Tonal Implications of Harmonic and Melodic T_n-Types

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Music composed of *tones* (in the psychoacoustical sense of *sounds that have pitch*) can never be completely atonal (Reti 1958). Consider any quasi-random selection of tones from the chromatic scale, played either simultaneously or successively. Most such sets generate associations with musically familiar pitch-time patterns and corresponding tonal stability relationships (Auhagen 1994). A pattern of pitch can imply a tonal centre simply because it reminds us of a tonal passage: it has *tonal implications* that depend on the intervals among the pitch classes (pcs) in the set.[1] The only clear exceptions to this rule are trivial: the null set (cardinality = 0)[2] and the entire chromatic aggregate (cardinality = 12). Since every interval, sonority and melodic fragment has tonal implications, even the so-called "atonal" music of composers such as Ferneyhough, Ligeti and Nono is full of fleeting tonal references: at any given moment during a performance, some pitches are more likely than other pitches to function as psychological points of reference. In the following, I will use the terms "tonal" and "atonal" in this broad, psychological sense.

A number of terms have been coined in an attempt to map out the diverse terrain that separates (major-minor or harmonic) tonality from (extreme) atonality, including pantonal, extratonal, atonical, neotonal and polytonal. Tonality is multidimensional in the sense that there are many different ways of bridging that gap that manifest as different styles (such as impressionism, bebop and minimalism). The present analysis attempts to map this complexity onto a single dimension. Instead of dividing music into "tonal" and "atonal", I conceive of *degrees* of tonality or atonality and imagine the possibility of evaluating the degree of tonality of a passage of music as a positive whole number between, say, 0 for completely atonal and 1 for completely tonal. That number should generally be higher for music that has clearer or longer-lasting tonal anchors.

The "atonal" repertoire avoids tonal references by favoring pc-sets with relatively weak tonal implications. A well-known exception is Berg's violin concerto, a work that is usually regarded as 12-tone but can barely be regarded as "atonal", because the row at the beginning of the first movement begins with a minor triad. It follows from this exceptional example that, as a rule, "atonal" composers deliberately avoid consonant intervals between successive notes and prefer tonally weak or ambiguous pc-sets. From a logical viewpoint, they may find "atonal" pc-sets in two main ways:

[1] The term "pitch" in "pitch-class set" is misleading, because a pc-set is primarily a configuration of *intervals*. Each set is defined by the number of times each interval class (of which there are six: 1, 2, 3, 4, 5 or 6 semitones) occurs in the set. This set of 6 numbers is called the *interval vector* (Forte 1973). For example, a major or minor triad contains no semitone, no tone, one minor third, one major third, one fourth (fifth) and no tritone, so its interval vector is [001110].

[2] The cardinality of a pc-set is simply the number of members in the set.

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either by *borrowing* them - consciously or unconsciously - from the existing “atonal” literature, or by *discovering* them by aurally guided exploration and trial and error, exploring the various possibilities creatively and listening carefully. Since the “atonal” repertoire presumably includes all possible pc-sets, it is no longer possible to find “new” ones, suggesting that these two strategies cannot be separated.

How might a composer best find pc-sets corresponding to a given desired degree of tonality or atonality? Composers in atonal idioms (including serial approaches) have intuitively favored pc-sets that avoid perfect intervals (fifths, fourths) and favor tritones and semitones. This paper presents a new method by which composers can systematically and quickly seek and find pc-sets of any specified cardinality and strength of tonal implication. This aim is appropriate given the large number of possible pc-sets from which a composer can choose and the long history of constructive interaction between composition and music theory. My approach is intended to shed light on three areas simultaneously: perception, analysis and compositional practice.

The tonal implications of a sounding musical fragment depend not only on the underlying pc-set but also, of course, on its musical realization. The realization of a pc-set has several aspects: properties of individual tones (duration, loudness, timbre, temporal envelope); whether melodic (successive) or harmonic (simultaneous); if melodic, the order of the tones (especially important in 12-tone music) and which tones are repeated; if harmonic, voicing (octave register of each tone, spacing between the tones, doubling in different octaves) and onset synchronicity. A tone is more likely to be perceived as a tonal center if it is repeated (or doubled), has a longer duration, or is simply louder than other tones (Oram and Cuddy 1995; Parncutt 1988; 1997). Here, I assume that it is possible to separate effects associated with the intervals within a set from effects of the set’s specific realization, and focus only on the former. While this assumption may not be entirely valid, it is a good first approximation and a useful starting point.

In the following analysis of the tonal implications of pc-sets, I make use of Forte’s convenient and well-known method of enumerating all possible pc-sets within given constraints. While Forte’s method is often referred to as pc-set *theory*, in the present approach it is no more than a systematic classification system or *taxonomy*, because the taxonomy itself does not generate predictions that can be empirically tested – unlike the perceptual theory with which it is combined in this paper. While the music-analytical application of Forte’s taxonomy is usually confined to “atonal” music, there is no reason why it should not be applied to any music composed within the confines of the 12-tone chromatic scale.

T_n-types of cardinality 3

Rahn (1980) broke down Forte’s pc-sets into *types*. One such type is the *transpositional type* or *T_n-type*. A T_n-type is a pc-set that is invariant under transposition but not inversion. The subscript *n* refers to the size of a transposition in semitones, and *T_n-type* refers to all 12 possible transpositions of a given collection of pcs.

The mathematical jargon sounds complicated, but the concept is fundamentally simple. The major and minor triads are both examples of T_n-types. A major triad is a set of three pcs: a root and two further tones, 4 and 7 semitones above the root – 047 for short. The intervallic inversion of the major triad is the minor triad 037, and both belong to the same pc-set, whose *prime form* (Forte 1973) is 037. Because 037 is the 11th in Forte's list of pc-sets of cardinality 3, it is also referred to as 3-11. When the two T_n-types corresponding to this pc-set are separated, the minor triad is labelled 3-11A and the major 3-11B.

Table 1. All T_n-types of cardinality 3 (after Rahn 1980)

set name	3-1	3-2	3-3	3-4	3-4	3-6	3-7	3-8	3-9	3-10	3-11	3-12
prime form	012	013	014	015	016	024	025	026	027	036	037	048
inversion		023	034	045	056		035	046			047	

For purpose of argument, let us begin by enumerating all possible T_n-types of cardinality 3. There are 19 of them, and they are presented in Table 1. In the table, "set name" corresponds to "name" in Appendix 1 of Forte (1973); the number before the dash is the cardinality, and the number after the dash is the set's position in a list of all possible sets of that cardinality. The prime form "012" corresponds to C-C#-D in all chromatic transpositions, "013" to C-C#-D#, and so on.

Some of the pc-sets (prime forms) in Table 1 are symmetrical and some are not. For example, 012 is symmetrical, but 013 is asymmetrical. An asymmetrical set may be broken down into two T_n-types, which are labelled A and B: e.g. 013 is labelled 3-2A, and 023 is labelled 3-2B.

The tonal implications of the 19 T_n-types of cardinality 3 vary markedly. At one extreme, the major and minor triads have strong tonal implications; the major is the more strongly tonal, since its root is perceived more clearly (Parncutt 1988). At the other extreme, 012 has almost no tonal implications – by which I mean that the tones sound about equally important and no other (virtual) tones are strongly implied. (Even this is not quite true: when 012 is presented harmonically, 0 and 2 are more audible than 1, due to masking.) All other T_n-types of cardinality 3 have tonal implications with various degrees of strength. For example, 023 may be heard as either the 1st, 2nd and 3rd degrees of a minor scale or the 6th, 7th and 8th degrees of a major scale, suggesting that either the 0 or the 3 in 023 may be heard as a point of reference. The major-third (4-semitone) interval embedded within 014 suggests that its reference pitch is 0, regardless of whether the pattern is heard as Neapolitan, Arabic or Flamenco; in Terhardt's approach, both pitch-salience patterns and cultural associations are learned, but since pitch-salience patterns are ultimately based on universal aspects of pitch perception in speech, they are expected to vary less than cultural associations across listeners and musical contexts.

Given the wide range of tonal implications within T_n-types of cardinality 3 (and any other cardinality for that matter), it is surprising that many pc-set theorists tacitly consider all pc-sets *a priori* to be equivalent or value-free, as if they had no tonal implications – or as if tonal implications did not exist. Can the tonal implications that we learn from music simply disappear (which is psychologically implausible), or are they arbitrary (which is psychoacoustically and ethnomusicologically implausible)? It may be possible to make tonal implications disappear in a magical, ideal world of mathematics located in a far-off galaxy and inhabited by aliens, but in real music heard by real human beings, pc-sets will always have tonal implications. Moreover, the appeal of so-called atonal music may be due not to an absence of tonal implications, but to their multiplicity, fluctuation and intangibility.

The tonal implications of a T_n-type may be understood and quantified by first evaluating the *perceptual salience* of each chromatic scale degree in the context of that T_n-type. By "salience" I mean the (subjective) importance of something for a listener, or the (objective) probability that a listener will notice or become consciously aware of something - in this case, a tone at a given chromatic scale degree. The perceptual profile of a T_n-type is a set of 12 values, one for each of the 12 chromatic scale degrees. Each value reflects the perceptual salience of that scale degree in the context of (i.e. during or following presentation of) that T_n-type. In the following, I will distinguish between two kinds of perceptual profile, harmonic and tonal, and present separate algorithms for calculating these profiles that are based on contrasting empirical data and perceptual-cognitive[3] theory.

The harmonic profile

The harmonic profile of a T_n-type is a vector of twelve values, each of which is an estimate of the perceptual salience of a pc. In Parncutt (1988; 1989), I assumed the salience of pitches in chords to be proportional to the probability that a pitch will function as the root when the tones are sounded simultaneously (i.e., as a sonority). I assumed that when a given T_n-type is heard repeatedly in different voicings and contexts, the probability increases that a certain pitch will be heard as a reference – a long process that involves learning, history and culture (Parncutt 2005). I then developed a simple algorithm for pc-salience within harmonically presented T_n-types that was based on the virtual pitch algorithm of Terhardt et al. (1982) and the chord-root model of Terhardt (1982). The model was tested by presenting chords of octave-complex tones (OCTs, Shepard tones) followed by individual OCTs and asking listeners how well the single OCT fits with the chord (Parncutt 1993). In that experiment, and many other experiments reported for example by Krumhansl (1990), OCTs are used to operationalize the music-theoretical concept of a pc (which is equivalent to the music-psychological concept of chroma).

Terhardt assumed that the root of a chord is a virtual pitch. By that, he meant that the root corresponds to the fundamental of an approximately harmonic series of audible pure-tone components (partials). Those components, which are a subset of all the chord's audible[4] partials, generally include harmonics of different chord tones. There are usually several possible candidates for the root of a chord; "the" root may be the one corresponding to the most salient virtual pitch, but may also depend on the music with which a listener is familiar, and thus indirectly on the history of musical

[3] There is no clear boundary between "perceptual" and "cognitive". Terhardt's theory tends to be regarded as perceptual or psychoacoustical, but it is also cognitive in the sense that it involves information processing (or better: his algorithm to predict the pitch salience profile of a complex sound involves information processing). Krumhansl's approach is explicitly cognitive, but it is based on empirical data obtained from perceptual or psychoacoustical experiments.

[4] By "audible" I mean present in the running spectral analysis of the sound which is performed physiologically by the basilar membrane and transmitted to the brain along the auditory nerve. The initial masking stage of Terhardt's algorithm predicts what is "audible" in this sense and what is not, and assigns spectral pitches to all audible partials.

syntax and implicitly learned conventions of music theory. Whichever way you look at it, the root is assumed to be *learned* and enters culture when listeners are repeatedly exposed to consistent patterns of pitch relationships within musical sonorities.

Table 2. Root-support intervals (after Parncutt (1988))

Root-support interval	diatonic notation	P1, P8…	P5, P12…	M3, M10…	m7, m14…	M2, M9…
	size in semitones	0	7	4	10	2
root-support weight		10	5	3	2	1

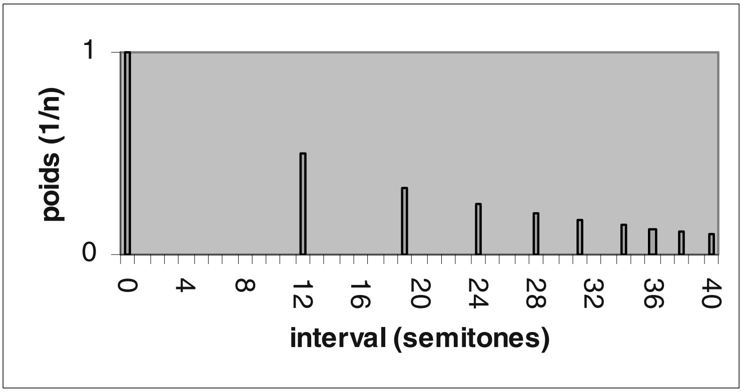
Abbreviations: P=perfect, M=major, m=minor

According to Terhardt (1982), the virtual pitch at the root of a chord is generated by the chord's tones and that the intervals octave/unison, perfect fifth, major third, minor seventh and major second/ninth determine the root. I call these intervals *root supports* (see Table 2; Parncutt 1988). They are octave generalizations of the intervals between spectral and virtual pitches in typical harmonic complex tones such as voiced speech sounds (i.e., between harmonic overtones and the fundamental).

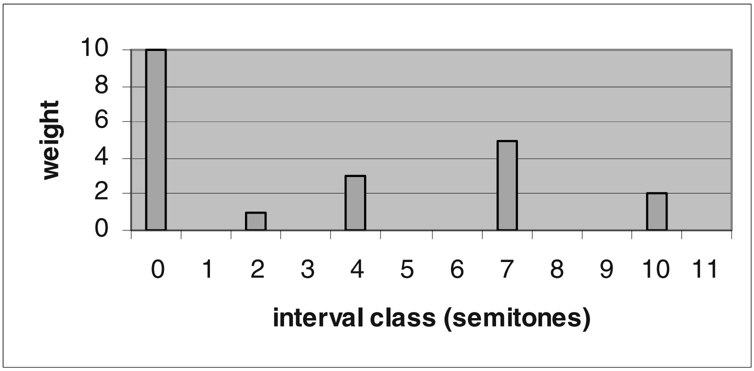
The chord-root model includes free parameters called *root-support weights.* These are quantitative estimates of the influence of each root-support interval on the salience of the virtual pitch at the lower tone of the interval, and hence on the perceived root of a chord. The weights used in the present calculations are presented in Table 2 and Figure 1 (b, c). They are assumed to depend on the position of the corresponding element in the harmonic series: intervals that occur early in the series are assumed in Terhardt's approach to be more familiar to the ear and therefore to play a more important role in the determination of virtual pitches and chord roots. The values in Table 2 have been tested by studying the predictions of the model and comparing them with both music-theoretic intuition and various published sources of empirical data. The predictions of the chord-root algorithm (including an additional masking procedure) were tested experimentally in Parncutt (1993) for a limited set of chords of octave-complex tones; when Krumhansl and Kessler (1982) asked how well octave-complex probe tones follow single chords (rather than short progressions), they obtained essentially the same results (that is, the correlation coefficients between the two sets of profiles are highly significant). The predictions of the chord-root algorithm may also be considered to apply to a typical or average voicing[5] of a given T_n-type when it is realized as regular musical tones (harmonic complex tones).

Note the absence of the minor third from the root-support intervals presented in Table 2 and Figure 1 (b, c). In this approach, the m3 is not assumed to have any *direct* influence on the root. First, it is not found in the lower reaches of the harmonic series between an element of the series and the fundamental. Second, the root of the minor

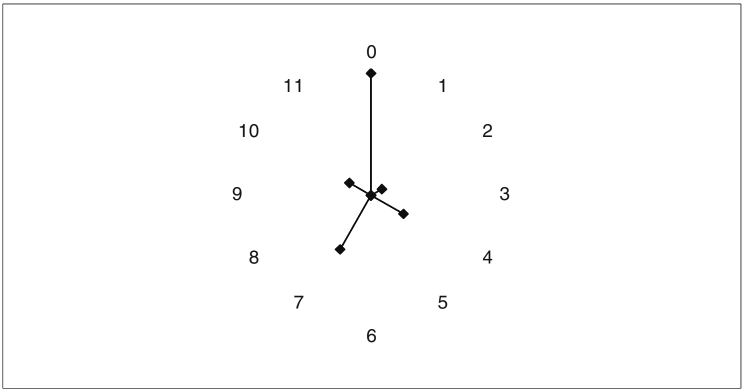
[5] The idea of a "typical or average voicing" could be quantified by documenting all voicings of a given T_n-type in a given musical repertoire using software such as David Huron's *Humdrum*.



(a) The template assumed by Parncutt (1989), in which weights are set to the reciprocal of harmonic number.



(b) The octave-generalized template used in the present calculations (similar to that of Parncutt (1988), but without the m3 interval).



(c) A circular representation of the same template; the numbers are intervals above the root in semitones.

Fig. 1. The harmonic series template for calculating virtual pitch salience

triad can be explained solely in terms of the P5 interval between the fifth and root of the chord. Figure 2 (below) shows how the theory correctly predicts the root of the minor triad without explicitly including the minor third as a root-support interval.

Incidentally, the omission of the minor-third interval from the root supports does not contradict the relatively high salience of the third degree of the minor scale in the K-K profiles (see Figure 4 below). On the contrary: this model can *explain* why Krumhansl found the third degree of the minor scale to be more salient than the fifth: the strong minor third is also present in the pc-salience profile of the minor triad (Parncutt, in preparation).

Table 3. Matrix used to calculate the harmonic profiles of T_n-types

10	0	2	0	0	5	0	0	3	0	1	0
0	10	0	2	0	0	5	0	0	3	0	1
1	0	10	0	2	0	0	5	0	0	3	0
0	1	0	10	0	2	0	0	5	0	0	3
3	0	1	0	10	0	2	0	0	5	0	0
0	3	0	1	0	10	0	2	0	0	5	0
0	0	3	0	1	0	10	0	2	0	0	5
5	0	0	3	0	1	0	10	0	2	0	0
0	5	0	0	3	0	1	0	10	0	2	0
0	0	5	0	0	3	0	1	0	10	0	2
2	0	0	5	0	0	3	0	1	0	10	0
0	2	0	0	5	0	0	3	0	1	0	10

The harmonic profile of a T_n-type is calculated by a simple pattern-matching routine in which the octave-generalized template illustrated in Figure 1 (b) is compared with the pcs of the set, in all 12 transpositions around the pc cycle. One way to represent this routine is by matrix multiplication. The first column of the matrix in Table 3 corresponds to the template in Figure 1 (b). Successive columns are generated by rotating the template down, one element at a time. The T_n-type is expressed as a vector (1 row and 12 columns) of 12 numbers corresponding to 12 pcs (0 to 11), with the value 1 for each pc that is present and 0 for pcs that are absent. For example, the major triad (047) is denoted (1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0). This vector is then multiplied by the matrix in Table 3. The result after matrix multiplication is the T_n-type's harmonic profile.

The calculated harmonic profile of the major triad is (18, 0, 3, 3, 10, 6, 2, 10, 3, 7, 1, 0). According to this profile, the most salient pitches evoked by a C major triad are C (pc 0, predicted salience = 18), followed by E and G (pcs 4 and 7, salience = 10 in each case). Tones F (pc 5, salience = 6) and A (pc 9, salience = 7) are predicted to be strongly implied although they are not among the chord's notes. Similar results are obtained for the minor triad; one striking difference is that the difference in salience between the root and the third is smaller for the minor triad, which can explain why the minor triad is tonally more ambiguous and - in that sense - less consonant than the

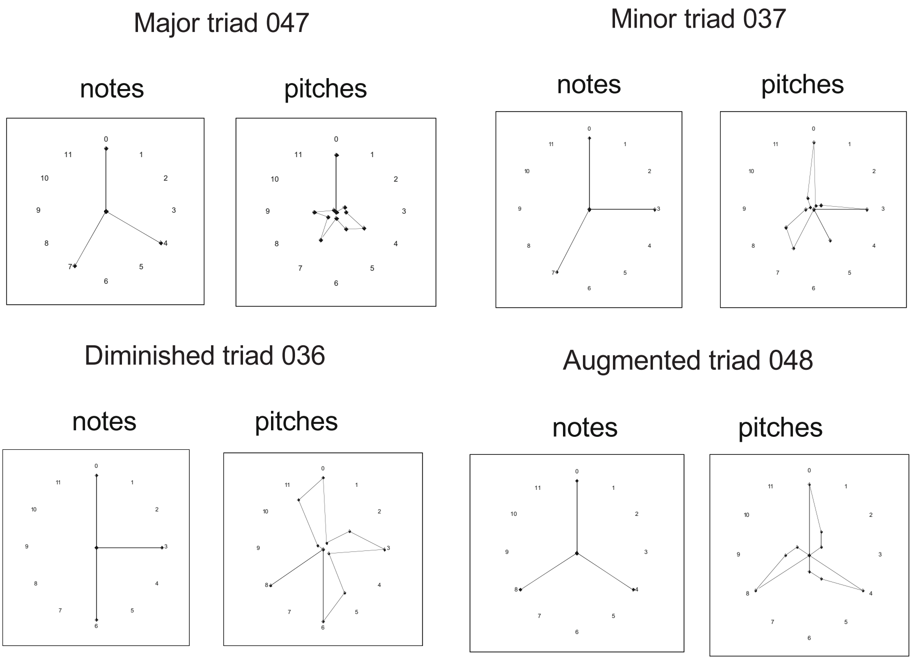


Fig. 2. Calculated harmonic profiles of four common triads

major. That can in turn explain why minor triads and tonalities are less prevalent and less stable than major triads and tonalities (Eberlein 1994).[6] The implied pitches at the 4th and 6th scale degree above the root (M6 for the major triad, m6 above the minor) can explain why chord progressions in which roots fall through fifth or third intervals are more prevalent in tonal music than progressions in the other direction (Eberlein 1994): the pitches that are implied by the first triad (the 4th and 6th) are realized as tones in the second (root and 3rd; Parncutt 2005).

The predictions of the model for major, minor, diminished and augmented triads are shown in Figure 2. Corresponding experimental data are presented in Figure 3. In Parncutt (1993), 27 listeners (mainly musicians) rated how well a probe tone went with a preceding chord. Both chords and probe tones were constructed from octave-complex (Shepard) tones. Trials were shuffled and rotated randomly around the chroma cycle. Filled diamonds in Figure 3 are mean experimental ratings; bars are 95% confidence intervals about those means.

[6] Eberlein calculated the frequency of occurrence of different sonorities including major and minor triads, and presented the results in an appendix. He consistently found more major than minor triads in the music of the 18th and 19th centuries - even in a sample that included equal numbers of pieces in major and minor tonalities. The reason is evidently that the dominant triad tends to be major in both modes.

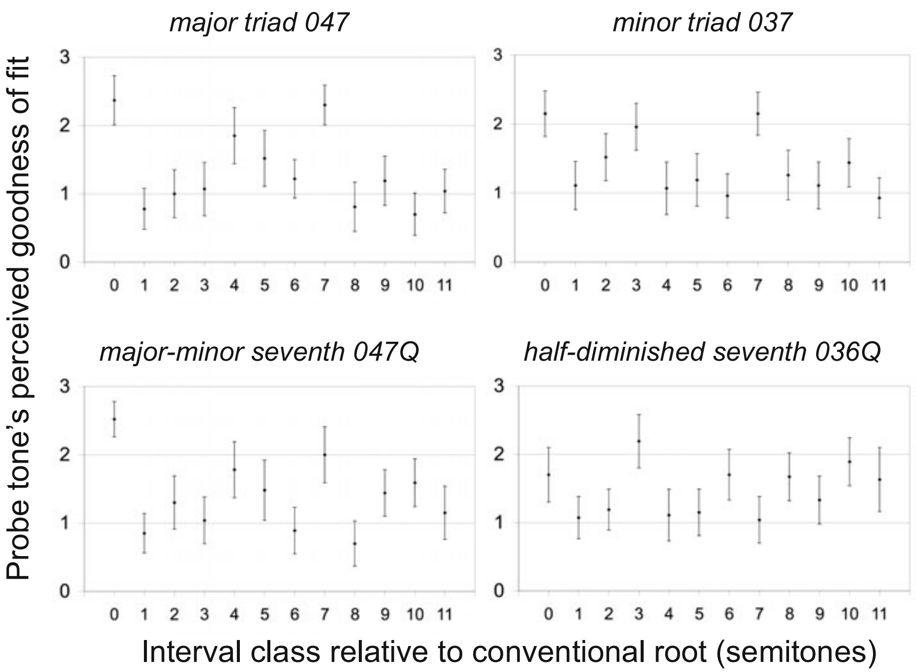


Fig. 3. Experimental data on the salience of pitches evoked by common musical sonorities composed of octave-complex tones (after Parncutt 1993). The diamonds denote the mean responses of 27 listeners; the error bars are the 95% confidence intervals. In each trial, listeners heard a chord of octave-complex (Shepard) tones followed by a single such tone. They were asked to rate how well the tone went with the chord on a scale from 0 (very badly) to 3 (very well). In the chord labels, the letter Q means 10.

The tonal profile

The *tonal profile* of a T_n-type is similar to the harmonic profile, but it is calculated in a quite different way. Each value is an estimate of the probability that a chromatic scale degree will be perceived as the *tonic* when the T_n-type is realized melodically (successively) in random order(s) and register(s). The calculation involves the major and minor key profiles of Krumhansl and Kessler (K-K) (1982), which are reproduced in Figure 4. They comprise 24 values that may be regarded as measures of the stability of chromatic scale steps in the context of major and minor keys (12 for each).

In the following, I will not consider Krumhansl's well-known explanation of the psychological distances between musical keys based on correlation coefficients between key profiles, nor will I develop the mathematical procedures based on the K-K profiles proposed by Temperley (e.g. 2007). Instead, I propose a new algorithm for the pc-salience profile of a T_n-type that is based on the assumption that listeners are familiar with the tonal stability relations within major and minor keys, as represented by K-K profiles. I begin by subtracting a constant (2.23) from all values in the profiles so that the minimum value becomes zero. I then estimate the probability that a given set of tones will occur in a given key by adding up the stability, according to the K-K profiles, of those tones in that key. For example, the probability that the set CEF# will

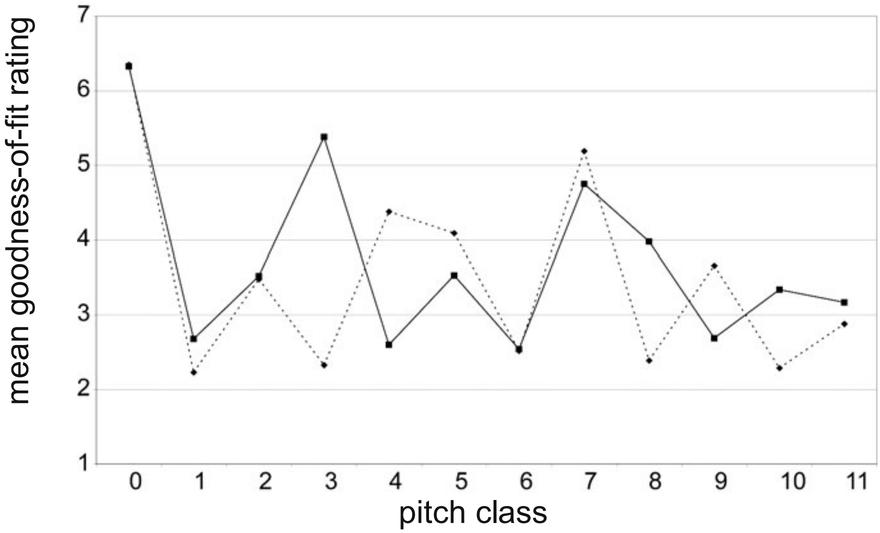


Fig. 4. The key profiles of Krumhansl and Kessler (1982). The full line denotes the (i.e. any) minor key (or tonality), the dotted line the major key.

occur in the key of C major is estimated by adding up the stability of C, of E and of F# in the C-major key profile. The novel aspect of this procedure is as follows: I then calculate the tonal profile of the T_n-type as a *weighted mean of all 24 K-K profiles* (one for each major and minor key), where the weights are the probabilities calculated in the previous step (i.e. how often we expect the T_n-type in question to occur in each key). The underlying idea is that any T_n-type can be heard in any key, but with different probabilities; the tonal profile of a T_n-type is therefore a weighted mean of all 24 key profiles, where each weight is the probability that a key will be cognitively instantiated when the T_n-type is heard. Finally, I normalize that weighted-mean profile so that its mean is 10; individual values are rounded to the nearest whole number. This new algorithm is conceptually simple, but the weighted mean of all 24 keys would be very time-consuming to perform by hand. Although the algorithm is based on the culture-specific assumption that tonality is limited to the Western major and minor modes, it yields intuitively reasonable results for all T_n-types (see appendix).

Table 4. Calculated harmonic and tonal profiles for C major and minor triads

pitch class	**in semitones**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
	as letter	**C**		**D**		**E**	**F**		**G**		**G**		**B**
major triad	harmonic profile	34	0	6	6	19	11	4	19	6	13	2	0
3-11B (047)	tonal profile	22	0	13	5	17	10	0	22	4	13	4	9
minor triad	harmonic profile	29	2	4	25	0	15	0	19	15	4	2	6
3-11A (037)	tonal profile	14	7	10	12	8	11	7	14	10	8	11	8

Table 4 compares the two kinds of perceptual profile, harmonic and tonal, for the C-major and minor triads according to these procedures. Both profiles have been normalized so that their mean is 10, and all entries have been rounded to the nearest whole number. The correlation coefficient between the harmonic and tonal profiles is quite high ($r = 0.84$ for both major and minor), although the two profiles have been calculated on the basis of quite different assumptions and using quite different procedures and numerical values. The results for the major and minor triads are also similar in the following ways. In both harmonic profiles, the most likely root is the conventional root, the third and fifth have relatively high salience, and the fourth and sixth (M6 in the major triad, m6 in the minor) are strongly implied. The approximately equal stability of root and fifth in the tonal profiles is consistent with the idea that the root of a chord does not generally (or even often) coincide with the tonic; for example, a repeated chord near the end of a classical development section is often perceived as a dominant rather than a tonic.

The perceptual profile of a T_n-type of cardinality between 1 and 11 always has peaks, which means that it is always to some extent tonal: the clearer the peaks, the clearer the tonality.[7] In Parncutt (1988), I developed a simple mathematical formulation of the "peakedness" of a pc-set's perceptual profile and called it *root ambiguity*. It was calculated by dividing the sum of the 12 values by their maximum and taking the square root of the result; the square root came from a model developed to account for empirical data on the number of tones simultaneously perceived in a set of musical and non-musical sonorities (their *multiplicity*) in Parncutt (1989). According to this procedure, the calculated harmonic ambiguity of the major triad is 1.87, which makes it the least ambiguous of all 19 T_n-types of cardinality 3 and is consistent with its ability to blend (to fuse perceptually).

Perceptual profiles, consonance and prevalence

The appendix presents the calculated perceptual profiles of all T_n-types of cardinality 3.[8] These data, when extended to include cardinalities, have interesting compositional and music-analytical applications: they can help composers to find T_n-types of any given degree of tonal strength and analysts to analyze the tonal strength of T_n-types found in the repertoire. However, things are not quite that simple, because the tonal strength of a T_n-type depends not only on the intervals in the set, but also on the prevalence of a set in the tonal literature and the contexts in which it normally appears. And that depends in turn on its consonance, or lack of dissonance.[9] This theory is not circular: the prevalence of a T_n-type is assumed originally to depend causally on just two factors, its (lack of) roughness and the peakedness of its tonal

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[7] My basic assumption is that the flatter the profile, the more ambiguous the tonal implications. I have not considered bitonality, that is, the possibility that a single profile can imply more than one root/chord or tonic/tonality. Bitonality may be regarded as an example of tonal ambiguity. I also deliberately fail to distinguish between ambiguity and multiplicity. A profile with two main peaks may cause a listener to perceive one peak or the other at different times (ambiguity), or both at once (multiplicity). That distinction is beyond the present scope.

[8] Profiles for T_n-types of larger cardinality may be obtained directly from the author.

[9] This idea applies regardless of how the term "consonance" is defined or understood. The rank order of consonance of common triads is presumably the same as their rank order of prevalence in tonal music: major, minor, diminished, augmented (Parncutt 2006).

profile (cf. Terhardt 1976). But the theory is complicated by the gradual historical evolution of tonal syntax (Parncutt, in preparation). Pitch patterns may be perceived as consonant because they are often heard in tonal music and are therefore familiar. A pitch pattern may also be performed and therefore heard more often because it is a subset of commonly-used scales. Since rough sonorities are generally less prevalent in tonal music, they may also have fewer or weaker tonal implications.

The roughness of a T_n-type may be predicted on the basis of the average roughness of the six interval classes (cf. Huron 1994). In a first approximation, the roughest interval is the minor second, followed by the major second and tritone (Plomp and Levelt 1965). These may be combined with the interval vector of each pc-set, which shows how often each interval class occurs in the set.

In the absence of a comprehensive table of such calculations, consider the interaction between roughness and the calculated ambiguity of the harmonic profiles in the appendix. The least ambiguous sets according to the appendix are 047 (major), 035 (part of a seventh chord), 027 (suspended), and 037 (minor), in that order. The reason why 037 is more prevalent in tonal music than 027 or 035 evidently involves the roughness of the major second interval within 035 and 027.

The most ambiguous T_n-types of cardinality 3 are predicted to be 036, followed by 012, 013 and 023, then by 014, 034, 046 and 048. The model predicts that 036 (the diminished triad) has four root candidates of approximately equal salience, making it highly ambiguous. None of its three tones is reinforced by a root-support interval (see Table 2), so all have approximately equal salience, and a non-chord tone - 8 relative to 036, or Ab relative to CEbGb - is reinforced by all three tones, which gives it the character of a "pitch at the missing fundamental". Why is 036 so prevalent in tonal music in spite of its tonal ambiguity? First, it is relatively smooth because it contains no major or minor seconds. Second, it is a subset of the prevalent major-minor (dominant) seventh chord (4-27B or 0368), which is the least ambiguous T_n-type of cardinality 4. Third, it is a subset of the standard major and minor scale sets (Parncutt 2006). Thus, it is both relatively smooth and relatively prevalent. The other listed sets are less prevalent because they contain rough second intervals. These sets may therefore be considered suitable for composition of "atonal" music.

Conclusion

In this paper, I have sketched a new, systematic approach to the enumeration and perceptual analysis of T_n-types. I have attempted to explain the relative tonalness, consonance and prevalence of T_n-types on the basis of the pitch-salience profiles and the roughness of corresponding musical sonorities. The preliminary findings are promising and the approach shows potential for future application in music analysis and composition.

This is not a new investigation in the sense that an answer is sought to a new question. Rather, I have considered the implications of existing empirical and theoretical work for music theory, analysis and composition. The novel aspects of this paper include the systematic application of the algorithm presented in Parncutt (1988) to all possible T_n-types, and consideration of the implications of that procedure for both the history of tonal-harmonic syntax and contemporary composition. Another original element is the development of a new algorithm for the pitch-salience of a T_n-type based on the K-K profiles.

The models that I have presented are incomplete in that they do not account for differences in the musical realization of T_n-types. It would be possible, but beyond the present scope, to account quantitatively in the presented models for parameters such as register, doubling, loudness, doubling and repetition.

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Appendix: Calculated Perceptual Profiles of All T_n-Types of Cardinality 3

Row 1: T_n-type labels, harmonic profile (12 values), ambiguity of harmonic profile (a)

Row 2: correlation between harmonic and tonal profiles (r), tonal profile (12 values), ambiguity of tonal profile (a)

3- 1 (012)	21	19	23	4	4	10	10	10	6	6	8	2	a=2.29
r = 0.72	11	10	11	9	10	11	9	11	9	11	10	9	a=3.26
3- 2A (013)	19	21	4	23	0	13	10	0	15	6	2	8	a=2.29
r = 0.75	11	11	8	12	8	11	9	9	12	8	11	8	a=3.11
3- 2B (023)	21	2	23	19	4	13	0	10	15	0	8	6	a=2.29
r = 0.75	12	8	11	11	8	11	8	12	9	9	11	8	a=3.11
3- 3A (014)	25	19	6	4	19	10	13	0	6	15	2	2	a=2.20
r = 0.75	11	11	9	9	12	10	9	9	11	11	8	10	a=3.13
3- 3B (034)	25	2	6	19	19	13	4	0	15	10	2	6	a=2.20
r = 0.75	12	9	9	11	11	10	8	11	11	9	9	10	a=3.13
3- 4A (015)	19	25	4	6	0	29	10	4	6	6	11	2	a=2.05
r = 0.83	13	11	9	10	8	14	8	9	11	9	11	7	a=2.97
3- 4B (045)	25	6	6	2	19	29	4	4	6	10	11	0	a=2.05
r = 0.83	14	8	10	9	11	13	7	11	9	11	9	8	a=2.97
3- 5A (016)	19	19	10	4	2	10	29	0	10	6	2	11	a=2.05
r = 0.73	10	13	8	10	10	10	12	8	11	10	10	10	a=3.08
3- 5B (056)	19	6	10	2	2	29	19	4	10	0	11	10	a=2.05
r = 0.73	12	10	10	10	8	13	10	10	10	10	11	8	a=3.08

3- 6 (024)	27	0	25	0	23	10	4	10	6	10	8	0	a=2.12
r = 0.75	12	8	12	8	12	10	8	12	8	12	8	10	a=3.11
3- 7A (025)	21	6	23	2	4	29	0	13	6	0	17	0	a=2.05
r = 0.85	14	7	12	9	8	14	7	12	8	11	11	7	a=2.95
3- 7B (035)	19	8	4	21	0	32	0	4	15	0	11	6	a=1.93
r = 0.85	14	8	9	12	7	14	7	11	11	8	12	7	a=2.95
3- 8A (026)	21	0	29	0	6	10	19	10	10	0	8	10	a=2.05
r = 0.59	11	9	12	9	9	10	11	11	8	11	9	10	a=3.14
3- 8B (046)	25	0	11	0	21	10	23	0	10	10	2	10	a=2.20
r = 0.59	11	10	9	9	12	9	11	10	9	11	8	11	a=3.14
3- 9 (027)	30	0	23	6	4	11	0	29	6	4	8	0	a=1.98
r = 0.90	14	6	14	8	10	11	7	15	7	11	10	8	a=2.82
3-10 (036)	19	2	10	19	2	13	19	0	19	0	2	15	a=2.51
r = 0.62	11	10	8	12	8	10	11	10	11	8	11	10	a=3.11
3-11A (037)	29	2	4	25	0	15	0	19	15	4	2	6	a=2.05
r = 0.84	14	7	10	12	8	11	7	14	10	8	11	8	a=2.95
3-11B (047)	34	0	6	6	19	11	4	19	6	13	2	0	a=1.87
r = 0.84	14	7	11	8	12	10	7	14	8	11	8	10	a=2.95
3-12 (048)	25	10	6	0	25	10	6	0	25	10	6	0	a=2.20
r = 0.74	12	10	8	10	12	10	8	10	12	10	8	10	a=3.15