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Pitch of Complex Tones*

R. Plomp

Institute for Perception RVO–TNO, Soesterberg, The Netherlands

The question of whether the pitch of complex tones is based either on the frequency of the fundamental or on the periodicity of the sound as a whole was studied. Fifteen naïve subjects compared the stimuli

$$A=\sum_{n=1}^{12} a_n \cos(2\pi nft) \quad \text{and} \quad B=\sum_{n=1}^{m} a_n \cos[2\pi n(0.9f)t]+\sum_{n=m+1}^{12} a_n \cos[2\pi n(1.1f)t]$$

and had to decide which of the two stimuli, presented successively during 200-msec intervals, was higher in pitch. The results showed that for $m=1$ (fundamental of B was 10% lower in frequency than that of A, and all other harmonics were 10% higher in frequency than A) the pitch changed according to the harmonics for $f<1400$ Hz and according to the fundamental for $f>1400$ Hz. For $m=2$, 3, and 4, the corresponding transition frequency was about 700, 350, and <125 Hz, respectively. This holds for both $a_n=1$ and $a_n=1/n$. The experiments support the view that the pitch of complex tones is based on periodicity rather than on frequency. The description of the experiments is preceded by a historical review.

INTRODUCTION

PERIODIC sound waves produced by musical instruments and the human voice usually consist of a large number of harmonics. As was discussed previously (Plomp, 1964), the organ of hearing is able to distinguish the first five to eight harmonics of a complex tone individually. This indicates that the auditory mechanism can be compared with a (continuous) frequency analyzer with a selectivity corresponding to the critical bandwidth.

In normal listening, however, we are not aware of the individual harmonics, but the complex tone is always perceived as a single tone with one definite pitch equal to the pitch of the fundamental.[1] The "classical" explanation attributes this to the high relative loudness of the fundamental, which either exists in the sound itself or is introduced by the ear's nonlinearity. There are indications, however, that the presence of the fundamental is not essential for hearing a corresponding pitch, but that pitch is related to periodicity. For that reason, it appeared to be of interest to investigate more extensively on which physical parameter the pitch of complex tones is based: on the frequency of the fundamental or on the periodicity of the soundwave as a whole.

I. HISTORICAL REVIEW

A. Controversy between Seebeck and Ohm

Not until the publication of a paper by Seebeck in 1841 was it realized that the pitch of complex tones might be a controversial question. Seebeck reported the results of some investigations on the conditions for hearing tones. His sound sources were rotating disks provided with a concentric ring of holes; the interspaces between the holes were, in most cases, larger than the diameters of the holes. Air was blown through a tube perpendicular to the rotating disk so that the air stream was interrupted periodically by the interspaces between the holes. With equidistant holes, these siren disks produced a tone with a pitch corresponding to the number of holes per second that passes the air tube. The use of these devices demonstrated that short, periodic pressure pulses give rise to tone sensation, so it is not surprising that Seebeck interpreted his findings in terms of periodicities perceived as tones.

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[1] To avoid any misunderstanding, by *pitch* is meant, in this paper, that attribute of auditory sensation in terms of which sounds may be ordered on a *musical scale*, or, otherwise stated, that attribute that constitutes *melody*. Wider definitions of pitch, as can be found in literature, give rise to confusion and have to be abandoned.

Seebeck used this siren to investigate the pitch for disks with different distances between the holes, namely with distances alternately equal to a, b, a, b, etc. and a, b, c, a, b, c, etc., respectively. His most important conclusions were that (1) the hearing organ is able to analyze such a sequence of irregular pulses into two and three sequences of periodic pulses, respectively; for only one pitch is heard equal to the pitch of a disk with equidistant holes at a distance of $a+b$ in the first example, and $a+b+c$ in the second one; (2) for small differences between the distances a and b, and a, b, and c, respectively, the ear overlooks the irregularities and a pitch is heard corresponding to $\frac{1}{2}(a+b)$ and $\frac{1}{3}(a+b+c)$, respectively.

Ohm criticized, two years later, Seebeck's interpretation of these experiments (Ohm, 1843). Up to now, so he wrote, he had accepted as an established truth that a sinusoidal wave given by $\sin(2\pi ft+\varphi)$ is required to hear a pitch corresponding to the frequency f. This is Ohm's famous definition of tone. In his opinion, Seebeck's conclusions were at variance with this definition, for they suggested that the pitch of tones is based on periodicities of acoustic pulses. Introducing Fourier's theorem on the resolution of a periodic function into sinusoids, Ohm demonstrated that the sounds produced by the siren disks contained sinusoidal components corresponding to the pitches heard. He tried in this way to reconcile Seebeck's experimental results with the "traditional" conception of the origin of tone pitch.

In the continuation of this interesting discussion, the difference between the views of Seebeck and Ohm about the physical correlate of pitch becomes more clear. In view of the fact that the pitch corresponding to the period of acoustic pulses of a siren disk with equidistant holes is heard much stronger than the individual harmonics, Seebeck (1843) concluded that the presence of a sinusoidal component with frequency f is not essential for hearing a pitch corresponding to this frequency. Ohm (1844) replied that this phenomenon might be due to "acoustical illusion" of the ear. Rightly, Seebeck (1844a) took this answer as a confirmation of his point of view, because—as he said—only the ear can decide in which way tones are perceived. Moreover, he indicated that only by assuming that the pitch of a periodic sound wave is based on the total of the sinusoidal components, can the existence of different timbres of tones with the same pitch be understood. In a closing paper (Seebeck, 1844b), he tried to prove quantitatively that periodic sound waves with very faint fundamental can be produced. He also found in those cases that it was difficult to distinguish the higher harmonics, but that the sound as a whole had a distinct pitch corresponding to the periodicity of the pulses.

B. Phenomenon of "Interruption Tones"

The fact that von Helmholtz (1863) strongly promoted Ohm's view did not mean the end of the controversy. König published in 1876 a detailed study on beats and combination tones, including also some investigations with siren disks. In one of these experiments, he rotated such a disk in front of a vibrating tuning fork. He found for an interruption rate of 128/sec and a tuning fork of $n\cdot 128$ Hz (n integer) that a pitch corresponding to 128 Hz could be heard. The strength and clearness of the pitch of this "interruption tone" increased for increasing n. In another experiment, König blew siren disks provided with equidistant holes varying periodically in diameter, and also, in this case, the variations resulted in a definite pitch. Although König did not touch the question of the Fourier spectrum of these sounds, it is evident that the stimulus in both cases consisted of a complex tone with harmonics much stronger than the fundamental.

König's experiments directed the attention of several other investigators to the phenomenon of interruption tones. Dennert (1887) repeated and extended these experiments and came to the same conclusion as König did. In his opinion, the existence of interruption tones is incompatible with Ohm's acoustical law. Hermann (1890), in a study on the frequency spectrum of sung vowels, determined that in many cases the fundamental is nearly absent; nevertheless, always a pitch equal to the pitch of this tone is heard. Pursuing his interest in this question, he carried out experiments similar to those of König and Dennert and found that when a tone is periodically interrupted, the tone disappears as the interruption rate increases and a tone with a pitch corresponding to this rate becomes audible. Apparently, so he concluded, the ear is capable of perceiving periodic variations in amplitude as a tone. Pipping (1895) made an interesting attempt to reconcile this phenomenon with von Helmholtz's hearing theory by introducing a distinction between the pitch of a tone and the pitch of a clang ("Klang" in German): for a complex tone, we may direct our attention to the partials (tone pitch), but it is also possible to pay attention to the total impression of the sound (clang pitch); the latter pitch is not influenced by the absence of the fundamental, a group of partials being sufficient to perceive this pitch.

Pipping also pointed to the fact that in cases for which the stimulus does not contain the fundamental, this tone will be reintroduced by nonlinear distortion in the hearing organ.

Everett (1896) and Schaefer (1899) suggested that the interruption tone itself may be owing to nonlinearity. They tried in this way to obviate the difficulty of how the existence of this tone could be squared with the frequency principle in hearing. For that reason, the origin of interruption tones was studied more extensively by Schaefer and Abraham (1901, 1902). On the basis of their experiments with siren disks, they found that in many cases the objective existence in the sound of a partial with a pitch equal to the pitch of the interruption tone could be demonstrated with the aid of an acoustic resonator, whereas they explained the latter tone in the other cases as a difference tone produced in the ear. Thus, in all cases, the interruption-tone perception was attributed to a simple tone in accordance with Ohm's law.

Meanwhile, more investigations on interruption tones were published. Zwaardemaker (1900) placed the tone source in front of a microphone and, by means of an electric interrupter, a strong interruption tone could be perceived. These experiments were repeated by Schaefer and Abraham (1904), who obtained a somewhat different result; they only heard a distinct tone corresponding to the interruption rate if the frequency of the tone source was a multiple of the number of interrruptions per second. Schulze (1908) explained these results in terms of von Helmholtz's hearing theory. Hermann (1912), however, pointed to the fact that the results contradicted Schaefer and Abraham's opinion that the interruption tone has to be considered as a difference tone. Interrupting a tone of f Hz g times per second gives rise to other tones of $f-g$, $f+g$, $f-2g$, $f+2g$ Hz, etc; every pair of adjacent partials will contribute, in the case of nonlinear distortion, to the production of a difference tone of g Hz, independent of whether $f=ng$ (n integer) or not. So, the fact that a distinct tone corresponding to g was heard only for $f=ng$ does not support Schaefer and Abraham's own interpretation that this tone was a difference tone.

Using similar equipment, Hermann performed new experiments and came to the interesting conclusion that sometimes, for $f\neq ng$, the pitch that is heard does not correspond to g, but to a value differing by 10%–20%. He was not able to explain this effect. For $f\neq ng$, the stimuli used by Hermann consisted of a set of inharmonic partials, separated by g Hz. Therefore, these experiments anticipated some much more recent experiments wherein similar pitch shifts were found. Without his realizing it, Hermann's findings represented one of the best demonstrations that the interruption tone is not a difference tone, but has to be understood as the result of periodicity perception.

Summarizing our historical review on the pitch of a complex tone up to about 1920, we may conclude that at that time the problem was still just as controversal as it was 80 years before. Most workers followed Schaefer and others in their attempt to reconcile the experimental evidence on the pitch of complex tones with the frequency principle by pointing to the fact that most often the fundamental is not completely absent and that the ear's distortion may reintroduce this tone. However, there were some investigators who did not accept this point of view, because it could not explain why the fundamental pitch of complex tones is so dominant. Perhaps it was not possible to solve this problem with the equipment available at that time.

C. Recent Investigations

Even after the introduction of electronic equipment in hearing research, it took many years to surmount the impasse. Fletcher (1924) used electric filters to eliminate the lower harmonics of complex tones and noticed that the pitch did not change. He studied synthesized complex tones consisting of multiples of 100 Hz and found that three consecutive harmonics were sufficient to give a clear musical tone with a pitch equal to the pitch of a simple tone of 100 Hz. Fletcher explained these results by assuming that in all these cases the missing fundamental is reintroduced by the ear's nonlinear distortion. Jeffress (1940) investigated the pitch of complex tones in which the fundamental was removed and obtained different results. His subjects found great difficulty in comparing this pitch with the pitch of a simple tone equal to the fundamental, and in most cases they preferred the octave of the latter tone for best match.

We now arrive at some pioneer investigations by Schouten, carried out in 1938–1940, the importance of which was not fully realized before about 1955. The experiments were carried out with the aid of an optical siren with which it was possible to produce periodic sounds with any desired waveform. In a first study, Schouten (1938) investigated the pitch of periodic pulses of finite width (1/20th of the repetition time of 1/200 sec) of which the fundamental of 200 Hz was cancelled completely. This was verified by the absence of beats when an additional tone of 206 Hz was introduced. The pitch of the complex tone, however, was exactly the same as the pitch of the periodic pulse with the fundamental included. In a second paper, Schouten (1940a) described more extensively the sensation corresponding to a periodic sound wave with many partials and he stated: "The lower harmonics can be perceived individually and have almost the same pitch as when sounded separately. The higher harmonics, however, cannot be perceived separately but are perceived collectively as one component (the residue) with a pitch determined by the periodicity of the collective waveform, which is equal to that of the fundamental tone." A third paper (1940b), dealing with the consequences of these investigations for hearing theory, reconciled periodicity perception with auditory frequency analysis.

Schouten's conclusion, in agreement with the views of some scientists mentioned above, that periodicity of waveform may lead to a definite pitch sensation was strongly opposed by Hoogland (1953) as a representative of the "traditional" view that a sinusoidal sound wave is necessary to hear a corresponding pitch. Although Hoogland's observations, indicating that the ear's nonlinearity does introduce the fundamental of a series of higher harmonics, are correct, his opinion that Schouten had perceived a difference tone shows that he missed the essence of Schouten's exposition.[2]

Other experiments, however, supported Schouten's results. Davis *et al.* (1951) led brief periodic pulses through a bandpass filter with a center frequency of 2000 Hz and asked subjects to match the pitch of this signal to the pitch of a simple tone. The listeners varied greatly in the accuracy with which the latter pitch was matched to either the period of the pulses (about 130 pps) or the bandpass frequency; errors of exactly one octave were particularly common.

A much more conclusive experiment was carried out by Licklider. He demonstrated in 1954 that a pitch corresponding to the periodicity of a series of high-frequency harmonics of a low-frequency (missing) fundamental was audible even when low-frequency noise, sufficiently loud to mask a possible fundamental created in the ear, was introduced (see his paper of 1956). He made a tape recording of pairs of tone pulses; the first pulse in each pair consisted of low-frequency sinusoids and the second pulse of high-frequency harmonics of the same low frequency. The first tone pulse was definitely louder than the complex of harmonics. The successive pairs of tone pulses progressed up and down in the scale of frequency. The tone pulses consisting of sinusoids disappeared by the introduction of the masking noise, but the low pitch of the other tone pulses could be heard right through the noise, going up and down. During Licklider's visit to Utrecht, in 1955, the writer had the opportunity to hear this tape recording; the demonstration was very impressive.

Thurlow and Small (1955) repeated (nearly at the same time) the experiments of Davis *et al.*, mentioned above, but they used bandpass filters with different center frequencies. For a 1000-Hz filter, the pitch corresponded most closely to the repetition rate of the pulses (100 pps) and no pitch corresponding to 1000 Hz was heard. For a 5000-Hz filter, both low and high pitches were heard. The stimuli were presented at low sensation levels, and spectral analysis showed that no low frequency corresponding to pulse rate was present.

In another experiment, Small (1955) used periodic tone pulses by interrupting tones of 1000, 2000, 4000, and 8000 Hz, respectively, with an interruption rate of 100/sec. Varying also pulse duration and rise–fall time, he asked listeners to match the pitch of the periodic tone pulses to the pitch of a sinusoidal comparison tone. The stimuli were presented at a very low sensation level of 20 dB in order to avoid audible tones at 100 Hz, corresponding with repetition rate. Two pitches were identified, one low pitch corresponding to periodicity, and a high pitch corresponding to the frequency of the interrupted tone. The periodicity pitch decreased in audibility with the increase of any of the following variables: frequency, pulse duration, and rise–fall time. These variables had the opposite effect on the perception of the high pitch, which generally was audible only for the longer pulse durations.

The stimuli used in this and the foregoing experiment can be regarded as consisting of a series of high harmonics of a (missing) fundamental. Apparently, the audibility of periodicity pitch for 100 Hz decreases when the frequency of the strongest harmonic shift from 1000 to 8000 Hz and increases when the stimulus contains more harmonics. Small found that periodicity-pitch perception is not limited to repetition rates that are submultiples of the frequency of the interrupted tone; varying the repetition rate of the tone pulses resulted in a corresponding pitch shift. In this case, the partials of the complex signal are no multiples of the repetition rate.

The perception of pitch of inharmonic complexes of tones was studied more extensively by de Boer (1956). In his experiments, based on an important observation by Schouten (1940c), he made use of amplitude modulation as described in the following example. A simple tone of 2000 Hz is modulated in amplitude by a signal composed of tones of 200, 400, and 600 Hz, resulting in a complex of tones with frequencies of 1400, 1600, 1800, 2000, 2200, 2400, and 2600 Hz. This stimulus has a clear pitch equal to the pitch of a 200-Hz tone, even for low sensation levels (20–40 dB was used). By shifting the carrier frequency from 2000 Hz up to, for instance, 2030 Hz, the originally harmonic complex of tones is transformed in an inharmonic complex with frequencies of 1430, 1630, 1830, 2030, 2230, 2430, and 2630 Hz. de Boer observed that this transformation results in a pitch shift from a value corresponding to a tone of 200 Hz to a value corresponding to about 203 Hz. This effect cannot be attributed to nonlinear distortion, for in both stimuli the frequency difference between adjacent partials is 200 Hz. de Boer demonstrated that the phenomenon can be explained by assuming that pitch sensation is caused by periodicities in waveform. It is interesting to notice that similar pitch shifts were already observed by Hermann (1912), as we saw above.

Later investigations showed that the audibility of periodicity depends upon both the frequency of the harmonics presented and the frequency of the missing fundamental. Flanagan and Guttman (1960) carried out experiments in which the pitch of a periodic train of very short pulses was matched to that of another train whose fundamental tone was removed. They found that, for pulse rates less than 100 pps, the pitches are matched to equal pulse rate, regardless of the polarity pattern of the pulses (positive and negative pulses alternated). For fundamental frequencies of 1000 Hz and higher, subjects tend to equate the fundamental of the matching tone to the lowest spectral component present in the matched stimulus. In between these ranges, in particular between fundamental fre-

[2] Recent experiments have shown that the ear's nonlinear distortion is very small and has to be abandoned as a source of perceiving a pitch corresponding to the missing fundamental (Plomp, 1965).

quencies of 200 and 500 Hz, there is a decided tendency to equate fundamental frequencies, so in this range the polarity pattern of the pulses is indeed taken into account.

The limit of periodicity pitch was studied in another way by Ritsma (1962) with a harmonic complex of three tones produced by modulating the amplitude of a simple tone with frequency f with another simple tone with frequency g. The minimal modulation depth required to hear periodicity pitch was determined for $f/g=4$, 5, 6, 7, etc., respectively, as a function of g. The results indicate that (1) periodicity is heard more clearly as modulation depth increases; (2) no periodicity pitch is audible for $f>$ about 5000 Hz nor for $g>$ about 800 Hz. The latter result agrees with the finding of Flanagan and Guttman, mentioned above.

From this review on the pitch of complex tones, we may conclude that experimental evidence has shown clearly that, over a wide range, the presence of the fundamental is not required in order to perceive a definite pitch equal to the pitch of the fundamental. Apparently, the ear is able to use periodicity as a basis of pitch perception.

II. EXPERIMENTS

The conclusion that the ear appears to be able to use the period of a sound wave as a basis of pitch perception deviates from the "classical" opinion that pitch is related to frequency. It seems rather unlikely, however, that the ear is provided with two pitch-detecting mechanisms, one using frequency and the other periodicity.

For a better insight into this question, it is of interest to know more about the relative contribution of the fundamental to the pitch of a complex tone. If experimental evidence should indicate that this pitch is mainly or exclusively based on the tone's periodicity, this would support the view that frequency is not a relevant parameter for pitch. In order to answer this question, the following experiments were carried out.

A. Method

If the pitch of a complex tone is mainly based on the frequency of the fundamental, we may expect that the pitch is much more influenced by a 10% frequency decrease of the fundamental than by a simultaneous 10% frequency increase of the second and higher harmonics. This was studied by presenting the ear, successively, with two stimuli:

Stimulus 1:

$$\sum_{n=1}^{12} a_n \cos[2\pi n f t],$$

Stimulus 2:

$$a_1 \cos[2\pi(0.9f)t]+\sum_{n=2}^{12} a_n \cos[2\pi n(1.1f)t].$$

The subject's task was to indicate whether the pitch of Stimulus 2 was higher or lower than of Stimulus 1.

Since it appeared that, over a large frequency range, the pitch of Stimulus 2 was definitely judged as higher than of Stimulus 1, demonstrating that pitch depends upon the periodicity of the second and higher harmonics together rather than on the frequency (or periodicity) of the fundamental, additional experiments were carried out in which Stimulus 2 was of the more general form

$$\sum_{n=1}^{m} a_n \cos[2\pi n(0.9f)t]+\sum_{n=m+1}^{12} a_n \cos[2\pi n(1.1f)t],$$

with $m=1$, 2, 3, 4, respectively. It was possible in this way to investigate to what degree pitch is related to the lower or the higher harmonics of a complex tone.

In one experiment, the amplitudes of all harmonics were equal to each other ($a_n=1$). Since this situation may not be representative for complex tones as used in practice, a circuit with a frequency–response characteristic of -6 dB/oct was introduced in a second experiment ($a_n=1/n$). The relative amplitudes of the harmonics agree in the latter case rather well with the mean situation for speech vowels and musical tones.

B. Apparatus and Procedure

Figure 1 represents a block diagram of the apparatus. Function Generator 1, described earlier (Plomp, 1964), was adjusted to produce the function

$$\sum_{n=m+1}^{12} \cos(2\pi n g t).$$

Function Generator 2 consists of a circuit that divides the frequency of the input signal by a factor of 3, followed by a circuit similar to Function Generator 1 but with 20 shift-register elements. This generator produced the function

$$\sum_{n=1}^{m} \cos(2\pi n g t).$$

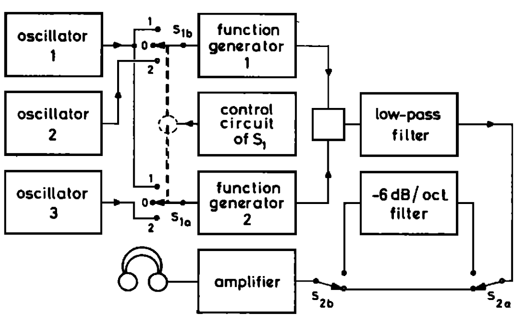

FIG. 1. Block diagram of the apparatus.

Periodic input signals of frequency $60g$ are required to produce these functions. These signals could be taken from three sine-wave oscillators (Hewlett–Packard 200 CD) by means of the electronic switch S_1. Oscillator 1 was adjusted to $60f$, Oscillator 2 to $66f$, and Oscillator 3 to $54f$. In Position 1 of S_1, both function generators are connected to Oscillator 1, so the superimposed output signals of the generators result in a signal equal to Stimulus 1 with $a_n=1$. In Position 2, the generators are connected to Oscillators 2 and 3, respectively, resulting in an output signal equal to Stimulus 2, also with $a_n=1$. Switch S_1 was controlled by an electronic circuit with two switching programs represented by A and B in Fig. 2. In both conditions, the stimuli were presented two times, with a stimulus duration of 200 msec. Possible frequency components above $12f$ were removed by means of a low-pass filter. A -6-dB/oct filter was introduced in the upper position of Switch S_2, corresponding with $a_n=1/n$. The headphone (Beyer DT 48) was provided with a correction filter after the design of Zwicker and Gässler (1952).

The equipment used for adjustment and control of the signals is not represented in Fig. 1. The amplitude of the fundamental of the signal produced by Generator 1 was in all cases about 40 dB or more lower than of the partials wanted. Every time that S_1 was switched to Position 1, a resetting pulse was applied automatically to both function generators, which assured that the phase relation between their output signals was always the same. These signals were controlled continuously during the experiments with the aid of a double-beam oscilloscope. In addition, the experimenter could check the stimuli presented to the subject by means of a loudspeaker.

The experiments were carried out for $f=125$, 175, 250, 350, 500, 700, 1000, 1400, and 2000 Hz, respectively. For each value of f, both the cutoff frequency of the low-pass filter and the tilting frequency of the -6-dB/oct filter, if used, were adapted to f. The stimuli were presented monaurally at a sound-pressure level of 60 dB.

The following procedure was used. The subject was presented with two identical pairs of successive tone pulses after the scheme reproduced in Fig. 2 (Condition A or B). After that, he had to indicate whether the first or the second tone pulse of the pair was higher in pitch. In order to avoid the influence of timbre, no comparison tone pulses were used, but the subject was asked to reproduce the pitch difference between the stimuli vocally by singing, humming, or whistling. He was told before that not the absolute pitch but only the shift of it was important.

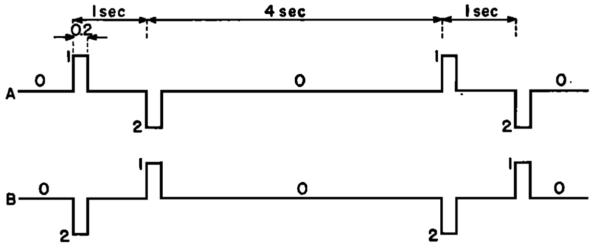

FIG. 2. Position of Switch S_1 for Conditions A and B. The positions 1 and 2 correspond with the presentation of Stimuli 1 and 2, respectively.

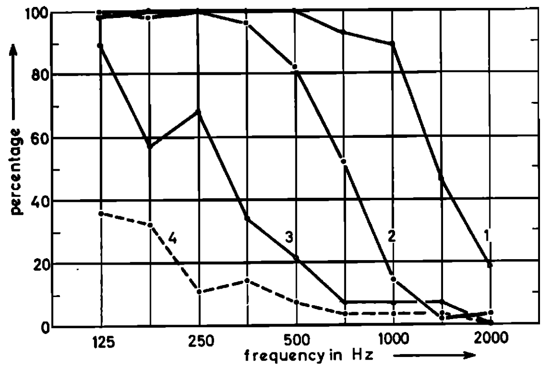

FIG. 3. Percentage of responses, averaged over 14 subjects, in which the pitch of the stimulus

$$\sum_{n=1}^{m} \cos[2\pi n(0.9f)t] + \sum_{n=m+1}^{12} \cos[2\pi n(1.1f)t]$$

was judged as higher than that of

$$\sum_{n=1}^{12} \cos[2\pi nft]$$

as a function of f, with m as a parameter ($m=1$, 2, 3, 4, as indicated).

In a typical test session, the subject had to judge the pitch shift two times for all values of f mentioned above, since the stimuli were presented once in Condition A and once in Condition B. These 18 presentations were given in random order, so the subject did not know *a priori* whether Stimulus 1 or Stimulus 2 preceded. In eight test sessions, the experiment was repeated for $m=1$, 2, 3, 4, with $a_n=1$ and $a_n=1/n$, respectively. Fifteen subjects participated in the experiments and care was taken that no one knew the physical properties of the stimuli and the feature of the experiments. The subjects were tested individually.

C. Results

Figures 3 and 4 present the results of the experiments. The percentage of responses, averaged over 14 subjects, in which the pitch of Stimulus 2 was judged as higher than of Stimulus 1 is plotted as a function of f, with m as a parameter. The data points in Fig. 3 were obtained for $a_n=1$, and the points in Fig. 4 for $a_n=1/n$. Stated differently, the percentages indicate how often the shift in pitch corresponded to the shift in frequency of the higher partials of the stimuli. From the original group of 15 subjects, one had to be omitted because he did not give reliable results even in the case of judging the direction of pitch shift for tones produced by a piano.

The curves show that, for $m=1$ and $m=2$, there is a distinct transition between a fundamental-frequency

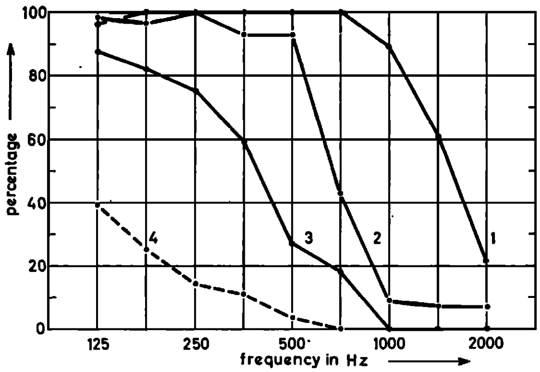

FIG. 4. Percentage of responses, averaged over 14 subjects, in which the pitch of the stimulus

$$\sum_{n=1}^{m}\frac{1}{n}\cos[2\pi n(0.9f)t]+\sum_{n=m+1}^{12}\frac{1}{n}\cos[2\pi n(1.1f)t]$$

was judged as higher than that of

$$\sum_{n=1}^{12}\frac{1}{n}\cos[2\pi nft]$$

as a function of f, with m as a parameter ($m=1$, 2, 3, 4, as indicated).

range over which the pitch shift follows the higher harmonics and a range over which it follows the fundamental. It is of interest to point out that, for frequencies within these ranges, the subjects considered their task as a very easy one; they did not notice that the harmonic complex of tones was split up in two parts with opposite frequency shifts but had the impression that the pitch of the whole stimulus was varied. Within the transition range, some subjects remarked that they had noticed both shifts indeed. In this respect, the experiments for $m=3$ and $m=4$ were judged as much more difficult than for the lower m values.

D. Discussion

The curves in Figs. 3 and 4 for $m=1$ show that, for fundamental frequencies below about 700 Hz, the responses agree with the frequency shift of the higher harmonics. This implies that the pitch of complex tones is determined in this frequency range by the harmonics and not by the fundamental. For higher frequencies, however, the situation is different. Beyond about 1400 Hz, the pitch followed the fundamental in the majority of cases. This result can be compared with the finding by Flanagan and Guttman (1960) that, for 1000 Hz and higher, pitch matches of complex tones are determined by the lowest partial of the stimuli. Furthermore, it is of interest to remember that, for harmonic complexes consisting of only three adjacent partials, Ritsma (1962) could observe periodicity pitch for (missing) fundamentals up to about 800 Hz; it is obvious that for more partials, as in our experiments, this limit shifts to a higher frequency.

For higher m values, the curves shift to lower frequencies. It is remarkable that there are only minor differences between the curves of Fig. 3 and Fig. 4, with $a_n=1$ and $a_n=1/n$, respectively, indicating that the relative amplitudes of the partials do not influence the pitch of complex tones over this range. It is also remarkable that even in the case for which the frequency of the first three harmonics of 125 Hz were lowered, still about 90% of the responses were determined by the higher harmonics.

In the writer's opinion, the experimental data are significant in answering the question of whether the pitch of complex tones is based on frequency or on periodicity. The finding that below about 1400 Hz the pitch is determined by the harmonics rather than by the fundamental strongly suggests that pitch is not derived from frequency but from periodicity. It is not very probable that the situation should be different in the frequency range above 1400 Hz. The highest pitch that can be produced by the human voice does not exceed this frequency, so it is not clear why we should have a second pitch mechanism that is never used in social life. The consequence of this reasoning is that also the pitch of simple tones is based on periodicity. The fact that beyond 1400 Hz pitch is related to the fundamental can be explained by the limit of the ear's ability to detect periodicities.

In the light of these results, we understand why, in practice, complex tones are always characterized by one definite pitch. If pitch is based on frequency, we should expect a number of different pitches to be audible, just as in the case of an inharmonic complex of tones. If, however, pitch is based on periodicity, the existence of one pitch, related to the period of the stimulus, is obvious.

The question can be asked how this view is compatible with the conception of the ear as a frequency analyzer. At first sight, it seems difficult indeed to reconcile preservation of periodicity with this analyzing process. We must realize, however, that the frequency-analyzing power is determined by the critical bandwidth, so it is rather limited. Moreover, previous experiments (Plomp, 1967) showed that simple tones do interfere over large frequency distances.

III. CONCLUSIONS

- For fundamental frequencies of up to about 1400 Hz, the pitch of a complex tone is determined by the second and higher harmonics and not by the fundamental, whereas beyond this frequency the opposite holds; this is the case both for tones with harmonics of equal amplitude and for tones with harmonics of which the amplitudes fall by 6 dB/oct.

- For fundamental frequencies of up to about 700 Hz, the pitch is determined by the third and higher harmonics; for frequencies up to about 350 Hz, by the fourth and higher harmonics.

- The experimental results strongly suggest that the pitch of complex tones is based on periodicity rather than on frequency; it is reasonable that this also holds for simple tones.

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