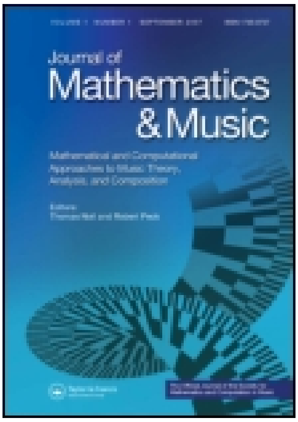


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# Mathematical and computational modelling within a music analysis framework: motivic topologies as a case study

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# Mathematical and computational modelling within a music analysis framework: motivic topologies as a case study

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This paper revisits a mathematical model of motivic analysis, together with its computational realization, from an interdisciplinary perspective, relating it to concepts and methods in the field of computational music analysis. Issues such as segmentation, motivic formation, knowledge representation, similarity and categorization, and interpretation of results are discussed. A further introspection on the approach, in the context of mathematics, computation, and music analysis as a large interdisciplinary field, reveals relations between the three: the mathematical model (motivic topologies), its computational counterpart (OM-Melos), and music analysis (Réti's and Nattiez' methods). In doing so, we stress the importance of neutrality, objectivity, and scientific rigour in the modelling part while at the same time preserving the freedom of the music analyst in order to create musically interesting results.

**Keywords:** motivic topologies; motivic analysis; mathematical modelling; computational music analysis

## 1. Introduction

The area of mathematical modelling in music has been gaining increased interest in the last few decades, addressing topics ranging from traditional music theory to more contemporary theories of gesture and performance. However, a mathematical model is an abstract construct, and in order to apply it to a piece of music, a computational model is called for. Computational Music Analysis (CMA), as a distinct area of research, uses computational means for music analysis purposes [1]. Its aims are, on the one hand, to produce musicologically interesting results and, on the other hand, to formalize the human analytical process while assisting the analyst with data, calculations, and making explicit analytical choices.

If the purpose of modelling is to reveal parts of musical structure, generally or as applied to a piece of music, as well as formalizing an analytical procedure, then the whole modelling process, be it mathematical and/or computational, in order to be meaningful needs to remain close to the main questions, challenges, and interests of the discipline of music analysis. However, the combination of these three elements, mathematics, computation, and music, is not always straightforward, for several reasons that are highlighted throughout this paper.

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Key issues in this enterprise are the need for scientific rigour and objectivity provided by a model and, at the same time, the freedom of the human analyst who needs to make explicit choices according to the piece in question and his or her preferences, interests, cultural background, and goals. The combination of the two aspects needs to be carefully designed, in order to provide the scientific objectivity while allowing the analytical freedom which would make a system useful and usable to the analyst.

There have been various critical reviews and meta-level discussions in the area of using computer models for music analysis from early on in the history of the field up to the present day [2–4], but without ever crossing the boundary towards mathematical music theory. Recently, Volk and Honingh [5,6] have brought the issue forward and correctly stressed the existing gaps in various aspects of the interdisciplinary collaboration, such as research topics and objectives, between subfields and between theory and experiment. They asked the following:

> …connections between different [mathematical and computational] approaches are often difficult to find. Does this result inevitably from the multifaceted nature of music – or do we limit the success of our research by not making the effort to reach across?

A significant initiative in bridging the gap between disciplines has also been the publication of the *Journal of Mathematics and Music*. However, a fundamental problem that we believe is paramount to the bridging of the areas is that similar concepts have often existed in parallel in the two disciplines, without ever having been fully connected and discussed in comparison. An obvious example of this, among others, is the concept of knowledge representation, which is a fundamental concept in CMA, existing but not explicitly brought forward in this terminology in mathematical modelling.

In this paper, we propose to approach the challenging issues mentioned above by revisiting a mathematical model of motivic analysis, that is, the motivic topology approach [7], in the light of CMA. The mathematical presentation, although very brief, is slightly adapted for the present purposes. In particular, it stresses the distinction between the model construct of motivic structure over the infinite set of theoretical motives and the modelling of score-specific motivic analysis schemes, for example, the identification of germinal motives and the paradigmatic categorization.

This paper builds on the preliminary work [8] done by the authors in which some initial issues were discussed in the context of a specific mathematical and computational model of music analysis. Here, the broader context of this large interdisciplinary field is discussed, attempting to go deeper into the connections between the mathematical model, its computational model counterpart, and the related music analytical methods (see Figure 1). The aim is not to provide

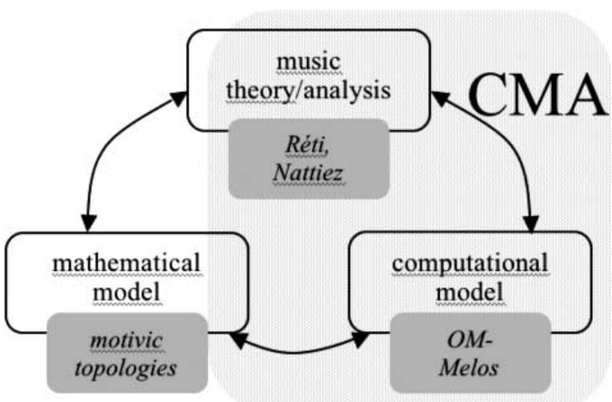


Figure 1. The topological method as an interdisciplinary example between mathematics, computation and music analysis, and thus an approach of CMA.

a comprehensive epistemological discussion, but rather to bring forward various observations, questions, and thoughts when investigating the topological approach as a case study. The paper is organized as follows. Section 2 briefly explains the mathematical and computational models. Due to space limitations, and because this is not the focus of the present paper, here only a summary is given, referring the reader to other more complete sources. Section 3 relates the concepts of the mathematical model to the concepts of CMA, by discussing the main analytical issues. Section 4 proposes a discussion of CMA in the context of the presented approach. The paper ends by a general overview of the challenges and implications of the approach, touching on issues of scientific rigour, objectivity, and other key points of mathematics and computation in music.



## 2. The mathematical and computational models

The mathematical and computational models address the formalization of an inductive approach to motivic structure and analysis of music compositions. When analysing a composition, motives (sequences of non-necessarily consecutive notes) are compared with one another in order to identify all repetition, imitation, and variation relations between them. Variation relations depend on a similarity threshold, thus making use of a similarity measure function. For each similarity threshold, the extent of a motive's presence in a composition (in terms of repetitions, imitations, and variations) determines its salience level for that similarity threshold. In both mathematical and computational models, all thresholds are considered and computed, providing a spectrum of results, evolving from strict imitation identification to more distant variations of motives as the similarity threshold increases. It is then the role of the analyst to select appropriate similarity threshold(s) for meaningful analytical results relating to the composition. This proposed immanent method of motivic salience points to Réti's approach [9], and the regrouping of variations at each similarity threshold points to paradigmatic organization [10].

### 2.1. *Motivic topologies: the mathematical model*

In this section, the main concepts of the mathematical model of *motivic topologies* are briefly recalled. For reasons of clarity, as well as space limitations, we restrict our attention to a minimal set-up, which is needed for the discussion that follows. For a more complete description of the model, together with more information on the motivation of the introduced concepts, the reader is referred to [7,11]; see also [12–14].

In the following, a topological space on the infinite set MOT of all theoretical motives is constructed and then restricted to a finite set of existing motives within a music composition, based on which three models of motivic analysis schemes are finally proposed: notes are parameterized by at least onset and pitch. A *motive* $M$ of cardinality $n$ is a non-empty finite set of $n$ notes with all different onsets.[1] A motive $N \subset M$ is called a *submotive* $N$ *of* $M$. The shapes of motives, $t(M)$, are calculated by introducing a set mapping $t$ on MOT; for example, $\mathrm{Com}(M)$ = the comparison matrix[2] of $M$ [15] or $\mathrm{Dia}(M)$ = vector of consecutive pitch differences (i.e. of consecutive intervals); see Figure 2. These set mappings $t$ are what we call *shape types*.

The motives are regrouped in (imitation) classes, called *gestalts*, from a group $P$ acting on their shapes, for example, the *affine counterpoint paradigmatic group* generated by translations in time, transpositions, inversions, and retrogrades. Pseudo-metrics $d_n$ on shapes of motives with fixed cardinalities $n$ are introduced, and the $(t,P)$-*gestalt distance*, $\mathrm{gd}_t^P(M,N)$, *between two motives* $M$ *and* $N$ *of cardinality* $n$ is then defined from retracting the distance of their respective shapes' class; that is, $\mathrm{gd}_t^P(M,N) := \inf_{p,q \in P} d_n(p \cdot t(M), q \cdot t(N))$, the latter also defining a pseudo-metric if $P$ is a group of isometries.

Figure 2. A motive in Bach's *Kunst der Fuge* as an onset–pitch representation: $M = \{(0, 2), (\frac{1}{2}, 9), (1, 5)\}$ if one chooses the note gauge with the first onset of the score set to 0 and one bar having a duration value of 1; $C_4$ has a pitch value of 0 and a chromatic semi-tone has a value of 1. Two examples of shapes: $\mathrm{Com}(M) = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$ represents the melodic contour within all notes of motive $M$ and $\mathrm{Dia}(M) = (7, -4)$ is the vector of consecutive chromatic pitch intervals.

This naturally leads to a topological space on each set $\mathrm{MOT}_n$ of motives of cardinality $n$. A crucial step in the model for regrouping motives of different cardinalities is the introduction of the $(t, P, d) - \epsilon$*-neighbourhood of a motive* $M$ for a given (similarity threshold) $\epsilon > 0$: $V_\epsilon^{t,P,d}(M) := \{N \in \mathrm{MOT} | \exists N^* \subset N \text{ s.t. } \mathrm{gd}_t^P(N^*, M) < \epsilon\}$. If the inheritance property is fulfilled [13,14], the collection of all $V_\epsilon^{t,d,P}(M)$ forms a basis for a topology $\mathcal{T}_{t,P,d}$ on MOT, and the space is called a *motivic space*. In such a space, one cannot separate motives with the same gestalt (i.e. a motive and its imitations) since their gestalt distance is 0. The topology $\mathcal{T}_{t,P,d}$ is thus of type $T_0$ 'up to gestalts'[3] [14]. The sets $\mathrm{Var}_\epsilon^{t,d,P}(M) := \{N \in \mathrm{MOT} | N \in V_\epsilon^{t,P,d}(M) \text{ or } M \in V_\epsilon^{t,P,d}(N)\}$, called $\epsilon$*-variations of a motive* $M$, in this space conceptualize variations and transformations of motives. When considering a composition $S$ and a finite collection of motives $\mathrm{MOT}(S)$ in $S$ satisfying the submotive existence axiom (SEA), that is, every submotive of a motive in $\mathrm{MOT}(S)$, down to an arbitrary minimal cardinality $n_{\min}$, is also in $\mathrm{MOT}(S)$, the *motivic space of composition S* is the relativization of $\mathcal{T}_{t,P,d}$ to $\mathrm{MOT}(S)$, and the resulting topology represents the motivic structure of composition $S$ [12,13].

Based on this generic topological space, three motivic analysis schemes are further modelled:

(1) The *identification of germinal motives*, motives that are omnipresent in a composition in terms of imitations, variations, and transformations, as proposed by Réti [9], is modelled by quantifying the $\mathrm{Var}_\epsilon^{t,d,P}(M)$ sets through a *weight* function [7] (see Figure 3). The *weight* function is a measure of the size of $\epsilon$-variations of each motif: the larger the $\epsilon$-variation set, the higher the weight of the motive (*interpreted as the more salient*). The germinal motive identification can also be visualized through the related *motivic evolution tree* [11,12], a graphic representation of germinal motives of a composition in function of the similarity threshold $\epsilon$ (see Figure 4).
(2) *Paradigmatic categorizing*, the first stage of semiotic analysis [10], that consists of organizing a collection of segments of a composition into paradigms, that is, categories of 'meaningful' musical units according to various similarity criteria, is realized in motivic spaces by the $\epsilon$-variations, though with a certain control[4] on the cardinality of motives grouped in the same paradigm [16]. This provides an overall structure of a composition in terms of overlapping paradigms (regroupings of 'similar' motives).
(3) An *indirect temporal distribution of motivic paradigms or germinal intensity*, as seen from a note perspective, is proposed by extending the weight function to notes [13]: given a motive similarity threshold $\epsilon$, sum up for each note the weights of motives that contain the note. This denotes the note's 'motivic strength' and can be thought of as a summary of the time-wise motivic evolution along a composition, for a given similarity threshold.

### 2.2. *OM-Melos: the computational model*

Applying the present mathematical model to motivic analysis means to first explicitly construct a motivic space of a composition and to further calculate the three analysis scheme models

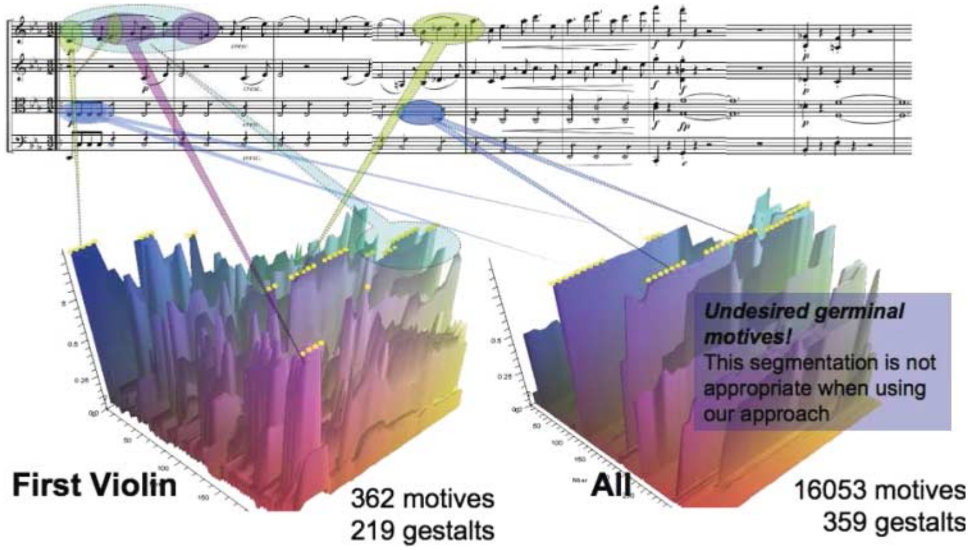


Figure 3. The weight function graphs for two analyses of the same piece, the 10 opening bars of Brahm's String Quartet Op.51 No.1 first movement, with the same analytical parameters ($t$ = Dia, $P$ = Id, $d$ = absolute value distance functions, and *weight* = Mazzola weight [19]), but with different segmentations. To the right, the segmentation considers all four voices, whereas to the left motives strictly stem from the first violin voice. Weight graphs, automatically produced by Maple implementation, give an overall picture of the dynamic motivic landscape: the peaks (white dots) correspond to gestalts (front axis) with highest weights (vertical axis), that is, germinal motives, as the similarity threshold grows (horizontal axis). The analysed score section has been manually added for clarity purposes. The actual motives with peaks in the weight graph are represented as notes in the score through the implementation in OpenMusic.

mentioned above. Due to the large computations involved, this can only be achieved through computer implementation. The following briefly describes the computational model implementation,[5] called 'OM-Melos' [16], which is further discussed in the next section in the context of CMA.[6] OM-Melos not only performs all the computations, but also provides visualization of multiple representations of the results. It is an improved, complete-model implementation, stand-alone version of the software module MeloRUBETTE® in RUBATO® [17,18] (implemented in 1996 by Mazzola and Zahorka [17]) and of the MeloTopRUBETTE, (implemented in 2002 by Buteau [19]).

(1) *Data representation.* The manipulation of data is symbolic (e.g. using the MIDI format). The piece ($S$) is reduced to its set of notes represented by onset and pitch and, if desired, also by duration and loudness.
(2) *Segmentation.* Given the segmentation preference (using large sections defined manually or a set time window) and the minimum and maximum motive cardinalities selected by the analyst, OM-Melos computes the set $\mathrm{MOT}(S)$ of motives of composition $S$ for the analysis.
(3) *Shapes* (*contour*) *of motives.* Given the shape type $t$ selected by the analyst, which involves the musical parameter(s) the analysis will focus on, OM-Melos computes the shape of each motive in $\mathrm{MOT}(S)$. This can be thought of as the representation of the motive in the chosen parameter(s).
(4) *Motivic grouping into gestalts.* Given the paradigmatic group $P$ selected by the analyst, motives are regrouped with their imitations (gestalts). For this grouping of motives into gestalts or categories, the analyst can choose to allow, for example, repetitions, transpositions (if these have not been allowed in the representation scheme), and retrograde forms of a motive.

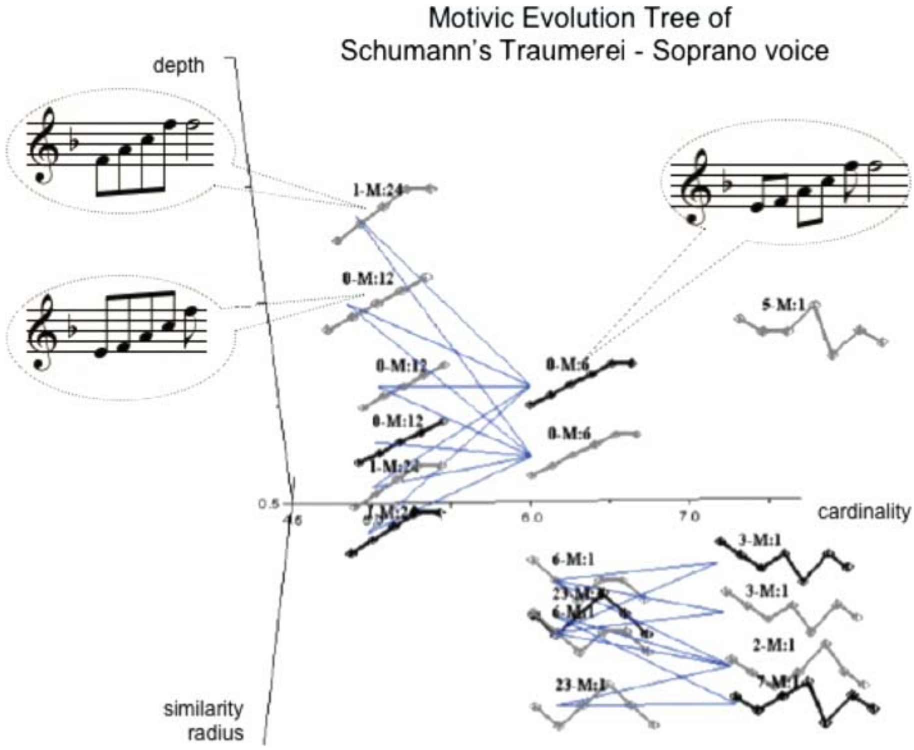


Figure 4. The motivic evolution tree three-dimensional representation of Schumann's *Träumerei* in which three significant motives were manually stressed, according to the topological approach with analytical setting: $t$ = Com, $P$ = transpositions and translations in time only, and $d$ =relative Euclidean distance [16] . The most significant motives are shown in black and the second most significant ones are shown in grey. Submotive relations are represented by lines linking the respective motives.

(5) *Motivic similarity.* Given a distance function $d$ selected by the analyst, the distance between any two gestalts of motives in MOT($S$) with the same cardinality is calculated in MOT.[7]
(6) Further motivic analysis procedures
   (a) *Calculation of weights.* Given a weight function selected by the analyst, the weight of each gestalt in MOT($S$) and the motivic note weight of each note in $S$ are calculated at each similarity threshold $\epsilon$.
   (b) *Paradigmatic categorization procedure.* Given a collection of motives in $S$ selected by the analyst, that is, a subset of MOT($S$), the paradigmatic categorization is calculated.
(7) *Result production.* Intermediate and final results (of diverse types, e.g. numerical, graphic, and in music notation) are returned to the analyst for interpretation. These can be viewed in *OpenMusic*/Maple/JAVA visualization system implementation. A text file with an exhaustive list of all numerical results, including intermediate calculations, is also made available.

OM-Melos currently offers a selection of nine shape types $t$ (Step 3), eight paradigmatic groups $P$ (Step 4), and three types of distance functions $d$ (Step 5); for details, see [16]. However, new additions to all these types by the analyst are facilitated by the system's overall design.

## 3. Related methodological issues in CMA

The procedure reported in Section 2.2 describes the steps of the present computational approach to motivic analysis of a music piece. This computational model was developed and presented

[16,17,19] in counterpart to the mathematical model for the purpose of constructing an explicit example of music analysis. In this section, we revisit the computational model stressing methodological key issues in the context of CMA: segmentation, concept of motive, knowledge representation, similarity and categorization, and interpretation of results.

### 3.1. *Segmentation*

Step 2 (in Section 2.2) relates to segmentation, which is the first crucial step in any analysis in order to obtain meaningful results. In practice, when using OM-Melos, the analyst gives the system his or her own segmentation as a collection of 'large', possibly overlapping segments (which can involve different voices). Then, the implemented segmentation procedure insures that all motives in these segments are considered in the analysis (i.e. are in MOT($S$)). If the analyst chooses instead not to impose a segmentation, an automatic procedure collects all motives within a set time window (e.g. a bar-long window moving along time).

However, according to the mathematical approach, any set of motives in a composition could theoretically be selected to serve as the set of motives for the analysis, that is, the set MOT($S$) – one could, for example, select all motives in a composition. But this would include potentially absurd note combinations that musically might be meaningless, for example, the two-note motive comprising the very first and last notes of the composition. Even though theoretically possible (and allowed in the computational model implementation OM-Melos), this is not of much interest in practice.

Furthermore, the segmentation has to take into account the nature of the applied approach. For example, two natural segmentation choices for the analysis of the 10 opening bars of Brahm's String Quartet Op.51 No.1 first movement (see Figure 3) are to either consider only the first violin voice and segment it or consider each four voices and segment them (without overlap). The result is clear: when applying the model with the same analytical parameters ($t$, $d$, $P$, *weight*), the segmentation with all four voices leads to absurd germinal motive results. Indeed, a repeated-note motive in the violin voice surpassed all motives in the first violin voice. As exemplified, the choice of the segmentation procedure is important for the rest of the analysis and, for an analysis aided with OM-Melos, it relies on the judgement of the analyst respecting the analytical context proposed by the model.

### 3.2. *The concept of motive*

As it has been mentioned above, the definition of a motive is here a particularly general one: in the theoretical construct, a motive can be any combination of notes taken from the piece, including non-consecutive ones. However, in the realization of the piece, only selected motives brought forward by the segmentation are taken into account. Furthermore, once a motive has been selected for the analysis, all its submotives,[8] down to a certain minimal motive cardinality (e.g. cardinality 2 if one wishes to consider intervals as motives for the analysis), are also necessarily considered in the analysis (SEA axiom; see Section 2.1).

The concept of motive in this approach stresses the inductive character of the method: to start with, motives are all unconnected and of the same importance, and it is only through similarity comparisons (see Section 3.4) that a hierarchical structure (topology) is built. The three proposed analysis schemes provide different representations or points of view of this hierarchy as further discussed below, Section 3.5. The model implementation, OM-Melos, returns the hierarchy in different representations, but also allows the analyst to track throughout the hierarchy a motive of his or her choice.

### 3.3. *Knowledge representation*

The representation of information (or knowledge) for use in intelligent problem-solving lies at the core of any computational modelling approach, almost forming an independent area of study within artificial intelligence. In music processing, this defines not only what musical information is represented, but also how this is done. There can be several parameters extracted, at different levels of abstraction. In this case, they are onset, pitch, duration, and loudness, which can be considered either alone or in combinations. Further derived representations, including interval and contour functions, are calculated on any of the parameters. Pairs or groups of parameters can be considered. Note relations in representations can be constructed between consecutive or non-consecutive notes. Finally, all motives are represented (i.e. shape of the motive – Step 3) according to the same selected scheme (i.e. shape type $t$).

Knowledge representation creates the first level of abstraction in the current approach, allowing motives with the same representation to be grouped together. In this case, the similarity of the musical surface (different motives) is based on the identity on the musical parameter level. The analyst is free to choose the representational parameter or parameters to work on for the further analysis of the piece. In terms of defining motivic closeness and similarity, other more general properties are taken into account (see below).



### 3.4. *The concept of similarity and categorization*

This approach allows for several levels of motivic groupings and categorizations, based on various types of similarity relations and criteria. The topic of similarity and categorization has been extensively studied in music analysis, music theory, and computational modelling, with many approaches ranging from more theoretical (e.g. [10] and all related references of categorization in pure music analysis and, more recently, on cognitive aspects; e.g. [20]), to empirical (e.g. [21–23]), to computational ones (e.g. [24–28] and many more). In this study, there are four separate levels of musical similarity to be found and one extension which is related to paradigmatic analysis applications. Thus, at the end of the section, these are linked back to the mathematical model, in order to clarify where the relations between the two lie.

- *Similarity level 1*: The first grouping of motives is realized through the different abstract knowledge representations, as explained in Section 3.3 (see also Step 3). Motives with the same representation are considered identical for the further analysis. This similarity level is thus the identity within the chosen representation. This is where the shape of each motive is calculated.
- *Similarity level 2*: The second level of similarity appears when constructing the gestalts (see Step 4). While transposition is often covered in the previous level by an adequately abstract representation, in the construction of a gestalt, an analyst may decide to allow, for example, inverted and retrograde forms of a motive to belong to the same gestalt. This similarity level is thus co-membership of a given gestalt.
- *Similarity level 3*: The third level is related to distances between gestalts, according to chosen distance function and threshold (see Step 5). In this level, which allows certain variations (on the musical parameters upon which the shapes were built), only motives with the same cardinality are considered, and distance values are, therefore, calculated on the shapes of motives with the same cardinality. Regroupings of motives represent clusters of motive variations for the same cardinality.
- *Similarity level 4*: The fourth level of similarity is directly related to the formalization of motivic variations (as involved in Step 6a). Motives with different cardinalities are now considered and grouped further in $\epsilon$-variations $\mathrm{Var}_\epsilon$ whenever they are variations of each other, according to a similarity threshold $\epsilon$. This includes variations as described in level 3, that is, when two

motives are similar to each other (depending on the distance function and threshold). But it is also involved when a motive contains (or vice versa: is contained in) a submotive similar to another motive.

- *Similarity level 4**: As an extension to level 4 comes the analytical procedure related to paradigmatic analysis: motives can be further grouped into paradigmatic categories (see Step 6b), but the cardinality difference for belonging to the same class is controlled.[9]

#### 3.4.1. *Mathematical structures derived from the similarity levels*

The similarity levels underline different mathematical structures in the model. Similarity level 1 gives a partition, that is, a non-overlapping regrouping of $\mathrm{MOT}(S)$, the set of all motives in composition $S$ considered for the analysis. At the second level, the similarity concept also provides a partition, though coarser than that in level 1, of $\mathrm{MOT}(S)$. These first two similarity levels thus involve an equivalence relation (reflexive, symmetric, and transitive binary relations) defined through a map (level 1) and group action (level 2) concepts. Similarity level 3 brings forward a different kind of structure. At this level, the similarity relation between two motives of the same cardinality depends on a similarity threshold: with a very small threshold, only motives with the same gestalt will be regrouped (or, in other words, the partition from level 2 will remain intact); with a large enough threshold, all motives with the same cardinality will be related (thus, will give $\mathrm{MOT}_n(S)$[10] for each cardinality $n$); in between, overlapping regroupings at each $\mathrm{MOT}_n(S)$ will be formed. At this stage, only motives of the same cardinality can be related through similarity of any level (1, 2, or 3). The computational approach deals with *all* similarity thresholds (which can be reduced to a finite number since $\mathrm{MOT}(S)$ is finite).

Similarity level 4 enfolds the three previous ones and brings together their related analysis parameters: shape type, allowed imitations, and distance function. This similarity concept regroups motives of different cardinalities. Even with a very small similarity threshold, all submotives of a motive $M$ are regrouped with $M$. The complete (overlapping) regrouping (i.e. by considering all similarity thresholds $\epsilon$) of $\mathrm{MOT}(S)$ is what is proposed as the formalization of the motivic structure of a composition (following Réti's [9] approach). Whereas the non-transitive binary relation described in level 3, stemming from the pseudo-metrics, yields an intuitive Euclidean geometry ($T_2$-topological space) on motives' gestalts, the binary relation in level 4, stemming from the asymmetric $\epsilon$-neighbourhoods $V_\epsilon^{t,P,d}$, yields a rather abstract geometry ($T_0$-topological space). Similarity level 4* simply leads to the intersection of some sets with the $\epsilon$-variations in level 4.

### 3.5. *Interpretation of results*

An inherent part of modelling, especially given the interpretative nature of music analysis, is the interpretation of results related to the context of work(s). Here, analytical results obtained by the computational model must be interpreted and musically evaluated and discussed – an essential step often forgotten in many computational musical tasks. With certain methodologies, especially where large amounts of data are presented by the algorithm, one can easily find himself or herself lost in large numerical results. Although not essential, a visualization of results can be particularly useful, even possibly a crucial aid to the analyst as a first step in assessing and interpreting the results obtained.

The motivic topology approach is an example of such a computationally complex model. Because of the large amount of data carried over and produced, and the computer limitations at the time, the initial computational model implementation (MeloRUBETTE) produced a unique hierarchy representation output (with visualization in RUBATO) that was the motivic weight of notes at a fixed similarity threshold. In a later version such as MeloTopRUBETTE, the implementation opened up and provided all the intermediate and final analytical results at all similarity thresholds (see e.g. Figure 3, which shows germinal motives for all similarity thresholds). But the data were large and very tedious to manipulate, and visualization of numerical results was done manually. Because of the generic aspect of the model (many analytical parameters) and segmentation issues, detailed analyses with the model were arduous, virtually impossible for compositions of reasonable size, and thus limited.

The latest implementation, OM-Melos, with integrated visualization of numerical results (see Step 7) makes it now possible to concretely analyse in detail music compositions of significant size or, in other words, to assess the meaningfulness of results interpreted according to the model. The analysis of Schumann's *Traümerei* according to the topological approach through the use of OM-Melos for all computations and the visualization of results can be given as an example [16]: based on the results represented through weight graphs, motivic evolution trees, and gestalt info functionalities in OM-Melos, the 'salient' (according to Repp [29]) opening E-F-A-C-F-F motive was characterized as playing a significant germinal role (see Figure 4). Furthermore, the resulting paradigmatic categorizing of the soprano voice compared well with Repp's manual analysis [16].

More precisely, this mathematical/computational approach suggests modelling Réti and Nattiez' methodologies. The numerical results can then be interpreted in the context of these traditional music analytical methods as applied to the analysed piece. However, the final results are presented 'for all similarity thresholds' and possibly according to different analytical parameters (when applying the approach with different parameter settings). As such, the approach provides a broader and deeper view (thus more complex, justifying the need to 'easily' navigate through the results in order to make sense of them) on the analytical methods applied to the analysed piece. Furthermore, it reveals intermediate results that are beyond the traditional 'manual' methods. For example, the weight function graph allows to identify the germinal motives (the heaviest ones), but also informs about how much 'more germinal' they are in comparison to others and about other significant motives (e.g. second or third heaviest ones). But computational approaches can usually also propose analytical results that have no direct correspondence of specific methodologies of traditional music analysis, for example, the motivic weights of notes (see Step 6a). In this case, one has to argue about the potential meaning of such an analytical scheme and the related numerical results in the context of the analysed music piece.

## 4. Mathematical modelling, computational modelling, and music theory

In the previous section, we revisited the topological approach of motivic structure and analysis of music in terms of CMA concepts. In this section, we discuss a further examination of the topological approach in the context of CMA *as an interdisciplinary field*, by drawing connections between the mathematical model (motivic topologies), its computational model counterpart (OM-Melos), and the related music analysis methods (by Réti and Nattiez): see Figure 1. By stressing these connections in the topological approach, we contend that this could be understood as a preliminary case study of a much more ambitious epistemological discussion.

The approach that is presented is a case where the mathematical model preceded the computational model. The mathematical model was initially developed as a formalization of Réti's approach to the motivic analysis of a music piece. As discussed in detail by Buteau and Mazzola [7], the model provides arguments and answers to controversial oppositions to Réti's approach by use of topological properties and concepts. The formalization also contributes objectivity to Réti's analytical method by detailing and refining all steps of the approach and by making explicit all assumptions that are to be made by the analyst throughout the method. In fact, the formalization may have made possible the realization of a computational model of Réti's approach, a realization that, as stated by Dunsby [30], did not seem possible since Réti's approach was described as '*certainly not scientifically reproducible*'. In addition, the mathematical model stresses the inductive character of Réti's method, which, due to his manual computation limitations, may sometimes have looked deductive. Similarly, the paradigmatic categorizing concept in the mathematical model contributes a formalization of Nattiez' paradigmatic analysis. As for the other proposed mathematical analysis scheme, which is the motivic weight to notes, there is no parallel analytical method in music theory, but this scheme could be interpreted as an indirect temporal distribution of motivic Q2 paradigms. As such, it could be seen as potentially extending an existing analytical scheme or contributing a new one to traditional music analysis.

One could argue that these connections could be said about a strict computational model and its related modelled music analytical method. The discussion presented below exemplifies some key aspects stemming from the mathematical model, which perhaps could not have been drawn within a strict computational approach.

The computational model was designed in line with the mathematical model. Transferring a mathematical model to a computational one can be rather straightforward, but depending on the programming language structure and the model, proper adjustments in the model design might be preferred. For example, the mathematical model is introduced on the infinite set MOT of theoretical motives rather than on the finite set $\mathrm{MOT}(S)$ of motives in a composition $S$. This is a necessary step, in line with Réti's method [7], as the proposed assignment of a contour similarity value between two given motives of the same cardinality (Similarity level 3) is carried out by fixing one of the two motives and by finding the theoretical imitation of the other one that is 'closest' to the fixed motive (in the composition). These imitations are, therefore, theoretical (e.g. fictive translations in time or transpositions) and might not be found in a composition. However, the computational model's starting point is a music composition to be analysed. It constructs gestalts of motives directly in the finite set $\mathrm{MOT}(S)$, but then computes the contour similarity values as if it were in the theoretical set MOT.

In addition to naturally providing a structure for the computational model, the mathematical model informs about computational efficiency. For example, the topological space of motives is reduced to a related topological space of motive gestalts (classes of motives): an application to Schumann's *Träumerei* led to 355,299 motives regrouped in 172 gestalts, and 30,186 similarity computations were performed on gestalts instead of 63,150,488,961 similarity computations[11] on motives.[12] Importantly, the mathematical model makes explicit what assumptions on parameter combinations (i.e. the analytical choices made by the analyst), for example, $t$, $P$, and $d$, are required to ensure a computational reduction that yields similar results as if it were done on motives. Here, the related similarity distance function defined on gestalts must be a metric (as opposed to strictly a pseudo-metric) and the inheritance property must be satisfied; it is the case of all implemented combinations, except for one due to failing the inheritance property: we discuss it below. In music analysis context, the metric assumption means that whenever the similarity distance between two motives is 0, these motives must have the same gestalt (i.e. are imitations according to the selected analytical parameter $P$).

This stresses that the mathematical and computational models are related, but not equivalent. Besides, in theory, when applying the mathematical model to identify germinal motives of a music piece $S$, one first builds the topological space by constructing in $\mathrm{MOT}(S)$ the $\epsilon$-neighbourhoods $V_\epsilon(M)$ and $\epsilon$-variations $\mathrm{Var}_\epsilon(M)$ for each motive $M$, before evaluating the weight function *weight* for each motive (with different gestalts). But in practice when using the computational model, since the aim is to model the motivic analysis schemes, the implementation does not construct the space but instead directly determines the desired numerical information. In addition, the mathematical model can be considered as a pure mathematical object of interest for further study. For example, it was extended, without an initial aim for applicability to music analysis, within a

Category Theory framework [14]. It turned out that one of the results could be interpreted in the context of music analysis: motivic evolution trees (representing germinal motives in function of the similarity threshold – e.g. see Figure 4) from two analyses can be compared whenever there exists a continuous function between their respective motivic spaces [14].

In the topological approach, the diastematic index shape type defined through vectors describing the melodic contour of motives (between consecutive notes) with 1's, 0's, and $-1$'s does not lead to a topology. The issue is that, for example, knowing that the three-note motive has shape $(+1, -1)$ indicates that the melody goes up and down; but what can be said about the melodic movement from the first to the third note? Nothing: it could be $(+1)$, $(-1)$, or (0). Thus, 'similarity closeness' does not carry over respective smaller parts of motives. In other words, two motives could be considered similar overall, but their respective parts would not necessarily be considered so. This can be thought of as counter-intuitive. In the mathematical model, this 'awkward' behaviour is discarded since the diastematic index shape type fails the inheritance property (similar motives must have respective similar submotives), a property that then ensures a topological space as introduced in the model.

This shape type is, therefore, not included as a possible analytical case in the general mathematical model.[13] However, this is included in the computational model: OM-Melos computes the same things as if resulting in a topological space, and the final results are in the same format. One could, therefore, interpret the results in a similar manner as for the other shape types. But something different is happening in the intermediate computations (reflecting the awkward behaviour mentioned above) and, as such, the interpretation of the final results should take this into account. In fact, because of this issue and since this melodic contour vector is often mentioned in music theory, the problematic case of the diastematic index shape type is 'solved' [11] in the mathematical model by considering the comparison matrix shape type Com (see Section 2.1), a knowledge representation that keeps slightly more information than the original vector, but also sufficient information to satisfy the inheritance property.

Finally, due to the computational complexity, the computational implementation of the motivic space approach is needed for any significant application of the model. Thus, it is only through the use of OM-Melos (or any other implementation of a computational model counterpart to the topological model) that the mathematical model makes sense and fits in CMA.



### 4.1. *Musical interpretation and evaluation of results*

As mentioned in Section 3.5, important aspects of CMA are the discussion and evaluation of analytical results obtained by the model when applied to a piece or pieces of music. The main reason is that music analysis is primarily an interpretative operation, with no apparent 'ground truth', and results in any method, computational or not, have to be evaluated and explained in order to acquire meaning and significance. In a computational approach, this evaluation can also subsequently inform aspects of the model, such as segmentation, choices of representations, or algorithmic procedures, which can then be fine-tuned and produce new results in a cyclic scientific informing procedure. New results can also prompt the analyst to reconsider relevant musicological viewpoints [31].

Result evaluation is directly related to the computational method used, its aims, and its closeness to the traditional field of music analysis. Attempts have been made to explicitly formalize the already established methods of music analysis, such as Nattiez' (e.g. [32]), Réti's (e.g. [7]), and Schenker's (e.g. [33]) methods. Others such as those of Pople [34] and Lartillot [35] and many more might stay close to an existing method, without attempting to fully adhere to it. Quite often though, computational approaches borrow elements from many methods and create new ways to study the music under examination. This can be especially true in cases of comparative analysis and analyses of large musical corpora, not previously possible to analyse manually. In this case, often encountered in the area of Music Information Retrieval, statistical approaches are often used, which can reveal interesting results emerging from the data, which would have otherwise been impossible to detect (many can be found in ISMIR proceedings [36] and various related journals [37–39] to mention a few).

Possible ways of evaluation, depending on the approach used, include a juxtaposition of the results to an existing analysis by a human (often a well-known published analysis), which can reveal interesting points (e.g. [32,40]); asking a musicologist or musicologists to give his or her expert opinion on the results obtained by a system (e.g. [41]); and employment of perceptual studies in order to reveal what a number of people perceive and whether this is in accordance with what has been produced by the computer (e.g. [42]).



## 5. Theoretical implications and conclusions

In this paper, we discussed connections between the mathematical and the computational models of motivic topologies, as applied to music analysis. We attempted a novel way of connecting the mathematical and the computational parts by discussing the topological approach in terms of CMA concepts, such as segmentation, definition of a motive, knowledge representation, different levels of similarity, and interpretation of computational results. We further elaborated on some aspects of the approach stressing their fine connections seen within three disciplines: mathematics, informatics, and music analysis. The contribution of this work is mainly of theoretical nature. On the one hand, it proposes a preliminary epistemological discussion about the area of mathematics and computation in music analysis, whereas the topological approach serves as a case study. In particular, the question of *why formally introduce a topology in the approach?* has been addressed. On the other hand, we also discussed a methodology from different points of view and how it can be related in a meaningful way to music analysis.

Apart from the already stressed potential inaccordance between the already existing concepts in the two disciplines, perhaps the most interesting challenge in this enterprise is to combine the objectivity and scientific rigour of mathematics and informatics, with the interpretative nature of music analysis. As a result, based on the interconnection between mathematics, computation, and music analysis, there are two main issues that still need to be addressed. Although at first view conflictual, these two issues reunite in the topological approach.

*Scientific rigour and objectivity.* The constructed mathematical structure presented in Section 2.1 is rigorous. In terms of the computational model, this paper focused on the implementation of OM-Melos. The analysis procedure is systematic and completely reproducible as long as the specific parameters used by the analyst are taken into account. As for the validity of the models proposed, the detailed formalization of Réti's approach [7], in addition to the juxtaposition of the analytical results to the existing analyses by humans, for example, the analysis of Schumann's *Träumerei* [40] in comparison with Repp's analysis [29], support the model validity. But the evaluation of results and, more broadly, the verification of model validity remain without a doubt a challenge in CMA, as discussed in Section 4.1. In [43], the authors asked

> Can this type of analysis [CMA] be closer to what Nattiez originally thought about the neutrality, objectivity and scientific nature of music analysis? Researchers working in CMA are called to address the issue. (pp. 75–76)

The question now is whether the process of music analysis, following a model, can also be thought of as 'neutral' (to use Nattiez' terminology) and objective. We comment on this question in view of the presented approach to motivic analysis. When looking at the mathematical model as described in Section 2.1, it first proposes an abstract construct on an infinite set of theoretical motives. It then carries the topological structure to 'an arbitrary finite set of motives' (with SEA property) stressing the immanent, objective character of the approach. Whether the actual choice of the 'arbitrary' set of motives for the analysis of a composition makes sense (see segmentation issue in Section 3.2) or the choice of analysis parameters is appropriate, the answer that we are inclined to give is that as long as the points of choice for the analyst are made explicit, the analytical process is formalized and replicable and therefore can be thought of closer to Nattiez' original claims.

*Musical freedom of the human analyst.* From the description of the computational procedure (see Section 2.2), it is apparent that the freedom of choice given to the human analyst in this approach is a significant aspect of the model in all steps. Since the model is of generic nature, one would wish to vary the parameters (as described in Steps 1–6 in Section 2.2) for a more complete and context-free analysis. More specifically, the points of choice given here relate to all steps in the procedure: score representation, segmentation and motive definition, knowledge representation (shape type parameter $t$), and definition of similarity in all four levels (shape type $t$, imitations $P$, and similarity distance function $d$ parameters). In practice, the points of choice for Steps 1–6 can be given at the beginning of the procedures, and the fully automatic computations follow thereafter. The human analyst recovers at the end his or her musical freedom when he or she is required to interpret and make sense of the numerical results (Step 7) in the context of the analysed piece of music. This important CMA methodological issue has been discussed at length in Section 3.5.

As in many CMA approaches, the proposed computational model involves a similarity threshold in its procedures. Should this analytical parameter also be made accessible to the analyst (although it is not clear in this case on which criteria the analyst would base his or her choice)? This turns out to be a key aspect of the topological approach: due to the diverse visualizations of results, the analyst does not have to select a similarity threshold *a priori*, but instead can use the overall motivic spectrum, for example, represented through the weight function graph, the graphic motivic evolution tree, or the paradigmatic categorization dynamic plots, and select, a posteriori, which similarity threshold(s) should be considered based on the meaningfulness of their corresponding results. This is a concrete example of the cyclic procedure mentioned in Section 4.1.

In conclusion, the enterprise of combining mathematics, computation, and music is a very interesting and challenging one, which has to be carefully thought out in order to be meaningful in all disciplines. Often, scientists in mathematical music theory and CMA talk about similar concepts, without relating them in an explicit manner. In this paper, we attempted to address this by taking one method and describing it from different points of view – which proved to be non-straightforward. We believe that future research needs to take a broader interdisciplinary perspective in order to bridge the existing gaps that have already been found to exist and promote music analysis work in many exciting new ways in the years to follow.



## Notes

1. This means that motives in this approach are strictly monophonic, though they can originate from polyphonic music.
2. If $M$ has cardinality $n$, the comparison matrix of $M$ is an anti-symmetric $n \times n$-matrix in which the $ij$-entry represents the pitch movement from note $i$ to note $j$, that is, it is 1 if $p_j - p_i > 0$; $-1$ if $p_j - p_i < 0$, and 0 if $p_j = p_i$, where $p_i$ and $p_j$ are the pitch values of note $i$ and note $j$, respectively.
3. The $T_0$ condition is actually satisfied on the quotient space of gestalts: given any two distinct gestalts of motives, there exists an open neighbourhood containing one but not the other.
4. For example, intersect the $\epsilon$-variations of motives $M$ with $\{N \in \mathrm{MOT}(S) | \min(\mathrm{card}(M), \mathrm{card}(N)) / \max(\mathrm{card}(M), \mathrm{card}(N)) \geq 70\%\}$, to ensure that the motives grouped together have cardinality not too far apart [16].
5. Details of Step 1 to Step 6(a) can be found in [19]; for Step 6(b) and Step 7, see [16].
6. The reader may choose to simultaneously read Sections 2.2 and 3. We favoured the current presentation in order to separately stress the CMA key issues of the method proposed.
7. It is emphasized here that the distance calculations are done on the theoretical set MOT of motives rather than on MOT($S$). Also, distances are calculated for *gestalts* (i.e. only the first motive occurrence in the piece, for each gestalt) instead of for all motives and so is the weight function. *This step relates to the pseudo-metric topologies on each set layer* $\mathrm{MOT}_n(S)$ *of motives of fixed cardinality n.*

8. Possibly containing non-consecutive notes.
9. For example, with the 70% minimum cardinality ratio restriction as described in Section 2.1.
10. $\mathrm{MOT}_n(S) = \{M \in \mathrm{MOT}(S)|\mathrm{card}(M) = n\}$.
11. For the same composition, different segmentation preferences, shape types, and paradigmatic groups can lead to very different overall totals of motives and gestalts. But then for the construction and manipulation of variations, the computations strictly depend on the number of gestalts (or motives if computations were at that level): if there are $n_i$ gestalts of motive cardinality $c_i$, $i = 1, \ldots, k$, then $\sum_{i=1}^{k}(n_i(n_{i-1})/2)$ gestalt distances, $\sum_{i=2}^{k-1} n_i(\sum_{j=i+1}^{k} n_i)$ small gestalt relations [14], and $3N(\sum_{i=1}^{k} n_i)$ adding weights computations are performed in total, where $N$ is the number of similarity thresholds analysed. In the example mentioned [19], $n_1 = 2, n_2 = 5, n_3 = 24, n_4 = 141$, and $N = 30$, whereas for motives, the figures then become $n_1 = 5,272; n_2 = 29,461; n_3 = 100,947; n_4 = 219,619$, and $N = 30$.
12. Computations in OM-Melos are done on gestalts, whereas in the original MeloRUBETTE (1996), computations were done on motives.
13. Note that if one considers for the analysis only motives of a fixed cardinality, this trivially leads to Euclidean topological spaces. In other words, if $\mathrm{MOT}(S)$ contains only motives of the same cardinality, then the resulting motivic topology, for any shape type, is Euclidean ($T_2$) and the $\epsilon$-variations are simply the $\epsilon$-neighbourhoods, that is, for any $\epsilon > 0$ and any $M \in \mathrm{MOT}(S)$: $\mathrm{Var}_\epsilon(M) = V_\epsilon(M)$. In this case, the weight function then trivially involves only counting the number of motives in each $\epsilon$-neighbourhood. But this trivial case of a fixed cardinality is not interesting from a music theoretical point of view.

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