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A CODED ELEMENT MODEL OF THE PERCEPTUAL PROCESSING OF SEQUENTIAL STIMULI[1]

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A parameter-free model is developed of how human subjects perceptually process sequential patterns of binary and trinary events, for example, aabbab; aabcacc; etc. Emphasis is on repeating and on short, nonrepeating patterns. The model's major characteristics are (*a*) coding of the pattern into a hierarchy of larger and fewer elements until the pattern itself is the final coded element. This coding is described with a set of coding axioms. And (*b*) evaluation of the information (*H* or statistical uncertainty) associated with the coded elements using Garner's multivariate uncertainty analysis. The total uncertainty of the coded elements of a pattern, called H_{code}, is defined as the pattern's complexity. H_{code} values of a large variety of binary and trinary patterns are used to predict such response indexes as accuracy of pattern recall, mean number of words to describe a pattern, and judged pattern complexity. Typical correlations ranged from .90 to .95. Some deviations from the predictions that imply changes or additions to the model are discussed.

The purpose of this paper is to present a model of how human subjects perceptually process sequential patterns of nonmetric stimulus events. The term "nonmetric" denotes stimulus elements that differ only in qualitative properties, that is, events on a nominal scale, such as a, b, c, etc. Although only binary and trinary patterns are treated, the model is applicable to patterns generated from a population of *n* different stimulus events. Emphasis is on periodic or repeating patterns such as abaabaab, abaabaab, · · · ; abaccc, abaccc, · · · ; etc, and on short nonrepeating patterns of the same kind.

There are two major characteristics of the model: first, the coding of the pattern into larger and fewer elements until the pattern is completely predictable; second, the evaluation of the information associated with the coded elements. The term "information" is used in the sense of statistical uncertainty as defined by the well-known Shannon-Wiener formula. (This usage is in sharp distinction to other information models of psychological processes which are represented in computer programs. These generally describe and operate on some class of elements, e.g., hypotheses, properties of a pattern, etc., but do not actually evaluate and respond to uncertainties. See Hilgard & Bower, 1966, Ch. 12.) One important derivation from the model is a measure of pattern complexity. This complexity measure is the sum of the total amount of uncertainty evaluated in processing the stimulus sequence.

The Coding Problem

It is well known that human subjects code or organize stimulus sequences into larger and "simpler" units. Recent interest in this problem stems from Miller's (1956) now classic article "The magical

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number seven, plus or minus two." He introduces the problem as follows:

> We must recognize the importance of grouping or organizing the input sequence into units or chunks. Since memory span is a fixed number of chunks, we can increase the number of bits of information that it contains simply by building larger and larger chunks, each chunk containing more information than before [p. 93].

More recently Dodwell (1961) has stressed the importance of developing an understanding of stimulus coding before one can effectively describe how stimulus-response connections are formed. He discusses data that suggest a hierarchy of coding principles, and he proposes that some sort of stochastic processing of the coded stimulus properties seems plausible. The model presented here incorporates both these features.

The problem of perceptual coding also is central to the study of perception and to studies of learning and memory that use stimuli which can be coded. Obviously, in studies of perception, if the subject recodes the stimulus and the new coded stimulus elements overlap only in part with the initial elements, then much of the effective stimulus is unknown and the experiment is being run with an ambiguous independent variable. Experiments on learning, such as the common probability-learning experiment, employ stimulus event sequences which have long been known to be coded. If one assumes that the effective stimuli are these coded stimuli, or chunks, then it seems reasonable to describe the learning and memory processes involved in terms of operations on these coded elements. But, only recently have probability-learning models reflected this issue. Restle (1961, 1966) and Gambino and Myers (1967) have developed models assuming that the subject codes the event sequence into runs and that probability learning is essentially a function of the subject's experience with runs. Gambino and Myers suggest that other simple patterns, such as alternations, also may be special coded elements.

The problem of perceptual coding is relevant to experiments on pattern or concept learning and to memory experiments which use such stimuli as strings of binary events, strings of decimal digits, etc. One recent model of binary pattern learning which assumes the subject codes the stimulus into runs has been proposed by Vitz and Todd (1967). Usually, however, the experimenter sidesteps the problem of coding by preselecting stimulus sequences so as to exclude runs, alternations, simple numerical progressions, and other obvious bases for coding. In most cases these precautions are probably adequate, but unfortunately there is no good rationale which guarantees the exclusion of coding. In addition, one can question the basis of developing models of learning and memory when perceptual coding, a fundamental characteristic of the natural processing of sequential input, has been excluded from the experimental situation.

Existing coding principles based on stimulus properties of a binary or trinary event sequence are not common. With the exception of the previously cited rule of coding into runs of like events, most suggestions about coding are based on purely cognitive rules such as grouping short sections into octal or verbal codes. Typically, subjects instructed in these procedures do no better or do even worse than uninstructed subjects (see Glanzer & Fleishman, 1967). In any case, there is no evidence that such cognitive rules unrelated to stimulus properties of the pattern are used in perceptual tasks by uninstructed subjects. A major purpose of the present model is to propose a set of rules (axioms) describing a perceptual coding process. Because of its emphasis on coded elements, the model will be referred to as a coded element processing system or CEPS.

Before dealing directly with the problem of coding, it must be made clear that the following discussion will be focused on the application of the coding principles by an ideal processing system. CEPS is ideal in the sense that it represents the perfect application of the coding principles and of all other features of the model. Obviously, subjects will only approach such efficiency, but for patterns of simple and moderate complexity such as those used here, it will be shown that it is safe to assume that the subject closely approximates the model. Because of this assumption of perfect processing, the model falls into the relatively uncommon class of parameter-free models.

The Coding Principles

It is assumed that the stimulus pattern is presented as a sequence of discrete events, each of which is separated by a constant no-stimulus interval. Almost all experiments investigating the perception of sequential patterns have used this kind of stimulus presentation. If the pattern is presented only once and not repeated, then it is assumed that the subject scans the pattern repeatedly from left to right. (Note: The model can be easily adapted to other consistent methods of scanning such as right-to-left scanning.) As examples of such experimental situations, the pattern abaabaab could be presented repeatedly one event at a time, for example, as a sequence of tones, or several repetitions of the pattern could be presented at the same time on a stimulus card, or the pattern could be presented without repetition in a single tachistoscopic exposure.

In the perception of a sequential pattern, CEPS proceeds through a hierarchy of levels of perceptual organization. At each level the pattern is represented as a sequence of coded elements. This sequence specifies a series of coded stimulus-element and coded response-element contingencies. Each response element at a particular point in the sequence serves as the stimulus element at the next point. The task for CEPS is to code the pattern into larger and larger coded elements until the repeating pattern itself is the only coded element. When this stage is reached, CEPS stops.

There are two basic properties or dimensions of the elements in the patterns considered here. The first and most important is the nonmetric qualitative difference between elements, for example, a versus b. This will be called the nominal property. One of the chief characteristics of the coding principle is that repetitions of identical events are grouped into runs of these identical events, for example, a, a, a, a becomes (aaaa). This grouping immediately introduces a quantitative or metric difference between elements based on the number of repetitions, that is, the element length. The presence of both nonmetric and metric differences among the elements poses a problem for any coding principles. For the patterns considered here, the coding principles assume that the nonmetric characteristics are the only bases for forming coded elements.

From the point of view of the subject, it is assumed that he predicts the elements in sequence. Even when the task does not require an overt prediction it is assumed that prediction occurs on a covert level. In this process, as indicated above, there are two characteristics of the next element which must be predicted: (*a*) what the element actually is and (*b*) the length of the run in which it occurs. The coding principles given below and the uncertainty analysis described in a later section will focus on these two properties. Both properties are characteristics of the stimulus and they are also characteristics of the response which is to be predicted. When one of these properties serves as a stimulus it is symbolized S, and when it is a response characteristic, that is, to be predicted, it is symbolized R.

The coding process is described by the following axioms:

Axiom 1: The initial coded elements with which the process begins are the discrete binary, trinary, $\cdots$, n-ary stimulus elements. These are called the Code Level 1 elements and symbolized e_1.

Definition 1: A run, e^n, is a string of n adjacent identical elements, preceded and followed by any element not identical with those constituting the run.

Axiom 2: The e_1 elements are grouped into runs. These are called the Code Level 2 elements and are symbolized $e_1{}^n$.

Axiom 3: Adjacent $e_1{}^n$ elements are sequentially grouped into a new element, termed a composite element and symbolized e_2. This grouping continues until an e_1 element already in the composite is encountered. At that point the composite element in the process of being formed is completed and a new composite is started. This means that no composite can contain more than one run of any e_1 element. If the application of Axiom 3 allows the pattern to be coded into more than one set of composites, then with one exception, the model does not distinguish between the alternative sets. The exception occurs if the alternative codings have different numbers of composite elements. In this case, the coding with the fewest composite elements is selected. A difference in the number of composites in alternative codings is uncommon; it arises when an alternative grouping ignores a possible run of composites.

Axiom 4: The e_2 elements are grouped into runs. (This repeats the same principle as in Axiom 2.) These are called Code Level 4 elements and are symbolized $e_2{}^n$.

Axiom 5: If necessary, new composite elements and new runs of composite elements are formed by applying the operations given in Axioms 3 and 4.

Axiom 6: The process of forming new coded elements stops when the repeating pattern itself is formed as a coded element.

For an examination of the coding process, the following repeating trinary pattern will be used: abcabcaccbb.

Code Level 1: Here the coded elements are the simple discrete n-ary events themselves in the example a, b, and c. Since the pattern has not been formed, CEPS proceeds to Code Level 2.

Code Level 2: Here the coded elements are the runs of identical e_1 elements. The pattern is now coded: a b c a b c a cc bb.

Code Level 3: At this level new composite elements are formed by grouping adjacent Code Level 2 elements, runs, until CEPS encounters a Code Level 1 element already in the grouping.

Axiom 3 may be applied beginning with any of the previous $e_1{}^n$ elements and in the present case this means the pattern may be coded in any of three ways:

Code Group 1: (abc) (abc) (accbb).
Code Group 2: (bca) (bca) (ccbba).
Code Group 3: (cab) (ca) (ccbba).

Code Group 3 can be immediately ruled out as a possible coding because it is an unstable set of coded composites. On the next repeat of the pattern, Axiom 3 will form the composite element (bca), and the process will then stabilize with Code Group 2. (In Table 1, Pattern 3 demonstrates at Code Level 3 another example of unstable grouping.) Since the remaining two alternative codings in the example are both stable and have the same number of composite elements, the model does not distinguish between them.

Observe that at Code Level 3 the repeating pattern has still not been formed, and so CEPS proceeds to Code Level 4 which consists of the runs of e_2 elements. At this level the pattern is coded into {(abc) (abc)} (accbb) or its equivalent. At the next and final code level, Level 5, the single composite which is the repeating pattern itself is formed.

In summary, the coding process begins at Code Level 1 with the initial events, e_1. At Code Level 2 CEPS forms runs of identical e_1 elements. At Code Level 3 the process groups together adjacent $e_1{}^n$ elements until it encounters an e_1 element which is already in the grouping. These composite elements at Code Level 3 are symbolized e_2; these elements are then formed into runs of identical composites. This repeats the process described at Level 2. If still higher elements are necessary, the coding process continues in the same systematic way, forming new elements, e_k, and grouping them into runs $e_k{}^n$.

It is important to note that the coding principles do not fix the set of coded elements in advance. Instead they are an algorithm which allows the specification or generation of a hierarchy of increasingly larger but fewer elements, the particular elements depending on the characteristics of the pattern. In addition, the types of elements generated are the same as, or very similar to, those described in the literature on human coding of binary and trinary patterns. Besides the literal events themselves, the most common coded elements reported are runs of like events, alternations, and runs of alternations (see Feldman, 1963; Gambino & Myers, 1967; Payne, 1966; Royer, 1967; Vitz, 1968).

Examples of the coded elements for a variety of binary and trinary patterns are shown in Table 1. A little study of them will quickly familiarize the reader with the coding principles and facilitate understanding of the uncertainty evaluation.

The Information Evaluation

The second major characteristic of the model is the method used in evaluating the uncertainties associated with the coded elements. Before describing the uncertainty evaluation in detail, a few necessary characteristics of such an uncertainty evaluation will be mentioned. The method of evaluation should be capable of representing the uncertainties associated with the two dimensions of the elements, that is, element quality and element length; it also should reflect the transition or conditional uncertainty arising from the sequential processing of the coded elements, and if possible the analysis should be a familiar and well-developed method and not a specially created one. Fortunately, Garner's (1962, Ch. 5) well-known multivariate uncertainty analysis meets all three of these criteria.

At each code level, the model evaluates two uncertainties: H_{max} and the joint uncertainty, H_{joint} (Garner, 1962, p. 148). These two terms represent two different characteristics of a pattern's complexity. Before *S-R* transitional uncertainties can be evaluated, the perceiving system must first independently evaluate the stimulus-element and response-element uncertainties. H_{max} represents the information associated with the different stimulus and response elements treated independently. Once H_{max} has been evaluated, it is possible to evaluate the uncertainty that remains after the nonindependence of the elements has been taken into account; this uncertainty is represented by H_{joint}. The reduction in uncertainty in proceeding from H_{max} to H_{joint}, Garner's $H_{contingent}$, is considered to be the reinforcement or reward for the processing system. While this principle of reinforcement is obviously a potentially important part of the model with a number of interesting implications, it is not directly tested in the context of the present experiments.

To more clearly suggest the properties of the coded elements, H_{max} will be symbolized $H_{max}(SE,SL,RE,RL)$ where S = stimulus, R = response, E = element, and L = length. Garner's formula for H_{max} becomes in the present notation:

$$H_{max}(SE,SL,RE,RL) = H(SE) + H(SL) + H(RE) + H(RL).$$

Similarly,

$$H_{joint} = H_{joint}(SE,SL,RE,RL) = H(SE) + H_{SE}(SL) + H_{SE,SL}(RE) + H_{SE,SL,RE}(RL).$$

Now consider again the pattern abcabcaccbb.

Code Level 1

The model begins by evaluating H_{max} at Level 1, $H_{max}{}^1$.

$$H_{max}{}^1 = H^1(SE) + H^1(SL) + H^1(RE) + H^1(RL).$$

For sequential patterns the *SE* and *RE* terms of H_{max} will always be identical, as will the *SL* and *RL* terms. The terms, $H^1(SE)$ and $H^1(RE)$ are just the uncertainties associated with the simple event probabilities. And since at this level we are dealing only with elements of unit length, there is no stimulus or response length uncertainty. Thus, $H^1(SL)$ and $H^1(RL) = 0$. In complete form:

$$\begin{aligned} H_{max}{}^1 &= H^1(SE) + H^1(SL) + H^1(RE) + H^1(RL) \\ &= H(3/11, 4/11, 4/11) + H(1,0) + H(3/11, 4/11, 4/11) + H(1,0) \\ &= 1.57 + 0.00 + 1.57 + 0.00 \\ &= 3.14. \end{aligned}$$

The model now evaluates H_{joint} at Code Level 1:

$$H_{joint}{}^1 = H^1(SE) + H_{SE}{}^1(SL) + H_{SE,SL}{}^1(RE) + H_{SE,SL,RE}{}^1(RL).$$

$$H^1(SE) = 1.57 \text{ (see above)}.$$

$H_{SE}{}^1(SL)$ is the uncertainty in an element's length given knowledge of the element. Again since there are only ele-

TABLE 1

EXAMPLES OF THE HIERARCHY OF CODED ELEMENTS FORMED BY APPLYING THE CODING AXIOMS OF CEPS TO REPEATING BINARY AND TRINARY PATTERNS

Code level	Repeating pattern			
	1	2	3	4
Code Level 1 e_1 (initial elements)	abbabbab	ababaababb	abcaabbaab	abcabbcabcabbcaabbc
Code Level 2 e_1^n (runs of e_1)	(a)(bb)(a)(bb)(a)(b)	(a)(b)(a)(b)(aa)(b)(a)(bb)	(a)(b)(c)(aa)(bb)(aa)(b)	(a)(b)(c)(a)(bb)(c)(a)(b) (c)(a)(bb)(c)(aa)(bb)(c)
Code Level 3 e_2 (composites of e_1^n)	1. (abb)(abb)(ab) 2. (bba)(bba)(ba)	1. (ab)(ab)(aab)(abb) 2. (ba)(baa)(ba)(bba)	1. (abc)(aabb)(aab) 2. (bcaa)(bbaa)(ba) (caabb)(aab)(ab)[a]	1. (abc)(abbc)(abc)(abbc) (aabbc) 2. (bca)(bbca)(bca)(bbcaa) (bbca) 3. (cab)(cabb)(cab)(cabb) (caabb)
Code Level 4 e_2^n (runs of e_2)	1. (abbabb)(ab) 2. (bbabba)(ba)	1. (abab)(aab)(abb) (ba)(baa)(ba)(bba)[b]	1. no change 2. no change	1. no change 2. no change 3. no change
Code Level 5 e_3 (composites of e_2^n)	[c]	[c]	[c]	1. (abcabbc)(abcabbcaabbc) 2. (bcabbca)(bcabbcaabbca) 3. (cabcabb)(cabcabbcaabb)
Code Level 6 e_3^n (runs of e_3)				1. no change 2. no change 3. no change
Code Level 7 e_4 (composites of e_3^n)				[c]

[a] Unstable method of coding.
[b] Grouping not acceptable on the basis of number of elements criterion.
[c] At this code level the only element is the repeating pattern itself.

TABLE 2

MATRICES CORRESPONDING TO $H_{SE}{}^1(SL)$, $H_{SE,SL}{}^1(RE)$, AND $H_{SE,SL,RE}{}^1(RL)$

p_i	SE	(SL) 1
3/11	'a'	1
4/11	'b'	1
4/11	'c'	1

$H_{SE}{}^1(SL) = 3/11\,H(1, 0)$
$+ 4/11\,H(1, 0)$
$+ 4/11\,H(1, 0)$

p_i	SE, SL	(RE) 'a'	'b'	'c'
3/11	a	0	2/3	1/3
4/11	b	1/4	1/4	1/2
4/11	c	1/2	1/4	1/4

$H_{SE,SL}{}^1(RE) = 3/11\,H(2/3, 1/3)$
$+ 4/11\,H(1/4, 1/4, 1/2)$
$+ 4/11\,H(1/2, 1/4, 1/4)$

p_i	SE, SL, RE	(RL) 1
2/11	a – 'b'	1
1/11	a – 'c'	1
1/11	b – 'b'	1
2/11	b – 'c'	1
1/11	b – 'a'	1
1/11	c – 'c'	1
2/11	c – 'a'	1
1/11	c – 'b'	1

$H_{SE,SL,RE}{}^1(RL)$
$= 11/11\,H(1, 0)$

Note.—Quotes around an element indicate that only the character of the element but not its length is known.

ments of unit length at Code Level 1 this uncertainty is 0.00. The matrix representing this somewhat trivial case is presented in Table 2.

$H_{SE,SL}{}^1(RE)$ is the uncertainty in the response element given the preceding stimulus element and its length. The matrix representing this case is in Table 2 also.

$$\begin{aligned} H_{SE,SL}{}^1(RE) &= 3/11\,H(2/3,\ 1/3) \\ &\quad +4/11\,H(1/4,\ 1/4,\ 1/2) \\ &\quad +4/11\,H(1/2,\ 1/4,\ 1/4) \\ &= 1.24. \end{aligned}$$

$H_{SE,SL,RE}{}^1(RL)$ is the uncertainty in the response length given the preceding stimulus element and its length, and the response element. This term is 0.00 and its appropriate matrix is in Table 2.

In summary, $H_{\text{joint}}{}^1 = 1.57 + .00 + 1.24 + .00 = 2.81.$

Next H^1 is defined as the *sum* of $H_{\max}{}^1$ and $H_{\text{joint}}{}^1$. This term represents the total amount of information processed at Code Level 1, and it can be considered as roughly analogous to the amount of work associated with processing the Code Level 1 elements. Since $H_{\max}$ and H_{joint} are not independent, it is clear that some uncertainties are duplicated in this sum. It is believed that it is plausible to assume that this kind of reprocessing of information occurs in perceptual organization. This lack of independence may seem to weaken the case for combining $H_{\max}$ and H_{joint}. However, rational and empirical grounds for discarding alternative measures based on various plausible and independent uncertainty terms are presented in the Discussion section.

The more familiar term, $H_{\text{contingent}}$, $(H_{\max} - H_{\text{joint}})$, already mentioned, represents the amount of reinforcement or reduced uncertainty for CEPS at any code level. In general $H_{\text{contingent}}$ will be nonzero, representing the reinforcement which normally occurs when one takes into account the nonindependence present in most sequential patterns.

To conclude the Code Level 1 analysis:

$$\begin{aligned} H^1 &= H_{\max}{}^1 + H_{\text{joint}}{}^1 \\ &= 3.14 + 2.81 = 5.95 \text{ bits.} \end{aligned}$$

Code Level 2

At this code level the same uncertainties are evaluated for Code Level 2 elements, the runs of e_1.

$H^2(SE)$ and $H^2(RE)$ are the uncertainties based on the five different runs a, b, c, bb, and cc. In the present pattern, runs of a's, b's, and c's occur equally often and therefore both these H values equal $H(1/3,\ 1/3,\ 1/3)$ or 1.58. $H^2(SL)$ and $H^2(RL)$ are the uncertainties of the run lengths which in this case take on the values

TABLE 3

MATRICES CORRESPONDING TO $H_{SE}{}^2(SL)$, $H_{SE,SL}{}^2(RE)$, AND $H_{SE,SL,RE}{}^2(RL)$

p_i	SE	(SL) 1	(SL) 2
1/3	'a'	1	0
1/3	'b'	2/3	1/3
1/3	'c'	2/3	1/3

$H_{SE}{}^2(SL) = 1/3\,H(1, 0) + 1/3\,H(2/3, 1/3) + 1/3\,H(2/3, 1/3)$

p_i	SE, SL	(RE) 'a'	(RE) 'b'	(RE) 'c'
1/3	a	0	2/3	1/3
2/9	b	0	0	1
1/9	bb	1	0	0
2/9	c	1	0	0
1/9	cc	0	1	0

$H_{SE,SL}{}^2(RE) = 1/3\,H(2/3, 1/3) + 2/3\,H(1, 0)$

p_i	SE, SL, RE	(RL) 1	(RL) 2
2/9	a – 'b'	1	0
1/9	a – 'c'	0	1
2/9	b – 'c'	1	0
1/9	bb – 'a'	1	0
2/9	c – 'a'	1	0
1/9	cc – 'b'	0	1

$H_{SE,SL,RE}{}^2(RL) = 9/9\,H(1, 0)$

Note.—Quotes around an element indicate that only the character of the element but not its length is known.

of 1 and 2 with probabilities of 7/9 and 2/9, respectively.

$H^2(SL)$ and $H^2(RL) = H(7/9,\ 2/9)$ or .76.

Thus,

$$H_{\max}{}^2 = H(1/3, 1/3, 1/3) + H(7/9, 2/9) + H(1/3, 1/3, 1/3) + H(7/9, 2/9) = 1.58 + .76 + 1.58 + .76 = 4.68.$$

$$H_{\text{joint}}{}^2 = H^2(SE) + H_{SE}{}^2(SL) + H_{SE,SL}{}^2(RE) + H_{SE,SL,RE}{}^2(RL).$$

$H^2(SE) = 1.58$ as evaluated above. $H_{SE}{}^2(SL)$ is the uncertainty in run length given the element composing the run. In this case, knowing that you are in a run of a's completely specifies its length, while knowing you are in a run of b's or c's, still leaves uncertainty about the length of the run. Table 3 shows the components of $H_{\text{joint}}{}^2$.

$$H_{SE}{}^2(SL) = 1/3\,H(1, 0) + 1/3\,H(2/3, 1/3) + 1/3\,H(2/3, 1/3) = 0 + .30 + .30 = .60.$$

$H_{SE,SL}{}^2(RE)$ is the uncertainty about the response element given the stimulus element and its length, that is, run and run length. In the present example, the only uncertainty is after an a which may be followed by a run of b's or c's; specifically, $H_{SE,SL}{}^2(RE) = .30$. Next $H_{SE,SL,RE}{}^2(RL)$ is evaluated, which in this case is zero, although this is not necessarily true for other patterns. In summary,

$$H_{\text{joint}}{}^2 = 1.58 + .60 + .30 + .00 = 2.48.$$

And

$$H^2 = H_{\max}{}^2 + H_{\text{joint}}{}^2 = 2.48 + 4.68 = 7.16 \text{ bits.}$$

Code Level 3

The same uncertainties on the Code Level 3 elements are evaluated. Only the first of the two possible equivalent codings previously mentioned, (abc) (abc) (accbb), will be considered. The element (abc) occurs with probability 2/3 and (accbb) with probability 1/3. The length of a composite is defined as the number of elements of the immediately preceding code level in the composite, that is, the number of runs in a composite is its length at Code Level 3.

At this code level:

$$H_{\max}{}^3 = H(2/3, 1/3) + H(1, 0) + H(2/3, 1/3) + H(1, 0) = .91 + .00 + .91 + .00 = 1.82.$$

The matrices representing the conditional components of $H_{\text{joint}}{}^3$ are presented in Table 4. Therefore,

$$H_{\text{joint}}{}^3 = H(2/3, 1/3) + H(1, 0) + 2/3\,H(1/2, 1/2) + H(1, 0) = .91 + .00 + .67 + .00 = 1.58.$$

$$H^3 = H_{\max}{}^3 + H_{\text{joint}}{}^3 = 1.82 + 1.58 = 3.40 \text{ bits.}$$

TABLE 4

CODE LEVEL 3 MATRICES CORRESPONDING TO $H_{SE}{}^3(SL)$, $H_{SE,SL}{}^3(RE)$, AND $H_{SE,SL,RE}{}^3(RL)$

p_i	SE	(SL)
		3
2/3	'abc'	1
1/3	'accbb'	1
$H_{SE}{}^3(SL) = 3/3\ H(1, 0)$		

p_i	SE, SL	(RE)	
		'abc'	'accbb'
2/3	abc	1/2	1/2
1/3	accbb	1	0
$H_{SE,SL}{}^3(RE) = 2/3\ H(1/2, 1/2) + 1/3\ H(1, 0)$			

p_i	SE, SL, RE	(RL)
		3
1/3	abc – 'abc'	1
1/3	abc – 'accbb'	1
1/3	accbb – 'abc'	1
$H_{SE,SL,RE}{}^3(RL) = 3/3\ H(1)$		

Note.—Quotes around an element indicate that only the character of the element but not its length is known.

Code Level 4

At this code level there are two elements (abcabc) and (accbb), each occurring with probability 1/2. The first is of Length 2 and the second of Length 1, the length being the number of like events, in this case composites. Therefore, $H_{max}{}^4 = H(1/2, 1/2) + H(1/2, 1/2) + H(1/2, 1/2) + H(1/2, 1/2) = 4.00$. The matrices representing the conditional components of $H_{joint}{}^4$ are presented in Table 5. Therefore,

$$H_{joint}{}^4 = H(1/2, 1/2) + H(1, 0) + H(1, 0) + H(1, 0) = 1.00.$$

$$H^4 = 4.00 + 1.00 = 5.00 \text{ bits.}$$

Code Level 5

At this level there is only one element, the complete pattern and all uncertainty terms are zero.

The Complexity Measure

As mentioned before, the total complexity of a pattern, H_{code}, is defined as the sum of all the uncertainties evaluated in the processing of the pattern. Recalling that H^i is the sum of $H_{max}{}^i$ and $H_{joint}{}^i$, then $H_{code} = \sum_{i=1}^{k} H^i$ where k is the final code level reached in the processing of the pattern.

For the relatively complicated pattern used as the example,

$$H_{code} = 5.95 + 7.16 + 3.40 + 5.00 = 21.51 \text{ bits.}$$

Most existing measures of the complexity of sequential patterns are response measures. For example, Glanzer and Clark (1963a, 1963b) use the mean number of words to describe a pattern as a measure of pattern complexity. Although useful, there are fundamental difficulties with such response measures. One important limitation is that complexity cannot be evaluated in advance. Because of the large number of patterns of experimental interest, having to run subjects to get a measure is a serious restriction. In addition, a correlation be-

TABLE 5

CODE LEVEL 4 MATRICES CORRESPONDING TO $H_{SE}{}^4(SL)$, $H_{SE,SL}{}^4(RE)$, AND $H_{SE,SL,RE}{}^4(RL)$

p_i	SE	(SL)	
		1	2
1/2	'abcabc'	0	1
1/2	'accbb'	1	0
$H_{SE}{}^4(SL) = H(1, 0)$			

p_i	SE, SL	(RE)	
		'accbb'	'abcabc'
1/2	abcabc	1	0
1/2	accbb	0	1
$H_{SE,SL}{}^4(RE) = H(1, 0)$			

p_i	SE, SL, RE	(RL)	
		1	2
1/2	abcabc – 'accbb'	1	0
1/2	accbb – 'abcabc'	0	1
$H_{SE,SL,RE}{}^4(RL) = H(1, 0)$			

Note.—Quotes around an element indicate that only the character of the element but not its length is known.

TABLE 6

H^i VALUES AND H_{code} FOR THE BINARY PATTERNS USED IN VITZ (1968), ROYER AND GARNER (1966), AND ROYER (1967), AND JUDGED COMPLEXITY VALUES FROM VITZ (1968)

Pattern number	H^1	H^2	H^3	H^4	H_{code} (ΣH^i = Complexity)	Judged complexity, mean rank (N = 15)
	e_1	e_1^n	e_2	e_2^n		
1. a	0.00				0.00	1.00
2. ab	3.00	3.00			6.00	2.40
3. aabb	4.00	3.00			7.00	5.60
4. aaab	3.11	5.00			8.11	5.40
5. aaaabbbb	3.81	3.00			6.81	5.33
6. aaaaabbb	3.64	5.00			8.64	7.07
7. aaaaaabb	3.17	5.00			8.17	5.93
8. aaaaaaab	2.14	5.00			7.14	3.60
9. aaababab	3.46	5.21	3.40	5.00	17.07	14.33
10. aaabbbab	4.00	6.00	3.00	3.00	16.00	14.00
11. aaaabbab	3.80	7.00	3.00	3.00	16.80	14.13
12. aaaaabab	3.11	5.62	3.00	3.00	14.73	10.80
13. aaaababb	3.80	7.00	3.00	3.00	16.80	15.60
14. aaaabaab	3.11	7.00	3.00	3.00	16.11	11.33
15. aaabaabb	3.80	7.00	3.00	3.00	16.80	15.40
16. aaabbaab	3.80	7.00	3.00	3.00	16.80	14.47
17. aaabbabb	4.00	7.00	3.00	3.00	17.00	14.13
18. aabababb	3.81	6.06	3.40	5.00	18.27	16.33
19. aababbab	3.81	6.40	4.74	4.74	19.69	17.80
20. aabaabab	3.46	5.73	3.40	5.00	17.59	15.33

tween two response measures is often hard to interpret causally. H_{code} being derived from the stimulus avoids these problems.

A recent and interesting stimulus measure of complexity has been developed by Alexander and Carey (1968). For any binary string their measure is obtained by counting the relative number of symmetric sub-sequences within the string. The fewer the number of symmetric sub-sequences the more complex the pattern. The conceptual simplicity of this measure has much to recommend it. However, the measure is developed and tested only for binary patterns; its computation gets lengthy for patterns of Length 12 or greater, for example, a pattern of Length 12 has 352 sub-sequences to be evaluated for symmetry; and the measure's relation to the issues of perceptual coding and processing is not developed.

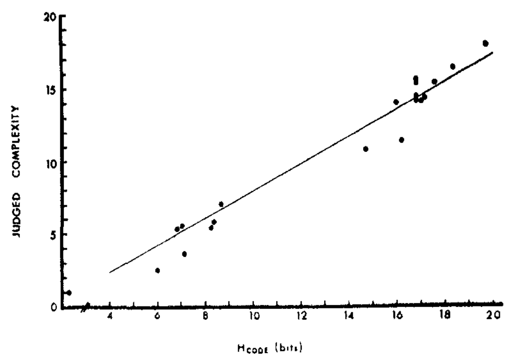

FIG. 1. Mean rank of judged complexity of 20 repeating binary patterns as a function of H_{code} (data from Vitz, 1968).

SUPPORTING DATA

Repeating Patterns: Binary

In a recent study, Vitz (1968) had 15 college subjects rank the complexity of the 20 logically possible repeating binary patterns of Length 8. These patterns are shown in Table 6 with their H^i and H_{code} values. All other possible eight-place patterns are equivalent either by complementarity or by cyclic repetition. Each pattern was typed on a 3 × 5 inch card and repeated until the string of binary characters was 32 in length. The subjects ranked the cards from simplest, Rank 1, to most complex, Rank 20. The scatter diagram for the correlation of .98 between the H_{code} and mean complexity ranks for these patterns is shown in Figure 1.

The same 20 repeating patterns were used in a study by Royer and Garner (1966). They presented the patterns as repeating series of two distinct tones. The subject listened to the pattern and at whatever time he believed he could follow the pattern he started predicting. Although the subjects had an equal opportunity to select any of a pattern's logically possible starting points, they usually showed marked preferences for particular starting points. Royer and Garner computed the uncertainty associated with the probabilities of starting to predict each pattern at the different possible starting points. This measure, called "response point uncertainty" (*RPU*), is proposed as a measure of subjects' difficulty in organizing a pattern and can be considered a measure of pattern complexity. The correlation of .93 between H_{code} and *RPU* is shown in Figure 2.

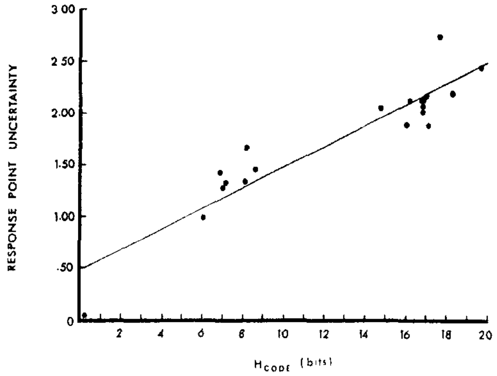

FIG. 2. Response point uncertainty of 20 repeating binary patterns as a function of H_{code} (*RPU* data from Royer & Garner, 1966).

The same 20 repeating binary patterns were used in a related study by Royer (1967). In this study the subject was shown the binary pattern on a card and his task was to reproduce the pattern by pressing two telegraph keys. The keys had to be pressed at a speed that kept pace with a preset auditory click, starting at one click per second and accelerating at a rate of .2 clicks per second every eight events. For each pattern, the subject's score was the maximum number of responses per second which he could make before an error was made or before failing to keep pace with the click rate. The correlation between H_{code} and maximum responses per second is $-.83$. The scatter diagram in this case is similar to Figure 2 and shows the same two clusters. The lower correlation appeared to be due to a generally greater dispersion about the regression line within each cluster and not to any systematic discrepancy from the predicted values.

One interpretation of this lower correlation is that Royer's response measure is not ideally suited for testing the finer discriminations between patterns made by the H_{code} measure. Specifically, this response measure may be primarily influenced by the most difficult point in the pattern, since this would seem to be the chief determinant of the first error. If so, such a measure would not discriminate well between patterns only having one, as compared with two or three, difficult points.

Repeating Patterns: Trinary

Experiment I

The following experiment was run to test H_{code} as a predictor of judged complexity for repeating trinary patterns.

Method. Two groups of 10 subjects were run. All subjects were undergraduates fulfilling an introductory psychology course requirement at New York University. Sixteen trinary patterns were used, representing all of the logically different repeating trinary patterns of Length 3, 4, 5, and 6. These patterns are shown in Table 7 with their corresponding H^i and H_{code} values. All other trinary patterns of these lengths are equivalent, either by complementarity or cyclic repetition. Two representations of these trinary patterns were constructed. In Set 1, the three elements were the uppercase letters X, B, and J. S, L, and P were used in Set 2. These elements were selected because they have quite discriminable visual and auditory properties, yet do not form words or other familiar combinations. The trinary patterns were typed, leaving a blank space between letters, on 5×7 inch index cards until the string of characters was 30 in length. The patterns of Length 4 were repeated eight times to form a string of 32 characters. (Previous studies indicate that slight differences in the number of repetitions of the pattern or in length of the string make no appreciable difference.)

Each subject was run individually and had the set of 16 cards arranged haphazardly on the desk in front of him. The subject's task was first to divide the cards into the eight simplest and eight most complex patterns. Then he ranked the eight

TABLE 7

H^i VALUES, H_{code}, AND JUDGED COMPLEXITY FOR THE TRINARY PATTERNS USED IN EXPERIMENT I

Pattern number	H^1 e_1	H^2 e_1^n	H^3 e_2	H^4 e_2^n	H_{code}	Judged complexity, mean rank ($N = 20$)
1. abc	4.74	4.74			9.48	1.7
2. aabc	5.00	6.56			11.56	5.6
3. aaabc	4.66	6.56			11.22	4.2
4. aabbc	5.36	6.56			11.92	6.6
5. aabbcc	5.74	4.74			10.48	4.2
6. aaaabc	4.29	6.56			10.85	4.4
7. aaabbc	5.17	7.90			13.07	6.8
8. aaabac	4.75	6.62	3.00	3.00	17.37	9.4
9. abcacb	5.74	5.74	3.00	3.00	17.48	10.3
10. aabaac	4.75	7.00	3.00	3.00	17.75	8.6
11. abbbac	5.17	6.62	3.00	3.00	17.79	11.7
12. ababac	4.84	4.84	3.40	5.00	18.08	13.0
13. abbaac	5.50	7.00	3.00	3.00	18.50	13.2
14. abbacc	5.74	7.00	3.00	3.00	18.74	9.8
15. ababcc	5.41	6.40	5.00	3.00	19.81	12.5
16. abcacc	5.17	6.80	5.00	3.00	19.97	14.1

simplest in order of complexity, and of these the five simplest were set aside and their ranks recorded. Next the subject selected the three most complex patterns and set them aside and ranked the eight remaining patterns from simplest to most complex. The five simplest of this ranking were set aside and recorded, and the remaining six were then ranked. This modified ranking procedure guaranteed that the subject attended to his judgments. The entire procedure lasted about 25 minutes.

Results. The correlation of .95 between H_{code} and judged complexity for these 16 repeating trinary patterns is shown in Figure 3. The complexity rankings of each set of 10 subjects were intercorrelated to get information on the consistency between subjects. These correlations ranged from 94 to .16. The median intercorrelation was .67 in both sets.

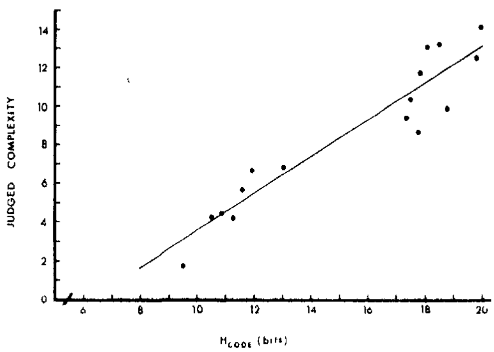

FIG. 3. Mean rank of judged complexity of 16 repeating trinary patterns as a function of H_{code} (data from Experiment I).

Nonrepeating Patterns: Binary

A typical nonrepeating binary pattern is one of a short, fixed length which is presented to the subject tachistoscopically. The subject has a task such as reproducing the pattern as accurately as possible, describing it with words, etc. CEPS can be applied to such patterns on the assumption that the subject repeatedly processes the pattern from left to right. It is assumed that when the end of the pattern has been reached, the processing is reset to the first element.

The application of CEPS to such patterns is straightforward with one exception. On a few patterns, the application of the rules of compositing elements previously outlined will form all of the elements into composites with the exception of a single element at the very end of the pattern. It seems unsatisfactory to have an element at the composite level which is not included in a composite, only because it happens to be the last element. To resolve this difficulty for nonrepeating patterns, a special axiom for these final elements has been added:

Axiom 3a: If the application of Axiom 3 to nonrepeating patterns leaves a single element at the end of the pattern, then that element is to be included as part of the immediately preceding composite and not considered as a separate element.

As an example of the application of Axiom 3a, consider the nonrepeating trinary pattern, Number 19, shown in Table 8. At the first composite level, Axiom 3 would lead to the following coding (abbc) (abbc) (bbc) (b). The last element, b, is the kind of element to which Axiom 3a applies. Axiom 3a codes this pattern as (abbc) (abbc) (bbcb).

An experiment by Glanzer and Clark (1963b) provides data to test the prediction of CEPS on nonrepeating binary patterns.[3] In their Experiment 2, they presented tachistoscopically the 256 binary numbers of Length 8. The subjects ($N=39$) had 30 seconds in which to write a description of each pattern. The mean number of words to describe a pattern, termed mean verbalization length or *MVL*, is used as a measure of pattern complexity.

Results. Of the 256 patterns, the *MVL* scores of logically equivalent patterns were combined. Thus, the scores of each pattern and its complement were averaged and the scores of each pattern and its right-left reversal, if it had one, were averaged. This procedure results in 72 different patterns and provides a more representative pattern score, since differences between the scores of equivalent patterns are due to random error, or to the easier perception of one of the binary elements, or to preexperiment perceptual habits.

There were three special patterns which required adjusted H_{code} values. These patterns were: 11001100; 11101110; 10101010. These contain short patterns that repeat two or four times. H_{code} is a measure of the fundamental pattern only and does not reflect the number of times the pattern is

[3] The authors wish to thank Murray Glanzer for graciously providing these data.

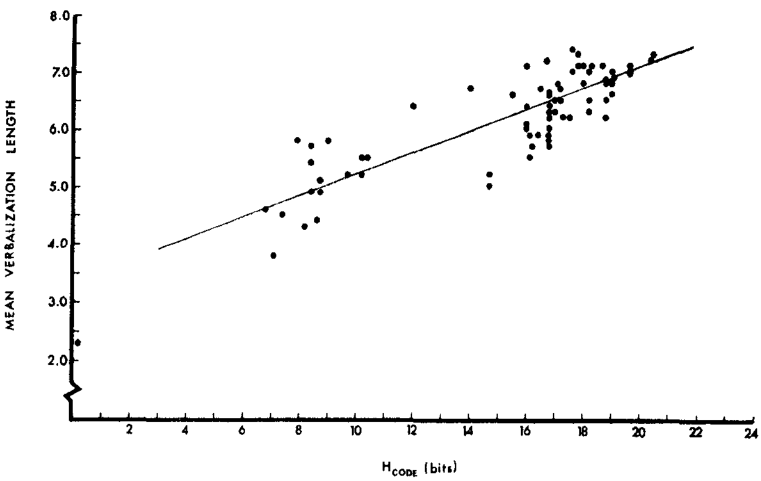

FIG. 4. Mean verbalization length of 72 nonrepeating binary patterns as a function of H_{code} (MVL data from Glanzer & Clark, 1963b).

repeated. Thus, H_{code} for 10 is the same as for 10101010. Obviously, adjustment for the number of times a pattern is repeated is needed. The adjustment used was to multiply the H_{code} value of the pattern by the number of pattern repetitions. The correlation of .87 between H_{code} and MVL for these 72 nonrepeating patterns is shown in Figure 4.

Glanzer and Clark also report an accuracy of recall score for each binary number. After a 1/2-second tachistoscopic exposure, 80 subjects wrote down the number, and the proportion of subjects whose recall was perfect served as an index of the number's complexity. The correlation between H_{code} and these accuracy of recall scores is $-.82$.

Experiment II

Experiment II was run in a manner similar to Experiment I in order to test the predictions of CEPS for nonrepeating trinary patterns. Fifteen New York University undergraduates ranked the

TABLE 8

H^i VALUES, H_{code}, AND JUDGED COMPLEXITY FOR THE TRINARY PATTERNS USED IN EXPERIMENT II

Pattern number	H^1	H^2	H^3	H^4	H^5	H^6	H_{code} (ΣH^i = Complexity)	Judged complexity, mean rank (N = 15)
	e_1	$e_1{}^n$	e_2	$e_2{}^n$	e_3	$e_3{}^n$		
1. abc	4.74	4.74					9.48	1.1
2. aabc	5.00	6.56					11.56	3.1
3. aabbc	5.36	6.56					11.92	5.1
4. aaabc	4.66	6.56					11.22	4.2
5. aaabbc	5.17	7.90					13.07	7.9
6. aabcab	5.17	6.80	5.00	3.00			19.97	12.4
7. aabacb	5.17	6.80	5.00	3.00			19.97	11.0
8. ababac	4.84	4.84	3.40	5.00			18.08	8.3
9. aaaaaabc	3.67	6.56					10.23	5.9
10. abababac	5.00	5.00	4.00	3.00			17.00	9.7
11. aaabaacb	4.75	6.73	5.00	3.00			19.48	13.9
12. abbabacb	4.98	6.60	6.56	4.74			22.88	16.5
13. aaaaabbbcc	4.56	6.56					11.12	7.7
14. abbabbaacc	5.96	6.99	3.40	5.00			21.35	14.2
15. abababacbb	4.68	5.89	4.66	5.00			20.23	14.3
16. abbacabacb	5.48	6.77	7.62	6.00			25.87	18.4
17. aaaabbbbcccc	5.55	4.74					10.29	5.6
18. aabbbbcaaccc	4.74	9.20	5.00	3.00			21.94	15.9
19. abbcabbcbbcb[a]	5.24	7.29	3.40	5.00			20.93	16.5
20. abbcababbcac	5.56	6.93	7.00	5.00	3.00	3.00	30.49	18.4

[a] Coded using Axiom 3a.

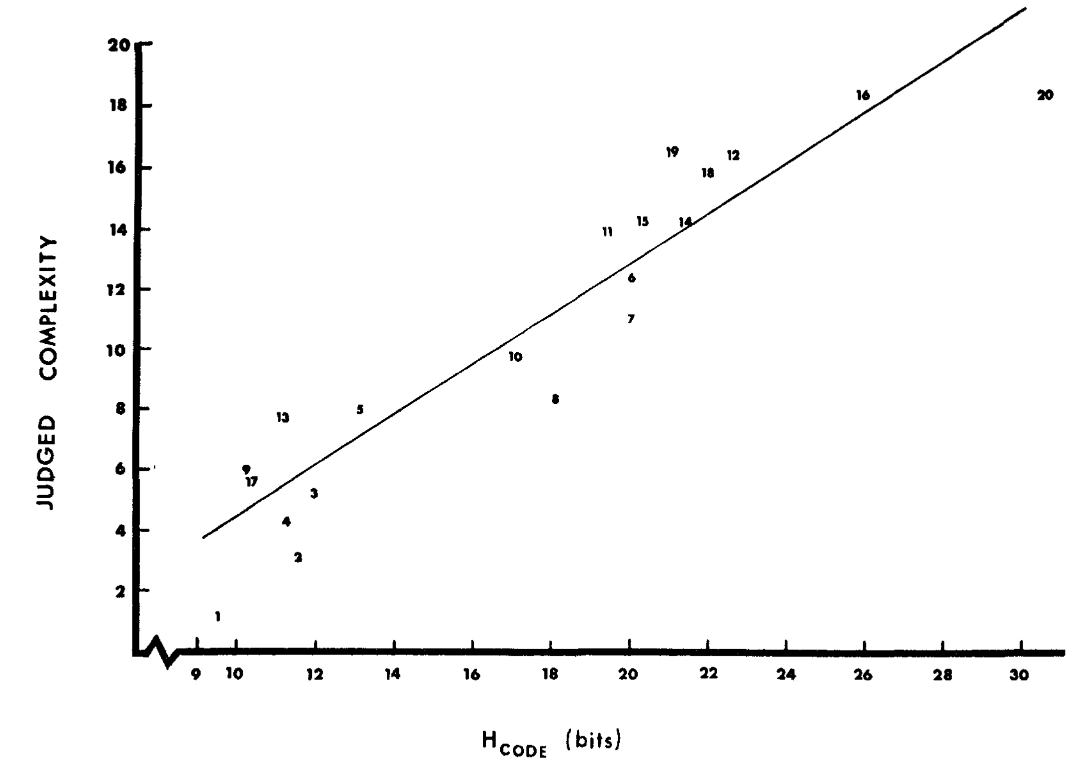

FIG. 5. Mean rank of judged complexity of 20 nonrepeating trinary patterns as a function of H_{code} (data from Experiment II).

complexity of the 20 nonrepeating trinary patterns shown in Table 8.

These patterns include all the logically possible trinary patterns of Length 6 or less, plus four patterns each of Lengths 8, 10, and 12 selected to cover the entire range of H_{code} values for these lengths. All the patterns, with the exception of Pattern 19, are possible repeating trinary patterns. The special significance of Pattern 19 was discussed previously. The subjects' rankings had a median intercorrelation of .84.

Results. The major results are shown in Figure 5 which shows the correlation of .94 between H_{code} and mean judged complexity. The numbers in Figure 5 correspond to the pattern numbers of Table 8. Since these numbers increase with increasing pattern length, it is clear from the deviations from the regression line that the complexity of short patterns is underestimated relative to the longer patterns. This regular deviation from the regression line suggests that prediction could be improved further by adjusting H_{code} for differences in pattern length.

Experiment III

This experiment was designed to test H_{code} as an index of perceived trinary pattern complexity.[4] All the stimuli used were of Length 9 in order to control for length differences.

Method. Thirty-six undergraduates at New York University fulfilling an introductory psychology course requirement served as subjects. Twenty-four different trinary patterns of Length 9 were used. These patterns are shown in Table 9. The trinary elements were the capital letters X, B, and J. Three sets of 24 patterns were constructed with the letters interchanged. These patterns were selected from the 3^9 possible trinary patterns of Length 9 on the basis of two criteria: one, that their H_{code} values did not cluster but were spaced in a reasonably uniform manner. Two, that these patterns on an intuitive basis represented the major classes of possible trinary patterns of Length 9.

The patterns were presented tachistoscopically for 700 milliseconds. The subject's task was to record the pattern as accurately as possible in a response booklet with 9 dashes for each pattern. The procedure was subject-paced, and each subject was run individually. Each subject saw two of the three representations of each pattern, for a total of 48 stimuli. Order of presentation and different stimulus sets were counterbalanced across subjects.

Results. Accuracy scores were obtained for each pattern by finding the mean number of errors made for each pattern. Each incorrect letter within a pattern was scored as an error, so error scores ranged

[4] This experiment was carried out with the assistance of Carol Silverstein.

TABLE 9

H^i VALUES, H_{code}, AND MEAN ERRORS OF RECALL FOR THE TRINARY PATTERNS USED IN EXPERIMENT III

Pattern number	H^1	H^2	H^3	H^4	H^5	H^6	H_{code}	Mean recall errors ($N = 36$)
	e_1	e_1^n	e_2	e_2^n	e_3	e_3^n		
1. aaaaaaabc	3.40	6.56					9.96	.69
2. abbbbbbbc	3.40	6.56					9.96	.95
3. aaabbbccc	5.65	4.74					10.39	1.18
4. acccbcccc	3.84	7.00					10.84	2.40
5. aaaaabbbc	4.75	7.90					12.65	1.44
6. aaabccccc	4.75	7.90					12.65	1.65
7. aaaabbbcc	5.47	7.90					13.37	1.43
8. aaaaaabac	3.84	6.62	3.00	3.00			16.46	1.47
9. abbcbbcbb	4.57	6.89	3.00	3.00			17.46	3.38
10. aaaaabaac	3.84	8.00	3.00	3.00			17.84	2.74
11. ababababc	4.53	4.53	4.73	5.00			18.79	2.83
12. aaabaaabc	4.48	6.90	5.00	3.00			19.38	2.58
13. aabbccabc	5.65	7.74	3.00	3.00			19.39	2.90
14. aabbaabbc	5.06	6.40	5.00	3.00			19.46	2.86
15. abbaccabb	5.56	7.21	3.40	3.40			19.57	3.43
16. aabaaabcc	5.27	8.00	5.00	3.00			21.27	3.32
17. aabbaaccb	5.77	8.00	5.00	3.00			21.77	3.49
18. abaaaabcc	5.27	9.30	5.00	3.00			22.57	3.25
19. abcabcabb	5.25	6.44	6.56	4.74			22.99	3.69
20. abacccabb	5.65	8.00	4.74	4.74			23.13	3.06
21. acbcbbccb	5.19	7.32	6.56	4.74			23.81	4.33
22. acabcbacb[a]	5.65	5.65	7.90	4.74			23.94	4.56
23. abbacabcb[a]	5.77	7.13	6.56	4.74			24.20	4.01
24. abacabacb	5.26	5.26	6.62	4.50	3.00	3.00	27.64	4.15

[a] Coded using Axiom 3a.

from 0 to 9. The correlation of .91 between H_{code} and a pattern's mean number of errors is shown in the scatter diagram of Figure 6.

The intercorrelation of error scores for each pair of subjects ranged from .85 to −.14 with a median of .48. These low correlations are not surprising since presentation order effects were present and since there was relatively little variability in error scores.

DISCUSSION

Before discussing H_{code} a word about three mathematically independent and possible alternative measures to H_{code} is called for. These three are the sum of the H_{max} terms $(\sum_{i=1}^{k} H_{max}{}^i)$, the sum of the H_{joint} terms $(\sum_{i=1}^{k} H_{joint}{}^i)$, and the sum of the $H_{contingent}$ terms $(\sum_{i=1}^{k} H_{contingent}{}^i)$. To evaluate the possible usefulness of these measures each of them was correlated with the dependent measures presented in Tables 6–9. The sum of the $H_{contingent}$ terms was markedly inferior to H_{code} since on the average it accounted for 32% less of the variance than H_{code}. The sum of the H_{joint} terms accounted on the average for 6% less of the variance, and since it is computationally as difficult as H_{code}, it has little to recommend it. The sum of the H_{max} terms, although always slightly inferior to H_{code},

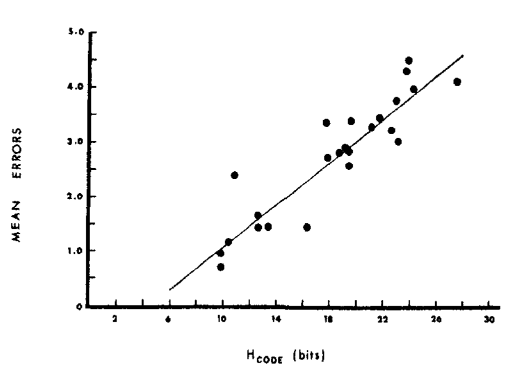

FIG. 6. Mean errors of recall of 24 nonrepeating trinary patterns as a function of H_{code} (data from Experiment III).

on the average accounted for only 3% less variance than H_{code}. Because $\sum_{i=1}^{k} H_{max}{}^{i}$ is considerably easier to compute, it may often be a useful substitute for H_{code}. Of course $\sum_{i=1}^{k} H_{max}{}^{i}$ ignores all sequential uncertainties, and for many tasks this is not defensible, but in cases where sequential evaluation is difficult, then $\sum_{i=1}^{k} H_{max}{}^{i}$ may be an adequate and, in some cases, even a superior measure.

The almost uniformly high correlations between H_{code} and the various response measures provide considerable support for the model and its fundamental assumption of an uncertainty measure applied to a hierarchy of coded elements. Nevertheless, there are a number of questions raised by the data which imply modifications of the model or areas in which further testing would be useful. Discussion will be focussed on these issues.

One characteristic of many of the scatter diagrams is the tendency for the data to fall into two distinct clusters. In the case of binary patterns, this is apparently an unavoidable consequence of the structure of the patterns. This suggests that further investigations of CEPS should include binary, trinary, $\cdots$, n-ary patterns in the same study which have been carefully selected to yield a fairly continuous and uniform distribution of H_{code} values.

One obvious difficulty of any parameter-free model such as CEPS is that it assumes equal weighting of the various aspects of the stimulus situation. This becomes a particularly serious problem when one attempts to apply a model to the variety of stimulus and response situations represented in the preceding experiments. The model makes no adjustment for the absolute length of the pattern or for the number of times a pattern is repeated. Although a simple adjustment for the number of repetitions was used, it is obvious that estimating parameters from the data would more effectively improve the model's fit. Perhaps of more significance is the assumption that each code level has an equal weighting. It seems likely that the lower H_{code} levels are evaluated more efficiently and that some estimate of this efficiency could be usefully incorporated. This differential efficiency would seem to depend strongly on the response required of the subject and on the way this response is summarized by the experimenter. One example was mentioned in the discussion of the experiment by Royer (1967), in which the response measure of a pattern was determined solely by the subject's first failure to keep pace. Such a measure would presumably not depend on higher order organization such as Code Level 3 and above. Evidence for this interpretation is that a measure presented by Vitz (1968) similar to H_{code}, but based only on the run structure, correlates $-.87$ with Royers' response measure, compared with $-.83$ for H_{code}.

The preceding interpretation also allows explanation of the relatively low correlation of $-.82$ between H_{code} and Glanzer and Clark's accuracy of recall measure. If the mean number of errors had been used in their study instead of the simpler proportion of subjects who got the pattern completely correct, presumably a higher correlation with H_{code} would have resulted.

One of the special implications of the model for such tasks as perceptual accuracy is that the perception of the pattern is not an all-or-nothing phenomenon but consists of a hierarchy of thresholds associated with a hierarchy of code levels. This "hierarchy of thresholds" aspect of the model is not directly tested by any of the preceding experiments, although as discussed above there is the suggestion that different response tasks may reflect different levels of this presumed threshold hierarchy.

The coding axioms are the other major characteristic of the model which need further testing. A direct test of the coding axioms would require an experimental situation in which the response more clearly reflected the subject's manner of coding.

There is some evidence in the preceding studies that implies that the present coding axioms are insufficient. Pattern 4 of Experiment III, acccbcccc, resulted in the largest discrepancy from the predicted value. The model predicts that this pattern will be coded into runs and then into a single composite which includes the last element. The authors believe that it is more likely that this pattern is coded into two composites at Code Level 3, (accc) (bcccc). This is one example where a plausible alternative to the model's coding axioms is based on metric properties of the elements. In this example the alternative coding is based on the similarity of the pattern of run lengths, (1–3) (1–4). This alternative would be still more likely if the run lengths were (1–4) (1–4). The present coding axioms are based exclusively on the nominal characteristics of the elements and cannot account for any coding based on metric properties. Another example of very probable coding on the basis of run lengths is Pattern 8 of Table 9, aaaaaabac. The present axioms code this as (aaaaaab) (ac) at Code Level 3. It seems quite likely that this pattern is coded (aaaaaa) (bac). The data in the present studies are not sensitive to such differences. In order to investigate more thoroughly coding strategies, it appears desirable to use metric stimuli, for example, strings of numbers or of intensities. If the coding of sequences of metric events were to be developed, the authors expect that the coding of the nonmetric sequential patterns treated here could be interpreted as a special degenerative case of metric stimuli.

REFERENCES

ALEXANDER, C., & CAREY, S. Subsymmetries. *Perception and Psychophysics*, 1968, **4**, 73–77.

DODWELL, P. D. Coding and learning in shape discrimination. *Psychological Review*, 1961, **68**, 373–382.

FELDMAN, J. Simulation of behavior in the binary choice experiment. In E. A. Feigenbaum & J. Feldman (Eds.), *Computers and thought.* New York: McGraw-Hill, 1963.

GAMBINO, B., & MYERS, J. L. Role of event runs in probability learning. *Psychological Review*, 1967, **74**, 410–419.

GARNER, W. R. *Uncertainty and structure as psychological concepts.* New York: Wiley, 1962.

GLANZER, M., & CLARK, W. H. Accuracy of perceptual recall: An analysis of organization. *Journal of Verbal Learning and Verbal Behavior*, 1963, **1**, 289–299. (a)

GLANZER, M., & CLARK, W. H. The verbal loop hypothesis: Binary numbers. *Journal of Verbal Learning and Verbal Behavior*, 1963, **2**, 301–309. (b)

GLANZER, M., & FLEISHMAN, J. The effect of encoding training on perceptual recall. *Perception and Psychophysics*, 1967, **2**, 561–564.

HILGARD, E. R. & Bower, G. H. *Theories of learning.* New York: Appleton-Century-Crofts, 1966.

MILLER, G. A. The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review*, 1956, **63**, 81–97.

PAYNE, B. The relationship between a measure of organization for visual patterns and their judged complexity. *Journal of Verbal Learning and Verbal Behavior*, 1966, **5**, 338–343.

RESTLE, F. *Psychology of judgment and choice.* New York: Wiley, 1961.

RESTLE, F. Run structure and probability learning: Disproof of Restle's model. *Journal of Experimental Psychology*, 1966, **72**, 383–389.

ROYER, F. L. Sequential complexity and motor response rates. *Journal of Experimental Psychology*, 1967, **74**, 199–202.

ROYER, F. L., & GARNER, W. R. Response uncertainty and perceptual difficulty of auditory temporal patterns. *Perception and Psychophysics*, 1966, **1**, 41–47.

VITZ, P. C. Information, run structure and binary pattern complexity. *Perception and Psychophysics*, 1968, **3**, 275–280.

VITZ, P. C., & TODD, T. C. A model of learning for simple repeating binary patterns. *Journal of Experimental Psychology*, 1967, **75**, 108–117.

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