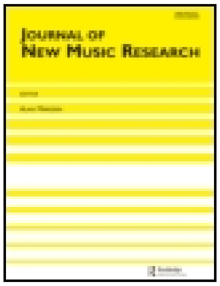

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Matthew Woolhouse [a]
[a] University of Cambridge, UK

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Modelling Tonal Attraction Between Adjacent Musical Elements

Matthew Woolhouse

University of Cambridge, UK

Abstract

A context-independent model of tonal attraction is presented based on the formal musical property of interval cycles. An interval cycle is the minimum number of additive iterations of an interval that are required for the original pitch classes to be re-stated. Interval cycles are conjectured to give rise to an abstract grouping property, *interval cycle proximity*, which in turn is responsible for tonal attraction. The model was tested using a probe tone experiment requiring subjects to rate the probe for strength of attraction/resolution with respect to a preceding context chord. The results, displayed as 'attraction profiles', agreed with the predictions of the model, and showed that even diatonic chords, such as dominant sevenths, can be heard chromatically. The results are discussed in relation to examples of real music and previous research within the field.

1. Introduction

> All musical phenomena rest upon kinetic processes and their inner dynamics.[1]
>
> (Ernst Kurth, 1931)

When listening to tonal music we perceive rhythmical groupings, temporal pitch patterns, distinctive timbres and fluctuating sound intensities. But what dynamic sensations do these elements create in combination with each other that are more than their sum? Why, for example, do we perceive a musical tone—which is simply an air-transmitted pressure wave with a given frequency—as possessing an energetic quality, a tendency to move towards or to attract other tones? Why, also, at the end of a musical phrase do some chords create a sense of rest or closure, while others seem to propel music (and us) forwards with increased momentum? Clearly, as Kurth recognized over seventy-five years ago, dynamic sensations are an axiomatic part of the musical experience. Without the quality of dynamism, musical pitches (including, crucially, tonal music's tonic element) would not have a capacity to 'resolve' or to 'attract' other pitches towards themselves. Indeed, it can be argued that music's pitch hierarchy, its tonality, is a consequence of the dynamic attraction between tones, a process that gives the tonic a dominating position within the hierarchy due to its pre-eminent ability to attract other tones. Without the 'kinetic processes' referred to by Kurth and others (see Rothfarb, 2002), our experience of Western music in the common practice period as being 'tonal' would be greatly diminished, if it were to exist at all.

This paper aims to model an important aspect of tonal music's dynamism, the tonal attraction between adjacent musical elements (pitches and/or chords), using a formal, context-independent model based on interval cycles. Following a short review of three context-dependent models of tonal attraction, my model and supporting hypothesis are set out, and then tested in an experiment employing a probe technique, similar to that used by Krumhansl and Shepard (1979).

1.1 Context-dependent models of tonal attraction

According to Rothfarb (2002, p. 949) the 'energiticist' concepts of Kurth were based on 'intuitively sensed intensities' in that '[t]he emphasis is on revealing and explicating qualitative characteristics and their psychic resonance rather than on quantifying or systematizing

[1]Cited and translated by Zuckerkandl (1956, p. 78).

Correspondence: Matthew Woolhouse, Centre for Music and Science, Faculty of Music, University of Cambridge, Cambridge, UK. E-mail: mhw31@cam.ac.uk

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them'. More recently however, energeticist ideas have influenced systematic approaches to pitch cognition amongst music analysts and psychologists, including Jamshed Bharucha, Steven Larson, and Fred Lerdahl.

Bharucha (1996) modelled the perceptual tendency of unstable pitches to resolve or move to stable pitches within a tonal context, a process he refers to as 'melodic anchoring'. The model is asymmetrical in that the temporal order is from unstable elements to stable elements, and not vice versa. The relationship between an unstable tone and its nearest anchors is modelled as a 'tonal force vector'. The strength of this vector is proportional to the strength of the activation of an anchor (for example, whether or not the anchor is the tonic of the prevailing key), and is inversely proportional to the distance in semitones between the unstable tone and the anchor (i.e. the further, the weaker). Within the model the tonal force vector is responsible for the creation of a listener's sense of expectation; that is, the expectation that an unstable pitch will move in a particular direction, either up or down. The overall directional expectation of an unstable tone is referred to as the 'yearning vector'. With reference to our sense of expectation and the psychological quality of yearning in music Bharucha states: 'All things being equal, the more stable a pitch class, the more highly expected it is . . . Expectation becomes specific or unambiguous when attention is directed to the immediate neighbourhood of a potential anchor and when alternative anchors within that neighbourhood are further away. This results in a strong, conscious expectation, a yearning' (1996, pp. 386–387).

Larson (2004) proposed that there are three musical forces that generate melodic completions, and therefore that determine tonal attraction, namely, 'gravity', 'inertia', and 'magnetism'. A computational model has been proposed based on an algorithm that quantifies these forces' interactions. According to Larson, musical gravity is 'the tendency of an unstable pitch to *descend*'; musical inertia is 'the tendency of a pattern of musical motion to continue in the *same* fashion; and musical magnetism is 'the tendency of an unstable note to move to the *nearest* stable pitch, a tendency that grows stronger the closer we get to a goal' (1997, p. 102; emphases, Larson's). Larson's (2004) model computes melodic magnetism by calculating (in a manner not unlike Bharucha, 1996) 'the magnetic pull towards the closest stable pitch as the difference between the pull of that pitch . . . minus the pull of the closest [stable] pitch in the other direction' (p. 463). In sum, Larson's model reflects his view that 'we experience musical motion metaphorically in terms of our experience of physical motions . . . the metaphor of musical motion is neither optional nor eliminable' (2004, p. 462).

Lerdahl's (2001) monograph *Tonal Pitch Space* (TPS), like his landmark work *A Generative Theory of Tonal Music* (GTTM) coauthored with Ray Jackendoff (Lerdahl & Jackendoff, 1983), is a synthesis of cognitive science and music theory, and like GTTM, TPS is predicated on rule-based, analytical processes. In GTTM there are four hierarchical structures that determine how a listener parses a piece of music (from the common practice period): grouping structure, metrical structure, time span reduction, and prolongational reduction. Each hierarchical structure (or domain) has a set of preference rules that allow the most plausible analysis to be derived with respect to the way in which an 'ideal' listener is expected to hear the musical work. In TPS Lerdahl proposes that GTTM's time span and prolongational reduction should be augmented and modified by the addition of 'stability conditions'. 'The stability conditions', writes Lerdahl, 'represent features of the tonal system as a whole rather than event structures in a specific piece. GTTM mostly ignores the stability conditions. They form the centerpiece of this study [TPS], treated through the concept of pitch space' (2001, p. 4). The pitch space to which Lerdahl refers is 'atemporal in that it represents more or less permanent knowledge about the system . . . This knowledge arises from the listening experience' (p. 41). Lerdahl first introduced the concept of pitch space, also referred to as 'basic space', in his article *Cognitive constraints on compositional systems* (1992).[2] Through a combination of vertical and horizontal distance the basic space is used to describe the relative distance of pitches, chords and 'regions' (Lerdahl's term for keys) to a local tonic. These distances are then combined with time span and prolongational reduction structures in order to calculate the tonal tension and attraction of a given passage of music. Lerdahl's model of tonal tension and attraction is also intended to provide a partial basis for understanding musical expectation and expression.

The models of Bharucha, Larson and Lerdahl are context-dependent in that each is based, to varying degrees, on music-theoretic descriptions of pitch organization; that is, they depend upon specifying the tonal context. For example, Bharucha's 'anchors' and Larson's 'magnetism' are calculated with respect to music-theoretically defined scale-step function and stability. In Lerdahl's pitch space model the relationship between levels is organized with respect to music-theoretic accounts of tonal organization and to empirically derived pitch hierarchies. The term 'context-independent model' refers to a model that does not rely upon descriptions of tonal music in order to model it. In this

[2]The basic space, as Lerdahl acknowledges, is very similar to that proposed by Deutsch and Feroe (1981), and empirically recovered by Krumhansl (1979), and Krumhansl and Kessler (1982). It has five levels: level *a* octave (or root) space; level *b* is fifth space; level *c* is triadic space; level *d* is diatonic space; and level *e* is chromatic space.

paper it is proposed that interval cycles—a formal, *context-independent* property of the cyclic group of cardinality 12—give rise to an abstract grouping property, which I refer to as *interval cycle proximity*. Furthermore, interval cycles are used to model the data from a perception experiment involving tonal attraction judgments.

The difference between context-independent and context-dependent models is important. The author presumes that tonal music is a by-product of principled cognitive processes that are subject to historical change, and not, for example, the result of a random assemblage of culturally idiosyncratic mental schemata. Whether or not this presumption is correct is of course a matter of debate and research; however, in order to elucidate these hypothesized cognitive processes it is not sufficient merely to catalogue their effects (i.e. to describe how tonal music is constituted), or build models that incorporate descriptions of these effects—more important is to uncover the root causes of the effects.

For example, a researcher wishing to make a predictive model of plant growth can adopt either a resultant or causal approach. In the first approach, the researcher might *observe* that growing plants are green, and therefore state in her model that vegetation that is green will grow. In the second approach she might *explicate* the molecular process of converting carbon dioxide into sugars using sunlight, photosynthesis. Neither model is necessarily incorrect; both will accurately predict growth. However, the levels at which the two models operate, the first at a resultant level, the second at a causal level, means that the utility and power of the latter is greater than the former (see McClamrock, 1991).

While it would be wrong to imply that Bharucha, Larson and Lerdahl's models are merely descriptive—akin to the resultant green-plant model referred to above—it could be argued that their explicatory power is reduced by the incorporation of music-theoretic observations. That is, the inclusion of context-dependent factors, such as the observation that the tonic is hierarchically important, means that the models are unable to show why or how certain tonal phenomena are instantiated at a cognitive level. Or put another way, a music-theoretic notion consciously built into a model, rather than being an outcome of it, will be beyond the power of the model to explain.

Interval cycles and interval cycle proximity are now discussed.

2. Interval cycles

An interval cycle can be defined informally as the minimum number of additive iterations of an interval that are required for the original pitch classes to be re-stated. For example, if the interval is a major third (four semitones) and the original pitch classes are C and E, the first additive iteration of an (ascending) major third states pitch classes E and G♯/A♭, the second additive iteration of a major third states pitch classes G♯/A♭ and C, and the third additive iteration of a major third states pitch classes C and E—the original pitch classes of the cycle. The interval cycle of a major third, therefore, is three. Note that enharmonic intervals are equivalent. That is, the interval cycle of a given interval is calculated with respect to its absolute size (in semitones), not with respect to its intervallic description; for example, C to E♭ = three semitones = C to D♯. Therefore, unlike interval descriptions such as 'major third' or 'perfect fourth', which are determined by harmonic context, interval cycles are independent of harmony, i.e. context-independent.

Interval cycles exist in music for the following reason. The word 'cycle' implies repetition, and in the case of musical pitch the repetition is of tones whose fundamental frequencies have a $1{:}2^n$ ratio, where n is an integer greater than 0. In the West's musical system two pitches separated by a frequency ratio of $1{:}2^n$ are said to be octave-related, and are usually perceived as equivalent to one another. Considerable empirical evidence exists to justify this assertion (see, for example, Allen, 1967; Thurlow & Erchul, 1977; Lockhead & Byrd, 1981; Demany & Armand, 1984; Dowling & Harwood, 1986), and Wright, Rivera, Hulse, Shyan, & Neiworth (2000) found evidence for octave generalization in rhesus monkeys.

2.1 Interval cycle proximity

The following section aims to show how interval cycles endow music with an abstract grouping property, referred to as interval cycle proximity. Figure 1 shows intervals of a given size iterated additively so that they appear as stacks. Each stack begins at zero on the y-axis (representing the originating note) and continues until it is exactly level with an octave-related pitch (an integer on the y-axis), at which point the interval completes its cycle and the stack terminates. The number of intervals in each stack is the interval cycle of the interval (shown in parentheses on the x-axis). For example, there are twelve intervals in the minor second stack (m2), six intervals in the major second stack (M2), four intervals in the minor third stack (m3), and so on. Note that some stacks only terminate after multiple octaves; for example, the perfect fourth (P4, iterated 12 times) requires five octaves for its stack to terminate.

The effect of octave equivalence—in which pitches separated by a frequency ratio of $1{:}\ 2^n$ are perceptually highly similar—can be shown by collapsing the multiple octaves on the y-axis in Figure 1, a process which leads to the 'compression' of interval stacks covering more

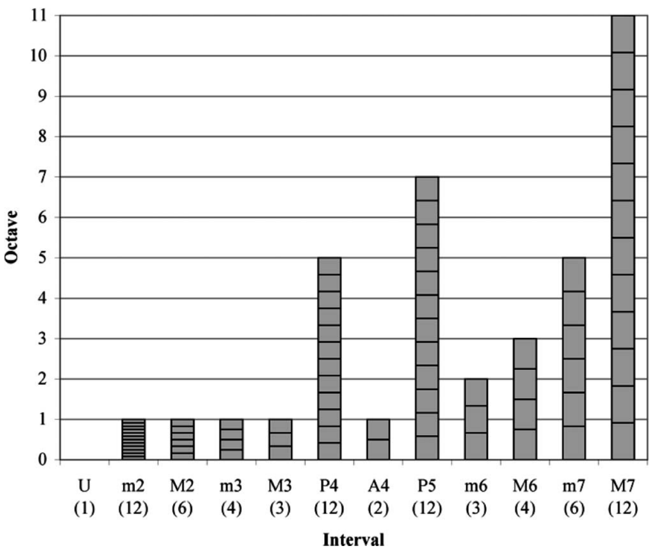

Fig. 1. Graph showing the minimum number of additive iterations of an interval that are required for a pitch bearing the same note-name as the originating pitch (represented by zero on the *y*-axis) to be repeated. Horizontal lines on the *y*-axis represent octave-related pitches. Interval cycle values are shown in parentheses on the *x*-axis.

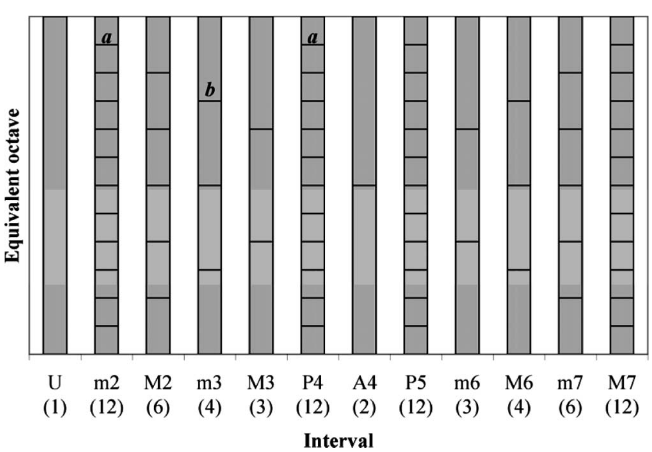

Fig. 2. The equivalent octave in which the interval stacks from Figure 1 are 'compressed' to the same height.

than one octave into a single 'equivalent octave'; see Figure 2.

The effect of collapsing the interval stacks can be seen on the relative sizes of intervals *a* and *b* in Figure 2. First, represented by the letter *a* , the minor second and perfect fourth intervals are the same size; second, the minor third, *b*, is larger than the perfect fourth, *a*.

This size-changing effect is not only limited to the minor second, the perfect fourth and the minor third, but is common to all intervals. For example, in the representation in Figure 2 any two pitches a minor third (m3) apart are relatively distant in comparison any two pitches a perfect fourth (P4) apart; any two pitches a minor seventh apart (m7) are relatively close in comparison to any two pitches an augmented fourth apart (A4), and so on. In the representation in Figure 2, interval cycle proximity is a result of listeners perceiving octave-related pitches as equivalent, and of being sensitive to the extent of the subdivision of the equivalent octave by interval cycles. The higher the interval cycle value, i.e. the higher the subdivision, the closer two pitches are in the equivalent octave; the lower the interval cycle value, i.e. the lower the subdivision, the more distant two pitches are in the equivalent octave.

From Figure 2 it is apparent that interval cycle proximity is not a discrete grouping process in which pitches are unambiguously associated with, or dissociated from each other; rather, pitches are associated by degree. For example, close pitches, such as those with interval cycle value 12 (m2, P4, P5, M7), are strongly associated, while distant pitches, such as those with interval cycle value 2 (A4/d5), are weakly associated. In this respect, interval cycle proximity is somewhat different to Gestalt Psychology's *prägnanz* law of proximity that states that physically or chronologically close elements are grouped and perceived as belonging together (Ellis, 1938). First, interval cycle proximity is not a measure of semitone distance in the rectilinear tone system; second, all pitches are associated within a spectrum ranging from relatively strong to relatively weak. In short, grouping implies 'either or', whereas association and imply 'varying degree'. Interval cycle proximity belongs to the latter case.[3]

2.2 Interval cycle proximity hypothesis

The interval cycle proximity hypothesis states that:

> Tonal attraction in music is proportional to the sum of the interval cycles formed between sequential pairs of tones and/or chords. Higher interval cycle values produce strong tonal attraction; lower interval cycle values produce weak tonal attraction.

The hypothesis is based on the idea that varying degrees of interval cycle proximity are perceptually integrated with feelings of attraction in the minds of encultured listeners (Woolhouse, 2007). The higher the interval cycle of two pitches, the closer their proximity in the equivalent octave (Figure 2), which in turn leads to higher tonal attraction; the lower the interval cycle of two pitches, the more distant their proximity in the

[3]For a more detailed explanation of interval cycle proximity, see Woolhouse and Cross (2010, in press).

equivalent octave, which in turn leads to lower tonal attraction. This hypothesis is tested in the experiment presented later in the paper.

The cognitive mechanism linking interval cycle proximity to tonal attraction is hypothesized to be as follows. Borrowing from the work of social scientist Howard Margolis and philosopher Mark Johnson, Brower (2000) proposes a cognitive theory of musical meaning based on the ideas that (1) cognition is comprised of mapping patterns of thought onto patterns of experience, and that (2) cognition involves the projection of bodily experiences onto patterns in other domains. In Brower's conception, pattern mapping implies that music takes on meaning with respect to itself (and other works within a listener's mnemonic musical corpus) as a result of a listener's ability to map in-coming musical patterns onto those stored in memory. And cross-domain projections explain how music acquires metaphorical meaning as a result of projecting musical patterns and/or schemas onto the physical experiences that a listener has in relation to his or her own body.

As a result of these processes, Brower hypothesizes that three types of memory-based representations give rise to musical meaning: (1) *intra-opus*—patterns within a work, such as the recapitulation of the First Subject in Sonata Form; (2) *musical schemas*—abstracted patterns, possibly involving pitch transition probabilities; and (3) *image schemas*—patterns abstracted from our knowledge of corporeal experiences, force and motion. Brower's central hypothesis rests on the idea that many aspects of tonal music, including melody, harmony, phrase structure and form, are based on image schemas derived from bodily experience. A primary motivation for Brower's work is to uncover the image schemas lying behind tonal music, thereby grounding music with respect to its bodily origins.

In relation to the hypothesis, I propose that tonal attraction is the result of cross-domain mappings between musical schemas, in which interval cycle proximity is a core structural component, and image schemas based on our real-world, physical experiences. This mechanism allows interval cycle proximity to impart dynamic characteristics onto music that as an inanimate, formal property of the chromatic octave it does not itself possess. Interval cycle proximity is the transcriptive information in music onto which are bound different image schemas, depending on the specificity of the information (or perhaps a single image schema varying in intensity). As a result, music is able to create different energetic sensations, many of which are experienced as different degrees of tonal attraction. Finally, it should be noted that image schemas derived from our experience of the physical world must by necessity involve certain basic features such as mass and motion, features that could be said to be synonymous with the experience of tonal music.

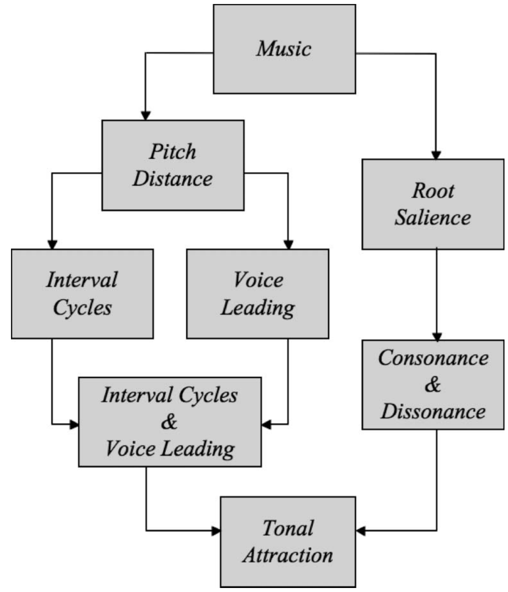

Fig. 3. Schematic diagram of the components of the interval cycle-based model of tonal attraction.

3. Model of tonal attraction

Figure 3 shows the components of the interval cycle-based model of tonal attraction. Within the model component *Interval Cycles* is the most important in terms of the extent to which it contributes to the calculation of tonal attraction. However, in addition to *Interval Cycles*, the model has a number of other components that are designed to modify the calculation of tonal attraction depending on the type of music being modelled. For example, if the attraction between functionally unambiguous diatonic chords is being modelled, i.e. between chords with identifiable roots such as in *God Save the Queen*, component *Root Salience* (a weighting method applied to the chords' root relationships) influences the calculation significantly. Alternatively, if the attraction between functionally ambiguous chromatic chords is being modelled, i.e. between chords with ill-defined roots such as at the opening of *Tristan und Isolde*, *Root Salience* does not influence the calculation.

The description of the components begins with the input to the model, *Music*, and ends with the output from the model, *Tonal Attraction*. Each component of the model is now formally described, and is accompanied by a commentary and illustrative example. The components are indicated using italic text at the beginning of the following subsections.[4]

[4]A version of the model without commentary and examples is presented in the Appendix.

3.1 *Music*

Given two successive chords (a chord is defined as a simultaneity containing one or more notes): past chord, X, and present chord, Y

$$X = \{x_1, x_2, \ldots, x|X|\}, \quad x_1 < x_2 < \ldots < x|X|$$

$$Y = \{y_1, y_2, \ldots, y|Y|\}, \quad y_1 < y_2 < \ldots < y|Y|$$

where $|X|$ *= size of set X,* $|Y|$ *= size of set Y, and where* x_i *and* y_j *are defined with reference to* $C_4 = 60$.

Component *Music* defines the number of pitches in the past chord (X) and the present chord (Y) and assigns each an integer value relative to C_4. For example, a G major first inversion triad to a C major root position triad:

X = G major triad (past chord) = $\{B_4, D_4, G_4\}$ = {59,62,67}
Y = C major triad (present chord) = $\{C_4, E_4, G_4\}$ = {60,64,67}

3.2 *Pitch distance*

Form matrix PD where

$$PD_{ij} = |y_j - x_i|, \quad i = 1, 2, \ldots, |X| \\ j = 1, 2, \ldots, |Y|$$

PD (pitch distance) is a pitch distance matrix measured in semitones. See Example 1.

Matrix PD		**Y (chord of C)**		
		60	**64**	**67**
X	**59**	1	5	8
(chord	**62**	2	2	5
of G)	**67**	7	3	0

Example 1.

3.3 *Interval cycles*

Form matrix IC where

$$IC_{ij} = \frac{12}{hcf(PD_{ij}, 12)} \quad for\ PD_{ij} \neq 0$$
$$IC_{ij} = 1 \quad if\ PD_{ij} = 0$$

where hcf(a, b) is the highest common factor of a and b, and a, b are whole numbers.

Distance values from matrix PD are converted into interval cycles in matrix IC (interval cycles). See Example 2.

3.4 *Voice leading*

Define a further matrix VL such that

$$VL_{ij} = \frac{\alpha}{PD_{ij} + \alpha}\ for\ some\ \alpha > 0$$

Euclidian distance between pitches is considered an important factor contributing to their perceived association. For example, experimental ratings of relatedness and similarity between pairs of tones have been found to depend on pitch distance (Levelt, Van der Greer, & Plomp, 1966; Krumhansl, 1979), as has the degree to which tones form discrete sub-groups (Bregman & Campbell, 1971; Dowling, 1973). Van Egmond, Povel, & Maris (1996) found that the perceptual similarity of a tonal melody and its transposition largely depended on two factors: first, the pitch distance between the original melody and its transposition; and second, key distance on the circle of fifths. The former variable was found to account for most of the variance in their experiments.

Various models of pitch perception have been proposed (notably those concerning forms of tonal attraction) in which pitch distance plays a crucial part. For example, as previously mentioned, Bharucha (1996) modelled melodic anchoring by calculating the sum of the linear distances between an unstable note and local points of stability. Lerdahl (1996) developed the prolongational component in GTTM (which assigns tensing and relaxing patterns to tonal sequences) by advancing a number of interrelated algorithms that incorporated pitch distance. These developments played an important role in Lerdahl's (2001) subsequent pitch space model.

Matrix PD		**Y**		
		60	**64**	**67**
X	**59**	1	5	8
	62	2	2	5
	67	7	3	0

→

Matrix IC		**Y**		
		60	**64**	**67**
X	**59**	12	12	3
	62	6	6	12
	67	12	4	1

Example 2.

Component *Voice Leading* controls the influence of pitch distance, i.e. semitone distance, using operand α. The rational for this component is that if the semitone distance between two pitches is small, the level of tonal attraction will increase, and vice versa. If α is large (e.g. $\alpha > 50$), the influence of component *Voice Leading* is relatively small; if α is small (e.g. $\alpha < 10$), the influence of component *Voice Leading* is relatively large. If no influence of pitch distance is required in the model, for example when modelling data produced in response to Shepard tones, the value of operand α can be set to infinity.

In Example 3, $\alpha = 50$.

3.5 *Interval cycles & voice leading*

Using entry-wise multiplication (array multiplication), combine matrix IC with matrix VL to give matrix ICVL, defined by

$$ICVL_{ij} = IC_{ij}VL_{ij}$$

Using entry-wise multiplication, matrix VL and matrix IC are combined to form matrix ICVL. This allows the influence of semitone distance to be combined with interval cycles to produce a single value per matrix entry. See Example 4.

3.6 *Root salience*

Form matrix RS1 where

$$RS1_{ij} = 1, \quad i = 1, 2, \ldots, |X| \\ j = 1, 2, \ldots, |Y|$$

If either chord has an identifiable root, let the row corresponding to the root of the past chord (X) be the m^{th} row and the column corresponding to the root of the root chord (Y) be the n^{th} column.

Form matrix RS2 where

$$RS2_{ij} = \begin{cases} RS1_{ij}, & \text{if } i \neq m \\ RS1_{ij}, & \text{if } j \neq n \\ \beta \times RS1_{ij}, & \text{if } i = m \text{ for some } \beta > 1 \\ \gamma \times RS1_{ij}, & \text{if } j = n \text{ for some } \gamma > \beta \end{cases}$$

such that root intersection entry $RS2_{mn} = \beta \times \gamma \times RS1_{mn}$. If neither chord has an identifiable root, form matrix RS2 where

$$RS2_{ij} = RS1_{ij}$$

Form matrix RS3 where

$$RS3_{ij} = \frac{RS2_{ij}}{\sum_{i=1}^{|X|} \sum_{j=1}^{|Y|} RS2_{ij}}$$

Matrix PD		Y		
		60	64	67
X	59	1	5	8
	62	2	2	5
	67	7	3	0

Matrix VL		Y		
		60	64	67
X	59	.98	.91	.86
	62	.96	.96	.91
	67	.88	.94	1.00

Example 3.

Matrix VL		Y		
		60	64	67
X	59	.98	.91	.86
	62	.96	.96	.91
	67	.88	.94	1.00

Matrix IC		Y		
		60	64	67
X	59	12	12	3
	62	6	6	12
	67	12	4	1

Matrix ICVL		Y		
		60	64	67
X	59	11.76	10.91	2.59
	62	5.77	5.77	10.91
	67	10.53	3.77	1.00

Example 4.

Component *Root Salience* is based on the idea that root relationships in functional tonal music are of greater perceptual importance, and must therefore be weighted more strongly than non-root relationships. This idea is supported theoretically (Piston, 1978)[5] and by Terhardt's (1974) virtual pitch model which proposes that the consonance of chords depends, to some degree, on the extent to which they create salient, harmonically related virtual pitches. For example, a major triad in root position produces a strong virtual pitch that is congruent with its root. Parncutt (1989) adapted Terhardt's model to chords constructed from octave-spaced sine tones. Parncutt found that the spectral composition of the major triad created the least ambiguous root psychoacoustically, followed by the minor triad.

The perception of tonal attraction depends strongly on music's temporal nature; that is, music is comprised of events in the perceptual past moving to events in the perceptual present. Studies into the nature of memory (for an overview see Baddeley, 2000) have found that the perception of an event is subject to temporal decay or interference, and therefore that events in the present are perceptually privileged with respect to those in the past (i.e. beyond working memory limits). The interval cycle proximity hypothesis specifies that tonal attraction occurs between sequential pitches/chords, and must therefore be tied closely to the way in which events in the past relate to events in the present. With respect to tonal attraction, this past/present relationship suggests that there is a perceptual inequality between a musical event occurring in the past, an image of which must be stored in memory (for it to be compared with the event in the present), and a musical event occurring in the present to which the listener has immediate, sensory access. This notion is modelled by stating that if both chords have roots, the root of the present chord (n of Y) must be weighted more strongly than the root of the past chord (m of X).

First, a new matrix, RS1, of identical dimensions to matrix PD (*Pitch Distance*), is formed in which all entries = 1. See Example 5. Second, the entries in row m (the past chord's root) are multiplied by some number greater than 1, β. Third, the entries in column n (the present chord's root) are multiplied by some number greater than β, γ. This ensures that the relationship between the roots of the chords has a greater weight than the relationship between the roots and non-roots of the chords, and that the present root has a greater weight than the past root (reflecting the time-dependent nature of present over past). This procedure also ensures that the relationships between

Matrix RS1		**Y**		
		(n) 60	**64**	**67**
X	**59**	1	1	1
	62	1	1	1
	(m) 67	1	1	1

Example 5.

the non-roots of chords have the least weight, and are therefore considered to have the least perceptual significance.

In Example 6, β, the operand of the root of the past chord (G), equals 4; γ, the operand of the root of the present chord (C), equals 8.

Matrix RS2 is scaled to form matrix RS3, such that the sum of entries of matrix RS3 is equal to 1 (the values have been rounded to two decimal places). Chromatic chords frequently do not have clearly identifiable roots and, as a result, unlike diatonic chords do not produce weightings in matrix RS2 greater than 1. The scaling from matrix RS2 to matrix RS3 ensures that diatonic and chromatic chords are treated equally with respect to tonal attraction. That is, it is assumed that the level of attraction in diatonic and chromatic music is broadly similar. If matrix RS2 were not scaled to matrix RS3, the weightings of diatonic chords would lead to higher calculated levels of attraction. Furthermore, it is assumed that the level of attraction between chords is broadly independent of the total number of notes in each chord. The scaling process ensures that chords made from different numbers of notes are treated equally with respect to tonal attraction.

3.7 *Consonance & dissonance*

(i) *If chords X and Y are both sensory consonances, or both are sensory dissonances, form matrix CD where*

$$CD_{ij} = RS3_{ij}$$

(ii) *If chord X (past) is dissonant and chord Y (present) is consonant, form matrix CD where*

$$CD_{ij} = (1+\delta) \times RS3_{ij} \quad \text{for some } \delta > 0$$

(iii) *If chord X (past) is consonant and chord Y (present) is dissonant, form matrix CD where*

$$CD_{ij} = (1-\delta) \times RS3_{ij} \quad \text{for some } \delta > 0$$

The order-dependent nature of the relationship between sensory consonance and dissonance has long been acknowledged. In the sixteenth century,

[5]'[T]he individual sonority of two chords' Piston asserts, 'is of less importance than the relation of the two roots to each other and to the scale from which they are drawn' (p. 20).

Matrix RS2		Y		
		60 (×8)	64	67
X	59	8	1	1
	62	8	1	1
	67 (×4)	32	4	4

Matrix RS3		Y		
		60 (×8)	64	67
X	59	0.13	0.02	0.02
	62	0.13	0.02	0.02
	67 (×4)	0.53	0.07	0.07

Example 6.

Zarlino (1558/1968) proscribed that a dissonance should always be quitted to a consonance by step, and that if this rule is broken 'the dissonance is made so noticeable . . . it can hardly be tolerated' (p. 95). Since the time of Zarlino the correct treatment of dissonance has been an important theme in music pedagogy (see, for example, Albrechtsberger (1790/1958), Marpung (1756/1958), and, in particular, Fux (1725/1965)). In the nineteenth century Helmholtz (1885/1954) provided tables of auditory roughness based on two factors: (1) the overall size of the interval, and (2) beating caused by the interference of adjacent harmonics in complex tones. In the early twentieth century Malmberg (1918) provided tables, similar to those of Helmholtz, ranking the consonance of musical intervals based on theoretical, mathematical and perceptual criteria.[6]

From a music-theoretic perspective, Piston (1978) proposed that unisons, octaves, fifths and fourths are perfect consonances, and that major and minor thirds and sixths are imperfect consonances; all other intervals are classified as dissonant. These assertions strongly agree with the psychoacoustic accounts referred to above, apart from one notable exception—the perfect fourth, which, Piston observed, is either consonant or dissonant depending on the musical context: 'the perfect fourth is dissonant when there is no tone below its lower tone. It is consonant when there is a third or perfect fifth below it' (p. 7). This observation is based on the fact that a chord in which there is a fourth above the bass has a tendency to resolve, as, for example, in a 6/4 to 5/3 progression. For Piston, therefore, dissonance was not only a matter of sensory beats and roughness, but of harmonic practice. And like Zarlino, over 400 years ago, this involves the treatment of consonance and dissonance as order dependent: 'unstable' dissonances move to 'stable' consonances.

Within the model consonant and dissonant interval categories broadly follow Piston's definitions, but with an important theoretical distinction. Among the dissonances listed by Piston are augmented and diminished intervals due to the fact that they require harmonic resolution irrespective of their sensory consonance or dissonance. For example, in the context of C major, the augmented triad {C, E, A♭} would normally resolve to {C, E, G}. In the interval cycle proximity model, however, consonance and dissonance depends not on the music-theoretic description of a chord but simply on the number of semitones between pitch classes. Two pitch classes that are zero, three, four or five semitones apart are considered consonant; two pitch classes that are one, two or six semitones apart are considered dissonant. Consonant intervals are, therefore, unisons/octaves, minor and major thirds and sixths, and perfect fourths and fifths (and their enharmonic equivalents); dissonant intervals are major and minor seconds and sevenths, diminished fifths and augmented fourths. Furthermore, for a chord to be considered dissonant by the model it needs to contain only a single dissonant interval, irrespective of the number of consonant intervals it might have. For example, the dominant seventh {G, B, D, F} contains six intervals in total (i.e. six pitch-pairs), four that are consonant, two that are dissonant: {G, F} and {B, F}. Within the model the dominant seventh is therefore a dissonant chord. If it precedes a consonant chord, the level of attraction increases, and if it follows a consonant chord, the level of attraction decreases.

To summarize, the effect of consonance and dissonance on tonal attraction is assumed to depend on temporal order:

(1) if both chords (X and Y) are consonant, or both are dissonant, then component *Consonance & Dissonance* does not affect the level of tonal attraction;
(2) if the past chord (X) is dissonant and the present chord (Y) is consonant, i.e. the progression is from dissonance to consonance, then component *Consonance & Dissonance* increases the level of tonal attraction; and
(3) if the past chord (X) is consonant and the present chord (Y) is dissonant, i.e. the progression is from

[6]For other related tables of consonance and dissonance, see Plomp and Levelt (1965), Kemeoka and Kuriyagawa (1969) Hutchinson and Knopoff (1978), and Burns and Ward (1982).

consonance to dissonance, then component *Consonance & Dissonance* decreases the level of tonal attraction.

In the example progression, G first inversion to C root position, both chords are consonant, and therefore component *Consonance & Dissonance* does not influence the level of tonal attraction (i.e. $CD_{ij} = RS3_{ij}$).

3.8 *Tonal attraction*

Using entry-wise multiplication (array multiplication), combine matrix ICVL with matrix CD to give matrix TA, defined by

$$TA_{ij} = ICVL_{ij}CD_{ij}$$

Sum the entries in matrix TA to produce the overall attraction value, A, where

$$A = \frac{\sum_{i=1}^{|X|} \sum_{j=1}^{|Y|} TA_{ij}}{12}$$

Using entry-wise multiplication, matrix TA (tonal attraction) combines interval cycle and voice leading information from matrix ICVL with root salience and consonance and dissonance information from matrix CD. Lastly, the entries of matrix TA are summed and scaled ($x/12$) to produce a single attraction value, A. See Example 7. The entries of matrix TA summed equal 8.80. This value when scaled ($x/12$) equals the attraction value of the progression of a G major first inversion triad to a C major root position triad: $A = 8.80/12 = 0.733$.

3.9 Limitations of the model

Although the model is sufficiently complex to model the types of experimental stimuli in this paper, it is limited in a number of respects. First, tonal attraction is calculated only between temporally adjacent events (occurring in the immediate past and present), and therefore is does not take into account events prior to the first chord, should the progression consist of more than two chords. Although this is unproblematic given the nature of the experiment stimuli to be modelled, empirical research strongly indicates that chords that are non-adjacent to the present, i.e. occurring before an immediately preceding chord, can influence the perception of later harmonic events (Tillmann & Bigand, 2004; Woolhouse, Cross, & Horton, 2006).

The model is also restricted in that there is no component for contextualizing tonal attraction with respect to a fixed tonic, a fact that is likely to be problematic when modelling larger tonal progressions. For example, although chord progressions G to C and C to F are transpositionally equivalent, the perception of tonal attraction that the two progressions produce when in the key of C major is different due to C being the tonic and F the subdominant (see Tillmann & Bigand, 2004). Although it is possible to contextualize the model with respect to a tonic and/or key using interval cycles,[7] there is no formal procedure within the model presented here that enables this to be done.

However, as has been said, despite these shortcomings the model was fit-for-purpose, especially given that the data to be modelled were produced from relatively simple experimental stimuli. The following section describes a pitch perception experiment designed to test the core component of the model, namely, *Interval Cycles*.

4. Experiment: Chord plus probe tone

An important aim of this experiment was to determine whether subjects' responses, when asked to rate the level of attraction and/or resolution between a chord and a following probe tone, significantly matched the model's predictions for chromatic pitches as well as for diatonic pitches. For example, the chord {C, E, G, B♭} might be perceived diatonically as the dominant seventh of F. However, it may also be perceived enharmonically and chromatically as a German sixth, {C, E, G, A♯}, i.e. as the flattened supertonic in the key of B. The model predicts that listeners will find both interpretations plausible, and respond accordingly when asked to rate the probe tone with respect to the preceding context chord.

Empirical research into chromatic harmony is relatively uncommon within the field of pitch perception; overwhelmingly, diatonic music has been the focus of study. There may be a number of reasons for this. First, chromatic chords are usually analysed within the theoretical framework of functional tonality (Gauldin, 2004), despite the fact that they often have ill-defined, ambiguous roots whose functional implications are not always clear.[8] As a consequence, modelling percepts such as tonal hierarchies produced by chromatic harmonies can be a difficult task: if we do not know what key a given chord implies, how is it to be interpreted? Second, the functional ambiguity of chromatic harmony may mean that data are produced that is difficult to explain using existing models of pitch cognition, be they the tonal hierarchy schema of Krumhansl and Kessler (1982), or event hierarchy models as advanced most notably by Bharucha (1984, 1994) and Bharucha and Krumhansl (1983). In short, the

[7]For example, if the tonal attractions of all 49 C-diatonic chord-pairs are calculated, those with the highest level of attraction have chord C in the terminal position, for example, G to C and b° to C.

[8]For a concrete example of this, see Lerdahl's (2001) two analyses of the opening of *Tristan und Isolde* (pp. 223–224).

Matrix ICVL		Y 60	Y 64	Y 67
X	59	11.76	10.91	2.59
X	62	5.77	5.77	10.91
X	67	10.53	3.77	1.00

Matrix CD		Y 60	Y 64	Y 67
X	59	0.13	0.02	0.02
X	62	0.13	0.02	0.02
X	67	0.53	0.07	0.07

Matrix TA		Y 60	Y 64	Y 67
X	59	1.53	0.22	0.05
X	62	0.75	0.12	0.22
X	67	5.58	0.26	0.07

Example 7.

theoretical problems surrounding chromatic harmony complicate an already complex field of research, and perhaps as a result have been approached empirically relatively infrequently. One notable exception, however, is Krumhansl and Schmuckler's (1986) examination of the effects of polytonality on listeners' perceptions of musical pitch.

Polytonality is a compositional method that employs materials from more than one key simultaneously. Krumhansl and Schmuckler used a well-known example of polytonality in Western music as the basis of one of their experiments, the beginning of the second *tableau* from Stravinsky's ballet, *Petroushka*. The music consists of two arpeggiated triads, one C major, the other F♯ major. Krumhansl and Schmuckler sought to identify whether listeners perceived this music as being composed from distinct keys or as a single, fused harmonic aggregate. Multiple regression analysis of their data, in which Krumhansl and Kessler's (1982) major tonal hierarchies of C and F♯, and the distributions of pitches in the context were the independent variables, showed that the distribution of tones in the context more strongly determined the ratings than the tonal hierarchies of the two keys. That is, no evidence was found to support the hypothesis that key-specific tonal hierarchies are operational with respect to the perception of polytonal music.

In a related study, Oram and Cuddy (1995) assessed the responsiveness of musically trained and untrained adults to pitch-distribution information in melodic contexts. The contexts consisted of a number of diatonic and non-diatonic, i.e. chromatic, melodies. As in Krumhansl and Schmuckler's (1986) study, probe tone ratings were significantly related to the frequency of occurrence of tones in the context (for both trained and untrained listeners). In addition, ratings decreased as the temporal distance between the probe tone and the tone matching the probe tone increased. For musically trained listeners, probe tone ratings for diatonic sequences also tended to reflect the influence of an internalized tonal schema, notably Krumhansl and Kessler's (1982) major-key tonal hierarchy.

In both of the above studies, subjects were required to rate the extent to which the succeeding probe tone fitted the preceding musical context. However, it is doubtful whether the paradigm 'goodness-of-fit' is particularly well suited to elicit from listeners the hypothesized effect of interval cycle proximity in music, which is the strength of tonal attraction between pitches and chords. The following experiment was devised (1) so that chords could be probed using a test paradigm that specifically matched the hypothesized effect of interval cycle proximity, and (2) so that the under-investigated area of chromaticism might be explored further.

4.1 Method

4.1.1 Stimulus design

The stimulus design used a single context chord followed by a single probe tone. The context chord and the probe tone each lasted two seconds. There was no temporal gap between the chord and probe tone; each stimulus therefore lasted four seconds. The context chords used in the study were a major triad {C, E, G}, a minor triad {C, E♭, G}, a dominant seventh {C, E, G, B♭}, a French sixth {C, E, G♭, B♭}, and a half-diminished seventh {C, E♭, G♭, B♭}. Each probe tone was one of the twelve chromatic pitches.

4.1.2 Subjects

The subjects were 11 adults, and included five females and six males with a mean age of 29 (SD = 8). All the subjects were members of the Faculty of Music at the University of Cambridge, England. The subjects in the study were largely involved with Western classical music, had received on average over 10 years' formal musical training, and professed to practising/playing/performing on average 6.6 h per week. The subjects' practical involvement with classical music was reflected in their listening habits which showed a 55% to 45% split in favour of classical music over popular/jazz/rock/folk music, and which was on average 7.4 h per week. The subjects received no financial remuneration for their participation in the study.

4.1.3 Apparatus

The stimuli were produced on an Apple Macintosh G5 computer using *SuperCollider*. Shepard tones were used for the presentation of the stimuli.[9] The subjects listened via headphones and adjusted the volume to a comfortable level before testing commenced.

4.1.4 Procedure

The subjects were required to rate on a seven-point scale (via a computer interface) the level of attraction and/or resolution they felt from the chord to the probe tone: seven for a high level of attraction, one for a low level of attraction.[10] Following each stimulus, subjects were given an unrestricted length of time in which to record their response.[11] The subjects repeated the experiment, and were therefore presented with each stimulus twice during the session, from which a mean rating was calculated. Prior to starting the experiment, each subject responded to ten practice stimuli. To counter order effects each subject was presented with the test stimuli in a different random order and transposition. Following the experiment session, participants completed a subject questionnaire.

4.1.5 Analysis

Prior to analysing the data, the stimuli presentation order for each subject was unscrambled and re-transposed in terms of a single pitch. In order to equalize subjects' use of the seven-point scale, the ratings were normalized and are presented as *Z*-scores. Factor *probe tone* was investigated with one-way within-subjects ANOVAs, and multiple pairwise comparisons were performed using the least statistical difference method (LSD, equivalent to multiple *t*-tests). Outliers were included in the analysis. The model's efficacy was assessed by treating the 12 probe tone values per context chord as 12-point profiles, and by calculating the Pearson correlation coefficient between the model's profiles and the data profiles.

4.2 Results

Table 1 shows the subjects' normalized mean rating for each probe tone following the context chord, either the major triad {C, E, G}, minor triad {C, E♭, G}, dominant seventh {C, E, G, B♭}, French sixth {C, E, G♭, B♭}, or half-diminished seventh {C, E♭, G♭, B♭}. Table 2 shows the model's tonal attraction values for each context chord (X, past event) to each probe tone (Y, present event). Due to the fact that the stimuli were presented using Shepard tones, all the pitch distances between the context chords and probe tones measured in semitones were equal. The operand controlling the effect of component *Voice Leading*, α, was therefore set to infinity; i.e. component *Voice Leading* did not influence the calculation of tonal attraction. Using a heuristic method, the root weight given to the major triad, minor triad and dominant seventh context chords in the model was 2 (i.e. $\beta = 2$). The weight ascribed to the following probe tone was 4 (i.e. $\gamma = 4$).[12] Due to the functional ambiguity of the French sixth and half-diminished seventh, no root weighting was given to these chords (i.e. $\beta = 1$). Similarly, because the probe tones were only single pitches, and therefore equally consonant, component *Consonance & Dissonance* did not change the level of tonal attraction. The results are discussed below; alternative possible variables were assessed using multiple regression analysis.

[9]Shepard tones (Shepard, 1964) consist of superimposed octave-related sinusoids in which the overall amplitude is controlled by a Gaussian function. The technique produces tones with approximately equal overall pitch height and with no clear pitch maxima or minima. Their use is intended to ameliorate voice leading effects.

[10]Requiring subjects to rate the probe for 'attraction/resolution' with respect to the preceding context chord links this experiment, albeit tangentially, to research on melodic and/or harmonic expectation. The literature on expectation in music is vast, and ranges from the pioneering theoretical work of Meyer (1956) to an extensive recent treatment of the subject by Huron (2006). From an empirical perspective, the work of Brown, Butler, & Jones (1994) is closely related to the current study. In a series of experiments Brown et al. required subjects to make 'goodness of completion' judgements for probe tones following short triadic contexts (arpeggio figures). Their results indicated that the temporal order of the context tones significantly affected subjects' responses.

[11]Prior to the experiment, the subjects were told that this was not an examination of their aural skills. Accordingly subjects were encouraged to respond based on their initial intuitive reactions to the stimuli.

[12]Although the root weight of each probe tone was notionally equal to 4, because this event was only a single pitch the value of operand γ did not affect the calculation. Had event Y contained more than one pitch, the value of operand γ would have affected the calculation (as shown in the model, Section 3).

Table 1. Subjects' normalized mean rating of each probe tone following one of the five context chords: Major triad {C, E, G}; Minor triad {C, E♭, G}; Dominant seventh {C, E, G, B♭}; French sixth {C, E, G♭, B♭}; Half-diminished seventh {C, E♭, G♭, B♭}. The standard deviations for each probe tone are shown in bold italic.

	Probe tone											
	C	D♭	D	E♭	E	F	G♭	G	A♭	A	B♭	B
Major triad	−0.824	0.358	0.003	−0.824	−0.410	1.718	−0.558	0.151	0.181	0.151	−0.322	−0.204
{C, E, G}	***0.858***	***1.055***	***0.816***	***0.702***	***0.863***	***0.668***	***0.675***	***0.625***	***0.799***	***1.029***	***0.908***	***0.951***
Minor triad	−0.647	0.033	0.477	−0.972	0.329	0.684	−0.410	−0.085	0.890	0.033	0.033	0.033
{C, E♭, G}	***0.689***	***0.744***	***0.785***	***0.698***	***0.726***	***0.821***	***0.356***	***0.858***	***0.776***	***0.607***	***0.657***	***0.478***
Dominant seventh	0.033	−0.115	−0.440	−0.529	−0.204	1.896	−0.736	−0.558	−0.588	0.506	−1.002	0.713
{C, E, G, B♭}	***1.006***	***0.805***	***0.523***	***0.484***	***0.750***	***0.446***	***0.439***	***0.520***	***0.789***	***0.750***	***0.549***	***0.827***
French sixth	−0.174	−0.071	−0.692	−0.263	−0.293	1.660	−0.174	−0.071	−0.692	−0.263	−0.293	1.660
{C, E, G♭, B♭}	***0.905***	***0.960***	***0.319***	***0.682***	***0.639***	***0.755***	***0.865***	***0.986***	***0.573***	***0.730***	***0.760***	***0.433***
Half-diminished seventh	−0.529	0.536	0.270	−0.410	0.299	0.831	−0.617	0.329	−0.233	−0.144	0.033	0.477
{C, E♭, G♭, B♭}	***0.540***	***1.005***	***0.740***	***0.564***	***0.849***	***0.740***	***0.523***	***0.997***	***0.743***	***0.504***	***0.981***	***0.868***

Table 2. The model's tonal attraction values from each context chord (X, past event) to a following tone (Y, present event). The context chords were: Major triad {C, E, G}; Minor triad {C, E♭, G}; Dominant seventh {C, E, G, B♭}; French sixth {C, E, G♭, B♭}; Half-diminished seventh {C, E♭, G♭, B♭}.

	Probe tone (Y, present event)											
	C	D♭	D	E♭	E	F	G♭	G	A♭	A	B♭	B
Major triad {C, E, G}	0.354	0.625	0.625	0.479	0.229	0.875	0.458	0.604	0.438	0.542	0.375	0.813
Minor triad {C, E♭, G}	0.375	0.667	0.750	0.250	0.458	0.750	0.417	0.583	0.625	0.333	0.583	0.625
Dominant seventh {C, E, G, B♭}	0.383	0.567	0.550	0.583	0.217	0.900	0.417	0.550	0.450	0.633	0.317	0.850
French sixth {C, E, G♭, B♭}	0.250	0.667	0.375	0.667	0.250	1.000	0.250	0.667	0.375	0.667	0.250	1.000
Half-diminished seventh {C, E♭, G♭, B♭}	0.271	0.708	0.500	0.438	0.479	0.875	0.208	0.646	0.563	0.458	0.458	0.813

Figures 4 to 8 show the subjects' mean responses (grey line) and the model's attraction values (black line) for each probe tone of the chromatic octave following a C major triad {C, E, G}, C minor triad {C, E♭, G}, dominant seventh {C, E, G, B♭}, French sixth {C, E, G♭, B♭}, and half-diminished seventh {C, E♭, G♭, B♭}.

In the ANOVA for the C major triad, factor *probe tone* was highly significant: $F(11,110) = 8.076$; $p < 0.005$, and the test of pairwise comparisons (LSD) revealed that F was rated significantly higher than any other pitch ($p < 0.005$). Below F came a group of pitches that were rated fairly equally: D♭, D, G, A♭ and A. These pitches, for the most part, were rated significantly higher than the group of pitches that were rated the lowest: C, E♭, E, G♭, B♭, and B. It is interesting to note that the chromatic pitches of the key of C major, i.e. D♭, E♭, G♭, A♭, and B♭, did not all receive the lowest ratings. The correlation coefficient between the model and data was

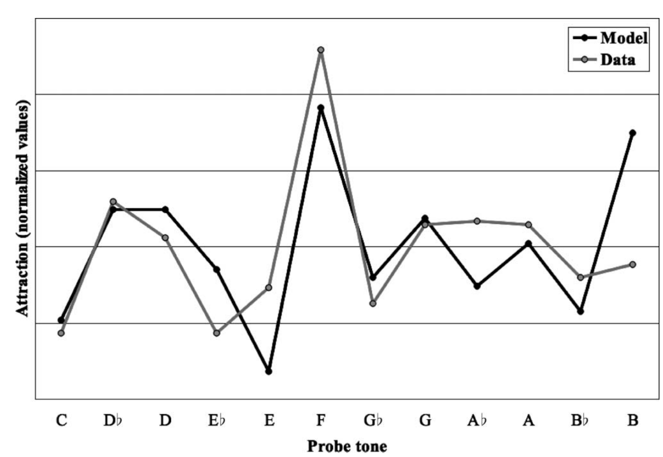

Fig. 4. Subjects' mean responses (grey line) and the model's attraction values (black line) for each pitch of the chromatic octave following a C major triad {C, E, G}.

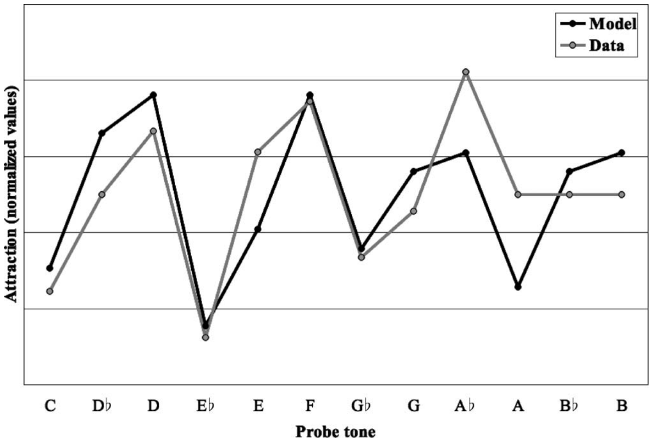

Fig. 5. Subjects' mean responses (grey line) and the model's predicted attraction values (black line) for each pitch of the chromatic octave following a C minor triad {C, E♭, G}.

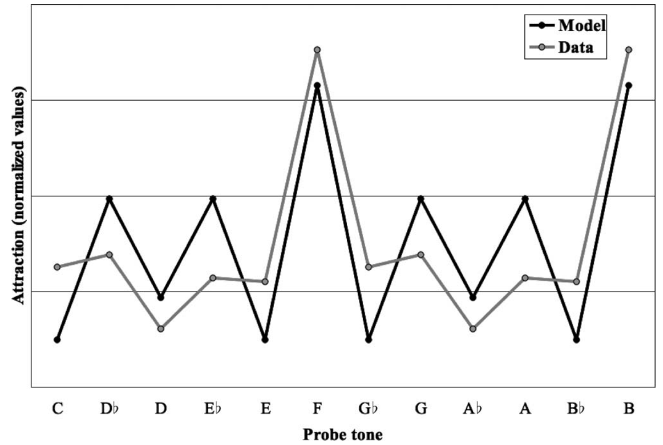

Fig. 7. Subjects' mean responses (grey line) and the model's attraction values (black line) for each pitch of the chromatic octave following a French sixth chord (C, E, G♭, B♭).

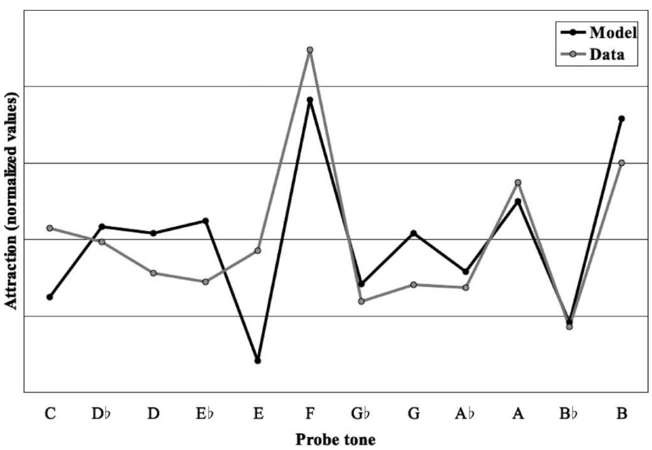

Fig. 6. Subjects' mean responses (grey line) and the model's attraction values (black line) for each pitch of the chromatic octave following a dominant seventh chord in the key of F {C, E, G, B♭}.

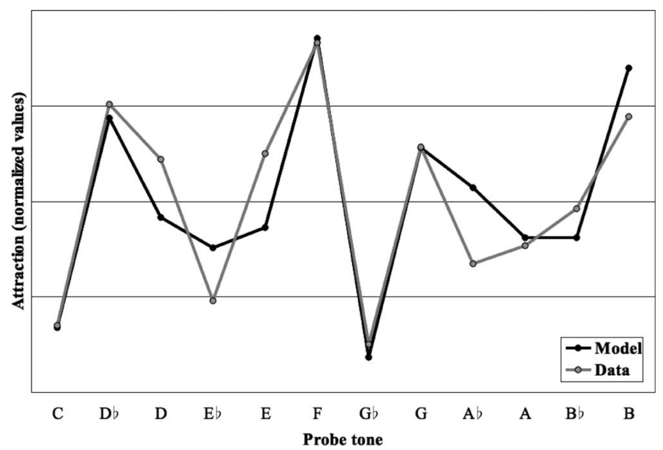

Fig. 8. Subjects' mean responses (grey line) and the model's attraction values (black line) for each pitch of the chromatic octave following a half-diminished seventh chord {C, E♭, G♭, B♭}.

significant: $r(10) = 0.69$; $p < 0.05$. The largest discrepancy between the data and model occurred for B, the seventh scale degree in the key of C major. Other discrepancies occurred for E and A♭; in both cases the subjects' ratings were higher than the values predicted by the model.

For the C minor triad, *probe tone* was highly significant: $F(11,110) = 6.449$; $p < 0.005$, and pairwise comparisons between the probe tones also showed significant differences. For example, A♭, the highest rated pitch, was significantly higher than nine other pitches, while E♭, the lowest rated pitch, was significantly lower than ten other pitches (C was the only pitch not significantly greater than E♭). The correlation between the model and data was highly significant: $r(10) = 0.76$; $p < 0.005$. The largest differences between the data and the model occurred for pitches E, A♭ and A, and in contrast to the model's predicted values the subjects' data showed very little variability between pitches A, B♭ and B. However, the model did correctly predict the two peaks in the data on D and F, and also the maximum trough in the data on pitch E♭.

For the dominant seventh {C, E, G, B♭} *probe tone* was highly significant: $F(11,110) = 14.372$; $p < 0.005$. Pairwise comparisons showed that F was rated significantly higher than any other pitch ($p < 0.005$). Below F, B and A were rated significantly higher than the other pitches, except for C. Below these pitches there was a large group of pitches that showed relatively few significant differences between their ratings. These included D♭, D, E♭, E, G♭, G and A♭. The correlation between the model and data was highly significant: $r(10) = 0.76$; $p < 0.005$. The model predicted the three peaks in the data in the correct order: first, F; second, B;

and third, A. The trough in the data on B♭ was also correctly predicted, although this was not the case for E, the pitch predicted to have the lowest rating by the model.

For the French sixth {C, E, G♭, B♭} *probe tone* was highly significant: $F(11,110) = 12.927$; $p < 0.005$. In the test of pairwise comparisons the data peaks on F and B were significantly greater than the remaining pitches.[13] Amongst these remaining pitches, D and A♭ were rated significantly lower than D♭, E♭, G♭, G and B♭; numerous other significant differences were also found. The correlation between the model and data was highly significant: $r(10) = 0.79$; $p < 0.005$. The model correctly predicted the two peaks in the data on F and B. The remaining data peaks and troughs were, to a large extent, also correctly predicted; however, the model predicted a greater degree of difference between these pitches than was found in the subject data.

Finally, for the half-diminished seventh {C, E♭, G♭, B♭} *probe tone* was highly significant: $F(11,110) = 3.782$; $p < 0.005$. In the pairwise comparisons test F, the pitch with the highest mean rating, was significantly higher than either C, E♭, G♭, A♭, A and B♭. G♭, the pitch with the lowest mean rating, was significantly lower than D♭, D, E, F, G, B♭ and B. Numerous other significant differences between pitches were also found, although not to the same extent as between F and G♭. The correlation between the model and data was highly significant: $r(10) = 0.89$; $p < 0.005$, and the three main data peaks on D♭, F and B were correctly predicted (although not quite in the correct order). The model also correctly predicted the two main troughs in the data on C and G♭. Less successfully modelled, however, were the data for D and E, which were higher than expected, and the data for E♭ and A♭, which were lower than expected. Despite these discrepancies, the fit of the data to the model was strong.

The analyses reported above are summarized in Table 3.

4.3 Discussion

The results appear to indicate that the task 'attraction and/or resolution' successfully captured the hypothesized effect of interval cycle proximity on listeners' perceptions of tonal attraction. The average correlation between the model and data across the five chords used in the experiment was 0.78, ranging from 0.69 for the major triad to 0.89 for the half-diminished seventh chord.

The data peak on F following the C major triad suggests that the subjects may have interpreted the context as a dominant harmony. It is possible, therefore, that the mental schema of the key of F (major) was primed when the subjects were asked to attend to the attraction/resolution potential of the C major triad. In order to ascertain whether the data and model were more closely related for the pitches of the key of F major than for the pitches lying outside F major, two separate correlations were run: first, using the pitches of F major ($n = 7$, see Figure 9); and second, using the pitches lying outside F major ($n = 5$, i.e. G♭, A♭, B, D♭ and E♭).

The correlation coefficient produced between the model and data for the seven pitches of F major was high: $r(5) = 0.90$; $p < 0.01$. In contrast, the coefficient produced between the model and data by the five pitches lying outside F major was low: $r(3) = 0.24$; n.s. If the interval cycle proximity hypothesis is correct, this evidence, together with the data peak on F, appears to support the view that the subjects perceived the major triad as the dominant harmony of F major.

Although the interval cycles of the C major triad indicate that it is most strongly attracted to the pitch a perfect fifth below its root, it should be noted that all major triads possess this property. The possible implications of this are now explored.

In the key of C major there are three major triads: C, F and G. As shown in Figure 4, for both the model and data the maximum level of attraction for a C major triad was to F (possibly to the key of F major). Similarly, the triad of F major is attracted to B♭, and G major is attracted to C, the tonic. Therefore, with respect to the major triads of C major (C, F and G) there is a greater tendency for attraction towards the subdominant (or flat keys) than towards the dominant (or sharp keys). Another way to conceptualize this is to think of the key of C major (or any major key) as a stable pitch set, albeit psychoacoustically (Huron, 1994). Within this pitch set there are a number of chords, for example, the C and F major triads, that are attracted towards pitches (and possibly towards keys) that are outside or beyond the key of C major. In relation to the model, the only major triad in the key of C major whose attraction is not destabilizing in this respect, i.e. that does not point outside C major, is G—the triad that plays a central role in key establishment and stabilization: 'The strongest tonal factor in music is the dominant effect. Standing alone, it determines the key much more decisively than the tonic chord itself' (Piston, 1978, p. 51). This suggests that a modulation to the dominant might be perceived differently from a modulation to the subdominant, in that the former will tend to be attracted *back* to the first key, whereas the latter will tend to create a clearer shift in tonal organization away from the first

[13] It should be noted that the data have been averaged between pitches a tritone apart. For example, the data for C and G♭/F♯ have been summed and divided by two. This is because (1) the French sixth is comprised of two major thirds a tritone apart, (2) the stimuli were presented using Shepard tones, i.e. in multiple octaves, and (3) the stimuli were randomly transposed. As a result, the French sixth is identical in relation to probe tones a diminished fifth apart.

Table 3. Summary of AVOVA and correlation analysis.

Chord	ANOVA Factor: Probe Tone	Pearson correlation coefficient between model and data
C major triad {C, E, G}	$F(11,110) = 8.076$; $p < 0.005$	$r(10) = 0.69$; $p < 0.05$
C minor triad {C, E♭, G}	$F(11,110) = 6.449$; $p < 0.005$	$r(10) = 0.76$; $p < 0.005$
Dominant seventh {C, E, G, B♭}	$F(11,110) = 14.372$; $p < 0.005$	$r(10) = 0.76$; $p < 0.005$
French sixth {C, E, F♯, A♯}	$F(11,110) = 12.927$; $p < 0.005$	$r(10) = 0.79$; $p < 0.005$
Half-diminished seventh {C, E♭, G♭, B♭}	$F(11,110) = 3.782$; $p < 0.005$	$r(10) = 0.89$; $p < 0.005$

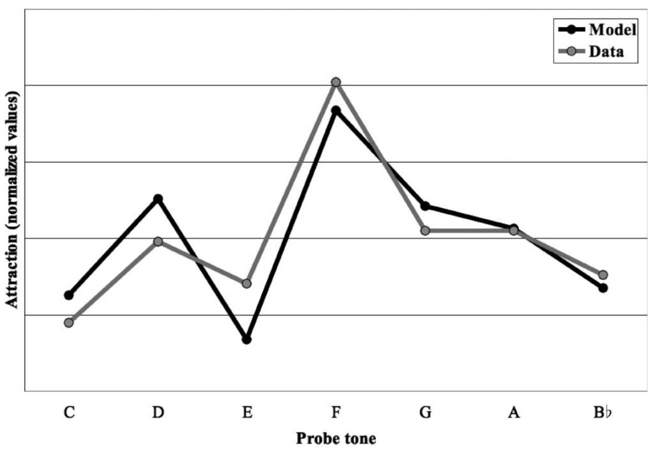

Fig. 9. Subjects' mean responses (grey line) and the model's attraction values (black line) for the pitches of F major following a C major triad.

key. Simply put, in a modulation to the dominant there is a tendency for the new tonic to be attracted to the pitch/key a fifth lower, i.e. to the original tonic; in a modulation to the subdominant there is reduced tendency for the new tonic to be attracted to the pitch/key a fifth higher, i.e. to the original tonic.

Perceptual evidence supporting this conjecture was found by Cuddy and Thompson (1992) who presented musically trained and untrained listeners with short chord sequences from Bach chorales that modulated to keys clockwise (sharp) and anticlockwise (flat) around the circle of fifths. The subjects were required to rate the 'goodness-of-fit' of probe tones to the preceding modulating sequences. Regression analysis revealed a directional asymmetry in the perceived key movement conveyed by the sequences. Cuddy and Thompson used Krumhansl and Kessler's (1982) key-distance map to represent their findings spatially. For a given modulation distance, modulations in a flat direction created a larger shift in the key-distance map towards the final key than sharp modulations. That is, modulations to the dominant tended to stay relatively close to or 'shadowed' the original tonic, modulations to the subdominant did not. This finding is consistent with the predictions of the model outlined above.

The data and model for the C minor triad presents a very different picture to that of the C major triad. Here the main peak in the data was on A♭. This suggests that the subjects may have interpreted the C minor triad as chord iii in A♭, the key of the sub-mediant. The aspect of the stimulus that influenced the subjects' judgments in this respect may have been the leading-tone progression G (in the chord) to A♭ (the probe tone). Two other peaks on D and F in the data and model are interesting and require comment.

First, the peak on D, which is more pronounced than for the C major chord, suggests that subjects perceived the stimulus C minor triad to D in a manner akin to a Phrygian cadence. In a sixteenth-century Phrygian cadence, the lowest part descended to the tonic (or *final*) by a semitone step; the highest part normally rose to the tonic by a whole-tone step. For example, if the lower part was E♭ to D, the upper part would be C to D. In the eighteenth century the term Phrygian cadence came to imply a ivb to V cadence in a minor key, and often occurred at the end of a slow movement or Introduction prior to an Allegro. For example, if the tonic was G minor, the Phrygian cadence would be a C minor first inversion chord (ivb) followed by a D major chord (V), as similar to the stimulus with the probe tone on D. Figure 10 is an example of a Phrygian cadence from J.S. Bach's *Brandenburg Concerto No. 4*, and is in the key of E minor.

Second, the peak on F, although not as pronounced as for the C major chord, suggests that subjects perceived the stimulus C minor to F as a modal perfect cadence. That is, despite the absence of the leading tone (E♮), subjects still experienced a relatively strong sense of attraction from the chord to the probe tone. Instances of modal perfect cadences are now relatively rare; however, the English hymn tradition has preserved a number of examples, as in the case from the end of the eighteenth-century Advent hymn, *Veni Emmanuel*. The hymn is in the Dorian mode, the modal perfect cadence is indicated by the asterisk in Figure 11.

The data and model produced in relation to the C major and C minor triads contrast the attraction properties of the two chords and puts forward reasons that may account, to some degree, for their different functions within music. These differences are more

Fig. 10. An example of a Phrygian cadence from J.S. Bach's *Brandenburg Concerto, No. 4*.

Fig. 11. The final phrase from the eighteenth-century Advent hymn, *Veni Emmanuel*. The harmonization is taken from *The New English Hymnal* (1986). The modal perfect cadence is indicated by the asterisk.

clearly observed when the major and minor-chord tonal attraction profiles produced by the model are overlaid in a single graph, see Figure 12.

By comparing the profiles in Figure 12 two observations can be made. First, according to the model the major triad has a higher level of attraction to F than the minor triad (0.875 versus 0.750). Second, the attraction maxima and minima are greater for the major triad than for the minor triad.

The first observation—that the attraction to the pitch a perfect fifth below the root is greater for the major triad than for the minor triad—might explain why the perfect cadence occurs more frequently than the modal perfect cadence: if the first chord of the cadence is major the attraction to the following (root) pitch is high; if the first chord of the cadence is minor the attraction to the following (root) pitch is lower.

The second observation—that there is greater variability (maxima and minima) in the levels of attraction produced by the major triad—is reflected in the standard deviations of the two profiles, which for the major triad $= 0.187$, and for the minor triad $= 0.165$. This suggests that the attraction of the major triad is more dynamic and changeable across the pitches of the chromatic octave than the minor triad. The difference in the variability of major and minor triads was also reflected in the standard deviations of the subjects' responses to the two chords, which for the normalized means of the major triad was 0.68, and for the minor triad was 0.53.

For the dominant seventh {C, E, G, B♭} the model correctly identified F as the pitch with the highest level of attraction, and in this respect the model and data support Piston (1978, p. 236) who stated, '[t]he regular resolution of V^7 [is] to I [and] is probably the most fundamental harmonic progression in music' (see also Schoenberg, 1978, p. 134). However, the model and data for the dominant seventh chord also had a peak on B. In the introduction to this experiment (Section 4) it was noted that the chord {C, E, G, B♭} could also be interpreted enharmonically (and chromatically) as the German sixth {C, E, G, A♮}. The resolution for a German sixth is usually via a chromatic downward step (in this case from C to B), as demonstrated in the excerpt by Schubert in Figure 13. The high rating given to B indicated that the subjects were also able to interpret the chord and probe tone chromatically, i.e. in a manner akin to a German sixth.

Typically, the French sixth functions as a chromatically altered secondary dominant chord prior to chord V, or as a chromatically altered dominant chord prior to chord I.[14] With respect to the French sixth used in the experiment {C, E, F♯/G♭, A♯/B♭}, if C is the root, the chromatic alteration involves lowering G♮ to G♭ to produce the chord {C, E, G♭, B♭}; if F♯ is the root the chromatic alteration involves lowering C♯ to C♮ to produce the chord {C, E, F♯, A♯}. The usual resolution of {C, E, G♭, B♭} is to F and the usual resolution of {C, E, F♯, A♯} is to B; see Figure 14. The peaks on F and B in Figure 7 show that the data and the model conformed well to the way in which the French sixth typically resolves within musical contexts.

The correlation between the data and model for the half-diminished seventh chord was strong ($r = 0.89$). In total, four data and model peaks coincided on pitches

[14] Opinions differ on this point. For example, Eric Taylor (1991) describes all augmented sixth chords as having arisen, 'because accidentals have been introduced into a simple progression: IVb to V' (p. 161). Whatever the contrapuntal origin, the function, however, is clearly as a dominant or secondary-dominant harmony (Piston, 1978).

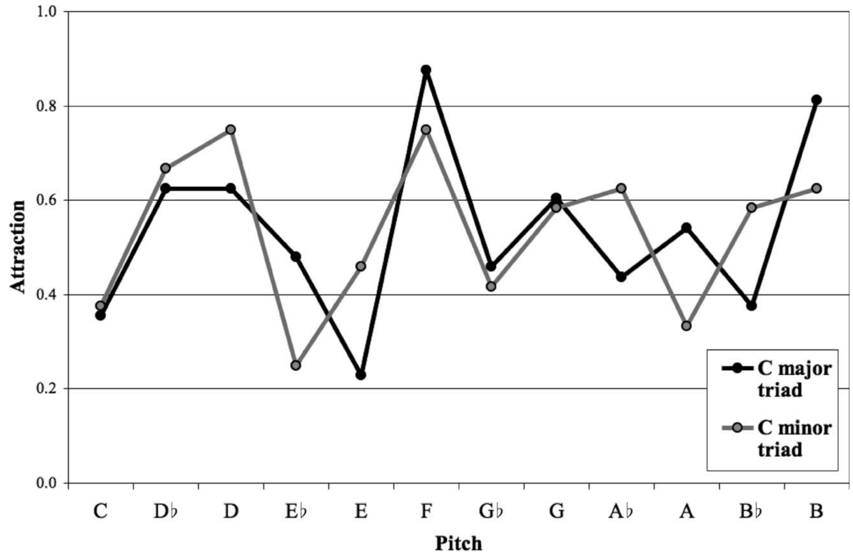

Fig. 12. The model's tonal attraction profiles for C major and C minor triads.

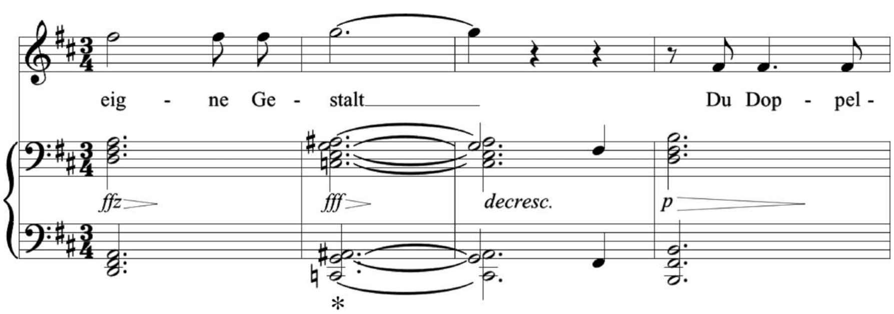

Fig. 13. Franz Schubert, *Der Doppelgänger*, bars 40–44. The German sixth {C, E, G, A♯} is indicated by the asterisk, and resolves via a downward chromatic step to B.

Fig. 14. Two enharmonically related French sixth chords. The usual resolution of chord {C, E, G♭, B♭} is to F; the usual resolution of chord {C, E, F♯, A♯} is to B. French sixths also frequently function as chromatically altered secondary dominant chords prior to the dominant.

D♭, F, G and B (see Figure 8). This discussion is limited to the first two data peaks, D♭ and F. The peak on D♭ suggests that listeners may have interpreted the preceding chord as an incomplete dominant ninth chord in D♭ major: {(A♭), C, E♭, G♭, B♭} to D♭, as in the chorale *Jesu, nun sei gepreiset* harmonized by J.S. Bach in C major (Riemenschneider, 1941, No. 11); see Figure 15.

The peak on F in Figure 8 suggests that listeners may also have interpreted the chord and the probe tone as chord ii^7 leading to the dominant of B♭: {C, E♭, G♭, B♭}→ {F}(V) → {B♭}(I), as in the chorale *Jesu, du mein liebstes Leben*, harmonized by J.S. Bach in G minor (Riemenschneider, 1941, No. 243), see Figure 16.

4.4 *Post hoc* analysis

Finally, the question arises as to whether, in addition to interval cycles, other variables are able to model accurately the probe tone ratings obtained in relation to the C major, C minor, dominant seventh, French sixth and half-diminished seventh chords.[15]

[15]Parncutt and Bregmann (2000, pp. 31–33) noted that the modelling of melodic effects could not be accounted for adequately using psychoacoustic models. Given that subjects were required to rate the level of attraction and/or resolution they felt from the chord to the probe tone, the experiment paradigm had an implied melodic dimension. Perhaps not surprisingly therefore, initial analysis of the data using Parncutt's (1993) chroma salience algorithm revealed no significant positive correlations with the data. This can perhaps be most clearly seen in Figure 5 in which, following a C minor triad, pitches C, E♭ and G were rated poorly, virtually the

Fig. 15. An example of a half-diminished seventh chord (*) in J.S. Bach's harmonization of the chorale, *Jesu, nun sei gepreiset* (Riemenschneider, 1941, No. 11).

Fig. 16. An example of a half-diminished seventh chord (*) in J.S. Bach's harmonization of the chorale, *Jesu, du mein liebstes Leben* (Riemenschneider, 1941, No. 243).

The additional variables included in the analysis were semitone distance and perfect fifth distance. Semitone distance was used in order to ascertain whether pitch proximity in terms of distance on the circle of semitones might account for the variability; perfect fifth distance in order to ascertain whether harmonic relatedness in terms distance on the circle of fifths might account for the variability.

For each probe tone, distances were calculated in relation to each pitch of the context chord and summed. The distances for these two variables were inverted so that pitches close together on the circle had higher values than pitches further apart. It was hypothesized that pitches close together on the semitone and perfect fifth circles would be associated with high levels of attraction, and vice versa.

The semitone and perfect fifth distances of the roots of the major triad, the minor triad and the dominant seventh were weighted so as to match the parameters of the interval cycle model, i.e. the weight given to the root was double that of the non-root pitches ($\beta = 2$, see Section 3.6). Figure 17 shows the semitone and perfect fifth distances for each probe tone from the C major triad.

Multiple regression analysis was used to assess the relative contributions of the variables to the probe tone profile of each chord. The independent variables were *interval cycle model*, *semitone distance* and *perfect fifth distance*; the dependent variable was the subjects' probe tone ratings for each chord. The analysis was carried out separately for each chord. Due to the fact that the semitone and perfect fifth distance profiles produced by the French sixth have no variability, this chord was omitted from the analysis. Table 4 is a summary of the regression analysis used to predict the probe tone profiles of the C major, C minor, dominant seventh and half-diminished seventh chords.

To summarize Table 4, *interval cycle model* was the only variable to have significantly influenced listeners' judgments of tonal attraction. Consideration of the beta-coefficients shows that this variable was consistently more important than the other variables; the *t*-statistics show that *semitone distance* and *perfect fifth distance* did not contribute significantly to the power of the regression equations.

Failure of the two distance variables may have occurred for a number of reasons. First, each probe tone distance was calculated by summing the distances of all the pitches in the context chord. While this method is objective, it may not accurately reflect auditory stream segregation processes (Bregman, 1991); that is, the cognitive operation whereby non-concurrent frequencies occupying a relatively narrow bandwidth are grouped together.[16] A better method for constructing these variables may therefore have been to take into account only those pitches that were relatively close in the first instance, and to disregard distances that were, for example, greater than a major third. Second, the stimuli were delivered using Shepard tones, i.e. in multiple octaves. Shepard tones are hypothesized to lessen the

opposite of the output of Parncutt's chroma salience algorithm. Further analysis of the data using psychoacoustic models was therefore abandoned.

[16]In a musical context, auditory stream segregation roughly equates to voice leading.

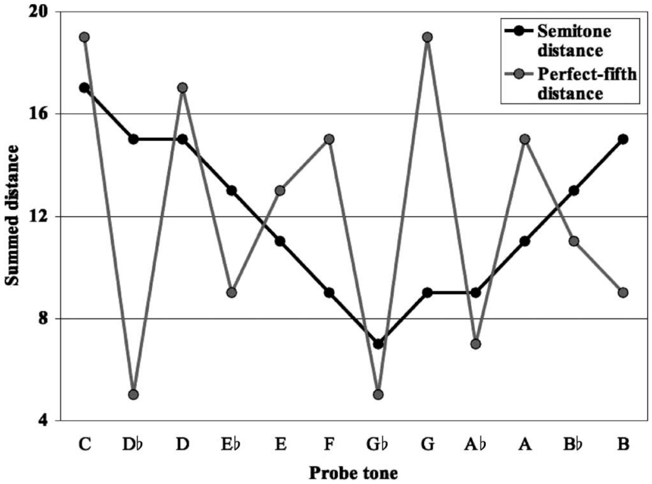

Fig. 17. Semitone and perfect fifth probe tone distances from the C major triad. The distances have been inverted; probe tones what were close to the pitches of the C major triad therefore have higher values.

Table 4. Summary of multiple regression analysis.

Factor	Pearson correlation coefficient	Beta-coefficient	t-statistic df = 11
C major triad			
Adjusted $R^2 = 0.47$			
Interval cycle model	0.69*	0.688	3.145*
Semitone distance	−0.33	−0.362	−1.628
Perfect-fifth distance	0.12	−0.166	0.747
C minor triad			
Adjusted $R^2 = 0.56$			
Interval cycle model	0.76**	0.750	3.718*
Semitone distance	−0.34	−0.301	−1.459
Perfect-fifth distance	−0.09	−0.057	−0.276
Dominant seventh			
Adjusted $R^2 = 0.46$			
Interval cycle model	0.76**	0.755	3.417*
Semitone distance	−0.03	−0.062	−0.276
Perfect-fifth distance	0.19	0.171	0.755
Half-diminished seventh			
Adjusted $R^2 = 0.78$			
Interval cycle model	0.89**	0.903	6.361**
Semitone distance	−0.02	0.060	0.406
Perfect-fifth distance	−0.16	−0.207	−1.395

*Significant at $p < 0.05$; **significant at $p < 0.005$.

effects of voice leading (Krumhansl, 1990). It is possible therefore that *semitone distance*—the variable designed specifically to model pitch distance—was confounded by the nature of the timbre, to some degree. More generally, an appropriate extension of the study would undoubtedly involve the use of more ecologically valid timbre, perhaps using sampled vocal or orchestral sounds.

5. Conclusion

In this experiment listeners were required to rate the level of attraction and/or resolution from a chord to a following probe tone. All the probe tone profiles correlated positively and significantly with the model's predictions for tonal attraction. In certain instances, depending on the probe tone, the ratings indicated that the listeners, all of whom were musicians, interpreted the preceding context chords chromatically. This was particularly so for chords that had less clearly defined roots, for example, the French sixth and half-diminished seventh chords. The model and subjects' tonal attraction profiles of the major triad were discussed in relation to the directional asymmetry in the perceived key movement found by Cuddy and Thompson (1992), and it was proposed that the strong attraction of the major (tonic) triad to the subdominant pitch (and key) might account for their findings. Finally, the attraction profiles were modelled using two additional variables, *semitone distance* and *perfect fifth distance*, neither of which contributed significantly to the multiple regression used to test their predictive power.

In the discussion, musical examples were used to show how the model could be used to interpret and perhaps even explicate real musical phenomena. For example, the model predicted the preference of the major perfect cadence (V to I/i) over that of the modal perfect cadence (v to I/i); the Phrygian cadence appeared also to be accurately modelled, as was the dominant seventh, the German and French sixths, and the half-diminished seventh.

However, although there are many similarities between the data and the model in the experiment, it is of course possible that other factors could better predict the dependent variable than those included in the *post hoc* multiple regression analysis, for example, Krumhansl and Kessler's major or minor-key tonal hierarchy profiles. The reason for not using data such as Krumhansl and Kessler's tonal hierarchies as independent variables in the multiple regression was because these data were themselves dependent variables, whose cognitive origin is still debated. This research intended to develop and test a context-independent model of pitch perception based on the formal property of interval cycles: behavioural data of a pre-existing experiment lacking full cognitive elucidation, however well it might have predicted the results, would not have provided a principled cognitive mechanism that might explain these data. The independent variables chosen in the *post hoc* analysis, *interval cycle model, semitone distance* and *fifths distance*, were therefore all context-independent.

An issue naturally arising within this field of research is whether or not interval cycles might be cognitively operational in a tonal system that uses a different division of the octave, such as an octave with 20 'semitones' rather than 12. Balzano (1980) proposed that structurally similar octave divisions to the equal tempered system can be created by $n = k(k+1)$, where k is an integer > 0, and therefore that viable alternatives include an octave divided into 20 ($k = 4$), 30 ($k = 5$) or 42 ($k = 6$) 'semitones'. However, any empirical study designed to investigate alternative octave divisions must first overcome the possible confounding effects of categorical perception; that is, the perceptual tendency to hear pitches belonging to the new octave division as mistuned members of our current, familiar tuning system. As yet it is unclear how this might be achieved convincingly.

Finally, the experiment presented in this paper did not test a number of the model's components. For example, using probe tones meant that it was not possible to explore component *Consonance & Dissonance*, and the Shepard tone timbre resulted in component *Voice Leading* being factored out of the model and probably the subjects' data also. Clearly further experimentation is required in order to test fully all the model's components and to ascertain their utility with respect to calculating tonal attraction. However, despite the forgoing limitations, in general the experiment was consistent with the central idea advanced in the interval cycle hypothesis, which is, that tonal attraction is actuated, to a large degree, by an abstract grouping process linked to interval cycles. Interval cycle proximity provides a theoretical and cognitive framework in which pitches separated by relatively large frequencies can be associated or grouped with one another. The application of the theory and model to the analysis of music may prove to be a very useful tool for elucidating hitherto intractable questions concerning musical pitch perception.

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Appendix: Model

1. Music

Given two successive chords (a chord is defined as a simultaneity containing one or more notes): past chord, X, and present chord, Y

$$X = \{x_1, x_2, \ldots, x|X|\}, \quad x_1 < x_2 < \ldots < x|X|$$

$$Y = \{y_1, y_2, \ldots, y|Y|\}, \quad y_1 < y_2 < \ldots < y|Y|$$

where $|X| =$ size of set X, $|Y| =$ size of set Y, and where x_i and y_j are defined with reference to $C_4 = 60$.

2. Pitch distance

Form matrix PD where

$$\mathrm{PD}_{ij} = |y_j - x_i| \qquad i = 1, 2, \ldots, |X| \quad j = 1, 2, \ldots, |Y|$$

3. Interval cycles

Form matrix IC where

$$\mathrm{IC}_{ij} = \frac{12}{hcf(\mathrm{IC}_{ij}, 12)} \qquad \text{for } \mathrm{PD}_{ij} \neq 0$$

$$\mathrm{IC}_{ij} = 1 \qquad \text{if } \mathrm{PD}_{ij} = 0$$

where $hcf(a, b)$ is the highest common factor of a and b, and a, b are whole numbers.

4. Voice leading

Define a further matrix VL such that

$$\mathrm{VL}_{ij} = \frac{\alpha}{\mathrm{PD}_{ij} + \alpha} \quad \text{for some } \alpha > 0$$

5. Interval cycles & voice leading

Using entry-wise multiplication (array multiplication), combine matrix VL with matrix IC to give matrix ICVL, defined by

$$\mathrm{ICVL}_{ij} = \mathrm{VL}_{ij}\mathrm{IC}_{ij}$$

6. Root salience

Form matrix RS1 where

$$\mathrm{RS1}_{ij} = 1; \qquad i = 1, 2, \ldots, |X| \quad j = 1, 2, \ldots, |Y|$$

If either chord has an identifiable root, let the row corresponding to the past root be the mth row and the column corresponding to the present root be the nth column. Form matrix RS2 where

$$\mathrm{RS2}_{ij} = \begin{cases} \mathrm{RS1}_{ij}, & \text{if } 2i \neq m \\ \mathrm{RS1}_{ij}, & \text{if } 2j \neq n \\ \beta \times \mathrm{RS1}_{ij}, & \text{if } i = m \text{ for some } \beta > 1 \\ \gamma \times \mathrm{RS1}_{ij}, & \text{if } j = n \text{ for some } \gamma > \beta \end{cases}$$

such that root intersection entry $\mathrm{RS2}_{mn} = \beta \times \gamma \times \mathrm{RS1}_{mn}$. If neither chord has an identifiable root, form matrix RS2 where

$$\mathrm{RS2}_{ij} = \mathrm{RS1}_{ij}$$

Form matrix RS3 where

$$\mathrm{RS3}_{ij} = \frac{\mathrm{RS2}_{ij}}{\sum_{i=1}^{|X|} \sum_{j=1}^{|Y|} \mathrm{RS2}_{ij}}$$

7. Consonance & dissonance

(i) If chords X and Y are both sensory consonances, or both are sensory dissonances, form matrix CD where

$$\mathrm{CD}_{ij} = \mathrm{RS3}_{ij}$$

(ii) If chord X (past) is dissonant and chord Y (present) is consonant, form matrix CD where

$$\mathrm{CD}_{ij} = (1 + \delta) \times \mathrm{RS3}_{ij} \text{ for some } \delta > 0$$

(iii) If chord X (past) is consonant and chord Y (present) is dissonant, form matrix CD where

$$\mathrm{CD}_{ij} = (1 - \delta) \times \mathrm{RS3}_{ij} \text{ for some } \delta > 0$$

8. Tonal attraction

Using entry-wise multiplication (array multiplication), combine matrix ICVL with matrix CD to give matrix TA, defined by

$$\mathrm{TA}_{ij} = \mathrm{ICVL}_{ij}\mathrm{CD}_{ij}$$

Sum the entries in matrix TA to produce the overall attraction value, A, where

$$A = \frac{\sum_{i=1}^{|X|} \sum_{j=1}^{|Y|} \mathrm{TA}_{ij}}{12}$$