A Structuralist Approach to the Diatonic Scale
Author(s): Ramon Fuller
Source: *Journal of Music Theory*, Vol. 19, No. 2 (Autumn, 1975), pp. 182-210
Published by: Duke University Press on behalf of the Yale University Department of Music
Stable URL: http://www.jstor.org/stable/843588
Accessed: 17/01/2011 17:28

A STRUCTURALIST APPROACH

Introduction

Certain workers in various fields ranging from psychology and anthropology to linguistics and mathematics are known as structuralists.[1] This seems to imply a closely-knit interdisciplinary school of thought, but such is not the case. Instead, researchers have tested certain general principles independently, found them applicable to their own fields, and established a loose kinship with other structuralists. The eminent Swiss psychologist Jean Piaget has described these principles[2] which, because of their generality, belong to *metatheory*,[3] or the theory of theories. We can expect structuralism, then, to provide, not so much a new music

TO THE

DIATONIC SCALE

RAMON FULLER

theory as a new perspective for old and new theories and a basis for criticizing and revising them.

Structuralism also provides new meanings for old words. It will be well, therefore, to discuss the word "structure" at the outset, since its meaning is obviously at the core of the whole movement. One of the most confusing aspects of this word as used by structuralists is that it does not necessarily refer to a single entity, such as "the structure of the first movement of Beethoven's Sonata, Opus 57." Even if it did, we would be talking, not about an *Urlinie* or other supporting framework, but about a set of substructures, their interrelationships and transformations. Thus, a structure is not a single entity but *an entity and all of its transformations.* A whole corpus of mythology, for example, might be a structure for an anthropologist, or all of Haydn's "London" symphonies for a musicologist, if warranted by the relationships between individual works. On the other hand the structure of a single work might be the set of its components as related by transformation. Since this paper deals primarily with the diatonic scale, it is important to remember that the structure of the scale is the complete set of all twelve transpositions of each of the seven principal modes (the plagal modes will come in through the back door, as it were). Since these scale-forms are studied in the abstract, i.e., for *potential* structural properties, little reference is made here to musical practice.

I *First-order Structural Analysis of the Diatonic System*

According to Jean Piaget, any structure must exhibit three properties: wholeness, transformation, and self-regulation.[4] The diatonic scale, considered as a subset of the twelve-tone equal-tempered scale,[5] has these properties. An underlying metatheoretical hypothesis is that all materials basic to any musical style must also have them in some recognizable way. These structuralist concepts might, therefore, guide both speculative theory and empirical research on new musical materials, once they are understood in their application to familiar materials.

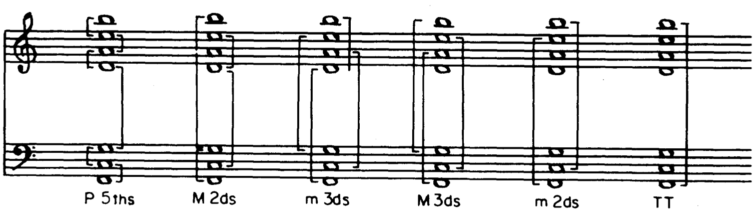
Wholeness is a broad concept which includes the ideas of completeness, coherence, and interdependence of parts.[6] These, in turn, imply laws by which elements are bound together and which determine the point of closure of a structure.

The *completeness* of the diatonic scale is due to two factors:

a) Pitch adjacency. When all tones of the scale are brought within the compass of an octave, we perceive that all gaps are filled in; the major and minor seconds are nearly equivalent steps.

b) Deep scale organization.[7] All interval classes of the chromatic scale occur at least once, and they are organized into a hierarchy of six perfect fifths, five major seconds, four minor thirds, three major thirds, two minor seconds, and one tritone (see Figure 1). When the last interval class, the tritone, is generated once by the chain of fifths, the structure is complete, and therefore closed. To continue the chain beyond the sixth perfect fifth is to disturb the hierarchy, since some interval classes will then occur the same number of times.

FIGURE 1



The *coherence* of the diatonic scale is due to two factors:

a) Octave equivalence. This identity relation makes it possible to close the scale at the octave;[8] to continue beyond the octave is to begin a new scalar cycle.

b) The strength of the generating interval, the perfect fifth. We perceive the perfect fifth to be harmonically the strongest interval of the diatonic system. It is the law of composition[9] of the system—a law which, combined with octave equivalence, is strong enough to bind the diatonic system strongly together, and make it a whole.

The *interdependence of parts* of the diatonic scale makes it a *system*, in which a change of one part has an immediate effect on all the other parts.[10] A good example of this is what happens when B-flat replaces B-natural in the C major scale: the whole scale is transposed up a perfect fourth, becoming an F major scale, and all tones of the scale change function.[11]

Transformation is of three types:

a) Permutation. The modes can be seen as circular permutations of the basic diatonic scale. This type of transformation is neither trivial (each mode has a unique quality) nor disruptive of the diatonic identity (all pitch adjacencies are preserved). One particular permutational transformation—change from major (Ionian) to minor (Aeolian)—retains all essential structural features[12] while effecting a striking change of color; it has therefore been a very useful transformation in diatonic music. It would be an overgeneralization, however, to say that any melody in any given mode could be mechanically transmuted into any other mode.[13] For example, transmuting a Dorian melody into the Lydian mode would change the perfect fourth, D–G, to the tritone, F–B, and might therefore imply a restriction on this particular transformation, depending on the musical style. Thus, in general, permutational transformations do not permit naive compositional usage, but like all transformations, require an understanding (or at least intuition) of the functional changes they bring about.

b) Mirroring. Each diatonic mode has a literal mirror[14] (see Figure 2):

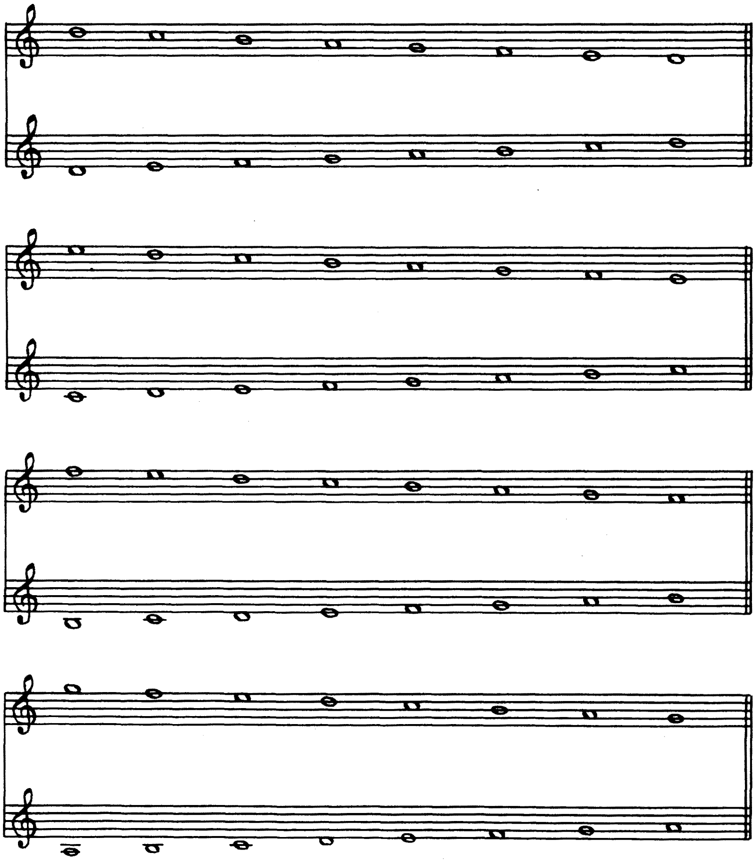
Dorian – Dorian
Ionian – Phrygian
Locrian – Lydian
Aeolian – Myxolydian

All the above mirror transformations are non-trivial, except with respect to the Dorian mode; and again, the identity of the system is preserved over the mirror transformation because the pitch adjacencies remain intact. As far as I can determine, this property (mirror transformation) of the diatonic system has not been used compositionally.

c) Transposition. This is perhaps the most common of all transformations in Western music. It involves a shift of all structural and coloristic properties (except those dependent on absolute pitch of course) to a different pitch level. Again, it is inherently neither trivial nor disruptive of the diatonic identity. However, we should note that transposition requires the chromatic scale as a superset of the diatonic scale, which permutation and mirroring do not.

One of the basic tenets of structuralism is conservation-over-transformation—i.e., that substructures[15] should retain their identity even though they pass through a change.[16] Transformations which are neither trivial nor destructive thus enlarge and enhance a structure, giving it flexibility and scope while preserving its identity. Perhaps the most significant thing to know about such musical transformations is that they make possible the articulation of large forms. This

FIGURE 2



is especially apparent in the classical sonata, in which relationships between keys are important structural factors. The change from the modal system to the major-minor system was an apparent impoverishment of structure, since it eliminated several transformations; however, restriction of the modal transformations to two was more than compensated for by the systematic exploitation of transpositional and polar relationships[17] in the classical style.

As we can see, transformation with identity conservation is the source of the flexibility and power of the diatonic pitch system. If similarly powerful structures are to be created from other sound materials such as the twelve-tone or other equal temperaments, noises, or vaguely-pitched clangorous sounds, then we must find the key to conservation in whatever transformations we intend to apply to these materials. In the case of pitch-related phenomena, that key seems to be *the transposability of relations*. All three of the transformations we have discussed in this paper are dependent upon this law, which is, therefore, basic to all traditional pitch-related music. It means that a given interval, say a perfect fourth, is perceived to be the same at any level of transposition, e.g., C–F is equivalent to E–A as an intervallic relation. This already is the transposition transformation, so we need to discuss only how mirroring and permutation are dependent on transposition.

If, say, the minor third D–F is played as a two-note motive, it can be mirrored by reversing the order to F–D. Obviously this operation retains the identity of the motive.[18] The generality of the mirror relation depends upon the operation of conservation over transposition, so that a mirror equivalence is perceived, for example, between D–F and C′–A. The mirror equivalence of longer motives is a chain of such two-note equivalences. Likewise, modal transformation (circular permutation) of the diatonic scale depends on both the octave equivalence between two pitches, e.g.,, C and C′, and on the equivalence of two different octave relations, e.g., C–C′ and D–D′. That is to say, conservation in the transformation from Ionian to Dorian depends on our perception that both octaves (C–C′ and D–D′) can represent a closure of the scale; likewise, there is an equivalence relation between the two dominant-tonic relationships: G-C is equivalent to A-D. The transformation, or quality shift, of the scale is brought about by the different relationships of the other scale members to these tonics and dominants. Thus, the transposability of relations is the basic principle of conservation for all pitch-related transformations. Conservation may well operate differently in the case of the transformations of vaguely pitched materials, but it must operate somehow if these newer musical resources are to articulate structures in the sense discussed here.

Self-regulation, according to Piaget, refers to closure of a structure (which was dealt with above, under completeness), and to the impossibility of errors, which are operationally excluded before they are made.[19] The latter seems to mean that no permissible transformation of a substructure generates elements really foreign to the larger structure to which it belongs. Thus a composite of all the above-mentioned transformations of the diatonic scale would yield nothing more than the chromatic scale (already defined as the superset of the diatonic scale). Equal temperament is, therefore, the key to the self-regulation of our pitch system. In this connection we should remember that the chromatic scale has the same law of composition as the diatonic scale—the cycle of fifths—and is therefore certainly not foreign to it. The important structural difference is that for the chromatic scale the cycle of fifths closes at the return to the octave rather than at the boundary of the deep scale. The diatonic scale, although a subset of the chromatic scale, retains its identity and its boundaries. It is not entirely swallowed up by the chromatic scale. The deep scale boundaries and scalar qualities of the diatonic scale are clear enough that even diatonic segments of only a few notes will stand out in a chromatic context. Likewise, chromatic alterations within a basically diatonic context are clearly perceived as such. This ability to retain its identity even though merged with a higher level structure is also a prime characteristic of any structure.[20]

Here may be added a fourth characteristic of structures: *the presence of several functionally-differentiated parts*. This is already implied, of course, when we say that transformations should be non-trivial; that is to say, the transformations of a structure whose parts are functionally differentiated yield new functions for the parts. This can be shown, for example, by comparing the variability of scalar quality of the seven modes of the diatonic system with the single quality of the six "modes" of the whole-tone scale. This is the point which distinguishes a step continuum[21] from a structure: a mere chain of identical parts is not a structure. Although one could perhaps conceive of such a thing as a "chain structure," it would be a structure without depth, since all its parts would share the same function; it would thus also be a structure without boundaries or non-trivial transformations.

Functional differentiation of parts is closely related to yet another aspect of structures: *hierarchical organization*. This form of organization is nearly universal in nature, including all phenomena from atoms to galaxies. Parts are either *contained within* larger wholes at higher levels of organization, or are *subordinated to* other parts at the same level of organization. Thus, notes are contained within the diatonic scale, which is contained within the chromatic scale. But, in tonal music, six of the seven diatonic notes are subordinated to the tonic. Likewise, most relationships are subordinated to the dominant-tonic relationship.

One can draw the distinction, as Meyer has done,[22] between degrees of hierarchical organization. Thus, there are highly-arched hierarchies, moderately-arched hierarchies, and flat hierarchies. The diatonic scale is highly arched, in terms of both the subordination of certain pitch classes to others and of certain interval classes to others (because of its deep scale properties). The "string of pearls" scale is a slightly-arched pitch hierarchy because it contains only two functionally distinct sets of pitch classes (see Figure 3). The whole tone and chromatic scales are flat hierarchies, since all their relationships apply equally to each of their pitch classes.

FIGURE 3

"String of pearls" scale and diminished 7th components

The two functionally-distinct sets of pitch classes

II *Second-order Structural Analysis of the Diatonic Major, Minor, and Ecclesiastical Modes*

The above first-order analysis of the diatonic system was useful for explaining certain fundamentals of structuralism, but it did not fully describe the structure of the complete diatonic system or the deeper relationships between the major and minor modes of the common practice period. To accomplish these tasks, we shall have to sharpen our structuralist tools and deepen our knowledge of structuralist concepts.

Let us begin with the idea of binary opposition. In his analysis of myths,[23] Lévi-Strauss makes use of the polar relation between such elements as culture/nature, sky/underworld, men/animals, life/death, domestic animals/wild animals, etc. These oppositions are more than contradictory elements of a given myth; they are, for Lévi-Strauss, *transform pairs,*[24] which enable him to relate groups of myths to each other by showing that some of their elements, or even complete myths, are paired as mutual inverses. Such opposition between the myths or their elements enriches the effect of the mythical corpus by enabling a society to treat certain problems (often its most central problems) from different, even opposing viewpoints. According to Jean Piaget, the ability to reverse an operation or a concept is a natural and necessary aspect of mature thought.[25]

Thus, it is natural for us to conceive the major and minor modes in music as mutually inverse; we think major is "bright," minor "dark"; major is "happy," minor "sad," etc. These generalities can be debated in many particular instances, but the perceptual-emotive tendencies we feel toward them are unmistakable. We shall not dwell on these purely subjective aspects, but instead investigate how, considered as a binary pair constructed from the same elements, the major-minor system is a structure in the Lévi-Straussian sense, and how the ecclesiastical modal system consists of similarly organized binary pairs.

But first we must note that analogy is as important as, and complementary to, the polar opposition relation. For example, Lévi-Strauss, in his analysis of the Oedipus myths,[26] points out that Oedipus' name means "swollen-foot," while his father's name, Laios, means "left-sided," and his grandfather's name, Labdakos, means "lame." All these names imply weakness or lameness; according to Lévi-Strauss, this refers to a mythical concept of the autochthonous origin of man—that he was born from the earth. We need not pursue this further here, except to note that such groupings of similar elements are important in structural analyses.

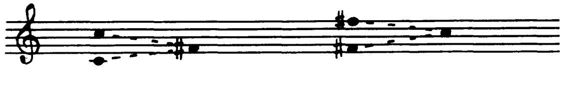
The limiting case of similarity is structural duplication or congruity. Two major scales with different tonics are congruent, while a major scale and a minor scale are, in some respects, members of a binary polar opposition, and in other respects, analogs. Binary opposition and analogy themselves form a binary opposition, similarity/dissimilarity, and are obviously aspects of *transformation.*

Once a polar opposition is formed, there is, according to Lévi-Strauss, a tendency for the human mind to look for a median between the two poles.[27] An example given by Leach is the polarity red/green used in traffic signals,[28] with yellow lying between red and green on the color scale and, therefore, fitting logically as a signal mediating between "stop" and "go." In music, an analogous process can be used to derive pitch scales and harmonic materials such as triads.[29]

Consider the octave as a polar relation, i.e., as two instances of the same pitch class mutually opposed by the relation high/low. As important as this purely registral polarity is, it cannot suffice; other pitch-related dimensions are needed.[30] To this polarity must, therefore, be added a harmonic polarity, which will also be a suitable median between the two octave-related poles. Let us first consider the perceptually symmetrical bisection of the octave by the equal-tempered tritone. This might seem to provide the ideal median, except for the following problems:

1. If we assume that all pitch classes generated can be used in more than one register, then it will be impossible harmonically

FIGURE 4 The tritone as median between octave registral poles



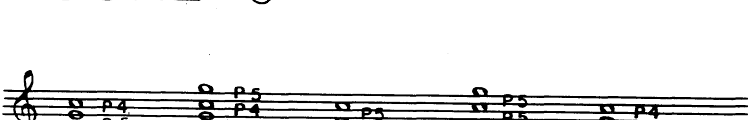
poles median poles median

to distinguish between our original poles and the median, since either pole can also be a median to the median (see Figure 4). Therefore *a perceptually asymmetrical bisection of the octave is necessary* to preserve the functional distinctions of the system we are constructing. Compositional technique can, in principle, create a polar relationship between any pair of musical elements; here we are attempting to recreate a system of relationships that are independent of compositional technique.

2. The tritone is a dissonance, and therefore cannot logically be introduced into the system until the concepts of harmony and consonance have been more fully developed. Therefore, *the bisecting interval must be a consonance.*

The interval which meets both of the above criteria is the perfect fifth which, being the arithmetical mean of the octave relation,[31] yields a not-quite equidistant bisection of the octave. This asymmetry preserves the distinction between the two octave poles and the perfect fifth median (which we may now name the tonic and dominant, respectively) because C does not bisect the octave G–G′ in the same way as G bisects C–C′ (see Figure 5). There are two such perfect fifths—one downward from the tonic and one upward. The downward fifth, or subdominant, also yields an asymmetrical division; when combined, the two perfect fifth divisions are symmetrical about the tonic (see Figure 5). This symmetry, however, still preserves the functional distinction of the three pitch-classes, since they are not balanced around either the circle of fifths or the circle of minor seconds.[32]

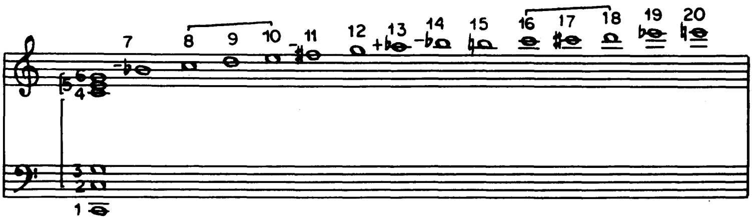
FIGURE 5



There are grounds in the overtone series for our perception of a functional polarity between the two tones of the perfect fifth. Referring to Figure 6, recall that the third partial lies a perfect fifth plus an octave above the fundamental. The bottom note of any perfect fifth is, therefore, heard in a sense

FIGURE 6

The overtone series



as the generator of the upper note. Since the upper note cannot generate the lower one,[33] the relation of a tonic to its dominant is *unidirectional*. This fact further strengthens the functional distinction between tonic and dominant.

The process of creating new polar relations from medians can be carried out just a few more steps with good logic, if not adherence to historical sequence or musical practice. Referring again to the overtone series shown in Figure 6, note that two pairs of partials—1 and 2, and 2 and 4—outline the octave registral polarity. Partials 2 and 4 are separated by the perfect fifth median, which generates the new polarity, tonic/dominant. If we raise this fifth an octave higher in the overtone series so that it is outlined by partials 4 and 6, we find a major third median (the fifth partial) which makes a major triad of the perfect fifth polar system. Another octave higher, partials 8, 9, and 10 form the major third tonic/mediant polarity and its major second median.[34] Likewise, partials 16, 17, and 18 outline the tonic/supertonic polarity and its chromatic semitonal median.

All these divisions of the various tonic polarities also yield other intervals not yet mentioned: partials 3 and 4 form the perfect fourth which, being the octave complement of the perfect fifth, is harmonically identical to it in many contexts; partials 5 and 6 form the minor third which is treated the same as its perfect fifth complement, the major third, in traditional harmonic practice; partials 9 and 10 form the minor tone (a small major second)[35] which is not found in the equal-tempered system because all major seconds there are the same size. Likewise, the minor second between partials 17 and 18 is effectively identical to that between partials 16 and 17. Thus, there is a real identity between both halves of each of the intervals we have bisected, and none higher in the overtone series than the perfect fifth retains its unequal bisection in the twelve-tone equal-tempered system. Perceptibly unequal bisections of the major third and major second in an equal tempered system would necessitate the introduction of microtones; the fact that this is unacceptable to most musicians in our culture may help to explain why only the perfect fifth has been widely used as a polar relation. Different ways of dividing an interval almost equidistantly make it easier to distinguish between different instances within the same scale—in the same way that we can distinguish between, say, the tonic and supertonic triads because one is major and one minor (i.e., because their fifths are divided differently).

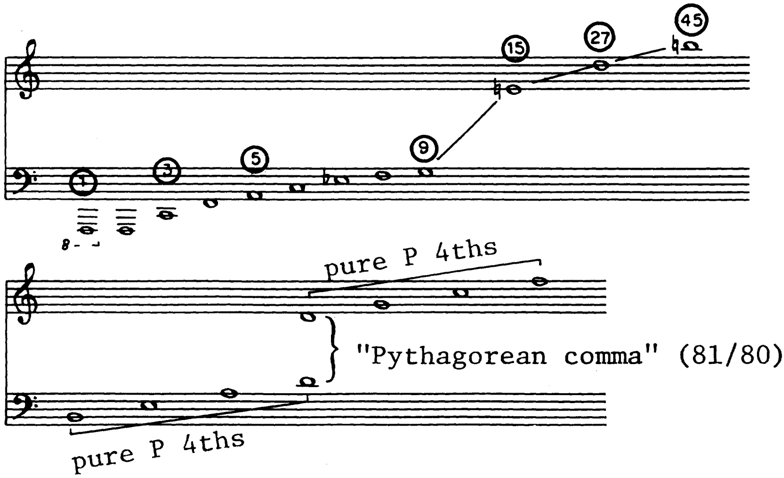
By applying the process of constructing perfect fifth polarities and their medians to the dominant and subdominant pitch-classes, we obtain the three primary (major) triads of the C scale, thereby arriving at the so-called just intonation system. Referring to Figure 7, note that this version of the C scale is really generated by the overtone series on F, which means that from this perspective the subdominant is the true generator of the diatonic scale.[36] However, due to the dilemmas encountered in working with the higher partials of the overtone series, a quasi-Pythagorean method (equal temperament) is much more suitable for our use. Since the 5th and 27th partials do not form a pure (expanded) perfect fourth, an adjustment must be made, either by having microtonal alternates for at least one pitch class (probably D)[37] or by using a tempered system. Rather than speculate further on various tuning systems, we shall proceed on the assumption that the twelve-tone equal-tempered system is truly basic to our present musical consciousness—which means that our diatonic scale is generated by a chain of fifths rather than by the overtone series. However, several interval relationships from the overtone series are still operative, since they are approximated in the equal tempered system.

Major/minor as a Polar Opposition

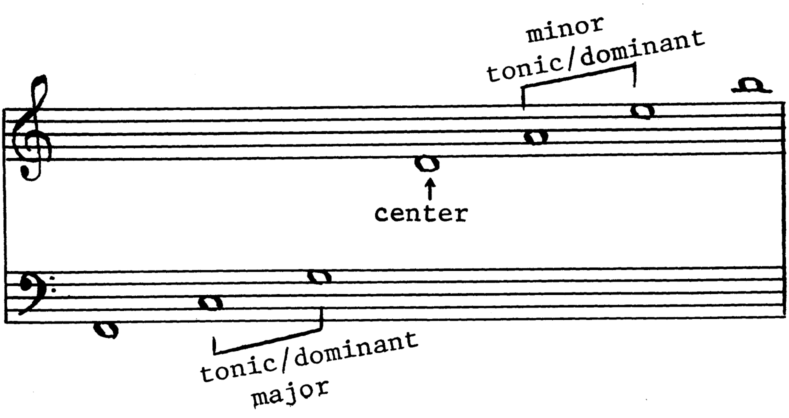
In Figure 8, which shows the C scale as a chain of perfect

FIGURE

7 The F overtone series and the "just" scale on C



8 Symmetry of the major/minor system



fifths, note that D is the dead center of the system, falling between the major and minor tonic/dominant polarities, C/G and A/E; the latter are therefore symmetrically arranged around the center. The two outer limits of the system, F and B, are both leading tones, B leading upward to C and F leading downward to E.

As we saw earlier, the mirror of the Ionian or major mode is apparently the Phrygian mode, not the Aeolian on which the minor scale is most easily based; one's first thought is that Ionian ought to be paired with Phrygian, rather than with Aeolian, as its nearest relative. Since this does not agree with musical practice, the close structuralistic relationship between a major key and its relative minor must be sought at a deeper level, which can be understood by combining the mirror principle with the unidirectionality of the tonic/dominant polarity. Thus E is not the tonic of an A/E subdominant tonic polarity, but, rather, the dominant of an A/E tonic/dominant polarity even though E is the mirror-correspondent of the major tonic C. Our modern perception of the Phrygian cadence confirms this; we have a strong tendency to hear it as a half cadence on the dominant.

The mirror operation on the major scale generates a new scale in which the function of each scale member is the inverse of the function of its mirror correspondent in the

FIGURE 9

Functional opposition of mirror correspondents

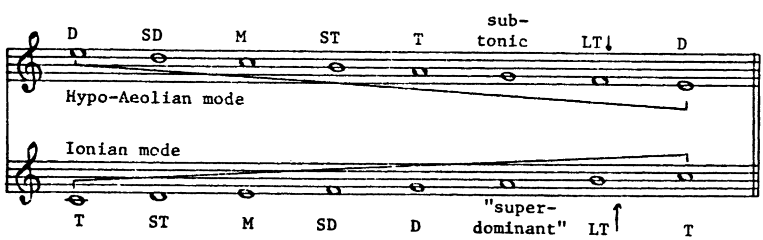
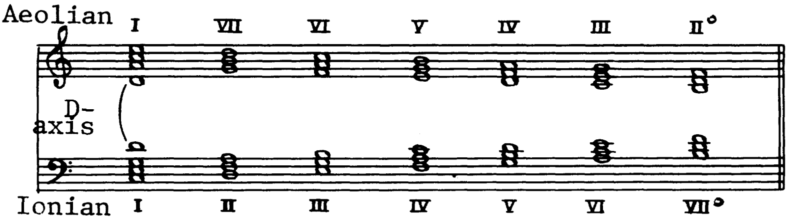


FIGURE 10

Triad mirror relationships in Ionian/Aeolian



major scale. In other words, the true structural mirror of Ionian is not Phrygian but Hypo-Aeolian (see Figure 9).[38] Thus the dominant of the minor scale is the mirror correspondent of the tonic of the major scale and vice-versa; the minor scale supertonic is the mirror-correspondent of the major scale subdominant. Similarly, the minor scale subtonic is the reflection of the major scale "superdominant"[39] (remember that "super" means above and "sub" means below); the minor scale mediant,[40] which is a major third below the dominant, is the reflection of the major scale mediant, which is a major third above the tonic; and the sixth degree of the minor scale is a leading tone down to the dominant, while its reflection, the seventh degree of the major scale, is a leading tone up to the tonic. Thus the mirror relation between scales implies more than a simple reflection of major by minor; there is also a deeper structural relationship—the opposition of the functional roles of mirror correspondents. This is now the structural reason why the Aeolian, and not the Phrygian mode, is the true binary opposite to the major mode.

Figure 10 shows the triads of the scales reflected around the central axis D. It is readily seen that the mirror relationship is quite exact, although a new complication appears: the roots of mirror-corresponding triad pairs are not mirror correspondents. This is because the root relationships between perfect-fifth diads exist between their lower notes, whereas mirror-correspondence relationships exist between the lower note of one diad and the upper note of the other.

For example, C, the root of the Ionian tonic triad, is not reflected by A, the root, but rather by E, the fifth of the Aeolian tonic triad. The root concept is a useful analogical device; it enables theorists and composers to treat minor triads like major ones. This conforms to musical practice and must be considered legitimate, but it may be that preoccupation with roots has obscured some structurally important relationships for theorists, so that we have not fully understood how major and minor comprise a binary pair.

Furthermore, as might now be anticipated, every ecclesiastical mode is paired with another whose triads are reflections of its own. So now the complete system is found to be composed of the modal pairs, Ionian/Aeolian, Mixolydian/Dorian, Lydian/Phrygian, and Locrian/Locrian, as seen in Figure 11, which shows the mirror relationship of each pair when each mode is seen as a cycle of thirds centered around its tonic triad. Of particular interest is the fact that the one mode

FIGURE 11

Second-order binary pairings of the modes

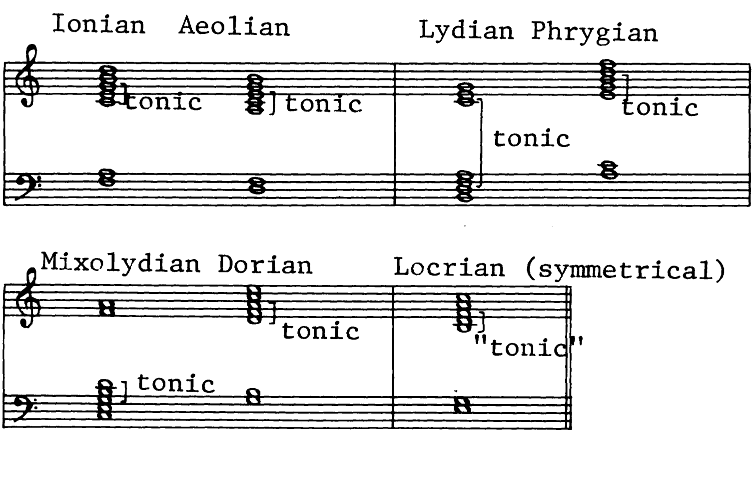
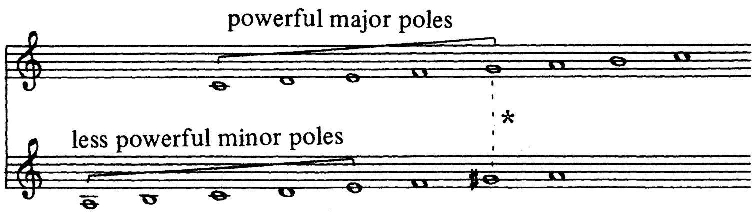


FIGURE 12 The leading tone in minor replaces the dominant of the relative major



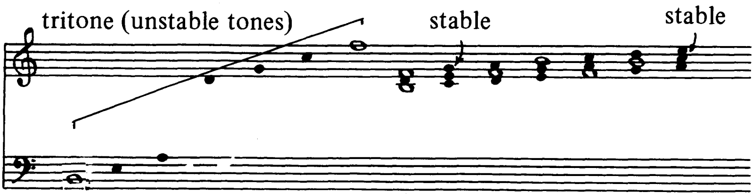
*G-sharp replaces G-natural, destroying the T/D polarity of the relative major

which was never used compositionally—Locrian—is now seen to be the only one whose mirror transformation is trivial at a deeper structural level; on the other hand, the mode which was used perhaps more than any other during the medieval period—Dorian—has a non-trivial transformation at a deeper structural level despite the self-mirroring property which we observed at the first-order level of analysis.

In order for the dominant/tonic polarity in minor to function as an analog to that of the major scale, the seventh degree of the minor scale must be raised. This brings about a departure from the strict mirror relation between the two modes. Besides providing a true leading tone and a major triad on the dominant, raising the seventh degree neutralizes the tonic/dominant polarity of the relative major by removing its dominant (see Figure 12). This operation is necessary because the major tonic, being more stable than the minor tonic, would easily overpower the latter, yielding at best a weak or confused key feeling in minor. The relative major tonic, when thus neutralized, becomes a true median between the tonic/dominant poles of the minor key.

At this point, we may raise an interesting question: Is there any structuralistic reason why the Ionian and Aeolian modes were preferred when tonality and harmony became the reigning norms in Western music? In other words, are they in any way superior to the other modes? If we allow three easily accepted postulates, there are several reasons, I

FIGURE 13 The most stable triads of the diatonic scale



believe, why the Ionian and Aeolian modes evolved into our traditional major and minor modes.

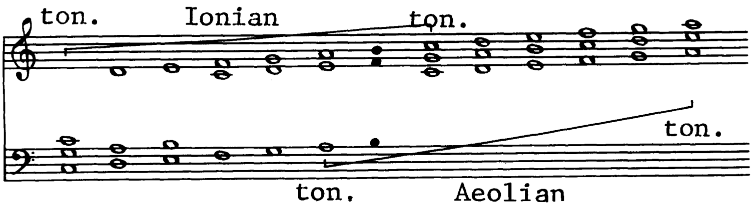
In Figure 13, note that the two tones at the endpoints of the diatonic chain of fourths generate the tritone, which is universally considered to be unstable in a diatonic setting. These two tones, F and B in a C scale, are unstable whether or not they are sounded together, simply because, when we have the complete scale in our consciousness, one note of the tritone will be heard in reference to the other tritone note, even if the latter is not sounded. This is the first postulate, and it finds ready expression in the familiar dicta of theory teachers that "the leading tone has a tendency to resolve up to the tonic," and "the fourth degree of the scale has a tendency to resolve down to the third." The first two reasons for the prominence of the Ionian and Aeolian modes then are:

1. As Figure 13 shows, only two triads of the C scale contain neither of these unstable tones–the two tonics of the Ionian and Aeolian modes, respectively. If the stability of all tones of the tonic triad is any criterion, then it follows that these two triads are most suitable as tonics. Note also that raising the seventh degree of the minor mode, G to G-sharp, creates a tritone with D, and not with any note of either tonic triad.

2. In Figure 14 we can see that, in the cyclic structure of the scale, B lies between C and A, and F lies between G and E. Therefore, when the notes of the scale are arranged as a series of octaves (or octave species) divided by the fifth/fourth

FIGURE 14

The tritone as a marker among the diatonic octave species



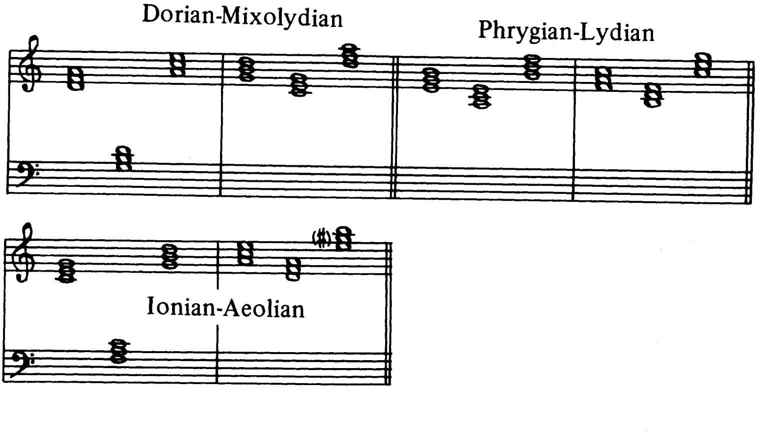
relationship, our two tonic octave species are both adjacent to the unstable tritone octave species. This is significant in two ways: (a) the proximity of instability to stability means that resolution of the former to the latter is easily accomplished by stepwise motion (albeit not usually by parallel fifth motion); (b) the most stable octave species (i.e. Ionian and Aeolian) are also significant endpoints in the scale, because they occur next to a unique marker of the scale–the tritone. Since the latter occurs only once in the pure diatonic scale, it is able to represent the end of a scalar cycle and point to the stable tonic as the beginning of a new cycle. Thus B signals the end of a downward cycle toward the tonic of the minor mode, or the end of an upward cycle toward the major tonic. F functions analogously for the two dominants E and G which, though less stable than the tonics, are equally significant points in the scale.

The second postulate is that a pair of modes ought to be related by analogy as well as by polar opposition; this is linked to a third postulate–that tonality in Western music is dependent on the functioning of the tonic, dominant, and subdominant triads. In their pure diatonic form, these triads in Ionian and Aeolian are beautifully related in both ways: all three are major in Ionian, and all three are minor in Aeolian. This is both a polar opposition (major vs. minor) and an analogy (each set consists of three congruent triads). Furthermore, chords of similar function are clearly opposed, e.g., a major subdominant in the major mode opposes a minor subdominant in the minor mode. When the leading tone is raised in minor, the two dominants function by exact analogy. But this, if anything, strengthens the opposition between major and minor, because the progression from a major dominant to a minor tonic accentuates the minor quality of the tonic. (See Figure 15) The other modal pairs are not related quite so clearly as either opposites or analogs. For instance, the two tonics of the Dorian-Mixolydian pair are opposed, but the two dominants and the two subdominants are similar. The Phrygian-Lydian pair is even worse from this standpoint: the tonics are opposed, but the dominants and subdominants are not comparable (see Figure 15). It is true that in every case the dominant of one mode is reflected by the subdominant of its opposing mode, but this relationship is not easily perceived, because of the human mind's tendency toward analogical relations.[41] The simpler relation between Ionian and Aeolian therefore makes them the inevitable choice, given the rise of triadic functional harmony.

Our structuralistic study of the diatonic scale is now as complete as it can be made without reference to actual musical compositions. Whether musical structuralism can provide the basis for new or improved analytical procedures must remain an open question until a significant number of works have been analyzed. The fundamentals discussed do thus far seem promising, in that they fit the properties of

FIGURE 15

The primary triads of the three principal modal pairs



the diatonic scale rather well. Since its concepts are very general in nature, structuralism may be a new metatheory under which such diverse techniques as set-theoretic, information-theoretic, and Schenkerian analysis can be brought together to yield fruitful new insights into both tonal and non-tonal idioms in many style periods, and by which new clues for the development of contemporary compositional techniques may be obtained.

REFERENCES

1. Especially Piaget in psychology, Lévi-Strauss in anthropology, de Saussure and Jakobson in linguistics, and "the Bourbaki" and Barbut in mathematics. See Lane, 1970 (table of contents) and Piaget, 1970, p. 23.
2. Piaget, 1970, Chapter 1, pp. 3–16.
3. Unification of traditional theory and practice with those of the twentieth century has long been my dream. The best hope of realizing this now appears to be at the metatheoretical level, via structuralism or some modification of it.
4. Piaget, 1970, p. 5.
5. The diatonic scale is of course a segment of the cycle of fifths, in the tempered system. In some other tuning systems this is not the case; for instance the C scale of the just intonation system must have two D's separated by a microtone, to yield a scale feasible for harmonic purposes (see note 37). The equal tempered system is really a compromise between the Pythagorean cycle of fifths and the necessity for harmony and a reasonably simple scale.
6. See Piaget, 1970, pp. 6–10, DeGeorge, 1972, p. xxv, and Lane, 1970, pp. 14, 24.
7. Gamer, pp. 39–41.
8. Scalar closure at the octave, as in a diatonic mode, should be distinguished from the closure at the octave which occurs at the twelfth perfect fifth in a cycle of tempered fifths.
9. The minor second is not a law of composition of the diatonic system, even though it is theoretically a perfect generator of the chromatic system, of which the diatonic system is a subset. In terms of practical acoustics, moreover, the equal-tempered minor second is not even a good generator of the chromatic scale, despite the superficial ease with which chromaticism is approached on the piano: a piano tuner would never, without special equipment, be able to tune to the tempered minor second, which has a ratio of $12\sqrt{2}$, yet he can readily learn to temper the fifths. There is thus good reason for considering the chromatic scale as an enlarged and completed diatonic scale.
10. DeGeorge, p. xxvi. My use of the word "system" here may trouble some readers, especially those familiar with general systems theory as described by Ludwig Von Bertalanffy (*General Systems Theory*, New York: George Braziller, 1968) and Ervin Laszlo (*Introduction to Systems Philosophy*, New York: Harper Torchbooks, 1972). The problem arises from the fact that these writers say that a cybernetic feedback loop is a necessary component of a system

(see Laszlo, pp. 38–47, and Von Bertalanffy, pp. 149–50, 160–163). Now it is obvious that a musical scale does not of itself possess any sort of feedback loop. If feedback occurs in connection with a scale, it takes place in the human mind, which is a full-fledged Bertalanffian system. The relation between systems and structures can be stated in two propositions:

a) *structures* are (non-physical) ideational or perceptual *systems* regulated by external, physical super-systems, and

b) *systems* are physical *structures* regulated by internal cybernetic feedback loops.

11. The word "function" is used several times in this paper, sometimes in unfamiliar, though not illogical ways. Presumably everyone is familiar with the usages, "tonic function," "dominant function," etc. But what characterizes the tonic function? Is it not a complex of relations which pinpoint a particular pitch class as the tonic of a given mode of a given scale? This thought can be extended just slightly to include such less familiar "functions" as, say, the third pitch class of a given tone row—which would probably not have a function name. But since such a pitch class would fit in a particular complex of serial and pitch relationships, any transformation of the row would be likely to change its relation-complex and therefore its function—even though serial functions are unnamed (as they must be unless a very new light is thrown upon serialism). This example should be kept in mind whenever the word "function" occurs in this paper.
12. This can of course be done only through a chromatic alteration of the minor (i.e. Aeolian) mode—putting a leading tone in place of the natural seventh degree. The other structural features such as the dominant/tonic and subdominant/tonic relationships are of course left unchanged by the major/minor transformation.
13. "Transmute"—this word was chosen because it is related to "transpose," but different enough not to be confused with it. It is of course nonsensical to try to "transpose" a given melody into a different mode, since this term means to preserve all the original intervals (and thus the original mode) of a melody.
14. These mirror transformations are applicable only to purely melodic aspects of the modes.
15. A substructure is a member of a set of similar entities which, at a particular level of hierarchical organization, are related to each other by transformation. The Dorian mode of the C scale is an example of such a substructure. The set of all such substructures (circular permutations or modes of a given diatonic scale) is the next higher level of structure.
16. DeGeorge, p. xxv, and Piaget, 1970, pp. 20, 21.
17. See Part II of this paper.
18. At first sight, this might appear to be a serious lapse of logic—using permutation to help explain the mirror principle when both mirroring and permutation are supposed to depend on transposition. The following considerations should be kept in mind in this connection.

There is a vast difference between applying the principle of permutation to, say, two or three pitch classes and applying it to seven, as in the diatonic scale. In the case of the smaller numbers of pitch classes, permutation cannot alter the basic shape or sound enough to undermine conservation. This is manifestly not true where as many pitch classes are involved as in the diatonic scale, except for the one special case—circular permutation. But even here, as shown in the body of this article, conservation is dependent on transposition. Thus, permutation is an independent, identity-preserving transformation only for small numbers of pitch-classes. In the present example only the reversal of the order of the two pitch classes is required to make the basic point. Since the contrary motion entailed by mirroring will usually generate new pitch classes, the further operation of conservation depends on transposition. That is, a given interval will be recognized in the mirror if the hearer can recognize the interval in both directions and recognize it when transposed.

19. Piaget, 1970, pp. 13–15.
20. Piaget, 1970, pp. 13–14.
21. A "step continuum" is an entity which, though divided into discrete steps, does not change in quality from one step to the next, thus being a continuum despite its step-like formation.
22. Meyer, 1967, pp. 304–312.
23. Leach, 1970, chapter ii; also Lévi-Strauss, 1955 (in DeGeorge), pp. 169–194.
24. This terminology is borrowed from mathematics, and refers to any two ideas or entities related as transformations of each other. Also included in this concept is the idea that a transformation is effected by a mathematical or logical algorithm (i.e., by an intellectual operation in Piaget's sense). See Piaget, 1966, pp. 32–37; note especially Piaget's insistence that operations form systems.
25. Piaget, 1966, pp. 40–41.
26. Lévi-Strauss, 1955, in DeGeorge, pp. 177–179.
27. Lévi-Strauss, 1955, in DeGeorge, p. 188.
28. Leach, pp. 16–20.
29. It is not claimed that pitch scales were historically derived this way. A plausible genetic description is, however, at least a useful aid for understanding a phenomenon, especially if it shows it in a new light.
30. If absolute pitch and pitch distances or registers represented the only pitch-related dimension in music (as some mathematically inclined theories seem to assume in their present form), music would be far less subtle and expressive than it is. The harmonic dimension is real, and while it is obviously dependent on absolute pitch distances, it offers much more to our perception than is implied by a strict consideration of pitch distance information only. It therefore deserves to be included in any theory of music. The question is, once we leave the familiar triadic domain, how can harmonic phenomena operate?

31. The arithmetical mean of two numbers is the familiar average, obtained by dividing the sum of the two numbers by two. In the overtone series, any partial is the arithmetical mean of its two immediate neighbors on either side. For example, the third partial is the arithmetical mean of the second and fourth partials. Since the ratio 4/3 is smaller than the ratio 3/2, the octave is not bisected into equal halves, by taking its arithmetical mean.
32. Functional distinctions between pitch classes distributed equidistantly around the circle of fifths or the circle of minor seconds are always vague or lacking, apart from an appropriate context. For example, no functional distinction can be made between the pitch classes of a diminished seventh chord on the basis of the chord structure alone. Only a tonal context can establish one tone as a prime (i.e. as a leading tone) and thus as functionally distinct from the other chord members, because they are all equally spaced across the octave. On the other hand, the spacing of the tonic, dominant, and subdominant roots, though symmetrical, is not equidistant around either the circle of fifths or the circle of minor seconds. These pitch classes are therefore functionally distinguishable, even without reference to a context.
33. It is true that the overtone series is not an infinitely useful explanatory vehicle for music theory. It is used here because it does contain, among others, all the useful "just" intervals arranged in the order of their mathematical simplicity and structural importance for tonal music; because, as we shall see, the division of musical pitch space by successively higher partials yields relationships that structuralists would hope for; and because there is a unidirectional relation of the overtone series which proceeds from the fundamental to the partials: that of generator to generated. Thus, for instance. a C fundamental can generate a G partial, but a G fundamental cannot generate a convincing C partial until so high in the overtone series as to be out of the realm of practicality. So for our purposes, generation of overtones is strictly unidirectional—and this corresponds to our perception of both the perfect fifth as an interval, and of the dominant-tonic progression, which we perceive to be more restful than the tonic-dominant progression.
34. This polarity, like the ones mentioned later, is not as strong as the dominant/tonic polarity. It has nevertheless proven useful, for instance, in those compositions of the 19th century in which modulation to the mediant replaces that to the dominant.
35. By translating the ratios of the minor and major whole tone into decimal notation, we can see that the latter is really larger than the former: 9/8 = 1.125 or 204 cents, while 10/9 = 1.111 or 182 cents. The difference of 22 cents is almost an eighth tone, certainly large enough to be audible. All such differences are eliminated by the equal tempered system. A very useful table of intervals is found in Helmholtz, 1954, pp. 453–56.
36. If partials up to the 45th were perceptible in their relation to the fundamental, this would greatly alter our perception of scales—the

subdominant, being the fundamental, would have to be the tonic. Likewise, a "dominant" harmony would become the tonic if we were to include partials no higher than the 11th. Therefore, since neither of these cases holds true, there is only a loose analogical relation between the fundamental of an overtone series and the tonic of a scale.

37. By raising the fifth partial up three octaves ($5 \times 2 = 10$, $10 \times 2 = 20$, $20 \times 2 = 40$), we obtain the 40th partial; the ratio 40/27 is about 1.485, whereas the ratio of the perfect fifth, 3/2, is exactly 1.5. This difference, which is again 22 cents, is very audible, creating a sour fifth between D and A in the just C-scale as generated by the overtone series on F. By having two D's, one in tune with the just A, and one in tune with the just G, the problem is solved so that the three primary (major) triads and the three secondary (minor) triads are all available in their pure form. Equal temperament of course obviates all of this.
38. The distinction between the plagal and the authentic modes is of course a moot point as soon as polyphony is considered, since the assignment of some voices to the plagal and some to the authentic mode will be made for reasons of range only. Our conception of unity demands that we consider the voices to be in the same mode in such cases. Therefore, for our present purpose the Hypoaeolian and Aeolian modes are identical.
39. The coining of such new terms is not meant as an effort to build a new theory system, which would further overload an already confusing field, but rather to show, with terminology, the mirror relations explained. Thus "sub" and "super" are mutual reflections, as are "tonic" and "dominant," in this case.
40. It seems entirely fitting that the two mediants, in reflecting each other, do not represent a polar opposition of functions as do the other mirror pairs. The mediant is, after all, a median between the two poles, tonic and dominant, and as such is a neutral position between them.
41. That is to say, it is easier for us to compare chords of similar function when comparing modes; i.e., we are more likely to compare the two subdominants of a modal pair than to compare the dominant of one mode with the subdominant of the other, even though they are properly related as mirrors. The mirror relation is a useful operational method of deriving the polar opposite of a mode (or of seeing the relationships between two modes), but once this is done the tendency is to proceed by analogy in the treatment of the modes, i.e., to treat them in the same way, rather than attempt to exploit the mirror-relations. The latter seem to be more compositionally useful in atonal than in tonal music.

BIBLIOGRAPHY

Bertalanffy, Ludwig Von. *General System Theory*. New York: George Braziller, 1968.

DeGeorge, Richard T. and DeGeorge, Fernande M., eds. *The Structuralists: from Marx to Lévi-Strauss*. New York: Anchor Books, 1972.

Gamer, Carlton. "Some Combinational Resources of Equal-Tempered Systems." *Journal of Music Theory*, 11 (1967): 32–59.

Helmholtz, Hermann. *On the Sensations of Tone* (trans. Alexander Ellis). Dover Publications, Inc., 1954.

Lane, Michael, ed. *Introduction to Structuralism*. New York: Basic Books, Inc., 1970.

Laszlo, Ervin. *Introduction to Systems Philosophy*. New York: Harper Torchbooks, 1972.

Leach, Edmund. *Claude Lévi-Strauss*. New York: The Viking Press, 1970.

Lévi-Strauss, Claude. "The Structural Study of Myth." *Journal of American Folklore*, 78 (1955): 428–44. Republished in DeGeorge, 1972, pp. 169–94.

Meyer, Leonard B. *Music, the Arts and Ideas*. Chicago: University of Chicago Press, 1967.

Piaget, Jean. *Psychology of Intelligence* (trans. Malcolm Piercy and D. E. Berlyne). Littlefield, Adams and Co., 1966.

Piaget, Jean, *Structuralism* (trans. Chanina Maschler). New York: Harper Torchbooks, 1971.

Sutherland, John W. *A General Systems Philosophy for the Social and Behavioral Sciences*. New York: George Braziller, 1973.