Reading group on Laplacians

Session 1: Laplacians: definition and relations to graph Laplacian
In the first session we will define the Laplacian in dimension k using the operator definition and the combinatorial definition, and see that in dimension 0 this is just the graph Laplacian.

- Boundary and co-boundary operators, inner product on the space of cochains [1]
- Definition of the k-Laplacian as an operator (using boundary and co-boundary) ([1] beginning of 2.2)
- Combinatorial definition of the Laplacian (upper and lower adjacency) ([2], p.5 from "as mentioned earlier…")
- Example in dimension 0: this is just the graph laplacian L=D-A , give intuition of averaging over adjacent vertices ([2] from "as mentioned earlier…")
- Example in dimension 0,1,2: entries are 0 if two simplices are both lower and upper adjacent, intuition about the flow. ("[2] example B)

Session 2: Hodge decomposition
In this session we will give some properties of the Laplacian and study the Hodge decomposition, following the thesis [1].

- properties (symmetric, real non-negative eigenvalues..)
- Spectrum is not an invariant ([1] ex 2.6)
- Theorem 2.5 : the spectrum is independent of the choice of orientation
- Characterization of the kernel of the laplacians ([1] lemma 2.7)
- Decomposition of the space of cochais (hodge decomposition) ([1] lemma 2.9)
- Kernel is isomorphic to the homology [([1] 2.11)

Session 3: Harmonic representatives
In this session we will study the harmonic representatives, which are representatives of homology with a certain minimality property.

- Definition of harmonic representatives ([5] section 2.2,)
- Harmonics are minimal representative in their class with respect to the L2-norm ([5] section 2.2,
- On graphs: harmonics are constant functions on connected components (can be derived by computing eigenvectors associated to 0 eigenvalues of graph Laplacian)
- Intuition of the harmonics not having minimal perimeter but minimal energy ([5] example2.2.1 and [2] example B page 6-7)

Session 4: Graph Laplacian and spectral clustering
This session will be specific to graph spectral properties.

- Quickly recall the definition and properties of the graph Laplacian ([4] def Laplacian and normalized laplacian)
- Spectral clustering algorithm ([4] section 4 algorithm 1)
- Mincut problem ([4] section 5)
- Relation between mincut problem and spectral clustering==>Fiedler vector is an approximate solution of the mincut problem ([4] 5.1 and 5.2)

Session 5: Diffusion processes on simplicial complexes ([2 Section iv])
This will be the first application of Laplacian on simplicial complexes.
- diffusion processes on graphs and simplicial complexes
- convergence to harmonic representatives

Session 6: Random walks (Details coming soon)
Second session on applications of the laplacians
- define random walks on graphs
- convergence of random walks
- applications (choose paper)

[1] "Some new results on the combinatorial Laplcian" , see attachment
[2] "Control using Higher Order Laplacians in Network Topologies", see attachment
[3] http://www.math.ucsd.edu/~fan/research/revised.html
[4] https://arxiv.org/pdf/0711.0189.pdf
[5] https://arxiv.org/pdf/1708.01907.pdf