Pitch-Class Set Genera and the Origin of Modern Harmonic Species
Author(s): Allen Forte
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PITCH-CLASS SET GENERA AND THE ORIGIN OF MODERN HARMONIC SPECIES

Allen Forte

I. Introduction

Efforts to classify musical materials and to study their characteristics form a part of music theory, one that is ancient in origin and exemplified in many high musical cultures, even though sometimes lacking both with respect to explicit purpose as well as clear method.[1] This tradition extends up to the present era, as witness the diversified corpus of writings made up of the works, for example, of Haba, Hindemith, and Yasser in the 1930's and 1940's and, more recently, including the sophisticated studies by Clough and Myerson, Eriksson, the very large-scale microtonal investigation by Gamer, and the cosmic effort by Schaeffer.[2]

The present study approaches the question of the large-scale organization of tonal materials from very elementary and simple premises. On the other hand, the goal of the study is somewhat grandiose, and it is claimed that the system of genera presented has implications beyond the merely lexical. I have indicated the most important of these in the second part of the title: I feel that study of the pitch-class set genera may illuminate certain general developments in harmonic usage, perhaps evolutionary in nature, that can be seen in relatively recent music.

In addition, the system of genera offers an objective frame of reference for harmonic materials, one that is independent of any particular compositional practice, in the specific sense that none of the genera are derived empirically from actual music, but, true to the Pythagorean heritage, are constructed entirely on a logical basis from a few primitives.

The study begins by forming the genera according to certain rules, after which the special characteristics of each genus are discussed. Using a numerical measure called the difference quotient, the genera are then compared. Conglomerates of genera, genera with a natural basis of association, then enter the picture. These structures, called supragenera, also undergo comparison. Finally, examples of genera in operation in actual musical compositions serve as a basis for a discussion of certain details of generic connections and as transitional material to a brief concluding section in which further directions are suggested.

II. The System of Pitch-Class Set Genera

Because we posit the intervallic content of pitch-class sets as the fundamental basis of the genera, which provides a foundation independent of the vicissitudes of pitch-class constituency, it seems most fruitful to take the trichord as the set of least cardinal number which, unlike the dyad, offers more than one representative of an interval class and which therefore is capable of being compared meaningfully with others of its kind.[3] The results of this decision, as compared to those which would ensue if we began with sets of other sizes—for example, with hexachords—will become evident as we proceed. What is more important is the effect this decision has upon the 12 pitch-class system as a whole, which will become clear below when we consider the relation between dyads, representing the six interval classes, and their six decadal complements.

To return to the main argument, comparison of all twelve trichords yields the results summarized in Table 1, which reveals that the world of trichords, unfortunately, is not as neatly partitioned according to interval-class distribution as one would hope. Indeed, there is no apparent symmetry, nor is there an immediately obvious sense of association among the trichords that are grouped together on the table.[4] The symmetry of the entire system, however, exists along an entirely different axis.

More interesting and more suggestive with respect to the development of a theory of pitch-class set genera are the following observations. First, no trichord shares more than two (non-empty) interval classes—not necessarily in the same number—with any other. Second, and proceeding from this observation, we derive the notion of interval-class congruence. Two trichords will be called "interval-class congruent" if they share two (non-null) interval classes, not necessarily in the same number. Table 2 displays all such congruent pairs.[5]

Table 1
Summary of Non-Empty Interval Classes for Trichords

One interval class (1)	
	3-12: [000300]
Two interval classes (4)	
	3-1: [210000]
	3-6: [020100]
	3-9: [010020]
	3-10: [002001]
Three interval classes (7)	
	3-2: [111000]
	3-3: [101100]
	3-4: [100110]
	3-5: [100011]
	3-7: [011010]
	3-8: [010101]
	3-11: [001110]

Table 2
Interval-Class Congruent Trichords

Interval Classes	Pitch-Class Sets		Interval Classes Exchanged
1 & 2	3-1	3-2	
1 & 3	3-2	3-3	2 & 4
1 & 4	3-3	3-4	3 & 5
1 & 5	3-4	3-5	4 & 6
2 & 3	3-2	3-7	1 & 5
2 & 4	3-6	3-8	
2 & 5	3-7	3-9	
3 & 4	3-3	3-11	1 & 5
3 & 5	3-7	3-11	2 & 4
4 & 5	3-4	3-11	1 & 3

Although it would be convenient if Table 2 presented the essentials of genus formation, leaving only a few cosmetic details to be completed, that is not the case, unfortunately. Several further steps are required, of which Table 3 displays the first. If two interval classes a and b are represented uniquely in a single trichord, such that no other trichord exhibits this pairing of interval classes, the trichord may be said to be "unique with respect to a and b." Notice that in Table 3, b always has the value of interval-class 6.

Table 3 documents the fact that only three trichords exhibit the property "unique with respect to a and b": 3-5, 3-8, and 3-10. Further on in this article we will show that it is precisely those trichords which are the progenitors of the three genera that comprise the first supragenus, Supragenus I. More than a hint of their strong association is given here.

The significance of having taken interval content rather than pitch content as basic to the formation of the genera becomes evident when we traverse the entire 12 pitch-class universe and approach the decads at the outer extreme.[6] Table 4 summarizes the situation with respect to the 9-element subsets of the six decads, numbered 10-1 through 10-6. The natural symmetry of the 12 pitch-class system now becomes evident, for each list of nonads corresponds exactly to a list of trichords whose interval content is designated by the ordinal number of the decad. For example, on Table 2 the trichords which generate interval class 1 are 3-1, 3-2, 3-3, 3-4, and 3-5. These correspond one-for-one with the nonads on the list for decad 10-1: 9-1, 9-2, 9-3, 9-4, and 9-5, reflecting the symmetry of subset and superset that is a universal property of the 12 pitch-class system.

From these simple observations it is apparent that the basic notion of interval class in combination with the pitch-class inclusion relation would satisfy three basic desiderata for a system of pitch-class set genera: (1) completeness; (2) symmetry; (3) internal consistency.[7] A fourth desideratum, instantiation in actual musical compositions, will have to await later confirmation, as is the case with many a music-theoretic endeavor, both ancient and modern.

III. Procedures and Rules of Genus Formation

On the basis of interval-class singularity, each of the trichords listed on Table 3, those with unique interval-class representation, will tentatively assume the role of progenitor. Similarly, each of the trichordal pairs listed on Table 2, those exhibiting the property of interval-class congruence, will serve as progenitors. (Not all of these produce self-standing genera, as will be pointed out; two will prove to form subgenera.)

The rules for genus formation, given the trichordal progenitor(s), are two:

Table 3
Trichords with Unique Interval-Class Representation

Interval Classes		Pitch-Class Set
a	b	
1	6	3-5
2	6	3-8
3	6	3-10
4	6	3-8
5	6	3-5

In addition, there is one (and only one) pitch-class set class that forms only one interval class: pitch-class set 3-12, the genetically deprived "augmented triad," which has proved itself to be such a remarkably resilient musical object.

Table 4
Nonadal Subsets of the Six Decads

Decad	Nonads
10-1	9-1,9-2,9-3,9-4,9-5
10-2	9-1,9-2,9-6,9-7,9-8,9-9
10-3	9-2,9-3,9-7,9-10,9-11
10-4	9-3,9-4,9-6,9-8,9-11,9-12
10-5	9-4,9-5,9-7,9-9,9-11
10-6	9-5,9-8,9-10

1. Each member of the genus as well as its complement must be a superset of (must contain) the progenitor(s).

This rule satisfies an "intuitive" musical requirement, namely, that each component of a genus remain in contact, as it were, with the progenitor. It also preserves the symmetry of the genus, since it guarantees that any set is reflected by its complement in the same genus. As will be shown, this severely limits genus membership in some instances.[8]

2. In addition to satisfying Rule 1, each pentachord must contain at least one of the tetrachords in the genus and each hexachord must contain at least one of the pentachords and at least one of the tetrachords in the genus.

This rule assures continuity and internal consistency within the genus. Thus, for example, 6-14 is a member of Genus 4, but not all the tetrachords and pentachords of 6-14 are members of that genus.

CONSTRUCTION OF GENUS 4 AS AN EXAMPLE. Genus 4, the smallest of the genera, is based upon trichord 3-12, the augmented triad, as progenitor. The tentative members of the genus, before applying Rules 1 and 2 above, may be viewed in Table 5, which displays all the supersets of 3-12 up through the nonads. The cardinal number of the set is followed by a colon, and after that the ordinal numbers of the sets are given. Thus, for example, 3-12 has only two tetrachordal supersets, 4-19 and 4-24. Comparison of Table 5 with Table 6, a matrix representation of Genus 4 in its entirety, shows that the winnowing-out process engendered by Rules 1 and 2 begins at the level of hexachordal subsets.[9] Specifically, 6-z17 is omitted because its complement, 6-z43, is not a superset of 3-12 (Rule 1), and the same rule eliminates the other sets whose ordinal numbers are preceded by "z," with the exception of 6-z19 and 6-z44, which are complement-related. Only those septads whose ordinal numbers have counterparts on the list of pentachords (such as 7-13) are retained as bona fide members of the genus, and the same rule (Rule 1) eliminates all but two of the octads and all but one of the nonads. Note that the matrix representation, Table 6, assumes that all the sets listed are supersets of the progenitor 3-12 and that 3-12 therefore does not appear on the matrix.

The matrix contains more information than can be discussed here. For example, the genus constituents differ with respect to their inclusion relations. Five inclusions is the maximum for hexachords and two (for 6-35 only) is the minimum. This is hardly surprising in view of the fact that one ordered form of 6-35 is the familiar whole-tone scale, a notoriously antisocial creature. In addition, the matrix does not display certain information important to understanding the intricacies of genus organization. For example, all the septadal supersets both of 6-35 and of 6-20 are members of Genus 4. To many students of early twentieth-century music, hexachord 6-

Table 5
Potential Members of Genus 4 (3-12)

9:1,2,3,4,5,6,7,8,9,10,11,12.

8:2,3,4,5,7,8,11,12,14,z15,16,z17,18,19,20,21,22,24,25,26,z29.

7:3,6,8,9,11,13,15,16,z17,z18,20,21,22,24,26,27,28,30,32,33,
34,z37,z38.

6:14,15,16,z17,z19,20,21,22,z24,z28,31,34,35,z37,z39,z44,z48.

5:13,z17,21,22,26,30,33,z37.

4:19,24.

3:12.

Table 6
Genus 4 (3-12)

	4-19	4-24	5-13	5-z17	5-21	5-22	5-26	5-30	5-33	5-z37
5-13	o	o								
5-z17	o									
5-21	o									
5-22	o									
5-26	o	o								
5-30	o	o								
5-33		o								
5-z37	o	o								
6-14	o			o	o					o
6-15	o	o	o		o		o			
6-16	o	o	o		o			o		
6-z19/44	o			b	o	o				o
6-20	o				o					
6-21	o	o	o				o		o	
6-22	o	o	o					o	o	
6-31	o	o			o		o	o		
6-34	o	o					o	o	o	
6-35		o							o	

20, which has only four distinct pitch-class forms, will be familiar from the works of many composers.

SYMMETRY OF THE GENUS. Although the matrix (Table 6) shows only sets of cardinals 4, 5, and 6—tetrachords, pentachords, and hexachords—the entire range of set sizes is included in the genus by virtue of Rule 1 concerning complementation. Thus, if 4-19 is listed, its complement, 8-19, enjoys membership in the genus, and it is understood that it includes, for example, both 5-13 and 7-13, just as 4-19 was a subset of both those sets. In this very specific respect the genus meets the desiderata of symmetry and internal consistency established at the outset.

CONSTRUCTION OF GENUS 5. Genus 5, the genus which we will style "chroma" (on Table 10) may be taken as exemplary of the situation in which there are two progenitors, making construction of the genus somewhat more intricate. For example, although the septadal complements of all thirty pentachords of both 3-1 and 3-2 are supersets of one or the other trichord, respectively, only ten meet the requirement of joint membership (Rule 1). Rule 2 also considerably reduces the hexachordal membership of the genus. In particular, trichord 3-2 has a total of forty-four hexachordal supersets. The complements of a large number of these, however, are not supersets of 3-2. As a result of the application of Rule 2, Genus 5 contains only fifteen hexachords, including their (possibly trivial) complements. To indicate just how selective Genus 5 is, let us consider the unfortunate case of hexachord 6-18, which contains both progenitors, 3-1 and 3-2, but which contains none of the pentachords of Genus 5 and therefore, in the absence of the appropriate lineage, is excluded.

Table 7 provides essential information about Genus 5, the hybrid "chroma-dia" genus. Among the salient features that deserve attention is the presence of members that are unique to the genus, of which there are only two in this instance, the chromatic trichord 3-1 (not included on Table 7) and the chromatic tetrachord 4-1. The two other fully chromatic sets, 5-1 and 6-1, cultivate other generic affiliations as well: 5-1 is a member of Genus 6 (3-2 and 3-3), while 6-1 belongs to four genera, Genera 5, 6, 7, and 8. Hexachord 6-1 is most firmly entrenched both in Genus 5 and in Genus 6, however, by virtue of the fact that all its pentachordal subsets are also members of those genera, while that is not the case with the two other genera to which it belongs.

A particularly interesting member of this genus, one that has to do with the "origin of harmonic species," is the hexachordal z-pair 6-z3/36, whose subsets within the genus number nine in all, exceeding the count for the subsets of any other constituent. The "almost chromatic" structure of 6-z36

Table 7
Genus 5 (3-1 & 3-2)

	4-1	4-2	5-1	5-2	5-3	5-4	5-5	5-8	5-9	5-11	5-13	5-z36
5-1	o	o										
5-2	o	o										
5-3		o										
5-4	o											
5-5	o											
5-8		o										
5-9		o										
5-11		o										
5-13		o										
5-z36		o										
6-1	o	o	o	o	o							
6-2	o	o	o	o		o		o	o			
6-z3/36	o	o	b	a	a	o	b			b		b
6-z4/37	o	o	b		a		b		a		b	
6-5	o					o	o					
6-8	o	o		o						o		
6-9	o	o		o			o		o			
6-z10/39	b	o		b	a	b		a		a	b	
6-z11/40	b	o		b	a		b					o
6-z12/41	b	a				b	b		a			a
6-14		o			o					o		
6-15		o			o						o	
6-16		o								o	o	
6-21		o						o	o		o	
6-22		o							o		o	

is evident in the number of "b" entries for row 6-z3/36 on Table 7; hexachord 6-z3 does not exhibit this chromatic character quite so strongly, as is apparent from its prime form, {0,1,2,3,5,6}, compared with the prime form of 6-z36, which is {0,1,2,3,4,7}. The reason 6-z3 and 6-z36 are particularly important in this genus, with respect to harmonic species, is that they are fundamental components of the post-tonal music of Arnold Schoenberg, in particular, of his atonal masterwork, *Pierrot Lunaire* (1912) as well as of the early 12-tone composition, the Suite, Op. 25, where the two hexachords partition the linear 12-tone row.[10] In this specific sense, therefore, the harmonic species of which 6-z3 and its complement, 6-z36, partake can be understood to emanate from the fundamental chromatic genus, Genus 5. The importance of that genus to the post-tonal, modern harmonic idiom is even more dramatically manifest in the 12-tone works of Anton Webern, many of which feature hexachords 6-1 or 6-2. Webern's predilection for the special hexachordal pair 6-z3/36 is evident in three compositions: Op. 18/1, Op. 26, and Op. 31. (See Maegaard 1985.)

In Genus 5 as in the other genera, we find sets that are somewhat loosely affiliated with the genus—in this case, 6-5, 6-14, 6-15, and 6-16 have only three entries each. This means that their primary associations are with other genera. In this context they enjoy only "shirttail" status.

Coming from a somewhat different direction, we may be momentarily startled to find 6-21 and 6-22 as members of Genus 5, since those hexachords are "almost whole-tone." They too are "fringe members" of this genus, whereas in Genus 2, the wholetone genus, they occupy much more significant, perhaps even preeminent positions. Also 6-21 and 6-22 belong to a small group of hexachords that are very gregarious. I will return to this feature of hexachord distribution below.

Finally, it must be said that from the standpoint of participation in actual musical practice, Genus 5 contains some very low-class pitch-class sets, indeed, among which I am obliged to mention 5-4 and 5-11, pentachords for which it is exceedingly difficult to find any musical instantiation whatsoever.

INTERVAL CONTENT OF MEMBERS OF GENUS 5. Table 8 offers a dramatic and highly significant view of the relation between the interval content of members of Genus 5 and the interval content of the progenitors. Because of the arrangement of interval vectors in descending numerical order we can easily see a more or less regular progression of interval content, one that reflects a process that is evident in the progenitors themselves. Thus, the vector of trichord 3-1 groups two intervals into class 1 and has one in class 2, to produce the array [210000]. The vector of trichord 3-2 represents a right shift with respect to that of trichord 3-1. That is, the third position in the vector, for interval class 3, now receives the entry 1 and the

Table 8
Intervallic Profiles of Constituents of Genus 5

Set Name	Interval Vector 1	2	3	4	5	6
3-1	2	1	0	0	0	0
3-2	1	1	1	0	0	0
4-1	3	2	1	0	0	0
4-2	2	2	1	1	0	0
5-1	4	3	2	1	0	0
5-2	3	3	2	1	1	0
5-3	3	2	2	2	1	0
5-4	3	2	2	1	1	1
5-5	3	2	1	1	2	1
5-8	2	3	2	2	0	1
5-9	2	3	1	2	1	1
5-11	2	2	2	2	2	0
5-z36*	2	2	2	1	2	1
5-13	2	2	1	3	1	1
6-1	5	4	3	2	1	0
6-2	4	4	3	2	1	1
6-z3/36	4	3	3	2	2	1
6-z4/37	4	3	2	3	2	1
6-5	4	2	2	2	3	2
6-8	3	4	3	2	3	0
6-9	3	4	2	2	3	1
6-z10/39	3	3	3	3	2	1
6-z11/40	3	3	3	2	3	1
6-z12/41	3	3	2	2	3	2
6-14	3	2	3	4	3	0
6-15	3	2	3	4	2	1
6-16	3	2	2	4	3	1
6-21	2	4	2	4	1	2
6-22	2	4	1	4	2	2

*5-z36 here occupies the position of its "z-correspondent," 5-z12, which has the same interval content, in order to maintain the correct numerical ordering of the vectors.

first position in the vector, for interval class 1, is reduced by 1 because one of the minor seconds of trichord 3-1 (icl) is now a major second (ic2) and one of the major seconds (ic2) is now a minor third (ic3) in trichord 3-2.

A similar process, the right shift, can be seen in the progression from 4-1 to 4-2. With tetrachord 4-2, interval class 4 is brought into operation. In short, as we proceed down the list there is a gradual shift of interval content from left to right, so that with 5-11 and 6-z10/39 the progressive redistribution of interval classes produces an almost flat intervallic profile.

The musical significance of this process with respect to the genus as a whole is as follows. The members of the genus represent different degrees of divergence from the pattern of interval classes established by the progenitors. Viewed in the orderly fashion displayed in Table 8, they can be seen gradually to accrue to themselves new interval classes in different numbers. What this means is that the members of the genus that diverge more than do others from the pattern established by the progenitors are intersecting with other genera as a result, while still retaining their membership in Genus 5. The extreme cases in Genus 5 are 6-21 and 6-22, two of the three "almost whole-tone" hexachords, which, as mentioned above, seem out of place in Genus 5, a genus characterized (on Table 10) as chroma, in view of its general chromatic nature. Of course, Genus 5 is no exception: all the genera exhibit a process of intervallic diversity, which lends an attractive richness to the generic structures and provides a splendid sonic resource for the development of modern harmonic species.

INTERVAL CONTENT OF GENUS 4. In Table 9 we see a situation that is rather different from that shown by Table 8. Because the progenitor of this genus, Genus 4, forms intervals entirely of the same class, ic4, we would expect to see that characteristic interval featured in all the constituents of the genus, which is clearly the case, as can be seen by inspecting the table, especially in the extreme instances of 6-20 and 6-35, both of which produce 6 intervals of class 4. Characterization of the process of differentiation here is not as simple as it was in the case of Genus 5, however. It involves the interaction of odd and even interval classes, as in the relation between the two tetrachords of the genus, where the three additional intervals (with respect to the three intervals of the trichord) are distributed equally over the odd interval classes in the vector of 4-19 and over the even interval classes in the vector of 4-24. Hexachords 6-20 and 6-35 exhibit a similar interchange. What remains fixed is the number of representatives of ic4: 3 for tetrachords, 3 or 4 for pentachords, and 4 or 6 for hexachords.

Does this mean that any set of cardinal 5 with 3 or 4 in position 4 of its interval vector is a member of Genus 4? Yes. And the same is true for tetrachords and hexachords. An interesting case is the interval vector of 6-z19/44, where the odd interval classes are all equally represented and the

Table 9
Intervallic Profiles of Constituents of Genus 4

Set Name	Interval Vector					
	1	2	3	4	5	6
3-12	0	0	0	3	0	0
4-19	1	0	1	3	1	0
4-24	0	2	0	3	0	1
5-13	2	2	1	3	1	1
5-z17	2	1	2	3	2	0
5-21	2	0	2	4	2	0
5-22	2	0	2	3	2	1
5-26	1	2	2	3	1	1
5-30	1	2	1	3	2	1
5-33	0	4	0	4	0	2
5-z37	2	1	2	3	2	0
6-14	3	2	3	4	3	0
6-15	3	2	3	4	2	1
6-16	3	2	2	4	3	1
6-z19/44	3	1	3	4	3	1
6-20	3	0	3	6	3	0
6-21	2	4	2	4	1	2
6-22	2	4	1	4	2	2
6-31	2	2	3	4	3	1
6-34	1	4	2	4	2	2
6-35	0	6	0	6	0	3

even are all different. (Remember to count double for ic6.) With respect to the development of modern harmonic species these are historically very significant sets, for 6-z19 is the complement of Schoenberg's motto, 6-z44 (Es-C-H-B-E-G), and both are source harmonies in a number of his atonal compositions. Hexachord 6-z19, in particular, is everywhere in early 20th-century avant-garde music as well.

THE 12 GENERA AND 2 SUBGENERA. From the theoretical material presented at the beginning of the preceding section, it might appear that there would be fourteen genera in all: ten based upon the pairs displayed in Table 2 and four based upon the single trichords with unique interval-class representation shown in Table 3, namely, 3-5, 3-8, 3-10, and 3-12. Fortunately, that is not the case, and the correct number of genera is twelve, the correspondence of which with the number of pitch classes can certainly not be merely fortuitous. The reason for this count is that two of the possible genera based upon trichordal pairs are subgenera of larger genera. Specifically, the candidate genus based upon 3-4 & 3-5 is actually a subgenus of Genus 1; the candidate genus based upon 3-6 & 3-8 proves to be a subgenus of Genus 2. As one peculiar result of the latter subsumption, trichord 3-6 does not appear among the trichords on Table 10, which excludes the subgenera.

The format of Table 10, which may be regarded as the main reference display for the pitch-class set genera, is intended to be straightforward as well as informative. In the column headed "Genus," the arabic numbers assigned to the genera are aligned in ascending order, according to the arrangement that is apparent in the column headed "Progenitor(s)": The single progenitors are given first, in accord with the ordinal number of the trichord, then, beginning with Genus 5, the progenitor pairs are listed, again, for consistency, in ascending numerical order according to ordinal number. This arrangement has the distinct advantage over a number of other possible arrangements in that the members of the four supra genera are then adjacent: As indicated in the leftmost column of Table 10 and with the assistance of the symbol >, Supragenus I comprises Genera 1, 2, and 3; Supragenus II comprises Genera 5 and 6; Supragenus III comprises Genera 8, 9, and 10, and Supragenus IV comprises Genera 11 and 12. An odd and fortuitous result of this arrangement is that the most familiar of the genera and the most ancient, diatonic Genera 11 and 12, occupy one end of the list, while the most modern, Supragenus I, occupies the other.

In the column headed "Type" on Table 10 I have provided very informal descriptive terms so that the genera might seem more accessible and familiar to the reader, at least in some cases. It is hoped that these will become more meaningful as this exposition proceeds.

The rightmost column of Table 10 shows a simple array that indicates

Table 10
The Pitch-Class Set Genera

	Genus	Type	Progenitor(s)	Counts (#3/#4/#5/#6)*	
SUPRA I	>1	atonal	3-5	1/9/24/29	63
SUPRA I	>2	whole-tone	3-8	1/9/24/30	64
SUPRA I	>3	diminished	3-10	1/5/16/21	43
	4	augmented	3-12	1/2/8/9	20
SUPRA II	>5	chroma	3-1 & 3-2	2/2/10/15	29
SUPRA II	>6	semichroma	3-2 & 3-3	2/3/16/24	45
	7	chroma-dia	3-2 & 3-7	2/3/15/25	45
SUPRA III	>8	atonal	3-3 & 3-4	2/3/15/21	41
SUPRA III	>9	atonal-tonal	3-3 & 3-11	2/3/15/21	41
SUPRA III	>10	atonal-tonal	3-4 & 3-11	2/3/15/21	41
SUPRA IV	>11	dia	3-7 & 3-9	2/2/10/15	29
SUPRA IV	>12	dia-tonal	3-7 & 3-11	2/3/16/24	45

*Each pair of z-related hexachords receives one count.

the size of the genera in terms of its constituents of cardinal numbers 3, 4, 5, and 6, reading from left to right. Thus, Genus 4, the smallest of the genera, has one trichord, two tetrachords, eight pentachords, and nine hexachords. Remember, however, that a genus is symmetric with respect to the complementation of its set members. Thus, the two tetrachords also stand for their two octadal complements, the eight pentachords for their eight septadal complements, and the nine hexachords for their nine hexachordal complements.

Finally, and for democratic as well as theoretical reasons, it needs to be said that none of the 220 pitch-class sets of sizes 2 though 10 is excluded from the system of genera, although, as will be pointed out from time to time, some have had more distinguished careers than others. The Appendix at the end of this article provides a complete list of the members of each genus and supragenus.

IV. The Genera from Two Perspectives

DISTRIBUTION OF PITCH-CLASS SETS OVER THE TWELVE GENERA. In this section we will view the genera from two different perspectives in order to see how they relate to other ways of organizing the 12-pitch classes, beginning with the pattern of distribution created by trichords, tetrachords, pentachords, and hexachords when they are placed in correspondence with the genera. I remind the reader that the arrangement of pitch-class sets within a genus is symmetrical. For example, set 9-x has the same relations within the genera as set 3-x. The same holds for tetrachords and pentachords. Only the z-hexachords vary with respect to the designation that follows the hyphen, in which case it is necessary to know the names of the z-correspondents. (See Forte 1973, 19–24)

DISTRIBUTION OF THE TRICHORDS. Table 11 provides an overview of the twelve trichords in relation to the twelve genera, giving the number of genera that correspond to each trichord progenitor preceded by a colon that follows the trichord name in column 1. As is evident upon inspecting the table, the number varies between 1 and 3. Seven trichords participate in the formation of only one genus: 3-1, 35, 3-8, 3-9, 3-10, and 3-12, with 3-6 a special case as described in the note at the bottom of Table 11 and mentioned earlier. One trichord, 3-4, participates in the formation of two genera, while the remaining four trichords (3-2, 3-3, 3-7, and 3-11) are progenitors of three genera each. I conclude from this that the trichordal basis of the genera presents quite a spartan pattern of behavior: four trichords are totally self-monogamous, while eight are monandrous.

Table 11
Distribution of Trichords over the Twelve Genera*

	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
3-1:1					o							
3-2:3					o	o	o					
3-3:3						o		o	o			
3-4:2								o		o		
3-5:1	o											
3-6:1		o										
3-7:3							o				o	o
3-8:1		o										
3-9:1											o	
3-10:1			o									
3-11:3									o	o		o
3-12:1				o								

*Progenitors 3-6 & 3-8 form a genus that is a subgenus of G2. Thus, of all the trichords only 3-6 does not appear as a progenitor. In this and other matrices in which it occurs 3-6 is always assigned to G2.

DISTRIBUTION OF THE TETRACHORDS. As is the case with the trichords, the tetrachords are fairly selective in their generic affiliations, which range in number from 1 to 4. Perhaps the most remarkable of all the tetrachords in this respect is 4-19, virtually a hallmark of early 20th-century music, which is alone in enjoying membership in four genera. What this implies for the development of harmonic species remains to be seen.

Seventeen of the twenty-nine tetrachords, a considerable majority, have singular generic associations. As might be expected, this exclusiveness is apt to have a significant impact upon procedures of analytical interpretation when we come to essay a practical application of the genera, since, if several tetrachords are operative in a particular work, it is likely that some of them will be of the singular type and thus skew an analysis in which a genus or genera unrelated to them are, in fact, principal contributors to the music under consideration.

On the other hand, we can see from Table 12 that three of the genera, Genus 1, Genus 2, and Genus 3 contain a large number of tetrachords, 15 out of 29, in fact. Thus, there is the possibility that these genera, which comprise Supragenus I (Table 10), may exert a possibly undesirable hegemony in the analytical domain. Therefore, ways must be found to lessen their influence.

Of special interest with respect to the development of modern harmonic species are tetrachords 4-z15 and 4-z29, the "all-interval" tetrachords. Because they belong both to Genus 1 and to Genus 2 and because they are very likely to be compositional sets in any number of avant-garde works of the early 20th century, those genera are apt to be invoked, willy-nilly, by many analyses. Again, strategies of interpretation will be developed to cope with these problems.

Also of considerable historical significance are the four tetrachords which hold memberships in three genera: 4-12, 4-13, 4-18, and 4-27. Table 13 shows these tetrachords in compact form together with the unique tetrachord 4-19 (with respect to distribution over the genera). The horizontal lines joining the matrix entries highlight the theoretical connections between genera afforded by these special tetrachords. Thus, 4-27 (the dominant/half-diminished seventh chord), which is a hallmark of late 19th-century experimental music, connects Genus 2, the whole-tone genus, with Genus 3, the "diminished" genus, and, at the far end of the spectrum, joins the traditional "dia-tonal" genus, Genus 12. In fact, from a historical vantage, the progression should follow the right-to-left path across the matrix, corresponding to the transplantation of the familiar half-diminished seventh (in particular) from its native diatonic clime to the exotic habitat of the diminished and whole-tone genera.

Perhaps the most striking feature of the structure displayed in Table 13 is the convergence of four of the five tetrachords on Genus 3 as a kind of

Table 12
Distribution of Tetrachords over the Twelve Genera

	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
4-1:1					o							
4-2:2					o	o						
4-3:1						o						
4-4:1								o				
4-5:2	o	o										
4-6:1	o											
4-7:1								o				
4-8:1	o											
4-9:1	o											
4-10:1							o					
4-11:1							o					
4-12:3		o	o			o						
4-13:3	o		o				o					
4-14:1										o		
4-z15:2	o	o										
4-16:2	o	o										
4-17:1									o			
4-18:3	o		o						o			
4-19:4				o				o	o	o		
4-20:1										o		
4-21:1		o										
4-22:2											o	o
4-23:1											o	
4-24:2		o		o								
4-25:1		o										
4-26:1												o
4-27:3		o	o									o
4-28:1			o									
4-z29:2	o	o										
Counts:	9	9	5	2	2	3	3	3	3	3	2	3

Table 13
Tetrachords with Broad Distribution Patterns

	G1	G2	G3	G4	G6	G7	G8	G9	G10	G12
4-12		o	●		o					
4-13	o		●			o				
4-18	o		●					o		
4-19				o			o	o	o	
4-27		o	●							o
	2	2	4	1	1	1	1	2	1	1

node. Again, the historical implications of this phenomenon, presented here in abstract form, are intriguing. There is, in fact, some musical evidence, only a suggestion of which will be given later in Table 34 (Messiaen) and Table 36 (Stockhausen), that Genus 3 plays a major role as a harmonic resource for certain kinds of 20th-century music.

Totally omitted from the system of tetrachordal connectors shown in Table 13 are Genus 5 (chroma) and Genus 11 (dia). One obvious conclusion to be drawn from this observation is that those genera, which have occupied historically important positions in compositional practice, are relatively isolated with respect to tetrachordal constituents. They find their connections to other genera through sets of larger cardinality.

DISTRIBUTION OF THE PENTACHORDS. As might be expected, the pattern of distribution of pentachords over the genera is more complicated, more diversified than that of the tetrachords. A quick survey of the numbers following the colons on Table 14 will show that the number of genera associated with the pentachords ranges from one (5-15) to nine (5-26).

These extremes offer an opportunity to make a very informal comment on the pentachords in actual compositional practice, with respect to their generic associations. There is apparently no correspondence between the number of such associations and the popularity of the set as a harmony. For example, pitch-class set 5-21 is everywhere in early 20th- century music, yet it has only a modest number of genera associations (4). Pentachord 5-15, representing one of the extrema, has only 1 association and is unique among the pentachords in that respect. A famous occurrence of this set is as the last of the series of three chords that pervades the music of Act I, scene 2 of Berg's *Wozzeck.* (See Jarman, 70, Ex. 94.) And at the other extreme, pentachord 5-26, with the greatest number of genera associations (9), and also unique in that respect. Although not one of the most common atonal sonorities, it is also prominent in *Wozzeck,* occurring throughout the work as the harmonic symbol associated with the "Earrings" episode. (See Jarman, 51, Ex. 58.)

What is important, however, with respect to compositional practice, are the genera, and, in particular, the supragenera to which the pentachord belongs. Table 15, offers a non-objective view of fifteen popular pentachords of early 20th-century avant-garde music and their affiliations, complete with boxes to show how certain pentachords group within the supragenera. It shows the preponderant supragenus to be Supragenus III, comprising Genera 8, 9, and 10. Supragenus I is next in prominence, and the tonal genera comprising Supragenus IV follow last.

Table 14
Distribution of Pentachords over the Twelve Genera

	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
5-1:2					o	o						
5-2:4					o	o	o	o				
5-3:4					o	o	o	o				
5-4:7	o	o	o		o	o	o	o				
5-5:4	o	o			o					o		
5-6:3	o	o						o				
5-7:2	o	o										
5-8:4		o	o		o	o						
5-9:5	o	o			o	o	o					
5-10:5	o	o	o			o	o					
5-11:7					o	o		o	o	o	o	o
5-z12:3	o		o				o					
5-13:8	o	o		o	o	o		o	o	o		
5-14:4	o	o						o			o	
5-15:1	o											
5-16:5	o	o	o			o			o			
5-z17:5				o		o		o	o	o		
5-z18:7	o	o	o			o		o	o	o		
5-19:5	o	o	o				o		o			
5-20:3	o	o								o		
5-21:4				o				o	o	o		
5-22:6	o		o	o				o	o	o		
5-23:4							o			o	o	o
5-24:5	o	o					o				o	o
5-25:5	o	o	o				o					o
5-26:9		o	o	o		o	o	o	o	o		o
5-27:4							o			o	o	o
5-28:5	o	o	o			o						o
5-29:7	o	o	o				o			o	o	o
5-30:8	o	o		o				o	o	o	o	o
5-31:7	o	o	o			o	o		o			o
5-32:5	o	o	o						o			o
5-33:2		o		o								
5-34:4		o	o								o	o
5-35:2											o	o
5-z36:8	o		o		o	o	o		o		o	o
5-z37:5				o				o	o	o		o
5-z38:7	o	o	o					o	o	o		o
	24	24	17	8	10	16	15	15	15	15	10	16

Table 15
Pentachords Common in Early 20th-Century Harmonic Species

	SUPRA I				SUPRA II			SUPRA III			SUPRA IV	
	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
5-7:2	o	o										
5-10:5	o	o	o			o	o					
5-16:5	o	o	o			o			o			
5-z17:5				o		o		o	o	o		
5-z18:7	o	o	o			o		o	o	o		
5-21:4				o				o	o	o		
5-22:6	o		o	o				o	o	o		
5-30:8	o	o		o				o	o	o	o	o
5-31:7	o	o	o			o	o		o			o
5-32:5	o	o	o						o			o
5-34:4		o	o								o	o
5-35:2											o	o
5-z36:8	o		o		o	o	o		o		o	o
5-z37:5				o				o	o	o		o
5-z38:7	o	o	o					o	o	o		o
	10	9	9	5	1	6	3	7	11	7	4	8

DISTRIBUTION OF THE HEXACHORDS. No hexachord belongs to every genus, but six hexachords belong to every genus but one. Table 16 shows that these are 6-z11, 6-15, 6-21, 6-22, 6-31, and 6-34. Hexachords 6-15 and 6-21 do not belong to Genus 11, while hexachords 6-31 and 6-34 do not belong to Genus 5. Hexachord 6-z11 does not belong to Genus 4, and 6-22 does not belong to Genus 3.

Five of these hexachords will be very familiar to students of 20th-century music. The "almost whole-tone" hexachord 6-34, one of the main harmonic-melodic components of Berg's Wozzeck, has been identified by several writers, including Douglas Jarman, who calls it "cadential chord B." (Jarman 47, Ex. 49). Hexachord 6-31, dubbed "cadential chord A" by Jarman, also plays an important role in that work. (Jarman 47, Ex. 49).[11] The "almost whole-tone" hexachords 6-21 (*Pierrot Lunaire*) and 6-22 have been mentioned above in several contexts and are very standard items in the vocabulary of atonal music. And 6-z11 is familiar to students of Stravinsky's serial music as one of the main sets in his masterwork, *Agon* (for example, in the "Bransle Gay").

Again, we can foresee an analytical problem here: if one of these gregarious hexachords turns up in a composition it is very apt to belong to every genus represented, which, once more, implies that strategies of interpretation will have to be developed to ensure a meaningful reading of generic organization.

It is obvious that this problem is less and less serious as the number of a hexachord's genus memberships decreases. Thus, the three hexachords at the bottom of the scale, 6-z6, 6-7, and 6-35, each with only two memberships, should not prove to be recalcitrant in an analytical application. Not entirely incidentally, the grouping of these three hexachords by minimal genus membership offers an opportunity to make an observation about the structure of the genera. While it might seem intuitively apparent that 6-35, the whole-tone hexachord, would probably have very restricted genus membership because of its meager interval content, it is not immediately obvious why 6-z6/38 has the same number of genus memberships as 6-35. The reason, of course, has to do with the trichordal subsets of the former complement-related pair. By the rules of genus formation set forth in Part III, above the level of the trichord, subset relations take precedence over interval content. The same holds for 6-7, whose retinue of trichords includes trichords of classes 3-1, 3-4, 3-5, 3-8, and 3-9 only. By the rules of genus formation only 3-5 and 3-9 may serve as progenitors; hence, 6-7 is assigned to Genus 1 and to Genus 2.

There is, however, an observable general pattern, for as the number of genus memberships increases, the hexachords tend to be of the non-symmetric, non-redundant type, so that hexachords that have 9, 10, or 11 genus memberships are all of the non-redundant type and can generate 24 distinct pitch-class forms each.

Table 16
Distribution of Hexachords over the Twelve Genera

	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
6-1:4					o	o	o	o				
6-2:7	o	o	o		o	o	o	o				
6-z3:7	o	o	o		o	o	o	o				
6-z4:5	o	o			o	o		o				
6-5:9	o	o	o		o	o	o	o	o	o		
6-z6:2	o	o										
6-7:2	o	o										
6-8:8					o	o	o	o	o	o	o	o
6-9:9	o	o			o	o	o	o		o	o	o
6-z10:10	o	o	o		o	o	o	o	o	o		o
6-z11:11	o	o	o		o	o	o	o	o	o	o	o
6-z12:8	o	o	o		o	o	o				o	o
6-z13:6	o	o	o			o	o		o			
6-14:9				o	o	o	o	o	o	o	o	o
6-15:11	o	o	o	o	o	o	o	o	o	o		o
6-16:10	o	o		o	o	o		o	o	o	o	o
6-z17:8	o	o	o			o		o	o	o		o
6-18:9	o	o	o				o	o	o	o	o	o
6-z19:7	o	o	o	o				o	o	o		
6-20:4				o				o	o	o		
6-21:11	o	o	o	o	o	o	o	o	o	o		o
6-22:11	o	o		o	o	o	o	o	o	o	o	o
6-z23:6	o	o	o			o	o					o
6-z24:10	o	o	o			o	o	o	o	o	o	o
6-z25:7	o	o	o				o			o	o	o
6-z26:5	o	o								o	o	o
6-27:7	o	o	o			o	o		o			o
6-z28:6	o	o	o			o			o			o
6-z29:6	o	o	o				o		o			o
6-30:7	o	o	o			o	o		o			o
6-31:11	o	o	o	o		o	o	o	o	o	o	o
6-32:4							o			o	o	o
6-33:6	o	o					o			o	o	o
6-34:11	o	o	o	o		o	o	o	o	o	o	o
6-35:2		o		o								
	29	30	21	10	15	24	25	21	21	21	15	24

There are other regularities, as well, which are of interest with respect to particular repertories. For example, the four octatonic hexachords of the z-type (6-z13/42, 6-z23/45, 6-z28/49, and 6-z29/50) all have six genus memberships. Both of the remaining octatonic hexachords, 6-27 and 6-30, have seven genus memberships. This situation is a reminder, of course, that the mere occurrence of one of the "octatonic" hexachords in a work does not necessarily mean that the octatonic collection is operative.

THE DISTRIBUTION OF FOUR COLLECTIONS OVER THE GENERA. I have emphasized several times during the previous exposition that genera are not based upon scales. Nevertheless it is instructive to look at familiar scales, or, more correctly, collections of pitch-class sets that occur as subsets of familiar scales, in relation to the system of genera, the prismatic operational effect of which may suggest that conventional stereotypes should be reexamined critically, if not rejected outright, in the search for historically illuminative modes of description.

It is important to make a basic distinction here—one which has profound implications for the general topic of this paper—a distinction between the ordered scale and the unordered collection or set of which it is one form (permutation). The tables in the present section explore the total resources of the unordered set with respect to the pitch-class set genera and do not extend to a consideration of any particular ordering of the set. Thus, some of the sets listed on the matrices may not be linear subsets of any familiar scalar form of the set. These surface formations vary with the ordering of the set, except in the case of extremely redundant sets, such as the whole-tone collection. As an example, an important Polish folk mode which Karol Szymanowski used in his music (See Kosakowski, 21) is based upon pitch-class set 7-34. The interlocking linear hexachords of the mode are 6-z24 and 6-33. Because of its ordering, the ascending melodic minor scale, also based upon pitch-class set 7-34, presents interlocking forms of hexachords 6-33 and 6-34. Yet both are produced by the same pitch-class set and may be viewed as ordered variants of the same fundamental entity; the linear hexachords in both instances represent a selection from the total resources of the pitch-class set. (See note 16.)

THE DIATONIC COLLECTION (7-35). The diatonic collection 7-35 spans seven genera. Indeed, five of those genera are also engaged by 7-31, indicating that the two sets intersect significantly not only at the pitch-class level—which we knew already—but also at the level of the largescale genera. However, as can be seen on Table 17, Genus 3 (diminished) is the least populated of all in the case of the diatonic collection, whereas it is one of the three genera most strongly represented in the octatonic collection.

Table 17
The Diatonic Collection (7-35) in Relation to the Genera

	G1	G2	G3	G7	G10	G11	G12
4-8:1	o						
4-10:1				o			
4-11:1				o			
4-13:3	o		o	o			
4-14:1					o		
4-16:2	o	o					
4-20:1					o		
4-21:1		o					
4-22:2						o	o
4-23:1						o	
4-26:1							o
4-27:3		o	o				o
4-z29:2	o	o					
5-z12:3	o		o	o			
5-20:3	o	o			o		
5-23:4				o	o	o	o
5-24:5	o	o		o		o	o
5-25:5	o	o	o	o			o
5-27:4				o	o	o	o
5-29:7	o	o	o	o	o	o	o
5-34:4		o	o			o	o
5-35:2						o	o
6-z25:7	o	o	o	o	o	o	o
6-z26:5	o	o			o	o	o
6-32:4				o	o	o	o
6-33:6	o	o		o	o	o	o
	12	12	7	12	10	12	14

Because the basis of the system of genera is not scalar, but inclusional, based upon trichord progenitors, eight of the tetrachordal subsets of 7-35 are missing from diatonic Supragenus IV, comprising Genera 11 and 12: 4-8, 4-10, 4-11, 4-13, 4-14, 4-16, 4-20, and 4-22. This leaves only four of 7-35's tetrachords in Supragenus IV: 4-22, 4-23, 4-26, and 4-27.

Most of the tetrachords not in Supragenus IV (comprising Genera 10, 11, and 12) in Table 17 are, in fact, unfamiliar as discrete components of "diatonic" compositions. Take 4-8: {0,1,5,6}, for example. This set could occur in a traditional diatonic composition only under the most extreme relaxation of the syntactic rules that govern the construction of self-standing vertical harmonies or harmonies that result from the conflation of linear motions. Table 21, which will be discussed below, represents an effort to collect all the harmonies that commonly occur in the tonal work, including those that result from appoggiatura configurations and suspensions, such as 4-18 and 4-19. Excluded from that table are the following tetrachords of Table 17: 4-8, 4-10, 4-11, 4-13, 4-14, and 4-16. These sonic objects, although "diatonic" in the sense that they are subsets of the diatonic scale, have found their historical destinies not in traditional tonal music, but in essentially non-tonal musics of other kinds, including, for example, the atonal music of Schoenberg and Berg, and the octatonic-diatonic music of Stravinsky. (See Table 39.) Thus, the distribution shown in Table 17, which might at first seem counter-intuitive, actually comes closer to the musical-historical facts than does a reading of the diatonic collection that takes it as an integral and meaningful musical structure rather than as a pedagogically convenient referential arrangement of pitch class representatives. In this connection, it should be stressed that some of the common "appoggiatura" figures found in ordinary tonal music are reducible to such pitch-class sets as 4-18 and 4-19, mentioned immediately above, which are not subsets of 7-35, the diatonic scalar collection.

THE WHOLE-TONE COLLECTION (7-33). Most of the subsets of 7-33 (22 of 27) belong to Genus 2, the whole-tone genus, which is hardly surprising. Looking to the left and right of the matrix in Table 18 with respect to the G2 column, and with the aid of the counts following the colons after the set names, we see that many of the whole-tone sets have a narrow range. The number 2 is clearly the most prevalent, and many of those cases reflect the intersection of Genus 2 and Genus 1. On the other hand, Genus 2 and Genus 12, the dia-tonal genus, intersect almost completely. Indeed, the only set in Genus 12 that is not also in Genus 2 (for this selection of pitch-class sets, not in general) is 4-22. Thus, in some sense, the alliance of diatonic and whole-tone structures is "natural."

Table 18
The Whole-Tone Collection (7-33) in Relation to the Genera

	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
4-2:2					o	o						
4-5:2	o	o										
4-11:1							o					
4-12:3		o	o			o						
4-z15:2	o	o										
4-16:2	o	o										
4-19:4				o				o	o	o		
4-21:1		o										
4-22:2											o	o
4-24:2		o		o								
4-25:1		o										
4-27:3		o	o									o
4-z29:2	o	o										
5-8:4		o	o		o	o						
5-9:5	o	o			o	o	o					
5-13:8	o	o		o	o	o		o	o	o		
5-15:1	o											
5-24:5	o	o					o				o	o
5-26:9		o	o	o		o	o	o	o	o		o
5-28:5	o	o	o			o						o
5-30:8	o	o		o				o	o	o	o	o
5-33:2		o		o								
5-34:4		o	o								o	o
6-21:11	o	o	o	o	o	o	o	o	o	o		o
6-22:11	o	o		o	o	o	o	o	o	o	o	o
6-34:11	o	o	o	o		o	o	o	o	o	o	o
6-35:2		o		o								
	13	22	8	10	6	10	7	7	7	7	6	10

THE CHROMATIC COLLECTION. Comparison of Table 19 and Table 17 will reveal marked similarities between diatonic and chromatic collections. Both are distributed over Genera 1, 2, and 3 (Supragenus I) to the same degree, but not in the same patterns, and both are represented in Genus 7 in the same way. Here the resemblances cease, for while 7-35 engages atonal-tonal Genus 10 and diatonic Genera 11 and 12, 7-1 engages atonal Genus 8 and chromatic Genera 5 and 6. Indeed, and as might be expected, Supragenus II, which comprises Genera 5 and 6, is predominant in Table 19, with Genus 6 capturing the greatest number of subsets of 7-1. Moreover, Genus 5 is required for only one pitch-class set, albeit an important one, and that is 4-1.

Perhaps the most striking and obvious feature of Table 19 is the absence of connections between the chromatic collection 7-1 (5-1) and the genera of diatonic and diatonic-tonal character, Genera 11 and 12. On the other hand, the chromatic collection has significant connections to Genus 8, one of the principal contributors to modern harmonic species. In particular, tetrachord 4-7 and pentachord 5-6 may be cited as commonly occurring sonic materials of music in the atonal idiom. Compared to the whole-tone collection 7-33 (Table 18), the chromatic collection has somewhat more extensive connections to genera other than its principal one, Genus 6. As the counts at the bottom of Table 19 indicate, it is particularly attracted to atonal Genus 1, as compared to the whole-tone collection 7-33, which has only one "essential" connection with that genus, through pentachord 5-15, which, despite its eccentricity, has had a prominent career in 20th-century music.

THE MELODIC MINOR SCALE (7-34). The septad 7-34 has enjoyed a number of external forms in various musics, both composed art music and folk music.[12] Table 20, a generic profile of this structure, illustrates the preponderance of Genus 2, the whole-tone genus, and the scattered pattern of relations which it exhibits with respect to five genera outside Supragenus I, many of which are brought about by tetrachoral subsets of 7-34 with singular associations, such as 4-7, which belongs only to Genus 8 (Table 12).

Although septad 7-34 maintains significant connections with the dia and dia-tonal genera through three tetrachords and two pentachords, including the ultra-diatonic pentachord 5-35, its non-tonal connections are clearly stronger than those, indeed, markedly so, in view of the overriding importance of Genus 2 in the matrix, Table 20. Small wonder that 7-34 played such an important role in the harmonic species of such innovative 20th-century composers as Busoni, Bartok, and Ravel (see note 12).

Table 19
The Chromatic Collection (7-1) in Relation to the Genera

	G1	G2	G3	G5	G6	G7	G8
4-1:1				o			
4-2:2				o	o		
4-3:1					o		
4-4:1							o
4-5:2	o	o					
4-7:1							o
4-8:1	o						
4-10:1						o	
4-11:1						o	
4-12:3		o	o		o		
4-13:3	o		o			o	
4-z15:2	o	o					
4-21:1		o					
5-1:2				o	o		
5-2:4				o	o	o	o
5-3:4				o	o	o	o
5-4:7	o	o	o	o	o	o	o
5-6:3	o	o					o
5-8:4		o	o	o	o		
5-9:5	o	o		o	o	o	
5-10:5	o	o	o		o	o	
5-z12:3	o		o			o	
6-1:4				o	o	o	o
6-2:7	o	o	o	o	o	o	o
6-z3:7	o	o	o	o	o	o	o
6-z4:5	o	o		o	o		o
	12	12	8	12	14	12	10

Table 20
Distribution of 7-34 over the Genera
Reduced Representation

	G1	G2	G3	G6	G7	G10	G11	G12
4-3				o				
4-10					o			
4-11					o			
4-12		o						
4-13	o		o					
4-14						o		
4-z15		o						
4-16		o						
4-19						o		
4-21		o						
4-22							o	
4-23							o	
4-24		o						
4-25		o						
4-26								o
4-27		o						
4-z29		o						
5-10		o						
5-z17						o		
5-23								o
5-24		o						
5-25		o						
5-26		o						
5-28		o						
5-29		o						
5-30		o						
5-33		o						
5-34		o						
5-35								o
6-z23		o						
6-z24		o						
6-33		o						
6-34		o						
	G1	G2	G3	G6	G7	G10	G11	G12
	1	21	1	1	2	3	2	3

TRADITIONAL TONAL HARMONIES. In this section we confront what should be a very large subtopic, but comments will necessarily be restricted in pursuit of the major thrust of this part of the presentation: the brief examination of familiar tonal scales and harmonies as they pass through the filter of the pitch-class set genera.

Although I have made an effort to include many constructions that might occur as traditional tonal harmonies, including suspension, linear, and common scalar formations (such as 6-32 and 6-33), it is likely that I have omitted some of the more exotic combinations that one might find in complex music of the later 19th century by composers such as Mahler, Strauss, Debussy, and Schoenberg. Nevertheless, the coverage is deemed adequate for the present purpose.[13]

Table 21, which depicts a very broad landscape, demonstrates the extent to which the genera are context insensitive and require further interpretation—a feature overlooked until now, but which may have attracted the attention of the astute reader. This is perhaps most apparent when we notice that the denizens of both Genus 2 (whole-tone) and Genus 3 (diminished) are more than amply represented here, which suggests that their credentials should be inspected. Trichord 3-8, for example, is the "Italian Sixth Chord" in its most customary mode of occurrence. Pitch-class set 4-21, the archetypical whole-tone tetrachord, occurs, albeit infrequently, as a linear or suspension chord. Tetrachord 4-25 is the "French Sixth Chord," while 4-z29, one of the two all-interval tetrachords and a favorite of Chopin and Mahler, among others, occurs most often as an elaborate suspension formation. Other constituents of Genus 2 in the matrix of Table 21 have additional affiliations as well. For example, septad 7-32, discussed above, belongs to the other two genera of Supragenus I and to the dia-tonal genus, Genus 12. Note once more, however, that it belongs to the atonal genus, Genus 9, which, as we shall see in Part VIII of this study, is a basic contributor to several modern harmonic species.

In the main, the diatonic tonal harmonies fall into Supragenus IV, as might be expected. In fact, the pentatonic set 5-35 as well as tetrachords 4-26, 4-23, 4-22, and trichord 3-9 occur only within one or both of the two genera that comprise Supragenus IV. Many of the tonal sets not within that supragenus are singletons that represent other genera. However, the exceptions to that situation are especially interesting, for they suggest connections between the tonal genera and genera which are characteristic of other musics, non-tonal musics, in particular.

Perhaps the prime examples of this are 4-18 and 4-19, highly characteristic of the atonal music of the early 20th century which also occur in certain kinds of experimental music in the 19th century, in some of the music of Liszt, for example. (See Forte 1987.)

Not altogether unexpectedly, two genera are totally unrepresented in Table 21: Genera 5 and 6, the chroma and semichroma genera. This is be-

Table 21
Tonal Harmonies and the Pitch-Class Set Genera

	G1	G2	G3	G4	G7	G8	G9	G10	G11	G12
3-5	o									
3-7					o				o	o
3-8		o								
3-9									o	
3-10			o							
3-11							o	o		o
3-12				o						
4-18	o		o				o			
4-19				o		o	o	o		
4-20								o		
4-21		o								
4-22									o	o
4-23									o	
4-25		o								
4-26										o
4-27		o	o							o
4-28			o							
4-z29	o	o								
4-z12	o		o		o					
5-23					o			o	o	o
5-24	o	o			o				o	o
5-25	o	o	o		o					o
5-32	o	o	o				o			o
5-34		o	o						o	o
5-35									o	o
6-z25	o	o	o		o			o	o	o
6-32					o			o	o	o
6-33	o	o			o			o	o	o
	9	11	9	2	8	1	4	7	11	14

cause the constituents of these genera have primarily melodic functions determined by the syntax of tonal music. If the chromatic hexachord 6-1 were to be added to the repertory of pitch-class sets in the matrix of Table 21, it would still be relatively isolated and would not form extensive connections to the other genera. Indeed, as we will see in Part VIII, the two chromatic genera do not appear to have contributed as substantially to the development of modern harmonic species as have other genera, contrary to what certain dearly held platitudes of music appreciation would lead us to believe.

V. Comparison of Genera

The previous section of this presentation concentrated upon the genera as they relate to pitch-class sets of three cardinalities, tetrachords, pentachords, and hexachords, and upon familiar scales and harmonies profiled against the background of the generic structures.

The present section undertakes a full-scale comparison of the genera, on the assumption that it is useful, if not actually essential, to have some understanding of the extent to which a given genus differs from the other genera. To this end, I develop a simple calculus, called the difference quotient, which is a fixed index number for each of the sixty-six pairs of genera.

THE DIFFERENCE QUOTIENT. Given two genera GA and GB, we observe that intersection and symmetric difference (exclusive or) partition their union. To illustrate, Table 22 displays four cases. The entries in column GA display integers that represent the constituents of one hypothetical genus; The entries in column GB do the same for a second hypothetical genus. The entries in column INT give the integers common to the GA and GB entries; the entries in column SYMDIF list the symmetric differences of the GA and GB entries, the integers in GA but not in GB combined with the integers in GB but not in GA. In the column labelled Interp. are given evaluations of the similarity of the two genera GA and GB.

Thus, Case 1 represents one of the two possible extreme cases: the two genera are maximally different. Case 2 represents the other extreme: the two genera are identical. In Case 3, the two genera share two constituents and differ with respect to one, a situation that can be described as near-minimum difference. In Case 4, on the other hand, the two genera share only one member and differ with respect to five: a condition of near maximum difference.

From the numerical standpoint, the determining factors in the construction of a measure of difference are the difference of the cardinal number, symbolized by #, of the difference and intersection sets with respect to the

Table 22
Examples of Difference

Case	GA	GB	INT	SYMDIF	Interp.
1	0 1	4 5	nil	0 1 4 5	max
2	0 1 2	0 1 2	0 1 2	nil	min.
3	0 1 2	0 1 3	0 1	2 3	median
4	0 1 2 3	3 4 5	3	0 1 2 4 5	near max.

Calculations
Case 1 (4 - 0) / (4 - 0) = +1.0
Case 2 (0 - 3) / (6 - 3) = -1.0
Case 3 (2 - 2) / (6 - 2) = 0.0
Case 4 (5 - 1) / (7 - 1) = 0.6

Table 23
Genera Comparison: Difference Quotients

1	2	3	4	5	6	7	8	9	10	11	12
1	.01666	.31385	.85452	.72629	.52671	.46394	.61339	.50088	.61339	.72629	.52671
	2	.32592	.76919	.73197	.46969	.54117	.62096	.57727	.62096	.73197	.46969
		3	.85454	.79192	.42934	.40259	.67970	.44696	.70384	.81409	.48604
			4	.87058	.81746	.87012	.47443	.47443	.47443	.87058	.81746
				5	.13112	.46759	.55681	.76515	.73951	.79227	.78591
					6	.31459	.32608	.35584	.67803	.78591	.60740
						7	.65641	.65641	.65641	.46759	.31459
							8	.17564	.14333	.73951	.67803
								9	.17564	.76515	.35584
									10	.55681	.32608
										11	.13112

total size of the genera without duplicates. On this basis, we construct variables X and Y, as follows:

$$\text{Let } X = \#\text{Symdif} - \#\text{Int}$$
$$\text{Let } Y = \#(GA + GB) - \#\text{Int}$$

Then the difference quotient Difquo is the ratio calculated by the expression

$$\text{Difquo} = X/Y$$

The hypothetical range of decimal values of Difquo is from -1 (representing minimum difference or identity) to +1 (maximum difference—no members in common). Sample calculations are shown on Table 22 for the four exemplary cases.

However, for the genera an additional step in the calculation is necessary in order to take into account the cardinalities of the constituents. Given two genera A and B, the intersection of A and B with respect to hexachords is apt to be greater than that of other cardinalties, just because hexachords may have accumulated a large number of relations at their location in the generic hierarchy. Thus, to adjust for the possible discrepancies between difference quotients among the four cardinalities (3, 4, 5, and 6) that comprise a genus, the equation is extended to compute the arithmetic mean of X divided by Y. Thus, in its final form,

$$\text{Difquo} = (X / Y) / 4$$

This is a quantitative measure; it does not indicate the difference of two genera in terms of pitch-class set membership. Nevertheless, it does provide a simple and straightforward index, one that reflects important general properties of the genera that will be of interest when we see them in operation in actual music in Part VIII of this essay.

Table 23 offers a concise overview of the difference quotients for the sixty-six pairs of genera. The genus number is indicated in the top row. To determine the difference quotients for any genus, simply read down the appropriate column to its end, then across the row. For example, the difference quotient for Genus 2 and Genus 1 is .01666, for Genus 2 and Genus 3 .32592, for Genus 2 and Genus 4 .76919, and so on across the row.

The reader should bear in mind that the larger the number, the greater the difference. Thus, Genus 2 (whole-tone) is not very different from Genus 1 (atonal), since Difquo = .01666, whereas it is very different from Genus 11 (dia), since Difquo = .73197. In all cases, the numbers should correspond to our intuitive perception of the character of each genus as reflected by the informal type assigned to it.

Table 24 provides an overview of the difference quotients from another

Table 24
Values of Difquo

Difquo	Genera	Supragenus Membership
.0166667	G1 & G2	SI
.1311275	G5 & G6/G11 & G12	SII/SIV
.1433333	G8 & G10	SIII
.175641	G8 & G9/G9 & G10	SIII/SIII
.313857	G1 & G3	SI
.3145927	G6 & G7/G7 & G12	SII & G7/G7 & SIV
.325926	G2 & G3	SI
.326087	G6 & G8/G10 & G12	SII & SIII/SIII & SIV
.3558489	G6 & G9/G9 & G12	SII & SIII/SIII & SIV
.4025974	G3 & G7	SI & G7
.4293478	G3 & G6	SI & SII
.4469661	G3 & G9	SI & SIII
.4639499	G1 & G7	SI & G7
.4675926	G5 & G7/G7 & G11	SII & G7/G7 & SIV
.469697	G2 & G6/G2 & G12	SI & SII/SI & SIV
.4744318	G4 & G8/G4 & G9/G4 & G10	G4 & SIII
.4860427	G3 & G12	S1 & SIV
.5008817	G1 & G9	S1 & SIII
.5267137	G1 & G6/G1 & G12	S1 & SII/SI & SIV
.5411765	G2 & G7	SI & G7
.5568182	G5 & G8/G10 & G11	SII & SIII/SIII & SIV
.5772727	G2 & G9	SI & SIII
.6074074	G6 & G12	SII & SIV
.6133919	G1 & G8/G1 & G10	SI & SIII/SI & SIII
.6209677	G2 & G8/G2 & G10	SI & SIII/SI & SIII
.6564102	G7 & G8/G7 & G9/G7 & G10/	G7 & SIII
.6780397	G6 & G10/G8 & G12	SII & SIII/SIII & SIV
.6797083	G3 & G8	SI & SIII
.7038462	G3 & G10	SI & SIII
.7262931	G1 & G5/G1 & G11	SI & S2/SI & SIV
.7319749	G2 & G5/G2 & G11	SI & SII/SI & SIV
.7395105	G5 & G10/G8 & G11	SII & SIII/SIII & SIV
.7651515	G5 & G9/G9 & G11	SII & SIII/SIII & SIV
.7691964	G2 & G4	SII & G4
.7859195	G5 & G12/G6 & G11	SII & SIV/SII & GIV
.7919255	G3 & G5	SI & SII
.7922705	G5 & G11	SII & SIV
.8140929	G3 & G11	SI & SIV
.8174603	G4 & G6/G4 & G12	G4 & SII/G4 & SIV
.8545259	G1 & G4	SI & G4
.8545455	G3 & G4	SI & G4
.8701298	G4 & G7	G4 & G7
.8705882	G4 & G5/G4 & G11	G4 & SII/G4 & SIV

vantage. The leftmost column of the table consists of an ordered list of all the difference quotients (of which there are 43), the next column to the right specifies the generic pair or pairs to which that number is attached, and the rightmost column displays the supragenera to which the genera in the middle column belong. The information in the latter column will be more meaningful to the reader after the discussion of the basis of the supragenera in Part VII below.

GENERAL OBSERVATIONS ON TABLE 24 (DIFFERENCE QUOTIENTS). The extrema on Table 24 and the genera with which they are associated are of interest. Take the three difference quotients at the high end of the scale. Each of these involves Genus 4. Specifically,

.87058	Genus 4 (augmented) & Genus 5 (chroma)
	Genus 4 & Genus 11 (dia)
.87012	Genus 4 & Genus 7 (chroma-dia)
.85454	Genus 4 & Genus 3 (diminished)

In short, Genus 4 differs extensively from the diatonic genus, the chromatic genus, the hybrid chroma-dia Genus 7, and the diminished genus. Table 23 shows that the pairs formed by Genera 3, 5, 7, and 11 also differ considerably from each other.

At the other end of the difference quotient scale, the first three indices associate with genera as follows:

.01666	Genus 1 (atonal) & Genus 2 (whole-tone)
.13112	Genus 5 (chroma) & Genus 6 (chroma-dia)
	Genus 11 (dia) & Genus 12 (dia-tonal)
.14333	Genus 8 (atonal) & Genus 10 (atonal-tonal)

It should not be surprising that each of the pairs listed above belongs to the same supragenus. The reason for this will become even more evident in Part VII below, where the basis of association for genera within a supragenus as the intersection of pitch-class set constituents is covered in greater detail.

VI. Supragenera

BASIS OF THE SUPRAGENERA. During the foregoing presentation I made frequent mention of the supragenera, often with some comment to the effect that further discussion of those entities would take place at a later point in the paper. That moment has now arrived: Genera are grouped into supragenera on the basis of intersection of their constituents—the extent to which they hold pitch-class sets in common.

SUPRAGENUS I (GENERA 1, 2, 3). Consider the special relations between the three genera which comprise Supragenus I. Although the three genera differ, to a greater or lesser degree, with respect to their tetrachordal and pentachordal constituents (and, obviously, with respect to their trichordal progenitors), they intersect to a remarkable degree with respect to hexachords. In symbolic terms,

Expr. A $\quad G3h < G1h < G2h$

In this expression, Expr. A, h is attached to the abbreviated genus name to signify the hexachords of that genus. The inclusion symbol $<$ represents the (transitive) relation between the sets of pitch-class sets designated. Thus, Genus 2 is the focal structure of Supragenus I in the specific sense that it contains the hexachords of the other two genera. Conversely, and obviously,

Expr. B $\quad G2h \nless G1h \nless G3h,$

where $\nless$ signifies "is not included in."

Thus, at the hexachordal level, the three genera enjoy a close relationship, one that none holds with any of the other genera in the system.

The relationship may be seen as still more intimate when it is pointed out that the Expr. A above implies

Expr. C $\quad \cdot(G3h,G1h) = \cdot(G3h,G2h),$

where $\cdot$ symbolizes intersection and the comma separates the two intersecting operands enclosed in parentheses. Among other things, what this means is that the 21 hexachords of Genus 3 form a nucleus of sets that links the hexachords of the two other members of the supragenus.

Certain analytical implications of this nesting may be apparent to the reader. If all three genera are operative in a given work, Genus 2 will necessarily assert its hexachordal hegemony with respect to Genus 1 and Genus 3, and therefore the analyst must be aware of this natural situation as he interprets relations, with due consideration given to actual musical context. (There will be occasion at the end of Part VII to illustrate this very circumstance.)

SUPRAGENUS II (GENERA 5 & 6). For the two chromatic genera of this supragenus, hexachordal intersection is again determinative. In symbols,

Expr. D $\quad G5h < G6h$

Moreover, the pentachords of Genus 5 intersect with those of Genus 6 in all but one case: pentachord 5-5 belongs to Genus 5 and not to Genus 6. Since 5-5 is not a pentachord of the better class, the exception is relatively inconsequential.

Although Genus 6 differs substantially from Genus 5, both with respect to hexachords and pentachords, the difference quotient of .13112 for the two genera reflects their very close association, an association most immediately apparent in their shared progenitor, trichord 3-2. Only one difference quotient is less, that formed by Genus 1 and Genus 2. (See Table 24.)

SUPRAGENUS III (GENERA 8, 9, AND 10). The basis of Supragenus III is most conveniently approached directly by surveying the intersections among the constituents of Genera 8, 9, and 10. For tetrachords (t) and pentachords (p), the situation is summarized in the following expressions:

Expr. E $\quad \cdot(G8t,G9t) = \cdot(G8t,G10t) = \cdot(G9t,G10t)$

Expr. F $\quad \cdot(G8p,G9p) = \cdot(G8p,G10p) = \cdot(G9p,G10p)$

Thus, the tetrachords of all three genera share the same set of pitch-class sets, namely, tetrachord 4-19, which, as remarked earlier, is one of the primary hallmarks of atonal music. For pentachords over the three genera, the situation is the same: the intersection set consists of the same ten pitch-class sets.

If it were not for hexachord 6-9, the hexachords of the three genera would exhibit the same correspondence. However, 6-9 occurs in the intersection of Genus 8 and Genus 10, whereas that set is not represented in the other two intersections. Supragenametaphorically speaking, hexachord 6-9 is something of a black sheep, which may be one reason that its career as a component of modern harmonic species has been thwarted.

In terms of intersection, therefore, the constituents of Supragenus III are firmly associated over sets of cardinals 4, 5, and 6. And significant connections are provided by the trichordal progenitors, each of which serves once to link each of the three pairs.

SUPRAGENUS IV (GENERA 11 & 12). Perhaps the simplest and most significant connection between Genus 11 and Genus 12, the diatonic and tonal genera, is provided by the common progenitor trichord 3-7, just as trichord 3-2 links the two genera that comprise Supragenus II. The most extensive linkage, however, occurs at the hexachordal level, since

Expr. G G11h < G12h

Moreover, all but one of the pentachords of Genus 11 are also pentachords of Genus 12. The exception is 5-14.

A single tetrachord joins Genus 11 and Genus 12, tetrachord 4-22, which occurs in tonal music as a fragment of the pentatonic scale: in prime form, [0,2,4,7].

GENERA 4 AND 7 AS SUPRAGENERA. Two supragenera combine three genera: Supragenus I and Supragenus IV; two supragenera combine two genera: Supragenus II and Supragenus III. For numerological as well as aesthetic reasons it would be attractive to partition the system of genera into six supragenera instead of four plus two singletons, by designating Genera 4 and 7 as supragenera. This temptation has been resisted, however, since Genera 4 and 7 could only be regarded as supragenera in the most trivial sense and, more important, no significant theoretical consequences appear likely to ensue as a result of such a taxonomical decision.

COMPARISON OF SUPRAGENERA. Table 25 provides a convenient display for a discussion of the relations between the supragenera. For the convenience of the reader, a summary of the informal descriptors (types) is also given.

As can be read from the table, the greatest difference obtains between Supragenus II and Supragenus IV, which is neither surprising nor counter to musical experience, since these genera represent chromatic and diatonic domains, respectively.

What is somewhat unexpected, however, is that the least difference obtains between Supragenus II (chromatic) and Supragenus III (atonal-tonal) and between Supragenus III and Supragenus IV (diatonic-tonal). Indeed, the difference quotient is precisely, and remarkably, the same number in both instances. In terms of the origin of modern atonal harmonic species this might be interpreted as shown by the small diagram at the bottom of Table 25: both chromatic and diatonic supragenera contribute to the atonal supragenus.

Two additional pairs of supragenera exhibit the same difference quotient: Supragenus I and Supragenus II and Supragenus I and Supragenus IV. Again, and with respect to the development of the modern harmonic species derived from the hybrid Supragenus I, these relations may be inter-

Table 25
Supragenera Comparison
Table of Difference Quotients

	SII	SIII	SIV
SI	.47222	.38293	.47222
	SII	.35625	.61379
		SIII	.35625

SI: atonal, whole-tone, diminished (atonal hybrid)

SII: chroma, semichroma

SIII: atonal, atonal-tonal

SIV: dia, dia-tonal

Relation between chromatic, atonal and diatonic supragenera

Chromatic (SII) Atonal (SIII) Diatonic (SIV)

Relation between chromatic, atonal hybrid, and diatonic supragenera

Chromatic (SII) Atonal Hybrid (SI) Diatonic (SIV)

Relation between atonal and atonal hybrid genera

Atonal (SIII) Atonal Hybrid (SI)

preted as shown in the bottom portion of Table 25: both chromatic and diatonic supragenera contribute to the other primarily atonal supragenus, here called the "hybrid atonal supragenus" because of the three distinct types it contains, Supragenus I.

Finally, the other non-duplicated difference quotient .38293, formed by the atonal and hybrid atonal Supragenera I and III, reflects the special relation between these two structures which will be evident in a number of ways in many analytical applications of pitch-class set genera, some of which are demonstrated in Part VII of this article.

Because Supragenera I and II form the same difference quotient as Supragenera I and VI, one might guess that the counts of the separate difference (and symmetric difference) sets correspond exactly, which is the case. For example, the symmetric difference of the tetrachords of Supragenus I and Supragenus II consists of 17 pitch-class set names and so does the symmetric difference of the tetrachords of Supragenus I and Supragenus IV. This regularity extends to the counts of difference and symmetric difference sets for Supragenera II and III and Supragenera III and IV. For example, the symmetric difference of the hexachords of both pairs consists of 10 pitch-class set names.

However, and perhaps obviously, given the distinctive nature of the supragenera involved, the sets that comprise the corresponding difference and symmetric difference sets are not the same, with two exceptions: (1) the difference of the tetrachords of Supragenus III with respect to the tetrachords of Supragenus IV (those in SIII but not in SIV) is identical to the difference of the tetrachords of Supragenus III with respect to the tetrachords of Supragenus II:

4-4, 4-7, 4-14, 4-17, 4-18, 4-19, 4-20;

(2) the difference of the hexachords of Supragenus II with respect to the hexachords of Supragenus III is identical to the difference of the hexachords of Supragenus IV with respect to the hexachords of Supragenus III:

6-z12, 6-z23.

Further study of the structure of the system of 12 pitch-class set genera may reveal the significance of these evidently special circumstances.

RELATION OF PITCH-CLASS SET GENERA TO Kh SET COMPLEXES. The set complex designated Kh is a collection of related pitch-class sets formed according to the following definition:

Expr. H $\quad S/\bar{S} \text{ e } Kh(T,\bar{T}) \text{ iff } S >< T \ \& \ S >< \bar{T}$

In set-theoretic language, this expression states that a pitch-class set S and its complement are members of the set complex Kh about a set T and its complement if and only if S contains or is contained in T and S contains or is contained in the complement of T.[14]

The effect of this definition is to create a relatively exclusive club of pitch-class sets, the Kh collection, headed up, as it were, by some (any) individual set. For example, the set complex about 4-19, symbolized Kh(4-19) consists of four trichords, seven pentachords, and nine hexachords. Because of the strict requirements for membership imposed by the "and" of Expr. E above, many of the supersets of 4-19 are excluded for want of proper pedigree.

Since the set complex and, in particular, the set complex of the Kh type are often useful in interpreting set relations analytically, it would seem advantageous to retain them as theoretical tools. At the same time, it seems essential, at least in the context of the present study, to understand how they relate to the system of 12 pitch-class set genera, the topic of this section.

Because of the "chaining" effect of the definition of pitch-class set genus and the large-scale structures it creates, the set complex Kh when placed against the grid of the generic system will show the results of what has been termed a prismatic effect; that is, the members of the set complex may retain their unity to a certain extent within the genus, but, at the same time they are apt to be distributed or scattered over a number of genera, perhaps even highly diversified genera.

The set complexes Kh about trichords 3-12, 3-10, 3-8, and 3-5 produce the simplest and most straightforward pictures of the relation between Kh and genera. Table 26, a matrix display of all the pitch-class set members of Kh(3-12), provides an introductory example. To facilitate reading of the matrix brackets and names of supragenera and genera have been added at the bottom of the matrix, below the frequency counts.

Every member of Genus 4 is represented in the matrix in Table 26, a fact that cannot be ascertained simply by looking at the table, without comparing the sets listed there with the list for Genus 4 in the Appendix. It can be fairly said that Genus 4 is coextensive with Kh(3-12). However, the converse is not true: Kh(3-12) is not coextensive with Genus 4; its members form affiliations that extend over all 12 genera. Observe, in particular, the regular pattern formed over Supragenus III, a second home for Kh(3-12) outside its normal habitat in Genus 4. Because Supragenus III appears to be one of the basic reservoirs from which many modern harmonic species are drawn, Table 26 dramatizes the role of trichord 3-12 and Kh(3-12) in basic metamorphic processes.

To sum up, genera with a single trichordal progenitor T are coextensive with the Kh set complex about T, and the greatest number of genera memberships for the constituents of that Kh will occur within the supragenus to

Table 26
Genus 4 (3-12) and Kh(3-12): Distribution Over the 12 Genera

	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
3-12				o								
4-19				o				o	o	o		
4-24		o		o								
5-13	o	o		o	o		o	o		o	o	
5-z17				o		o		o	o	o		
5-21				o				o	o	o		
5-22	o		o	o			o	o	o			
5-26		o	o	o		o	o	o	o	o		o
5-30	o	o		o				o	o	o	o	o
5-33		o		o								
5-z37				o				o	o	o		o
6-14				o	o	o	o	o	o	o	o	o
6-15	o	o	o	o	o	o	o	o	o	o		o
6-16	o	o		o	o	o		o	o	o	o	o
6-z19	o	o	o	o				o	o	o		
6-20				o				o	o	o		
6-21	o	o	o	o	o	o	o	o	o	o		o
6-22	o	o		o	o	o	o	o	o	o	o	o
6-31	o	o	o	o		o	o	o	o	o	o	o
6-34	o	o	o	o		o	o	o	o	o	o	o
6-35		o		o								
Counts:	10	13	7	21	6	10	7	17	17	17	6	10
	[SI		]	G4	[SII	]	G7	[SIII		]	[SIV	]

which the genus based upon T belongs, thus clearly associating the concept of the set-complex Kh with the concept of supragenus, an association that will have important ramifications when we consider analytical applications of the system of pitch-class set genera in Part VII.

From the preceding material, it should be evident that counts of genus membership for a set of pitch-class sets are highly significant since they reflect not merely an abstract frequency statistic, but the operation of cohesive subunits of structure, the Kh set complexes.

ADDITIONAL PERSPECTIVES ON Kh SET COMPLEXES AND GENERA. The relation between Kh set complexes and genera is somewhat more complex in cases other than that of the Kh about a single trichordal progenitor. In order to make this clearer, the following examples have an additional feature. For each genus represented in the matrix a real number serves as an index of its relative strength. The index, to be called the Status Quotient, or Squo ("S-Quo"), is calculated as follows:

$$\text{Expr. I} \qquad \text{Squo(Ga)} = ((X \;/\; Y) \;/\; Z) \cdot 10.$$

The variable X is the number of representatives of Genus a (Ga) in the matrix; Y is the total set count for the matrix (all genera), and Z is the total size of Ga. The constant 10 shifts the resulting decimal number one place to the left for greater legibility. In case a genus is completely represented in the matrix ($X = Z$), an index of 1 is assigned to it.

The Status Quotient index is a further step toward refining and interpreting the matrix structure for the analyses of actual compositions which will be presented in Part VII of this study.

Thus, Table 27 shows the interesting situation in which the intersection of two genus progenitors, 3-1 and 3-2, the progenitors of Genus 5, are mapped onto the genera. As we might expect, the result is not as comprehensive and straightforward as in the case of the single trichordal progenitor. Notice, however, that the number associated with Genus 5, on the list of Squo indices at the bottom of Table 27, is the largest of the entire list. The two gaps in the column headed G5 represent the absence of pitch-class sets 6-z17 and 6-18, hexachords that fulfill the requirements for membership in the two Kh set complexes but do not fulfill the requirements for membership in Genus 5. We conclude, tentatively, that there is a very close relation, but not a perfect correspondence, between the intersection of Kh(3-1) and Kh(3-2) and Genus 9. For the remaining cases of dual progenitors, the situation is comparable.

The indices at the bottom of Table 27 represent a considerable refinement of the data consisting of raw counts, since they are scaled in accord

Table 27
Intersection of Kh(3-1) and Kh(3-2) Mapped into Genera

	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
4-1					o							
4-2					o	o						
5-1					o	o						
5-2					o	o	o	o				
5-3					o	o	o	o				
5-4	o	o	o		o	o	o	o				
5-5	o	o			o					o		
5-8		o	o		o	o						
5-9	o	o			o	o	o					
5-11					o	o		o	o	o	o	o
5-13	o	o		o	o	o		o	o	o		
6-1					o	o	o	o				
6-2	o	o	o		o	o	o	o				
6-z3	o	o	o		o	o	o	o				
6-z4	o	o			o	o		o				
6-5	o	o	o		o	o	o	o	o	o		
6-8					o	o	o	o	o	o	o	o
6-9	o	o			o	o	o	o		o	o	o
6-z10	o	o	o		o	o	o	o	o	o		o
6-z11	o	o	o		o	o	o	o	o	o	o	o
6-z12	o	o	o		o	o	o				o	o
6-14				o	o	o	o	o	o	o	o	o
6-15	o	o	o	o	o	o	o	o	o	o		o
6-16	o	o		o	o	o		o	o	o	o	o
6-z17	o	o	o			o		o	o	o		o
6-18	o	o	o				o	o	o	o	o	o
6-21	o	o	o	o	o	o	o	o	o	o		o
6-22	o	o		o	o	o	o	o	o	o	o	o
Counts:	18	19	12	6	26	25	18	21	13	15	9	13
	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
	[SI		]		[SII	]		[SIII		]	[SIV	]

Counts	SQUO Indices in Descending Order with Genera
26	.320: G5 (chroma)
25	.198: G6 (semichroma)
21	.182: G8 (atonal)
18	.142: G7 (chroma-dia)
15	.130: G10 (atonal-tonal)
13	.113: G9 (atonal-tonal)
9	.110: G11 (dia)
6	.107: G4 (augmented)
19	.106: G2 (whole-tone)
13	.103: G12 (dia-tonal)
18	.102: G1 (atonal)
12	.099: G3 (diminished)

with the three quantities, X, Y, and Z, in Expr. I above, to give a more accurate picture of the contribution of the intersection of Kh(3-1) and Kh(3-2) to the constituent genera of the matrix. The most extreme example of the adjustment made by the index can be seen in connection with Genus 1. Whereas the count of Genus 1, which has 18 representatives in this matrix, places it fifth on a list of simple frequencies, the Squo number associated with Genus 1, .102, stands next to the lowest on the list of indices.

VII. Genera and Harmonic Species in Musical Compositions

RULES FOR THE INTERPRETATION OF GENERIC RELATIONS. In order to study the generic structure of a particular composition it seems essential to establish rules by which the complexity of relations, as represented in the by-now familiar matrices, may be reduced and made comprehensible. To that end I offer below five rules for the analysis of genera matrices, followed by examples drawn from the works of diverse composers, primarily of the twentieth century.

1. *Rule of greatest status quotient* determines the genus with primary role, unless the representatives of that genus are a proper subset of a genus with a greater Squo, in which case Rule 2 takes effect. If more than one genus enjoys a particular Squo, Rule 1 associates the relevant pitch-class set with it (them) as well, unless there is a third candidate genus which has been invoked by Rule 4, the Rule of Singleton Extension, in which case the latter genus receives the pitch-class set.
2. The *Rule of Intersection* omits genera which are proper subsets of other genera with higher Squos.
3. The *Rule of Completion* completes the generic matrix in case the genus with the highest "operational" Squo (if Rule 2 has been placed in effect) does not account for every set, by invoking the genus with the next highest Squo to provide a setting for the vagrant pitch-class set(s).
4. The *Rule of Singleton Extension* causes pitch-class sets which are attached to only one genus ("singletons") to engage that genus in its entirety. Genera so engaged may incorporate other pitch-class sets not yet situated in the matrix by Rules 1 or 3. Rules 1 and 3 apply if more than one genus is a candidate.
5. The *Rule of Reduction* omits genera, "passive genera," which do not contribute to the generic profile of the composition, as determined by Rules 1, 3, and 4, and produces the reduced matrix representations which are the main illustrations in this part of the article, representations in which

each pitch-class set in the matrix is assigned to only one genus.

To see the application of the rules in detail, let us consider a case in which we know something about the analytical outcome in advance, the Kh(4-5) generic distribution.

Table 28 provides the required overview, indicating in the leftmost column the rule or rules that have been applied to assign each pitch-class set to a genus. Thus, for example, Rule 3, the Rule of Completion, assigns trichord 3-4 to Genus 8, since Genus 8 has the next-to-highest Squo of .144. Tetrachord 4-5 has a choice between Genera 1 and 2 by the Rule of Singleton Extension, but selects Genus 1 by the further application of Rule 1, the Rule of Greatest Status Quotient.

Above eight of the genus designations underscored in the topmost column of the matrix in Table 28 are indicated the rules which have caused those genera to vanish from the reduced matrix shown in Table 29. The latter matrix reveals the very skimpy, but essential structure created by the pitch-class set constituents of the Kh set complex about tetrachord 4-5 when subjected to the rules of interpretation set forth above. Notice, however, that the order of the counts now corresponds more closely to the order of the Squos, with Genus 5 acquiring the lion's share of the pitch-class sets (15), Genus 1 next (7), followed by Genus 8 (4) and, finally, Genus 2, with only 1 representative. Among other things, this tells us that Kh(4-5) is primarily a chromatic structure, with strong atonal characteristics and a modest infusion of essence of whole-tone.

PITCH-CLASS SET GENERA AND HARMONIC SPECIES. With the preceding discussion as technical background we can now look at pitch-class set genera as they relate to actual compositions. To that end I have selected for analysis ten works by a wide variety of composers, many of whom ostensibly have nothing in common. Some of the works will be familiar to the reader while others may not. It should be said that the primary criterion for selection was the author's interest in the music, mainly for its unusual features. There has been no conscious effort to span a certain repertory, to cover a particular period, or to select a work in order to make a point. Indeed, the "point" comes only with the interpretation of the generic relations. Only with further study of the genera system in relation to real music can more extensive insights develop, a matter to which I shall return at the conclusion of this presentation.

In order to focus on the genera and the harmonic species which derive from them I have not attempted to present the segmentations that result in the particular pitch-class set vocabulary associated with an example-work.

Table 28
Kh(4-5) Distributed over the Genera and Interpreted

				R2	R2		R2	R2		R2	R5	R2	R2
		G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
R1	3-1					o							
R3	3-4								o		o		
R4	3-5	o											
R4	3-8		o										
R4,1	4-5	o	o										
R1	5-4	o	o	o		o	o	o	o				
R1	5-5	o	o			o					o		
R4,1	5-6	o	o						o				
R4,1	5-7	o	o										
R1	5-9	o	o			o	o	o					
R1	5-13	o	o		o	o	o		o	o	o		
R4	5-15	o											
R3	5-z38	o	o	o					o	o	o		o
R1	6-2	o	o	o		o	o	o	o				
R1	6-z3	o	o	o		o	o	o	o				
R1	6-z4	o	o			o	o		o				
R1	6-5	o	o	o		o	o	o	o	o	o		
R4,1	6-z6	o	o										
R4,1	6-7	o	o										
R1	6-9	o	o			o	o	o	o		o	o	o
R1	6-z12	o	o	o		o	o	o				o	o
R1	6-15	o	o	o	o	o	o	o	o	o	o		o
R1	6-16	o	o		o	o	o		o	o	o	o	o
R3	6-z17	o	o	o			o		o	o	o		o
R3	6-18	o	o	o				o	o	o	o	o	o
R1	6-21	o	o	o	o	o	o	o	o	o	o		o
R1	6-22	o	o		o	o	o	o	o	o	o	o	o
	Counts:	24	23	10	5	15	14	11	16	9	12	5	9
		G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
		[SI		]		[SII	]		[SIII		]	[SIV	]

SQUO Indices in Descending Order with Genera

.191: G5 (chroma)
.144: G8 (atonal)
.141: G1 (atonal)
.133: G2 (whole-tone)
.115: G6 (semichroma)
.108: G10 (atonal-tonal)
.092: G4 (augmented)
.090: G7 (chroma-dia)
.086: G3 (diminished)
.081: G9 (atonal-tonal)
.074: G12 (dia-tonal)
.063: G11 (dia)

Table 29. Kh(4-5) in Reducted Representation

	G1	G2	G5	G8
3-1			o	
3-4				o
3-5	o			
3-8		o		
4-5	o			
5-4			o	
5-5			o	
5-6	o			
5-7	o			
5-9			o	
5-13			o	
5-15	o			
5-z38				o
6-2			o	
6-z3			o	
6-z4			o	
6-5			o	
6-z6	o			
6-7	o			
6-9			o	
6-z12			o	
6-15			o	
6-16			o	
6-z17				o
6-18				o
6-21			o	
6-22			o	
Counts:	7	1	15	4
	G1	G2	G5	G8

Squo Indices in Descending Order with Genera

.191: G5 (chroma)
.144: G8 (atonal)
.141: G1 (atonal)
.133: G2 (whole-tone)

SCHOENBERG, DREI KLAVIERSTÜCKE, *OPUS 11, NO. 1 (1909).* Table 30 offers a complete survey of the genera relations for this famous atonal work, the pitch-class set vocabulary of which consists of exactly six hexachords, six pentachords, two tetrachords, and two trichords. (See Forte 1981.)

Interpretation of the matrix is relatively uncomplicated. Rule 1 determines Genus 4 as the primary genus. By Rule 3 Genus 8, which contains the only singleton, 4-7, completes the matrix, except for 6-z13, which is located in Genus 9 by interative application of Rule 3.

From Table 31, which presents the reduced matrix for the movement, we can see that Genus 4 and Genus 8 are the predominant components of this harmonic species. The only representative of the other genus, Genus 9, is the sometime "octatonic" hexachord, 6-z13, which Genus 8 excludes because it does not possess the proper genealogical history; in particular, trichord 3-4 does not appear at the base of its family tree. To generalize still further, it can be seen that the special genus, Genus 4, combines with two of the genera of Supragenus III to create this very unusual, very compact, and very typically Schoenbergian harmonic species. From the earlier material in Part VI and from Table 23, in particular, the reader will recall that Genus 4 is strongly related to Supragenus III, forming the same low difference quotient, .47443, with each of its genera.

Beginning with the next example, Table 32, the generic matrices are all of the reduced type. Consequently the reader will have to accept as an article of faith the fact that the author has properly and completely observed the five rules—in particular Rule 2 and Rule 5—for the interpretation of generic relations set forth at the beginning of this part of the study. I will make comments on certain features of the missing complete matrix as the occasion arises.

RAVEL, TROIS POEMÈS DE MALLARMÉ, *NO. 3 (1913).* From one standpoint, the generic structure of this remarkable song is strikingly different from that of the Schoenberg piano piece just examined. Here the harmonic species is based overwhelmingly on Genus 3, the progenitor of which is the diminished triad, 3-10. The obvious preponderance of Genus 3 constituents is especially noteworthy in view of the large number of pitch-class sets in this music, a circumstance which would normally result in far greater generic diversity. Despite the strong differences which this matrix exhibits when compared with the matrix in Table 31, there is a significant intersection of the two with respect to Genus 4. In fact, there are some shared pitch-class sets, as a comparison will reveal. But these are offset to a certain extent by the contribution which Genus 12 makes to this special harmonic species, representing a tonal residue, as it were, which is very evident at certain moments at the very surface of the music.

Table 30
Schoenberg, Op. 11/1

	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
3-3						o		o	o			
3-4								o		o		
4-7								o				
4-19				o				o	o	o		
5-13	o	o		o	o	o		o	o	o		
5-z17				o		o		o	o	o		
5-z18	o	o	o			o		o	o	o		
5-21				o				o	o	o		
5-z37				o				o	o	o		o
5-z38	o	o	o					o	o	o		o
6-z3	o	o	o		o	o	o	o				
6-z10	o	o	o		o	o	o	o	o	o		o
6-z13	o	o	o			o	o		o			
6-16	o	o		o	o	o		o	o	o	o	o
6-z19	o	o	o	o				o	o	o		
6-21	o	o	o	o	o	o	o	o	o	o		o
6-z43	o	o	o			o		o	o	o		o
Counts:	10	10	8	8	5	10	4	16	14	13	1	6
	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
	[SI		]		[SII	]		[SIII		]	[SIV	]

Squo Indices in Descending Order with Genera

.235: G4 (augmented)
.229: G8 (atonal)
.200: G9 (atonal-tonal)
.186: G10 (atonal-tonal)
.130: G6 (semichroma)
.109: G3 (diminished)
.101: G5 (chroma)
.093: G1 (atonal)
.091: G2 (whole-tone)
.078: G12 (dia-tonal)
.052: G7 (chroma-dia)
.020: G11 (dia)

Table 31
Schoenberg, Op. 11/1

	G4	G8	G9
3-3		o	
3-4		o	
4-7		o	
4-19	o		
5-13	o		
5-z17	o		
5-z18		o	
5-21	o		
5-z37	o		
5-z38		o	
6-z3		o	
6-z10		o	
6-z13			o
6-16	o		
6-z19	o		
6-21	o		
6-z43		o	
Counts:	8	8	1
	G4	G8	G9

Squo Indices in Descending Order with Genera

.235: G4 (augmented)
.229: G8 (atonal)
.200: G9 (atonal-tonal)

Table 32
Ravel, *Trois poèmes de Mallarmé,* No. 3

	G2	G3	G4	G12
3-10		o		
3-11				o
4-12		o		
4-13		o		
4-z15	o			
4-18		o		
4-19			o	
4-26				o
4-27		o		
4-28		o		
4-z29	o			
5-z17			o	
5-z18		o		
5-21			o	
5-22		o		
5-26		o		
5-28		o		
5-29		o		
5-30			o	
5-31		o		
5-32		o		
5-33			o	
5-34		o		
5-35				o
6-z12		o		
6-15		o		
6-z23		o		
6-z24		o		
6-27		o		
6-z28		o		
6-z29		o		
6-30		o		
6-34		o		
6-z44		o		
6-z49		o		
Counts:	2	25	5	3
	G2	G3	G4	G12

Squo Indices in Descending Order with Genera

.161: G3 (diminished)
.138: G4 (augmented)
.129: G12 (dia-tonal)
.104: G2 (whole-tone)

Taking the large view of this harmonic species, it can be seen to represent an astonishing combination of genera: the diminished and augmented genera, Genus 3 and Genus 4, respectively, which have a very high difference quotient (Table 23); the whole-tone genus, Genus 2, here represented by the two all-interval tetrachords, 4-z15 and 4-z29; and, as noted above, the genus which includes much of the harmonic-melodic material of traditional tonality, here represented by 3-11, the triad itself, by the "minor seventh chord," 4-26, and by 5-35, the pentatonic collection.

MUSORGSKY, BORIS GODUNOV, *ACT II, "CLOCK SCENE" (1874).* As is the case with all the music in this part, a more complete analysis would show how the generic elements that comprise the harmonic species combine or are set in opposition as the work unfolds. These phenomena are of course highlighted in a dramatic work as strong and unique as *Boris Godunov,* and, in the so-called clock scene, they are expressed in the most stunning and effective way.

However, some aspects of the sonic matrix can be inferred from the generic matrix shown in Table 33. First, I should point out that in the complete matrix from which this reduced matrix was derived two genera are not represented at all, Genus 5 (chroma) and Genus 11 (dia), reflecting the composer's choice of harmonic materials that lack traditional associations. This predilection – one might with justification say "plan" – is fully evident in the constituents of the harmonic species. Again we see Genus 4 at work, a small but powerful genus, closely associated with Genus 9 on the complete matrix. Of the supragenera, only the non-tonal Supragenus I is represented on the matrix. The Squos of its constituent genera are very close numerically, as are, indeed, the five genera at the top of the list at the bottom of Table 33. Thus, the discrepancy between raw counts and Squos is not so great as it appears.

Readers conversant with pitch-class sets will realize that the elements of the harmonic species in this case are those of an atonal composition. Only in the case of Genus 10, which is represented by one pitch-class set, 4-20, can there be said to be a reference to traditional tonal materials. But in the context of the actual music, 4-20, the "major seventh chord," has none of the attributes so familiar to us from standard tonal music. Thus, the harmonic species in this case, and in virtually every other case in this section as well, is a very general, but nonetheless telling indicator of style, in the very broad sense of the composer's selection of musical materials.

CHOPIN, MAZURKA IN B♭ MINOR, OP. 24/4 (1835). A brief look at a highly "chromatic" work, to use the jargon of music appreciation, is in-

Table 33
Musorgsky, *Boris Godunov,* "Clock" Scene

	G1	G2	G3	G4	G6	G9	G10
3-10				o			
3-12				o			
4-3					o		
4-9	o						
4-16	o						
4-18						o	
4-19				o			
4-20							o
4-21		o					
4-25		o					
4-z29	o						
5-31						o	
5-32						o	
6-7	o						
6-20				o			
6-30						o	
6-z45	o						
Counts:	5	2	1	3	1	4	1
	G1	G2	G3	G4	G6	G9	G10

Squo Indices in Descending Order with Genera

.088: G4 (augmented)
.086: G9 (atonal-tonal)
.084: G1 (atonal)
.082: G2 (whole-tone), G3 (diminished)
.052: G6 (semichroma)
.043: G10 (atonal-tonal)

structive, so that without wishing to make any historical pronouncements, but simply to display a single case which has interesting implications, I direct the reader's attention to Table 34, a generic matrix of the earliest work to be considered in this section.

The inclusion of suspension formations, which accounts for the inclusion of pitch-class sets not on Table 21, produces interesting results, not least of which is the unusual harmonic species drawn from the special genera, Genus 4 and Genus 7, The dia-tonal genus, Genus 12, the whole-tone genus, Genus 2, and Genus 6, the semichroma genus—surely an eclectic combination.

Notice that Genus 7 and Genus 12 have the same Squo, an instance of the second part of Rule 1, which may be termed dual membership. What this suggests is that a traditional tonal formation—or, better said, a formation that is possible in traditional tonal music—belongs to an essentially non-tonal genus as well, suggesting, but not certifying an evolutionary process of some kind.

It is perhaps worthwhile to point out that although one of the two "chromatic" genera that comprise Supragenus II is represented in the matrix of Table 34, the representative is the singular 5-1, the chromatic pentachord. This implies, once more, that certain traditional nomenclature is far from satisfactory in explaining the actual harmonic content of an elaborate work in which chromatic notation is extensive.

MESSIAEN, CATALOGUE D'OISEAUX *6: "LE MERLE DE ROCHE" (1956–58).* Although Table 35 displays the generic structure of only a small portion of this lengthy work (bars 1–34), it is, in fact, more representative than it appears to be, due to the extensive repetition of pitch-class sets stated in the opening music in the remaining 311 bars. Strictly speaking, however, the matrix reading applies only to the opening music.

With this proviso in mind, it is nevertheless possible to state that the habitat of the "Merle de Roche" (Rock Thrush) has a distinctly atonal cast, as indicated in the matrix of Table 35 by the predominance of Genus 1, with its 16 entries. Indeed, all the entries not in Genus 1 are singletons, which accounts for the sparse representation of Genera 2, 10, 11, and 12.

Because of the prominence of Genus 1, with its very high Squo, and because of the relatively large number of singletons, reduction from the complete matrix is uncomplicated. Of the seven genera at the top of the Squo list, two do not survive the Rules of Interpretation: Genus 3 (.065) is absorbed by Genus 1 (Rule 2) and Rule 5 banishes Genus 9 (.048).

The difference quotients formed between Genus 1 and the other genera in the matrix are all high, not only as regards those particular relations, but in general. (See Table 23.) In the music Genus 1 serves as a central core or

Table 34
Chopin, *Mazurka* in B♭ minor, Op. 24/4

	G2	G4	G6	G7	G12
3-11					o
4-11				o	
4-19		o			
4-24		o			
4-25	o				
4-27					o
4-z29	o				
5-1			o		
5-26		o			
5-27				o	o
5-31	o			o	o
6-1				o	
Counts:	3	3	1	4	4
	G2	G4	G6	G7	G12

Squo Indices in Descending Order with Genera

.125: G4 (augmented)
.092: G7 (chroma-dia), G12 (dia-tonal)
.078: G2 (whole-tone)
.074: G6 (semichroma)

Table 35
Messiaen, *Catalogue d'oiseaux,* No. 6: "Le Merle de Roche," bars 1–34

	G1	G2	G10	G11	G12
3-11			o		
4-6	o				
4-8	o				
4-9	o				
4-13	o				
4-14			o		
4-z15	o				
4-16	o				
4-18	o				
4-20			o		
4-21		o			
4-23				o	
4-25		o			
4-26					o
4-z29	o				
5-7	o				
5-z12	o				
5-15	o				
5-20	o				
5-z36	o				
6-z19	o				
6-z41	o				
6-z43	o				
Counts:	16	2	3	1	1
	G1	G2	G10	G11	G12

Squo Indices in Descending Order with Genera

.101: G1 (atonal)
.062: G2 (whole-tone)
.058: G10 (atonal-tonal)
.044: G12 (dia-tonal)
.041: G11 (dia)

reservoir of generically-related pitch-class sets against which the sets drawn from other genera are deployed, supported by a plethora of non-pitch musical features, notably, accentual patterns created by abrupt changes in register and/or dynamics.

WEBERN, FÜNF STÜCKE FÜR ORCHESTER, OP. 10/5 (1911–13). This work, which is representative of the composer's mature atonal music, employs a large pitch-class set vocabulary in a relatively short composition (about the same size as the Messiaen piece above, which is several magnitudes longer than it). The generic picture is also considerably more diversified than that of the Messiaen composition, as can be seen in Table 36.

The harmonic species of this music is dominated by members of the augmented Genus 4, a major contributor to non-tonal harmonic species, as we have seen. Genus 6 and Genus 12 have the same Squo index, .088 and they also form the same difference quotient with G9, namely, .35584 (Table 23). In short, they enjoy the same generic status in the music and relate to the secondary genus, Genus 9, in the same (quantitative) way. But any excitement that might attend the discovery of these relations must be tempered by the further observation that pitch-class set 5-35 is the sole representative of Genus 12 in the matrix. In the actual music, this pentatonic object plays a very special and limited role, one that qualifies as a programmatic and possibly biographical (Mahler?) motif.

Genus 1 and Genus 10 also enjoy the same status in the matrix of Table 36, although, again, Genus 10 has only one member present and is greatly outnumbered by Genus 1, with its five representatives. The outstanding feature of this harmonic species is therefore provided by Genus 4, whose special characteristics, discussed in Part VI above, render it especially distinctive with respect to the other genera in the system, with one exception, the atonal genera that comprise Supragenus III, two of which, Genus 9 and Genus 10, appear in this reduced matrix.

STOCKHAUSEN, NR. 2 KLAVIERSTÜCKE, *II (1954).* A few words about segmentation are in order, since this music has not been written about extensively in a useful technical way. (See Wörner.) In order to avoid complications of detailed segmentation I have included the pitch-class sets associated with the very distinct short sections, later to be called "moments" in Stockhausen's music, some of which appear to have fixed functions. (See Kramer.) For example, 5-15 is almost always reserved for a "cadential" role.

Although it is tempting to embark upon a discussion of some of the

Table 36
Webern, Op. 10/5

	G1	G4	G6	G9	G10	G12
4-3			o			
4-8	o					
4-9	o					
4-12			o			
4-z15	o					
4-17				o		
4-18				o		
4-19		o				
4-z29	o					
5-6	o					
5-7	o					
5-10	o					
5-16				o		
5-21		o				
5-23					o	
5-32				o		
5-35						o
5-z38				o		
6-14		o				
6-15		o				
6-16		o				
6-z19		o				
6-21		o				
6-22		o				
6-31		o				
Counts:	7	9	2	5	1	1
	G1	G4	G6	G9	G10	G12

Squo Indices in Descending Order with Genera

.180: G4 (augmented)
.136: G9 (atonal-tonal)
.107: G1 (atonal), G10 (atonal-tonal)
.088: G6 (semichroma), G12 (dia-tonal)

more unusual aspects of this music, I will remain with its generic organization, in accord with the central purpose of the present report.

A reading of the matrix, Table 37, presents few problems, since Genus 1 is clearly in the ascendant, with six pitch-class sets distributed over the other four genera. Of those, the complete matrix (not displayed) shows that all but 4-12 and 4-27 are singletons, a somewhat unusual situation which is difficult to interpret, given the very tight organization of the music in terms of pitch-class set relations – complements, in particular being deployed strategically throughout. This notwithstanding, the predominance of Genus 1 is beyond dispute, as is the relatively straightforward organization of a large part of the generic matrix, which exhibits no traditional tonal (Supragenus IV) attributes at all, in comparison, for example, with the other matrix we have seen in which Genus 1 was the prominent structure, namely, the matrix for the Messiaen composition, Table 35.

CARTER, CONCERTO FOR ORCHESTRA, *I (1969).* For this work we have the composer's own catalogue of harmonic materials (see Carter) as well as an extended discussion of them using both the jargon of pitch-class set theory as well as Carter's own systemic nomenclature (see Bernard 1983). While this by no means confirms that the issue of detailed segmentation is totally solved, it does provide the analyst with an unusual degree of security when it comes to making observations about the harmonic species of the work in question.[15]

As shown in Table 38, the main generic components of the harmonic species of the movement are Genus 4 and Genus 8. In fact, on the complete matrix, Genus 4, with one exception, is a subset of the representatives of Genus 8 – the exception being the augmented triad itself, 3-12.

The generic structure is more "unified" than might appear at first, since Supragenus III is completely represented and the uniform relation of Genus 4 to each of the genera of that supragenus is familiar to the reader from the discussions in Part VI of this study. Only Genus 1 remains something of an anomaly; both in the complete matrix and in the reduced matrix of this work its sole inhabitant is the symmetric tetrachord 4-8.

An interesting feature of the generic structure of this work is that the "tonal" and "diatonic" genera, Genus 11 and Genus 12, which comprise Supragenus IV are excluded from the reduced matrix by Rule 2, since both are included in the predominant Supragenus III. Thus, although pitch formations that might be considered part of the traditional vocabulary of diatonic-tonal music occur, they are subsumed by the essentially atonal, or possibly hybrid, Supragenus IV, which we have seen in operation so often in the preceding tables.

Another striking aspect of the generic structure of this movement, construed in the broadest, unreduced sense, is the extreme range of the Squo

Table 37
Stockhausen, *Nr. 2 Klavierstück*, II

	G1	G2	G3	G7	G8
3-2				o	
3-5	o				
3-7				o	
4-4					o
4-6	o				
4-9	o				
4-12			o		
4-13	o				
4-z15	o				
4-18	o				
4-21		o			
4-27			o		
5-4	o				
5-7	o				
5-15	o				
5-20	o				
5-25	o				
5-28	o				
5-31	o				
6-z3	o				
6-z6	o				
6-15	o				
6-16	o				
6-z29	o				
6-z47	o				
Counts:	19	1	2	2	1
	G1	G2	G3	G7	G8

Squo Indices in Descending Order with Genera

.115: G1 (atonal)
.107: G3 (diminished)
.096: G2 (whole-tone)
.085: G7 (chroma-dia)
.046: G8 (atonal)

Table 38
Carter, *Concerto for Orchestra* I

	G1	G4	G8	G9	G10
3-3			o		
3-4			o		
3-12		o			
4-4			o		
4-7			o		
4-8	o				
4-14					o
4-17				o	
4-19		o			
4-20					o
5-6			o		
5-11			o		
5-z17		o			
5-z18			o		
5-21		o			
5-22		o			
5-26		o			
5-z37		o			
5-z38			o		
Counts:	1	7	8	1	2
	G1	G4	G8	G9	G10

Squo Indices in Descending Order with Genera

.184: G4 (augmented)
.179: G8 (atonal)
.154: G10 (atonal-tonal)
.141: G9 (atonal-tonal)
.041: G1 (atonal)

indices, which is not visible, of course, on the reduced matrix in Table 38. These range from .011 for Genus 7, through 0.58 for Genus 6, up to .184 for Genus 4, a very rich harmonic panorama, indeed, and one that is brought into focus, effectively, it is hoped, by the reduction presented in Table 38.

STRAVINSKY, SYMPHONY OF PSALMS *I (1930).* It would be difficult to imagine a generic structure that contrasts to a greater extent with that of the Carter movement just examined than does the matrix shown in Table 39, which is based upon a famous composition by Stravinsky. By the Rules of Interpretation given at the beginning of this part of the study, pitch-class sets common to Genus 7 and Genus 12 are inseparable, Siamese twins, as it were, since both enjoy the same Squo value, .106. Thus, we have a striking instance of dual genus membership involving pitch-class sets 3-7, 5-25, 5-31, 6-27, and 6-32.

It will be recalled that the special Genus 7 and Genus 12 of Supragenus IV have a close relation in the abstract, as indicated by the low difference quotient of .31459 which they form (Table 23). The particular situation here suggests a somewhat different picture of the "octatonic" aspect of Stravinsky's music in the general and somewhat informal sense that the notion of operative harmonic species which has been developed in the course of this presentation may appear to be at odds with current views of the significance of the role of that feature. (See van den Toorn.)

This is not the place to argue an issue which could not even be clearly defined without a considerable digression. Suffice it to say that the relation to the genera of traditional scalar concepts, including the octatonic and diatonic, which are clearly apparent in the foreground of Stravinsky's music, has been discussed earlier, albeit briefly. Nevertheless, it is interesting to see that the Rules of Interpretation, strictly construed, place such familiar octatonic formations as 6-z13, the first hexachord of the ordered octatonic scale, in Genus 7, rather than Genus 3. Less strictly construed, dual membership for 6-z13 in both Genus 7 and Genus 3, the diminished genus and therefore intuitively the most attractive setting for 6-z13, might be allowed, for the Squo value of Genus 3 (Table 39) does not differ markedly from that of Genera 7 and 12. There is evidently room for flexible interpretation of the generic matrices.

And once again, it seems useful to remind the reader that the abstract interpretations presented in the generic matrices are subject to qualification and refinement depending upon analysis of the foreground of the music which they purport to represent. In the Stravinsky example under consideration the whole-tone tetrachord 4-21 provides a case in point, since it appears only under very special circumstances in the music.

Table 39
Stravinsky, *Symphony of Psalms* I

	G2	G3	G5	G6	G7	G10	G11	G12
3-1			o					
3-2					o			
3-7					o			o
3-11								o
4-3				o				
4-10					o			
4-11					o			
4-12		o						
4-13					o			
4-18		o						
4-20						o		
4-21	o							
4-23							o	
4-26								o
4-27								o
4-28		o						
5-10					o			
5-23					o			
5-24					o			
5-25					o			o
5-28								o
5-31					o			o
5-35								o
6-z13					o			
6-27					o			o
6-32					o			o
6-z49								o
Counts:	1	3	1	1	13	1	1	11
	G2	G3	G5	G6	G7	G10	G11	G12

Squo Indices in Descending Order with Genera

.106: G7 (chroma-dia), G12 (dia-tonal)
.103: G3 (diminished)
.076: G11 (dia)
.074: G6 (semichroma)
.063: G2 (whole-tone)
.036: G10 (atonal-tonal)
.025: G5 (chroma)

DEBUSSY, LA TERRASSE DES AUDIENCES DU CLAIR DE LUNE (1913). The beautiful surface of this extraordinary piano piece, from the *Preludes,* Volume 2, displays a large number (35) and a wide variety of pitch-class sets, yet it is remarkable that so many of them (19) belong to a single genus, the diminished genus, Genus 3, as shown by Table 40.

On the complete matrix (not shown) Genus 2 actually has the greatest number of representatives (24), but the Squo measure reduces the contributory role of that genus to a position below that of Genus 3, with which it shares many pitch-class sets (16 out of 19). As can be seen in the matrix, Table 40, Genus 2, the whole-tone genus, still plays an important role in the harmonic species, as does the dia-tonal genus, Genus 12. Members of the remaining genera, Genera 6, 7, 9, and 11, play important and special roles in the music, as will be demonstrated.

In order to give the reader a somewhat broader picture of the generic structure of the work, Table 41 shows all the genera relations, omitting only those excluded by the Rule of Intersection (Rule 2) and the Rule of Reduction (Rule 5). Here we can see that many important sets whose primary affiliation is with Genus 3, for example, also belong to other genera in the matrix. Indeed, one of the basic pentachords and four of the most prominent hexachords in the excerpt belong to Genus 3 as well as to Genera 2 and 12: 5-28, 6-27, 6-z28/49, 6-z29/50, and 6-30. However, Genus 3 is usually the primary structure represented at the surface of the music, for good contextual reasons, having to do, for example, with thematic configurations and secondary, underlying, aggregates, the octatonic collection, in particular.

Example 1a reproduces the complete notation for the excerpt to be discussed, while Example 1b, an analytical graph, provides the basis for the discussion that follows, whose purpose is to explicate the role of the genera and their constituents at various levels of structure in the music.

A few preliminary comments are in order. First, the graph in Example 1b represents only a portion of the piece; therefore, not all the sets on Tables 40 and 41 appear on the graph, but either belong to later parts of the music or have been omitted from the analysis in order to reduce the amount of detail, which is already considerable. Second, the graph includes linear analysis at, usually, two levels. To avoid unnecessary digresssions, no attempt will be made to explain the decisions involved in the construction of these linear features. I will refer to them only in connection with the pitch-class sets they form as members of specific genera. Finally, the music exhibits a great wealth of motivic detail, discussion of which will be restricted to a few instances related to the main topic.

The question of key naturally will arise in the minds of many readers, and I would like to be unequivocal about the position represented by the graphic analysis (Example 1b). In addition to the very real key signature, which of course is purely a notational device and one that, in fact, is almost

Table 40
Debussy, *La terrasse des audiences du clair de lune*

	G2	G3	G6	G7	G9	G11	G12
3-7							o
3-8	o						
3-10		o					
3-11							o
4-3			o				
4-11				o			
4-12		o					
4-13		o					
4-16	o						
4-18		o					
4-19				o			
4-21	o						
4-23						o	
4-24	o						
4-25	o						
4-27		o					
4-28		o					
4-z29	o						
5-1			o				
5-10		o					
5-z18		o					
5-24	o						
5-25		o					
5-28		o					
5-31		o					
5-32		o					
5-34		o					
6-z13		o					
6-z19		o					
6-z23		o					
6-27		o					
6-z28		o					
6-z29		o					
6-30		o					
6-32							o
6-33							o
Counts:	7	19	2	1	1	1	4
	G2	G3	G6	G7	G9	G11	G12

Squo Indices in Descending Order with Genera

.116: G3 (diminished)
.098: G2 (whole-tone)
.093: G12 (dia-tonal)
.081: G7 (chroma-dia)
.077: G9 (atonal-tonal)
.070: G6 (semichroma)
.054: G11 (dia)

Difference Quotients

	G2	G12	G7	G9	G6	G11
G3	.32592	.48604	.40259	.44696	.42934	.81409

Table 41
Debussy, *La terrasse des audiences du clair de lune*

	G2	G3	G6	G7	G9	G11	G12
3-7				o		o	o
3-8	o						
3-10		o					
3-11					o		o
4-3			o				
4-11				o			
4-12	o	o	o				
4-13		o		o			
4-16	o						
4-18		o			o		
4-19					o		
4-21	o						
4-23						o	
4-24	o						
4-25	o						
4-27	o	o					o
4-28		o					
4-z29	o						
5-1			o				
5-10	o	o	o	o			
5-z18	o	o	o		o		
5-24	o			o		o	o
5-25	o	o		o			o
5-28	o	o	o				o
5-31	o	o	o	o	o		o
5-32	o	o			o		o
5-34	o	o				o	o
6-z13	o	o	o	o	o		
6-z19	o	o				o	
6-z23	o	o	o	o			o
6-27	o	o	o	o	o		o
6-z28	o	o	o		o		o
6-z29	o	o		o	o		o
6-30	o	o	o	o	o		o
6-32				o		o	o
6-33	o			o		o	o
Counts:	25	19	12	14	12	6	16
	G2	G3	G6	G7	G9	G11	G12

Squo Indices in Descending Order with Genera

.119: G3 (diminished)
.096: G2 (whole-tone)
.091: G12 (dia-tonal)
.079: G7 (chroma-dia)
.075: G9 (atonal-tonal)
.068: G6 (semichroma)
.053: G11 (dia)

immediately contradicted by the multiple accidentals on the score, there are moments, important formal junctures, at which one can read a functional tonal harmony in the traditional sense. I have included five of the most obvious of these, indicating them by the usual Roman numerals but enclosed in brackets to register the fact that the context in which they occur does not support an unambiguous functional interpretation. Also it should be said that the operation of Genus 12 in this music represents a general tonal climate, albeit one very much in the background. The work ends with an F# major triad and a dominant-tonic bass motion in the last two bars, confirming the composer's interest in a referential tonic at certain points throughout, an interest which did not prevent him from making the most imaginative and startling excursions into atonal territories, however.

The principal generic constituents of the music are designated above the upper stave at the level of the bar numbers. Each time a change in the generic composition of the music changes, that change is reflected in the genera names. Parenthesized genera names are intended to indicate that the genera are operative, but in a secondary capacity.

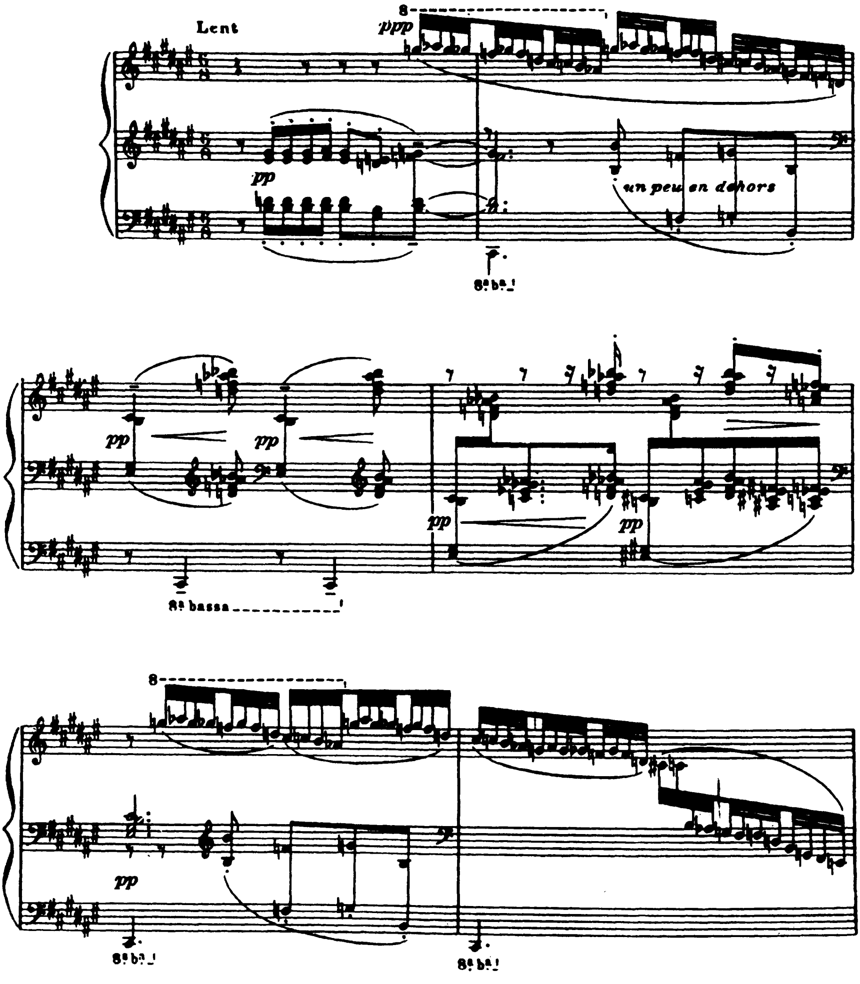
The linear structure of largest scale in the upper voice projects hexachord 6-z50, the complement of 6-z29. This extends from $g\sharp^1$ in bar 1 through bar 6, up to the beginning of the new section at bar 7 and consists of the pitch-classes, in order of entrance, 8-7-2-1-10-5. This pitch-class set is well known to afficionados of octatonicism, for it represent one of the six hexachordal classes of the octatonic collection.[16]

This projection of 6-z50 corresponds to the persistence of Genus 3 over this entire part of the music, although the genus is well represented by other pitch-class sets over other spans of structure, including vertical formations ("chords").

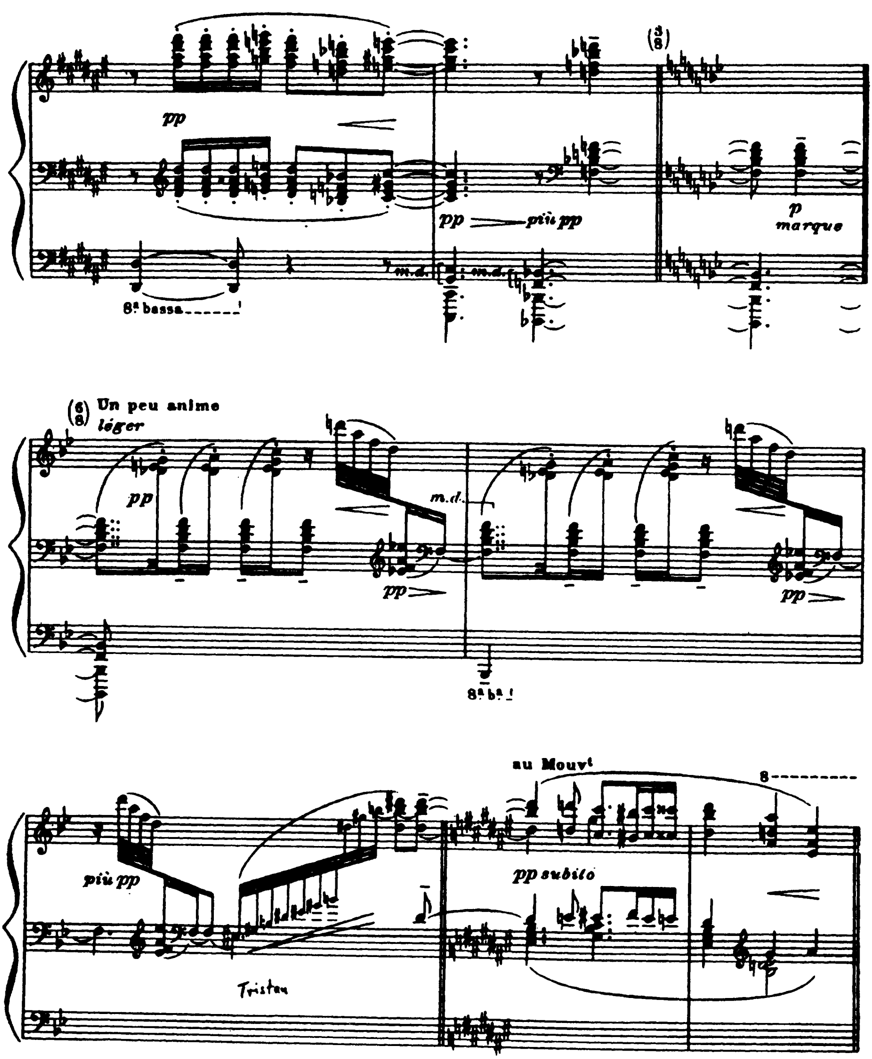
As 6-z50 slowly unfolds in the descant, other linear structures develop, the most prominent of which is the elaborate repeated line that begins on g^3 (Example 1a). Here the chromatic pitches connect the components of the more basic structure, beamed on Example 1b, which is a projection of hexachord 6-30, a respected member of the octatonic clan and an outstanding resident of Genus 3.[17]

Certain details of the very opening music demand our attention now, beginning with the melodic theme in the descant $g\sharp^1$-$a\sharp^1$-$g\sharp^1$-e^1-g^1. This thematic motive, a form of pitch-class set 4-12, belongs primarily to Genus 3, although it is affiliated with Genus 2 as well (Table 41). On the graph the motive is further analyzed in terms of two small cells, the whole step, designated alpha and the minor third, designated beta. Both these ideas are developed in later contexts, one of which will be pointed out below.

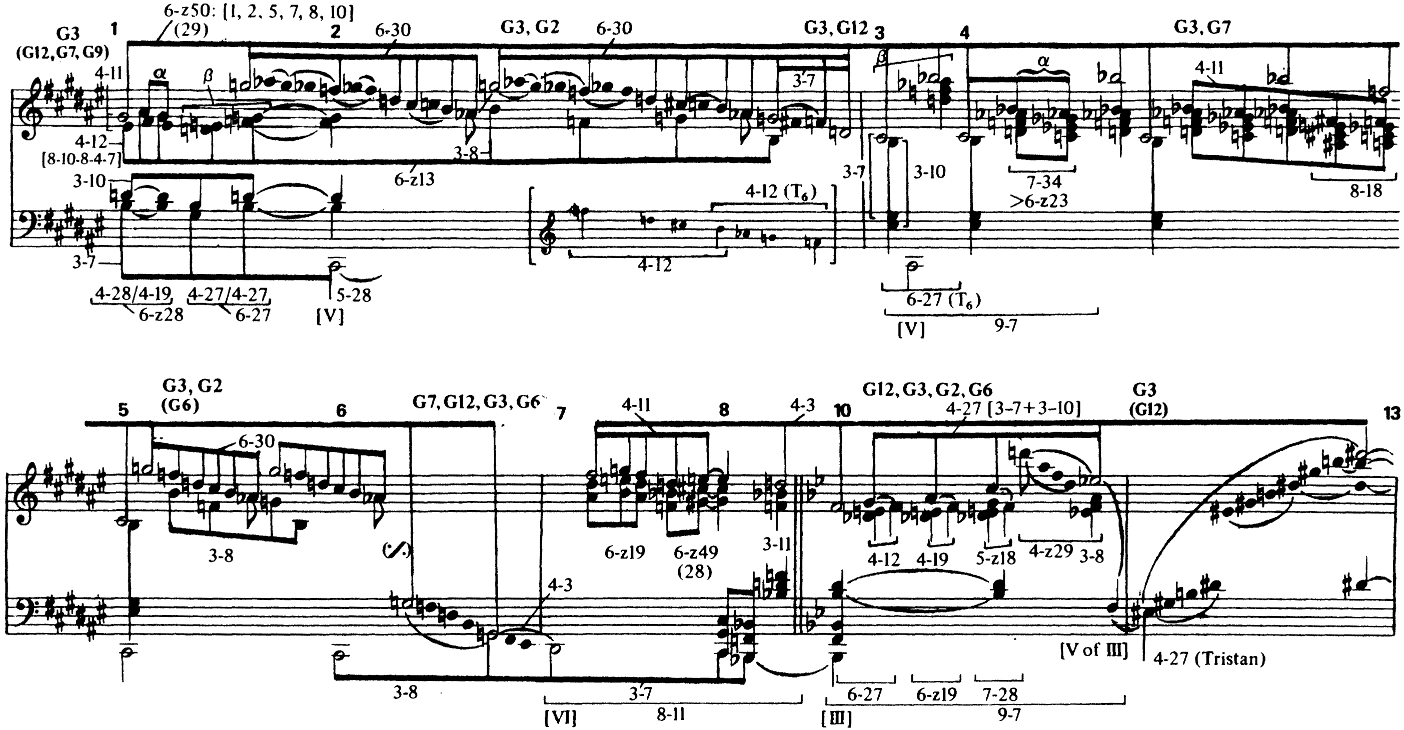
When the antecedent thematic motive, a projection of trichord 3-8, enters in bar 2 the entire thematic configuration is understood as a form of 6-z13, another octatonic hexachord. At the same time, trichord 3-8 signals the onset of Genus 2, which here combines with hexachord 6-30 of Genus



Example 1a. Debussy: *La terrasse des audiences du clair de lune*



Debussy, *La terrasse des audiences du clair de lune*



Example 1b. Debussy: *La terrasse des audiences du clair de lune,* analytical graph

3 to further enhance what will eventually be the very rich harmonic species of this composition.

To return to the opening bar, three other genera are represented in the music there: Genus 12 by the large-scale bass motion B-G♯-C♯, a form of trichord 3-7, Genus 7 by tetrachord 4-11, which is a composite of alto and soprano, bracketed on Example 1b, and Genus 9, represented by the vertical form of tetrachord 4-19, the second chord in the work.

Among the verticals of the opening music, however, the two occurrences of tetrachord 4-27, the "dominant seventh chord" are perhaps most striking, and, of course, it is these that establish the idiomatic pattern of parallel pairs of such harmonies as they occur in the rest of the work. Whenever the transposition that relates the two 4-27's is by minor third, hexachord 6-27 is formed. Transposition by whole step creates septachord 7-34, as in bar 4, and transposition by half step produces the large octad 8-18.

The elaborate descant figure commences with the arrival on the "G7" chord at the end of bar 1, and that harmony is sustained while low C♯ enters in the bass to complete the total accompanying sonority, pentachord 5-28, which might be read as a dominant function, taking into account, however, that it does not fulfill its role by proceeding to a tonic.

To complete the reading of the pitch-class sets in the opening music of this work, I point out that the substructure of the descending figure that expresses hexachord 6-30 relates directly to the tetrachordal antecedent of the theme since, as shown on the lower stave in brackets, it consists of two interlocking and ordered forms of 4-12, transpositionally related at the tritone, a situation in which the pitch classes 5 and 11 are held in common (invariant).

The section that extends over bars 3 to 5 is considerably less complicated than the opening music. The principal component here is the minor-third (motive beta) related tetrachordal pair "C♯7" and "B♭7," which forms 6-27, as remarked above. As before, these seventh chords are in the 6/5 position, which emphasizes the interlocking trichords 3-7 and 3-10, the one a progenitor of Genus 12, the other the sole progenitor Genus 3, exactly the the primary generic constituents of this section.

In bar 4 a new pairing of 4-27's appears, based upon the whole-step transposition, which produces septachord 7-34. (Recall Table 20.) Here the descant whole step b♭^{1}-a♭^{1}, a pitch-specific reference to motive alpha of bar 1, complements the preceding replication of beta's minor third, so that this section now appears to be, at least in part, a development of the initial melodic material.

Bar 3 and the first half of bar 4 comprise the large set 9-7, the complement of diatonic 3-7, and thus a large-scale representative of Genus 12.

In the second part of bar 4 the melodic dyad b♭^{1}-a♭^{1} extends to incorporate f♯^{1}-f^1, and the total melodic figure is then a form of 4-11, not just an arbitrary form of 4-11, but a pitch-specific replication of that tetrachord as it appears in descant and "alto" at the very beginning of the piece. It is this formation that brings Genus 7 strongly into the foreground, as can be seen from both Tables 40 and 41, where 4-11 is unabashedly a singleton attached to that genus only.

Bars 5 and 6 contain a modified repetition of the opening two-measure section to complete a miniature ABA form: statement-development-restatement. In terms of generic structure, however, a major reorientation occurs at the end of bar 6, where 3-8 (Genus 2) is brought into play in the lower register and shown as a beamed configuration on the graph, Example 1b. And at the very end of bar 6, connecting to the first note of bar 7, D♯, the descant pattern breaks to bring in tetrachord 4-3, the only representative of Genus 6 in the work thus far.

The new music of bars 7 and 8 involves three genera, Genus 12, Genus 7, and Genus 3. Of the representatives of these three, the most prominent foreground manifestation occurs in the descant, where 4-11 (G7) assumes the contour and rhythm of the intial thematic motive 4-12. This transformation, which represents a shift from Genus 3 to Genus 7, is truly remarkable, since 4-11 originated as a subsidiary voice-leading structure within a context dominated by Genus 3 (bar 1), then entered in the shape of a new melody in the descant of bar 4, associated with the original 4-12 motive through the dyad b♭^{1}-a♭^{1}, and now appears as a "variant" on the original motive.

The parallel and quasi-parallel motions in the upper part of bars 7 and 8 creates two hexachords, one new, one derived from an earlier hexachord. The new hexachord is 6-z19, which often represents Genus 9 in Debussy's music, but which Table 40 assigns to Genus 3, and the derived hexachord is 6-z49, one of the six "octatonic" hexachords, which is the complement of 6-z28, the sums of 4-28 and 4-19 at the beginning of the music in bar 1.

Here (bar 8), the common triad, 3-11, which is one of the progenitors of Genus 12, appears in pristine form for the first time. Indeed, it appears just as a linear form of the other progenitor of Genus 12, 3-7, completes itself in the bass: D♯-C♯-B♭.

However, the predominant genus of this section would appear to be Genus 7, for not only is 4-11, its sole representative on Table 40, in the foreground of the descant, but the complement of 4-11, the octad 8-11, is the sum of all the pitch-class representatives over this section (bars 7-8).

Finally, it is important to notice that Genus 6, hitherto represented very skimpily is now beginning to assert its influence in this section through the large-scale upper voice motion, notated in open noteheads on Example 1b, f♯^{2}-d^2. In this next section, this large-scale upper voice progression continues with f^1-e♭^{2} (d♯^{3}) to produce a form of 4-3, the tetrachord that first appeared in the miniscule "transition" at the end of bar 6.

In the last section of the excerpt, beginning with bar 10, another radical change occurs in the foreground, reflected by a change in generic content. The predominant genus over this section is Genus 12, since the total pitch-class content of the section sums to a form of 9-7, complement of the progenitor 3-7. Within this total formation are several distinct pitch-class sets, which can be read from Example 1b. The upper voice here is particularly elegant and subtle. As indicated by the beam that connects the filled-in noteheads, the descant outlines a form of 4-27, now in its "half-diminished seventh" guise as the inversion of the "dominant seventh" of the opening music. Specifically, it is the inversion of the "G7" chord at the end of bar 1, with pitch-class 7 (G) as the axis of inversion. This new form of 4-27 proves to be a preparation for the first climactic moment in this extraordinary piece, which occurs with the arpeggiation of 4-27 in bar 12 (Example 1a), a form which is pitch-class identical to the Tristan Chord and beyond doubt a conscious latter-day quotation by Debussy that tells us something about the hidden program of the prelude.

At the level of detail, it is interesting to observe that the first upward projection of 4-27 is so ordered that trichord 3-7 (g^1-a^1-c^2) and 3-10 (a^1-c^2-$e\flat^2$) interlock, just as they did in the vertical arrangement of the "dominant seventh" forms of 4-27 in the previous music. And, although not directly relevant to the generic constituents of the music here, it is of analytical significance to know that the two forms of 4-27 under discussion are transpositionally related in such a way that there is only one pitch class held in common between them, namely, pitch-class 3, here expressed by letter-names E♭ and D♯.

Thus, in the final portion of the excerpt presented in Example 1b Genus 3 resumes its ascendant position. However, Genus 12 lurks in the background because the situation is somewhat in flux at this moment, with only one tetrachord sounding, albeit a highly charged one.

VIII. Conclusion

WHAT THE GENERA MATRICES SUGGEST: FURTHER RESEARCH DIRECTIONS. After a lengthy and what some might regard as an overly detailed presentation, I will close with very brief thoughts about scholarly work that seems to me worth pursuing in connection with the theoretical material contained in the present study. Two possibilities among many follow.

Further investigations involving the pitch-class set genera might well focus upon the development of "harmonic styles" over entire repertories or over the works of individual composers, particularly of avant-garde composers of the 20th century, some of whom are represented by the tables at the end of the foregoing study. Such undertakings would be essentially "historical" in nature.

More analytical and speculative would be studies of linear structures as they relate to the genera—in particular, studies of levelled linear structures such as those shown in the graph, Example 1b, where pitch-class sets projected over various spans have attached to them prolongational configurations which may exhibit a diversity of generic composition.

Also intriguing is the role played by various mappings in inter-genera relations as they are expressed in real musical artifacts. For example, the prominent tetrachords 4-12 and 4-27 in the Debussy work discussed at the end of Part VII map onto each other under the M5 mapping mentioned earlier, thus effecting a specific connection between Genera 12 and Genera 3 and 2 (Table 41).

Finally, hierarchies and other structures within the genera would be of interest—perhaps along the lines of the Kh set complexes discussed in the present article.

Appendix: The Pitch-Class Set Genera

GENUS 1 (3-5)
Tetrachords
5 6 8 9 13 15 16 18 29
Pentads
4 5 6 7 9 10 12 13 14 15 16 18 19 20 22 24 25 28 29 30 31 32 36 38
Hexachords
2 3 4 5 6 7 9 10 11 12 13 15 16 17 18 19 21 22 23 24 25 26 27 28 29 30 31 33 34

GENUS 2 (3-8)
Tetrachords
5 12 15 16 21 24 25 27 29
Pentads
4 5 6 7 8 9 10 13 14 16 18 19 20 24 25 26 28 29 30 31 32 33 34 38
Hexachords
2 3 4 5 6 7 9 10 11 12 13 15 16 17 18 19 21 22 23 24 25 26 27 28 29 30 31 33 34 35

GENUS 3 (3-10)
Tetrachords
12 13 18 27 28
Pentads
4 8 10 12 16 18 19 22 25 26 28 29 31 34 36 38
Hexachords
2 3 5 10 11 12 13 15 17 18 19 21 23 24 25 27 28 29 30 31 34

GENUS 4 (3-12)
Tetrachords
19 24
Pentads
13 17 21 22 26 30 33 37
Hexachords
14 15 19 20 21 22 31 34 35

GENUS 5 (3-1 & 3-2)
Tetrachords
1 2
Pentads
1 2 3 4 5 8 9 11 13 36
Hexachords
1 2 3 4 5 8 9 10 11 12 14 15 16 21 22

GENUS 6 (3-2 & 3-3)
Tetrachords
2 3 12
Pentads
1 2 3 4 8 9 10 11 13 16 17 18 26 28 31 36
Hexachords
1 2 3 4 5 8 9 10 11 12 13 14 15 16 17 21 22 23 24 27 28 30 31 34

GENUS 7 (3-2 & 3-7)
Tetrachords
10 11 13
Pentads
2 3 4 9 10 12 19 23 24 25 26 27 29 31 36
Hexachords
1 2 3 5 8 9 10 11 12 13 14 15 18 21 22 23 24 25 27 29 30 31 32 33 34

GENUS 8 (3-3 & 3-4)
Tetrachords
4 7 19
Pentads
2 3 4 6 11 13 14 17 18 21 22 26 30 37 38
Hexachords
1 2 3 4 5 8 9 10 11 14 15 16 17 18 19 20 21 22 24 31 34

GENUS 9 (3-3 & 3-11)
Tetrachords
17 18 19
Pentads
11 13 16 17 18 19 21 22 26 30 31 32 36 37 38
Hexachords
5 8 10 11 13 14 15 16 17 18 19 20 21 22 24 27.28 29 30 31 34

GENUS 10 (3-4 & 3-11)
Tetrachords
14 19 20
Pentads
5 11 13 17 18 20 21 22 23 26 27 29 30 37 38
Hexachords
5 8 9 10 11 14 15 16 17 18 19 20 21 22 24 25 26 31 32 33 34

GENUS 11 (3-7 & 3-9)
Tetrachords
22 23
Pentads
11 14 23 24 27 29 30 34 35 36
Hexachords
8 9 11 12 14 16 18 22 24 25 26 31 32 33 34

GENUS 12 (3-7 & 3-11)
Tetrachords
22 26 27
Pentads
11 23 24 25 26 27 28 29 30 31 32 34 35 36 37 38
Hexachords
8 9 10 11 12 14 15 16 17 18 21 22 23 24 25 26 27 28 29 30 31 32 33 34

SUPRAGENUS I (G1 + G2 + G3)
Tetrachords
5 6 8 9 12 13 15 16 18 21 24 25 27 28 29
Pentads
4 5 6 7 8 9 10 12 13 14 15 16 18 19 20 22 24 25 26 28 29 30 31 32 33 34 36 38
Hexachords
2 3 4 5 6 7 9 10 11 12 13 15 16 17 18 19 21 22 23 24 25 26 27 28 29 30 31 33 34 35

SUPRAGENUS II (G5 + G6)
Tetrachords
1 2 3 12
Pentads
1 2 3 4 5 8 9 10 11 13 16 17 18 26 28 31 36
Hexachords
1 2 3 4 5 8 9 10 11 12 13 14 15 16 17 21 22 23 24 27 28 30 31 34

SUPRAGENUS III (G8 + G9 + G10)
Tetrachords
4 7 14 17 18 19 20
Pentads
2 3 4 5 6 11 13 14 16 17 18 19 20 21 22 23 26 27 29 30 31 32 36 37 38
Hexachords
1 2 3 4 5 8 9 10 11 13 14 15 16 17 18 19 20 21 22 24 25 26 27 28 29 30 31 32 33 34

SUPRAGENUS IV (G11 + G12)
Tetrachords
22 23 26 27
Pentads
11 14 23 24 25 26 27 28 29 30 31 32 34 35 36 37 38
Hexachords
8 9 10 11 12 14 15 16 17 18 21 22 23 24 25 26 27 28 29 30 31 32 33 34

NOTES

1. Palisca (37–50) has provided an excellent overview of the Greek system of harmony with attendant genera as they underwent various interpretations in the Italian Renaissance and later. The reader may rest assured that the present article, although it partakes of that tradition in some remote sense and must invoke some basic theoretical concepts and terms, will avoid introducing anything comparable to the dreaded proslambanomenos. For an original and highly informative historical picture of one large collection of harmonies, the diatonic, with reference to several cultures see Gauldin.
2. As bibliographical oddities in what would certainly comprise a large list if fully constituted, see the two fin de siècle studies by Goldschmidt (which involves crystallography) and by Klauser.
3. In order to avoid any commitment, explicit or implicit, to a traditional view of music, whether Riemannian, Schenkerian, or Pistonian, and thereby bias and constrict the field of inquiry, I will use the terminology of pitch-class set theory. By "intervallic content" is meant the total of intervals formed by a "pitch-class set." "Interval class" is a number from 1 through 6, representing the semitone through the tritone, respectively, while "cardinal number" is the number of elements in the set. For example, the pitches C,E,G are reducible to the pitch-class set 0,4,7, the intervallic content of which consists of one each of classes 3, 4, and 5: in traditional tonal parlance a minor third, a major third, and a perfect fifth. See Forte 1973 for more extensive explanations.
4. Here I introduce pitch-class set names. For readers unfamiliar with this nomenclature, an example will suffice. The set name 3-1 refers to the first pitch-class set on a list of sets of cardinal three, the twelve trichords. The array of numbers enclosed in square brackets on Table 1 is called the interval vector. The first entry in this array gives the number of intervals of class 1, the second entry the number of intervals of class 2, and so on. Thus, the interval vector for 3-11, one manifestation of which is the common triad of tonal music, displays the numeral 1 in positions 3, 4, and 5, reflecting the presence of the familiar minor third, major third and perfect fifth.
5. In the third column of Table 2, each non-empty entry exemplifies the so-called R1 similiarity relation. See Forte 1973, 46–60. Trichordal pairs 3-2 & 3-7 and 3-3 & 3-11 are also associated by means of the so-called M5 mapping, as is apparent from the exchange of interval classes 1 and 5. See Rahn 53–6.
6. From the standpoint of interval content there is only one 11- element pitch-class set and only one 12-element pitch-class set. In this respect the decad represents the pitch-class set of largest size that is of interest with respect to genera formation.
7. By "inclusion relation" I refer to the notion of subset and superset. See Forte 1973, 24–8.
8. Genera with single progenitor p exhaustively include the set complex Kh about p. See below and Forte 1973 96–100.
9. The matrix in Table 6 as well as matrices throughout this article are read in the following way. To determine the relation—here the inclusion relation—between an item listed in the leftmost column and an item listed in the topmost row, find the entry at the intersection of the appropriate column and row. For example, hexachord 6-35 does not include tetrachord 4-19 because there is an empty entry at the intersection of column 4-19 and row 6-35. Hexachord 6-35, however, does contain 4-24 and 5-33. Hexachords not of the "z" type are self-complementary. That is, 6-1 is the comple-

ment of 6-1. All z-hexachords form complementary pairs. In the genera tables in this article z-hexachords are combined on a single line with a virgule separating the ordinal numbers, as 6-z19/44 in Table 6. In the row designated by any such pair one of three different entries may appear: "o," "a," or "b." The letter o means that the inclusion holds for both hexachords of the z-pair; the letter a means that the inclusion holds for the first hexachord listed (the one with the lower number), while b means that the inclusion holds for the second hexachord listed (the one with the higher number).

10. See Maegaard 1976 for a valuable discussion of the pitch-class content of all of Schoenberg's 12-tone rows. With regard to Webern's rows, Maegaard makes the following observation: "To the overall characteristics of the 31 rows belongs first and foremost the tendency toward an independent twelve-tone condition, one freed from any tonal centricity. Among other things, a certain chromatic density serves this end, evident primarily in the configuration of unordered tone-groups, but also in short row segments." (my translation)
11. I cite Berg's Wozzeck often because the harmonic components of the work have been identified by several writers of different persuasions, including Jarman and Schmalfeldt, thus relieving me of the responsibility of "segmenting" the work and, in the process, inadvertently making what might be regarded as convenient selections, thus distracting from the topic at hand.
12. For 7-34 as the "Podhalean mode," a Polish folk-music scale used by Karol Szymanowski, see Kosakowski. Russom discusses 7-34 in the early music of Ravel, while Antokoletz gives detailed consideration to an ordering of 7-34 (not part of his nomenclature) which he calls a "non-diatonic mode found in Hungarian peasant music" and which is endemic to Bartok's music. (Antokoletz 205) This mode is a circular permutation of the melodic minor scale, for example: C-D-E♭-F-G-A-B becomes Antokoletz's "non-diatonic mode" when reordered circularly as A-B-C-D-E♭-F-G. Pitch-class set 7-34 is represented several times among the "Heptatonic Arpeggios" in Slonimsky. See, for example No. 1088, p.155, which is identified with Busoni's Fantasia Contrappuntistica. For a global discussion of "mode" see Powers.
13. I have included in the matrix one pitch-class set which is idiomatic to modern jazz, 6-z25, described by Larson in one of his examples as an "eleventh" chord on C. Larson 382, Ex. 5.16.
14. Lists of all Kh for tetrachords (octads), pentachords (septachords), and hexachords are given Forte 1973, Appendix 3, 200–208.
15. The harmonic content of each of the four movements of the Concerto for Orchestra was outlined by the composer on a sketch page which has been published in facsimile. See Carter. Each movement has a different generic structure, in the language of the present study, because of the varied and carefully controlled intervallic arrangement established by the composer in each case.
16. See van den Toorn for details of octatonic scalar organization. The unordered octatonic set, 8-28, is very prominent in this work altogether. Indeed, the primary pitches of bars 1-2 comprise a complete statement of one of the three forms of the octatonic set, the one whose ordered form begins on pitch-class 1: 1-2-4-5-7-8-10-11, which van den Toorn calls Collection I.
17. It is interesting to recall that Stravinsky had used this sonority in a famous work somewhat earlier: the so-called Petroushka chord, formed in that work in many ways, the most familiar of which is the juxtaposition of C major and F♯ major triads. The cor-

responding conflation in this work involves the G major and C♯ major triads, the elements of which can be read as alternating in the descending configuration: G-F-D-C♯-B-A♭. I regard this correspondence as purely coincidental, however, since the detail of the line is composed in such as way that it expresses two forms of the initial thematic motive, as explained below and shown on Example 1b on the lower stave beneath the first repetition of 6-30 in the descant.

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