

Akadémiai Kiadó

The Permutational Matrix in J. S. Bach's Art of Fugue. The Last Fugue Finished?

Author(s): Zoltán Göncz

Source: *Studia Musicologica Academiae Scientiarum Hungaricae*, T. 33, Fasc. 1/4 (1991), pp. 109-119

Published by: Akadémiai Kiadó

Stable URL: <http://www.jstor.org/stable/902439>

Accessed: 06/06/2009 19:00

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/action/showPublisher?publisherCode=ak>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is a not-for-profit organization founded in 1995 to build trusted digital archives for scholarship. We work with the scholarly community to preserve their work and the materials they rely upon, and to build a common research platform that promotes the discovery and use of these resources. For more information about JSTOR, please contact support@jstor.org.



Akadémiai Kiadó is collaborating with JSTOR to digitize, preserve and extend access to *Studia Musicologica Academiae Scientiarum Hungaricae*.

<http://www.jstor.org>

The Permutational Matrix in J. S. Bach's Art of Fugue

The Last Fugue Finished?

Zoltán GÖNCZ

*Ce qu'on appelle la postérité,
c'est
la postérité de l'œuvre.*
Marcel Proust

I

The closing Contrapunctus of the *Art of Fugue*, the “Fuga a 3 Soggetti” (Contrapunctus 14¹ = BWV 1080: 19), that only remained in fragments, is one of the most controversial composition in the history of music. Some² even question that it belongs to the cycle, mentioning that the principal subject does not appear in the piece. Others³ recognized the first theme as the simplified variation of the principal subject of the *Art of Fugue*. The strongest argument proving that the composition is an organic part of the cycle was first given in 1881 by Gustav Nottebohm, the famous researcher of Beethoven's œuvre.⁴ His research in counterpoint made clear that Bach composed the movement as a quadruple fugue and the missing fourth theme would have been the principal subject itself.

In his monography⁵ published in 1977 Kolneder speaks about exactly twenty completions. Most of these are based on the Nottebohm-clue. There are many who — taking the contents of Bach's obituary⁶ in literal sense — use

¹ In relation to numbering: Gregory Butler, *Ordering Problems in J. S. Bach's Art of Fugue Resolved*. = The Musical Quarterly LXIX (1983), pp. 44–61.

² Moritz Hauptmann, *Erläuterungen zu Joh. Seb. Bach's Kunst der Fuge*. Leipzig 1841, 1925.

Philipp Spitta, *Joh. Seb. Bach*. Leipzig 1873–80.

Wilhelm Rust, *Vorwort zur Ausgabe der Kunst der Fuge durch die Bach-Gesellschaft*. Jhg. XXV/1. Breitkopf & Härtel, Leipzig 1875.

³ Marcel Bitsch, *J. S. Bach: L'art de la Fugue* = L'Education musicale 1971, pp. 164–168, 204–207, Bernhard Martin, *J. S. Bachs letzte Fuge* = Die Musik 1941, pp. 409–412.

⁴ Gustav Nottebohm, *J. S. Bach's letzte Fuge* = Musik-Welt 1880/1881, pp. 232–236, 244–246.

⁵ Walter Kolneder, *Die Kunst der Fuge. Mythen des 20. Jahrhunderts*. Wilhelmshaven 1977, pp. 301–327.

⁶ H. J. Schulze (publishing company), *Bach Dokumente* Vol. 3. Leipzig-Kassel 1972, 86: Seine letzte Krankheit, hat ihn verhindert, seinem Entwurf nach, die vorletzte Fuge völlig zu Ende zu bringen, und die letzte, welche 4 Themata enthalten, und nachgehends in allen 4 Stimmen Note für Note umgekehrt werden sollte, auszuarbeiten. (His last illness prevented him, according to his draft, from bringing the next-to-the-last fugue to completion and working out the last one, which was to contain four themes and to have been afterward inverted note for note in all four voices.)



further on the *inversa* framing of *all the four themes*. But their success is doubtful. These combinations sound unfamiliar and they do not carry enough conviction either. Besides, in the closings there are often inadequate elements of style, forced strettì, inconsistencies of form.⁷

About the framings of the themes can confidently be stated that, following the broken m. 239, Bach did not intend to treat the *inversa* framings of the three themes that he had already exposed and elaborated.⁸ (a) Until theme 2 enters (m. 114) theme 1 appears 16 times in *recta* framing, and 8 times *inversa*, and when combines with theme 2 and 3 (another 6 phases following m. 147) it is *only* used in *recta* framing. (b) The *inversa* framing of theme 2 (9 phases) — which is fairly bizarre — *does not at all* appear in the torso. (c) The balance between the two framings of theme 3 is even more extreme: there can be found 10 *recta* framings but only 2 *inversa*. (d) The themes in the polythematic fugues of the *Art of Fugue* are exposed or elaborated — as a counterfugue — in *recta*- and *inversa* framings, still when they are later on combined with the principal subject in strettì, they appear in *one form only*. (Contrapunctus 10 and 11).

With theme 4, before looking at the contrapunctual possibilities, we have to revise the monothematic conception of the cycle and decide: Does it fit in with the logic of the complex work that, while both forms of the principal subject (*recta*- and *inversa* framings) turn up equally in the fugues before Contrapunctus 14 (among the first four simple fugues two include *recta* framings, two *inversa*, the three stretto counterfugues include *both* forms, the polythematic

⁷ Erich Bergel, *Bachs letzte Fuge*. Max Brockhaus Musikverlag 1985, pp. 196–203.

⁸ Bergel: op. cit., pp. 219–222.

fugues: Contrapunctus 8 and 10 only include *inversa*, Contrapunctus 9 only the *recta*, Contrapunctus 11 *both*, the two mirror fugues naturally include *both* framings), Contrapunctus 14, the last of the series, only includes the *recta* framing of the principal subject?

Regarding the structure of the existing fragment, the contrapuntal device of the completed movements, the conception of the cycle through the way the different forms of the principal subject are treated, and regarding the out-of-style stretti with the inversions of all the four themes, it is presumable that Bach planned *another* stretto combination — again achieved by superposing the four themes — in the fourth section of the quadruple fugue, besides the one Nottebohm described. Here the *recta* framings of the first three themes would combine with the *inversa* framing of the principal subject. But is a combination like that possible at all?

There is a possibility for a contrapuntal combination even under the condition mentioned above.⁹

Ex. 2

In comparison to theme 2 and theme 3 it can be seen that the *inversa* framing of the principal subject (4 inv) enters at the same place as the *recta* framing in Nottebohm's combination, but theme 1 here appears one bar earlier and takes the *comes* (answer) position whereas there the *dux* (subject). Examining the two combinations it can clearly be found that the twice two forms of the principal subject or theme 1, nearly take each other's place at some points as they follow each other's outlines.

⁹ It should be made clear that in Bach's polyphonic movements neither moving from augmented fifth (1st music example, m. 3, soprano, alto) nor from diminished fifths (2nd music example, mm. 3–4, bass — tenor) to perfect fifths are forbidden parallels. The latter can be observed in the soprano and alto voices in the Ctp. 6 m. 15, and in the tenor and soprano voices in m. 18.

Ex. 3



In consequence of the correlation of the two strettos shown above, an opinion that turned up earlier in a different context, saying that theme 1 is a variation of the principal subject, becomes true in another perspective.

The unification of the two strettos creates a new combination where the simultaneously delivered recta- and inversa framings of the principal subject, plus the three other themes appear. In this case theme 1 enters at a different pitch (invertible counterpoint at the twelfth), due to the parallels shown above, so the stretto obviously grows with superfluous voices (quintuple counterpoint over a tonic pedal point).

Ex. 4



The simultaneously delivered recta- and inversa framings of the principal subject always come to pass at a remarkable moment: in the Picardian coda of Contrapunctus 5 enlarged with superfluous voices (mm. 86–90), and at the end of Contrapunctus 11 (triple fugue) (mm. 157–168). Knowing these former devices, it is presumable that, because of the superfluous voices and the contra-

puntal concentration, the simultaneously inversed stretto (above) would have appeared in the last phases of the composition, following the quadruple counterpoints of the four themes.

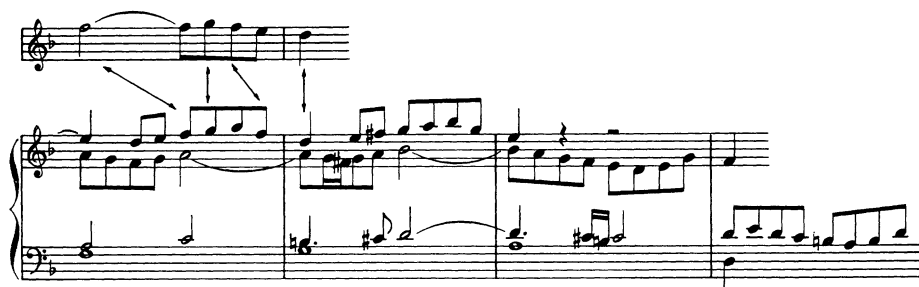
It turns out from the music examples given above, that the closing fugue was to include — changing slightly Nottebohm's idea¹⁰ — the dotted variation of the principal subject increased with passing-notes, and not the lapidary one (that appears in the four simple fugues and Contrapunctus 9). It is also significant — though not more decisive than the contrapuntal conclusions — that Bach, who always attached great importance to numerical symbolism in his former compositions, at the end of his last work to be presented to the *Sozietät der musikalischen Wissenschaften* in Leipzig, of which he was the 14th member, in Contrapunctus 14, where the musical equivalent of his name appears explicitly as a theme (the sum of the alphabetical serial numbers of the letters B—A—C—H = 2 + 1 + 3 + 8 = 14), he doubtlessly planned to treat a 14-note-variation of the cycle's principal subject.

Some points of the *Art of Fugue* bear the marks of parts that follow only later on. In the case of such a huge monothematic cycle (like this), where the composer spent years working on each different movement — returning all the time to certain parts — it is natural to have a good number of similarities, samenesses and overlaps. The reason is, besides composing parallel, that the principal subject clearly determines the countervoices. The different changing countervoices, unchanging counter-subject or themes, are all plastic backgrounds of the principal subject, and they act as “negative copies”.

But this quality is also true *vica versa*, since the Nottebohm-clue — that is the principal subject — itself is immanently included in one of the remaining fragments of the piece — in the last bars of the unfinished manuscript. (*Beilage 3 of P 200*) The three themes develop the outlines of the principal-subject-copy as a negative.

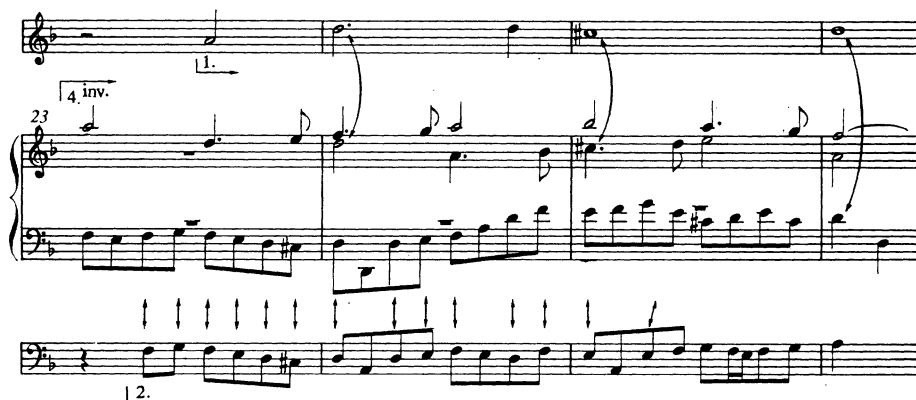
Ex. 5

¹⁰ Nottebohm also mentions this possibility. (Nottebohm, 4, p. 23.)



More explicit “navigation” marks can be found in Contrapunctus 10.

Ex. 6



The special thing about the extract is that it links — as a bridge — the earlier and later variations of the cycle.¹¹ In the earlier, written around the first half of the 40-ies (“6th Fugue” — at that time = BWV 1080 : 10a), the exposition of the principal subject starts with the inversa framing delivered by the soprano — in one voice, of course. In the later form (Contrapunctus 10) this follows the exposition and elaboration of the other theme. Complementary voices — not put in the earlier version — go with the first phase of the unchanging exposition of the principal subject, in order to keep the pulsation of quavers. Presumably, these later parts and the closing Contrapunctus — which is not in the earlier version — were composed around the same time, and this can also be the cause of the motivic and harmonical similarities.

¹¹ Douglas Seaton, *The Autograph: An Early Version of the “Art of Fugue”* = *Current Musicology* no. 19 (1975), pp. 54–59.

The fragment indicates, confirming the above statements, that in the closing Contrapunctus of the *Art of Fugue* following the broken m. 239 (after the further stretti of the three themes), Bach intended to treat not only the *recta* but the *inversa* framing of the principal subject in a contrapuntal combination where the first three themes appear in *recta* framing.

II

Knowing the structure of the completed fugues, there are several possibilities for the two framings of the principal subject to appear. However, the special form of the movement's remaining fragment tells — as we will see — that Bach did not intend to expose the principal subject separately in the closing Contrapunctus, but it will immediately enter with the first three themes.¹² (Both the gigantic dimensions of the movement and the monothematic characteristic of the cycle give reasons for this method.)

In Contrapunctus 14 we can detect some serial tendencies — surprising and unusual in baroque music. In the exposition of the first three themes in this fugue (theme 1: mm. 1–21; theme 2: mm. 114–141; theme 3: mm. 193–207) Bach uses a predestinated entry order of the voices and (in the terminology of the second half of our century) thus he determines the time- and space parameters of the thematic phases.

		time →											
s p a c e	soprano	theme 1				theme 2				theme 3			
	alto				1		2				3		
	tenor			1					2	3			
	bass	1						2				3	

The three expositional matrixes placed on one another predict and develop — as in the case of a negative copy — the entry order of the voices of theme 4.

		2	3	1
2	3	1		
3	1		2	
1		2	3	

4			
			4
		4	
	4		

¹² A similar solution can be seen in Contrapunctus 9.

As the four themes are superposed, they build up a structure most typical of vocal fugues in Bach's oeuvre: the *permutational fugue*.¹³

4	2	3	1
2	3	1	4
3	1	4	2
1	4	2	3

The permutational fugue combines the structures of both fugue and canon, the voices rotate the themes with special concentration in a *canon-like rotation without episodes*.

In relation to the structures of the permutational fugue and the quadruple fugue Werner Neumann's statement is specially remarkable: "Jede Permutationsfuge ist Tripel-, Quadrupel- oder Mehrfach-Fuge, oder wenigstens ein Ansatz dazu."¹⁴

The theme-series of the *permutational matrix* of Contrapunctus 14 (4, 2, 3, 1) horizontally determines temporal procedures, and determines spatial stratifications vertically; in this way, the successive order of the themes coincides with the simultaneous, spatial texture.¹⁵

However, Bach's "serialism" does not only determine the order of the themes but also the exact pitch of each of them (except for one occasion). In the first permutational phase the three themes enter at the same pitch as in their first expository phase, and this is consequently applied further on in relation to the expositions of themes and the permutational matrix. (The exception is theme 1 in the second phase. The reason of appearance of theme 1 at the fourth is to avoid a second inversion chord on a downbeat in m 5. of this phase.)¹⁶

The superposition of the three expositions determines the form of the principal subject just as its place and pitch to enter. The order of permutations clarifies that according to the Nottebohm-combination only the *recta* framing of the principal subject can appear in this part, because the *inversa* framing would lead to wrong constellations, forbidden parallels and harmonies.

¹³ On Permutational Fugues: Werner Neumann: *J. S. Bach's Chorfüge*. Breitkopf & Härtel, Leipzig 1950. Zoltán Gárdonyi: *J. S. Bach Kánon- és fúgaszerkesztő művészete. [J. S. Bach's art in composing canons and fugues.]* Zeneműkiadó 1972, pp. 122–132. Carl Dahlhaus, *Zur Geschichte der Permutationsfuge* = Bach-Jb. XLVI, 1959.

¹⁴ W. Neumann, 13, p. 42.

¹⁵ There are similar devices in Cantata no. 50. (Nun ist das Heil), and in the closing chorus (Lob und Ehre und Preis und Gewalt) of Cantata no. 21. (Ich hatte viel Bekümmernis). In these two movements — as in the 14th Ctp. — the themes rise in tiers, gradually moving upwards from below.

¹⁶ G. Nottebohm, 4, pp. 234–235.

Ex. 7

In the permutational matrix the principal subject does not have a separate exposition, but it is exposed immediately in four voices combined with the earlier three expositions; thus *the exposition of the principal subject coincides* — in a specially concentrated way — *with the stretti of the four themes*. The permutational section includes a third characteristic of form: a unique comprehensive *recapitulation* in which, besides the principal subject of the cycle, the expositions of the other three themes appear again.¹⁷

Though the following coincidence may not be significant, it is worth looking at a “characteristic” of numerical symbolism in the matrix. Taking the pitch of the B—A—C—H notes into consideration, it can be seen that in the matrix the serial numbers of the themes turning up in the places of the cross-configuration add up to 14.¹⁸

		C	
			H
B			
	A		

		3	
			4
3			
	4		

Knowing the possibilities for further combinations of the four themes,¹⁹ it is presumable that after the permutational section shown above, the stretti with the inversa framing of the principal subject and the first three themes

¹⁷ Da Capo appears in some of the earlier fugues: for example, in Fugue in C minor, BWV 537, or in Fugue in E minor, BWV 548.

¹⁸ Number 14 was the “numerical symbol” of the cycle, especially the closing quadruple fugue. Another possible interpretation of the numerical symbolism of the permutational matrix is to add up the serial numbers of the four themes ($1 + 2 + 3 + 4 = 10$) — which is very common in the numerical symbolism of baroque music — and correlate it with the strict canonic structure (canon = rule, measure). Number 10 is often the symbol of Ten Commandments while canon is the symbol of follow and obedience. (See for example Clavier-Übung III: Dies sind die heil’gen zehn Gebot’ BWV 678.)

¹⁹ See: 2nd and 4th music examples.

would follow next, then the two simultaneous (direct and inverted) forms of the principal subject would combine with the other three themes, creating a quintuple counterpoint.

The permutational section proves *Christoph Wolff's* hypothesis²⁰ saying, that Bach completed the closing Contrapunctus, at least no doubt that the combinatorial section (fragment x) was finished, but then, following the composer's death, it got lost as did manuscripts of other movements and parts. Even if it seems a paradox, the logic in the composition of the quadruple fugue demonstrates, that the work's *last* combinations with all the four themes were already composed and completed in the *first* compositional period, since the possibility for the stretto of the four themes is a *sine qua non* of constructing the whole quadruple fugue. Consequently the *fragment x* (or a section of it) is the *permutational matrix* itself that was ready at the genesis of the composition.

The speciality about Contrapunctus 14 is, that in this movement Bach *constructed stretti by superposing full expositions*: He "programmed" the late permutational stretti in the expositions of the first three themes, then he (would have) used the expositions as "programs — operational orders".

This structure, in the history of music, is a specific Janus-type one. The roots of permutational compositions go back to the 12th century (rota, rondellus), the use of permutational structure in thus instrumental piece, with the marks of *stile antico*,²¹ makes the work particularly archaic. At the same time the entries of the themes previously determined in space and time and the logical and consequent fixing of the elements (both their order and pitch), resulting in the permutational matrix (as an outcome, culmination), all project the compositional devices of our century.

The *Art of Fugue* remained to the posterity — due to the composer's illness, death and the heirs carelessness or ignorance, but definitely against Bach's intentions — as a torso, the greatest puzzle of his oeuvre. The extremely strict principle of the serial structure made it possible to reconstruct a lost part of the piece, the central seed itself: to find a part of the *temps perdu*.

Ex. 8



²⁰ Christoph Woff, *The Last Fugue: Unfinished?* = Current Musicology no. 19 (1975), pp. 71–77.

²¹ Christoph Wolff, *Der stile antico in der Musik Johann Sebastian Bachs*. Wiesbaden, 1968.

The musical score is presented in six systems, each with a treble and bass staff. The key signature is one sharp (F#), and the time signature is 4/4. The notation includes various musical symbols such as notes, rests, accidentals, and dynamic markings. The first system begins with a treble staff starting on G4 and a bass staff on G3. The second system continues the development. The third system features a more complex texture with sixteenth-note runs in the bass. The fourth system shows a continuation of the themes. The fifth system has a treble staff with a long note and a bass staff with a sixteenth-note pattern. The sixth system concludes the page with a treble staff on a whole note and a bass staff with a sixteenth-note pattern.