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Source: *Music Theory Spectrum*, Vol. 15, No. 1 (Spring, 1993), pp. 36-58
Published by: University of California Press on behalf of the Society for Music Theory
Stable URL: http://www.jstor.org/stable/745908
Accessed: 30/10/2009 11:22

Early Indian Heptatonic Scales and Recent Diatonic Theory

John Clough, Jack Douthett, N. Ramanathan, and Lewis Rowell

This article describes and evaluates an application of recent diatonic theory to one of the most intractable problems in music history: How and why have large numbers of unrelated musical cultures selected similar heptatonic scales as their basic melodic resources, and what similar features of "diatonicism" do these scales exhibit? We examine the scale system of ancient and medieval India, building upon the approach developed by John Clough and Jack Douthett in their recent study, "Maximally Even Sets."[1] More specifically, we address two questions: (1) By what path did ancient Indian musicians make their way from a "chromatic" universe of 22 microtonal divisions of the octave (the *śrutis)* to a "diatonic" set of seven degrees (the *svaras)?* and (2) What features do the resulting scale structures have in common with later versions of the diatonic scale in the West? The answer to the first question must remain somewhat conjectural, but we believe we have specific and surprising answers to the second question.

The first part of the article consists of a brief exposition of the ancient Indian system of pitch, focusing on the musical gamut and its three main variables—rotation, alteration, and omission. The level of detail here is sufficient only to introduce the problem. The second and more formal part of the article is an explanation of method and a presentation of findings. From a methodological point of view, we see this joint effort as a potentially valuable intersection of two very different fields of study, each with its own style and standards of verification.

Musicians have long been intrigued both by the microtonal intricacies reported by ancient authors (in both East and West) and by the widespread tendency for diverse musical cultures to distill similar heptatonic collections therefrom. The apparent reason for the latter was to construct scale systems that were at the same time simpler, more elegant, more consistent, and more flexible than their chromatic sources. These objectives are clearly incompatible, and trade-offs were often required. We examine the selection principles that appear to have guided early Indian musicians and explore some of the remarkable features and implications of their solutions. We consider briefly some of the explanations that have been offered for these selection principles, among them the influence of "lurking" universals, ease of calculation, the tendency toward "leveling" (of anomalous forms) that is characteristic of both musical and linguistic systems, the roles of various classes of intervals, the presence and function of

The original version of this paper was presented at the Annual Meeting of the Society for Music Theory, Cincinnati, 1991.

[1]John Clough and Jack Douthett, "Maximally Even Sets," *Journal of Music Theory* 35 (1991): 93–173.

ambiguities, and the need for internal structure within the scale.

In the case of the quasi-diatonic scales (the *grāmas)* set forth with such precision in the earliest layer of Indian musical treatises and practiced in much the same form for at least a millennium,[2] one encounters not only a set of provocative problems in musical tuning, but also interesting evidence for (1) the insertion of "altered" scale degrees, (2) the recognition of consonant and dissonant relationships among the scale degrees, and (3) the derivation of a limited number of hexatonic and pentatonic variants by the systematic and lawful omission of single notes and note pairs. In particular, we address the significance of the number 22, a number that has notably failed to capture the fancy of the builders of other world scale systems.

In the following exposition we avoid Sanskrit terms to the extent possible, but a few are inescapable and will appear frequently: *grāma* (scale), *śruti* (microtone), *svara* (scale degree), and the sol-fa names for the seven basic degrees of the scale—*sa, ri, ga, ma, pa, dha,* and *ni* (see Table 1). Other terms are defined as needed, and a glossary of terms appears at the end of the article. We use the term *scale* throughout the article, in full knowledge that many readers will prefer the term *collection.* In the context of this article, *scale* will always refer to a particular pattern of intervals arrayed in consecutive order, never to specific pitches or pitch classes. We regard it as the "scaling" of the tonal spectrum, in the same sense that the pattern of parallel lines on a ruler establishes the scale for its linear space.

[2]Part 1 of this article draws upon the information presented and conclusions reached in Lewis Rowell, *Music and Musical Thought in Early India* (Chicago: University of Chicago Press, 1992), chap. 7. Certain examples have been reproduced by kind permission of the University of Chicago Press. In subsequent footnote references, the book will be identified as MMTEI. For a detailed listing of sources, editions, and their chronology, see MMTEI, 18–22. "The earliest layer of Indian musical treatises" refers to a group of six or seven treatises (not all of which present the full details of the set of issues addressed in this article) spanning roughly a millennium between A.D. 200 and the middle of the thirteenth century: from the *Nāṭyaśāstra* (ascribed to the legendary sage Bharata) and the *Dattilam,* both dating perhaps as early as the second century A.D., to Śārṅgadeva's monumental *Saṅgītaratnākara* (ca. 1240). Despite minor differences and inconsistencies of presentation, there are no essential disagreements in this corpus of literature with respect to the topics expounded and analyzed in the present article. As one might expect, the latest of these documents contains the most complete presentation, and most Indian scholars tend to apply Śārṅgadeva's detailed specifications reflexively in an attempt to amplify the terse expositions of the earlier texts. By his time it is clear that the system had become fossilized and could no longer be said to represent current musical practice. It is evident that the authors cannot claim that Śārṅgadeva's exposition can speak with unchallenged authority on behalf of a musical practice that flourished a thousand years earlier. But since his conclusions are consistent with those of his predecessors, neither have we any firm basis on which to reject them. For the sake of readers who wish to compare the relevant portions of the above three treatises (the only ones so far to be translated into English), we provide the following references, each with chapter and verse citations for the topics covered in part 1 of the present article, and inclusive page references for the best and most accessible English translations: (1) *The Nāṭyaśāstra,* trans. Manomohan Ghosh, vol. 2 (Calcutta: Asiatic Society, 1961): scale degrees *(svaras),* chap. 28, v. 21, p. 5; microtones *(śrutis),* no separate discussion; the basic scales *(grāmas),* chap. 28, vv. 23–28, pp. 7–9; sonance *(vādi),* chap. 28, vv. 21–23, pp. 5–7; rotation *(mūrcchanās),* chap. 28, vv. 28–34, pp. 9–11; alteration *(sādhāraṇa),* chap. 28, vv. 34–37, pp. 13–14; omission (the pentatonic and hexatonic *tānas),* chap. 28, v. 34, pp. 12–13. (2) Mukund Lath, *A Study of Dattilam: A Treatise on the Sacred Music of Ancient India* (New Delhi: Impex India, 1978): scale degrees, vv. 10–11A, pp. 207–17; microtones, vv. 8–9, pp. 197–206; scales, vv. 11A–15, pp. 218–25; sonance, vv. 18–19, pp. 230–34; rotation, v. 21, pp. 237–46; alteration, vv. 16–17 and 46–47, pp. 226–29 and 262–64; omission, vv. 30–35, pp. 247–54. (3) *Saṅgīta-Ratnākara of Śārṅgadeva,* trans. R. K. Shringy and Prem Lata Sharma, vol. 1 (Varanasi: Motilal Banarsidass, 1978): scale degrees, 1.3.23–25b, pp. 130–35; microtones, 1.3.8–22, pp. 115–29; scales, 1.4.1–8, pp. 160–67; sonance, 1.3.47c–51, pp. 148–52; rotation, 1.4.9–26, pp. 167–79; alteration, 1.5.1–11b, pp. 229–33; omission, 1.4.27–90, pp. 179–227.

Table 1. The seven scale degrees *(svaras)*

degree	*name*	*abbreviation*	*derivation*	*meaning*
1	*ṣaḍja*	*sa*	*ṣaḍ* (six) + *ja* (born)	born of the six organs of utterance
2	*ṛṣabha*	*ri*	*ṛṣabha* (bull)	bellowing like a bull
3	*gāndhāra*	*ga*	*gandha* (fragrance)	the fragrant note
4	*madhyama*	*ma*	*madhya* (middle) + the superlative suffix *ma*	"middlemost"
5	*pañcama*	*pa*	*pañca* (five)	the fifth note
6	*dhaivata*	*dha*	perhaps from *dhī* (perceive, think)	unclear
7	*niṣāda*	*ni*	*ni* (down) + *sada* (seat)	the final note

I

THE GAMUT

The gamut of early Indian music, as recorded in sources dating approximately from the second to the thirteenth centuries A.D., may be described as a single diatonic system with a theoretical range of three full octaves, with seven nonspecific degrees in each octave and authorized scales beginning on three of these degrees (1, 3, and 4).[3] Each of the three scales was available in a complete set of rotations; two degrees were subject to alteration (upward inflections of 3 and 7); and one or two degrees could be omitted. We take up each of the three variables in turn in the following pages.

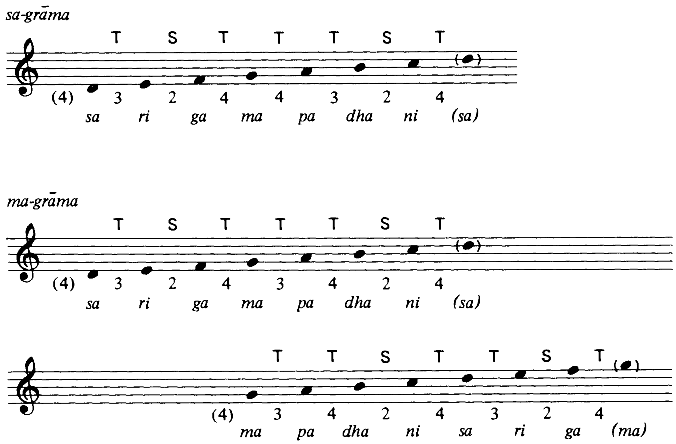
The simplest place to begin is with the seven scale degrees, as illustrated in Table 1. Note in particular the sol-fa names for the scale steps and their derivation, an obvious parallel to the Western solmization syllables. The scale is obviously an upward construction, if *pa* is the fifth note and *ni* the last. (The ancient Vedic scale ran downward, like the scales of ancient Greece.) More important, these degrees do not represent specific interval relationships; unlike the Western syllables, they are *generalized* scale degrees that can be inflected in several different ways when actualized in a particular *rāga* or one of the mode-classes ancestral to the later *rāga* system. It must be emphasized that throughout this study we are not analyzing individual *rāgas*, but rather the background collections from which a specific selection is made for a particular composition or performance. It will soon be apparent that there are several levels of these "background collections."

Example 1 is a display of the most important of these levels—the two basic *grāmas*.[4] Both are diatonic collections that resemble the church modes of medieval Europe; or perhaps it is more accurate to compare them to the general diatonic system that was the common territory of all the church modes. The octave within which each is here displayed should therefore not be construed as an *ambitus*. Further, functional tonal meanings have not yet been assigned. Tonal function arises only after a particular note has been designated as the *aṁśa* or tonic note; all other melodic functions follow from the choice of this note.

[3]We mean by *diatonic* a seven-note scale consisting of two small and five larger steps, with maximal separation between the small steps.

[4]The word *grāma* means "village," in the sense of an assembly or collection of any kind.

Example 1. The two basic scales (the *grāmas)*



Thus the basic collections comprise one scale running from *sa* (the first degree) and one running from *ma* (the fourth degree): in Western terms, a D-mode and a mode on either D or G, depending on where one begins. This was and still is a "fixed-do" system, so *sa* remains *sa* in all contexts. The *ma-grāma* has been illustrated in both the D- and G-octaves for purposes of comparison. The most interesting feature is the interval pattern as measured in *śrutis*—either 2, 3, or 4 of these microtones between each adjacent pair of scale steps. We address the tuning problem below, but by Western standards the intervals of 3 and 4 *śrutis* clearly count as "tones," and the 2-*śruti* interval is just as clearly a "semitone."

The scale was conceived with an interesting and typical Indian twist: the 2, 3, or 4 *śrutis* that comprise each interval are specifically located *below* each of the scale degrees. They were regarded as a seamless continuum of sound that becomes manifest only at the upper edge of this band of sound, when the accumulation of *śrutis* has become complete.[5] So the scale step *sa,* from an Indian perspective, is not the point D but a band of 4 *śrutis* below D that becomes manifest to sense perception only when its upper limit has been attained.

Notice further that, when the two *grāmas* are compared within the same octave, their pattern of *śrutis* differs only in one particular: the intervals between *ma* and *pa* (4 and 5) and between *pa* and *dha* (5 and 6) have been exchanged (that is, 4 + 3 *śrutis* in the *sa-grāma* becomes 3 + 4 in the *ma-grāma).* Students of historical tunings may be tempted to think of these scales as comparable to two particular tuning selections within the system of just intonation.

Early treatises mention one other scale, the *ga-grāma* as displayed in Example 2. We find this scale a less believable construction, and one of the early phonetic manuals asserts that it was practiced "nowhere else than in heaven."[6] And indeed it seems likely that the expertise of the celestial musicians would be required to negotiate its unusual interval pattern. It contains only one proper semitone, and the sequence of six "tones" of dissimilar size virtually rules out any precise transcription into Western staff notation. These *śruti* values were first reported in the thirteenth century, so it is entirely possible that the original structure (if, indeed, there is any meaning to the concept of "original structure" in a musical tradition of this antiquity) may have been somewhat different. Students of Indonesian music may see here a parallel to the seven-note scale named *pélog.*[7]

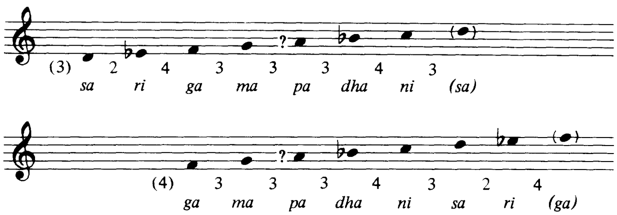
We illustrate this bizarre scale for an important purpose. If the interval pattern is correct as reported, then this interval set is the *only* such set with three consecutive step sizes (that

[5]For further background and analysis, see MMTEI, 151–52.

[6]*Nāradīyaśikṣā* 1.2.6.

[7]For the structure of *pelog,* see R. Anderson Sutton, "Asia/Indonesia," in *Worlds of Music: An Introduction to the Music of the World's Peoples,* 2d ed., ed. Jeff Todd Titon (New York: Schirmer Books, 1992), 270–71.

Example 2. The hypothetical *ga-grāma*



is, with 2, 3, and 4 *śrutis)* that sums to 22—apart from the interval set found in both the *sa-* and *ma-grāmas* and their altered versions. This is the "consecutivity" feature described in part 2 of this article.

We now address the question of tuning. In Table 2 we compare the only five meaningful intervals in the ancient Indian system to their equivalents in just intonation and twelve-tone equal temperament.[8] The interval of 2 *śrutis* is a good approximation of the Western semitone, but the discrepancy between the intervals of 3 and 4 *śrutis* exceeds by far the range of deviation permitted for whole tones in any known historical tuning. It strains belief to think of them as equivalent "steps" of the scale. The only appropriate comparison is to the "major" and "minor" tones of just intonation, but the comparison works only on the conceptual level.

We have avoided until now the question of whether the *śrutis* were identical in size, roughly similar, or quite different. The answer is not known, and the issue is hotly argued in modern Indian journals with no easy solution in sight. We hold that this was *not* twenty-two-tone equal temperament. The best guess is that the *śrutis* were determined on the basis of oral instruction and were never more than rough approximations, except within a particular *sampradāya* (tradition).

[8]For further analysis, see MMTEI, 145–49.

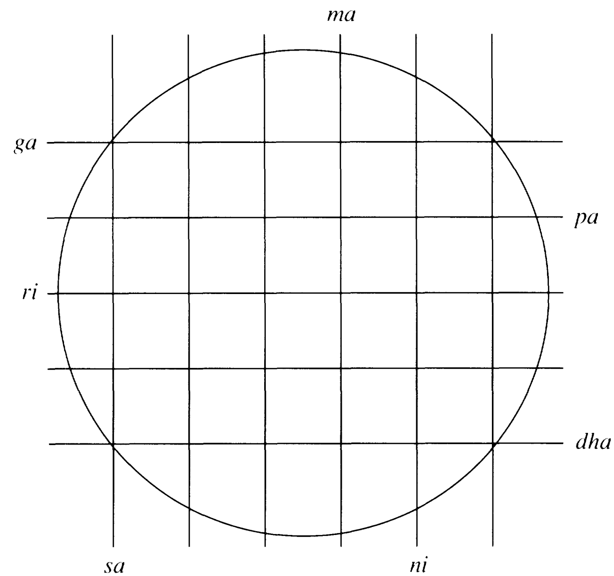
Table 2. Interval sizes in *śrutis* and centitones

22-tone equal temperament		*just intonation*	*12-tone equal temperament*
2 *śrutis*	= 110	16:15 = 112	100
3	= 164	10:9 = 182 (the "minor" tone)	200
4	= 218	9:8 = 204 (the "major" tone)	200
9	= 491	4:3 = 498	500
13	= 709	3:2 = 702	700

What *is* known is that in later Indian practice, dating probably from the thirteenth century, this ancient tuning was set aside in favor of a close approximation of twelve-tone equal temperament, from which the notes of a given *rāga* were selected, and with the possibility that certain notes could be shaded up or down from one of the twelve basic positions. In this way some of the microtonal flavors of preexistent *rāgas* could be preserved. Our argument in this article does not depend upon tuning assumptions, but it is well to keep in mind the present limits of our knowledge.

The only other significant intervals are remarkably precise approximations of the Western perfect fourth and fifth, a discovery that will surprise no one. These were the only *consonant* intervals; all the others were dissonant or neutral and had no separate conceptual identity. When we later refer to the concept of *sonance*, it will be with respect to the consonant interval that links the tonic note (the *aṁśa)* and one other scale degree that is situated either 9 or 13 *śrutis* above or below the *aṁśa.* Only one such relationship was possible in a given *rāga.* The moral here (and it is not an obvious one) is that the quality of consonance was located not in an abstract interval but in a specific relationship between a pair of notes. It was neither an automatic nor a standing relationship, but

Figure 1. A circular display of the *sa-grāma* (from Mataṅga's *Bṛhaddeśī)*



the result of a deliberate choice from among several alternatives.

VARIABLE 1: ROTATION

We turn to the three major variables of the system. Figure 1 has been included to demonstrate that ancient musicians conceived of a musical scale as a circle—the *svaramaṇḍala* ("circle of *svaras")*; it was constructed by superimposing a circle upon a nexus of eleven intersecting lines.[9] The circular diagrams in part 2 of this article are therefore very much in harmony with this early concept of the musical scale. Each successive *śruti* is represented by the tip of one of the lines.

Rotation was one of the main procedures for deriving variants of the basic *grāmas,* as illustrated in Example 3. The *mūrcchanās* were sets of systematic downward rotations of each of the *grāmas,* as shown by brackets A, B, and C in the example. In Western terms, they are octave species, not independent modes. For readers who may wonder why the brackets enclose sevenths instead of octaves, this is a consequence of the relative disinterest in the octave, as interval, in Indian musical thinking. Early musicians were more interested in its contents than in its role as a boundary interval, and consequently they referred to the complete scale as *saptaka* (a set of seven)—in contrast to the Attic Greek *diapason* ("through all [the notes]").

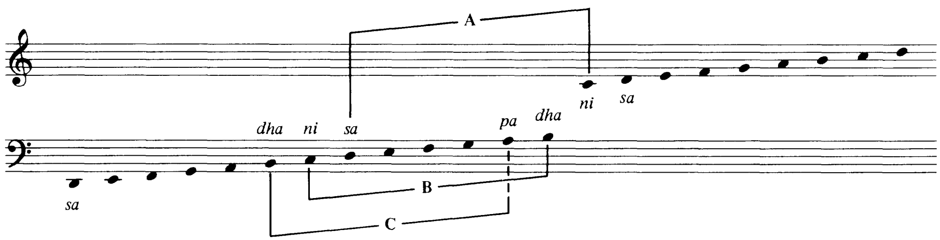
VARIABLE 2: ALTERATION

Alteration is the second major variable of the system. The altered scale degrees were known collectively as *sādhāraṇa* ("occupying a mean position," that is, between two adjacent degrees). The only two authorized scale alterations are displayed in Example 4. Their names mean "in-between" *ga (antara-ga)* and "soft and sweet" *ni (kākalī-ni)*. Each alteration merely rearranges the *śrutis* within one of the 4-*śruti* intervals, that is, 4 + 2 in place of 2 + 4 (compare Ex. 1). Readers will be reminded of the phenomenon of *musica ficta* in the European Middle Ages. But while these two are clearly similar to sharps, there is absolutely nothing in the ancient Indian system comparable to the Western flat (although there *is* in today's music).

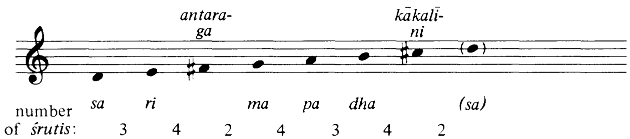
Alterations are a crucial part of our argument, and once again it will be useful to point out what is not known along with what is known. With respect to musical practice, it is not always possible to determine the exact role of altered degrees

[9]According to Mataṅga, *Bṛhaddeśī* 44–47.

Example 3. Rotations (the *mūrcchanās* or "octave species")



Example 4. The two altered scale degrees (A and K)



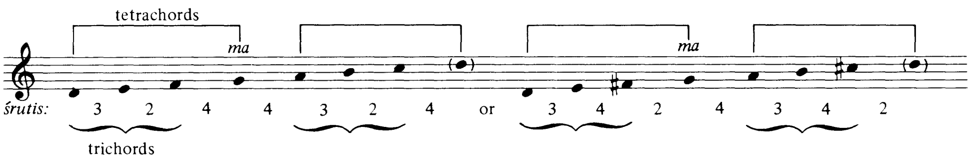
—whether, for example, they were simply permissible ornaments, coequals with their diatonic counterparts, or substitutions. What *is* known is that their status as full substitute members of the scale increased in actualized constructions such as the *jātis* and *grāmarāgas.*[10] There is solid evidence that both alterations were often used together, and also that *antara-ga* (that is, sharp 3) frequently appeared alone. It is less certain that *kākalī-ni* (sharp 7) could appear without *antara-ga.* Any and all of these combinations are heard in *rāgas* today, but this is no guarantee of their antiquity and the evidence of the early treatises is ambiguous.

Several early authors recognized the tendency toward tetrachordal balancing within the scale, as illustrated in Example 5, both in the unaltered *sa-grāma* and in the version with both altered degrees. The same symmetry cannot exist in the unrotated form of the *ma-grāma,* which may help to explain the slight preference for the *grāma* on *sa.* In this connection, it is important to point out that *ma* (the fourth degree) was the indispensable note in the scale; it could neither be omitted nor altered. It may perhaps be a misleading comparison to think of *ma* as the equivalent of the ancient Greek *mese,* but in the early Indian system the focal tone in any actualized melodic construction was more likely to be a central, pivotal tone than a "ground tone" to which all musical activity would eventually subside. In this respect the scales of ancient India seem closer in concept to the medieval European psalm tones than to the tonal structure of the free chants. By the end of the Indian Middle Ages all this had changed, and *sa* had become installed not only as the automatic tonic note of each *rāga,* but also of the entire system of *rāgas.* The prominent role of *ma* in the ancient and medieval system, along with the relative subordination of *pa* (the fifth degree), means that the tonal basis for the system can more accurately be described as *plagal,* rather than *authentic.* We recall Curt Sachs's claim that plagal organization appears to be of greater antiquity than authentic in the early history of world scale systems.[11] Our material seems to validate his contention.

[10]For the *jātis* and *grāmarāgas,* see MMTEI, pp. 166–77.

[11]Curt Sachs, *The Rise of Music in the Ancient World—East and West* (New York: Norton, 1943), 65.

Example 5. Trichords and tetrachords in the *sa-grāma*



Example 6, a display of the seven original *grāmarāgas* (one of the two oldest layers of the *rāga* system), will provide some solid evidence on the question of the two altered degrees and their status within the scale. These scales are mentioned in several sources between A.D. 500 and 1000, and they were also known in China during the Northern Zhou Dynasty (ca. A.D. 570) under Chinese transliterations of their original Sanskrit names.[12] This set of seven scales demonstrates that the theorists have already been hard at work, in an apparent attempt to construct a closed system characterized by both regularity and completeness (a typical pair of incompatible objectives). It is not a total success: finals have been restricted to the fourth and fifth degrees (*ma* and *pa),* and there is now only one possible tonic note for each *grāmarāga.* It is notable that the roles of tonic, initial, and final are separate functions. The compilers of this system seem to have had two primary objectives: instead of constructing a cluster of seven scales by systematic rotations of the diatonic octave, they sought (1) to limit the number of possible interval sequences, and (2) to establish three of the scale degrees (*sa, ma, and pa*—degrees 1, 4, and 5) as nodal degrees with specialized roles.

The *grāmarāgas* displayed in Example 6 include three familiar arrangements of the diatonic octave: in Western terms, two Dorians, two Mixolydians, and two Ionians, in addition to a hexatonic version that may have been added to fill out the system to the desired number seven. *Grāmarāgas* were also differentiated from one another by secondary features such as strong and weak degrees, oscillations between degrees, tremolo, slides, and other categories of ornament. Taken as a set, the *grāmarāgas* demonstrate a greater and more systematic application of the two altered degrees than any other early Indian scales. The altered notes may have been passed over lightly, shaken, treated as unstable lower neighbors, or even as alternates to the regular degrees, but at least within the context of this system they were specified among the features that gave each of the *grāmarāgas* its distinctive flavor and tonal color. Note also their tendency to serve as lower neighbors to notes with particular degree functions (final, initial, and/or tonic).

VARIABLE 3: OMISSION

One of the important goals of this study has been to locate the algorithm(s) by which the heptatonic scales were converted into hexatonic and pentatonic versions by the omission of one or two members. To put it bluntly, having gotten from 22 to 7 (a pathway that is addressed in part 2), how did early musicians then proceed to 6 and 5? We assume that this process was both systematic and lawful. A clear consensus on results exists in the early textual evidence, but rarely has any

[12]R. F. Wolpert, "Lute Music and Tablature of the Tang Period" (Ph.D. diss., Cambridge University, 1975), 106–11.

Example 6. A conspectus of the seven primary *grāmarāgas*

emboxed notes = *aṁśa*
F = final
I = initial
from S = derived from *sa-grāma*
from M = derived from *ma-grāma*

of the authors suggested any reasons for these choices—perhaps because of the need to compress maximum information into minimal space, and perhaps also because of a preference to impart this type of information in face-to-face instruction to those who had proved themselves worthy to receive the full details.

The rules for omission (that is, for converting the 14 *mūrcchanās* of the *sa-* and *ma-grāmas* into the 84 hexatonic and pentatonic *tānas)* are as follows:

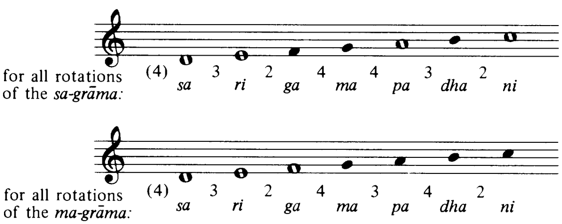
for the hexatonic versions,
in the *sa-grāma,* drop *sa, ri, pa,* or *ni;*
in the *ma-grāma,* drop *sa, ri,* or *ga;*

for the pentatonic versions,
in the *sa-grāma,* drop one of the pairs *sa/pa, ga/ni,* or *pa/ri;*
in the *ma-grāma,* drop one of the pairs *dha/ri* or *ga/ni.*

The numbers are not difficult to explain: each of the two basic *grāmas* can appear in seven rotations, and this accounts for the 14 *mūrcchanās.* The rules listed above specify 12 options for omission; further, each of these options is limited either to the *sa-* or *ma-grāma* and applies to all seven possible rotations of one or the other scale. Therefore, 7 rotations × 12 options = 84. This accounts for the numbers, but why these options and not others? The considerable discrepancies between the two source scales cannot be explained solely on the basis of the single-*śruti* discrepancy in their tuning.

First the problem of the hexatonics, as illustrated in Example 7; the formal explanation will follow in part 2. The relative priorities assigned to the various scale degrees must have been among the criteria for inclusion and exclusion: *ma,* for example, could never be dropped from any of the scales because it was situated 9 *śrutis* above, and 13 *śrutis* below, *sa.* But how then can one explain the fact that *sa* itself could be dropped from either scale? And similarly, *pa* could be omitted in the *sa-grāma.* The status of *pa* in the *ma-grāma* is interesting and affords a glimpse of Indian logic: in this *grāma, pa* may not be dropped, for the following reason. Although it was now located 12 *śrutis* above *sa* (instead of 13, as in the *sa-grāma)* and was therefore no longer in consonant agreement, *pa* was nevertheless held to be indispens-

Example 7. Dispensable and indispensable notes in the hexatonic *tānas*



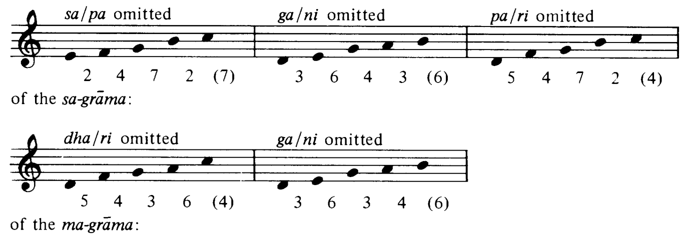
blackened noteheads = indispensable *svaras*
empty noteheads = *svaras* subject to omission

able because it had moved into a 9-*śruti* consonant relationship with *ri,* the second scale degree. Readers may well find this logic as elusive as we do.

The pentatonics are displayed in Example 8; here the algorithm by which the permissible sets are derived is somewhat clearer. The note pairs identified for possible omission exclude, as might be expected, the indispensable *ma.* More important, all but one of the pairs are in consonant agreement; the single exception is the pair *pa/ri* in the *sa-grāma,* spanning an interval of 10 *śrutis.* Three of the authorized pentatonic sets are versions of the familiar anhemitonic pentatonic, the scale that has dominated the musical history of East Asia.

This survey of the gamut and its variables concludes with two informal remarks on these specifications, which were undoubtedly reached on aesthetic grounds. First, the requirement that only note pairs separated by intervals of 9, 10, or 13 *śrutis* may be omitted insures that the resulting pentatonic sets will be balanced; that is, they will contain two nonadjacent gaps of minimal size, instead of two consecutive gaps or a single large interval.[13] And second, the authorized pen-

Example 8. Permissible pentatonic sets



tatonic sets, because of their derivation as subsets of a larger referential set, will be more likely to be conceived and perceived as "gapped" scales. This is generally not the case with other versions of the same pentatonic scales from around the world, in which the larger intervals are often conceived and heard *not* as gaps or skips, but as "steps" equivalent to the smaller intervals of the scale.

COMMENTARY

Readers of this journal may recall Robert Gauldin's article on "The Cycle-7 Complex" in which he remarked that he would not be surprised to discover the diatonic scale "alive . . . and well on some distant planet."[14] We prudently decided to confine our investigation to this planet, but we believe we have come several steps closer to identifying common scale-derivation mechanisms shared by very different cultures. These mechanisms, which we will explain in the following section, cannot be attributed to any theory of cultural diffusion, but we cannot exclude the possibility of ethnic or racial preferences. The scales of East Asia, for example, seem to represent an entirely different musical world.

[13]A more formal definition of a maximally even set will follow in part 2.

[14]*Music Theory Spectrum* 5 (1983): 53.

But what are the principles that influence the selection and construction of scales covered in this study? We are unwilling to exclude the possible influence of certain "universal" or "near-universal" tendencies, for example, the desire for tonal focus and gravitation (whether toward the center or the bottom), for ease of calculation, for balance, for regularity, for completeness, and preferences for certain interval classes. Any and all of these have probably contributed to the problem in one way or another, and must therefore be a part of an eventual explanation. We have had to remind ourselves on several occasions that the material examined here has come far beyond the stage of raw data. It represents a highly sophisticated stage in the efforts of theorists to assemble a comprehensive system from a diverse collection of musical phenomena. Some of the most interesting choices have already been made.

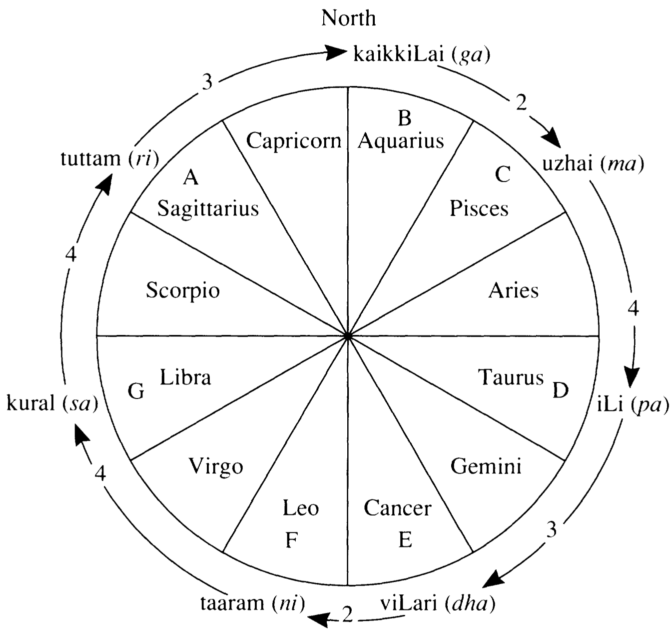
In their efforts to erect a rational framework for their colorful heritage of melody (which we assume to have included both chromatic and diatonic constructions of various kinds), the musical scholars of early India appear to have settled upon the diatonic system as the most appropriate, effective, and flexible template with which to organize the subtler pitch distinctions they evidently heard and relished in their music. But it obviously did not suit enough of their material to survive as the exclusive basis for the musical system. Eventually, as in the West, a more chromatic system was required, and by the thirteenth century a rough system of twelve-tone equal temperament had begun to replace the older system of 22 *śrutis*. This is convenient for our theory, as we postulate, in one of the three generation schemes, an intermediate set of 12 microtonal divisions as part of the pathway from 22 to 7. (We do *not* claim that this "pathway" is anything more than an interpretation of what seems to have happened, still less that it represents any conscious intentions or historical imperatives.) However, no evidence from early Indian literature has been presented here in support of a division of the tonal spectrum into 12 parts, and indeed there is no such evidence from the central Sanskrit tradition prior to the thirteenth century.

But the ancient Tamil musical tradition, the music of the Draviḍa people who inhabited the southern tip of the subcontinent (including the modern states of Kerala and Tamil Nadu), presents solid evidence for a cyclical 12-note system as early as the second century A.D.[15] In this system, 22 microtones (called, in Tamil, the *maattirais)* were distributed among 12 chromatic degrees in more-or-less regular pattern, and a rotating set of diatonic scales was then selected from this 12-note chromatic universe.

Figure 2 is a display of the scale known as *paalaiyaazh*, a scale that evidently served as the base for the modal rotations of the Tamil system in much the same way that the *sa-grāma* functioned for the Sanskrit system. This scale corresponds to the Western Mixolydian on G, as shown by the pitch-class names inside the circumference of the circle; the Tamil names for the scale steps and their Sanskrit equivalents appear outside the circumference of the circle. This circular projection of the scale is the only such projection that is fully explained in the *Cilappatikaaram*, but the text also mentions linear, triangular, and square projections. This is a tantalizing piece of information, but there is no basis for speculation on what the results of these projections might be. The numbers along the circumference of the circle indicate the number of *maattirais* between each of the scale steps; the overall pattern of these microtonal divisions duplicates the disposition of *śrutis* in the *sa-grāma* (compare Ex. 1). The scale proceeds in a clockwise direction along the 12 houses of the zodiac,

[15]In these paragraphs, we draw upon the information and interpretations presented in S. Ramanathan, *Music in Cilappatikaaram* (Madurai: Madurai Kamaraj University, 1979). We are also grateful to Sister Margaret Bastin of the University of Madras for assistance with the details of the ancient Tamil musical system.

Figure 2. *Paalaiyaazh* in the ancient Tamil musical system

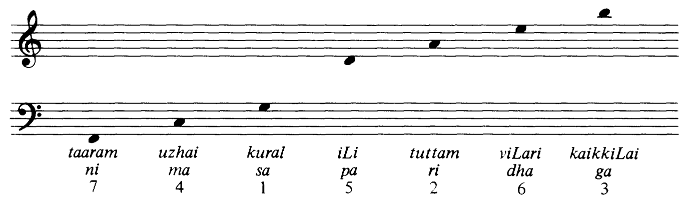


thus lending an additional symbolic dimension to the musical system.[16]

What tuning deductions can we draw from this intriguing diagram? First of all, it is evident that this system cannot represent both 12-tone and 22-tone equal temperament at one and the same time! It *looks* like 12-tone equal temperament, but how then can we explain away the fact that in-

[16]In addition to these cosmic coordinates between the zodiacal houses and the musical notes, the four major modes (*paNs)* are equated with and named after the four types of terrain in South India, that is, the cultivated, desert, mountainous, and seashore regions.

Example 9. *Paalaiyaazh* by fifth generation (*iLikkiramam)*



tervals of 3 and 4 *maattirais* turn out to be identical? S. Ramanathan has interpreted them as just intervals (10:9 and 9:8), which is convenient but which also destroys the symmetry of the circular projection and the equidistant spacing of the 12 houses.[17] There are no easy answers.

The notes of the heptatonic scale were apparently derived by fifth projection (*iLikkiramam*) by tuning the strings of the *yaazh,* a type of harp. Example 9 illustrates the process of fifth generation for *paalaiyaazh,* in which each successive note is said to have been "born" in its predecessor, and in which each new scale degree is attained at the fifth note and eighth house.[18] If the fifth needed to close the circle was never calculated, this may well have provided a satisfactory method of tuning. But what this interpretation does *not* solve is the distribution of the 22 *maattirais* among the 12 chromatic degrees. The suspicion remains that what we have been examining is an ingenious attempt to reconcile two competing systems, with the 12-note system as the eventual winner. What strikes us as important is that the diatonic scale was clearly the ultimate goal, no matter by what means it was approached.

Indian scholars are only now beginning to take the Tamil musical system seriously, and we have reservations about the

[17]Ramanathan, *Cilappatikaaram,* 13–17.

[18]That is, counting the note and house in which the new note was born.

tidy interpretation set forth in the preceding paragraphs. But this evidence places the pathway from 22 to 12 to 7 on Indian soil at a very early date, fully as early as the oldest extant treatises in the Sanskrit tradition. It also provides solid evidence for scale generation by perfect fifths, evidence that is lacking entirely in the early Sanskrit texts. Just as the ethnographic history of ancient India was determined by the cultural encounter between the indigenous Dravidian people and the Indo-Aryan invaders from Central Asia, so may some of the secrets of early Indian music be unlocked by further study of the mutual influences and creative friction between these two ancient musical traditions.

II

In the second part of this study we compare two heptatonic scale systems: the early Indian system described above and the Western diatonic. We also address briefly the problem of deriving the Indian 6-note and 5-note scales from their 7-note parents.

For the present purpose, we regard a scale as a particular division of the octave, based on 22 *śrutis* or 12 semitones. We assume that all *śrutis* are equivalent, likewise all semitones, but we make no assumptions about tuning. Thus our approach is essentially combinatorial, as opposed to acoustical. We reckon intervals in terms of ordinal distances, not frequency ratios, and we are concerned with how smaller collections of notes may be derived from larger ones. That the Indian consonance of 9 or 13 *śrutis* and its Western counterpart the perfect fifth play important and parallel roles in these derivations may be due ultimately to their acoustical qualities, but these intervals have special status here mainly because of their combinatorial properties.

In dealing with tonal music, a particular diatonic scale is commonly conceived as a member of a class including all the diatonic scales, to which it bears a hierarchical set of relationships. It might be interesting to treat *sa-grāma* and *ma-grāma* in this way; however, nothing in the early Indian literature indicates the concept of classes of distinct but transpositionally equivalent scales. For comparative work, then, one can deal directly with the *interval* structure of the various scales, without reference to particular pitch classes or set classes. As an identifier for a scale, we use the cyclic interval pattern of the scale in modular space—that is, essentially, Robert Morris's CINT_1 function—here called the *interval pattern* of the scale.[19] The interval pattern of the Western diatonic in semitones is, of course, 2212221, and the patterns of *sa-grāma* and *ma-grāma* in *śrutis* are those given in Example 1.

Since interval patterns are conceived as inherently cyclic structures with no first or last note, rotation of pattern (producing, loosely, a different *mūrcchanā* or mode) is not defined. But the retrograde of a pattern (producing, loosely, the inversion of a scale) is defined in the obvious way, and is considered distinct from the original pattern, except for cases of inversionally symmetrical scales.

One may represent an interval pattern as a string of numbers (2212221 or whatever) by beginning with *any* interval in the pattern and going through one complete cycle. There are many alternative and equivalent notations for the same pattern—these are of course rotations of one another. Since we shall examine only a few interval patterns, there is no need to establish a convention for normal form (that is, a convention for selecting a particular rotation). We choose freely among various equivalent rotations in order to highlight this or that feature of a pattern.

Our method will be to enumerate features of the early Indian heptatonics and the Western diatonic—features both shared and unshared between the two scale systems—and to

[19]Robert D. Morris, *Composition with Pitch Classes* (New Haven: Yale University Press, 1987), 107–08.

search for sets of common features which generate the scales of both systems and only those scales.

A nettlesome question arises at the outset: Which of the Indian scales (or which versions of them) shall we take as members of the group of scales to be generated in parallel with the Western diatonic? Certainly we wish to include *sa-grāma* and *ma-grāma,* but what of their altered versions? And what of the inversions of *sa-grāma* and *ma-grāma?* For the scales under discussion, the most telling features are invariant under inversion. (For example, if a scale has exactly one tritone, its inversion also has exactly one tritone.) Therefore, it will be unfortunate if inversional forms turn out to be illegitimate. In that event our machinery—that is, the sets of common features—will churn out unwanted scales along with the desired ones.

Since the Western diatonic scale, considered as a set of pcs, has no distinct inversional form, whether or not we recognize distinct inversions of this scale will have no effect on results. The situation is different in the Indian case, where both *sa-grāma* and *ma-grāma* have distinct inversional forms. Fortunately, the interval patterns of these inversional forms turn out to be identical to those of certain altered forms of the same scales, namely the forms with *ga* raised and with both *ga* and *ni* raised.

Shown in Figure 3 are correspondences among *sa-grāma, ma-grāma,* their altered forms, and inversions, based on interval patterns. Placed clockwise along the *inner* circles of parts a and b of the figure are *sa-grāma* and *ma-grāma,* respectively. Counterclockwise along the *outer* circles are their inversions. As indicated in tables beneath the circles, the inversions correspond to altered scales, and *ma-grāma* itself corresponds to an altered form of *sa-grāma.*

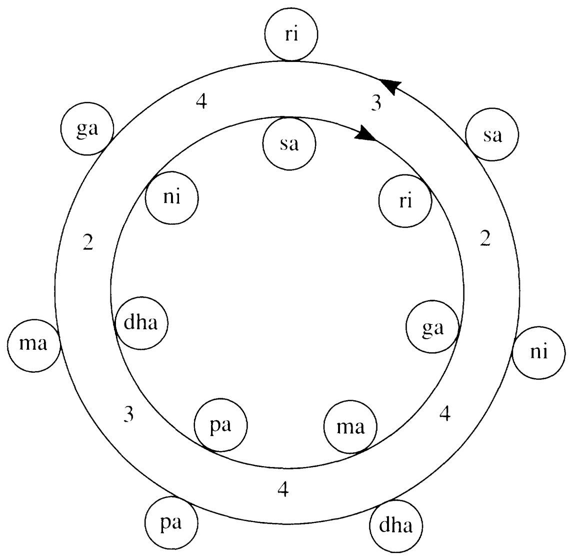
These correspondences of altered scales to inverted and uninverted forms of *sa-grāma* and *ma-grāma* (which, as far as the authors know, have not been previously noticed) are happy ones, for the particular altered forms involved, with *ga* raised or both *ga* and *ni* raised, are, as noted in part 1, precisely those given preferred status in the Indian literature, while the altered forms with only *ni* raised, whose patterns do *not* coincide with *sa-grāma, ma-grāma,* or their inversions, are not mentioned in the literature. Consequently, if we recognize, in addition to *sa-grāma* and *ma-grāma,* their preferred altered versions, the result is a set of six scales corresponding to just four interval patterns—those of *sa-grāma* and *ma-grāma* and their inversions—and the question of whether to include inversional forms becomes moot.

The strategy adopted here—that is, our answer to the nettlesome question raised earlier—is to designate the six scales of Figure 3 as the set of Indian scales to be generated in tandem with the Western diatonic. To the extent that the set is seen to be incomplete or fuzzy with respect to the early Indian literature, the results will have to be seen as similarly incomplete or fuzzy—a circumstance which may be tolerated in the interest of gaining some understanding of the striking resemblances between two culturally and chronologically separate musical systems.

Henceforth we refer to the designated set of Indian scales or interval patterns as "the *grāmas,*" and to the Western diatonic scale or its interval pattern as "the diatonic scale," also to the *grāmas* and the diatonic scale together as "the two scale systems."

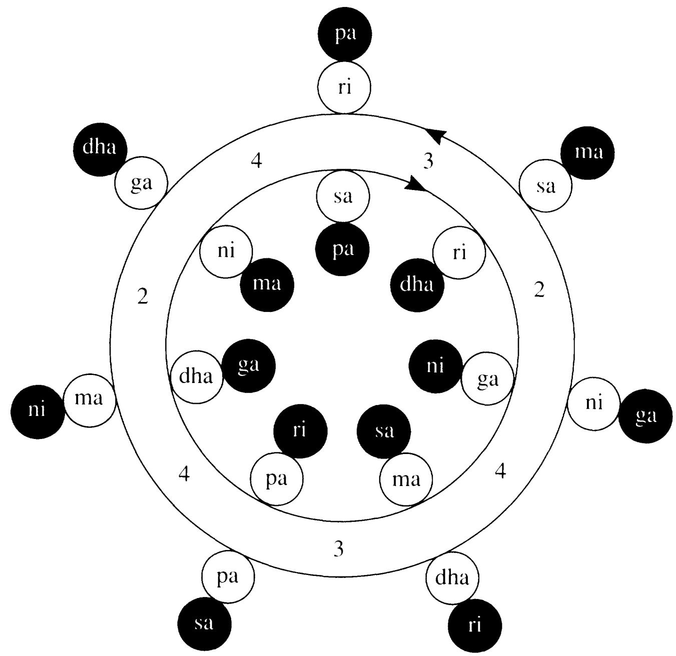
We now proceed to list features of the two scale systems, some of which may be immediately verified with reference to Figure 3. A preliminary list of common features is given in Table 3. Feature C1 is self explanatory. Feature C2, the solitary tritone (that is, a half-octave—11 *śrutis* or 6 semitones), occurs in the *grāmas* as a sequence of step intervals including two 4s and one 3, or as a sequence including the remaining step intervals: two 2s, one 3, and one 4. (Sequences with two 4s and one 3 can be seen at the bottom of each circle in Figure 3.) The chromatic distance of 11 *śrutis* corresponds to both

Figure 3a. Correspondences between *sa-grāma* and *ma-grāma*



	scale(s)	tetrachords	circles of 3rds	circles of 4ths
inner circle:	*sa-grāma*	324 or 432	5857676	9 9 9 11 9 10 9
outer circle:	*ma-grāma, ga# ni#*	234 or 423	7585676	9 11 9 9 9 9 10

Figure 3b. Correspondences between *sa-grāma* and *ma-grāma*



	scale(s)	tetrachord	circles of 3rds	circles of 4ths
inner circle				
unshaded:	*ma-grāma*	243	5767676	9 9 9 11 9 9 10
shaded:	*sa-grāma, ga#*			
outer circle				
unshaded:	*sa-grāma, ga# ni#*	342	7675676	9 11 9 9 9 10 9
shaded:	*ma-grāma, ga#*			

a 4th *and* a 5th, as does the chromatic distance of 6 semitones in the diatonic scale. Using the definitions given by Jay Rahn,[20] we find that there are no other *ambiguities* in the *grāmas*. That is, no chromatic interval other than 11 *śrutis* corresponds to more than one generic interval, and likewise

[20]Jay Rahn, "Coordination of Interval Sizes in Seven-Tone Collections," *Journal of Music Theory* 35 (1991): 33–60.

Table 3. Features in common between the *grāmas* and the diatonic scale

(C1)	7 notes
(C2)	exactly one tritone, no other ambiguity, no contradiction
(C3)	dual tetrachords—two 4-note scale segments of identical structure
(C4)	each nonzero diatonic interval (2d, 3d, . . . , 7th) comes in two or more consecutive chromatic sizes (C4)
(C5)	sonance—all notes in consonant relationship with at least one other note
(C6)	first- or second-order maximally even, out of 12-note parent
(C7)	partitioning—specific chords (reckoned chromatically) are unambiguous in their generic membership (reckoned diatonically)

for the tritone in the diatonic scale. Further, no scale in either of the two systems contains a *contradiction.* That is, all 2ds are smaller than or equal to all 3ds, all 3ds are smaller than or equal to all 4ths, and so forth.

Feature C3, dual tetrachords, is indicated in the middle column of Figure 3. In each of the four interval patterns, there are one or two *repeated* tetrachordal subpatterns. Each subpattern consists of three intervals in sequence, in every case a permutation of the intervals 2, 3, 4. Note that all six permutations appear as tetrachords in the four patterns. Two things follow logically: first, specification of the repeated tetrachord is sufficient to imply the complete interval pattern of the scale (two identical tetrachords with an additional interval of 4); and second, all four patterns have the same composition in terms of step intervals—two 2s, two 3s, and three 4s. The well-known dual tetrachord feature of the diatonic scale requires no comment here.

Feature C4, the consecutivity property, is easily checked for the *grāmas:* that step sizes are consecutive is evident from the preceding discussion; consecutivity of 3ds and 4ths can be seen in the circles of 3ds and 4ths given in the next columns of Figure 3; consecutivity of interval sizes for 5ths, 6ths, and 7ths follows as a consequence of consecutivity of their smaller, complementary generic intervals. Feature C5, the sonance condition, requires, for the *grāmas,* that every note be linked to one or both of its neighbors on the circle of 5ths by a consonant interval of 13 *śrutis;* this feature can be deduced from the circles of 4ths in Figure 3. Again, these features of the diatonic scale need no comment here.

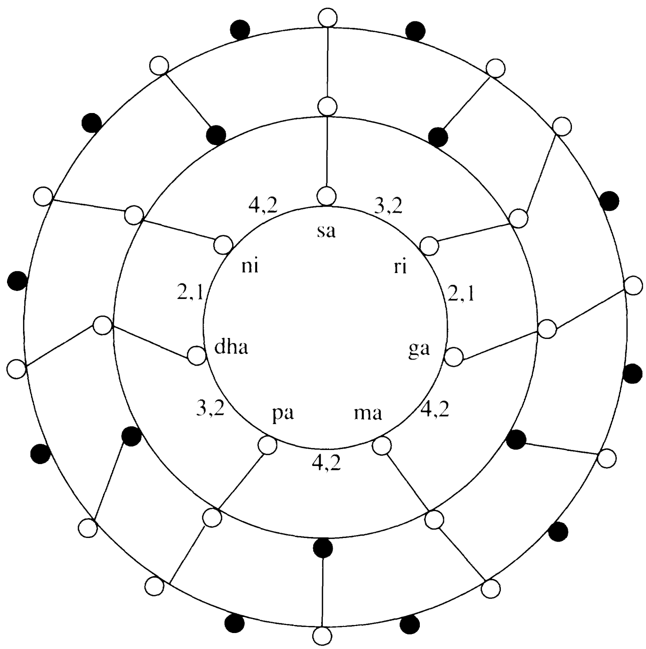
Feature C6 requires a bit more explanation. As defined by Clough and Douthett, a maximally even set is one in which every generic interval—2d, 3d, 4th, etc.—comes in just one size or in two consecutive sizes. Clearly the Western diatonic is maximally even with respect to the 12-note chromatic. If the diatonic scale is then regarded as a "chromatic" resource, the scalar triads—major, minor, and diminished—may be derived from the diatonic as *second-order* maximally even sets. Derivation of the *grāmas* proceeds similarly, but here the sequence of cardinalities is 22, 12, 7, instead of 12, 7, 3. This derivation is shown for the case of the *sa-grāma* in Figure 4. Thus the Western diatonic is (first-order) maximally even with respect to a 12-note "parent," while the *grāmas* are second-order maximally even with respect to the "grandparent" system of 22 *śrutis* and a hypothetical 12-note parent. If the 12-note parent of *sa-grāma* and *ma-grāma* is regarded as "chromatic," then these scales have the same relationship to their parent as does the Western diatonic to its 12-note parent.

Finally, feature C7, "partitioning," described by Clough and Myerson,[21] is that any particular chord corresponds to a

[21]John Clough and Gerald Myerson, "Variety and Multiplicity in Diatonic Systems," *Journal of Music Theory* 29 (1985): 249–70. A different version of this paper appeared in *American Mathematical Monthly* 93 (1986): 695–701.

Figure 4. *Sa-grāma* as a second-order maximally even set

Number pairs represent intervals between steps in *sa-grāma* measured in *śrutis* (outer circle) and in "semitones" of intermediate 12-note scale (e.g., "3,2" indicates an interval of 3 *śrutis* and 2 "semitones").



single chord class (for example, in the diatonic scale, chromatic interval pattern 435 [the major triad] corresponds only to diatonic interval pattern 223 [the triad]). This property is taken for granted, but it is easy to construct scales that do not have it. The *grāmas,* however, do have it.

What abstract properties have the *grāmas* that are *not* shared by the diatonic scale, and vice versa? Returning to the list of common features, note that feature C4 can be restated to reflect a *difference* between the two systems instead of a similarity: in the *grāmas* each diatonic size corresponds to three or four consecutive chromatic lengths, while in the diatonic scale each diatonic size comes in two consecutive chromatic sizes. Feature C6 may be similarly bifurcated to show a distinction between the two scale systems. Apart from these two distinctions, however, the *grāmas* seem to lack meaningful quantitative features *not* shared by the diatonic scale.

In contrast, the diatonic scale has features not found in the *grāmas:* these are listed in Table 4. Feature W1, "cardinality equals variety," also described by Clough and Myerson, is actually a consequence of that which they define as Myhill's property: every diatonic interval comes in precisely two sizes. Feature W2 is the famous property of unique multiplicity of chromatic intervals first noted by Milton Babbitt and later studied by Carlton Gamer.[22] As for feature W3, note that the *grāmas* lack one or two chromatic intervals (*sa-grāma* lacks 1/21; and *ma-grāma* lacks 1/21 and 8/14, etc.), while the diatonic scale contains all possible chromatic sizes. There may well be additional important properties of either or both scale systems which go unnoticed here. From the above analysis, though, it would appear that the intervallic features of the *grāmas* are loosely encompassed by those of the diatonic scale.

Having arrayed the features of the two scale systems, we now use these features to construct the two systems in parallel. As a first step in the construction we inquire: is the set of shared features {C1, C2, . . . , C7} sufficient to isolate the two systems in their respective chromatic universes, or are there scales outside the systems in question to which all seven

[22]Carlton Gamer, "Some Combinational Resources of Equal-Tempered Systems," *Journal of Music Theory* 11 (1967): 32–59; and idem, "Deep Scales and Difference Sets in ETS," *Proceedings of the American Society of University Composers* 2 (1967): 113–22.

Table 4. Features of the diatonic scale not shared by the *grāmas*

(W1) cardinality equals variety—Every generic chord comes in a number of species equal to its cardinality.
(W2) deep scale
(W3) all chromatic intervals present

features apply as well? The answer is that the set of seven shared features applies to all the scales of the two systems and only to those scales.

Are all seven shared features necessary? That is, are there smaller, irreducible sets of shared features which generate all the desired scales and no other scales? To present the affirmative answer to this question, we revise the preliminary list of shared features as follows, deleting whole features and parts of features not destined to appear in the irreducible feature sets: Feature C1, 7-note cardinality, is removed from the list and will be put as a global condition, along with the cardinalities of the two chromatic universes. Feature C2 is pared down to the feature of the single tritone. Feature C4, consecutivity for all generic sizes, spawns two separate features implied by that feature—consecutivity for steps and consecutivity for 5ths; consecutivity for 3ds is omitted. Finally, feature C7, partitioning, is omitted. The resulting set of features—we call them *defining* features—appears as Table 5a.

What are the irreducible sets of features which generate both systems? We have found three such sets, shown in Table 5b. For any of the sets E1, E2, E3, every scale in the two systems has all the features of the set, and no *other* 7-note scale in either the 12- or 22-pc universe has all of them. Further, for any proper subset of E1, E2, or E3, the above statement is false. Thus, any of the three feature sets is sufficient to isolate the *grāmas* and the diatonic scale from the other 7-note scales in their respective universes, and all of the features in the set are indispensable to this dual generation.

Table 5.

a. Defining features
 (D1) exactly one tritone
 (D2) dual tetrachords
 (D3) consecutive step sizes
 (D4) consecutive fifth sizes
 (D5) sonance
 (D6) first- or second-order maximal evenness, from 12-note parent
b. Irreducible, generating feature-sets
 E1 = {D1,D2,D3}
 E2 = {D1,D4,D5} (feature D4 is unnecessary in the diatonic case)
 E3 = {D2,D6} (feature D2 is unnecessary in the diatonic case)

Feature sets E2 and E3 are slightly weaker than E1 in that, for the isolation of the diatonic scale, not all of their features are necessary. (Specifically, feature D4 may be dropped from set E2, and feature D2 may be dropped from set E3.) Thus feature set E1 is stronger than E2 or E3 in that it is sufficient to isolate *either* the *grāmas or* the diatonic scale from the other 7-note scales in its universe, and all of the features in the set are indispensable to *each* of these purposes.

It is possible to construct other feature sets which isolate *either* of the systems but not both. For example, the pair of features "exactly one tritone" and "first-order maximally even" isolates the diatonic scale, and it is not even necessary to specify the cardinality of the scale in this case. But E1, E2, and E3 are special in that any one of them isolates the scales of both systems. That even one such lean feature set should exist seems remarkable; that at least three exist is surprising.

Each feature of an irreducible feature set may be thought of as specifying a collection of heptatonic scales including those of the *grāmas* and the diatonic scale and other scales as well. The two systems, then, lie at the intersections of the collections specified by the various features in the feature set. This is the *logic* of irreducible feature sets. The *process* of constructing them is largely empirical.

Suppose, for example, that we begin with feature C4, the consecutivity feature (each nonzero diatonic interval comes in two or more consecutive chromatic sizes). This feature is clearly true for both systems, but it is true of many other scales as well, including the hypothetical *ga-grāma* mentioned in part 1. Noting that some of the unwanted scales in both the 22- and 12-note worlds lack dual tetrachords, we adjoin feature C3, dual tetrachords. This leaves the desired scales intact and filters out all but the small residue of undesirables shown in Table 6 with dual tetrachords exposed: from the 22-note world, the boring scale given as part a, and various permutations of the interval set of *ga-grāma,* given as part b; and from the 12-note world, the scales of part c. All the scales of Table 6 have either zero tritones or two tritones, so we jettison these scales by adjoining feature C2: exactly one tritone, no other ambiguities, no contradiction. Now it turns out that only the "single tritone" part of feature C2 is needed, which yields feature D1, and feature C4 can be pared down to call for consecutive steps only, which gives feature D3. The result is feature set E1, which is now irreducible for both scale systems.

To see how feature set E2 may be induced for the 22-note world, the sonance condition, D5, may be taken as a starting point. This requires that all seven notes of the scale be attached in one or both directions to a consonant 5th of 13 *śrutis.* In terms of 4ths (where the numbers are a little smaller), at least four 4ths of 9 *śrutis* are needed. (If there are as few as three, then we can join together at most six notes.) To simplify matters, we immediately adjoin feature D4—consecutive sizes of 5ths (which implies consecutivity for 4ths as well). These two conditions limit the possible unordered combinations of 4th sizes to the two shown in Table 7a. Since combination a(ii)—five 9s, one 10, and one 11—fits

Table 6. Scales with consecutivity feature and dual tetrachords

a. 333 333 4		
b. 334 334 2	343 343 2	433 433 2
c. 113 113 2	131 131 2	

Table 7.

a. combinations of 4ths:	(i) 9 9 9 9 10 10 10	(ii) 9 9 9 9 9 10 11
b. sonant circles of 4ths:	(i) 9 9 9 9 10 9 11	(ii) 9 9 9 10 9 9 11

the *grāmas,* we close the door on combination a(i) by adjoining (yet again!) the single-tritone feature. It remains to arrange the intervals in the proper order. In order to preserve sonance, the 10 and 11 may not be next to each other in the circle of 4ths; this would imply that one note of the scale is unconnected by consonance. Considering rotations and retrograde orderings (which imply scale inversions) to be equivalent for the moment, there remain essentially two orderings—those given in Table 7b. These produce, respectively, the *sa-grāma* and its inversion and the *ma-grāma* and its inversion, which is to say, the *grāmas.* Hence, feature set E2, which turns out to be not only sufficient but irreducible in the case of the *grāmas,* and sufficient (but reducible to two of its three features, as noted above) in the case of the diatonic scale.

More insight on the necessity of particular features may be gained from Table 8, where each row lists one of the three feature sets minus one of its features, together with examples of "illegitimate" scales from the 12- and 22-note universes that are consistent with the reduced feature set. The two Xs indicate the reduced features sets discussed above that are sufficient for isolation of the diatonic scale.

Table 8. "Illegitimate" scales

	"illegitimate" scales from	
reduced feature set	*22-note universe*	*12-note universe*
E1 − D1 = {D2,D3}	3343342	1131132
E1 − D2 = {D1,D3}	4342333	1112223
E1 − D3 = {D1,D2}	1271272	1111116
E2 − D1 = {D4,D5}	3333334	2113113
E2 − D4 = {D1,D5}	2162416	X
E2 − D5 = {D1,D4}	5333332	3121131
E3 − D2 = {D6}	3441442	X
E3 − D6 = {D2}	3333334	1211214

Table 9. Indian hexatonic scales: interval structure

sa-grāma, *omit*		*paired with*	ma-grāma, *omit*	
ri	*544324*		*sa*	*543424*
ga	364324		ni	363424
ma	328324		dha	327424
pa	*324724*		pa	324724
dha	324454		ma	324364
ni	*324436*		*ga*	*324346*
sa	*244327*		*ri*	*243427*

Permissible scales are in italics. Tritone (= 11 *śrutis)* spans of 2 or 3 steps are underlined.

We offer next some brief observations on the hexatonic and pentatonic scales derived from the Indian heptatonics. Regrettably, we are unable to specify compact algorithms for these "6-out-of-7-note" and "5-out-of-7-note" cases, as we have done for the "7-out-of-22-note" case, but there are some interesting regularities to be noted.

A table of the fourteen possible hexatonics is given in Table 9, where scales are paired so that each pair includes one scale derived from *sa-grāma* and one from *ma-grāma,* and the two omitted notes, one in each scale, are symmetrical around the diatonic center *pa.* Permissible scales (according to the rules cited in part 1) are shown in italics. When the scales are paired in this way, either both scales of a pair are allowed, or neither is allowed, with the sole exception of the pair where *pa* is omitted from both scales—which is also the only pair in which both scales have the same interval pattern. (These are in fact the only two among *all* the hexatonics that have the same pattern.) It is conceivable that there was a reluctance to sanction the use of two scales of different parentage but with the same interval pattern. Of passing interest also is the fact that all the permitted scales include a tritone (tritone spans are underlined in Table 9), and all the tritones in the permitted scales span a 4th/5th, with the sole exception of that in *sa-grāma*-omit-*pa* (from the same exceptional pair mentioned above), which spans a 3d/6th. On the other hand, none of the disallowed scales includes a tritone spanning a 4th/5th.

It is easier to account for the allowable pentatonic scales. To begin with, they are all maximally even with respect to the parent heptatonics. This means that the two notes specified for omission are as far apart as possible, in terms of scale steps—a 4th/5th apart, not a 2d or a 3d—as described in part 1. This completes a path based on maximal evenness all the way from the 22 *śrutis* through the hypothetical 12-note intermediate scale to the heptatonics and then the pentatonics. The hexatonics may also be included in this scheme, as shown in Figure 5.

Table 10 provides a table of the fourteen pentatonic scales which are maximally even with respect to *sa-grāma* and *ma-grāma.* The two pentatonic scales in each row of the table omit the same pair of notes from *sa-grāma* and *ma-grāma.* Here interval patterns of permissible scales (in this case five) are shown in italics. The interval patterns are labelled A through E for those found in permissible scales, and V through Z for other patterns. (Like patterns are given in the same rotation to facilitate comparison.) In contrast to the case

Figure 5. Paths to the Indian pentatonics, via maximal evenness

$$\begin{tikzcd} 22 \arrow[r] & 12 \arrow[r] & 7 \arrow[r] \arrow[rr, bend right] & (6) \arrow[r] & 5 \end{tikzcd}$$

Table 10. Indian pentatonic scales: interval structure

omit	sa-grāma		ma-grāma	
sa-pa	*24727*	A	24727	A
ri-dha	54454	V	*54364*	B
ga-ni	*36436*	C	*36346*	D
ma-sa	28327	W	27427	X (A inverted)
pa-ri	*54724*	E	54724	E
dha-ga	54364	B	36436	C
ni-ma	32836	Y	32746	Z

Permissible scales are in italics. Tritone (= 11 *śrutis)* spans of 2 steps are underlined.

of the hexatonics, few of the pentatonics—six of fourteen—have unique interval patterns. The redundant patterns appear twice each in the remaining eight scales; however, no redundancy appears among the patterns of the permissible scales (also true of the hexatonics). We make no conjecture as to why a *particular* scale from each identically structured pair was designated.

If redundancy accounts for four of the nine impermissible scales, what accounts for the others? Four of the five would exclude the indispensable *ma.* This leaves only *sa-grāma*-omit-*ri-dha,* whose 54454 pattern is a likely clue. One sees at a glance that it is much more even than any other of the 14 possible pentatonics: it has only two "step" sizes and they are consecutive numbers. In fact this scale is a *first-order* maximally even set with respect to the universe of 22 *śrutis.* There is here an interesting parallel to the Western case,

Table 11. Irreducible generating feature set for Indian pentatonics

(P1) maximally even with respect to the parent heptatonics (omitted notes separated by a 4th/5th)
(P2) not maximally even with respect to the 22 *śrutis*
(P3) ma included

where the 3- and 4-note sets that are first-order maximally even with respect to the 12-note universe (that is, the augmented triad and the diminished-seventh chord) are treated very differently than the 3- and 4-note sets that are second order maximally even with respect to the parent diatonic (that is, the major and minor triads and assorted functional seventh chords).

So without the excessive contortions, we can account for the accepted pentatonics. They must satisfy the three conditions given in Table 11. These conditions do not account for choices which avoid duplications—that is, choices between two heptatonic scales as sources for a particular interval pattern. But the conditions, all of which are necessary, are sufficient to isolate the interval patterns of the five permissible pentatonics.

Our brief analysis of the Indian pentatonics and hexatonics should be taken as a speculative excursion, and we offer no comparisons to Western scales of the same cardinalities. For heptatonic scales, however, we have provided a comparison showing the deep affinities between the Indian and Western systems, and three generative schemes which yield the scales of *both* systems.

We conclude with some reflections on these findings and the method by which they were reached. We have surveyed the pitch structure of early Indian music from a distant perspective—the only one available to us—and we have taken pains to indicate that our concern has been for results, not motivations, intentions, and the like. It has also been clear to us that our findings reflect the preferences and decisions of a diverse clientele, not a solidly united bloc.

Diatonicism is a fact. That there are different, exquisitely different, versions of and approaches to diatonicism is also a fact. Do our findings confirm the existence of real similarities between the ancient Indian *grāma* system and Western diatonicism? We believe that they do, but readers will judge for themselves. Do they tell us what were the most flavorful, meaningful, or powerful features of early Indian musical practice and the mental processes of its performers and theorists? Surely not.

We believe our material reveals traces of deeper levels of musical instincts, instincts that have evidently led people in many different parts of the world to impose order upon a rich selection of raw sound data by channeling the sounds they heard into leaner, more economical, and more structured pitch collections. In the process, these musical thinkers have gravitated toward scale systems that (1) are balanced in certain predictable ways, (2) avoid large gaps, (3) can be manipulated without confusing their essential structure, (4) are comprised of intervals like enough to function as regular steps and unlike enough to differentiate the structures of small scale segments, and (5) have midway intervals that fall unambiguously into either a single defining dissonance or a smoothly blending consonance. There are strong indications, supported in the present article, that certain numerical pathways are more efficient than others in achieving these objectives. The numbers themselves possess no magical power, merely their own technical properties. In the same way, duodecimal currency systems arose independently in many different parts of the ancient world, because of the five divisors of 12.

Somewhere along the way (and here is the magic) members of a culture say "We like it." And when we confront this magic, we recognize the limits of musical explanation.

We hesitate to speculate further on the significance of the affinities between the two heptatonic scale systems we have examined, but our findings appear to confirm many of the claims made on behalf of diatonicism in the Western literature of musical speculation. Aristoxenus testified that "there is a marvelous orderliness in the constitution of melody," adding that "of all the objects to which the five senses apply not one other is characterized by an orderliness as extensive and so perfect."[23] His conclusion would have been endorsed by the musical thinkers of that great culture situated 4000 miles to the southeast, and it merits our endorsement today.

ABSTRACT

Two questions are addressed: (1) By what path did ancient Indian musicians make their way from a "chromatic" universe of 22 microtonal divisions of the octave (the *śrutis)* to a "diatonic" set of seven degrees? and (2) What features do the resulting scale structures have in common with later versions of the diatonic scale in the West? The authors examine the selection principles that appear to have guided these musicians, and explore some of the remarkable features and implications of their solutions.

The two basic early Indian heptatonic scales *(sa-grāma* and *ma-grāma)* share a number of features with the Western diatonic scale: (1) distinct step sizes which are consecutive integers, (2) dual tetrachords, (3) exactly one tritone, (4) distinct sizes of fifths which are consecutive integers, (5) a maximal number of consonant fifths (consistent with feature 4), and (6) first- or second-order maximal evenness (as defined by Clough and Douthett). There are three smaller feature sets, each including two or three of the above features, that serve to define both the early Indian heptatonics *and* the Western diatonic: given the appropriate chromatic cardinality (22 or 12), each feature set corresponds either (a) to all and only the heptatonics specified in the early Indian treatises plus two permissible altered versions or (b) uniquely to the Western diatonic.

[23]Aristoxenus, *Harmonics* 2.42 (trans. Henry Stewart Macran).

GLOSSARY OF SANSKRIT AND TAMIL TERMS

SANSKRIT

aṁśa ("denominator"), the tonic note

grāma, scale (in the sense of "collection")

grāmarāga, a scalar mode } the two earliest-known layers of the *rāga* system

jāti, a mode-class } the two earliest-known layers of the *rāga* system

mūrcchanā ("spreading"), rotations of the basic scales, that is, "octave species"

rāga, a mode

sa, ri, ga, ma, pa, dha, ni (Table 1), sol-fa names for the seven scale degrees

sādhāraṇa, altered scale degrees

saptaka, the set of seven [scale degrees]

śruti ("heard"), the 22 microtonal divisions of the octave

svara, a scale degree

tāna, a hexatonic or pentatonic version of one of the basic scales

TAMIL

iLikkiramam, a sequence of perfect fifths

maattirais, the 22 microtonal divisions of the octave

paalai, (1) the desert region, (2) a heptatonic scale

paalaiyaazh, a harp tuned to the "desert" mode

paN, one of the four major modes

yaazh, a harp with 7, 14, 19, or 21 strings