Tonal Implications of the Diatonic Set*

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This paper is an attempt to arrive at an explanation, of sorts, for some observed aspects of musical behavior. Its conclusions are arrived at by an informal kind of logic: seeing that certain "facts" seem to exist (in, say, the intervallic arrangement of the diatonic set), one asks, "Why?"--"What might they account for?" This paper appears in the company of two others, in which some evidence may be found in favor of its conclusions. In the recent literature on psychological experimenting with musical behavior, Hantz finds some likelihood that the concept of "pattern matching" may be useful and Brown and Butler's work seems to support the idea that "position finding" is a function of intervallic rarity. Our common goal is to arrive at a notion of what people are doing when they act tonally.

Tonal music and tonality are very large notions. In my attempt to make some precise observations about the diatonic set, I do not mean to imply that statements about the set control, or are identical with, statements about tonal music. The latter is an extraordinarily complex human activity. I will list just a few of the facets of tonal music which seem to need explanation or discussion:

(1) Its complexity under investigation vs. the apparent ease of our real-time processing of it.

(2) Its extraordinary amount of repetition (which somehow seems not to become boring) and the concomitant sense of predicting a great many things very rapidly, some to be checked immediately, some never.

(3) The fact that we learn it early in life without being taught and that we can apply whatever it is we have learned to first hearings of new pieces with success.

(4) The fact that duality (the sense that relationships are often, perhaps always, full of yes and no) seems perfectly natural, even necessary. This is coupled with a sense that some things are more important than others, that tonal music directs and re-directs our attention and our strategies, that we see the existence of many levels constantly being invented, discarded, corroborated, and improved; and thus the realization that music seems like life, or better, like living.

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*These ideas have been developed in classroom teaching at Yale University and The University of Michigan and in conversation with many colleagues. I wish to note in particular the influence of David Kraehenbuehl, Lloyd Rodgers, John Clough, Edwin Hantz, and William Benjamin.

These observations (and many more) remind me of attempts to characterize human psychological activity more broadly. In particular, I am reminded of some of the notions subsumed under the word *Gestalt*: above all, the idea that processes interior to an apperception, a judgement, are not necessarily observed during the *Gestalt*-making process, nor are they necessarily those which *are* observed, then or later. To seek to understand a mental activity by using the very brain which so effortlessly performs the activity may be impossibly circular. But attempts at observation, characterization, and logical explanation (for which read "It probably has to be this way or that") are all we have, and they seem valuable in themselves.

If we consider the enormous total activity we call tonality, it seems to be comprised of tens, more probably scores, of rather complex operations. It is conceivable that the intersection of tonal music with music in general, not to say with human mentality, will almost always look like a mosaic and only occasionally like a system. In this paper I have tried to focus on one area of tonal behavior, describable at first in two metaphorical, experience-derived ways.

First, it seems to me that in tonal music one is constantly doing something which I call *position finding*. The interior monologue, at blinding speed, asks questions such as these:

(1) "What's going on?"
(2) "Where are we now?"
(3) "Why did that come out right (or wrong)?"
(4) "How should I note what just happened in terms of *X*--and how long should I wait to see if following up that train of thought will be useful or enjoyable?"

The touchstone, the phenomenological observation, is that one seems to be able to *care*, just as in life, about the past and the future. One has the sense that position finding is something one is fairly good at and thus is, almost by necessity, inescapable. The thinking going on seems very rapid, but not as rapid as it is at some further, unobservable level. It thus attracts our attention--one may be glimpsing, in tonal music, one's own brain at work.[1]

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1. For another, complimentary view of music and the process we call "thinking" see Lewis Thomas, "On Thinking About Thinking" in *The Medusa and the Snail* (New York: Bantam Books, 1979), pp. 125-128.

An inverse aspect of the operation of position finding, of deciding upon the role or function of events vis-à-vis other, prior or only envisaged, events, is my second term: *pattern matching*. The monologue says things like:

(1) "Is this the same as that?"
(2) "Could this become more or less like that?"

Upon reflection, it becomes evident that the game of pattern matching remains interesting because, and only so long as, the underlying patterns *do not* match. Too much. Only if the patterns are *inherently incommensurable*, so that the patterns can never fully match, can one have an infinity of attempted solutions, enabling one to enter into the construction of abstractions, the recognition and classification of transformations, the effort of extraploation, and thus receive the benefit of a partly manageable experience which confronts the existence of ambiguity and calls into playful question the reality of memory--even the reality of thought.

An examination of the patterns and identifiable positions of objects in the diatonic set follows. However, several technical limitations and disclaimers should be entered at this point.

(1) I treat the diatonic set as a set of seven pitch-classes, thus 21 interval-classes, drawn from a 12-pitch-class equal-tempered universe, using notions set forth by Milton Babbitt and developed by (among others) Allen Forte for the study of atonal and serial music. Forte's pitch-class terminology is used.[2]

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2. See Allen Forte, *The Structure of Atonal Music* (New Haven: Yale University Press, 1973), William E. Benjamin, "*The Structure of Atonal Music* by Allen Forte," *Perspectives of New Music* 13/1 (Fall-Winter 1974): 170-190, and Richmond Browne "Review," *Journal of Music Theory* 18/2 (Fall 1974): 390-409. In the latter review, I raised some of the issues central to this paper: the notion of pitch- and interval-class as being clearly applicable to tonal as well as atonal music, the idea of tonal contextuality, and the enumerating of a group of musical "equivalence strategies."

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(2) Many interesting aspects of the set cannot be treated in this paper: its size, evolution, and relation to other sets (particularly the harmonic minor) receive little mention here.

(3) Much of this paper is drawn from classroom presentations where more ample demonstrations can be given. The elementary nature of some statements is, I think, deceptive; the most difficult problem, often, is to state clearly what may be going on behind the seemingly most basic "givens" of a powerful system. Even in a class, however, the goal remains the same: to reach an adequate description of the process of differentiation and association in tonal music.

The notions of *interval content* and *intervallic context* will serve as vehicles for my examination of the position finding and pattern matching operations in tonal usages of the diatonic set. Content and context, like position finding and pattern matching, are in some senses dialectically related ways of looking at the same data. *Content* evokes the things contained in a set. *Context* refers to the surroundings of those things. Sets (and subsets) "have" content; pitches (and subsets) "have" context, and derive their musical functions from their relations with their context as well as from their intrinsic content.

The Interval Content of the Diatonic Set

Perhaps the most striking property of the diatonic set is seen in its interval-class content. Each of the six possible interval-classes occurs a different number of times (called the unique multiplicity property).[3]

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3. There is only one other seven-note set in the 12-pitch-class equal-tempered universe with this property: the "chromatic cluster." Carlton Gamer calls sets like these "deep scales" and a number of not-so-obvious tonal behaviors are based upon this curious intervallic distribution. Two articles by Gamer will provide the reader with a technical and historical background for this paper: "Deep Scales and Difference Sets in Equal-Tempered Systems," *Proceedings of the American Society of University Composers* 2 (April 1967): 113-122; and "Some Combinational Resources of Equal-Tempered Systems," *Journal of Music Theory* 11/1 (Spring 1967): 32-59.

Using the usual tonal designations for intervals rather than interval-class language (when it does not matter), one may recall that there are two "minor seconds" and five "major seconds" in the set (the definition of "second" being "two tones with no intervening tone in the scalar arrangement of the set"). Every tone has a "second" of some sort adjacent to it; it follows that there are thus seven "seconds" in a seven pitch set. Similarly, there are seven "thirds" in the set, four "minor" and three "major" (a "third" being defined as "two tones with *one* intervening tone in the scalar arrangement, etc."); six "fourths" and one "augmented fourth" (the tritone being a "fourth" if it contains *two* intervening tones and a "fifth" if there are *three*). Forte calls this listing of the interval content of a set its "vector" and writes it <254361>.

A strange and wondrous thing is <254361>, when stared at long enough! For starters, every interval occurs at least once. This means that there is no interval which does *not* occur in the set; no inherently non-diatonic interval; no marker, at this level, for "leaving the set." As one expands this examination to look at the distribution of larger subsets (asking "How many three-note sets are there, and of what types?"), the principle of unique multiplicity bends but does not break. There are some three-note sets (and many larger subsets) which are not contained in the diatonic collection. A second consequence of the set's containing every interval at least once involves the intersecting of the set with its transpositions. There is no transposition of the set which contains all new tones; some degree of pitch intersection under transposition always obtains.

A more striking observation, however, is that the diatonic set contains a full range of intervallic ubiquity. The six interval-classes occur from one to six times, and each of them a unique number of times. This constitutes a full spread of possibilities from "rarity" to "common-ness"--a maximum possible hierarchization. This fact has important implications because the rare and the common intervals have vastly different musical usages. Rare intervals aid position finding; common intervals maximize successful pattern matching. When one hears a tritone, or a minor second, one's tonal "knowledge" offers a greater sense of the possible "places one may be in" than when one hears a relatively common interval (like a fourth or a major second) which could hold any one of a number of "places" in the diatonic field. (The "field" actually consists, behaviorally, of these correspondences between event and interpretation-of-event.) Other sets which display the the unique multiplicity property fail to be "tonal" (e.g., the chromatic cluster) because they lack other tonal necessities which lie outside the scope of this paper (but presumably not outside the scope of eventual description. Remember the "tens" and "scores" of sub-matrices in my opening remarks.). It therefore comes as no surprise that Brown and Butler (pp. 39-55) found trichords containing a tritone most effective in locating a correct tonic. A couple of reasons might be put forth:

(1) There are only five diatonic trichords which contain a tritone--because there are only five tones left to form a trichord with. Each of the non-tritone pitches holds a unique position with respect to the pitches of the tritone. Therefore each unique manner of "filling" a tritone can be heard as functionally different; the "other" five scale degrees are differentiated securely and with least effort. The tones of the tritone itself, ambiguous as to which is $\hat{4}$ and which is $\hat{7}$, are differentiated by the addition of only one tone; all five possible positions are used, and each only once. The five diatonic positions within a tritone:

(a) half-step above the lower tone	C in B-F	tonic
(b) whole-step above the lower tone	G in F-B	dominant
(c) in the middle of the tritone	D in B-F	supertonic
(d) whole-step below the upper tone	A in F-B	submediant
(e) half-step below the upper tone	E in B-F	mediant

Note that the five positions produce the other five scale degrees in circle-of-fifth order--another reflection of my observation (below) that scale degrees a fifth apart are most similar with respect to intervallic *context*.

(2) The 30 remaining trichords (which do not contain the rare tritone) all occur at least twice in the set. There are six different types (by interval content); two of them are symmetrical (have no inversion) and four have inversionally equivalent forms. Thus there are ten types of non-tritone trichords which occur in 2, 3, 4, or 5 transposed forms. Thus, there are from two to five possible interpretations of these trichords--which accounts for the spread of Brown and Butler's responses when dealing with non-tritone trichords. (Incidentally, Balzano, as cited by Brown and Butler, fails to recognize the importance of the fact that the B-F and F-B tritones have differing internal constituents. That is precisely what differentiates them: their (full) patterns do not match--and that makes position finding possible. It seems to be a general rule of tonality that identical objects can always be disambiguated on some other level.)

Rare intervals have great importance in the definition of position and the restriction of pattern matching. A *new* rare interval (i.e., one not contained in the diatonic collection of the moment) has great valence as a finder of (new) position, as the imparter of (new) function. The most important product of a "new" pitch is the rare (rarest) interval(s) it creates.

Rare intervals aid position finding. Common intervals do not. When applied to the minor system (the harmonic minor set), by the way, the apparatus produces many congruent observations and a few quite discrepant ones. Without going further than a single long sentence will allow, it may be said that, in the harmonic minor set (vector: <335442>), the rare diatonic intervals are less rare, the common less common; the principle of unique multiplicity of interval class is lost; the reliability of both position finding and pattern matching is somewhat decreased (though there are compensatory mechanisms); and ambiguity is thereby, I would say, increased. A fifth, or fourth, by itself, cannot define a position in a tonal field because it can be interpreted in too many ways. (That an isloated fifth "feels like" a tonic/dominant fifth is an example of the human *Gestalt* propensity for removing ambiguity by supplying, if necessary, contextual data in order to interpret an "empty" object according to some useful or familiar mode.) The common intervals, however, provide the obverse of position finding: *pattern matching*. There are many words in our musical vocabulary which refer to pattern matching and most of them describe *not quite literal* matches; that is, they describe the ability to "do the *same* thing" without actually "doing the same *thing*." Pattern matching may be called sequence or imitation, either literal or non-literal (and the distinction here is absolutely crucial).

The critical point is that because the intervals occur with differential ubiquity, their function in literal imitation (the most basic kind of pattern extension) is fully hierarchized and, somehow, every "tonal person" has this system thoroughly learned at an early age. To extrapolate with security, to move by a fifth from one note to another, and then by fifth again, and yet again, is possible six times before a note appears which is not a member of the diatonic set. Using the next most common interval, the major second, one can "generate" three strings of "double whole-steps" (e.g., in C major, CDE, FGA, and GAB) and one longer string (FGAB) before the tonal listener, with intimate knowledge of the content and layout of the diatonic field, will realize that now a non-literal continuation (N.B., a "rare" minor second instead of a common major second) is required *if* the field is to be preserved. This subtle requirement for the listener to know the crucial points where a non-literal continuation is needed and to know the means by which it may be effected constitutes a powerful act of involvement on the part of the listener. To succeed, the listener must be aware, however subliminally, of the following, seemingly paradoxical, axiom system:

Literal imitation → → → → → pitches outside the field
(non-change) (change)

produces
(usually, sooner or later)

Non-literal imitation → → → → → pitches still within the field
(change) (non-change)

But the "opposite" of the above would be trivial indeed; change would signal change and literal continuation would beget continuity (as is the case, of course, in simple systems). In the tonal usage of the diatonic set, literal imitation (the "easiest" *Gestalt* continuation) produces profound re-orientation; specific forms of non-literal imitation are necessary and sufficient to subvert loss of position. The "usually, sooner or later" phrases in the axiom are important; no tonal "law" fails to contain the possibility of its own negative, its useful and necessary removal.[4]

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4. To the observation that literal imitation of, say, the whole-step, will eventually carry one outside the diatonic set, a codicil can be added. Observe, in the opening *Grave* of Beethoven's Sonata, Op. 13 (*Pathétique*), measures 5-9, that in the upper voice a progression from Eb to F to G is unfolded (in mm. 5-6) as a wholly legitimate succession in the then prevailing key of Eb major. But through the persistence of whole-step progression to A-natural, and thence to B-natural (in m. 7), the key of Eb is swept away. The tonal listener, casting about for an interpretation of this wrenching passage, has a resource at hand: the non-diatonic but thoroughly tonal system of the melodic minor, in this case, that of the *home* key. Looking backward from the B-natural, a series of five tones embodying four consecutive whole-steps in the same direction can only be read, tonally, as $\hat{3}-\hat{4}-\hat{5}-\natural\hat{6}-\natural\hat{7}$, and thus a tonicization of C minor--a return from Eb major--has been effected. Breaking out of one system to enter (re-enter) another by using the most obvious form of raw continuance, literal imitation, is an extension of a negative (breaking) action to the point where it becomes a positive (linking) device.

One can carry the above a bit further, noting that only the next-most-common interval, the minor third, can be imitated only once (e.g., as in BDF), and that, in so doing, one creates the rarest trichord: the position-finding "diminished triad." The principle is that rarity controls position finding by controlling pattern matching.

The fact that the interval content of the diatonic set exhibits the unique multiplicity property has at least one other strong implication. Tonality possesses an extended reference system whereby a given diatonic set can be related to, even in a sense "control," other versions of itself. The tonal practice of hierarchizing the various transpositions of a (diatonic/major) set in a conceptual arrangement is often referred to as "nearly relatedness."

"Nearly relatedness" is an excellent topic in itself, involving many kinds of partial equivalence. Which pitches of the referential key can be tonicized (i.e., given the context of a tonic)? How and why are the major and minor tonicizations interleaved? The aspect of nearly relatedness which impinges upon the unique multiplicity property of the diatonic set is the distribution of common tones under transposition. A now well-known "law" states that transposition of any set to a given interval-class "distance" produces as many pitch-class intersections (i.e., "common tones") as there are occurences of the interval of transposition in the set's vector (double for tritones). So, because of its distribution of intervals, the diatonic set provides the fullest possible range of rare and non-rare pitch intersections with which to hierarchize the various transpositions of the set, which thereby hierarchizes nearly relatedness. (For example, the C major set, containing two minor seconds, produces two common tones when transposed by a minor second to either B or Db.) If, as seems to be the case, maximizing common tones is one credential of nearly relatedness, then a further refinement exists in the diatonic set by virtue of its content, namely that the various transpositions are hierarchically related to the referential set by their various common-tone distributions.

In discussing set *content*, I have dealt mostly with the two-note intervals, but the distributional principles of rarity and ubiquity also apply to the larger subsets. As I take up the subject of *context* (the surroundings of the various individual pitches), the heuristic is extended to deal with the contexts of pitch events. The third portion of this paper deals with the distribution of *all* the various subsets of the diatonic set, and with the contexts of each (i.e., the intervals contained in the "voice-leading" between each set and its complement).

The Intervallic Contexts of Pitches in the Diatonic Set

I now turn to the second of my working concepts: *intervallic context*. It is clear that to call a single pitch a "tonic" is to award it some quality which is not inherent in single frequencies. (But remember my earlier remark about what people do to impart functions to isolated objects.) It is thus careless, though a widespread practice, to say things like "leading-tones *have* tendencies." But I think it is a permissible extension of the sense of the word "have" to say that a pitch, or a triad, when thought of as mentally placed in specified intervallic relation to certain other pitches, may be said to "have" the *context* of, say, a tonic. This voice-leading relation to surrounding pitches can be fully described, though I know of no previous description which is complete. The usual handling of the notion "diatonic," as we all know from the literature and from our students, invokes the diatonic *scale* (an incomplete picture of a set) and describes the intervallic arrangement (and thus the context of each scale degree) only in terms of scale steps. But one can and should describe the complete context for each diatonic degree. For that purpose, I ask the reader to look at Example 1a (without, for the moment, looking at Ex. 1b, which is the solution to the problem posed by Ex. 1a).

The grid of Example 1a presents vertical columns labelled, at the top, for the seven scale degrees of the major key usage of the diatonic set. It will become clear in a moment why they are arranged in "circle-of-fifths" order. The twelve horizontal rows are labelled, at the left, with the various intervallic contextual possibilities which a given scale degree may or may not possess. The reason for the ordering of those rows will also become clear, I think, in a moment. The "+" and "-" signs are used in the sense of "above" or "below." Thus "+m2" means "this scale degree 'has' a m2 above it" (which must be distinguished from the opposite

Example 1a: The Interval Context of Each Diatonic Scale Degree

	$\hat{4}$	$\hat{1}$	$\hat{5}$	$\hat{2}$	$\hat{6}$	$\hat{3}$	$\hat{7}$
tt							
-m2							
+M3							
-m3							
+M2							
-P4							
+P4							
-M2							
+m3							
-M3							
+m2							
tt							

Example 1b: The Interval Context of Each Diatonic Scale Degree

	$\hat{4}$	$\hat{1}$	$\hat{5}$	$\hat{2}$	$\hat{6}$	$\hat{3}$	$\hat{7}$
tt	B						
-m2	E	B					
+M3	A	E	B				
-m3	D	A	E	B			
+M2	G	D	A	E	B		
-P4	C	G	D	A	E	B	
+P4		F	C	G	D	A	E
-M2			F	C	G	D	A
+m3				F	C	G	D
-M3					F	C	G
+m2						F	C
tt							F

case). The notation "-P4" means "this 'has' a P4 *below* it," etc. In a full chromatic setting, the eleven other pitches would all be there to provide each pitch with all possible contexts. But, as one fills the boxes of the diatonic grid, the six "other pitches" will provide six "yes" and five "no" answers. The "no" answers are just as important as the "yes" answers, as things which are not, and cannot, be part of a given degree's context. The "tt" (tritone) is given two rows for reasons of visual clarity.

One begins to fill out the grid at the upper left corner by asking "Does the subdominant, $\hat{4}$, 'have' a tritone?" (i.e., "In the C major system, is there a pitch which forms a tritone with F?"). "Yes, there is." So we put the name of that pitch, B, in the box at the intersection of column "$\hat{4}$" and row "tt." We proceed down the column. "Does F in C major have a minor second below it?" We put "E" in the box below "B" (-m2). If the answer is "No," we leave the box empty. The chart may be made somewhat more "general" by labelling the rows (at the left) with Forte's interval-class symbols, and filling the boxes with scale degree numbers rather than pitches in a single key.

The grid is completed in Example 1b. (I have provided Ex. 1a as a working diagram only.) Each of the seven columns now contains six filled boxes and five empty ones (disregarding the double listing of the tritone). For each scale degree, certain intervals are present as context--and necessary, though not all are equally necessary. And for each degree, certain intervals are *not* present--and shouldn't be, though not all missing intervals are equally dangerous to the scale degree function of a given pitch.

A number of observations can now be made. The arrangement of the scale degrees in "circle of fifths" order does have a systematic aspect. The arrangement of intervallic possibilities along the left side, the labelling of the horizontal rows, is also systematic. The following statements can now be elicited from a class:

(1) Each scale degree has a unique intervallic context. The intervals one hears, the "things one can do," when approaching or leaving each degree, are to some extent unique to each.
(2) Scale degrees a fifth apart (e.g., $\hat{4}$ and $\hat{1}$, or $\hat{1}$ and $\hat{5}$) have the most similar intervallic contexts:
   (a) The surroundings of, say, $\hat{4}$ and $\hat{1}$ are as close to being identical as is possible without *being* identical;
   (b) their differentiation rests upon a singularity involving the set's rarest interval: the presence or absence of a tritone; and
   (c) thus a $\hat{4}$ can be turned into a $\hat{1}$, or vice versa, by the most economical means: the addition or removal of a single pitch which either provides or removes the crucial rare interval. Possessing or losing common intervals matters much less; the rare ones make the difference.
(3) Scale degrees may share, or not share, their contexts to a fully hierarchized extent. Note, for example, that $\hat{4}$ and $\hat{7}$, at the extreme, share no aspect of musical context (save, of course, the ever-present loophole which I mentioned above when discussing

the way in which tonal "laws" seem always to contain the possibility of being not quite total. $\hat{4}$ and $\hat{7}$ do "share" the tritone, but in a way which requires levels of discussion beyond the scope of this paper). Scale degrees $\hat{4}$ and $\hat{5}$ share some pieces of context; $\hat{4}$ and $\hat{2}$ fewer and less rare ones; etc.

(4) The distinction between any two degrees always rests on the presence or absence of the rarest interval(s) either possesses--and that quantity is a different one for each pair of closely-similar scale degrees. Thus, between $\hat{4}$ and $\hat{1}$ a tritone is at stake. But between $\hat{1}$ and $\hat{5}$, neither having a tritone, the minor second below makes the difference.

So we may say that the rare intervals control position finding. The degrees are constantly matched in terms of their *presumed contextual patterns*.

Other utilizations of the grid must be passed over quickly in a paper of this size. If one relabels the columns, the grid can depict the various diatonic modes: $\hat{2}$ is the Dorian final, $\hat{3}$ the Phrygian, etc. One can depict the effect of adding a Bb or an F# to the C major set by writing the new pitch in the appropriate boxes. The new pitch then creates or destroys some critical rare interval. For example, Bb creates a new tritone for E, making it a $\hat{7}$, and costs F its tritone; it creates a new minor second for A and removes C's "leading-tone." That Bb creates a new Perfect fourth for F is much less important than that it removes F's tritone. (Such an elegant game deserves a full description.) Finally, the inversional property is also displayed in the grid: the set inverts literally around $\hat{2}$.

The foregoing discussion of the context differences which "individuate" the various diatonic scale degrees can be summarized in this way. Differentiation and similarity both rest on the sharing or non-sharing of rare intervals; similarity/differentiation are both fully hierarchized in terms of (a) the amount of sameness/difference in contexts and (b) the contextual agent of difference/sameness (always a rare interval, but always a different one, above and below); and, most importantly (and as a basis for the last section of this paper), any subgroup of the set is characterizable by, is differentiated by, derives its uniqueness and thus its function from, not just its intrinsic content, but from *its contextual relation with its complement*.

Diatonic Subsets as Differentiated by their Contexts and Complements

The final portion of this paper addresses three propositions:

(1) The meaning of notions like unique multiplicity, rarity and ubiquity, context, and complementary differentiation can be extended to subgroups of all sizes.

(2) In particular, it is the case that *repeated* subgroups (e.g., the three major triads of the diatonic set) *always* possess different, and thus individuating, complements and thereby, by subtraction, differing intervals in their respective voice-leading contexts. We have already seen that $\hat{4}$ has a context which differentiates it from $\hat{1}$. It is also true that the complement

of $\hat{4}$ (the six-note set CDEGAB) differs from the complement of any other pitch/degree by virtue of interval content (or, if not on that basis, by virtue of inversion of the duplicated complement). Similarly, the context and complement of, say, a tonic triad will differ from those of the other two major triads.

(3) By adding the interval content of a subset to that of its complement, we arrive at a sum which may be subtracted from the total content of the set, leaving what I will call a *subtraction vector*, i.e., the intervals in the total voice-leading between a set and its complement, the "content of a set's context," if you will.

Example 2 (p. 16) shows the distribution of the one-note set: seven of them, all of one type (the only type there is) and of the seven six-note sets: seven of them, of four different types (by content). Example 2C lists the pairs of one- and six-note sets. The principle established here is that the seven pitches are differentiated by having different six-note complements. If any complement appears twice, it appears in both of its inversions (shown in Ex. 2, 3, and 4 by vertical square brackets).

Example 3 (p. 17) displays the distribution of the 21 dyads (the six types occurring with unique multiplicity) and the distribution of the 21 complementary pentachords (9 types, with a fair range of multiplicity). The 9 types are drawn from the 38 possible Forte pentachords (including both forms of the Z-sets). Again, note that repeated subgroups, e.g., the three major thirds CE, FA, and GB, are differentiated by their respective complements. Note also that some pentachords appear as complements to (i.e., have as their complements) differing dyads. The tables also work backwards.

Example 4 (pp. 18-19) presents the distribution vectors of the trichords and tetrachords. The 35 trichords occur with a range of multiplicity of from 1 to 8 times, providing 9 of the 12 possible chromatic trichords. The 35 tetrachords are of 13 types (out of the 29 chromatic possibilities) and range in multiplicity from 1 to 6 times. Again, repeated instances of a given set are differentiated by their respective complements. The three major triads, for instance, have different "seventh chords" surrounding them. Some tetrachords act as complements to strikingly different trichords. A full examination of the 35 trichords is extremely useful in explicating their roles in tonal music, which rests heavily on the intricate interplay of these small subgroups.

Example 2:

A: Single Pitch Distribution

Forte	1		
	7	=	7 of one type

B: Hexachord Distribution

Forte	6-Z25	6-Z26	6 - 32	6 - 33		
	2	1	2	2	=	7 of four types

C: Seven Complementary 1/6 pairs*

	Forte			Forte
1.	1-1	A	BCDEFG	6-Z25
2.		B	CDEFGA	6 - 32
3.		C	DEFGAB	6 - 33
4.		D	EFGABC	6-Z26
5.		E	DCBAGF	6 - 33
6.		F	EDCBAG	6 - 32
7.		G	FEDCBA	6-Z25

*stated in the key of C and arranged to show inversional symmetry

Example 3:

A: Dyad Distribution

Forte	2-1	2-2	2-3	2-4	2-5	2-6	
	2	5	4	3	6	1	= 21 of six types

B: Pentachord Distribution

Forte

5-Z12	5 - 20	5 - 23	5 - 24	5 - 25	5 - 27	5 - 29	5 - 34	5 - 35	
1	2	4	2	2	4	2	1	3	= 21 of nine types

C: 21 Complementary 2/5 Pairs*

	Forte			Forte
1.	2-1	BC	DEFGA	5 - 23
2.		FE	DCBAG	5 - 23
3.	2-2	AB	CDEFG	5 - 23
4.		CD	EFGAB	5 - 24
5.		AG	BCDEF	5-Z12
6.		ED	CBAGF	5 - 24
7.		GF	EDCBA	5 - 23
8.	2-3	AC	BDEFG	5 - 25
9.		BD	CEFGA	5 - 27
10.		FD	ECBAG	5 - 27
11.		GE	FDCBA	5 - 25
12.	2-4	FA	GBCDE	5 - 27
13.		CE	ABDFG	5 - 34
14.		BG	AFEDC	5 - 27
15.	2-5	GC	ABDEF	5 - 29
16.		AD	BCEFG	5 - 20
17.		BE	CDFGA	5 - 35
18.		FC	EDBAG	5 - 35
19.		GD	FECAB	5 - 20
20.		AE	GFDCB	5 - 29
21.	2-6	BF	ACDEG	5 - 35

*stated in the key of C and arranged to show inversional symmetry

Example 4:

A: Trichord Distribution

Forte

3-1	3-2	3-3	3-4	3-5	3-6	3-7	3-8	3-9	3-10	3-11	3-12
0	4	0	4	2	3	8	2	5	1	6	0

= 35 of nine types

B: Tetrachord Distribution

Forte

4-8	4-10	4-11	4-13	4-14	4-16	4-20	4-21	4-22	4-23	4-26	4-27	4-Z29
1	2	4	2	4	2	2	1	6	4	3	2	2

= 35 of 13 types

C: 35 Complementary 3/4 Pairs*

	Forte			Forte
1.	3-2	ABC	DEFG	4 - 10
2.		BCD	EFGA	4 - 11
3.		FED	CBAG	4 - 11
4.		GFE	DCAB	4 - 10
5.	3-4	GBC	DEFA	4 - 14
6.		BCE	DFGA	4 - 22
7.		FEC	DBAG	4 - 22
8.		AFE	DCBG	4 - 14
9.	3-5	BCF	DEGA	4 - 23
10.		FEB	DCAG	4 - 23
11.	3-6	GAB	CDEF	4 - 11
12.		CDE	ABFG	4 - 21
13.		AGF	EDCB	4 - 11
14.	3-7	GAC	BDEF	4 - 13
15.		ABD	CEFG	4 - 14
16.		ACD	BEFG	4-Z29
17.		CDF	EGAB	4 - 22

Example 4: (cont)

18.	(3-7)	EDB	CAGF	4 - 22
19.		GED	FCBA	4-Z29
20.		GFD	ECBA	4 - 14
21.		AGE	FDCB	4 - 13
22.	3-8	ABF	CDEG	4 - 22
23.		GFB	EDCA	4 - 22
24.	3-9	ABE	CDFG	4 - 23
25.		ADE	BCFG	4 - 16
26.		ADG	BCEF	4 - 8
27.		GDC	FEBA	4 - 16
28.		GFC	EDBA	4 - 23
29.	3-10	BDF	ACEG	4 - 26
30.	3-11	FAC	GBDE	4 - 26
31.		GBD	ACEF	4 - 20
32.		ACE	BDFG	4 - 27
33.		GEC	FDBA	4 - 27
34.		AFD	GECB	4 - 20
35.		BGE	AFDC	4 - 26

*stated in the key of C and arranged to show inversional symmetry

My last point involves the simple calculation of the subtraction vector: the intervals which occur in the total voice-leading between a set and its complement. Again, repeated instances of a given group, say the three major triads which have the same interval content, are differentiated by the fact that their respective complements have slightly, but crucially, different contents. It follows, then, that the intervals in the voice-leading motions between each triad and its "seventh chord" complement must be slightly but critically different. Example 5 shows the operation which results in a subtraction vector for the I, IV, and V triads in major. The differences are small but noticeable, *and they involve rare intervals*. For instance, one cannot produce a tritone in the voice-leading when moving from I to vii^7. One can do so when moving from IV to iii^7, and from V to IV^7 (but it is the same tritone taken in reverse). One cannot produce a major third when moving from V to IV^7. One cannot avoid producing all the intervals when moving from IV to iii^7.

I have been proceeding on a course which states diatonic facts and attempts to correlate them with tonal usage. In reverse, one might look at a usage, even one which is merely a "feeling" long noted, and attempt to provide the structural differentiation which might account for that usage in terms of stateable "facts." It seems clear that an event, when perceived or imagined in context, is somehow enriched by its context.[5]

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5. My colleague James Dapogny, upon looking at these statements about the three triads in a major key, immediately said, "But of course! Students have always insisted that the I chord doesn't 'sound like' the IV chord or the V chord--even though they are all obviously major triads." Yes, they are, but when one is hearing tonally, one is hearing the tritone and those two minor seconds "out there, wherever they are."

This paper has attempted to sketch out some of the crucial issues in a technical description of the process whereby the various elements of the diatonic set are hierarchized, associated, and differentiated on the basis of their external referents, their contextual surroundings, and the content of those surroundings. I have also tried to show that the relative rarity and ubiquity of, even the unique multiplicity of, the subsets of the diatonic set account in part for the differeing function of each element in the tonal usage of the set. Tonality may be a maximally well-made construct, *if* our purposes are to make a game of position finding and pattern matching which will never come to a reflexive closure. Though I certainly have not proved it here, I think, nevertheless, that tonality is a magnificent expression of human mental process, a great *Gestalt* comprised of many marvelous--and describable--little *Gestalts*.

It is increasingly clear that tonal music is still a phenomenon in our time. Those who speculate about its logic stimulate others to test stateable propositions. Those who live in its membranes seem productive. William E. Benjamin, in a remarkable article fusing musical and intellectual

Example 5:

Interval content of I (in major)	CEG		<001110>
Interval content of I's complement	BDFA		<012111>
Sum			<013221>
Subtract sum from diatonic content		<254361>	$<\underline{241140}>$
Interval content of IV (in major)	FAC		<001110>
Interval content of IV's complement	EGBD		<012120>
Sum			<013230>
Subtract sum from diatonic content		<254361>	$<\underline{241131}>$
Interval content of V (in major)	GBD		<001110>
Interval content of V's complement	FACE		<101220>
Sum			<102330>
Subtract sum from diatonic content		<254361>	$<\underline{152031}>$

The vectors with the singularities highlighted

I	$<2\ 4\ 1\ 1\ \underline{4}\ \underline{0}>$
IV	$<2\ 4\ 1\ 1\ 3\ 1>$
V	$<\underline{1}\ \underline{5}\ \underline{2}\ \underline{0}\ 3\ 1>$

currents, sees tonality[6] as an everlasting form of humanity, surpassing the contradictory efforts of its greatest theorist to date. Theorists can only try to set forth clear descriptions of the activity which so fascinates us that we devote ourselves to creating, experiencing, and explaining its manifestations.

6. William E. Benjamin, "Schenker's Theory and the Future of Music," *Journal of Music Theory* 25/1 (Spring 1981): 155-173.

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