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PERCEIVED DISTANCE AND THE CLASSIFICATION OF DISTORTED PATTERNS [1]

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This work is a continuation of efforts to develop a psychophysics of form similarity appropriate to the study of concept learning. 5 configurations of dots (Triangle, Diamond, M, F, and Random) were studied. The psychophysical functions relating perceived distance from the original to each level of distortion were linear. The level of distortion was calculated from the statistical rule generating the distortions and expressed in terms of uncertainty. It also reflected the mean distance that each dot actually gravitated over random samples of the rule. The perceived distance between any pair of distortions increased with the level of the more distorted from the original. The level of distortion of a sample of patterns was related to the rate at which Ss learned a common response to that sample. Rate of learning in classifying these patterns, like multivariate concept learning, is a function of the uncertainty within a category.

Multivariate concept learning (Garner, 1962) has lent itself to an informational analysis (Hunt, 1962). Several types of experimental situations have shown linear relations between the amount of information which must be contained within a category and performance (Posner, 1964a). This holds true, at least as a first approximation, both for tasks which vary information by adding irrelevant dimensions (Archer, Bourne, & Brown, 1958) and for those which vary the relevant dimensions to which *S* must attend (Shepard, Hovland, & Jenkins, 1961).

The problem of the classification of familiar perceptual patterns by a common name, such as a set of geometrical forms being labeled "triangle" also represents a type of concept task (Kendler, 1961), since a number of related stimuli are given a single response. This particular aspect of concept learning has often been called pattern recognition (Hunt, 1962).

The difficulty of learning to classify separate instances clearly depends upon their similarity. White (1962) studied this question by distorting dot patterns resembling letters and digits by means of successive random walks of varying sizes. He found that in a recognition task the number of errors was linearly related to the logarithm of the number of random walks, with the absolute level increasing with step size. In this case already well-learned concepts were used by *S*s to recognize the distorted patterns.

Posner (1962, 1964b) distorted nonsense dot patterns by means of three different types of statistical rules. The level of distortion was calculated from the number and probability of the cells to which a given dot could move and was expressed in terms of uncertainty. All the rules correlated the number of cells to which a dot could move with

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distance. Thus the higher the uncertainty of the distortion, the greater on the average was the distance between a given dot in the original pattern and in the distortion. Linear relations between level of distortion and subjective distance were obtained for all three types of rules. This result indicates that the subjective distance between an original pattern and its distortions is a logarithmic function of the average distance which a dot moves, and agrees with the results which White (1962) obtained for recognition errors. Moreover, since three rather different type of statistical rules provided the same result, it apparently matters relatively little how the distance is distributed across the dots. That is, perceived distance is about the same whether the rule requires every dot to move a little or a few dots to move a great deal, provided only that the average amount of movement is constant.

While the average distance describes the amount of distortion for a given pattern, the amount of uncertainty summarizes the probability values which constitute the rule generating a given level of distortion. The use of the uncertainty measure provides the opportunity to explore the applicability of formulations derived from multivariate concept studies, where the degree of uncertainty within a category is related to the number of levels and dimensions and not to the degree of spatial distortion. The goal is to extend the general framework of concept experiments into the area of simple perceptual patterns where the dimensionality is not easily specified.

This paper reports three experiments. The first seeks to obtain psychophysical relations between five original configurations of dots and their levels of distortion. In Exp. II the question of perceived distance between the levels of distortion is considered. In Exp. III the rate of learning to classify these patterns as a function of the level of distortion is explored.

Experiment I

The purpose of this experiment was to obtain psychophysical relations between level of distortion and perceived distance for meaningful patterns. Magnitude estimates were obtained for five configurations of dots. For a triangular pattern ratings were made at nine levels of distortion and using two different procedures. For the other three meaningful patterns (Diamond, M, F) and the random pattern, ratings were made at five levels of distortion.

Method

Subjects.—The *S*s were 72 undergraduate students at the University of Wisconsin who worked for course points. Each *S* was run individually. The first 24 *S*s were run in the simultaneous condition using the triangles. The next 18 *S*s rated triangles in the sequential condition. The final 30 *S*s were used to rate all five patterns.

Material.—All figures were constructed using standard 20 squares per inch graph paper. The matrix sufficient for any distortion consisted of a 50 × 50 cell area. The original figures were constructed by placing filled dots in the central 30 × 30 cell area. There were five originals. The original triangle is shown in the upper left-hand corner of Fig. 1 and the other originals are shown in Fig. 2.[3] Each original was systematically distorted by a statistical rule. For the triangles, distortions of 1, 2, 3, 4. 5, 6, 7.7, 8.6, and 9.7 bits/dot were used. For the other figures only distortions of 1, 3, 5, 7.7, and 9.7 were used.

The level of distortion in bits/dot was defined in the following way. A template of 400 cells was constructed with the center cell called zero, the surrounding eight cells numbered 1–8, the next ring of cells numbered 9–24 and the remaining cells numbered in a spiral fashion out to a total of 399.

[3] The relative size of the dots in Fig. 1 has been slightly enlarged to increase the clarity for reproduction.

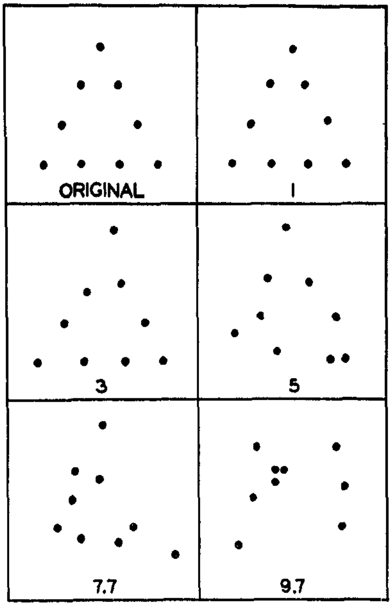

Fig. 1. Original and five levels of distortion for set of triangles.

Five areas were designated consisting of the central position (zero), Positions 1–8, Positions 9–24, Positions 25–99, and Positions 100–399. For each level of distortion a probability was assigned to any given dot moving to any of the five areas. Within an area all of the cells were equally likely. The probabilities were assigned in such a way that the total uncertainty of each dot conformed to the nine levels discussed above. For example, the 1 bit/dot distortion[4] had probabilities of .88 of staying in place (Area 1), .10 of moving to Area 2, .015 for Area 3, .004 for Area 4, and .001 for Area 5.

The probability values for all levels of distortion are shown in Table 1. The values are selected to obtain the overall levels of distortion discussed above. In addition, the probability of moving to a cell declines as the distance of the cell from the dot's original position increases. The only exception to this is in the 6- and 7.7-bit levels for which the probability of not moving at all is zero. This property guarantees that the average distance moved per dot increases with level. The actual distance moved of each dot for every level of distortion was calculated for all sets and the means are shown as the last column of Table 1. The values are in units of 1/20 of an inch which represents the grain used for these patterns. The logarithm of the average distance between dots is a linear function of the level of distortion. The 1-bit level point lies off this linear trend, having slightly too little distortion. The 9.7-bit distortion was unrelated to the original, since any dot had an equal probability of being in any of the 900 cells of the original matrix.

Eighteen sets of triangles, each containing a random sample of the rule for all levels of distortion, were made. For the other patterns 6 sets were made each consisting of five levels of distortion. Once the basic pattern was produced, the dots were transferred to ditto and reproduced.[5]

Procedure.—The two conditions involving triangles alone were run first. In the simultaneous exposure condition the 10 patterns of each set were presented in random order on facing pages of a notebook with the original in the upper left-hand corner of each double page. The *S* was given a score sheet which had lines for rating 12 sets of patterns. For each set the 4-bit distortion

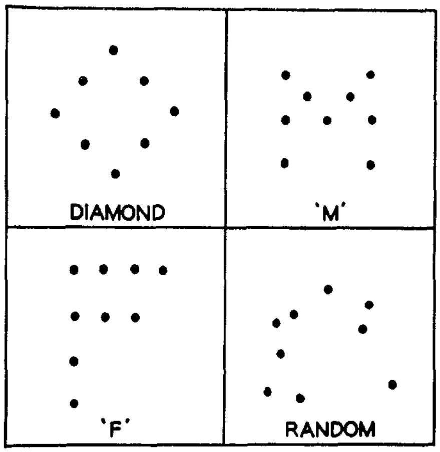

Fig. 2. Original patterns of Diamond, M, F, and Random.

[4] The rules governing construction of the patterns do not allow any two dots to occupy the same position. The uncertainty calculations do not include this restriction, but the correction would be exceedingly small.

[5] One of the 7.7-bit distortions was constructed with eight rather than nine dots. The data obtained from the ratings of that pattern do not seem to differ in any important way and no correction was made for this.

TABLE 1

PROBABILITIES OF MOVING TO EACH AREA FOR ALL LEVELS OF DISTORTION

Level of Distortion (Bits/Dot)	Area					Average (Distance/Dot)
	1	2	3	4	5	
1	.88	.1	.015	.004	.001	.23
2	.75	.15	.05	.03	.02	.60
3	.59	.20	.16	.03	.02	.66
4	.36	.48	.06	.05	.05	1.27
5	.2	.3	.4	.05	.05	1.91
6	0	.4	.32	.15	.13	2.89
7.7	0	.24	.16	.3	.3	4.56
8.6	Equally Probable Adjacent 400 Cells					7.02
9.7	Equally Probable any of 900 Cells					10.61

served as the modulus and had the number 30 already filled in. The *S*s were instructed to give each of the patterns a number which represented its distance from the original in similarity of shape or form. If a pattern appeared twice as similar to the original as the pattern already designated as 30, *S* was told to call it 15; if twice as distant it was to be called 60. The *S*s were told that any numbers at all could be used, but that they should use the 30 relation as a guide. Each *S* worked individually through 12 sets. The 18 sets were arbitrarily broken into three groups of 6 sets and each of the *S*s was assigned two of these groups such that there were 16 ratings of every set.

In the sequential procedure each of the 180 patterns was mounted on 5 × 7 in. cards and all except the 18 originals were placed in a notebook. Three of the 4-bit distortions were designated as moduli and given a value of 30. One-third of the *S*s rated the patterns using each modulus. During the ratings *S*s were allowed to keep a card with the original and their modulus in front of them. They were instructed to rate each card in the same way as described above, going on immediately to the next card and not returning to any prior ratings. Prior to running each *S*, all the cards were reshuffled so that each *S* had a new random order.

Thirty new *S*s were used to rate all five figures. The *S*s received instructions similar to those given in the previous experiment. Each *S* rated all 150 individual patterns divided into five groups of 30 according to the form of the original. Each group contained 6 sets with five levels of distortion per set. The order of the patterns was governed by three random 5 × 5 Latin squares with two *S*s assigned randomly to each row. Each Latin square was assigned one of three distortions of 4 bits/dot as a modulus (30). For each type of pattern the order of the stimuli was rerandomized for each *S*. The name of each pattern (triangle, etc.) was printed on the original which *S* could see during the rating.

Results

The basic results of this experiment for triangles are shown in Fig. 3. In this graph the median perceived distance from the original is plotted as a function of the level of distortion for the two different psychophysical techniques. The standard errors of the median for the simultaneous method are

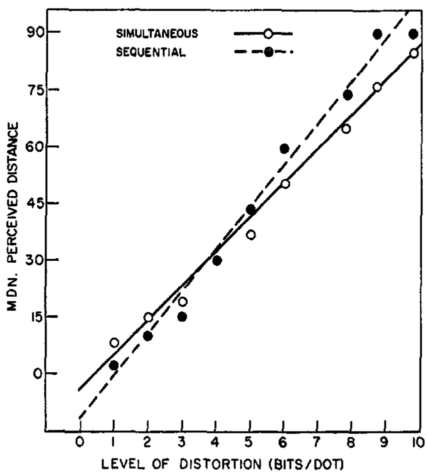

FIG. 3. Median magnitude estimates as a function of degree of distortion.

1.7, 3.2, 1.8, 1.8, 3.9, 7.7, 10.2, and 15.8 for the various levels. The standard errors for the sequential technique are slightly larger. The functions are quite linear, with the equations of the least-square fits being 11.2×-12.4 for the sequential method and 9.1×-3.9 for the simultaneous method.

If the logarithm of the values listed in Table 1 for average distance moved are substituted for uncertainty as the independent variable, the resulting functions are linear. Thus perceived distance can be viewed as a logarithmic function of mean objective distance in accord with suggestions for a metathetic continuum (Stevens, 1957). The use of distance as the independent variable does not eliminate the tendency shown in Fig. 3 for the lowest levels of distortion to lie above the overall trend.

The shape of curves for individual *S*s is much like the group curve, though the range of numerical values differs among *S*s. Thus the 9.7 bit/dot level was rated as high as 375 and as low as 60.

The results for all five figures are shown in Table 2. The median perceived distance for each pattern was computed for each level of distortion over all *S*s. The three moduli gave rather different values for individual levels particularly for the 1-bit distortion. Table 2 shows medians over all moduli. Combining the three moduli seems to be in order since they represent three independent samples of the 4-bit level. It might be argued that the three moduli were not a sufficient sample. The similarity of these functions to those found using the simultaneous method of Exp. I argues against this and also gives substantial replication to the results obtained with the triangles. The slope for the triangles is slightly below that shown in Fig. 3. The functions obtained do not seem especially sensitive to characteristics which may differ between forms, such as number of sides or meaningfulness. The random form does show a smaller slope than the others. This could be due to unique aspects of this particular random form. However, when the results for this form are compared with a large sample of random forms used in a previous study (Posner, 1964b), the slopes are quite similar. The slope for the present form is 8.5 as compared to an average value of 8.8 in the previous study. The closeness of these values, despite the coarser grain and different procedure used in the previous study, indicates the relative insensitivity of the slopes to these aspects of the stimulus situation.

Experiment II

The scales discussed so far refer only to the perceived distance of each pattern from the original. Two different views are possible with respect to the perceived distance between two levels of distortion.

TABLE 2

Median Magnitude Estimates and Slopes for Figures of Exp. II

Figure	Distortion (Bits/Dot)					Equation y =
	1	3	5	7.7	9.7	
Triangle	4	19	34.2	74.5	90	10.4×-10.4
Diamond	8.3	28	40	74.5	90	9.5×-2.2
M	7	25	43.7	81.3	87.5	9.9×-3.3
F	.7	22.5	35.7	71.3	82.5	9.7×-8.4
Random	9.7	24.7	35.5	66.8	81.7	8.5×-1.1

If *S*s are asked to make independent judgments of the distances of any two distortions from the original, the difference between the two judgments is clearly a function of the difference between the two levels of distortion. Therefore, if *S* is required to judge directly the distance between two distortions of a triangle, and if he judges along a single dimension reflecting their "triangularity," his perceived distance ought to be a function of the difference in their levels of distortion.

On the other hand, the average distance between corresponding dots in two distortions is a function mainly of the level of the more distorted and does not get smaller as the less distorted departs from the original. For example, a given dot of Level 7.7 is, on the average, no closer to the corresponding dot at Level 5 than it is to the corresponding dot in the original. Thus, if *S* judges within a multidimensional space, the perceived distance between distortions will not depend upon the difference between their levels of distortion. It will instead depend upon the average distance between dots which increases with the level of the more distorted pattern.

In order to determine which view is upheld it is necessary to have direct estimates of the perceived distance between distortions. This experiment uses the method of quadrads to obtain such estimates.

Method

Subjects.—The *S*s were 10 students of the summer session recruited to run in experiments for 4 wk. at $1.00 per hr. This experiment was run during the first week for half the *S*s and during the second week for the other half. Neither half had been exposed to the patterns prior to this week. Of the 10 *S*s only 9 are included in this report since 1 was unable to complete the full week for reasons unrelated to the study.

Material.—The materials consisted of all the original patterns and distortions of 1, 3, 5, 7.7, and 9.7 bits used in the previous experiment.

Procedure.—Each *S* was run individually seated at a desk viewing a display board. In the center of this display board was a cutout containing room for four separate patterns. The upper two patterns were the standard pair and consisted of an original and 4-bit distortion, while the lower two were changed from trial to trial and served as the comparison pair.

Each *S* was run for 5 days, 1 day on each of the figures. The order was controlled by a random Latin square with two *S*s assigned to each row. On a given day each *S* rated all six sets of a given pattern. All six levels of distortion were rated twice, once in each position, against each of the other levels for a total of 30 comparisons for each set. For a given set of 30 comparisons the two top (standard) figures remained constant. They consisted of the original and the 4 bit/dot distortion from each set.

Each *S* was presented with the following instructions:

> This is an experiment on judgment of form similarity. You will see two patterns of dots in the upper half of the opening. These are the standard pair. The standard pair is given as being 20 units apart in similarity of shape or form. You will see a pair of patterns in the bottom half of the opening which will be constantly changing. I want you to give this comparison pair a number which represents how far apart the two patterns are in similarity of shape or form. If, for example, they are the same distance apart as the standard pair, they should be called 20. If they are farther apart than the standard they should have a number higher than twenty . . . twice as far, 40, three times, 60, etc. If they are more similar than the standard patterns they should have a number less than 20 . . . twice as similar, 10, three times, about 7, etc. You may use any number that you feel is appropriate to express the similarity of the comparison patterns, remember to use the standard pattern's distance of 20 as a guide.

Results

For each *S* the median number assigned to each pair of comparison patterns regardless of position and set was computed. There were 12 such comparisons for a given figure, two for each

TABLE 3

Median Magnitude Estimates of All Pairs of Distortions

		Figure					
D_1	D_2	Diamond	Random	F	M	Triangle	Mean
0	1	15	9	2.5	12.5	7.5	9.3
0	3	15	16.5	16	17.5	12.5	15.5
1	3	13	17.5	15	15	13.5	14.8
0	5	20	20	24	27.5	22	22.7
1	5	20.5	20	25	25	22	22.5
3	5	22.5	21	25	30	22	24.1
0	7.7	35	35	45	40	35	38
1	7.7	39	35	47.5	39.5	35	39.2
3	7.7	40	40	47	40	35	40.4
5	7.7	40	40	52.5	41.5	37.5	42.3
0	9.7	49	40	45	30	40	40.8
1	9.7	45	40	50	46.5	38	44
3	9.7	49.5	40	59.5	42	40	46.2
5	9.7	48	45	53	47	39.5	46.5
7.7	9.7	60	55	60	43.5	40	51.6

set. The median of these values over the nine *S*s was then obtained. These values are shown in Table 3 for each of the patterns and for each pair of levels.

The first thing to note is the relation between each original pattern and its distortions. For each figure the perceived distance from the original increases regularly with the level of distortion. Levels 1 and 3 for the diamond and 7.7 and 9.7 for the M and F show reversals. The final column of the table shows the perceived distance averaged over all figures. The equation relating these values to the level of distortion is $3.7 \times +5.7$. The slope of this function is less than half as great as those shown in Table 2. This reduction in slope is probably due to setting the modulus at "20" rather than at "30" as was done in the previous experiments. The increased variability and reversals in the data are due in part to the fewer judgments collected and probably also to the greater overall difficulty in making judgments between levels of distortions instead of only from the original as was done in Exp. I.

Table 4 shows the average physical distance between a given dot at each level of distortion and the corresponding dot of other patterns at the same level and of every other level. The 9.7 level is not included since those patterns were statistically unrelated to all others and, therefore, the notion of a corresponding dot is rather arbitrary. In this table it is clear that the physical distance between corresponding dots at a given level increases regularly with the level of distortion from the original. In comparing any two different levels the distance depends almost completely

TABLE 4

Mean Distance[a] between Corresponding Dots at Each Pair of Levels

Level of Distortion Pattern D_2	Level of Distortion Pattern D_1				
	0	1	3	5	7.7
0		.23	.66	1.91	4.56
1		.12	.61	1.53	4.07
3			.97	1.68	4.17
5				2.5	4.6
7.7					6.3

[a] Units of 1/20 in.

upon the level of the more distorted pattern. For example, the dots of Level 7.7 are, on the average, 4.56, 4.07, 4.17, and 4.6 units from Levels 0, 1, 3, and 5, respectively.

Table 3 shows that the perceived distances between distortions are closely related to the average physical distances discussed above. The perceived distance, like the physical distance, increases with the level of the more distorted pattern. There is a small tendency for this perceived distance to increase with the level of the less distorted, but this differs between patterns.

These results indicate that the perceived distance between two patterns does not depend upon the difference between the two levels of distortion of the statistical rules which generated them. For this reason the function plotted in Fig. 1 can not be thought of as a single dimension along which the judgments are made. It represents only a set of distances from the original to each level of distortion and the difference between any two points on the line does not predict their perceived distance when compared with each other. However, the perceived distance between a sample of patterns at any level and a sample at the same or any other level may be predicted simply from the level of the more distorted pattern, which also represents the average physical distance between corresponding dots.

Experiment III

Most studies of concept identification have used multivariate stimuli which are formed of dimensions specified by *E* and readily identified (Hunt, 1962). Frequently such tasks involve the learning of responses to a number of levels of relevant dimensions which are embedded in noise through the addition of irrelevant dimensions. The addition of irrelevant dimensions increases the overall information within a category. The patterns used in our studies do not have readily discriminated dimensions and noise is added through distortion. This experiment is designed to determine whether learning to classify these patterns will be systematically related to the amount of noise, as it is for multivariate concept learning.

Method

Subjects.—The *S*s were 54 students obtained in the same way as in Exp. I. The data from 6 *S*s was discarded due to equipment or *E* error. A replication study involved 36 new *S*s.

Materials.—Dot patterns from the triangle, M, F, and Random sets discussed previously were photographed and mounted on 2 × 2 slides. Six sets of 1-, 5-, 7.7-, and 9.7-bit distortions were used for each pattern. These were randomly divided in half, forming two different lists of slides at each level. Each list consisted of three triangles, Ms, Fs, and Randoms at a given level. Two replications of a list in different orders constituted the materials for one *S*.

After completion of the study it was found that the triangles at Level 5 were actually taken from Level 6. This error was corrected in the replication.

Procedure.—The 48 *S*s were randomly assigned to one of the four levels and to one of the two arbitrary lists of slides within each level with the restriction that each condition have 6 *S*s.

Each *S* was run individually. He was seated at a table with panel of four microswitches and four colored lights in front of him. At a distance of approximately 18 in. and at head height was a ground-glass screen onto which the slides were projected. The four buttons were assigned randomly to the four stimulus classes and the assignment was constant over *S*s and conditions.

During the experiment *S* was first presented with a slide which remained on until he pushed one of the switches. The switch pulsed a light on *E*'s panel, presented *S* with a feedback light, turned off a clock which began at the time the slide was presented, and activated a timer which pulsed the Airequipt projector after a delay of 2.4 sec. The exposure time of the stimulus was under the control of *S*, the feedback light which was programmed by *E* remained on for 2.4 sec. during which the slide remained present.

TABLE 5
Mean Errors to Criterion as a Function of the Level of Distortion

Level (Bits/Dot)	Original Study	Replication
1	6.8	4.6
5	26.2[a]	12.2
7.7	77.8	71.1
9.7	133.8	—

[a] Triangles are at Level 6 (see text).

Each S was presented with two random orders of the 12-slide list which were repeated until he made either two consecutive perfect trials in which each pattern was indentified with the correct button, or completed 20 trials. Intertrial intervals were 15–20 sec. Total errors to criterion and time per trial were recorded. After the session S was questioned concerning the verbal labels which he used.

The replication experiment was identical except that only Levels 1, 5, and 7.7 were used. Twelve Ss were assigned to each level.

Results

The exposure time for each stimulus was left under the control of S since the perceptual complexity of the various patterns were likely to be rather different and the use of fixed presentation rates might have hampered identification of the various slides. The average presentation time per stimulus was 2.96, 3.15, 4.0, and 3.7 sec. for the four levels.

The primary dependent variable of interest was average number of errors *prior to termination.* These values are shown in Table 5 for each of the four levels. Preliminary analysis had shown no significant differences between the two lists. Thus, it is reasonable to combine them as is done in Table 5. The combined data were submitted to trend analysis using coefficients proper for the particular spacing of the stimuli along the distortion dimension (Grandage, 1958). The overall effect of levels is highly significant, with significant linear and quadratic components. The relation between level of distortion and errors is positively accelerated.

After completion of the analysis it was found that the triangles used for Level 5 were actually from Level 6. The replication data also shown in Table 5 corrected this error. In general, the replication Ss show better performance than those of the original study. This is particularly true at Level 5 where the error apparently increased the difficulty. The replication data gave an even more marked positive acceleration than the original data. This seems to indicate that noise below about 5 bits/dot distortion has relatively little effect upon learning. This finding is in marked contrast with the relatively uniform effect of distortion upon judged similarity.

The errors to criterion for the four types of patterns were highly similar. These values averaged over all levels of distortion were 63.1, 62, 54.5, and 62.7 for the triangles, random, M, and F. It should be borne in mind that only one assignment of the patterns to keys was used and this might have biased the values for individual patterns somewhat, if Ss had systematic preferences in guessing. However, it is clear that there is not much difference between patterns and in particular the random pattern is not more difficult than the others.

The increase in errors with addition of noise is in qualitative accord with data from experiments in which noise is increased by adding irrelevant dimensions. However, the positively accelerated shape of this curve differs from the usual finding of linearity (Bourne & Restle, 1959). This experiment probably underestimated the amount of the positive acceleration both because time per stimulus increases with level and because the number of Ss reaching criterion decreases with level. At the 9.7-bit level no Ss reach criterion within 20 trials. In fact, so difficult is the task of classification at this level that many *S*s reported that they paid no attention to the patterns and attempted to learn the sequence of 24 responses over the two independent orders. The positive acceleration is much more like the results obtained when relevant information is added in multivariate concept situations (Walker & Bourne, 1961). This result is interesting since in this experiment the noise which is added cannot be ignored in the way in which it can when its addition is through independent dimensions.

The reports of *S*s with respect to the verbal labels given to the patterns illustrate that the low levels of distortion were effective in eliciting common verbal labels. At Level 1, all 12 *S*s used the verbal label M, 10 identified F, 9 the triangle, and 7 expressed appropriate labels for the random pattern (e.g., glob, amoeba, random, etc.). This declined uniformly so that at the 7.7 level only 2 *S*s identified triangle and none the other patterns. However, it is clear from their reports that at the 7.7 level *S*s were still sensitive to certain properties of the pattern such as the relative density of dots at various locations which are preserved by the distortion rules. This is reflected also both in the psychophysical and in the learning data.

Conclusions

Price (1953) has summarized two related but distinct philosophical positions about the basis of pattern recognition. In the Theory of Universals, each instance of a triangle is conceived of as having a characteristic called "triangularity." Triangularity is a quality the dimensions of which must be separately defined. The Philosophy of Resemblances holds, however, that a number of individual objects form a nucleus of key exemplars of triangularity. New input is compared against such standards and if it falls within defined limits is accepted as a new instance of a triangle.

While these views are not mutually exclusive, the distinction between them is reflected in two rather different empirical methods of investigating similarity and its role in the learning of classifications. The first view requires a definition of form which consists of something like a set of psychologically relevant dimensions along which the pattern varies and which can form the basis for judgments. Hake (1957) summarizes work on this approach as follows: "It seems clear that even in judgments of relatively simple shapes or forms judgments are complex in that they are made along multiple dimensions which tend to combine in unique ways from judgment to judgment [p. 62]." Efforts have been made both to work out such dimensions (Attneave, 1957; Hake, 1957) and to discover the rules for their combination (Attneave, 1950; Shepard, 1964). The second view of pattern recognition is represented in contemporary psychology by so-called template matching (Hunt, 1962). Here the problem becomes one of measuring the distance from some specified ideal or template to various input stimuli. This may allow one to proceed without a specification of dimensions because even if the input forms vary within a multidimensional space the distance from the template to each form may produce a simple scale of perceived distance.

The results of these studies give some encouragement to the development of a view of concept learning based upon the distance between exemplars and not depending upon the specification of dimensions. Experiments I and II show that it is possible to predict the perceived distance between these patterns. This prediction depends upon the average distance moved by a dot in undergoing various degrees of distortion and can be summarized by the level of uncertainty of the statistical rule which generates the distortion. In Exp. I it was shown that the level of uncertainty or the logarithm of the average distance moved per dot was linearly related to perceived distance.

In Exp. II the perceived distance between any pair of distortions was shown to be a function of the objective average distance between corresponding dots in the two levels. This objective distance depends primarily upon the level of uncertainty of the more distorted from the original.

The use of uncertainty as a measure of distortion emphasizes the relation between these studies and those of multivariate concept learning in which dimensions are well defined. In Exp. III noise was added by the distortion of the originals and the rate of concept learning studied. The results indicate that the level of distortion is systematically related to the rate of learning. This is similar to results obtained when noise is added in multivariate concept learning by increasing irrelevant dimensions.

This result may seem puzzling at first. Since *S*s are not required to learn the name of the originals and since one of the originals is itself a nonsense pattern, why should the distance of the patterns from the original be related to the rate of learning? The answer lies in the increase in the distance between patterns within a given level as the level of distortion increases (See Table 4). Since uncertainty, as used here, is a logarithmic function of objective distance, the uncertainty within a category increases with the level of distortion. The average distance between corresponding dots in patterns at any level increases regularly with the level and is about the same as the distance between patterns at that level and the original (see Table 4). Thus the uncertainty within a level is the same as the level of distortion itself. Just as in the multivariate case, the rate of learning appears to be an increasing function of the uncertainty within a category. In the case of multivariate concept learning uncertainty is related to the number and levels of discrete dimensions, in this study uncertainty is related to the average distance between corresponding dots, in both cases uncertainty within a category is a predictor of the rate of learning the classification.

REFERENCES

Archer, E. J., Bourne, L. E., & Brown, F. G. Concept formation as a function of irrelevant information and instruction. *J. exp. Psychol.*, 1958, **56**, 262–270.

Attneave, F. Dimensions of similarity. *Amer. J. Psychol.*, 1950, **63**, 516–556.

Attneave, F. Physical determinants of the judged complexity of shapes. *J. exp. Psychol.*, 1957, **53**, 221–227.

Bourne, L. E., Jr., & Restle, F. Mathematical theory of concept identification. *Psychol. Rev.*, 1959, **66**, 278–293.

Garner, W. R. *Uncertainty and structure as psychological concepts.* New York: Wiley, 1962.

Grandage, A. Orthogonal co-efficients for unequal intervals. *Biometrica,* 1958, **14**, 387–389.

Hake, H. W. Contributions of psychology to the study of pattern vision. *USAF WADC tech. Rep.*, 1957, No. 57–621.

Hunt, E. B. *Concept Learning.* New York: Wiley, 1962.

Kendler, T. Concept formation. *Ann. Rev. Psychol.*, 1961, **12**, 447–472.

Posner, M. I. An Informational Approach to Thinking. *USAF Off. Scient. Res. tech. Rep.*, 1962, No. 2635.

Posner, M. I. Information reduction in the analysis of sequential tasks. *Psychol. Rev.*, 1964, **71**, 491–504. (a)

Posner, M. I. Uncertainty as a predictor of similarity in the study of generalization. *J. exp. Psychol.*, 1964, **68**, 113–118. (b)

Price, H. H. *Thinking and experience.* Cambridge: Harvard Univer. Press, 1953.

Shepard, R. N., Hovland, C. J., & Jenkins, H. M. Learning and memorization of classifications. *Psychol. Monogr.*, 1961, **75**(13, Whole No. 517).

Shepard, R. N. Attention and the metric structure of the stimulus space. *J. math. Psychol.*, 1964, **1**, 54–87.

Stevens, S. S. On the psychophysical law. *Psychol. Rev.*, 1957, **67**, 153–181.

Walker, C. M., & Bourne, L. E., Jr. The identification of concepts as a function of amounts of relevant and irrelevant information. *Amer. J. Psychol.*, 1961, **74**, 410–415.

White, B. W. Recognition of familiar characters under an unfamiliar transformation. *Percept. mot. Skills,* 1962, **15**, 107–116.

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