Two Hybrid Point Flexagons

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Introduction

Point flexagons previously described have all been made from regular convex polygons. These can be replaced by non regular convex polygons with the same number of edges. This changes the appearance without having any significant effect on the dynamic properties. However, if the polygons are replaced by convex polygons in which one or more of the vertex angles are 180°, then the dynamic properties do change. The use of edge vertices is common in finite element analysis, where they are known as nodes. It is convenient to adopt this terminology, and to refer to all the vertices as nodes.

Figures 1 and 2 show two examples of polygons with edge nodes. Figure 1 shows a square which has been transformed into a 4-node 45°-45°-90° (silver) triangle. Two side lengths and one angle are as in the original square. What was one of the vertices of the square is now an edge node midway along the hypotenuse. The other three vertices of the square have become vertices of the silver triangle, one with the original 90° vertex angle, but the vertex angles of the other two are 45°. Following finite element terminology all the vertices are shown as nodes in the figure. Four-node silver triangles can be used to replace the squares of any square point flexagon.

Figure 2 shows a regular convex hexagon which has been transformed into a 6-node equilateral triangle. Alternate vertices of the hexagon have become edge nodes midway along the edges of the equilateral triangle. The other three vertices of the hexagon have become vertices of the equilateral triangle, but the vertex angles are 60° instead of 120°. All 6 edge lengths of the equilateral triangle, measured between nodes, are identical, as were the edge lengths of the original regular hexagon. Six-node equilateral triangles can be used to replace the hexagons of any hexagon point flexagon. There are numerous other possibilities.

Theoretically, point flexagons made from polygons with edge nodes can have point hinges at edge nodes replaced by edge hinges. Hence, the flexagons are hybrids in the sense that they contain both point and edge hinges. In practice, paper models work more smoothly if point hinges are retained at edge nodes. Both theoretical and practical nets are given for the two examples below. In the theoretical nets some triangles have been truncated to make the connectivity of point hinges clear.

A Four Node Silver Triangle Hybrid Point Flexagon

The net for the four node silver triangle hybrid point flexagon was obtained from the net for the fundamental square point flexagon (Figure 3) by replacing the squares by 4-node silver triangles. This is the simplest possible hybrid point flexagon. Of course the silver triangles could be replaced by other types of triangle. The fundamental square point flexagon was first described by Scott Sherman, but not under that name. The theoretical version of the net for a four node silver triangle point flexagon is shown in Figure 4, and the practical version in Figure 5. As for the fundamental square point flexagon, it can be flexed round the 4-cycle shown in the intermediate position map (Figure 6). However, there is an additional main position, 3(1), which can be visited. Intermediate positions have the appearance shown in Figure 7(a). Main positions 2(1), 3(2) and 3(1) have the first main position appearance (Figure 7(b)), and main positions 4(3) and 1(4) the second main position appearance (Figure 7(c)). It is also possible to regard the flexagon as a variant of the tetrahexaflexagon in that the Tuckerman diagram is identical (Figure 8). From this viewpoint there are two linked 3-cycles, marked A and B in the figure.

A Six Node Triangle Hybrid Point Flexagon

The net for the six node triangle hybrid point flexagon was obtained from the net for the fundamental hexagon point flexagon (Figure 9) by replacing the hexagons by 6-node equilateral triangles. The theoretical version of the net is shown in Figure 10, and the practical version in Figure 11. The appearances of intermediate and main positions are shown in Figure 12. As for the fundamental square point flexagon, it can be flexed round the 6-cycle shown in the intermediate position map (Figure 13). In addition, it can also be flexed around a 3-cycle. It is also possible to regard the flexagon as a variant of the hexahexaflexagon in that the Tuckerman diagram is identical (Figure 14). From this viewpoint there are four linked 3-cycles.

Concluding Remarks

The intriguing and unexpected result for these two hybrid point flexagons was the similarity of their dynamic properties to those of the tetrahexaflexagon and the hexahexaflexagon. Obviously, there are numerous other possibilities.

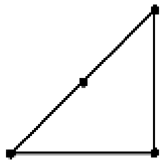


Figure 1. Square transformed into a 4-node 45°-45°-90° (silver) triangle. Dots represent nodes.

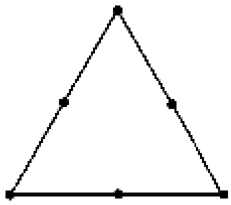


Figure 2. Hexagon transformed into a 6-node equilateral triangle. Dots represent nodes.

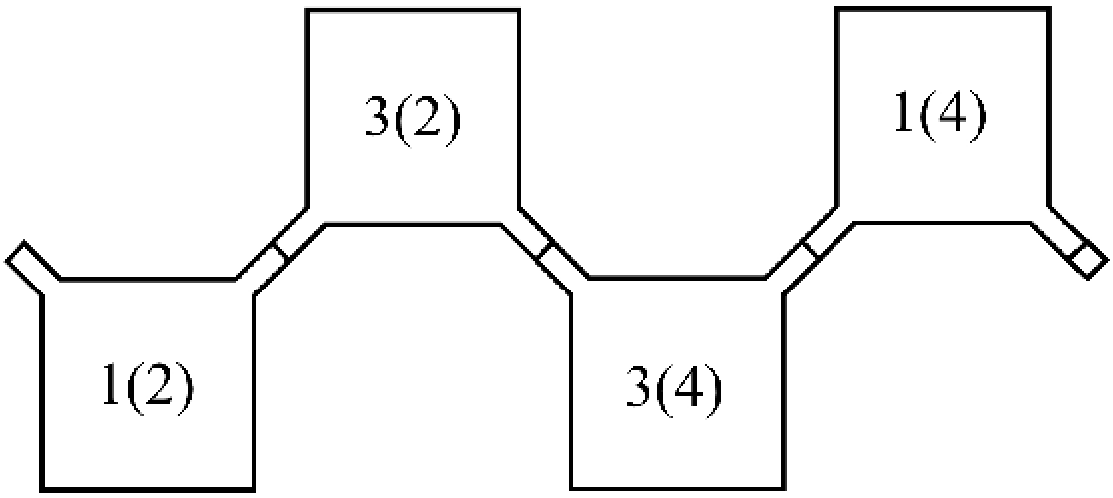


Figure 3. Net for the fundamental square point flexagon.

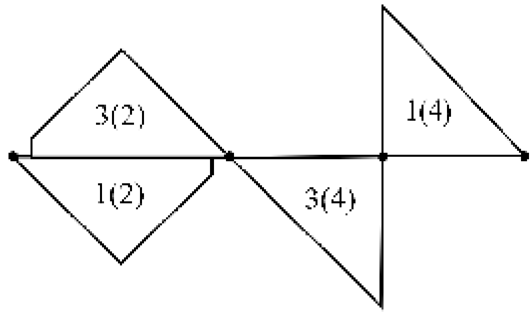


Figure 4. Theoretical net for a four node silver triangle hybrid point flexagon. Point hinges are indicated by dots.

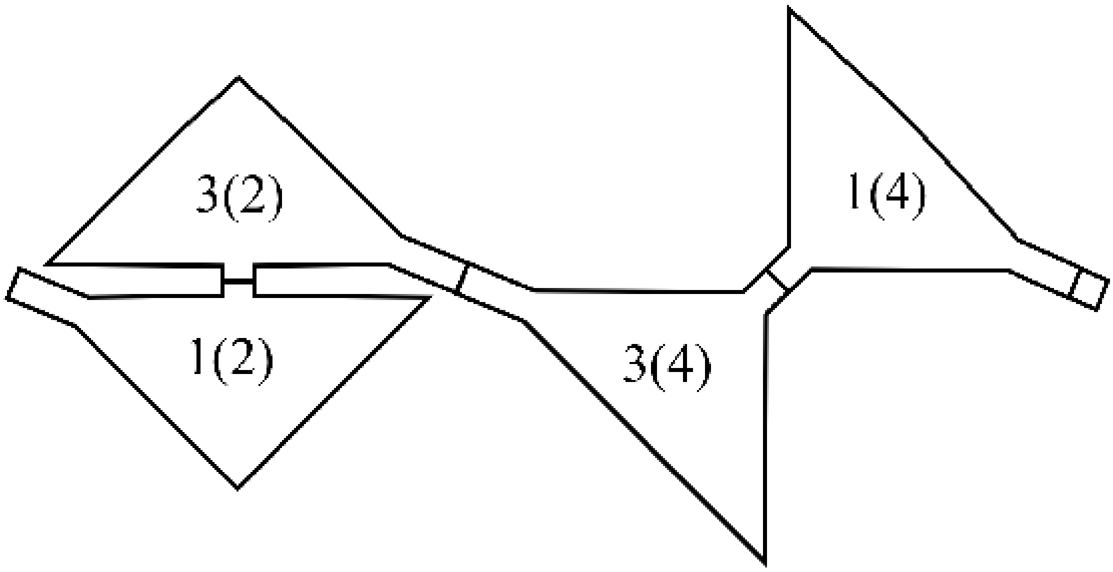


Figure 5. Practical net for a four node silver triangle hybrid point flexagon.

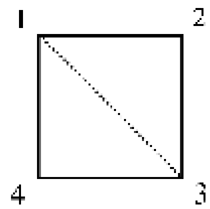
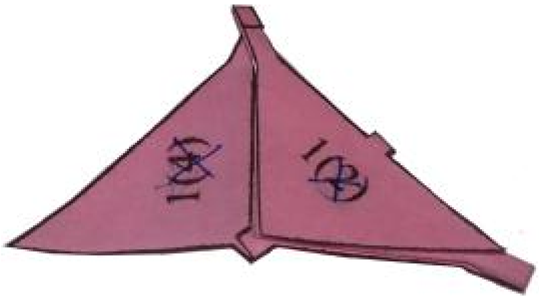


Figure 6. Intermediate position map for a four node silver triangle hybrid point flexagon.

(a)

(b)



(c)

Figure 7. A four node silver triangle hybrid point flexagon. (a) Intermediate position. (b) First main position. (c) Second main position.

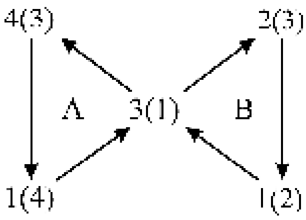


Figure 8.. Tuckerman diagram for a four node silver triangle hybrid point flexagon.

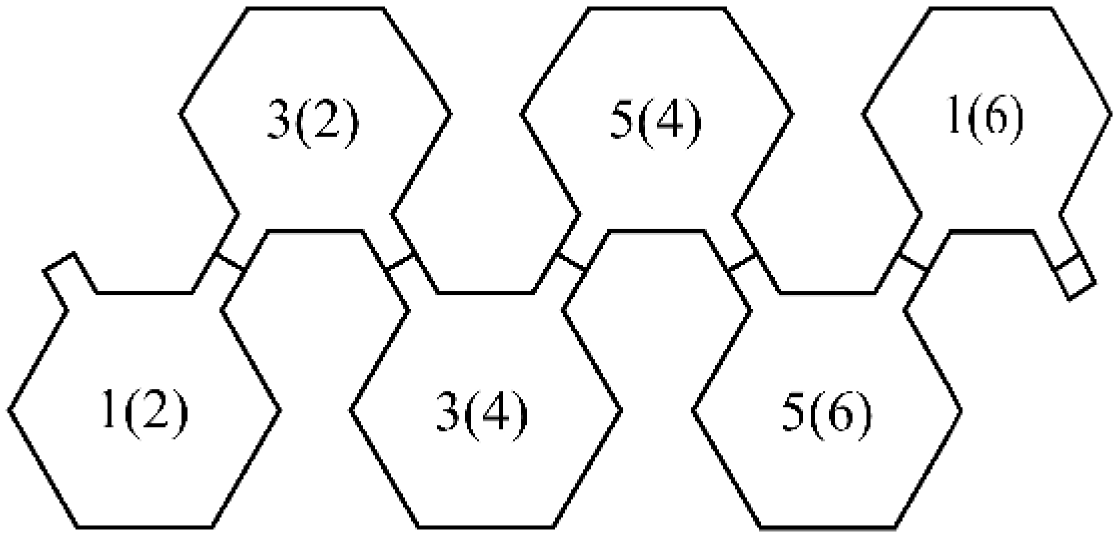


Figure 9. Net for the fundamental hexagon point flexagon.

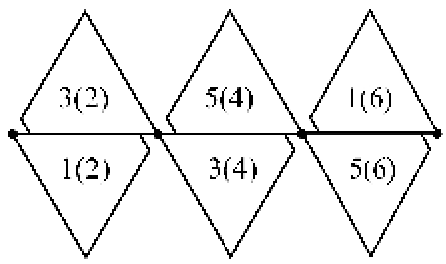


Figure 10. Theoretical net for a six node triangle hybrid point flexagon. Point hinges are indicated by dots.

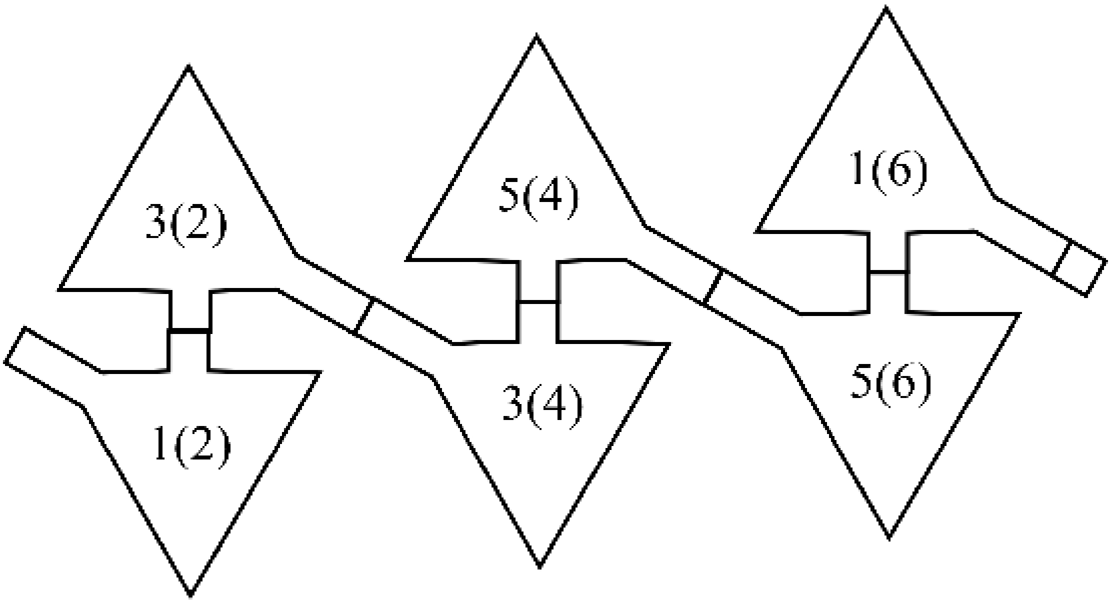
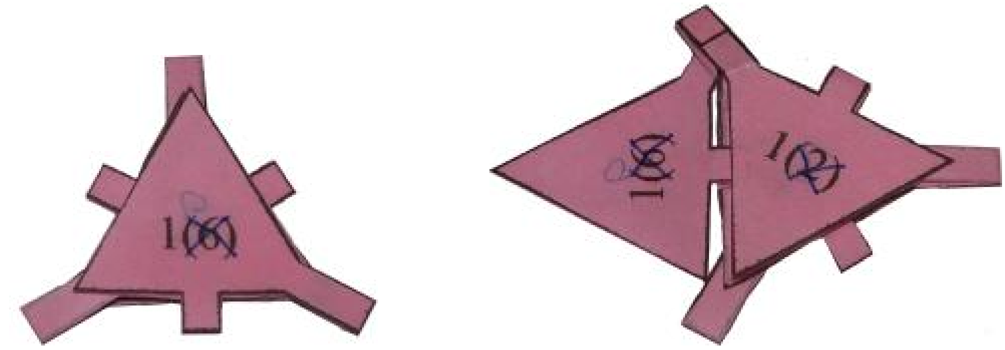


Figure 11. Practical net for a six node triangle hybrid point flexagon.



(a) (b)

Figure 12. A six node triangle hybrid point flexagon. (a) Intermediate position. (b) Main position.

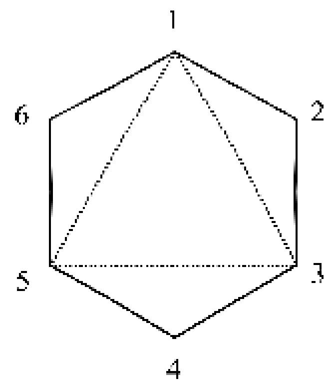


Figure 13. Intermediate position map for six node triangle hybrid point flexagon.

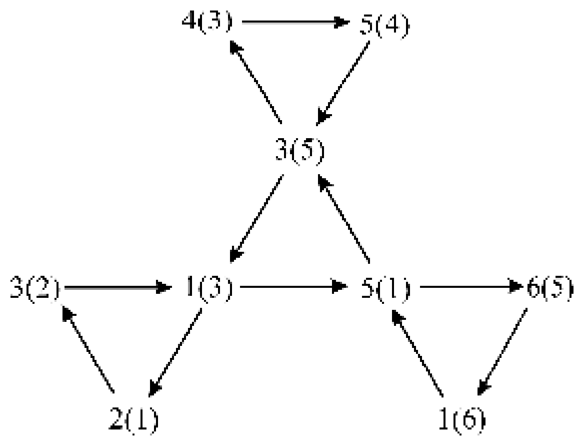


Figure 14. Tuckerman diagram for a six node triangle hybrid point flexagon.