Perle's Cyclic Sets and Klumpenhouwer Networks: A Response

Dave Headlam

Each of the papers in this issue addresses how elements of George Perle's compositional system and analytical methodology intersect with features of Klumpenhouwer networks (K-nets), drawing on Perle's own communication on the subject as well as his writings.[1] This commentary is intended both to clarify Perle's position and to provide additional context from his writings. Lewin and Lambert directly refer to Perle's materials. Lewin compares K-nets and "Perle cycles" (Perle calls them cyclic sets) as models for compositional events, and Lambert describes his K-net families in terms of Perle's cyclic sets.[2]

Perle was initially influenced by Berg's *Lyric Suite,* where the row is comprised of two interlocking inversionally-related interval-5 or -7 cycles and thus is comprised of adjacent dyads from two sums, 9 and 4, as shown in Example 1, where sums of 9 are bracketed on top, and those of 4 on bottom. Perle calls such a construct a "cyclic set." A cyclic set is identified by its interval cycles and its sums; the cyclic interval is the difference of the sums (9–4 = 5 or 4–9 = 7; either 5 or 7 may be used). Each imbricated trichord in the set is comprised of a middle "axis note" with surrounding "neighbor notes." The trichord is the smallest unit that can identify a cyclic set, as it contains the two sums and the cyclic interval. In K-net terms, these trichords are all related by strong isography, with the same configuration of Ts and Is.[3] For instance, {8,1,3}, {6,A,B} and {4,0,9} can all be interpreted as $T_{5/7}$, I_9/I_4 K-nets. The given row-form of P_5 (rows in Example 1 are rotated by one hexachord to create continuous cyclic sets) shares one sum with related forms at sum 9 (row-form I_4), and sum 4 (row-form I_B). These three rows vertically create the same group of K-nets related by strong isography when aligned. Cyclic sets that share one sum, such as the sums-9/2 and -9/4 sets, are called "cognate sets" by Perle.

Cyclic sets are part of a "cyclic set complex" that includes all alignments of the two interlocking interval cycles. Example 2(a) reproduces Lewin's Example 1.1, with T_1 and T_B cycles in an offset alignment and sums 6,7 found on the diagonals. In later examples, Lewin maintains this offset arrangement, showing trichords as triangles between the two sets, and does not represent them in one line, as Perle does in his cyclic sets. This example is rewritten as a cyclic set of sums 6/7 in Example 2(b). It is placed within a cyclic set complex, where the sets are based on interval cycles 1/B. The interval-B cycle in boldface shifts one to the right in

[1] See the citation list for pertinent material.

[2] Lewin writes that Perle suggests that K-nets "can best be viewed" in terms of cycles, but Perle only notes that they "may be defined and efficiently and economically represented in this way." Perle 1993, 300.

[3] Tracing this type of row would open a twelve-tone component to K-nets.

Example 1. Berg's *Lyric Suite* row (all rows rotated by one hexachord)

cyclic set 9/4 (P_5): (8) **8** 1 **3** 6 **A** B **5** 4 **0** 9 **7** 2 (2)

sum 9: 56789A
43210B

sum 4: 2345678
210BA98

sum 9	(I_4)	1 8 6 3 B A 4 5 9 0 2 7	sums 9/2
	(P_5)	8 1 3 6 A B 5 4 0 9 7 2	sums 9/4
sum 4	(I_B)	8 3 1 A 6 5 B 0 4 7 9 2	sums B/4

combined 0123456789AB
9876543210BA = $T_5/_7$ I_9/I_4 K-nets
789AB0123456

each set, causing the sums to increase by 1 in each cyclic set: 6/7, 7/8, 8/9, etc. Each cyclic set is extended to its full length here; the second half is a T_6 of the first half and is in a retrograde relationship as well. The second half therefore contains the trichords of the first half in reverse, and so the underlined ⟨4,2,5⟩ T_1 sums-6/7 K-net has a reverse pair in the same cyclic set: a ⟨5,2,4⟩ T_B sums-7/6 K-net. Thus, the cyclic sets also contain all the "reverse" K-nets in Lewin's examples from Webern's op. 27, first movement, which he relates to Webern's compositional process.

Lewin's Examples 1.1(a) (E,D,F) and (b) (F♯,C♯,G) are underlined in the cognate cyclic sets from sums 6/7 and 7/8. Moving along one cyclic set is analogous to Lewin's "CURSOR" (two steps along the cyclic set) and "FLIP" (one step along the cyclic set) transformations. Moving within the complex is analogous to his "SLIDE" transformation. In terms of K-nets, positive isographies result from SLIDE and CURSOR transformations, and negative isographies result from SLIDE and FLIP transformations. For instance, K-net (4,2,5) with T_1 and sums 6/7 is related by positive isography ⟨T_1⟩ to K-net (6,1,7) with T_1 and sums 7/8, and by negative isography ⟨I_4⟩ to K-net (3,6,2) with T_B and sums 9/8. All three are underlined in Example 2(b).

The group of cyclic sets in the complex is called a "family" by Lambert. Any one cyclic set in the complex contains its own group of transpositions by T_6. For instance, in the sum-6/7 set, the underlined notes ⟨4,2,5⟩ have T_6 ⟨A,8,B⟩ further along the set. In this way, any cyclic set in the complex is related by transposition within a group of six sets, and by inversion in a group with the remaining sets, as shown from the sum-6/7 set in Example 2(c). The cyclic sets are rotated to show the initial T and I relationships; the second T or s (for inversional sum) can be seen by transforming the sets to their T_6 rotations. K-nets from Lewin's Webern Examples 2.3, 2.4, 2.7–10 are underlined in the intervals-1/B sums-x/x+1 complex shown in Example 2(c), with his "motions within the space of a P-cycle" as motions within the complex.

The multiple orderings of K-nets mentioned by Stoecker as his "SHIFT" operation can also be represented by cyclic sets. In Perle

Example 2. Lewin's Example 1.1 as part of a Perle cyclic-set complex; Lewin's Examples 2.3 and 2.4 underlined in (c)

(a) Lewin's notation, with sums 6/7 on the diagonals

T_I	3	4	5	6	7	8	9	A	B	0	1	2
T_B	**3**	**2**	**1**	**0**	**B**	**A**	**9**	**8**	**7**	**6**	**5**	**4**

(b) cyclic set complex: interval-1/B generators

sum 6/7	3	**3**	4	**2**	5	**1**	6	**0**	7	**B**	8	**A**	9	**9**	A	**8**	B	**7**	0	**6**	1	**5**	2	**4**
sum 7/8	3	**4**	4	**3**	5	**2**	6	**1**	7	**0**	8	**B**	9	**A**	A	**9**	B	**8**	0	**7**	1	**6**	2	**5**
sum 8/9	3	**5**	4	**4**	5	**3**	6	**2**	7	**1**	8	**0**	9	**B**	A	**A**	B	**9**	0	**8**	1	**7**	2	**6**
sum 9/A	3	**6**	4	**5**	5	**4**	6	**3**	7	**2**	8	**1**	9	**0**	A	**B**	B	**A**	0	**9**	1	**8**	2	**7**
sum A/B	3	**7**	4	**6**	5	**5**	6	**4**	7	**3**	8	**2**	9	**1**	A	**0**	B	**B**	0	**A**	1	**9**	2	**8**
sum B/0	3	**8**	4	**7**	5	**6**	6	**5**	7	**4**	8	**3**	9	**2**	A	**1**	B	**0**	0	**B**	1	**A**	2	**9**
sum 0/1	3	**9**	4	**8**	5	**7**	6	**6**	7	**5**	8	**4**	9	**3**	A	**2**	B	**1**	0	**0**	1	**B**	2	**A**
sum 1/2	3	**A**	4	**9**	5	**8**	6	**7**	7	**6**	8	**5**	9	**4**	A	**3**	B	**2**	0	**1**	1	**0**	2	**B**
sum 2/3	3	**B**	4	**A**	5	**9**	6	**8**	7	**7**	8	**6**	9	**5**	A	**4**	B	**3**	0	**2**	1	**1**	2	**0**
sum 3/4	3	**0**	4	**B**	5	**A**	6	**9**	7	**8**	8	**7**	9	**6**	A	**5**	B	**4**	0	**3**	1	**2**	2	**1**
sum 4/5	3	**1**	4	**0**	5	**B**	6	**A**	7	**9**	8	**8**	9	**7**	A	**6**	B	**5**	0	**4**	1	**3**	2	**2**
sum 5/6	3	**2**	4	**1**	5	**0**	6	**B**	7	**A**	8	**9**	9	**8**	A	**7**	B	**6**	0	**5**	1	**4**	2	**3**

(c) same complex divided into transpositions and inversions of sum 6/7

Transpositions

sum 6/7	(T0/6)	3	**3**	4	**2**	5	**1**	6	**0**	7	**B**	8	**A**	9	**9**	A	**8**	B	**7**	0	**6**	1	**5**	2	**4**
sum 8/9	(T1/7)	4	**4**	5	**3**	6	**2**	7	**1**	8	**0**	9	**B**	A	**A**	B	**9**	0	**8**	1	**7**	2	**6**(	3	**5**
sum A/B	(T2/8)	5	**5**	6	**4**	7	**3**	8	**2**	9	**1**	A	**0**	B	**B**	0	**A**	1	**9**	2	**8**(	3	**7**	4	**6**
sum 0/1	(T3/9)	6	**6**	7	**5**	8	**4**	9	**3**	A	**2**	B	**1**	0	**0**	1	**B**	2	**A**(	3	**9**	4	**8**	5	**7**
sum 2/3	(T4/A)	7	**7**	8	**6**	9	**5**	A	**4**	B	**3**	0	**2**	1	**1**	2	**0**(	3	**B**	4	**A**	5	**9**	6	**8**
sum 4/5	(T5/B)	8	**8**	9	**7**	A	**6**	B	**5**	0	**4**	1	**3**	2	**2**(	3	**1**	4	**0**	5	**B**	6	**A**	7	**9**

Inversions

sum 6/7		3	**3**	4	**2**	5	**1**	6	**0**	7	**B**	8	**A**	9	**9**	A	**8**	B	**7**	0	**6**	1	**5**	2	**4**
sum 7/8	(s7/1)	**4**	4	**3**	5	**2**	6	**1**	7	**0**	8	**B**	9	**A**	A	**9**	B	**8**	0	**7**	1	**6**	2	**5**(	3
sum 9/A	(s8/2)	**5**	5	**4**	6	**3**	7	**2**	8	**1**	9	**0**	A	**B**	B	**A**	0	**9**	1	**8**	2	**7**(	3	**6**	4
sum B/0	(s9/3)	**6**	6	**5**	7	**4**	8	**3**	9	**2**	A	**1**	B	**0**	0	**B**	1	**A**	2	**9**(	3	**8**	4	**7**	5
sum 1/2	(sA/4)	**7**	7	**6**	8	**5**	9	**4**	A	**3**	B	**2**	0	**1**	1	**0**	2	**B**(	3	**A**	4	**9**	5	**8**	6
sum 3/4	(sB/5)	**8**	8	**7**	9	**6**	A	**5**	B	**4**	0	**3**	1	**2**	2	**1**(	3	**0**	4	**B**	5	**A**	6	**9**	7
sum 5/6	(s0/6)	**9**	9	**8**	A	**7**	B	**6**	0	**5**	1	**4**	2	**3**(	3	**2**	4	**1**	5	**0**	6	**B**	7	**A**	8

1993 (301), in reference to Lewin 1990, the author shows ⟨E♭, E, A⟩ in a sums-7/1 set reinterpreted as ⟨E, D♯, A⟩ in a sums-7/0 set. In general, the six orderings of trichords can be represented by three cyclic sets, as shown in Example 3, since the T_n and T_n-complement interval can be found within the same cyclic set, stated forwards or backwards. As mentioned previously, trichords of cyclic sets have a central "axis" note with a surrounding cyclic "neighbor" dyad; if the axis note is reassigned, changes to the T and I values result. A particular reordering can result in motion to a cognate set (two of the three cyclic sets in Example 3 are cognates, sharing the sums of A or 9) and/or a "modulation through reinterpretation" to a cyclic set based on different intervals. In Example 3, the intervals-2/A cycles change to intervals-1/B cycles.

Stoecker's "axial isography" from one shared I-number can be expressed with an adaptation of Perle's cognate sets. These are cyclic sets from the same intervals that share one sum, but the concept can be expanded to just the shared sum without maintaining the same intervals. Example 4 shows some of the possibilities.

While cyclic sets were initially generated from complementary intervals, such as intervals 7/5 or 1/B, Perle later worked with Paul Lansky on using two or more intervals to create "derived sets." Lewin's Example 1.2 shows tetrachordal K-nets (C♯, E♭, G, E) and (C, D, A♭, F) and how they can be defined by two interlocking cycles of generating intervals 3/A and 9/2, as shown in Example 5. These sets are elaborations of T_B/T_1 cycles (9+2, 3+A) that Lewin defines later along with 5/7 cycles as "particularly useful" intervals, since they exhaust the aggregate within one cycle. By intertwining cycles (cycle B as 5 **2** 4 **1** . . . and cycle 1 as 5 **3** 6 **4** . . .), their sums form B and 1 cycles (76543 . . . and 89AB01 . . .). A group of tetrachordal K-nets related by strong isography (or by symmetry) is embedded in the derived set. An alternate representation of the K-nets as "sum scales" from aligned dyads at sums 5/A and sums 7/7 is also shown; this type of alignment of sum and difference scales is also used by Perle in *Twelve-Tone Tonality,* and is pointed out in Perle 1993 (p. 302).

In his Example 2.5, Lewin shows a "larger level" K-net between K-nets, in a recursive transformation: with ⟨T_1⟩, ⟨I_8⟩, ⟨I_9⟩ among K-nets considered positively isographic with T_1, I_5, I_6 within a K-net (his K-net alpha at ⟨T_3⟩). Such recursion, Lewin claims, is not part of Perle (or Lansky) cycles. We may comment on this assertion in two ways. First, a cyclic-set complex contains a complete series of Ts and Is of each cyclic set and thus of each segmental K-net. Thereby, the K-net of K-nets shown in Lewin's Example 2.5 can be expressed as the complex of cyclic sets defined by intervals 1/B and sums x/x+1. In my Example 2, Lewin's alpha K-net is from sums 5/6, his beta from sums 6/7, and his retgamma from sums 3/2 (in reverse order). The higher level ⟨T_1⟩, ⟨I_8⟩, ⟨I_9⟩ is contained within the complex, but defined as s0/6 (6/7 to 5/6), T4/A-sO/6(2/3 to 5/6), and T4/A (6/7 to 2/3).

A second representation of this recursive relationship is found by combining the types of derived sets shown in Example 5 with the corresponding next higher level in Perle's compositional

Example 3. Multiple orderings of a trichordal K-net

```
⟨4,5,6⟩    ⟨6,5,4⟩    = ints 2/A, sums 9/B
⟨4,6,5⟩    ⟨5,6,4⟩    = ints 1/B, sums A/B (cognate)
⟨5,4,6⟩    ⟨6,4,5⟩    = ints 1/B, sums 9/A (cognate)

sum 9/B   4 5 6 3 8 1 A B 0 9 2 7(4
sum A/B   4 6 5(5 6 4 7 3 8 2 9 1 B (cognate)
sum 9/A   5 4 6 3 7 2 8 1 9 0 A B(B (cognate)
```

Example 4. Axial isography as Perle cognate sets, from Stoecker Example 4

```
(a) Perle cognate sets

0 0 7 5 2 A 9 3 4 8 B 1 6 6   7/0
0 0 5 7 A 2 3 9 8 4 1 B 6 6   5/0

take sum 0 dyads and shuffle

0123456
0BA9876
```

(b) expanded to different interval systems (Stoecker ⟨A, E, B♭⟩ and ⟨G♯, E, C⟩ are underlined)

```
0 0 7 5 2 A 9 3 4 8 B 1 6 6                        sums 7/0  ints 5/7
0 0 8 4 4 8 0 / A 2 6 6 2 A   / 7 5 3 9 B 1/       sums 8/0  ints 4/8
4 5 7 2 A B 1 8   /   6 3 9 0                      sums 9/0  ints 3/9
0 A 2 8 4 6 / B 1 9 3 7 5                          sums A/0  ints 2/A
6 5 7 4 8 3 9 2 A 1 B 0                            sums B/0  ints 1/B
6 0 / 3 9 / 4 2 A 8 / 1 5 7 B                      sums 6/0  ints 6/6
```

system. To derive his compositional materials, Perle doesn't use just one cyclic set; he combines two cyclic sets in different alignments called "arrays." Example 6(a) shows a sum array and difference array using cyclic sets from Example 2. Perle typically extracts dyads, tetrachords, and hexachords from these alignments. An extracted "axis-dyad chord" is shown to the right of the array: 342 over 435. It is a hexachord consisting of a vertical axis dyad (4,3) with surrounding cyclic chord (3,2 and 4,5) and two sum tetrachords (3,4,4,3, and 4,2,3,5). Lambert's Example 31 shows an array from aligned sum-7/0 and -A/3 sets, given here in Example 6(b). The sets have the same axis notes aligned (0, 7, 2, 9 . . .), and therefore the cyclic chords surrounding these axis notes can be defined as tetrachordal K-nets related by positive isography.

With the idea of an array introduced, Example 6(c) reinterprets the derived set from Example 5 as two cyclic sets, sums A/9 and sums 5/4, embedded in pairs. To complete the recursive relationships shown in Lewin's Example 2.5, we create two derived sets, from sums-5/6 and -2/3 cyclic sets combining into sum-5/4/2/4 derived set, and from sums-5/6 and -6/7 cyclic sets combining into a sum-5/7/6/5 derived set. We may then align these two derived sets in an array that contains the recursive relationships within expanded axis-dyad chords, which contain a trio of K-nets with relationships by sums 5/6, 3/2, and 6/7—the ones shown in Lewin's Example 2.5. Lewin's representative K-nets {E, C♯, F}, {G♯, B♭, A}, and {D, C, E♭} are bracketed in the example. Different sums can be related by creating a derived set complex. Thus, as we made the analogy between K-nets and cyclic-set segments, we can do so again between recursive K-net relationships and array segments.

In his second main commentary on Perle cyclic sets, following the discussion of recursion, Lewin finds the Perle cyclic-set repre-

Example 5. Lewin's Example 1.2 with Perle cycles

```
interval 3/A, 9/2 generators

A/7/5/7   5 5 2 3 4 6 1 4 3 7 0 5 2 8 B 6 1 9 A 7 0 A 9 8 B B
9/2-cycle 5   2   4   1   3   0   2   B   1   A   0   9   B
A/3-cycle   5   3   6   4   7   5   8   6   9   7   A   8   B

TB-cycle  5       4       3       2       1       0       B
TB-cycle      2       1       0       B       A       9

T1-cycle    5       6       7       8       9       A       B
T1-cycle        3       4       5       6       7       8

K-nets T2/TA T3/T9 I5/IA (verticals)    K-nets T2/TA T3/T9 I7/I7

               sum  scales              sum  scales
5241302B1A09B  sA    543210B            s7        56789A
5364758697A8B        56789AB                      210BA9
241302B1A09B5  s5    210BA98            s7        345678
364758697A8B5        3456787                      43210B
```

sentation is lacking when the generating cycles are not 1, 5, 7, or B, and so the complete possibilities for pitch-class realization of sums are not present within one cyclic set. (The 1, 5, 7, and B cycles complete the twelve notes and so are "particularly useful" in generating complete lists of K-nets.) With intervals 6/6 as the generating intervals, for instance, the three cyclic-set complexes for sums 3/9 consist only of four members in the cyclic sets, as shown in Example 7(a), and thus present limited possibilities for Lewin's SLIDE, CURSOR, and FLIP transformations. The strongly isographic K-nets (0,9,6) and (5,4,B), for instance, cannot be obtained from one cyclic set, and thus the Perle cycles do not "communicate with one another" in Lewin's words. However, if we look at Lewin's Examples 3.2 and 3.3, it is clear that the two pc representations in beta, (G, C♯, F♯) and (A, E♭, E), are collapsed into one K-net, which is represented in Example 3.3 by T_6 and sums 1/7. In cyclic sets, there are analogously several representations for the same sums, as shown in Example 7(b); the resulting cyclic sets are called "sub-collections" by Perle. We may regard the representations for each sum as existing on a continuum, as shown in Example 7(c), which in this case (using interval 6) is also a derived set from intervals 6/1 and 6/B cycles, with sums 9/3/9/A.[4] With a derived-set complex built around this larger set, and motion within the complex regarded as "communication," we can perform the Lewin CURSOR, FLIP, and SLIDE transformations to connect each of the K-nets. In pc-set terms, the 4-9[0167]/4-28[0369] types can communicate in this way with the 2-6[06]/4-25[0268] types. Similar continua, but in terms of interval 3, can be created for all instances of Lewin's Example 4, as

[4]This type of extension of cycles is also found in Lewin 1966.

Example 6. Perle array and combined derived cycles to show recursion

```
(a)
Array of sum 7 (Key 2,2, from 6+8, 7+7), with axis-dyad hexachord shown
sum 6/7   3)3 4 2 5 1 6 0 7 B 8 A 9(9        3 4 2
sum 7/8   4)4 3 5 2 6 1 7 0 8 B 9 A(A        4 3 5

Array of difference B (Mode 9,B, from 6-9, 7-8)
sum 6/7   3)3 4 2 5 1 6 0 7 B 8 A 9(9
sum 8/9   4 4 5 3 6 2 7 1 8 0 9 B A(A

(b) Lambert demonstration of Perle arrays

sums 7/0  7 0 0 7 5 2 A 9 3 4 8 B 1 6
sums A/3  A 0 3 7 8 2 1 9 6 4 B B 4 6

Cyclic chords and Knets related by positive isography: 70A3, 0538, 5A81, A316, etc.


(c) reinterpretation of derived set from Example 5

A/7/5/7   5 5 2 3 4 6 1 4 3 7 0 5 2 8 B 6 1 9 A 7 0 A 9 8 B B
sA/9      5 5     4 6     3 7     2 8     1 9     0 A     B B
s5/4          2 3     1 4     0 5     B 6     A 7     9 8

Lewin Fig. 2.5 recursive sets embedded in derived-set array

5/4/2/4   3 2 2 0 4 1 3 B 5 0 4 A 6 B 5 9 7 A
s5/6      3 2     4 1     5 0     6 B     7 A
s2/3          2 0     3 B     4 A     5 9

5/7/6/5   3 2 5 1 4 1 6 0 5 0 7 B 6 B 8 A 7 A 9
s5/6      3 2     4 1     5 0     6 B     7 A
s6/7          5 1     6 0     7 B     8 A     9


array
5/4/2/4   3 2  2 0 4 1 3 B 5  0 4 A 6 B 5 9 7 A
5/7/6/5   6 B  8 A 7 A 9 9 8  9 A 8 9 8 B 7 A 7
```

Example 7. Intervals 6/6, sums 9/3 cyclic sets, multiple representations of same sums

(a)

9/3   5 4 B A (5 4
3/9   5 A B 4 (5 A

9/3   7 2 1 8 (7 2
3/9   7 8 1 2 (7 A

9/3   0 9 6 3 (0 9
3/9   0 3 6 9 (0 3

Knet T_6, I_9/I_3:   (096) (54B)

(b)

sums	pcs	sums	pcs	sums	pcs
9/3	5 4 B A	1/7	1 0 7 6	2/8	1 1 7 7
9/3	2 1 8 7	1/7	9 4 3 A	2/8	5 9 B 3
9/3	0 3 6 9	1/7	2 5 8 B	2/8	4 A A 4
				2/8	2 0 8 6

(c)

continuum: for sums 9/3

ints 6/1: 5 B 0 6 7 1 2 8 9 3 4 A
ints 6/B: 4 A 9 3 2 8 7 1 0 6 5 B

set 9/3/9/A: 5 4 B A 0 9 6 3 7 2 1 8 2 7 8 1 9 0 3 6 4 5 A B
set 3/9/8/9: 5 A B 9 0 3 6 2 7 8 1 7 2 1 8 0 9 6 3 5 4 B A A
etc.

Example 8. Lewin's Example 4.7 redesigned in Perle terms

```
sums
B/2  0B386592  4/7: 04316A97  3/6: B4215A87  0/3: B12A5784
     11A4774A       225B88B5       0330699        03966930

continuum complex (partial)
     0 3 6 9  7 A 1 4 7 A 1   B 2 5 8
      B 8 5 2  4 1 A 7 4 1 A   0 9 6 3

     0 3 6 9  7 A 1 4 7 A 1   B 2 5 8
      4 1 A 7  4 1 A 0 9 6 3   B 8 5 2

     0 3 6 9  7 A 1 4 7 A 1   B 2 5 8
      4 1 A 0  9 6 3 B 8 5   2 4 1 A 7

     0 3 6 9  7 A 1 4 7 A 1   B 2 5 8
      1 A 0  9 6 3 B 8 5 2 4   1 A 7 4
```

shown in my Example 8, allowing the cyclic sets of Lewin's Example 4.7 to communicate within a continuum complex. Lewin calls this communication "hyperwarping." A third continuum complex appears in Example 9 (matching Lewin's Example 5.4), allowing for communication among these interval-3 combinations. These combinations do not translate easily into cyclic-set notation, however, and the final continuum in Example 9 requires a shift in orientation of the cycles in the bottom row. Nonetheless, these continuum complexes avoid the types of elaborate cycles Lewin creates and maintain the "Principle of Contiguity" in cyclic-set materials.

To accommodate Example 5.4 without resorting to my continua, Lewin posits a derived set of intertwined cycles $T_{1,3,3}$ and complements $T_{B,9,9}$, creating a derived set of sums 3/1/3 and 4/4/6 (Example 10), which may also be labeled by the consecutive sums 3/4/1/4/3/6. The example also sets up a central premise of Lewin's article: if any network can be divided into two T-only-related networks with I-only relationships between networks (where the networks are well-formed and connected), then the T-only networks can be represented by two cyclic interval patterns corresponding to the patterns of Ts. These T networks can be used to create aligned cycles, corroborating Perle's assertion that any K-net can be represented by cyclic sets.

In addition to the continua shown in Example 9, we may add another explanation for Lewin's Example 5.5 based on Perle's cyclic-set arrays. Possible axis dyad hexachords that generate the pentachord-based networks from Lewin's Example 5.2 are shown in Example 11(a); they derive from interval-3-based cyclic sets aligned in arrays. The boldface indicates a duplicated note to fill out the hexachord from the core pentachord. The sums in the upper trichords are from sum-7/A cyclic sets, and the sums in the lower trichords of each hexachord are from three source cyclic

Example 9. Lewin Example 5.4 redesigned in Perle terms

```
         P                  Q1                 q1                 p

4/7   9704316A   9/0   81BA2754   2/5   5986B320   7/A   9A073461
      225B88B5         09366390         114A77A4         25528BB8
```

```
continuum complex (partial)

      9 0 3 6   8 B 2 5 8 B 2 5   7 A 1 4
       7 4 1 A   8 5 2 B 8 5 2 B   9 6 3 0

      9 0 3 6   8 B 2 5 8 B 2 5   7 A 1 4
       2 B 9 6   3 0 7 4 1 A 8 5   2 B 8 5

      9 0 3 6   8 B 2 5 8 B 2 5   7 A 1 4
       5 2 B 8   5 2 B 9 6 3 0 7   4 1 A 8

      9 0 3 6   8 B 2 5 8 B 2 5   7 A 1 4
       A 7 4 1   B 8 5 2 B 8 5 2   0 9 6 3
```

Example 10. Lewin derived set of Example 5.5, rewritten

$T_{1,3,3}$ = ints 1-3-3
sums 3/4/1/4/3/6: **3** 0 **4** 9 **7** 8 **A** 5 **B** 2 **2** ...
$T_{9,B,9}$ = ints 9-B-9

sets, with sums 0/3, B/2, 7/A. These sets are all in the same complex, given the continuum idea described above in connection with Example 9 (compare with Example 11[b]); and so, grouping the different alignments of cyclic sets and the complex of each cyclic set, we may use Perle's system to describe these pentachords in a "connected" way. The notes of pentachord "P" are underlined in their hexachord at (b). These hexachords may also be used for realignments (also Stoecker's SHIFT), such as that shown in Lewin's Example 5.7; networks P and Q1 are rewritten (with different doubled notes) as aligned sets of sums 1/4 and two

Example 11. Axis-dyad hexachord representations for the K-nets in Lewin's Example 5.2

```
(a)
P           Q1          Q9          p           q1          q9

s7/A        s7/A        s7/A        s7/A        s7/A        s7/A
4 3 7       8 B B       2 8 B       A 0 7       8 B B       8 2 5
9 3 0       A 2 1       5 6 8       4 3 7       6 5 9       B 1 2
s0/3        s0/3        sB/2        s7/A        sB/2        s0/3

(b)
complex of 0/3 includes B/2, 7/A

3/0   9 3 0 0 3 9 6 6   /   A 2 1 B 4 8 7 5
B/2   9 2 0 B 3 8 6 5   /   A 1 1 A 4 7 7 4

3/0   9 3 0 0 3 9 6 6   /   A 2 1 B 4 8 7 5
7/A   9 A 0 7 3 4 6 1   /   5 2 8 B B 8 2 5

(c)
realignments (from Lewin's Ex. 5.7)
s1/4        s1/4
4 9 7       2 2 B
3 9 0       A 8 1
s0/9        s6/9
```

cognate sets of sums 0/9 and 6/9, from the same type of expanded complex, as shown in Example 11(c).

This commentary is intended to fill in some of Perle's terminology and constructs, partly in answer to K-net terminology (such as "families" for cyclic-set complex), and partly in answer to Lewin's assertions about the lack of any representation for recursion in Perle's system and the problem with cycles that are not 1, 5, 7, or B in producing connected structures. The latter required the notion of continua, allowing all cycles at given sums to be represented in a kind of extended cyclic set. The cyclic basis of K-nets, from Perle's writings, will hopefully continue to be a source for directions in K-net research.

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