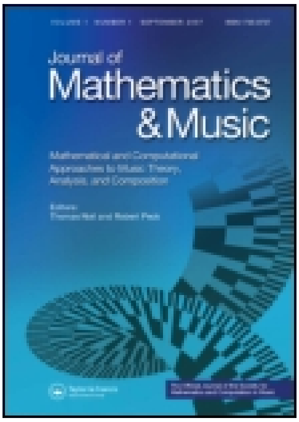

Journal of Mathematics and Music: Mathematical and Computational Approaches to Music Theory, Analysis, Composition and Performance

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tmam20

Music, mind and mathematics: theory, reality and formality

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Version of record first published: 06 Jul 2012

To cite this article: Geraint A. Wiggins (2012): Music, mind and mathematics: theory, reality and formality, Journal of Mathematics and Music: Mathematical and Computational Approaches to Music Theory, Analysis, Composition and Performance, 6:2, 111-123

To link to this article: http://dx.doi.org/10.1080/17459737.2012.694710

Journal of Mathematics and Music
Vol. 6, No. 2, July 2012, 111–123

POSITION PAPER

Music, mind and mathematics: theory, reality and formality

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I consider the nature of music as a cognitive and cultural construct and discuss the relationship between mathematical systems and musical ones. I propose that mathematical modelling of certain kinds may be appropriate to model certain musical relationships, but this is so because of underlying cognitive principles. The conclusion is that to model music mathematically is essentially to model (parts of) cognition mathematically, which means that to model music in the abstract, as though it were itself a mathematical construct, divorced from its source in the human mind, is misleading. For a true understanding, the power of mathematics should be applied to the *process* of musical behaviour, not merely to its product. That process lies in the embodied human mind.

Keywords: music; mathematics; modelling; cognition

1. Exposition: theory, music theory and theory-theory

1.1. *Introduction: theory*

'Theory' is a confusing word. It is defined by the New Oxford American Dictionary (2nd Edition) as meaning the following four related, but distinctly different things:

(1) 'a supposition or a system of ideas intended to explain something, especially one based on general principles independent of the thing to be explained : *Darwin's theory of evolution*',
(2) 'a set of principles on which the practice of an activity is based : *a theory of education* | *music theory*',
(3) 'an idea used to account for a situation or justify a course of action : *my theory would be that the place has been seriously mismanaged*',
(4) 'MATHEMATICS a collection of propositions to illustrate the principles of a subject'.

In contrast, Narmour [1] gives a precise and clear definition of 'theory' as used in humanities research in general, but the abstraction required to achieve this generality serves only to highlight the disparity of 'chosen approaches':

> *Theory* in the humanities means any kind of more or less fixed and hypothetically identifiable top-down approach to a text or a score, where the target of the literary or musical analysis is mapped onto the phenomenon in accordance with the chosen approach. [p. 4, original italics]

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ISSN 1745-9737 print/ISSN 1745-9745 online

http://dx.doi.org/10.1080/17459737.2012.694710
http://www.tandfonline.com

To a scientist, the startling aspect of this definition (whose original context is a discussion of the development of music cognition research in North America) is its dissociation of the activity described from agreed or objective principles. In paraphase, it says ‘there must be a stated framework and it must be applied properly’, but that is all.

In this paper, I examine what it might mean to theorize music, from a scientific, mathematical or computational standpoint, in the absence of an uncontroversial dictionary definition. I begin with an attempt to pin down the idea of ‘music theory’, which I then relate to the ‘theory-theory’ proposed in psychology. These two ideas are brought together into an argument that music theory *is* a folk psychology and that, that is why it works. In this context, the contribution of mathematics and computation is next considered. I conclude that a quasi-scientific endeavour towards a more objective and general theory of music is possible and desirable, but that *it will never be achieved* by mere application of mathematical methods to scales, scores and other examples of musical outputs, abstracted away from the musical mind/brain.[1] The key claim is that to study music properly, we must go beyond low-level music-theoretic structures and musical works to see the underlying causes of these things. Without doing so, we cannot have an explanatory theory, and, therefore, little is achieved.

Throughout the paper, I signal answers to the editors’ four questions with emboldened labels, and my first **challenge** thus indicated is the one of asking the right question, or the **failure** of not asking it: what is it we are attempting to study?

1.2. *First subject: music theory*

In the context of the diversity of definitions in Section 1.1 (and there are more where these came from), it is interesting to ask

‘What is *Music* Theory, really?’

The New Oxford tells us explicitly: ‘a set of principles on which the practice of [music] is based’, and Wiktionary (accessed 7 December 2011) has a loosely corresponding generic interpretation: ‘The underlying principles or methods of a given technical skill’. However, a quick look over the music theory section of any good library will quickly determine that this is either an absurdly over-narrow, or a wildly over-general, definition of the word, most obviously because it uses the word ‘music’ (whose ‘theory’ is related to its ‘practice’ in a rule-based way) while glibly assuming that it is clear what ‘music’ is meant. Wikipedia, as of 7 December 2011, defines *music theory* in terms only of Western Common Practice, with a strong bias towards tonal music. It makes no mention at all of, for example, the well-developed theories of Indian [2] or Persian [3] music, and only hints at rigorous approaches for discussing non-tonal Western music. There is little consensus, and coverage is patchy, at least partly because of the spurious historical compartmentalization of Western academic and educational musical discourse, into Western music and ‘ethno’ music, and because of elementary-level teaching materials that present a very narrow and old-fashioned view of music. There is a major **challenge** for **interdisciplinary discourse** to be met in educating future thinkers with sufficient depth and subtlety in *both* of Snow’s *Two Cultures*[2] [4], and this is nowhere clearer than in interdisciplinary music research. So it seems hard to find a good definition of music theory, even one that is narrowed to Western Tonal Music. Perhaps considering its historical development will help.

Historically, the post-mediaeval trajectory of music theory is, in some ways, not so different from that of science. In the early days, there tended to be very creative, extraordinary individuals who studied certain aspects of the physical world, in their own (often multi-disciplinary) way, with little or no knowledge of each other, and in something of a philosophical vacuum.[3] Thus, Leonardo da Vinci (qua scientist), Galileo Galilei and Isaac Newton might be said to correspond with such music theorists as Rameau [5], Schenker [6][4] or Riemann [7]: individuals, making hugely significant contributions, in relative isolation.

In science, organizations sponsored by national leaders (e.g. the Royal Society in the UK, the Academie Française in France) began to form networks which served to bind science together, presumably at least partly because of perceived commercial and military advantage from doing so. The resulting information exchange, coupled with a common frame of reference – the observable world[5] – meant that scientists could collaborate more effectively to piece together a more complete understanding of that world, a methodological opportunity which led ultimately to the scientific philosophies of Popper [8], Kuhn [9] and Lakatos [10]. Regrettably, no such impetus has driven music theory, as a research field, towards the quasi-scientific goal of *generality* – perhaps ironically, it seems often to be the case that a deep-seated horror of *reductionism*[6] drives many music theorists who desire a holistic view directly away from the unified, methodical research programme which might achieve one. Mutual understanding in this area is often a **failure** and always an important **challenge** for **interdisciplinary discourse**.

This lack of drive towards a *truly* holistic theory was exacerbated by the rejection of comparative musicology as a socially acceptable construct, following the discrediting of eugenics after World War II. The baby thrown out with this peculiarly abhorrent bathwater was the opportunity to understand differences and similarities between musical cultures, but without making subjective value judgements, and to move thence towards a general theory of music. Only recently has the search for musical universals been rehabilitated, in the form of evolutionary musicology [11], which presents interesting **challenges** to formal approaches to music research: the target begins to move, as one learns that not all musics work in the same way, thus highlighting the requirement for meta-theory developed in this paper.

The final intrinsic factor in music theory's non-achievement of generality is the particular aim of its users. Western music theory as practised has two primary functions: first, to facilitate teaching and to bootstrap the associated musical introspection;[7] and, second, as per Narmour [1], to underpin music analysis. It is significant that both of these functions, as usually practiced, rely strongly on the ability to *describe*; they both require a means of expressing opinions, either the received opinion of music theory as taught to students, or the individual opinion of the analyst who aims at a deeper understanding of a composer's method, or of a listener's experience. I return to this descriptive facility, which is a great potential **benefit** of formalized approaches, below; on the analytical side, there is a danger of presenting formalized music analysis as somehow more significant, simply because of its greater formality – it is a **failure** of philosophy to reason that formal description, alone, of a theory makes it a better theory: its predictions must also be appropriately tested.

First, however, there is a rather larger *extrinsic* problem for the idea of a general music theory. It lies in the fact that music, unlike the vast majority of things about which theories are made, or of which theory is developed,[8] is a *purely abstract* creative practice: it has no agreed denotation with reference to the observed world, nor does it need one [13]. This is not to deny that musical communities exercise intersubjective consensus, ascribing extra-musical connotation to certain musical devices, nor that music can refer quite directly to extra-musical sound, onomatopoeically.[9] Nevertheless, it is the case that music is not denotationally grounded in the way that the other key sonic communication method, language, is grounded: to see this, simply attempt to write a melody that says, 'The book is on the table'. Notwithstanding the fact that language abounds in directly ungrounded concepts (examples are 'fact', 'language', 'refer', 'ungrounded', 'concept' and 'example'), the human capacities for metaphor and abstraction allow us to build complex, distant meanings from simple, immediate ones [16]. Those simple, immediate meanings come from reference to the real world; even where the agreement is ineffably intersubjective, such as in colour perception, it is mediated by a real-world reference.[10] Notwithstanding ungrounded, learned intersubjective agreement about many musical constructs (e.g. pitch, harmony, timbre), music has no absolute grounding to apply a semantic brake to its content, and is therefore free to roam where language cannot go. It is this freedom to develop that simultaneously renders music the gloriously rich and fascinating construct that it is, and yet makes it very difficult to theorize in general. It is a consistent **failure** of researchers in music theory, mathematical, computational and otherwise, to write and reason as though music were a quasi-Platonic entity, existing in the world independently of the human mind – even when they actually do not believe this to be the case. A scientific approach to music research will accept the **challenge** of rising above the trees (particular examples of music) to be able to see the forest (of human musical mechanism), and for this a better **interdisciplinary discourse** is required.

This brief developmental overview would be incomplete without mention of the European school of Systematic Musicology [17], which applies rigorous methods to the study of music, and equally of the newly arrived, related discipline of Empirical Musicology [18–21]. These two related traditions apply substantially more rigorous methods than those necessarily entailed by Narmour's definition, above, to the study of music. This is particularly so in the latter case, where there is an emphasis on empirical measurement of music *as performed*. This, however, points the finger even more clearly towards the **challenge** at the heart of this section: what *exactly* is it that is being measured?

Now I return to the issue of description raised earlier. Aside from the 'set of principles' questionably introduced by the New Oxford, music theory necessarily does give us a *language* (or a set of languages) in which principles or rules can be – to a degree of precision – stated. This is the vocabulary of music practice: music, whether composed, performed, improvised or notated. In the Western classical tradition, it is based on the essential unit of the note, or musical event. All else is ultimately constructed from this: even a construct as abstract and large scale as a Sonata form is ultimately traceable down to its individual notes, their properties and the relations in which they stand with respect to one another. In the Jazz tradition, the vocabulary has a different dialect, but is based on the same principles. Comparable vocabularies exist in other traditions.

When instantiated by music-theoretic vocabulary, Narmour's definition of theory (above) describes music *analysis* very clearly, and the dichotomy between his definition and the New Oxford's becomes sharper. Narmour's definition is about retrospective inference of the actions of a composer, or about the 'workings' of a piece of music, which firmly contradicts the notion of 'a set principles' – the aim of Narmour's 'theory' is to elucidate the behaviour of an individual composer, not to provide a basis for musical practice in general.

Perhaps a better practical characterization of the aspects of music theory not already captured by Narmour's description is

> A vocabulary for small-scale and large-scale musical constructs, allowing relationships between musically significant structures to be described, coupled with constraints, specified in terms of that vocabulary, that characterise common practice in a given repertory.

Part of the point here is to emphasize taxonomy, which is completely omitted from the other definitions. Part of the point is to make explicit the multiplicity of possible music theories, depending on period, social milieu and so on, lack of which leads the New Oxford astray. This definition, then, covers the pedagogical aspects of music theory and the ontological aspects, while Narmour covers use and development of the latter in analytical research. A formalized approach to the development of taxonomy is a huge potential **benefit**, but it carries with it the **challenge** to resist a potential **failure**: the danger of missing or ignoring the line between aspects of music that may be treated objectively and those that must be viewed as subjective. A theory presupposing uniformity between listeners is doomed to **failure**; equally, a theory presupposing musical constructs as givens, unless they are shown to be so, is misleading.

My slightly sharper and more localized definition begs the question of generality to which I referred earlier. Given an ability to make statements about different music at different times, it is natural to ask, 'what are the relationships between them?' In beginning to answer this question, one must first ask about the basis of the different music theories: their vocabularies. To understand the higher level inter-theoretic relationships, one must first understand the low-level ones – at limit, those between the taxonomies. Here, again, is a significant **challenge** for the **interdisciplinary discourse**, both between formal approaches to taxonomy development and musicians, and between musicians themselves, on account of the artificial boundaries imposed by intra-cultural music education.

In everyday language, one can say the word and point at the object it denotes; but in music that is not the case, because one cannot assume that the frame of reference of each vocabulary is the same. Where, then, does the fundamental intersubjectivity required to support this inquiry come from?

1.3. *Second subject: theory-theory*

The theory-theory [22,23] is a cognitive-scientific theory of intersubjective understanding and learning. Specifically, this hypothesis proposes that one human understands and predicts another's behaviour by means of their own internal theory (hence the name 'theory-theory') of human behaviour, explicitly represented in their mind/brain. Using this theory, we predict the reactions of others to our actions and events in the world, and also infer information about the world from others' interactions with it. The theory-theory is one of several notions covered by the term *Folk Psychology* [23], and I use the terms interchangeably here, the latter, once disambiguated, being more appropriate to my purpose. Of course, the idea is expanded in considerably more detail than this is in the psychology literature; for the current purpose, however, the simple notion, made precise in locus, is adequate: an internalized theory, or model, of the perception and cognition of another, with respect to a certain class of behaviours.

Ravenscroft [23, Section 2.1] lists 10 interesting philosophical questions that arise from the notion of the theory-theory, three of which are directly relevant here:

> …
>
> C 'We can ask about the content of folk psychology. What states and properties does it quantify over, and what regularities does i[t] postulate?'
>
> D 'We can ask questions about the structure of folk psychology. Is it a 'proto-scientific' theory with a structure akin to that of scientific theories, or does it take some other form?'
>
> …
>
> F 'We can ask about the development of folk psychology in young children. Does it exhibit a characteristic developmental pattern?'
>
> …

The third of these, F, is included here more for completeness than centrality: Gopnik [24] proposes the theory-theory as an alternative to Chomsky's [25] theory of innateness during cognitive development. This relates to my point in Section 1.2 about the role of music theory qua descriptive and/or developmental tool in education, supporting the general line taken here. Here is an opportunity for **benefits** from formalized music research: computational systems can help support musical learning, especially if properly founded in developmental psychology.

Ravenscroft's Questions C and D, when specialized to music, relate directly to my argument and to the opportunities offered by formalized by the study of music. They are the subject of the next subsequent sections.

1.4. *Codetta: music theory as folk psychology*

Western music theory begins with the description of musical constructs, usually notes, in terms of their distinct features, pitch, time, duration and volume. In some cases, it encodes articulation (e.g. *staccato*, *martellato*) and instrument-specific technique markings (e.g. *pizzicato*, *gestopft*). There is well defined, sometimes context-dependent notation for all of these, culminating in the Western score. Given the basic units, it is possible to describe and discuss underpinning musical structures, such as scales, chords, melody, harmony and so on, as well as the particular instantiation of such structures and the relationships between them in music as composed or improvised.

As mentioned earlier, these structures are often discussed and used as though they were the definitive blocks of all music,[11] and this is a philosophical **failure**, since it is not the case for all of them. Some, such as Western tonality, are culture-specific, and require learned percepts, in order that their full detail can be perceived. Some, however, such as the existence of notes, seem to be so universal that their universality is never questioned, and here is another **failure** of scientific enquiry. It is worth pausing to consider why.

A musical tone, or, more generally, *event*, is a combination of sine waves (except in the unique case of electronic synthesis where a single perfect sine wave can arise in isolation). Particularly in the case of acoustic resonance, these sine waves are really present, and not merely a mathematical device: we can account for them in terms of the physical behaviour of the instrument. What is more, they are detected individually in the organ of Corti [26]. Nevertheless, they are *fused* into a single percept [27], and we do not generally hear them: to do so requires very skilled listening indeed, and is normally only possible when particular partials are unusually prominent in the context.[12] This is true of all humans, not just musicians: experience tells us that others perceive notes in the same way that we do (notwithstanding the associatively mediated intersubjective agreement required, as for colour, above). To put all this in another way: the perceived thing which is a tone is in the mind, while the waves that correlate to stimulate that percept are all that is in the world: the tone itself is not.

Thus, the most fundamental piece of Western music theory, the unit in terms of which all else is expressed, is an entity which has no existence in the physical world, but is in fact a *percept* which is composed of *individually imperceptible* correlated physical stimuli; it is the *correlation* which allows us to perceive the tone. This percept is discussed in music theory, exactly as though we all know exactly what it is, as though it were real, and as though it were constant.[13] To discuss notes and their corresponding tones with another human is to presuppose a mutual understanding of what a tone is; in other words, to have a theory of their musical mind, or a folk psychology of tones. It follows, therefore, that the higher structures of music theory must also be theory-theoretical, because they are composed of theory-theoretical components; there is no other route by which we can arrive at the indubitably-extant consensus of meaning. To understand what a musical structure is, by inference from another human, is the equivalent of learning the language via the theory-theory as proposed by Gopnik [24].

I argue, therefore, that, by definition, music theory is not any of the theory things described in the New Oxford's definition in Section 1.1 – least of all the 'set of principles on which the practice of [music] is based'; nor is it really the thing described by my improved definition, above. Rather, *it is a folk psychology*, which describes the model a human might have of another human's understanding of music as experienced in a shared culture. Nor is its *nature* well described by Narmour's otherwise useful definition, because this definition generalizes away the unusually perception-centred aspect of music theory, so as to capture its *application* in research, alongside other more referential humanities studies. This means that in nature, music theory is fundamentally perceptual, mutable and subjective, and it is a **failure** not to treat it so.

Ravenscroft's Question C, above, is therefore, in part, answered by music theory: it 'quantifies over' a range of perceptual constructs, such as notes, scales and chords, and 'postulates' regularities such as modality, tonality, consonance, harmonic function, melodic implication and so on. These abstract concepts can be identified perceptually in performances or theoretically in scores, and they can be used by composers to create new music. The clear **benefit** of a formalized approach to studying these entities is that they can be understood, taught, used, experienced more effectively, and, perhaps, automatically. The **challenge** is consistently to understand music theory as a *folk psychology* theory, and not as a *scientific* one, nor indeed as a *humanistic* one – if there was a way of saying what that would be.

However, music theory is not a naïve folk theory. It has been externalized, taxonomized, codified, developed and refined over centuries, in a truly remarkable way, and, indeed, it is the enduring theory-theoretical nature of the thing that allows it to be so. Perhaps it is a candidate to be the most formalized folk theory available. Nevertheless, multiple theories of music exist in mutual isolation, surrounded and separated by the lacunae of unexplored possibility. The **benefit** of a general, overarching theory of music, tying all musical cultures together through time, is surely a compelling **challenge**, and this is a reason to seek a *scientific* meta-theory in terms of which such a theory might be expressed.

So far, then, in summary: I have argued that music theory, as it currently is, is not an abstract, independent, theoretical entity, but an ontology which is intersubjectively agreed amidst common human experience by reference to psychological means available for general learning in all humans, known as folk psychology. As folk psychologies go, it is unusually well-specified and formalized. This ontology allows patterns and structures which arise frequently in music to be labelled and described, and thus 'rules' about their occurrence may be formulated. Again, it is important to understand that these 'rules' are not laws (like the so-called Laws of Physics) that are demonstrably repeatable and reliable in all specified contexts: they are merely statements of *what musicians do* and have no compulsory force whatever – except in badly taught high-school harmony lessons. What, therefore, does it mean to formalize them in mathematics?

2. Development: theory, reality and formality

2.1. *The big picture*

Having argued that music theory is almost entirely dependent on human psychology for its concepts (for that is the source of any perceptual intersubjective ontology), I now proceed to develop the consequences of this conclusion.

Ravenscroft's [23] Question D, quoted above, is a general meta-theoretical question, which may usefully be applied to music theory qua folk psychology. Is this particular branch of folk psychology 'proto-scientific'? Note, first, that 'proto-scientific' is not the same as 'quasi-scientific', or, fortunately, 'pseudo-scientific'. A proto-scientific theory is one with the capacity to develop into a scientific one: General Relativity was one such for a period of time, until it ceased to be untested. Like music theory, a proto-scientific theory will have precise terminology, and it will be empirically based. There are important, big differences, however. As explained above, music theory *describes*; it does not predict. This means that the empirical method espoused in scientific theories simply does not apply. Therefore, a **challenge** is that each of the particular, non-scientific, music theories will need to be expressible in terms of that larger (meta-)theory, and it must have the power to make falsifiable predictions. What is more, the meta-theory will have to be able to account for subjectivity, much as psychology in general – another opportunity for **interdisciplinary discourse**.

There are broadly two ways to proceed in this endeavour: on one hand, attempt to understand as much as possible about each music in isolation, and then compare them, building a new theory around the combination; or, on the other, attempt first to identify the common aspects of all known theories, and piece together the generality working from that fundamental level. The latter of these resonates with my description of music theory qua folk psychology above: those things that are genuinely common to all musics are those minimal musical things that are common to all humans; and since we are discussing essentially perceptual constructs, it is important to remember that these things will be, broadly construed, *musical behaviours*, and not *music in the abstract* [13,28]; again, **failure** to take this into account can render formal music research misleading. The reason for this is that music is *defined* by the human mind and its mechanisms.

2.2. *The basic units of music*

The only thing that seems to be an absolute musical universal in this sense is the perception of sequence in time, since some musics do without pitch, and some do without rhythm.[14] Sequence, and particularly *entrained* sequence [30], is, however, presented by evolutionary musicologists as a fundamental feature of musical behaviour. Perception of sequence can be explained in terms of a broader theory of expectation [31], based on an evolutionary pressure to predict one's environment [32].

The essential components of music are sonic events, real or imagined. The essential components of those essential components are time and timbre. These form an *inseparable* complex of perceptual dimensions, in the sense of Gärdenfors [33]: in these terms, a sonic event is a *natural concept*. Since pitch is a percept that derives from particular harmonic structures, one might suggest that it is a special aspect of timbre, and this corresponds naturally with the sensations of the quasi-pitch that arise from, for example, filtered noise. Timbre is famously difficult to formalize [34], so let us consider pitch. Here is something that can, perhaps, be treated rigorously with mathematics.

First, we can measure pitch – or rather frequency, which is not in strict correlation with pitch because intensity intervenes in some cases. But assuming the approximation that pitch is the logarithm of the fundamental frequency of a harmonic series, we have a continuous percept, which can be objectively measured and modelled as a straight line, finite at both ends. Presumably thanks to the operation of the afferent nerves from the organ of Corti, which respond statistically in phase with their stimulating sine waves [26], there is a perceptual equivalence between pitches corresponding to frequencies in a ratio of 2:1, which we call the *octave*. This fundamental equivalence is also perceptible by non-human animals [35] and bends our straight line model into a spiral [36].

However, humans are not restricted to absolute pitch; rather, they learn relative pitch encoding at the expense of the absolute pitch memory with which they seem to be born [37]. They are, therefore, capable of learning pitch equivalence classes, partitioning the spiral. What is more, relative pitch hearers can categorize intervals [38], dividing the continuous octave into some number of categories that, perceptually, maintain 'distance' uniformly around the spiral (though not all musical scales have this property). All the above steps confer biological or cognitive evolutionary advantage, most particularly the categorization, which enables more efficient cognitive representation in memory. It is also the case that a cognitively efficient way to represent this percept *corresponds with*[15] the mathematical construct called a *linear Abelian group* – at least as far as the endpoints of the line [39]. The asymmetrical nature of the majority of scales in use in music confers another advantage of efficiency: rotational symmetry around the centre of the pitch spiral allows equivalence between occurrences of the same scale starting on different degrees, admitting the ability to store musical structures with reference to a tonal centre, making relative representations even more efficient.

2.3. *The group theory of scale and harmony*

The correspondence between scales and groups is an interesting coincidence. It has led many researchers to ask 'What is the group theory of pitch?' in one way or another, looking at various ways to divide the octave into categories, and comparing the group so produced with the practice exhibited in a corpus, or a piece, of music. However, since musical structure perception is self-evidently learned from exposure to one's culture, what is startling is that the phenomenon is essentially group-theoretic at all. The key question is not 'what' but '*why* is there a group theory of pitch?' Here, the mathematical formulation opens a new **challenge** to understand the very *cause* of perceptual effects, and perhaps scientific understanding of music can confer a **benefit** back to general psychology, with a predictive theory. Regrettably, though, this route is not generally taken; Noll [40] is exceptional in proposing it.

Gärdenfors [33] already supplies a convincing hypothetical answer to this question, suggesting that geometrical cognitive representations arise naturally because we have evolved to survive in an essentially geometrical world; the fact that a group is information-theoretically efficient representation of this kind of structure may be a concomitant bonus. The evolutionary expense of a mechanism that can learn such structures creates pressure to use it wherever possible. Most importantly, group-theoretic formulation does not lend advantageous status to a pitch system: Western, Indian and Chinese pitch systems are groups *because they are perceived that way* – if, and only if, the perceiver has had sufficient exposure to learn the cognitive representations. Some African scales work differently [41]. It has taken a long time for Western musical culture to develop its particular scale system, the traditional Chinese one is different, because, historically, it happened upon a different categorization and therefore a different group, and the Indian ones are different again. These differences are chance happenings in socio-cultural evolution, amplified by positive feedback in the social loop between performer/teaching and composer/pupil. To ascribe Platonic mathematical significance to them is to protest altogether too much.

This said, of course, it is musicologically interesting to know, for example, that a certain composer applied a certain method of pitch generation in a certain piece. But this is a theory *about* music (in fact, about a composer), not theory *of* music, and so is a different issue from the one I attempt to address here.

2.4. *Theorizing in the wrong space*

Now, I illustrate the consequences of theorizing in the wrong theoretical space. I take examples from one of the most important musicians of the twentieth century, who, I argue, drew inferences that were incorrect because he theorized about music in isolation, and not about music perception.

Schoenberg [42] introduced two music-theoretic innovations. First, and famous, was the idea of '12-note'[16] composition, in which the essential musical material is a row of 12 unrepeated semitones, and the essential compositional operations are inversion and reversal (retrograde), and in which the use of a tonal centre is deliberately avoided; thus, he *removed* the efficient group-theoretic cognitive representation conferred by tonality. He did so in what he considered to be the next logical step beyond the intense chromaticism of the late romantic period: progressively adding new chromatic scale degrees would 'top out' when all of them were present in equal measure. Schoenberg was disappointed to learn that his listeners tended to hear tonality, even where there was none intended [43], and exhorted his pupils to take measures against this – which they did in varying degrees. I propose that the reason for this difficulty, and for the failure of 12-note music, in general, to catch on in the public imagination, is that it *removes* a means of efficient memory representation (the equivalence classes conferred by tonal organization), instead of defining and/or refining the representation needed to remember it, as new musics had, in sequence, more or less reliably, previously. In other words, people learn to hear tonality; once they have learned to hear it, they cannot help but hear it; and if a composer resists their attempts, the music becomes *hard to process* for that very reason. Many listeners to Schoenberg's music agree that they listen to it in a different way from tonal music, and that it is necessary to learn to listen to it. It is not clear what new geometrical or group-theoretic structure one might introduce beyond tonality, as a substitute for Schoenberg's subtractive idea; but my suggestion is that taking one away is not an option. We might hypothesize that the problem lies in the tone rows themselves; however, at least for strictly serial music, this is not the case: listeners can recognize tone rows in inversion and in retrograde [44], suggesting that the row is not the primary problem.

Schoenberg's second innovation was that of the *Tonfarbenmelodie*, in which tone colours are used instead of pitch to articulate a melody. This, he wrote Schoenberg [43], was a failure: people simply could not perceive the intervallic structure necessary to make the 'melodies' work as such. Timbre, of course, is a much more complex perceptual space than pitch, and it is therefore logical to ask whether this might be the cause of the problem. Tillmann and McAdams [45] have studied electro-acoustic composers, whose perception is particularly focused on timbre. Indeed, these timbre specialists are able to identify statistical structure in timbral sequences, perhaps forming the basis for a scale-like categorization.

The point of these two anecdotes is this: there are **benefits** of formal theorizing about music (mathematical or otherwise), so long as it is carried out within a fully informed framework of music perception. Otherwise, **failure** is likely.

3. Recapitulation: describing and theorizing the musical mind

In this paper, I have proposed that music theory, as first practised by collaborating musicians, and then formalized by abstract thinkers, is not ultimately a theory of music itself. Rather, it is a descriptive model [46] of music *perception* as collectively learned in a particular culture, rooted in folk psychology that allows mutual understanding of a shared experience. This **challenge** applies to music theory of all kinds, including mathematical and computational theories.

I have suggested that understanding music, as a general human phenomenon, requires a meta-theory that transcends Western, Indian, Chinese and all other music theories, formal or otherwise. Such a theory will have as its basic units universal perceptual constructs such as pitch (in basic form), timbre and time. This means that a broader understanding of these concepts than is common in Western music theory will be required, and that understanding must be fundamentally perceptual, before it can be musical. A mathematical theory of music would have the added extrinsic advantage of interfacing gracefully with mathematical theories of physics. Since behaviours are involved, it may be that the best mathematical theories are those that are computational, since they can be tested by comparison with human musicians. Achieving these **benefits** will require substantial **interdisciplinary discourse**.

To understand music in this way will entail evolutionary arguments, since it is extremely unlikely that a phenomenon as complicated as music could arise by chance. To allow such a research programme [10] to be placed in non-evolutionary terms would be to admit mysticism – a **failure**. Since past evolutionary processes are not open to direct scientific scrutiny, computational simulation methods offer the best possible means of rigorous testing of evolutionary hypotheses, and for this, a mathematical formulation is strictly necessary. Here is a broad and exciting **challenge** for scientific methods of various kinds [47].

Mathematical and computational methods have much to offer as a meta-theory of music, in the same way that they offer meta-theory to physicists and engineers. However, it is crucial to avoid the quasi-Platonic supposition that it is the mathematics that *defines* the music; this is philosophical **failure**. Rather, the **challenge** is for mathematics or computation to *model the* *perceptual processes* that underpin both individual musical behaviour and shared musical culture. This will allow testable hypotheses, which concern music perception and cognition, and *not* music as data, to be formed. An open **challenge** to understand the regularities of music can perhaps contribute to understanding the regularities of human cognition in general.

Computational methods have the particular **benefit** that models can be very rigorously tested; parameters exhaustively explored; and experiments run often quickly and efficiently. They are demanding and **challenge** researchers to be very precise in stating their theories. Some researchers are worried that computational modelling presupposes the strongly cognitivist view of mind, but this is not the case, because the nature of the simulator need not determine the nature of the simulation. The **challenge** is to formulate mechanistic models of behaviour which account for observed regularities in the musical data, and not merely to model the regularities themselves.

4. Coda: artists and scientists; generality and objectivity

Stravinsky, in conversation with Craft [48], addresses the relationship between science and art. His conclusion, from the perspective of one of the most accomplished musicians in human history, is not optimistic.

> As to generality, the problem is that whereas the arts interpreted, symbolized, adorned general ideas – religious beliefs, philosophies – in past cultures, today the general, no less than the particular, ideas of science are incomprehensible to artists and are likely to remain so. [48, p. 259]

It is certainly true that better education systems are needed, in which students are allowed to study in both of Snow's *Two Cultures* [4], and so to become truly interdisciplinary. But Stravinsky and Craft [48, p. 263] is right when he says, 'predictions are dangerous'. Given an appropriate education, it is not clear to me that someone of an artistic persuasion could not in principle be amenable to and expert in the scientific method. That is indeed what is required.

Ultimately, a scientific understanding of those parts of music (as a general human phenomenon, not just within a given culture) that can be studied objectively can only benefit music as a human pursuit and help understand humanity. A better understanding of how to treat the subjective parts of music scientifically, or at least rigorously, is likely also to assist humankind in its attempts to understand itself better, and to deepen our understanding of the amazing construct that is music. To omit psychology from the musico-mathematical equation would not only be a **failure** to grasp this opportunity; it would be a **failure** to understand music itself.

Acknowledgements

I gratefully acknowledge the positive influence of my colleagues Ollie Bown, Tim Crawford, Ian Cross, Jamie Forth, Justin London, Daniel Müllensiefen, Marcus Pearce, Keith Potter and Alan Smaill on the thinking and thoughts presented in this paper.

Notes

1. Use of this rather clunky epithet allows us to avoid confusion and tangential debate over where brain ends and mind begins.
2. Snow bemoans the gulf in education and practice, in Anglo-Saxon cultures at least, between people channelled into artistic and humanistic pursuits and those educated in science. He argues that there is little chance of these two cultures understanding each other until common language and background is better developed. He suggests that this is damaging to academic study and to society as a whole.
3. This is not to denigrate the spectacular contributions of early natural philosophers such as Roger Bacon or William of Ockham, but it has to be remembered that the writing of these luminaries was probably not widely available for a long time after it was first written.
4. As least, up to the appearance of his supremacist ambitions, which are best forgotten.

5. For the scientist, the object of study, the observable world, remained more or less the same, while composers such as Haydn, Beethoven and Wagner were regularly and deliberately moving the music-theoretic goalposts. This aspect of music creates a special *challenge* for research: a good theory must account for change.
6. In which a large problem is carefully dissected into smaller, self-contained ones, which are then studied *and their solutions equally carefully assembled to elucidate the whole*.
7. An interesting **challenge**, which has not to my knowledge been studied, is the effect of music-theoretic *knowledge* on the development of the associated perceptual skills. There is evidence that language affects colour categorization [12], and so top-down influence of learned terminology on auditory perception might not be so surprising. A positive answer to this question could have thoroughgoing effects on music pedagogy.
8. Note here that there is a difference between what one might describe as a *theory about music* and what one might describe as a *theory of music*, using the definitions above. Theories about music, or *musicology*, can be epistemologically unproblematic, because they subsist in philosophical systems outside music itself.
9. Indeed, the *concrète* [14] music tradition is based on the musical reuse of non-musical sound and, conversely, the very name of the *acousmatic* [15] music tradition refers to the dissociation of sound from its source.
10. That is, we can agree on the stimulus that causes the percept, on the association between it and the percept, but not on the percept itself.
11. The reason for this may be simply that music theory is often taught, from the start, as though it were absolute, and so people can grow up with a deep-seated belief in its 'truth'. Perhaps the author of the New Oxford entry suffered from such an education.
12. For example, the third harmonic of the bassoon is particularly prominent in some registers.
13. It is not. If you 'play the same note again', you are playing a different one.
14. Some might argue that Cage's 'silent piece', *4'33"*, is even without sequence. However, my interpretation of this work contradicts this: I contend that part of its point is that one becomes aware of environmental sounds during the performance, and one cannot help but incorporate them into one's sequential understanding of it. Cage's [29] conundrum, 'Which is more musical: a truck passing by a factory or a truck passing by a music school?' [p. 40], seems to support my claim.
15. Note the emphasis here: London buses and post-boxes correspond in colour, but this does not mean that they are the same thing.
16. Schoenberg did not call his work 'atonal', but insisted on the notion of dodecaphony instead [43].

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