stretched ICV is plainly not essential for drawing out what is characteristic about the interval-class content of Palestrina's music. Before delving into applications of the stretched ICV that appear more interesting from a music-analytical perspective, it is therefore useful to take a brief look at the degree of rough stylistic categorisation which can already be assembled from this global level of description with reference to a number of appropriate tools as outlined below.

The Bias Vector

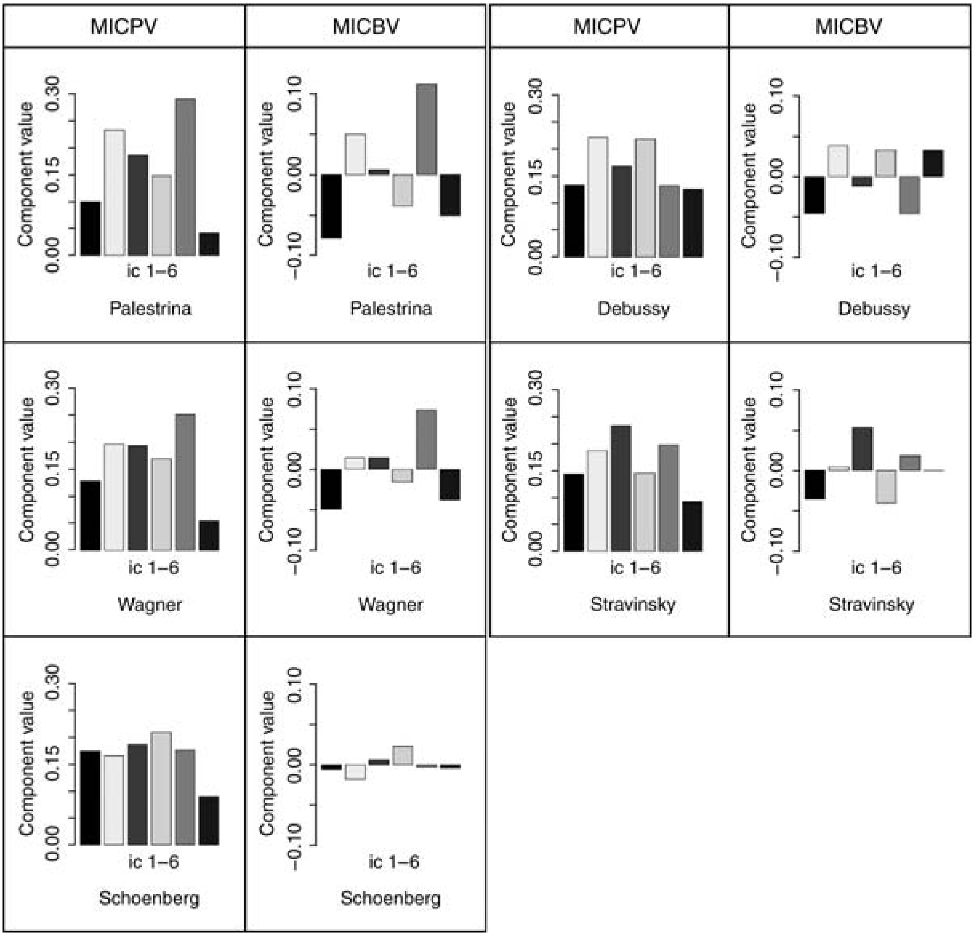
The mean interval-class vector (MICV) is a simple yet effective instrument in itself, but as a general-purpose tool we prefer its somewhat more elaborate by-product. The first step in further refining the MICV is simply to scale the MICV entries as percentages, which provides for comparability of results despite changes in set cardinality. This gives us the *mean interval-class percentage vector*, or MICPV.[8] In the context of the tail-segment-array method, the MICPV is calculated simply by converting the MICV for each bar to percentages.[9] The MICPV for the *Missa Papae Marcelli* Kyrie, for example, is [0.097, 0.237, 0.190, 0.143, 0.284, 0.049]. Although this may already provide us with some useful information, the percentage vector may strike the analyst as somewhat difficult to interpret in more complex music. Let us say, for example, that we are applying a hexachordal tail segmentation and wish to compute overall measures of interval-class content for a complete musical work. Both the MICV and its scaled version the MICPV still manifest a specific problem: in certain musical contexts it may be difficult to judge whether there is a positive or negative bias with respect to some of the constituent interval classes.

Consider, for example, the Act I Prelude to Wagner's *Parsifal*, an example we will explore more extensively later on. According to an analysis by hexachordal tail-segment arrays, the MICPV for the whole overture turns out to be [0.130, 0.197, 0.195, 0.170, 0.253, 0.055]. Such a percentage vector should obviously provide answers to questions concerning the relative emphasis acquired by the different interval classes on the surface level of the composition. But some relevant questions do not seem to have self-evident answers. For example, is ic 4 emphasised or de-emphasised in this instance? In order to provide some relevant guidance, the MICPV values must be judged against the whole set of possibilities provided by a tone system with twelve distinct pitch classes. The clearest way to establish a solution is to consider the ways in which the observed MICPV values differ from values corresponding to some random selection of hexachords. We thus first postulate an average MICPV for all T_n-type, similar-sized set classes of our tone system (a system with twelve pitch classes, regardless of temperament). For the set of all T_n-type set classes of cardinality 6, we get the MICPV [0.180, 0.182, 0.180, 0.185, 0.180, 0.093]. Second, we construe the observed values – say, the values within the Prelude – as deviations from this imaginary 'neutral' situation. The differences between the observed values and such 'neutral' values now constitute what we call the *mean interval-class bias vector* (MICBV).[10]

Fig. 5 presents the MICBV of the Prelude along with bias vectors for a sample of four other compositions representing diverse yet well-known compositional styles. In order to make the figures comparable, all of the bias vectors have been based on hexachordal tail segmentations. The bias vectors can be read as indicating the positive and negative biases with respect to the six interval classes in the compositions in question. In other words, the MICBVs show the extent to which each piece deviates from an imaginary situation in which all of the 80 T_n-type hexachords might have been employed with equal probability. To be sure, this kind of notional construct represents an abstraction that has little to do with actual compositional method, least of all Palestrina's. But this is exactly the point: each separate stylistic profile can be seen to represent a subset of possibilities within a pitch system which, theoretically at least, provides twelve more or less equal steps within the octave and thus also 80 T_n-type hexachords. The system as we know it today surely includes Palestrina as one of its subsets (how else could we know that he favoured what we call set class 7–35?), and thus it provides a backdrop against which the average intervallic characteristics of his music can be brought into sharper relief. (Of course, Palestrina is included here principally to give an intuitively accessible point of reference for other, more significant readings.)

The bias vectors interpret the average intervallic features of a piece as either positive or negative biases in relation to an imaginary neutral situation. The interpretative advantage which the bias vector displays with respect to the percentage vector should be clear from Fig. 5. Through the MICBV, some general stylistic features are immediately made more obvious. In Debussy's 'Ce qu'a vu le vent d'ouest', for instance, the correlation between the interval-class content and the whole-tone scale is brought to the fore: in other words, the three zeros in the ICV of the whole-tone scale [060603] correspond to three negative biases in this particular MICBV. More significantly, it should be noted

Fig. 5 Mean interval-class percentage vectors and mean interval-class bias vectors for: Palestrina, *Missa Papae Marcelli*, Kyrie; Wagner, Act I Prelude to *Parsifal*; Schoenberg, *Pierrot lunaire*, Op. 21 No. 5, 'Valse de Chopin'; Debussy, *Préludes*, Book I, 'Ce qu'a vu le vent d'ouest'; and Stravinsky, *Symphony of Psalms*/i (the tail-segmentation cardinality is 6)



that the percentage vector of the 80 T_n-type hexachords taken as a model of the imaginary average content provides a value for ic 6 which is only about half that of the other interval classes. Whereas the percentage vector of the Debussy example gives somewhat similar values for interval classes 1, 5 and 6, the bias vector shows that tritones (and their octave compounds) have, in fact, been favoured in relation to the two other interval classes mentioned. Furthermore, ic 3, which exhibits the third-largest value in the Debussy percentage vector, is shown as receiving a negative bias from the bias vector. In the case of the

Prelude to *Parsifal*, we may arrive at a similar conclusion in respect of ic 4: with regard to the possibilities of our tone system, ic 4 is negatively biased.

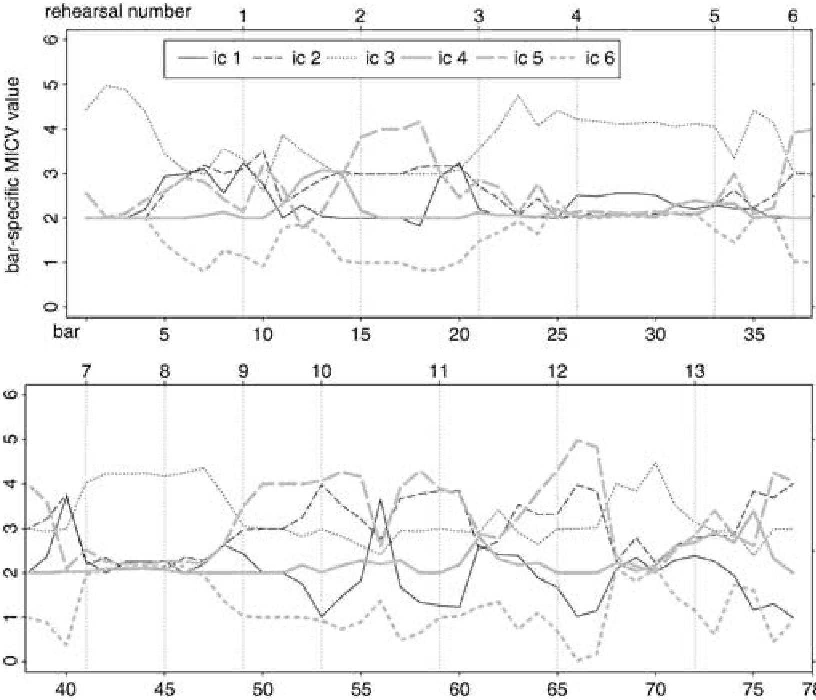
In short, the difference between the MICPV and the MICBV is the difference between a description of the average contents of the piece and a description of how these average contents deviate from those things which, say, a computer might be capable of generating with no musical purpose and no knowledge beyond the structure of the tone system. Much of what is shown in Fig. 5 is easily grasped in terms of an elementary knowledge of the history of musical composition. The fully diatonic interval-class hierarchy characteristic of Palestrina is somewhat levelled in Wagner's case, although the familiar hierarchical relationships of the interval classes do remain intact in the percentage vector (compare [254361], the ICV of set class 7–35). From the perspective of interval-class bias, the Schoenberg example upsets this hierarchy: the bias vector is now flat, with the exceptions of a positive bias for ic 4 and a negative one for ic 2 (suggesting that the positive ic 4 bias is not due to stacked major seconds). Whereas in this comparison Schoenberg's compositional technique brings the historical process of interval-class democratisation to a close, so to speak, Debussy and Stravinsky noticeably deviate from the same tendency on account of their pursuit of alternative kinds of intervallic bias. Borrowing a term from the statistical analysis of linguistic texts (Dolezel 1969, p. 20), we might dub these individual biases the 'subjective style characteristics' typical of each composer. Rough stylistic differentiations of this kind become possible when an average measure such as the MICBV is used in conjunction with tail-segment arrays.

The Stretched ICV as an Analytical Tool

Despite the promising outlook of Fig. 5, the average measures developed in the previous section are of limited value as music-analytical tools: they lose much of their descriptive power in music that is less internally consistent than the Palestrina Kyrie. When pitch organisation becomes more varied, it is advisable to turn back to the stretched ICV. We will confine our discussion of this point to one analytical example taken from the first movement of Stravinsky's *Symphony of Psalms*. Our stretched-ICV analysis of the complete movement is shown in Fig. 6. The analysis has been generated by the same procedure as that which produced the Palestrina analysis in Fig. 4, with the exception that we have switched to a hexachordal tail segmentation. For easier reference to the score, bar numbers are shown on the horizontal axis, and rehearsal numbers are indicated as vertical lines. As in the Palestrina example, the vertical scale on the left-hand side of the figure gives the MICV values; for now, the other vertical scale shown on the right-hand side may be ignored.

The graph illustrates very effectively the striking contrasts of interval-class content which characterise Stravinsky's writing in this opening movement. Both the broadly diatonic and octatonic areas are clearly discernible. The strict diatonic hierarchy of interval classes which Stravinsky has reserved for the

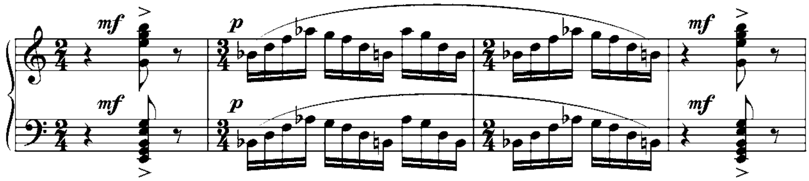
Fig. 6 A stretched ICV for Stravinsky, *Symphony of Psalms*/i, derived using a hexachordal tail segmentation



dynamic climax of the movement at rehearsal number 12 is already familiar from our earlier Palestrina example. Note too that the maximal hierarchisation of interval classes which is characteristic of set class 7–35 is also a feature of the ICV [143250] of the diatonic heptachord 6–32, which is *de facto* the set class Stravinsky uses here.

Apart from this passage, there are further points at which hexachordal tail segmentation reveals the presence of additional hexachordal subsets drawn from the diatonic scale. For instance, the Locrian and Aeolian hexachords, 6–Z25A [0, 1, 3, 5, 6, 8] and 6–Z25B [0, 2, 3, 5, 7, 8], are disclosed by their common ICV [233241] at rehearsal numbers 2, 6 and 9. Passages which are strictly diatonic, in the sense that they conform to 7–35, are also present, most markedly at rehearsal number 10. In addition to 6–32 (which exemplifies the diatonic interval-class hierarchy), tail-segment arrays also include other hexachordal subsets of 7–35. When their ICVs are averaged, the characteristic interval-class hierarchy is retained, although the curves do not exactly reflect the relative density of the six entries in the familiar heptachordal ICV [254361]. The reader may notice how sensitively the analysis succeeds in revealing anomalous events within an otherwise diatonic environment: the

Ex. 2 Stravinsky, *Symphony of Psalms*/i, bars 1–4



sudden rise of the ic 1 curve in bar 56 reflects an interruption of diatonicism by a compound of ten different pitch classes.

One of the broadly diatonic sections of the piece is also significant in that it demonstrates an important limitation in the use of tail-segment arrays. The music at rehearsal number 9, bars 50–51, is made up exclusively of notes belonging to the pentachord {C, D, E, A, B}. Consequently, each of the tail segments defined by the notes in these two bars is required to reach back to include a sixth pitch class, F, found in bar 49. Retrieving this pitch class completes the Aeolian hexachord for each of the tail segments formed for these notes; hence the graph mistakenly gives the impression of a hexachord with the ICV [233241]. Thus we learn that the stretched ICV at a given location is accurate only to the extent that the number of distinct pitch classes used in the music does not fall below the cardinality of the tail segmentation. Using a pentachordal tail segmentation would solve the problem in bars 50–51, but would elsewhere hinder the direct identification of hexachords on the basis of the stretched ICV.

Another conspicuous feature of the *Symphony of Psalms* analysis is its manifestation of octatonic materials at rehearsal numbers 4, 7 and the latter part of 12. At numbers 4 and 7 especially, Stravinsky exhausts the complete octatonic scale within each bar. With its exclusive emphasis of ic 3 in these areas, the stretched ICV reflects the contour of the octatonic ICV [448444]. Following the hexachordal tail segmentation, the graph actually approximates [225222], the ICV for set classes 6–27A and 6–27B, both of which are subsets of the octatonic scale. Likewise, the beginning of the work, reproduced here as Ex. 2, consists of octatonic subsets: the combined B♭ and G dominant seventh chords in bars 2–3 comprise 6–27B [0, 2, 3, 5, 6, 9], which taken together with the initial E minor triad produces 7–31B [0, 2, 3, 5, 6, 8, 9], a further subset of the octatonic scale. Notice how the tail-segmentation method accommodates such details: in bars 2–3, Stravinsky's hexachords are, of course, accurately reflected in the graph, but the E minor triad in bar 1 is overlooked. The first hexachordal tail segment in the work is completed only with the arpeggiated pitches of a B♭ major triad in bar 2, which produces 6–Z50 [0, 1, 4, 6, 7, 9], another subset of the octatonic scale and the initial set class depicted in the graph. In a pervasively octatonic context, the method's built-in 'triadic blindness' ought not to be regarded as a negative feature; after all, the E minor triad does indeed belong to the same form of 8–28 as the following material, and the curves similarly describe a clear subset of the octatonic scale. The problem may become more acute in music which contains more genuinely triadic passages, an issue which will be revisited at greater length in the following section of this study.

A stretched-ICV analysis delineates the basic proportions of a musical work only as far as its interval-class contents are concerned. With respect to the *Symphony of Psalms*, our method remains neutral with respect to, for instance, Wilfrid Mellers's opinion that the basic polarity of the work might be located between the keys of E♭ and C (whose dominants are juxtaposed at the outset; Mellers 1971). Neither does it help to evaluate Joseph Straus's more considered assertion that the work is organised around the tonal axis E–G–B–D (Straus 1982). Such claims reach beyond our analytical method in two respects: they require both a determinate segmentation and specific attention to the transpositional levels of the pitch-class sets. What our analysis does, conversely, is to highlight a further polarity which arguably carries still greater consequences for the way that the work sounds on the local level: the dichotomy between broadly diatonic and broadly octatonic materials. Indeed, the greatest average change in the values of the six curves in our Stravinsky analysis is situated immediately following the climax, between bars 67 and 68. Dramatic changes in local interval-class content such as this one between an explicitly diatonic and an unmistakably octatonic context are, indeed, visually obvious from the graph and indicate possible points of formal division. This suggests that our detector graphs, although based on a rather unorthodox and mechanical approach to segmentation in the first place, may themselves be used as a basis for more musically informed segmentations – that is, segmentations in the usual sense of the term.

IV

Comparison-Set Analysis

The analysis in Fig. 6 was generated by connecting the MICVs, which were first determined by averaging the lower-level MICVs of the tail-segment arrays, from bar to bar. We then sought to conceptualise the analysis in terms of 'stretching the ICV'. There is another useful way of looking at the same figure: our analysis also indicates the degree to which Stravinsky's composition answers to certain intervallic preferences. Applying the idea of 'weighting vectors' presented by Steven Block and Jack Douthett (1994), we may think of the lowest line in Fig. 6 as the result of successively taking a dot product between the ICVs of each of the consecutive hexachords and the weighting vector [000001]. More precisely, for each of the MICVs which describe the tail-segment arrays of the individual notes, the sixth MICV entry has been multiplied by 1, all of the other MICV entries having been multiplied by 0 and the products summed up, after which the mean of the resulting values for each bar is computed. Each of the six lines thus corresponds to a simple ‘set’ of interval-class preferences – that is, a positive preference for one interval class and a zero preference for the others. This alternative conceptualisation in terms of weighting vectors provides the pathway from stretched ICVs to a more sophisticated general approach in which each temporal station in a piece of music is judged against a reference point of some sort. In this approach, which we call *comparison-set analysis*, the temporal development of any piece is traced by comparing the intervallic properties of each successive pitch-class set with a comparison set – that is, a chosen set of intervallic properties. The sets of properties may be chosen either for their intuitive familiarity or in view of independent evidence that the properties in question might possess some analytical pertinence. The approach may easily be varied by choosing different comparison sets as well as by applying different functions in order to compare the properties in the comparison set with the properties of the pitch-class sets on the music’s surface.

It is important to remember that the motivation supporting the exploration of tail segments was the prospect of applying some function *f* in order to measure a given property of the pitch-class sets on the surface level of a composition. Tail-segment arrays were used to assign continuous local averages of these measurements to specific notes. Comparison-set analysis represents a special case of this technique: the function *f* now has one extra parameter, the comparison set. We prefer to think of this as a form of ‘referential contextualisation’: the successive events in a musical work are depicted with constant reference to some set of musical properties which may either be more familiar to us or intuitively more accessible than the piece in all its complexity. Local events are thus examined as if they were being observed through a filter which enables the observer to see only objects of a certain colour. In terms of the detector metaphor, to choose a comparison set is to programme the apparatus to be sensitive to an exclusive set of properties. The main difference with respect to our initial demonstration of ‘non-evenness’ is that the measurements are thus invested with greater musical meaning.

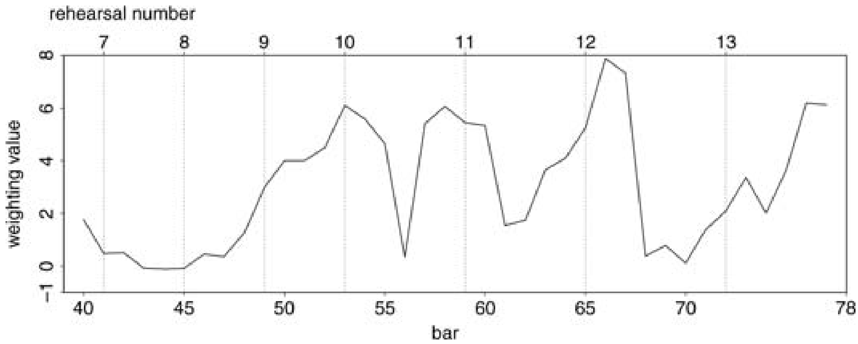
Using Weighting Vectors with Tail-Segment Arrays

Before presenting a more sophisticated version of comparison-set analysis, we will elaborate further on the use of weighting vectors in connection with tail-segment arrays. First, an important methodological observation: in different applications of the weighting-vector idea, it is advantageous to keep the sum of the preferences at 0. This prevents the higher-cardinality sets from acquiring disproportionately large values when pitch-class sets of different cardinalities are involved (as noted in Huovinen 2002, p. 341). Of course, this will not affect the result with tail segments of an unchanging cardinality, but it does permit the method to be generalised in order to accommodate additional situations in which alternative segmentation procedures may be applied. For the purpose of drawing a stretched ICV, the individual weighting vector for ic 2, for instance, would be modified from [010000] to [–0.2, 1, –0.2, –0.2, –0.2, –0.2]. Here ic 2 is once again given a positive weighting of 1, whereas the negative weights are divided evenly among the rest of the pitch classes and the whole set of weights sums to 0. For each interval class *i*, the *i*th entry in the appropriate weighting vector will similarly have the value 1, while the rest of the values will be –0.2. Such a modification affects only the scale of the diagram, even though each interval class has now been taken more substantially into account in the determination of each separate line. This can be verified by referring once again to the *Symphony of Psalms* analysis represented in Fig. 6: the vertical scale included on the right-hand side gives the preference values produced when employing weighting vectors of the above-described type. While the original scale shown on the left indicates the MICV values, thereby allowing direct inferences to be drawn in respect of the constituent pitch-class sets, the new scale complements this by facilitating judgements concerning positive versus negative intervallic bias (compare the MICBV of the previous section).

This change in perspective might appear cosmetic, but may be deemed important in so far as it creates a conceptual bridge from the stretched ICV to the full-blown comparison-set analysis developed below. In order to connect the two incrementally, we may next try using the weighting vector as a detector for more complex types of composite preferences, in a spirit which is perhaps closer to the original intent of Block and Douthett. Let us say, for instance, that we wish to achieve a composite measure of diatonicism which would detect the diatonic tendencies in musical units such as the first movement of the *Symphony of Psalms*. Intuitively, we might first construct a weighting vector such as [–1, 1, 0, 0, 1, –1], in which the broadly 'diatonic' ics 2 and 5 are positively weighted while the more 'chromatic' ics 1 and 6 acquire negative weights. The next step is to apply this set of intervallic preferences to the work as an elementary detector. For each ICV relating to every tail segment in the composition, the first ICV entry is multiplied by –1, the second by 1 and so on. The resulting values are then summed into a single preference value for the tail segments. The rest of the procedure is familiar: according to equation (1), each note in the composition is assigned a numerical value by averaging the preference values throughout its tail-segment array. The resulting values are once again averaged within each bar for improved legibility. Fig. 7 shows the results of applying this tentative 'diatonic detector' to the latter half (bars 40–78) of this opening movement.

Much of the information generated by the stretched-ICV analysis involving the relative number of broadly diatonic features appears to be reflected in the weighting vector analysis, here compressed into a single curve. To be sure, the latter analysis does not allow identification of the constituent pitch-class sets in the manner of the stretched ICV. However, the momentary decline in relative diatonicism in bar 56, the ultimate diatonic peak at the culmination of the movement in bars 65–67 and the rounding off of the composition by a relatively stable closing cadence are among the most clearly apparent features.

Fig. 7 An evaluation of Stravinsky, *Symphony of Psalms*/i, bars 40–78, using the weighting vector [–1, 1, 0, 0, 1, –1] (the tail-segmentation cardinality is 6)



Note too that ic 3, which assumed considerable significance in the stretched-ICV analysis, is effectively discounted by the chosen weighting vector. A metal detector is not sensitive to wood; in the same way, our intuitively constructed 'diatonic detector' is blind to Stravinsky's octatonicism as such. In the octatonic passages at rehearsal numbers 7 and 12, the curve does not, for instance, seem to convey any kind of negative signal: instead it just falls close to zero, meaning that no materials of the designated type have been detected (throughout the entire movement the lowest point of the curve occurs at bar 29, with the value –0.44). However, the range of values which the weighting vector [–1, 1, 0, 0, 1, –1] is capable of producing within the set of 80 T_n-type hexachords ranges from –1 to 8, and thus the lowest troughs in the curve might perhaps be said to represent 'non-diatonicism' in at least a relative sense.

The weighting vector [–1, 1, 0, 0, 1, –1] was selected to fit the intuitive idea of 'diatonic intervals at the expense of chromatic intervals'. The analysis that results from this particular combination of weights is reminiscent of Paul Hindemith's graphic demonstrations of 'harmonic fluctuation' (Hindemith 1940). One may also adopt a more systematic viewpoint for the selection of weighting vectors. In analysing post-tonal music, it is fair to say that the hexachordal tail segments usually include examples of most of the 80 different T_n-type hexachords. In such circumstances, therefore, it is appropriate to adopt weighting vectors which maximise the range or, preferably, the dispersion of values that are possible with reference to this total set of hexachords. For example, even though the Stravinsky example does not show much differentiation with respect to ic 4, it does not follow that weighting vectors which emphasise this interval class are generally incapable of showing variation in music that is segmented into hexachords. This may be confirmed by taking the simple weighting vectors of Fig. 6 (in which one entry has the value 1 while the others are set to –0.2) and processing the set of 80 T_n-type hexachords with them.

Taking the dot products between each of the simple weighting vectors and the 80 T_n-type hexachords in turn, we see that some of the vectors yield a greater dispersal of resulting preference values within the pool of T_n-type hexachords than others. In fact, the weighting vector which yields the greatest standard deviation of values (1.16) is the one which emphasises ic 4, that is, the vector [–0.2, –0.2, –0.2, 1, –0.2, –0.2].[11] Such a systematic approach to selection of the weighting vector is also readily applicable to compound weighting vectors, including our tentative 'diatonic' vector. Despite their somewhat artificial appearance, weighting vectors do have an unequivocal advantage: used as detectors, they measure precisely the properties which need to be accounted for while remaining insensitive to others.

Similarity Measures in Comparison-Set Analysis

Despite the apparent success of the weighting-vector method in detecting Stravinsky's diatonicism, for example, the interpretation of the curve in Fig. 7 as an indication of relative 'diatonicism' remains a matter of intuition. Put another way, how might we otherwise be sure that the features encoded in the weighting vector are indeed descriptive of what we commonly understand by the term 'diatonicism'? Not surprisingly, another fruitful approach involves restricting ourselves to dealing only with those sets of intervallic properties which may coexist in a real pitch-class set. One means of realising this principle might entail substituting the interval-class bias vectors of particular set classes for the intuitive weighting vectors explored in the course of the previous section. For instance, the bias vector for 7–35, the diatonic scale, is [–0.087, 0.056, 0.009, –0.039, 0.104, –0.043], which offers a more accurate model of 'diatonicism' that may be used in conjunction with weighting vectors in comparison-set analysis. For reasons of space, we will forgo a detailed discussion of these applications; in any case, the analytical procedure itself would be analogous to that used in the previous example. (As a matter of fact, the resulting curve also turns out to be all but identical to that shown in Fig. 7.) Instead, we will now turn to an alternative and methodologically fortuitous way of incorporating real set classes as comparison sets.

When a set class is employed as the comparison set, a wide range of possible comparison functions becomes available, namely the body of so-called similarity measures previously summarised by Castrén (1994), Richard Hermann (1994) and Tuire Kuusi (2001). In terms of standard pitch-class set-theoretic analysis, similarity measures tend to be conceptualised as tools for judging the abstract similarity between any given pair of pitch-class sets that appears within a composition. A familiar analytical intuition supporting this usage is that one should strive to unravel the hidden connections which account for a work's structural coherence. Quite naturally, perhaps, analytical enquiry has chosen to concentrate on the similarity relationships between pairs of set classes that occur successively. In evaluating such relationships, however, Thomas Demske has for instance observed that 'despite great effort in attending', it is difficult to discern any tangible correspondence between concrete musical events and the symbolic graph of chord-to-chord similarity measurements (Demske 1995, p. 15). From the point of view expressed in the opening section of this article, this should not seem altogether surprising: any quantitative measurement actually becomes meaningful only in relation to other similar measurements where what is being measured remains constant. In the case of applications such as Demske's, the burden of interpretation which arises from the sheer number of possible set-class pairs (and thus the consequent range of results) has apparently been too weighty to inspire a widespread commitment to the detailed tracing of similarity measures.[12] Such problems are alleviated in comparison-set analysis, which discards the idea of juxtaposing similarity measurements between distinct pairs of pitch-class sets within a specific musical context.

To reiterate, in comparison-set analysis, one of the two set classes that are being compared is held constant throughout the analysis. This 'comparison set' is taken as an unchanging standard against which each successive pitch-class set of the piece is evaluated. The basic idea has in fact been explored in various analytical and theoretical contexts, notably by Eric J. Isaacson (1992), who linked it to the topic of referential sets as a means of comparing the set classes associated with differing gestural elements. Other analysts too have maintained the same basic orientation towards 'compositional grammar'. Thus Castrén (1994) used his own similarity measure to define so-called RECREL regions, or areas within which the comparisons between a constant 'nexus set' and the set classes of any composition reach a certain level of similarity. Similarly Christopher Ariza (2000) has employed David Lewin's REL measurement as a means of comparing Stravinsky's successive vertical harmonies to a 'reference set'. In Ariza's study, the reference sets were made to corresponded to either the first or the last pitch-class sets of the harmonic succession itself, or, alternatively, to Stravinsky's horizontal row segments.

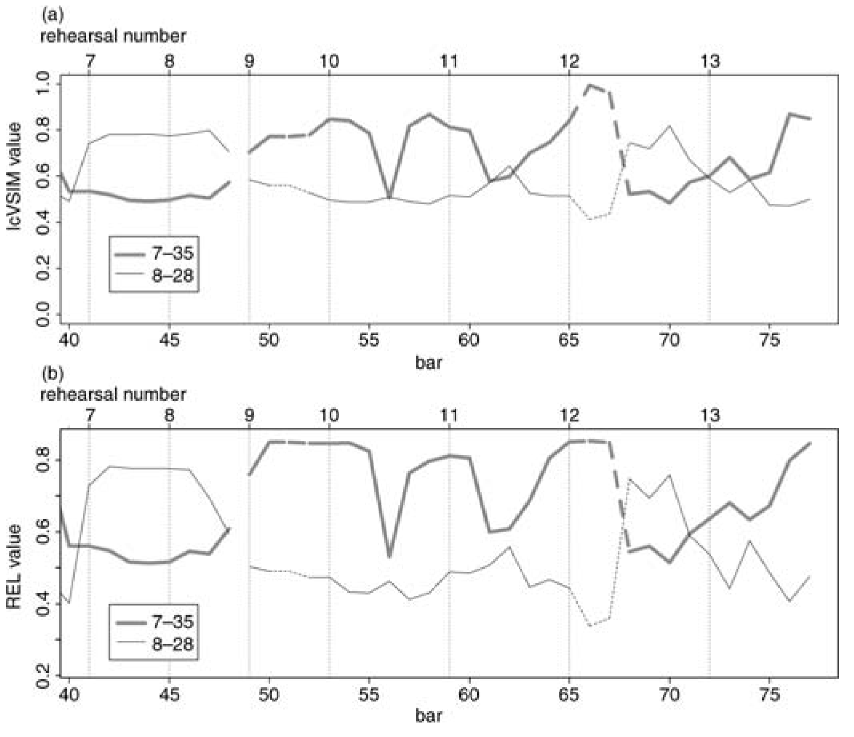
Using the term 'reference set class', Erkki Huovinen (2002) applied a similar idea in the context of experimental music psychology in order to characterise various (more or less unfamiliar) collections or 'domains' of set classes which had been chosen as the basis for experimental stimulus materials. In the absence of a musical work to be analysed, Huovinen employed a more top-down approach, in which the comparison set's purpose was to provide an intuitively accessible reference point for the understanding of a given set of pitch-class sets. The comparison sets themselves – including the diatonic heptad 7–35 and chromatic clusters such as 5–1 and 6–1 – were selected on the basis of their supposed familiarity. While the use of comparison sets in the present music-analytical context is otherwise reminiscent of the work of Isaacson, Castrén and Ariza, the rationale behind their selection is rather closer to that suggested by Huovinen's study. That is, we prefer to promote intuitive access to the materials being analysed by utilising familiar sets as reference points, even if these sometimes remain external to the piece under examination.

To proceed with comparison-set analysis proper, we may now substitute a similarity measure for function *f* in equation (1), embedding a comparison set of our choice into the function as a constant against which the successive tail segments are evaluated. The results can be illustrated with reference to the latter part of the first movement of the *Symphony of Psalms*, beginning at bar 40. Bearing in mind the earlier stretched-ICV analysis shown in Fig. 6, we have sound reasons for choosing two different comparison sets, namely the diatonic scale (7–35) and the octatonic scale (8–28). In order to illustrate the effects of selecting the similarity measure, we will compare two different measures: Isaacson's IcVSIM (1990; 1992) and Lewin's REL (1979–80b). The former effects a neat transition from our earlier demonstration since it is entirely based on the ICV. Given two ICVs, IcVSIM represents the standard deviation in the differences between the corresponding entries of the ICVs.[13] In other words, IcVSIM measures the variance among the differences by which two set classes differ from each other in terms of the six interval classes. For ease of comparison with the previous figures, the scale of the IcVSIM values has here been inverted in order to produce a measure of similarity rather than one of difference (which IcVSIM in actuality is). In addition, the range of possible IcVSIM values has been scaled between 0 and 1 in order to facilitate further interpretation. The second measure which is applied here, Lewin's REL, is a so-called total measure, which means that it takes the subset content of pitch-class sets into account in determining their similarity. Unlike IcVSIM, REL can also differentiate between the two inversional (A and B) forms of set classes.[14]

Before examining Stravinsky's *Symphony of Psalms* any further, the general appearance of Figs. 8a and b prompts the discussion of two methodological issues. First, some preliminary observations concerning the choice of similarity measures for comparison-set analyses can already be made on the basis of Fig. 8. Of the two measures applied here, Isaacson's IcVSIM has the advantage in that it is based on an established statistical procedure – the standard deviation – and is thus relatively easy to interpret. The near-identical appearance of the two comparison curves in Figs. 8a and b suggests that even a rather simple and straightforward measure such as Isaacson's IcVSIM may produce results which are not rendered significantly clearer by any more complex measures. In fact, this is not so surprising in the light of Ian Quinn's (2001) assertion that the results of IcVSIM and REL may produce rather similar categorisations among set classes, even though one of them is based on ICVs and the other on entire subset-class content. Following Quinn, we are inclined to think that the two well-known similarity measures in our example 'speak with a single extensional voice, regardless of what they profess to measure' (Quinn 2001, p. 155).

Accordingly, we have presented the comparison between Figs. 8a and b not so much to prolong the seemingly endless debate concerning the relative merits of and formal differences between various similarity measures as to emphasise that it may simply be more fruitful to find new ways of applying these measures. In light of Fig. 8, the issue concerning the relative merits of ICV-based

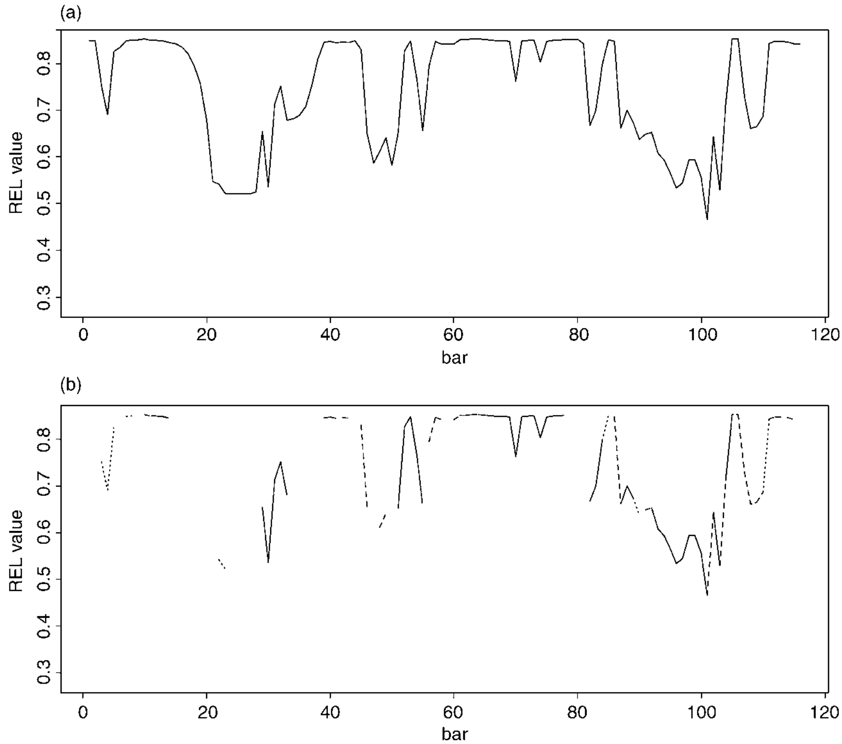
Fig. 8 Diatonic and octatonic measurements for Stravinsky, *Symphony of Psalms*/i, bars 40–78 (the tail-segmentation cardinality is 6)



measures and 'total measures' seems rather marginal with regard to the practical value of quantitative musical analyses. For the remainder of our discussion, we will use Lewin's REL, which has been more favourably judged by both Castrén and Kuusi. A major reason for preferring REL over IcVSIM is that it differentiates between inversional forms of Fortean set classes. In any case, comparison-set analysis is also capable of accommodating many of the other well-known similarity measures.

The second methodological issue we are obliged to address concerns the extent to which the comparison curves can in fact be relied upon as sources of music-analytical information. As noted in connection with the stretched ICV for the first movement of the *Symphony of Psalms* (see again Fig. 6), the hexachordal tail segments had occasionally to be extended in order to absorb a final pitch class. As a consequence, the curves in the analytical graph became somewhat skewed at these junctures. In the comparison-set analyses presented here, we have applied a practical solution to the problem by indicating such inaccuracies in the appearance of the curve itself. If the number of distinct pitch classes in a given bar falls short by one pitch class with respect to the

Fig. 9 Two versions of a REL(X, 7–35) analysis of Wagner, Act I Prelude to *Parsifal*, involving a hexachordal tail segmentation



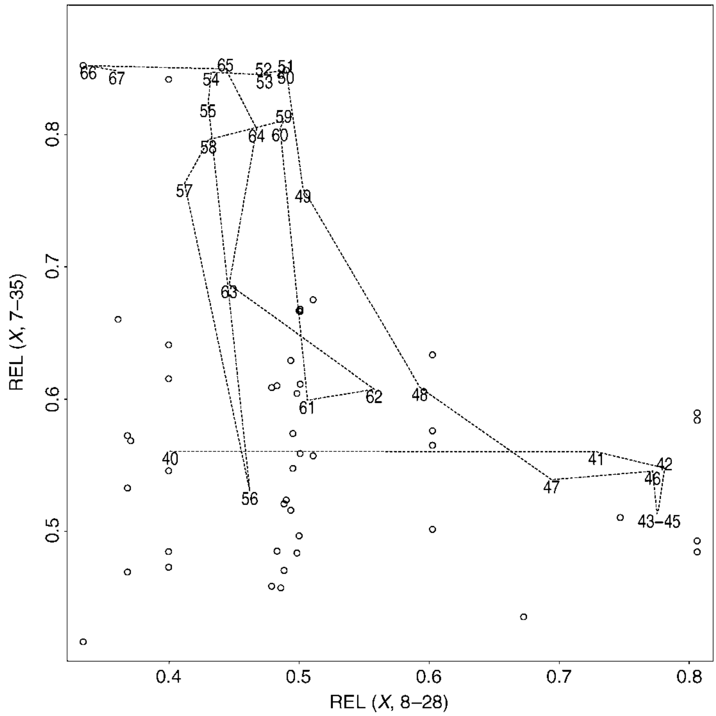
cardinality chosen for the tail segmentation, the curve is marked by a broken line from this point. Similarly, if the cardinality falls short by two pitch classes, the curve is then drawn as a dotted line. For still lower within-bar cardinalities, the curve is simply truncated. The appearance of the curve thus indicates the accuracy of the analysis. This *cardinality correction* will also be taken into account in all other calculations based on the comparison-set analysis: each bar in which the local pitch-class cardinality falls below the tail-segmentation cardinality is simply omitted from any further interpretation.[15]

The problem engendered by smaller cardinalities becomes somewhat more ambiguous in tonal music which relies on extended triadic prolongation. Consider again the Act I Prelude to *Parsifal*, which forms the basis of a comparison-set analysis using REL(X, 7–35) in Fig. 9a, together with a hexachordal tail segmentation that nonetheless lacks the cardinality correction. The diatonic areas throughout the Prelude are precisely delineated, while the more complicated harmonic textures towards the close are clearly indicated by the descending diatonic curve. Nevertheless, the graph shows a distinct anomaly: bars 25–28 and 33–38 should consist of nothing but the constituent pitches of a C minor triad, yet the curve does not seem to reflect such evident simplicity. Once again, this deviation arises from the fact that the tail segments for the notes within the C minor areas are made to extend backwards over several bars in order to reach the remaining pitch classes required to complete an entire hexachord. For all of the notes within such a triadic area, the tail segments consequently represent a single set class which is partly determined by whatever Wagner has written immediately prior to the passage in question. Thus, whereas we can be sure of the relative diatonicism of certain segments, genuinely non-diatonic passages cannot be distinguished from their purely triadic counterparts. As can be seen in Fig. 9b, the cardinality correction remedies the situation by indicating the bars in which the curve appears misleading. Admittedly the problem could be alleviated somewhat by choosing a still lower tail-segment cardinality. All the same, far from being a matter of mere practicality, the cardinality correction highlights an important aspect of the tail-segmentation method and hence of comparison-set analysis in general: our tools are appropriate only where the musical units at the bar-to-bar level equal or exceed the tail segments in set-class cardinality.[16] Put simply, the detectors only distinguish objects of a certain minimum size, although this minimum size may in turn be calibrated through the choice of tail-segment cardinality.

With this information in mind, we may take a closer look at the Stravinsky analysis in Fig. 8, which clearly underscores the contrasting alternation of broadly diatonic and broadly octatonic material. It is essential to note, however, that 'diatonicism' and 'octatonicism' are in no sense mutually exclusive properties. In fact, they can be thought of as independent of each other to the extent that it is theoretically possible for one of the curves to move up or down while the other remains stationary. In the Stravinsky analysis, this correspondence can be seen at bar 56, which was previously remarked upon in connection with the stretched ICV: here there is a drastic decrease in relative diatonicism but no parallel octatonic effect. Such relationships between comparison curves are perhaps better demonstrated using a two-dimensional *comparison-set space* in which the local comparison values are positioned according to two different comparison-set measurements (as noted in Huovinen 2002). In the case of the *Symphony of Psalms*, we might therefore use a space whose x-axis and y-axis are determined by the measurements REL(X, 8–28) and REL(X, 7–35) respectively.

The best way to visualise the trajectory of Stravinsky's pitch material within a $\mathrm{REL}_{8\text{–}28/7\text{–}35}$ space would arguably be in animated form. For Fig. 10 we have simply drawn a representative phase of the trajectory, averaging the values for each bar as before. The final measurements for the movement have been left out in order to avoid undue congestion. Notice the initial transition from bar 40 to bar 41, which is a typical case of non-octatonic/octatonic movement without a change in relative diatonicism. Corresponding non-diatonic/diatonic moves without an octatonic effect are seen between bars 55 and 57, as observed

Fig. 10 Stravinsky, *Symphony of Psalms*/i, bars 40–67, represented as a bar-to-bar trajectory within a two-dimensional comparison-set space in which the horizontal dimension corresponds to REL(*X*, 8–28) and its vertical equivalent to REL(*X*, 7–35) (the tail-segmentation cardinality is 6 and the small white circles indicate possible hexachord locations)



previously. The lateral movement at the top of the figure corresponds to more or less exact matches with the hexachordal subsets of the diatonic scale; according to REL(*X*, 8–28), these are variously distanced from the octatonic comparison set. For instance, at bar 66, no less than 96.7 per cent of the 1,107 tail segment instances involved represent set class 6–32 [0, 2, 4, 5, 7, 9], and thus the trajectory in two-dimensional space comes very close to the literal appearance of 6–32, thereby marking the pure diatonicism that Stravinsky reserves for the dynamic climax of the movement. At bar 50, the curve coincides with the location of another diatonic subset, 6–Z25B [0, 2, 3, 5, 7, 8], which is evaluated as being slightly more akin to the octatonic comparison set 8–28.[17]

Before moving on, we should note that the ranges of values returned by the similarity measures depend on the cardinality of the sets to be compared (as noted in Isaacson 1992). For this reason, our tail-segmentation method should not be seen merely as an opportunistic solution employed for want of a better method of 'musical' segmentation. On the contrary: a mechanical procedure for selecting equal-sized sets is in fact required in order for our chosen detectors to function properly. However, as we have already seen, when such a 'blind' tail segmentation is used in conjunction with an appropriate analytical function, the results can lead to propositions concerning 'musical' segmentations as well (as illustrated in Fig. 6). In this sense, the tail-segmentation method is best understood not as a rival to traditional segmentation procedures; rather, it provides the systematic groundwork for conceivable segmental divisions.

V

Intuitive Reference Points and Set-Class Referentiality

In the previous Stravinsky example, the choice of 7–35 and 8–28 as comparison sets was governed by rather self-evident facts concerning the pitch materials employed by the composer. These choices were presented as findings deriving from the stretched ICV in Fig. 6, but similar decisions could just as easily have been based on an elementary reading of the score. What makes comparison-set analysis such a versatile tool, however, is that the ways in which comparison sets may be selected are not exhausted by conforming to a composer's most obvious choices of scalar or chordal material. In this section we will discuss two alternative perspectives, in which the choice of comparison sets is made either more intuitive than in the case of our Stravinsky analysis or, conversely, more heavily dependent on complex music-analytical hypotheses.

Diatonicism and Non-diatonicism

The starting point for the applications in this section is the fact that most of us are more or less intuitively familiar with certain set classes. This is especially true of set class 7–35: most of us probably have some intuitive knowledge of how the diatonic scale sounds which is more or less independent of any particular progression. Such knowledge correlates directly with the comprehension of what it means for the diatonic comparison curves in one of our bird's-eye views either to ascend or descend. Of course, some music theorists may insist on the strictly quantitative character of our calculations, or of pitch-class set theory in general. However, for our purposes, the meaningfulness of a 'similarity measure' such as REL requires that its values exhibit an intuitive relevance when applied as a comparison function in conjunction with a familiar comparison set.

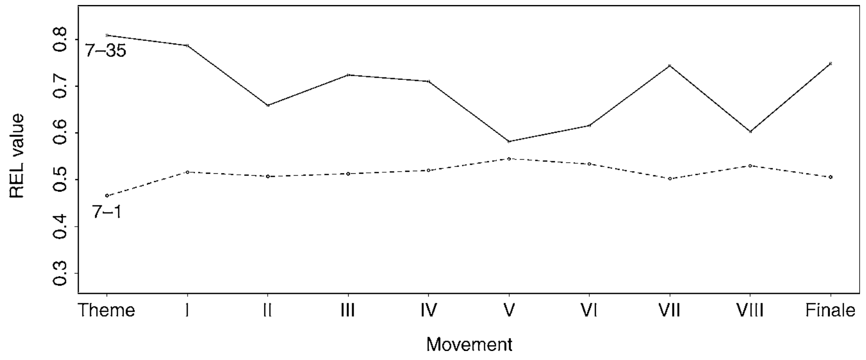
In consequence, it is reasonable to suppose that one's reading of the appropriate comparison curves will be qualitatively meaningful to the extent to which one is acquainted with the qualitative 'flavour' of certain set classes. Under this condition, comparison-set analysis may be used not only to focus on the specific manner in which composers deploy their preferred pitch materials, but also to grasp intuitively how any piece of music develops in relation to some familiar reference point. In the following analyses, we employ two such intuitively accessible set classes: 7–35 and 7–1. Among the heptads, 7–1 maximises the number of instances of ic 1 – an interval class which has been noted by Hindemith (1940) and Michael Friedmann (1990) among others as an important aural criterion for distinguishing between pitch-class sets. In order to simplify the following discussion, we will, somewhat simplistically, refer to REL(*X*, 7–35)-similarity and REL(*X*, 7–1)-similarity by the terms 'diatonicism' and 'chromaticism'. Admittedly, our particular usage deviates from the conventional connotations of these two terms. This should especially be kept in mind for our use of the term 'chromaticism'.

Let us first consider the case of diatonicism and chromaticism in a tonal composition such as Brahms's *Variations on a Theme by Haydn*, Op. 56a. The work consists of a theme, eight variations and a finale. Instead of continuous comparison curves, only average values will be considered here for each of the work's ten principal subdivisions. All were initially analysed in the customary manner using tail-segment arrays, wherein we identified a single numerical value for each note in the composition and then proceeded to average these values for each bar. The means for the principal subdivisions were subsequently calculated on the basis of such bar-specific values. In terms of levels of description, these means are thus akin to the MICV and its cognates as previously considered in the third section of this study.

As far as the question of diatonicism is concerned, the outcome of the analysis in Fig. 11 might serve as a rough schematic ground plan for the Haydn variations. As one might expect of a work of this type, the highest degree of diatonicism is associated with the initial theme, followed by the first variation. Similarly, the pastorale-like *grazioso* of the seventh variation along with the closing finale remains highly diatonic in character. In the fifth and sixth variations, the relatively low diatonic average is largely explained by the motivic content, which involves a series of lower chromatic neighbours. The fifth variation, for instance, begins with a sequentially descending triplet figure of third-related chromatic lower neighbour notes which ultimately exhausts the total chromatic. In the final variation, the relatively low value of the diatonic curve is accounted for by the specific character of this section, realised not only by the use of the raised leading note, but also through the introduction of distinctive chromatic lines and applied dominants. Note, however, that none of these features are very supportive of the particular type of 'chromaticism' which the other comparison curve is disposed to detect. Even though all twelve pitch classes may occasionally occur in close proximity to one another, they are nevertheless dependent on a basically diatonic framework which mostly prevents the individual tail segments from defining a complete 7–1 heptad.

This non-hierarchical foreground synopsis might seem to represent the antithesis of a descriptive common-sense analysis of the work (Stein 1976) as well as of a mainstream Schenkerian reading (Jackson 1999). Fig. 11 doesn't single out any features as being of primary importance and hence doesn't attribute a structure to the work in any usual sense. The neutrality of this

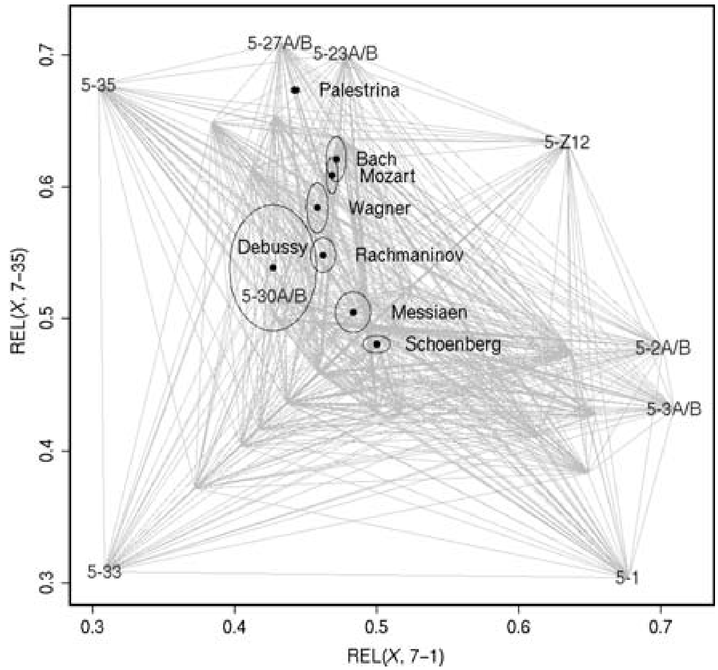
Fig. 11 Brahms, *Variations on a Theme by Haydn*, Op. 56a: average REL-similarity of the separate movements with respect to set classes 7–1 and 7–35 (the tail-segmentation cardinality is 6)



method with respect to structural interpretation is, however, primarily the reason that it should not be placed in opposition to more traditional modes of analysis. In this particular case, the choice of comparison sets is not linked to any significant conception of what Eugene Narmour (1990) has termed 'intraopus style', but was instead effected for the purpose of systematically characterising the work in relation to set classes which could be understood intuitively. Hence, this reading does not purport to be the most appropriate comparison-set analysis from the point of view of musical syntax (although the use of diatonic comparison sets for tonal music would seem to represent the most appropriate point of reference). Rather, we intend that it should furnish an appreciable overview of some well-defined but spontaneously accessible aspects of Brahms's compositional resources.

With paradigmatically 'diatonic' or 'chromatic' comparison sets, one may thus hope to acquire a bird's-eye view of some pertinent aspects of a compositional style without paying explicit regard to the particularities of compositional syntax. Let us now try to develop this same idea in the form of some more general comparisons between differing compositional idiolects. Earlier researchers (for example Fucks and Lauter 1965) have asserted the relevance of statistical evidence as a means of tracing stylistic development through time. For the purpose of tentatively demonstrating some comparable historical considerations with respect to the 'diatonicism' and 'chromaticism' of Western art music, we have selected ten compositions from a number of composers working across a span of several centuries: Palestrina, J. S. Bach, Mozart, Wagner, Debussy, Rachmaninov, Schoenberg and Messiaen (the complete list of works examined is reproduced here as an appendix).[18] To what extent, then, are the various stylistic changes reflected in comparison-set analyses employing set classes 7–1 and 7–35? Indeed, is there an evident transition from the 'diatonic' towards the 'chromatic' which reflects the development of Western music over time?

Fig. 12 Composer-specific centroids represented in a $REL_{7-1/7-35}$ space with pentachordal tail segmentation



The first thing we have to do to answer such questions is to set the tail-segment cardinality sufficiently low (to 5) so as to render the cardinality viable for those composers who rely on local pitch-class sets of smaller cardinality. Next, each work is subjected to two separate comparison-set analyses, one by REL(*X*, 7–35) and the other by REL(*X*, 7–1). Finally, the comparison values for each piece are averaged separately for both measurements. The results of this analysis are displayed in a two-dimensional $REL_{7-1/7-35}$ space in Fig. 12. Each individual stylistic corpus is here represented by a single ellipse whose centre is located at the mean of the two REL measurements and whose height and width represent the average standard deviations of the sets of ten compositions with respect to the two dimensions. Some indication as to the 'shape' of the space is given by the grey lines drawn between all possible pentachordal locations within the space. Set-class names have been included for the outermost pentachords (which thus serve to delineate the space) as well as for some other important collections which will be discussed below. In particular, the reader should note the locations of the 'diatonic' and 'chromatic' pentachords 5–35 and 5–1 as well as the whole-tone pentachord 5–33, which is uniformly dissimilar to both of the comparison sets.

It must be borne in mind that although the resulting centroids have been calculated from expanded numbers of individual tail segments, each of which corresponds to one of the pentachordal locations within the space, these centroids have no necessary connection to the most proximate pentachords. This can be clearly observed in the case of the Palestrina centroid, which represents a mean point among the pentachordal subsets of the diatonic scale. The figure should not be interpreted to mean that Palestrina's music would necessarily be most proximate to set class 5–29, although the tail-segment average in this case does in fact come near to the location of this particular pentachord. Theoretically, a centroid may represent an average of REL values for tail-segment pentachords which are not even close to it within the designated space. Another point worth mentioning is the symmetrical outlook of the $\mathrm{REL}_{7\text{–}1/7\text{–}35}$ space. Of the 2,178 different REL spaces which locate the T_n-type pentachords with respect to two different T_n-type heptachordal comparison sets, only 43 (2.0 per cent) exhibit comparable symmetry. These are precisely those set-class pairs for which the ICVs are otherwise alike but which have interchanged the entries for interval classes 1 and 5. Of all these pairs, 7–1 and 7–35 (with their attendant ICVs [654321] and [254361]) give the highest differences between the switched entries, and thus our $\mathrm{REL}_{7\text{–}1/7\text{–}35}$ space comes out as depicting the widest symmetrical space among the above-mentioned 2,178 two-dimensional spaces.[19]

What is immediately obvious from the composer space is the gradual process of non-diatonicisation, which neatly corresponds to the naïve sense of historical succession. Moreover, from Bach to Wagner, for instance, the figure also attests to a slight non-chromaticisation (in the respect of lessening REL similarity to 7–1). In our special definitions of the terms, such concurrent non-diatonicisation and non-chromaticisation is not as paradoxical as it may sound: as noted above, the ICVs of the two comparison sets, [254361] and [654321], differ only with respect to interval classes 1 and 5. Indeed, Forte would go so far as to say that they exhibit 'maximum similarity with respect to interval vector' (Forte 1973, p. 48). Thus, the string of centroids from Bach through Mozart to Wagner may be interpreted in terms of a gradually lessening tendency towards what is common to the two comparison sets. Of the eight composers whose works are mapped into Fig. 12, only Debussy is able to extend the process further towards the lower left corner – that is, towards the extreme non-diatonicism and non-chromaticism of the whole-tone scale. Notice, however, that our Debussy sample exhibits a far wider dispersion (reflected in the size of the ellipse) with respect to both measurement scales than any of the other samples included in the comparatively continuous chain from Bach to Schoenberg. In the $\mathrm{REL}_{7\text{–}1/7\text{–}35}$ space, this departure from the historical line of non-diatonicisation is obviously an indication of the broadly synthetic nature of Debussy's pitch materials (a consideration which is worth comparing with the bias vectors shown in Fig. 5). By contrast, a composer such as Schoenberg appears to favour a more homogeneous range of pitch-class materials from one work to another, a tendency which thus extends more consistently towards the corner occupied by the notionally 'chromatic' set classes.

Table 1 The five most frequently represented transpositional set classes among the tail segments sampled for each composer (the figures below the Forte numbers indicate the percentages within the overall set of tail segments while the right-hand column records the total percentages covered by the five commonest set classes)

Palestrina						Total %
	5–27A	5–23B	5–27B	5–35	5–23A	
%	20.7	13.8	11.0	9.5	8.8	63.8
Bach						
	5–23B	5–27A	5–23A	5–Z12	5–27B	
%	16.1	12.7	8.7	5.8	5.3	48.8
Mozart						
	5–27A	5–23B	5–23A	5–27B	5–29A	
%	16.3	9.7	7.9	5.1	4.6	43.7
Wagner						
	5–27B	5–27A	5–23A	5–Z17	5–34	
%	19.2	12.4	8.7	8.4	5.3	53.9
Rachmaninov						
	5–27B	5–27A	5–20A	5–35	5–Z17	
%	7.8	7.3	5.5	4.5	4.1	29.2
Debussy						
	5–33	5–35	5–28A	5–34	5–Z37	
%	11.6	10.8	9.9	6.5	5.4	44.1
Messiaen						
	5–23B	5–32A	5–32B	5–25A	5–33	
%	6.5	5.4	5.3	4.7	4.5	26.4
Schoenberg						
	5–30B	5–21A	5–22	5–21B	5–13A	
%	4.8	2.6	2.6	2.4	2.3	14.6

It is perhaps expedient to collate the centroids with some information concerning the extent to which various set classes are represented in the tail segments of individual compositional idiolects. For each separate corpus the five most common set classes were compiled into the form reproduced in Table 1 (together with figures indicating the percentage of tail segments covered by each of them). Despite the mechanical nature of the tail-segmentation method, relatively clear set-class predominance emerges in the cases of Palestrina, Bach, Mozart and Wagner, a feature less strongly delineated yet still syntactically comprehensible for Debussy, Rachmaninov, Messiaen and Schoenberg. Thus the commonest set class to feature among Debussy's tail segments, for instance, is none other than 5–33, the pentachordal subset of the whole-tone scale.

What is perhaps even more interesting is the difference between the predominant set classes which emerge both prior to and during the era of common practice tonality. 5–27A [0, 1, 3, 5, 8] is most firmly represented for Palestrina and Mozart, while for Bach (and interestingly for Messiaen as well) the tail segments more often revealed set class 5–23B [0, 2, 4, 5, 7]. One way of interpreting these outcomes would be to observe that both sets include, by transposition, the tetrachord [0, 2, 4, 7], and that this may often indicate a harmonic context with regard to the root of the major triad designated as pc [0]. In Bach's case, the tetrachord is more typically filled in with pitch class [5] to produce the lower half of the diatonic scale. Palestrina and Mozart, on the other hand, tend to substitute [11] for [5], which yields [0, 2, 4, 7, 11]. For this latter pairing it is relevant to note that the predominant set class thus incorporates two fifth-related major triads – a type which is also the second most prevalent in the cases of Bach, Wagner and Rachmaninov. Wagner's predominant set class may be read as [0, 4, 7, 9, 11], which facilitates its interpretation as an elaborated major triad, while Schoenberg's predominant set class [0, 2, 4, 7, 8] can also be construed in similar fashion. Of course, the pre-eminent set classes may have emerged in a multitude of ways, some of which have little to do with such informal harmonic designations. Furthermore, one should also bear in mind that the selected repertoire samples are far from exhaustive for any single representative. Still, we venture to claim that our interpretation does reflect some of the basic distinctions between the types of musical surface which constitute the individual styles in question.[20]

Fig. 12 further suggests that, Debussy notwithstanding, a predominant trend of average pitch content over the history of Western art music may be identified as a negative process of non-diatonicisation. In terms of tonal materials, there would appear to have been a stylistic progression from an overarching principle of diatonicism to a multitude of more or less different harmonic possibilities. As far as the direction of this progress is concerned, it is certainly reasonable to suppose that the general stylistic development of Western art music simply cannot be described as a gradual process towards any unified or homogeneous resource. Fig. 12 already shows that 'chromaticism' – in the rather special sense of relative similarity of the pitch-class materials on the musical surface to chromatic clusters – fails as a determinant of the historical process: even the centroid representing Schoenberg remains within the middle range of the chromatic measurement scale. In order to describe the various developmental branches from a more positive point of view, we need to take a closer look at the materials employed by each individual composer. This cannot simply be achieved by using intuitive comparison sets such as 7–35 and 7–1, or even the octatonic 8–28 of our previous Stravinsky example. Indeed, the exceptional dispersion of Debussy's oeuvre within the $\mathrm{REL}_{7-1/7-35}$ space attests to the individuality of this particular composer's preferred pitch materials, a feature which prompts us to choose one of his works as the basis of our final analytical example.

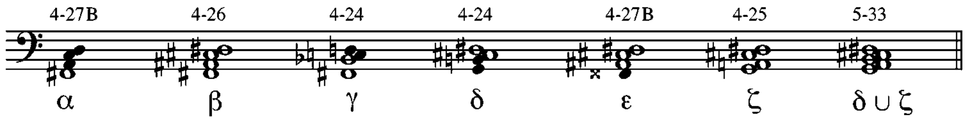
Quantitative versus Transformational Referentiality

A further extension of comparison-set analysis might be made either in conjunction with prolongational analyses or in terms of referential pitch-class sets. One should remember, though, that the techniques demonstrated so far have no bearing on the specific transpositional levels of pitch-class sets, although such techniques might additionally be developed. Of course, the claim that a pitch-class set is either prolonged or used referentially within a composition does not necessarily imply that the set will be a constant foreground presence. Although we do not want to commit ourselves to any specific theory of prolongation or referentiality, it should also be remembered that such forms of orientation may focus on deeper levels of musical structure which simply cannot be grasped using our present methods. Surface-feature detectors may nevertheless be useful for assessing the relevance of any putative referential constructs with respect to the musical surface, which in turn may help to determine the status of various theoretical claims concerning referentiality.

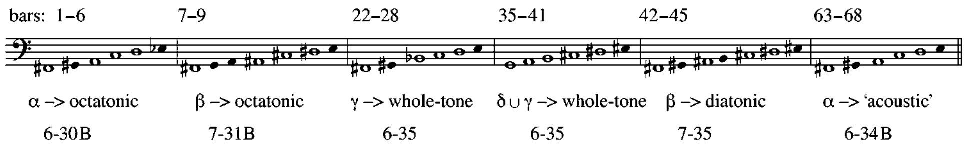
All of these considerations are amply demonstrated by the following example, which draws on Olli Väisälä's (2006) analysis of Debussy's 'Ce qu'a vu le vent d'ouest' from the first book of *Préludes*. Space does not permit a detailed examination of Väisälä's interpretation, but we may at least proceed to review some of his central arguments. According to Väisälä, the referential collection operative throughout the piece is {F♯, A♯, C♯, D♯}, which he terms chord β. Väisälä notes that this harmony does not appear 'as an explicit verticality' until the very end of the work (2006, p. 168); instead, 'considerable stretches of time' are governed by several other tetrachords which 'assume locally a referential status' but nevertheless 'remain subordinate to β in terms of large-scale organization' (Väisälä 2006, p. 169). These chords, all of which are related to β through semitonal motion, are shown in Ex. 3a. Ex. 3b reproduces Väisälä's analysis of the means by which the chords become members of Debussy's characteristic pitch-class set vocabulary over the course of the work. The black note heads in Ex. 3b indicate pitches to which Väisälä accords a non-harmonic status; in addition, set-class names are used to complement his rather less familiar chordal nomenclature.

Väisälä's conception of referential pitch-class sets is in fact transformational: thus the music's surface may be interpreted through formal operations such as semitonal movement – a process which may deprive the referential sets themselves of any surface prominence. His approach consequently differs markedly from that of Richard Parks (1989), who explicitly links pitch-class set referentiality to quantitative prominence at the musical surface. In principle, we see no reason why such different conceptions of referentiality may not be permitted to coexist assuming they are sufficiently well defined and the specific nature of their correspondence is taken fully into account. In order to illuminate the difference between a transformational and a quantitative conception of referentiality, therefore, Fig. 13 illustrates the application of a comparison-set analysis to 'Ce qu'a vu le vent d'ouest', Ex. 3 Väisälä's conception of referential pitch-class sets in Debussy, 'Ce qu'a vu le vent d'ouest' (2006, pp. 169–70)

(a) chord β and its semitonal relatives



(b) larger pitch-class set formations

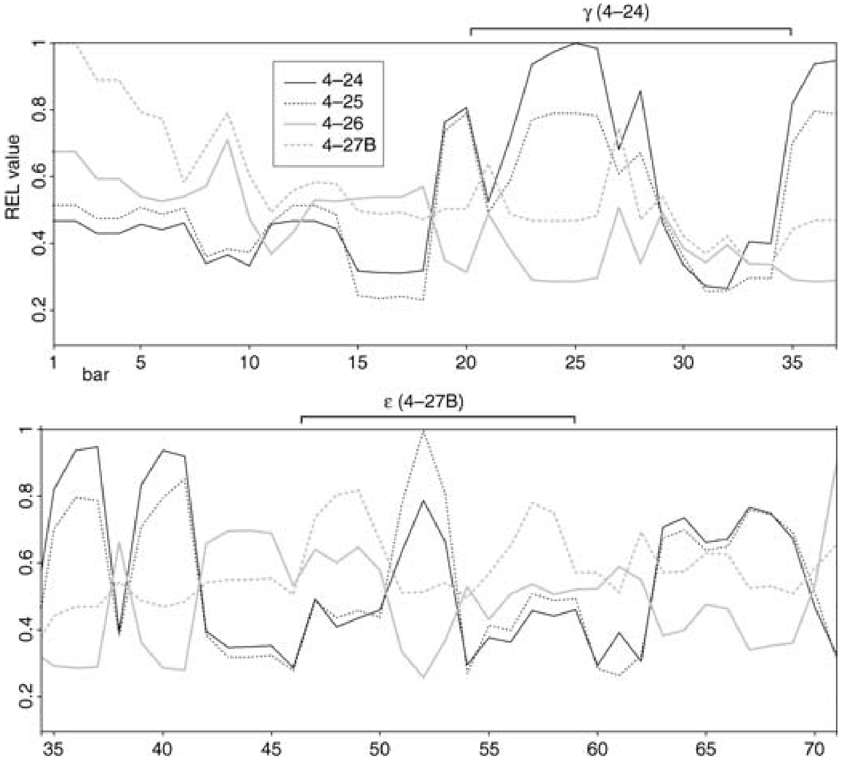


taking Väisälä's sequence of tetrachords as its starting point. A tetrachordal tail segmentation is used in order to make higher comparison values possible.

The analysis demonstrates that Väisälä's referential chord β, representing set class 4–26 [0, 3, 5, 8], is indeed relatively scarce as a foreground element. The concluding ascent of the associated comparison curve is largely due to the final chord of the piece. Additionally, there are only brief passages (at bars 15–18, 38, 42–46, 54 and 61) over which the 4–26 curve rises above the other three comparison curves. A similar observation can be made by taking a look at the distribution of T_n-type tetrachords in the tail segments themselves (analogous to the procedure summarised in Table 1 above). Here, set class 4–24 [0, 2, 4, 8] emerges as pre-eminent, covering 21.2 per cent of the tail segments, followed by 4–21 (8.7 per cent), 4–27B (7.8 per cent) and 4–25 (7.6 per cent). By comparison, no more than 2.0 per cent of the tail segments represent the supposedly referential 4–26. At first sight this may appear to confirm Väisälä's argument that the remaining tetrachords are treated as locally referential over and above chord β. However, one might then ask whether this local referentiality in fact has more to do with quantitative prominence than do Väisälä's assumptions in the specific instance of referential chord β. In any case, at least some of his analytical claims are confirmed by our quantitative analysis, if only to the extent that in certain locations none of the other tetrachords he mentions achieves a higher comparison value than the one designated as being locally referential. As a comparison between Ex. 3 and Fig. 13 reveals, the two analyses effectively converge over bars 1–6, 22–28 and 42–44, where the prominence of chord β is clearly apparent.

Aside from the approach summarised in Ex. 3, Väisälä also advances a series of Schenkerian-derived prolongational readings. For instance, he indicates a prolongation of chord γ (4–24) over bars 21–35 and a similar prolongation of

Fig. 13 A comparison set analysis of Debussy, 'Ce qu'a vu le vent d'ouest', involving set classes 4–24 [0, 2, 4, 8], 4–25 [0, 2, 6, 8], 4–26 [0, 3, 5, 8] and 4–27B [0, 3, 6, 8] (the comparison function is REL, and the tail-segmentation cardinality is 4)



chord ε (4–27B) over bars 47–58. These prolongational spans are indicated by brackets in our comparison-set analysis. In both cases, the supposed prolongations begin in the quantitative dominion of the appropriate set classes, while the latter passage also concludes with the prolongational set class being accorded a clear degree of surface prominence. The justifiable impression which might be formed is that whereas supposed referentiality on the level of the whole work gains quantitative foreground support only through the conventional idea of a 'tonic' ending, smaller-scale prolongational spans are triggered by initial quantitative prominence. However, Väisälä's analysis does not fare consistently well by these criteria: in bars 7–9 he adopts chord β as the locally referential element (see again Ex. 3b) even though the set class of the previous referential chord maintains pre-eminence throughout this section. This effect seems openly to contradict the principle of local prolongation suggested above: if initial quantitative prominence is sufficient to sustain a local prolongation despite the diminished role of the referential set, how is it that a new referential set may be introduced whose quantitative prominence does not in fact reach the level of the previous one?

The answer, of course, is that the continuing prominence of a referential set class does not say anything about the continuing prominence of a transpositionally determined pitch-class set. As a matter of fact, no positive prolongational claim can be properly supported by our comparison-set analysis, since the detector curves are not transpositionally restrictive. Thus Väisälä may be justified in supposing that bar 47 marks the beginning of a prolongation of the D♯ dominant seventh chord; yet while our analysis seems to concur with this view, it does so only superficially. Far from indicating the prolongation of a given dominant seventh chord, our reading reflects the fact that the whole texture of the composition is permeated by parallel dominant seventh chords at this point. Whereas a prolongational approach might be conjoined, as Boyd Pomeroy has suggested, with the common view that 'Debussy's surface chord successions typically serve ends of colouristic effect rather than tonal-syntactical coherence' (Pomeroy 2003, p. 158), we would therefore propose that it is the very homogeneity of the acoustic surface in such passages which carries the greatest responsibility for the music's manifest coherence. Even if comparison-set analysis reveals neither prolongational continuity nor 'tonal-syntactical coherence', it does at least succeed in uncovering another important plane of coherence.

Compared to our earlier Stravinsky analyses, the comparison-set reading of 'Ce qu'a vu le vent d'ouest' still leaves much to be desired: in short, there are areas throughout the piece in which none of Väisälä's referential chords shows much REL similarity with respect to the local tail segments. As far as referentiality in Parks's quantitative sense is concerned, one should obviously be able to find comparison sets yielding higher values even within these areas. Such points of localised primacy can be found simply by working through the composition with a larger set of detectors attuned to, say, all tetrachordal set classes, at the same time marking the comparison set which produces the highest value for each individual bar. With a tetrachordal tail segmentation allied both to Lewin's REL as the comparison function and the set of 43 T_n-type tetrachords, we may thus attempt to identify any other tetrachords which might assume quantitative prominence. Indeed, the mean REL values for the predominant comparison sets within each bar fall no lower than 0.62 under these conditions, an analytical return which marks a considerable improvement over the appreciable troughs apparent in Fig. 13. There are also continuous areas governed by a single pre-eminent collection which are not implied by Väisälä's analysis. Most important, bars 63–68 are exclusively governed by the comparison set 4–21 [0, 2, 4, 6], which is not represented by any of Väisälä's referential chords. To be sure, whole-tone scale fragments such as [0, 2, 4, 6] are such obvious features of Debussy's music that they might not merit attention as characteristic reference sets even if additionally tied to fixed-pitch formations.

All in all, ascribing structure to a work using concepts such as prolongation is likely to involve interpretative decisions which reach beyond a quantitative comparison-set analysis. Consequently we have not taken up Väisälä's interpretation simply in order to dismiss his transformational conception of referentiality.

Rather, our point is that such a conception can hardly remain completely independent of surface cues, which are often quantitative in nature. Moreover, there is clearly a need for operative guidelines capable of regulating the disparity between transformational analyses and the quantitative investigation of the musical surface.[21]

VI

Conclusion

According to Anthony Pople, set structural analysis 'rests on three fundamental principles: segmentation of a pitch structure, reductive classification of sets of pitches and making judgements about significance concerning the relationships between the various sets thus determined' (Pople 1983, p. 151). The methods of systematic pitch-class set analysis presented here imply a change of perspective with respect to each of Pople's three principles. First, determinate, interpretative segmentation is discarded in favour of a method which finds a tail segment of a given cardinality for each successive note within the event list that effectively represents the composition. Second, the procedure of reductively classifying the constituent sets is replaced by the task of assigning a numerical value to each note which reflects a given property of all those tail segments associated with it. The focus is thus shifted from the identification of structurally important pitch sets or pitch-class sets to the measurement of selected properties within the set of all locally available pitch-class sets.[22] Third, the detection of such properties either within the temporal course of a musical work or throughout a given class of works is substituted for the description of the relationships between structurally constitutive collections. Noting the divergence in basic premises between these structural and systematic views is important, since it allows us to regard their resultant findings as complementary rather than contradictory in kind.

As the Debussy example featured in the previous section illustrates, it may often be possible to achieve a fruitful relationship between a systematic overview of a piece on the one hand, and a more particularised reading on the other. Given the availability of such complementary methodologies, any apparent lack of sensitivity in the systematic detector to contextual specificity should be seen more as a strength than as a weakness. Indeed, it would be much more difficult to grasp the import of a single detector curve as a summary overview of a musical work were different stretches of it to result from different computational manoeuvres, each determined by the immediate musical context. As noted at the beginning of this article, quantitative measurement actually requires that the individual results be compared to other, similarly derived results. From this perspective, contextualisation of a local musical event with respect to its immediate surroundings is indeed concomitant with systematicity. As has been noted, this kind of contextual juxtaposition also operates on a larger scale when overviews of complete musical works (Fig. 5) or more extensive repertoire selections (Fig. 12) are examined comparatively. These comparisons motivate the application of a uniform analytical methodology to a wide variety of musical styles, ranging from Renaissance counterpoint to atonality. However, contextual juxtaposition across musical styles requires that the analytical apparatus be as stylistically neutral as possible, incorporating a minimum of style-specific concepts. It is only by way of such systematic methods, we submit, that the theories of tonality and atonality can be 'comparably linked', as proposed by Lerdahl (2001, p. 351), among others.

Apart from juxtaposed or, in J. H. Kwabena Nketia's phrase, 'situational' contextualisation (Nketia 1990, p. 81), we have also chosen to rely on what Nketia terms 'conceptual or notional' contextualisation. In an analytical project such as ours, this can be achieved in two ways. Most important for our present concerns, comparison-set analysis locates each local segment within the context of certain paradigmatic musical materials – in short, a comparison set whose properties can be expected to be familiar to the analytical community. This approach, referred to above as 'referential contextualisation', is responsible for our introduction of the detector metaphor. When using the diatonic heptad 7–35 as the comparison set, successive musical segments are thus assessed with reference to a recognisable collection of intervallic properties. In reading the resulting detector graph, the musical passage in question is, so to speak, perceived through the analyst's implicit knowledge of the properties belonging to the familiar comparison set. Note that in comparison-set analysis the referential mode of contextualisation is thus built into the analytical apparatus itself.

An alternative mode of conceptual contextualisation would be systematic in nature, in which some parameter values or other measurements are observed in relation to the whole set of relevant possibilities apparent within the system of twelve distinct pitch classes. Such considerations were for example taken into account when defining the mean interval-class bias vector (MICBV), or in analysing the Debussy piece in relation to the entire set of 43 T_n-type tetrachords. However, there are other, potentially more fruitful prospects which could be explored in connection with these present methods. To give one illustration, the choice of a comparison set could be motivated by a wish to maximise the dispersion of comparison values which are possible for the comparison set. Assuming a pentachordal tail segmentation such as that employed in establishing the centroids of Fig. 12, one might thus hope to replace the intuitively chosen comparison sets 7–1 and 7–35 with alternatives capable of achieving an even wider dispersion of values among the tail-segment pentachords. Taking Lewin's REL as the comparison function, we might thus compare each potential comparison heptad in turn with the set of 66 T_n-type pentachords. In this class of pentachords, the highest standard deviations are produced by the two subsets of the octatonic scale, 7–31A and 7–31B (SD = 0.097), followed by 7–1 and 7–35 (SD = 0.089 for both). Such an analysis shows that for the 66 pentachords of our tone system, the intuitive comparison heptads 7–1 and 7–35 are in fact among those which achieve the greatest dispersion of comparison values. In other words, these are among the strongest heptachordal detectors for investigating pentachordal objects. Viewing the chosen comparison sets against the background of all possible alternatives thus represents a further aid in trying to understand the quantitative results.

Despite their insensitivity to certain aspects of local musical context, the methods introduced in this article remain capable of incorporating different modes of contextualisation, namely through juxtaposition with other similarly obtained results, by making reference to paradigmatic musical materials and by embracing alternative possibilities within a given tone system. Quantitative results are thus rendered meaningful by providing links to other musical works, recognisable musical components or even the structure of the tone system itself. Many of the analytical results – for instance the 'diatonic' curves in the Stravinsky analyses of Fig. 8 – may thus appear to reflect pertinent aspects of our listening experience: simply stated, the detector graphs are often surprisingly easy to follow in the course of listening to music. As already noted, the processes of systematic pitch-class set analysis may nevertheless depart from the processes of human musical perception. Humans do not and cannot process music perceptually in the consequent algorithmic manner of a systematic pitch-class set detector.[23] Even if it might prove interesting to use comparison-set analyses in tandem with continuous human response to musical stimuli, any possible correspondences with respect to subjective judgement would not constitute evidence for the similarity of the underlying mechanisms. In themselves, our methods are only music-analytical tools; the analyst's decision on whether to use them depends on his or her aims. As we hope to have shown, however, this does not rule out the possibility of creating intuitively accessible representations of extensive musical surfaces.

All of the representational techniques presented in this article are founded on tail-segment arrays, sets of overlapping surface segments which all include a given pivotal note. These surface entities are found by counting backwards on an event list from each successive note to the (typically) fourth, fifth, sixth, or seventh distinct pitch class. The computational systematicity and sheer complexity of manipulating tail-segment arrays is the aspect of our analytical method which lies at the point of furthest remove from human cognition. Given its foundational status for our approach to pitch-class set analysis, tail segmentation is also likely to be the single aspect most vulnerable to criticism. For instance, the system cannot appreciate aspects of voice-leading in multi-voiced textures. Thus, in a homophonic chordal setting, the bass note of one chord will appear on the MIDI list next to the highest note of the previous chord while the succeeding notes in both parts will remain separated. The resultant disadvantages are somewhat ameliorated by the fact that tail segments are treated as unordered sets: that is, melodic intervals are not preserved. Nevertheless, the separation of voice-leading intervals means that the consecutive notes within a given registral range will not be grouped together as readily as one might wish. On this basis, one might well consider replacing tail segmentation with some other form of overlapping sequence involving a single pivotal note – for example, with segments of a given cardinality which include notes that are either registrally or temporally proximate to the pivotal note. However, such a solution requires weighting the two dimensions of proximity according to some formula, which would inevitably make it more difficult to derive the tail segments directly from the score.

A range of further music-analytical questions may be approached through these same methods of systematic, computer-aided pitch-class set analysis. Thus, the stylistic comparisons advanced in Fig. 12 suggest other historically oriented questions such as whether there may be overall stylistic differences between similar works composed several decades apart. Likewise it may well prove informative to carry out separate tail segmentations for distinct musical layers, thereby acquiring different stretched ICVs or different comparison curves for each. In a subsequent study we will seek to apply some of these methods for the purpose of analysing the average characteristics of temporal development in a large body of MIDI-recorded extemporisations by a professional keyboard improviser. The use of overlapping segments is especially helpful in this kind of musical context on account of the fact that the absence of a composed score (with its attendant tonal and metrical dimensions) would set extraordinarily high demands for any conventional segmentation. In all such applications, it is nonetheless important to remember that the focus of a detector-based analysis rests on the average characteristics of pitch-class material rather than on the specific ways in which the material has been arranged. In order to enquire into the workings of musical syntax altogether different methods are required.

Appendix List of works consulted (Fig. 12)

J. S. Bach

1. Chorale, *Es ist genug*, BWV 60:5
2. Aria 'Ach, mein Sinn', *St. John Passion*, BWV 245
3. *English Suite*, BWV 806
4. *Preludium e Partita del Tuono Terzo*, BWV 833
5. *The Well-Tempered Clavier I*: Fugue No. 24 in B minor, BWV 869
6. *Brandenburg Concerto* No. 1, BWV 1046
7. *Brandenburg Concerto* No. 4/i: Allegro, BWV 1049
8. *Brandenburg Concerto* No. 5/i: Allegro, BWV 1050
9. *Die Kunst der Fuge*, Contrapunctus XV, BWV 1080
10. *Die Kunst der Fuge*, Contrapunctus V, BWV 1080

Debussy

1. *Suite bergamasque*: 'Clair de lune'
2. *L'isle joyeuse*

3. *Images* 1: 2
4. *Préludes*, Book I: 'Voiles'
5. *Préludes*, Book I: 'Ce qu'a vu le vent d'ouest'
6. *Six épigraphes antiques* 1
7. *Six épigraphes antiques* 6
8. *Nocturnes*: *Sirènes*
9. *La mer*: I 'De l'aube à midi sur la mer'
10. *La mer*: III 'Dialogue du vent et de la mer'

Messiaen

1. *Le banquet céleste*
2. *Diptyque* No. 1, 'Essai sur la vie terrestre'
3. *Diptyque* No. 2, 'Le Paradis'
4. *L'ascension*: No. 4, 'Prière du Christ'
5. *La nativité du Seigneur*: No. 6, 'Les anges'
6. *La nativité du Seigneur*: No. 8, 'Les mages'
7. *Les corps glorieux*: No. 7, 'Le mystère de la Sainte Trinité'
8. Prelude for Piano No. 2, *Chant d'extase dans un paysage triste*
9. *Vingt regards sur l'enfant Jésus*: No. 10, 'Regard de l'esprit de joie'
10. *Vingt regards sur l'enfant Jésus*: No. 14, 'Regard des anges'

Mozart

1. Concerto for Bassoon/i: Allegro, K. 191
2. Twelve Variations on 'Ah! Vous dirai-je Maman', K. 300e
3. Concerto for Piano No. 14/i: Allegro vivace, K. 449
4. Sonata in C Minor/iii: Allegro assai, K. 457
5. *Fantasie*, K. 475
6. Symphony No. 40/i: Molto allegro, K. 550
7. Symphony No. 41/iv: Finale: Allegro molto, K. 551
8. *The Magic Flute*: Overture, K. 620
9. Concerto for Clarinet/i: Allegro, K. 622
10. Requiem: Kyrie, K. 626

Palestrina

1–7: *Missa Ave Regina Coelorum*
8–10: *Missa Papae Marcelli*: Kyrie, Gloria, Credo

Rachmaninov

1. *Second Suite*, Op. 17
2. Symphony No. 2, Op. 27/ii: Allegro molto

3. Piano Sonata No. 1, Op. 28/ii: Lento
4. *Etudes-tableaux,* Op. 33 No. 6
5. *Vocalise*, Op. 34 No. 14
6. *Etudes-tableaux,* Op. 39 No. 3
7. *Etudes-tableaux,* Op. 39 No. 6
8. *Rhapsody on a Theme of Paganini*, Op. 43: variations 7–10
9. *Russian Rhapsody*
10. *Oriental Sketch*

Schoenberg

1. *Drei Klavierstücke*, Op. 11 No. 1
2. *Drei Klavierstücke*, Op. 11 No. 2
3. *Drei Klavierstücke*, Op. 11 No. 3
4. *Sechs kleine Klavierstücke*, Op. 19 No. 1
5. *Sechs kleine Klavierstücke*, Op. 19 No. 3
6. *Pierrot lunaire*, Op. 21 No. 1, 'Mondestrunken'
7. *Pierrot lunaire*, Op. 21 No. 5, 'Valse de Chopin'
8. *Pierrot lunaire*, Op. 21 No. 8, 'Nacht'
9. *Pierrot lunaire*, Op. 21 No. 10, 'Raub'
10. *Pierrot lunaire*, Op. 21 No. 17, 'Parodie'

Wagner

1. *Tannhäuser*: Overture
2. *Götterdämmerung*, Act III: 'Trauermarsch'
3. *Tristan und Isolde*: Prelude
4. *Tristan und Isolde*, Act III: 'Liebestod'
5. *Die Meistersinger von Nürnberg*: Overture
6. *Parsifal*, Act I: Prelude
7. *Parsifal*, Act III: Introduction
8. *Siegfried Idyll*
9. *Wesendonck Lieder*: 'Träume'
10. *Wesendonck Lieder*: 'Im Treibhaus'

NOTES

1. In this article, the term 'set class' will refer to a collection of pitch-class sets that are equivalent under the transposition operation. That is, we are dealing with 'T_n-type' set classes, in contrast to Forte's 'T_n/I-type' set classes, each of which includes a collection of pitch-class sets that are equivalent under transposition and/or inversion (see Rahn 1980, pp. 75–6). Following Castrén (1989; 1994) and others, inversionally related set classes are here distinguished by adding the suffix 'A' or 'B' to the customary Forte names.

2. Throughout this article, our analyses have been computed using *R*, a free software environment for statistical computing and graphics (see http://www.r-project.org).

3. We would like to express our gratitude to Tuomas Eerola for his comments on an earlier draft of this paper, and to Tommi Viitanen for his indefatigable assistance with certain computational problems encountered during the initial phase of our work.

4. For example, the first segment represents the set class 7–Z36A with the prime form [0, 1, 2, 3, 5, 6, 8]. The list of successive pitch-class intervals, or 'successive-interval array' (Chrisman 1977), for this set class is 1–1–1–2–1–2–4. The relatively high standard deviation among the pitch-class intervals (1.11 as opposed to 0.49 of the diatonic scale) indicates that the pitch classes are not as evenly distributed around the pitch-class circle as might be the case.

5. In our later Wagner example, for instance, including all orchestral doublings would result in some tail segments comprising more than 900 notes. As far as pitch-class content is concerned, such extended segments would add nothing to our shorthand event lists. Moreover, they would also tend to skew our analytical results concerning the pitch-class organisation of Wagner's music in favour of passages with denser orchestral textures.

6. For heptads, the minimum and maximum values of standard deviation are marked by the maximally even diatonic scale (7–35), SD = 0.49, and the 'minimally even' chromatic heptachord (7–1), SD = 1.89.

7. The choice of cardinality for the tail segmentation will be examined in greater detail below.

8. The MICPV was introduced by Huovinen (2002, p. 69) in the rather different methodological context of experimental music psychology (there termed the 'interval-class *probability* vector'). The idea of a normalised ICV also occurs in Castrén's (1994, p. 4) '%-vector', which for a given set class and a given subset cardinality *c* gives the percentage of each subset type of cardinality *c* within the totality of *c*-cardinality subsets of the given set class.

9. When calculated directly from the MICVs for each note, the percentage vector would be influenced by the number of notes per time unit.

10. While working with well-defined collections of set classes or 'set-class domains', Huovinen (2002, pp. 69–71) labelled this the 'domain bias vector' (DBV). We now prefer the term 'mean interval-class bias vector' for its descriptive content as well as for its applicability in situations such as the present one. We likewise depart from Huovinen's original definition wherein only T_n/I type set classes were used to determine the 'neutral' values. Among other considerations, restricting the present context to T_n classes seems more appropriate as a means of remaining sensitive to the major/minor distinction. See also Huovinen (2002, p. 70) for a cardinality-neutral way of determining the 'neutral' values from which the MICBV is computed.

11. The standard deviations for the values produced by the six simple weighting vectors in the class of 80 T_n-type hexachords are {1.09, 1.15, 1.12, 1.16, 1.09, 0.85}.

12. Having assumed that each measurement between two set classes may be understood only in relation to the vast set of alternative measurements for other set-class pairs, theorists have understandably tended to focus less on the purported complexity of the measurement results and more on the mathematical properties of the similarity measures themselves (see, for instance, Quinn 2001).

13. Isaacson (1992, p. 75) formulates IcVSIM in terms of so-called interval-difference vectors, which enumerate the differences between corresponding entries in pairs of interval-class vectors. Thus, $\mathrm{IcVSIM}(X, Y) = \sqrt{(\sum_{i=1}^{6}(\mathrm{IdV}_i - \overline{\mathrm{IdV}})^2)/6}$ where IdV_i denotes the *i*th entry in an interval-difference vector and $\overline{\mathrm{IdV}}$ is the mean of the terms in the difference vector. Intuitively, we can see that where all of the differences between corresponding entries in two ICVs are the same (for example, the diatonic 7–35 [254361] as compared to 6–32 [143250]), the resulting IcVSIM value is 0, which means that the two set classes are judged to be as similar as possible.

14. REL is here defined in relation to a 'subset vector', which can in turn be defined for each T_n-type set class. The subset vector of any given set class is simply a 348-place vector consisting of its ICV, followed by its 3-class vector (giving the numbers of occurrence for each T_n-type trichord), its 4-class vector etc. and ending with its 10-class vector (see Castrén 1994, p. 3). For our version of the formal definition of REL, we additionally need the function sub(X, *i*), which selects the *i*th element from the subset vector of the set class X. For example, sub(4–9, 1) returns the value 2, because any member of set class 4–9 includes exactly two instances of subset class 2–1 whose index number *i* in the subset vector of 4–9 is 1. Set class 4–9 also includes T_n-type trichords among its subsets. There are, for instance, two instances of 3–5A, whose index number in the subset vector is 14; thus, sub(4–9, 14) = 2. Now, the REL similarity function between two set classes X_1 and X_2 is

$$\mathrm{REL}(X_1, X_2) = \frac{\sum_{i=1}^{348}\sqrt{\mathrm{sub}(X_1, i)\cdot \mathrm{sub}(X_2, i)}}{\sqrt{\sum_{i=1}^{348}\mathrm{sub}(X_1, i)\cdot \sum_{i=1}^{348}\mathrm{sub}(X_2, i)}}$$

For example, in order to calculate the REL similarity between set classes 4–9 and 4–Z15B, we first need their respective subset vectors. In this case, the subset vectors only indicate the presence of six interval classes, four trichords and one tetrachord (which is, of course, the set class itself). For 4–9 and 4–Z15B, we thus have, respectively:

[2 0 0 0 2 2 0 0 0 0 0 0 0 0 2 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 . . . zeros . . .]

[1 1 1 1 1 1 0 0 0 0 1 0 0 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 . . . zeros . . .].

Then,

$$\mathrm{REL}(4\text{–}9, 4\text{–Z15B}) = \frac{\sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2}}{\sqrt{11 \cdot 11}} \approx 0.51.$$

For slightly different, but formally equivalent definitions, see Lewin (1979–80a) and Castrén (1994, p. 89). For additional examples, see also Kuusi (2001, p. 213).

15. The cardinality correction was not applied in the preliminary demonstration of detector graphs (Fig. 3), in the stretched-ICV analyses (Figs. 4a and 6) or in the weighting-vector curve (Fig. 7). In the case of the stretched ICV, we abstained from using the correction only in order to avoid over-complicating the figures (which already contained six visually different curves). However, the mean results in Figs. 4b and 5 do employ the cardinality correction. In more ambitious instances, the correction should obviously be used regardless of the function applied to the tail segments.

16. In comparison-set analysis it is practicable to employ similarity measures that do not require the two pitch-class sets to be of the same cardinality. On this account, one can keep tail-segment cardinality conveniently low while still retaining access to larger comparisons sets such as 7–35.

17. Recall that the hexachords may not be completed within the bar itself (as noted in connection with Fig. 6).

18. The MIDI files for these works were selected from various Internet sources for the purpose of this demonstration and prepared in the usual way by arranging simultaneities as event lists in ascending order and removing doublings from them. Despite taking considerable care, it is clear that any serious comparative research project would need to use more critically encoded materials.

19. Recalling our preliminary examples referring to non-evenness, the reader may have noted that the two comparison sets used in Fig. 12, 7–35 and 7–1, represent maximal and minimal evenness among the heptachordal set classes. We should therefore perhaps point out that the two dimensions of the $REL_{7-1/7-35}$ space do not simply measure inverse properties. If this were the case, all pentachords would simply occupy a position along a straight line with the slope of –1; a brief glance at the shape of the space defined in Fig. 12 is sufficient to prove that this is not the case.

20. The reader should be reminded that the centroids in Fig. 12 are in fact computationally rather far removed from the tallies of Table 1. The table reports the set classes of the tail segments themselves. Calculation of the centroids, on the other hand, requires five more steps: (i) taking the REL measurements for the tail segments; (ii) averaging the tail-segment arrays in conformance with equation (1) in order to yield values for each individual note; (iii) averaging the note-specific values for each bar; (iv) averaging the bar-specific values for the entire composition and (v) averaging the values of the ten chosen compositions.

21. Of course, there may be good musical reasons for describing surface phenomena in ways that contradict our quantitative measurements. As mentioned in the first part of this article, surface phenomena are usually discussed from a much more particularised viewpoint.

22. Of course, the word 'all' is to be read in a qualified sense determined by the specifics of the tail-segmentation method.

23. From a perceptual point of view, the simple relationship between the data points and the means which represent larger sections of music also appears somewhat idealised. Certainly this is the case given that the overall perceptual effect of a piece of music on listeners may not be easily computed as an arithmetic mean of the strength of their momentary experiences (see Duke and Colprit 2001).

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ABSTRACT

This study presents a series of methods for the analysis of the average characteristics of pitch-class material on the surface level of musical works. The methods rely on so-called tail segmentation, the partitioning of a musical work into a large number of overlapping pitch-class sets of equal cardinality. The

resulting data can be used as a means of scanning a piece of music in order to detect changes in the local prevalence of some chosen pitch class–related feature. For example, scanning the musical surface for each individual interval class in turn results in a 'stretched interval-class vector' represented by six curves. In another application termed 'comparison set analysis', similarity measures for set classes are used to detect similarities with respect to some chosen set class. One may thus measure local changes in, for example, the relative degree of 'diatonicism' within a piece of music. Alternatively, one may compare whole corpuses of compositions in terms of their overall surface similarity with respect to a chosen comparison set. In such methods of systematic pitch-class set analysis, the focus is shifted from identifying structurally important pitch sets or pitch-class sets to measuring selected properties within the set of all locally available pitch-class sets.