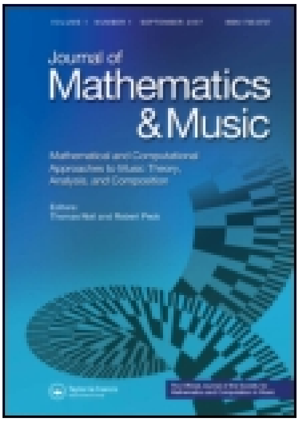

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On the correctness of imprecision and the existential fallacy of absolute music

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RESPONSE TO MARSDEN AND MAZZOLA

On the correctness of imprecision and the existential fallacy of absolute music

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I argued in my position paper (my first movement) that music theory is *de facto* a folk theory of mind and therefore that music is only amenable to study as a psychological or a cognitive entity; therefore, also, music is not modelled directly by mathematical methods, but its cognitive correlate may be. In this movement, I address issues raised by my co-authors, their examples and, importantly, the deep philosophical errors inherent in the notion that one can give a 'definition of music', let alone a mathematical one.

Keywords: music; mathematics; modelling; cognition

1. Prelude: what is it to formalize music?

The German word *Wissenschaft* is usually translated into English as 'science'. While this is often reasonable, it does not capture the sense of 'Musikwissenschaft', which is modified into 'Systematische Musikwissenschaft' to become rigorous. Wissenschaft means something more like 'scholarship' or 'deep knowledge'. 'Science' is, of course, that – but its connotation in English is, unless explicitly otherwise, in opposition to the Arts (including Music) and Humanities. This is the same Humanities/Science dichotomy that I mentioned in my first movement in this discussion (my position paper) [1] and is clearly apparent in the distinct falsificationist philosophy of science, also mentioned earlier [2–4].

My second movement departs from here because of a quote from Pierre Boulez, written in German, that appeared as a 'motto' in Mazzola's position paper. Translated, it says

> Because the mathematical method is the [science/Wissenschaft] which currently has the most advanced methodologies, I was keen to take them to a [model/Vorbild] that can help us to solve our current vulnerabilities. [[5], II, p. 71, cited by Mazzola [6]]

The difference between *Wissenschaft* (in German) and *science* (in English) precisely highlights the question of methodology. *The Scientific Method* refers to a particular set of methodological philosophies, which are intended to make rigorous the study of the world and to

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deny the possibility of naïve generalization. Wissenschaft does not *entail* those particular philosophies.

Mathematics is evidently one of the *Wissenschaften*: Gauss called it their 'Queen'. However, its status as a science is less clear. Hilbert's [7] definition is '… a conceptual system possessing internal necessity that can only be so and by no means otherwise', with the key idea here being *internal necessity*. Abstract mathematics has a 'Truth' like no other field of study: its truth is internally defined, up to incompleteness [8], *independent from the physical world*. It is not science, but formal abstract philosophy.[1]

Studying abstract mathematics does not normally require the Scientific Method that Popper and his successors intended for the study of the natural world. It requires deep knowledge, intuition, insight, creativity, rigour and, above all, proof – and the last one is *explicitly excluded* from the Scientific Method. It is not clear, because of the translation issues, if Boulez intended his words to refer to that Scientific Method or if he meant a more literally mathematical method, involving, for example, symbolic formalization, application of rules and development of proofs. But either way, there is a problem. First, music is not a physical artefact and is not directly governed by what are normally considered to be the laws of the physical Universe,[2] so the Scientific Method is not directly applicable. Second, music has no defined internal necessity, in Hilbert's sense; anything one can call 'truth' in it is metaphorical and subjective. Therefore, music *as a whole* does not naturally correspond with mathematics, even though *parts* of it demonstrably do so [9]. I think Boulez makes his suggestion at the level of methodology, calling for more formal *thinking*, and not at the level of method, calling for a literal interpretation of music as mathematical. The latter leads to confusion, examples of which I give below.

The second word of interest in the quote is *Vorbild* or 'model'. 'Few terms are used in popular and scientific discourse more promiscuously than "model"' [10, p. 171]. But *Vorbild* is more specific. It means 'archetype', 'pattern', 'paradigm' or 'ideal'. I have argued elsewhere [11] and in my position paper in this volume that mathematical formulation can help us to understand patterns in music and/or to describe musical archetypes. Mathematical thinking *has* produced new paradigms in music, exemplified in all three authors' initial contributions. However, in precise translation, it seems equally clear that Boulez did *not* intend to make the suggestion that mathematical concepts *define* musical ones, as implied, but not supported, by Mazzola in his position paper. This suggestion can be easily *dis*proven: the claim of *necessary* regularity merely challenges composers to demonstrate otherwise, as Marsden says. Refutation is easy because there is neither notion nor arbiter of truth or correctness in music, so anything goes: Schenker's [12] failed supremacism exemplifies the emptiness of the idea; clinging to Western Classical/Romantic Music Theory, quasi absolute, is equally empty. Mathematics does not *define* anything, except itself; to say otherwise is no less questionable than literal interpretation of Plato's [13] Theory of Forms.

Boulez is saying, I suggest, that mathematics can be a methodological archetype for a *way of thinking*, and I agree that it can be so *very* usefully. But, at a more literal level of method, for a model to convey any 'truth' about a process or an artefact in the world, it must be shown to *predict* successfully. Successful physical-scientific predictions are measured against the 'ground truth' of the objective world[3] and so should those of music science (and other social sciences) [14]. But music itself has no such objective ground truth [15]: it is a construct of embodied minds and must be treated as such.

The echo of 1960s Modernist fashion, embodied in the Darmstadt school, leads, via over-interpretation, to what a Freudian mathematician might call 'theorem envy'. It is depressing to see the same angst-driven academics chasing the wild goose of formality for its own sake in musical set theory (decried in the 'Proposals' section of Marsden's position paper) and in formalist, modernist composition (decried in Mazzola's position paper, Section 2.2). The historical question whether Boulez' metaphorical approach to formalization of music will succeed is still open. But as Marsden points out [15], the prognosis is not positive.

2. Minuet (first time): Marsden: 'music requires people, and people are messy'

In his position paper in the section on 'Metaphor, Art and Abstraction', Marsden discusses the relationship between musicology (interpreted broadly) and mathematics. For me, there is little to debate: his suggestions are supported by his argument. The quotation that heads this section, however, is of particular interest. First, along with numerous remarks in Marsden's musicologically motivated text, the emphasis on people supports my first movement's argument that music is primarily a psychological entity and that music theory as currently formulated is therefore a folk psychology (albeit a very advanced one). Then, it implies a question: how do we clean up the mess that the people inevitably cause?

This question has been studied in more than 150 years of rigorous psychological methodology, culminating in the computational cognitivist approach, which is, essentially, the use of operationalized mathematical models to simulate human behaviours, and the rigorous evaluation of those simulations. The messiness is managed by means of another mathematical construct, statistics, which also lends the elegance of an explicit method: we can see how the cleaning-up is done and evaluate *it*, along with the model. Ultimately, the method comes down to probabilities: the p-values that, used properly, estimate the likelihood that the effect observed did or did not arise by chance. All this information, complicated though it is, leads to the precision that Mazzola desires (position paper, Section 1.2 and title) and, elegantly, simultaneously admits the human variation that is Marsden's requirement (position paper, the section on 'Precise and imprecise concepts').

An example of the appropriate use of mathematics in music conveniently leads me to an important clarification of Marsden's report of some work of my research group. Marsden kindly refers to the IDyOM model of melodic expectation [16–19], which is a stochastic cognitive model based on multidimensional variable-order Markov chains [20–23]. In his analysis of the 'fiction of gap-fill', Marsden suggests that IDyOM might 'use a principle of gap-fill'. Unfortunately, this would be to misunderstand the motivation of this kind of mathematical model rather deeply. The theme of the paper cited [17] is precisely this: given an appropriate, general (i.e. not music-specific) learning mechanism, *no hard-wired, music-theoretic rules are necessary*, not even the Gestalt Principles, to capture the various effects attributed to them in more *ad hoc*, rule-based models. This work could not demonstrate more clearly that, at the level of abstraction where it is based (above auditory streaming [24]; below melodic grouping [25]), melodic structural expectation, as reported by human listeners, can be explained *just* in terms of their learning mechanism (which seems to be reasonably well modelled by the statistical model), the data to which they are exposed (simulated by around a thousand tonal melodies in the empirical work) and a simple prediction mechanism related to the statistics of the learned melodies and the entropy of the predictions made therefrom. Adding *ad hoc* music-theoretic rules to IDyOM would invalidate the conclusions of the work.

If there is a principle of gap-fill in IDyOM (and I suspect there is, though the analysis has not been carried out formally), it is implicit in the interaction of observed likelihoods in the model [16] and implemented in the process of estimating distributions as the model incrementally predicts the next note in a previously unseen melody. The relation between what is expected and what is heard is crucial, since that is a key aspect of musical tension: information content (to which I return below) not only drives the model, but also perhaps contributes to the elusive concept of musical salience [26].

So, IDyOM is an abstract mathematical system, using observed likelihood and information theory, that predicts an aspect of human musical intuition (viz. melodic expectation) better than any other predictor in the literature – even music-theoretic ones [17]. One reason that it does so is because it avoids the existential fallacy of music [27]: it does not suppose that music is a fixed thing with fixed rules, but rather it simulates, albeit at a fairly abstract level, a basic aspect of human cognition which is strongly implicated in musical listening: perceptual anticipation. When that model is given musical data, appropriately represented, it makes musical predictions, which have been favourably compared with human responses. Thus, this successful mathematical music-theoretical model is, by nature, psychological, and that is why it succeeds; scientifically, we must continue to attempt to falsify it.

3. Minuet (second time): Mazzola: 'music theory is not a branch of psychology'

In his position paper, Mazzola presents a set of claims about mathematics and music. Regrettably, there is no space to address all of them in this paper, so I have picked a selection of the ones in most need of correction, in an attempt to represent the range.

Consistency: Since my theme is that the psychological nature of music requires direct attention and that the appropriate treatment is mathematical, I begin with two important claims made in Mazzola's position paper, one of which heads this section. I believe that I have explained how music theory is indeed a branch of psychology [28]; but the issue here is philosophically more serious. I invite the reader to consider the following claim, also made by Mazzola (position paper, Section 2.3): great composers can 'intuitively find the right solutions' to the conundra allegedly posed by complex mathematical theories. Consider: what is this 'intuition'? Intuition is a (folk) psychological concept: the ability of humans to reason non-consciously and implicitly; 'intuition' is a broad name for a class of cognitive process. Thus, to argue against those (not me) who say his mathematics is too complicated, Mazzola *appeals to a psychological construct*. Composers, Mazzola says, *are* applying cognitive process, rather than mathematics, to make their music. They have replaced complex mathematical arguments by intuitive approximations, produced by that cognitive process. Thus, implicitly, Mazzola agrees with my claim that music is indeed a psychological construct. It follows, surely, that its study, as embodied in music theory, is therefore a branch of psychology, as I have argued.

The problem is that Mazzola holds *both* the explicit claim that music theory is *not* a psychological description *and* the implicit claim entailing that it *is* one to be true, simultaneously. The logical reader may judge what can thence be inferred; I turn elsewhere.

Method and inference: Scientific logic in mathematical study is highlighted in Mazzola's Section 1.3, concerning models of musical performance. Contributions, we are told, have been made that require 'intense mathematical and computational machinery'. The claim is that 'this theory cannot rely on algebraic methods, but needs heavy calculus, ...' Again, we can see an error of philosophy, as Mazzola claims that this advanced mathematics is there 'because the objective situation requires it'. This is not a scientifically supportable claim. What *can* be claimed is that mathematicians have so far failed to find a simpler explanation; one reason for this failure might be that they are looking in the wrong place, in the musical surface and not in the musical processor.

From the only scientifically supported viewpoint – that embodied brains are the source of music – there is no surprise in this complexity: brains are massively parallel, *analogue* processors, by nature inherently capable of more than the restricted symbolic computation of the digital computer, even if their eager capacity to categorize gives the illusion that they are, too, symbolic.

Mazzola continues this kind of argument into Section 1.4. He claims that his 'approach ...*proves* that [musical] gestures *must* be conceived in the framework of algebraic topology, topos theory, and homological algebra' (my italics). Again, in science, one case proves nothing: it merely demonstrates possibility. Proving that there is no alternative framework in which to conceive musical gesture would require quantification over, or empirical examination of, all possible frameworks, and since humankind has not yet quite finished the study of all possible mathematics, this option is not available. It would also require a proof that the domain of discourse for this reasoning was correct and complete and that cannot be given. I agree with my co-author that precision is a good idea, as much in methodology as elsewhere.

It is difficult to take strong conclusions away from the profoundly unscientific[4] 'arguments' in this section and the previous one. Instead, it may be better to systematically study the whole problem, from sound waves, via ears and neurons, up to symphonies, and piece together a scientifically defensible story, free of preconceptions, to explain the enormously complicated set of processes which combine to make the human experience of music. Mathematics will be hugely important at every level of this endeavour, but it will be *modelling* observed phenomena directly, neither predefining them nor presupposing their music-theoretic proxies (but perhaps supporting them). The models must be subject to empirical scrutiny, again verified statistically.

For example, predictions such as those of the IDyOM model, mentioned above, may be (and have been) tested both from the top down, in terms of reported human perception [25], and from the bottom up, by neurophysiological methods [29]. Through this kind of approach, mathematical methods lend the same opportunity for rigorous science that they offer throughout natural philosophy, and *this* is the kind of precision required for the future study of music, not the kind that focuses, for reasons unjustified, on a particular musical culture[5] and repeatedly makes unsupported claims that seem dangerously similar to the idealist and supremacist notions of the Romantic period about it. Reductionism cannot be practised arbitrarily: it must be defended, and even when it is properly justified, the schisms thus created must be rebridged later on. This is not to say that culture-specific musical phenomena do not exist or that they may not be studied separately: but it does mean that one needs to question and explain one's assumptions and ensure that they are not grounded in, and hobbled by, the narrow aesthetic philosophy of an earlier century.

Fact: Mazzola begins with the applied mathematics in physics and engineering. Here, there is no disagreement: the abstract power and elegance of properly applied mathematical models are undeniable, if one remembers that abstractions are abstractions, models are models, and neither is reality. However, even in this roll of mathematical honour, psychology rears its empirical head: while Fourier indeed invented the decomposition of complex waveforms used in mp3 compression, it is *psychoacoustic science* that allows the compressor to decide which information to omit.[6] Without this, mp3 would not work.

Evidence: My reference to the Information Dynamics of Music project (IDyOM), above, and in particular the work of Pearce [16] on which it is based, is in sharp contrast with the comment of Mazzola's Section 2.1, identifying information-theoretic analysis and composition of music in the style of Bach as a 'failure'. Mazzola definitively states that 'probability distribution is not the method with which Bach would build his works', but cites no evidence for this claim. Again, rigorous reasoning is undermined: there is indeed no evidence that Bach explicitly used distributions in designing his music; but the *absence of evidence is not evidence of absence*. More importantly, there *is* increasing evidence in the scientific literature for the statistical nature of both language [20] and music [18] processing. So, while it is indeed improbable that Bach systematically used a dice-game to construct music, it is plausible, and even likely, that he used *non-conscious* probability distributions to *experience* it. Therefore, it is an entirely reasonable proposition that he would (non-consciously) use such probability distributions to imagine the effect of the music on listeners. Therefore, the experimental study of probabilistic elements in musicological models is entirely defensible – indeed, much more defensible than an unsupported claim that very complicated mathematics is necessarily required – until the hypothesis that probabilistic models work for music is *refuted*. Attempts at such refutation have so far failed. So, Mazzola's claim in Section 2.1 of his position paper, that Draeger's [30] 'understanding' of music was 'too simplified', rebounds on its author, and Draeger seems rather perspicacious in the context of the current evidence, 50 years later.

4. Trio: three views; synthesis and antithesis

In his position paper, Marsden discusses *Art, Metaphor and Abstraction*, arguing that a musicologist's mode of thought is distinctly metaphorical: a reason for this is the lack of grounding for musical language, caused by the impossibility of pointing to the entity that is a musical gesture, feature or whatever one means, except via the highly abstract, and in many cases completely inadequate, device of notation. This is important here because it relates to Boulez' *Vorbild*, though at the lower level of individual mathematical structures and processes, rather than at the higher one, of mathematical *Wissenschaft*. It is what a theoretical physicist means by 'I have a model of stellar evolution' or an architect by 'I have modelled the stresses in these beams'. Each of these models is used to *explain* something.

Music theory, on the other hand, *explains* nothing. Rather, it names and describes structures: tones, chords, melodies, etc. It gives some simple calculi that seem to correspond with mathematical groups (and the key question of 'why?' languishes, unasked). Each music theory is limited to a particular range of style, time period and practice. Music theory does not predict the unobserved, but, at most, constrains. Its *de facto* single function is to admit discussion of agreed, perceived musical phenomena. Therefore, Mazzola is wrong to say (in Section 3 of his position paper) that we 'further *have to* develop music theory in order to *enable* new ways of making music' (my italics), because neither music theory nor mathematics can 'define' rules of music [30, Section 1.2], just as Mozart did not choose to use Sonata Form when writing his concerti: the music-theoretic *description* of the form was not conceptualized until well into the nineteenth century[7] [31]. Music theory, as constituted in the West, neither enables nor prevents anything in music; though, to be sure, it does *influence* common practice. To make this error, of supposing that descriptive music theory predicts future possibilities, is to misunderstand the fundamental nature of music theory and to impute to the word 'theory' the inappropriate quasi-scientific connotations against which I argued in my first movement. Composers do not need music theory, mathematical or otherwise, to tell them how to innovate, though, as I concede above, it does present them with tools that they *may, if they choose*, use to sculpt their work. As I said in my position paper, Schoenberg's 12-note experiment demonstrates that (quasi-)theoretical predictions do not work unless they are also cognitively and culturally supported.

The reason that music theory does not *define* music is because it cannot. It is only a descriptive model [32]. It contains no inference rules that can predict unknown structures, nor does it contain underlying mechanisms that might be able to generate them. It hires no police to enforce its rules. Ultimately, it is always retrospective, because perception of new musical possibilities *precedes* the need to name them, and that is because composers and culture define music, by *doing* it. Music theorists, on the other hand, write about that doing, *after* the event.

In his position paper, Section 3, Mazzola proposes that the first challenge of mathematics in music should be to 'penetrate into substantial concerns of composition, performance, and analysis', and I agree. He suggests that we have to 'make experiments, to test our understanding on real music', and I agree with this too. I even agree that mathematical models 'are the natural environment for extensions and generalizations'. The problem lies in an omission: mathematical models of *what*? Mazzola himself argues against the lack of semiotic culture inherent in arbitrarily mathematical approaches to composition, naming Schoenberg's 12-note method as one such (position paper, Section 2). Aside from the mischaracterization of Schoenberg's teaching of the 12-note method (more correctly described in my first movement), to attempt to apply mathematics without semiotics is always to risk the same failure.[8] But the semiotics of music is not inherent in the mathematical models of music in our literature, or even in the sound of music itself. The semiotics of music is in what is *experienced* when elements of a piece refer one to another, or to another piece, or to the real world, or to previous affective responses, in the interpretation of a listener. Music analysis requires a human to identify such effects, using the terms of music theory, and the human is introspecting about what is going on in the *mind* of a listener or a composer. The meaning of music – human affective response – is not referential semantics, as it is not denotational. It is only as subject to mathematical simulation as hunger, love or fear: it *requires* conscious experience. The answer, therefore, to the question of 'what?', above, is *cognition*.

5. Da Capo al fine: summary and conclusion

In this paper, I have argued that there is a very important place for mathematics in the study of music, but that it is not as an implementation of music theory. Rather, mathematical modelling can be applied, in numerous ways, to model the behaviours and percepts that music theory evolved to describe, rather than music theory itself, which is both a proxy and approximate. This has the advantage that it frees us from the implicit – and incorrect – ethnocentric assumption that Western Music Theory somehow *defines* music and allows us to go beyond, into the musical and linguistic universals that may yet reveal profound truths about the origins of music in human society [33].

To put this in another way: music, as we currently know it, did not spring into existence fully formed. It evolved, over a time scale comparable with human society, into many different forms using both coincident and differing principles. My claim is that to base one's mathematics exclusively on music theory as taught in many Western conservatories and schools until very recently is to base one's work on the philosophy of the Romantic era, with its value judgements and social, even supremacist, assumptions. The incorrectness of these assumptions is not only political, but very real, because they lead to the omission of the vast majority of the field of study, dismissed for so long as merely 'ethnic' music.

The precision of mathematics, naïvely applied, is in fact a counter-argument: human cognition is not precise. But the controlled imprecision allowed by statistical methods, so carefully applied in psychology, can allow models to be built. Further, the mathematics of information transmission can begin to supply inputs to the model of musical tension, which itself feeds musical affect, a part of consciousness firmly founded in human biology.

Finally, I revive an analogy from an earlier publication [15]: to attempt to study the physics of an ice floe by looking only above the waterline would be to omit 60% or more of the evidence available. Attempting the study of music from the perspective of music theory only, omitting the cognitive root cause, is comparable: one can see only the *effects* of what is beneath and not the *causes*. The value of a formal model is its predictions: and one needs a model which is both causal and complete to make predictions that are accurate and precise – in whatever sense.

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Notes

1. I must be absolutely clear that I do *not* mean that mathematics is *less than* science. I am saying that it is (except in the case of recent ideas about experimental mathematics) something *different*.
2. N.B.: Sound is just one aspect of music; it evidently *is* governed by physics.
3. Remembering that understanding of the physical world was the purpose of science, in Popper's [2] definition.
4. This is a strong accusation. But to hold contradictory ideas as simultaneously true and to commit naïve generalization from evidence are two of the most basic – and effective – ways of undermining the scientific method [2].
5. Of course, there is nothing wrong with studying a particular musical culture in isolation. But if one does so, one reduces the possibility of general understanding, because one cannot be sure which phenomena are peculiar to the

culture and which are general perceptual principles. In fact, this problem extends as far as music does, because we cannot argue that all possible musics are current in the world – but, to be scientific, we should do the best we can.
6. See http://www.iis.fraunhofer.de/en/bf/amm/diemp3geschichte/funktion/ for details.
7. Though the Minuet and Trio was established much earlier.
8. Schoenberg did not do this: he reasoned about the semiotics inherent in tonality, but he reasoned incorrectly, as I argued previously.

References

[1] C.P. Snow, *The Two Cultures*, Cambridge University Press, 1993.
[2] K. Popper, *The Logic of Scientific Discovery*, Routledge, Abingdon, 1959, translation of German original, 1934.
[3] I. Lakatos, *Falsification and the methodology of scientific research programmes*, in *Criticism and the Growth of Knowledge*, I. Lakatos and A. Musgrave, eds., Cambridge University Press, Cambridge, 1970, pp. 91–196.
[4] T.S. Kuhn, *The Structure of Scientific Revolutions*, The University of Chicago Press, Chicago, 1962.
[5] P. Boulez, *Penser la musique aujourd'hui*, Gonthier, Mayence, 1963.
[6] G. Mazzola, *Thinking music with precision, depth, and passion*, J. Math. Music 6(2) (2012), pp. 83–91.
[7] D. Hilbert, *Natur und Mathematisches Erkennen: Vorlesungen, gehalten 1919–1920 in Göttingen*, Birkhäuser, Basel, 1992.
[8] K. Gödel, *Über formal unentscheidbare Sätze der principia mathematica und verwandter Systeme, I*, Monatsh. Math. Phys. 38 (1931), pp. 173–98.
[9] G.A. Wiggins, M. Harris, and A. Smaill, *Representing Music for Analysis and Composition*, in *Proceedings of the Second Workshop on AI and Music*, M. Balaban, K. Ebcioğlu, O. Laske, C. Lischka, and L. Soriso, eds., AAAI, Menlo Park, CA, 1989, pp. 63–71.
[10] N. Goodman, *Languages of Art*, Hackett Publishing Company, 1976.
[11] A. Smaill, G.A. Wiggins, and E. Miranda, *Music representation – between the musician and the computer*, in *Music Education: An Artificial Intelligence Perspective*, M. Smith, G. Wiggins, and A. Smaill, eds., Springer, London, 1993.
[12] H. Schenker, *Das Meisterwerk in der Musik*, Drei Maksen Verlag, Munich, 1930, in 3 volumes, published 1925, 1926, 1930.
[13] Plato, *The Republic [Original 385BC, translated by J.L. Davies and D. J. Vaughan]*, Wordsworth Editions Ltd., Ware, Hertfordshire, 1997.
[14] H. Honing, *Computational modeling of music cognition: A case study on model selection*, Music Percept. 23(5) (2006), pp. 365–376.
[15] G.A. Wiggins, *Semantic gap?? Schemantic Schmap!! Methodological considerations in the scientific study of music*, Proceedings of 11th IEEE International Symposium on Multimedia, 2009, pp. 477–482.
[16] M.T. Pearce, *The construction and evaluation of statistical models of melodic structure in music perception and composition*, Ph.D. thesis, Department of Computing, City University, London, 2005.
[17] M.T. Pearce and G.A. Wiggins, *Expectation in melody: The influence of context and learning*, Music Percept. 23(5) (2006), pp. 377–405.
[18] G.A. Wiggins, M.T. Pearce, and D. Müllensiefen, *Computational modelling of music cognition and musical creativity*, in *Oxford Handbook of Computer Music*, R. Dean, ed., Chapter 19, Oxford University Press, 2009, pp. 383–420.
[19] M.T. Pearce and G.A. Wiggins, *Auditory expectation: The information dynamics of music perception and cognition*, Topics in Cogn. Sci. (in press).
[20] C.D. Manning and H. Schütze, *Foundations of Statistical Natural Language Processing*, MIT Press, Cambridge, MA, 1999.
[21] J.G. Cleary and I.H. Witten, *Data compression using adaptive coding and partial string matching*, IEEE Trans. Commun. 32(4) (1984), pp. 396–402.
[22] D. Conklin and I.H. Witten, *Modelling music: Systems, structure, and prediction*, J. New Music Res. 19(1) (1990), pp. 53–66.
[23] D. Conklin and I.H. Witten, *Multiple viewpoint systems for music prediction*, J. New Music Res. 24 (1995), pp. 51–73.
[24] A.S. Bregman, *Auditory Scene Analysis*, The MIT Press, Cambridge, MA, 1990.
[25] M.T. Pearce, D. Müllensiefen, and G.A. Wiggins, *The role of expectation and probabilistic learning in auditory boundary perception: A model comparison*, Perception 9 (2010), pp. 1367–1391.
[26] G.A. Wiggins, *Cue abstraction, paradigmatic analysis and information dynamics: Towards music analysis by cognitive model*, Musicae Scientiae, Special Issue: Understanding musical structure and form: Papers in honour of Irène Deliège, 2010, pp. 307–322.
[27] G.A. Wiggins, D. Müllensiefen, and M.T. Pearce, *On the non-existence of music: Why music theory is a figment of the imagination*, Musicae Sci. 5 (2010), pp. 231–255.
[28] G.A. Wiggins, *Music, mind and mathematics: Theory, reality and formality*, J. Math. Music 6(2) (2012), pp. 111–123.
[29] M.T. Pearce, M. Herrojo Ruiz, S. Kapasi, G.A. Wiggins, and J. Bhattacharya, *Unsupervised statistical learning underpins computational, behavioural and neural manifestations of musical expectation*, NeuroImage 50(1) (2010), pp. 303–314.

[30] H.-H. Draeger, *Analysis and synthesis of musical structures on the basis of information theory*, in *Bericht über den neunten Internationalen Kongress Salzburg 1964*, Vol. 2 of *Internationale Gesellschaft für Musikwissenschaft*, F. Giegling, ed., Kassel, Bärenreiter, 1966, pp. 261–268,
[31] C. Rosen, *Sonata Forms*, Norton, New York, 1988.
[32] G.A. Wiggins, *Models of musical similarity*, Musicae Sci. 4A (2007), pp. 315–338.
[33] N.L. Wallin, B. Merker, and S. Brown (eds.), *The Origins of Music*, MIT Press, Cambridge, MA, 2000.