Exploiting Flexibility in Coupled Electricity and Natural Gas Markets: A Price-Based Approach

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This document serves as an electronic companion for the paper "Exploiting Flexibility in Coupled Electricity and Natural Gas Markets: A Price-Based Approach" to be published in the proceedings of IEEE PES PowerTech 2017. It contains three sections that provide supplemental material relative to the mathematical formulation of the problem and additional results.

1 MPEC formulation of price-based coupled electricity and natural gas model (*P-B*)

In this section the bilevel *P-B* model is reformulated as a Mathematical Program with Equilibrium Constraints (MPEC) by replacing the linear, and thus convex, lower level problems by their Karush-Kuhn-Tucker (KKT) conditions. Then, the resulting MPEC is transformed into a Mixed-Integer Linear Program (MILP) in order to deal with the bilinear terms that arise from the complementarity conditions. We introduce a mapping $M_l^{i_g}$ of the natural gas-fired power plants i_g at pipeline l (entries are equal to 1 if NGFPP is connected to a pipeline and 0 otherwise).

$$\begin{aligned}\underset{\Theta^{\text{MUL}}}{\text{Min.}} \sum_{t\in T}\Big[\sum_{i_c\in I_c} C_{i_c} p_{i_c,t} + \sum_{k\in K} C_k g_{k,t} + \sum_{\omega\in\Omega}\pi_\omega\Big(\sum_{k\in K}(C_k^+ g_{k,\omega,t}^+ - C_k^- g_{k,\omega,t}^-) + \sum_{i_c\in I_c}(C_{i_c}^+ p_{i_c,\omega,t}^+ - C_{i_c}^- p_{i_c,\omega,t}^-)\\ + C^{\text{sh,E}} l_{\omega,t}^{\text{sh,E}} + C^{\text{sh,G}} l_{\omega,t}^{\text{sh,G}} + \sum_{j\in J} C^{\text{sp}} w_{j,\omega,t}^{\text{sp}}\Big)\Big]\end{aligned} \tag{1a}$$

subject to

$$-X \le x_t \le X, \ \ \forall t, \tag{1b}$$

$$\sum_{t\in T}\sum_{i_g\in I_g} \phi_{i_g} p_{i_g,t} x_t = 0, \tag{1c}$$

$$0 \le p_{i,t} \le P_i^{\max} : \underline{\mu}_{i,t}^{\text{P}}, \overline{\mu}_{i,t}^{\text{P}}, \ \ \forall i,t, \tag{1d}$$

$$0 \le w_{j,t} \le \widehat{W}_{j,t} : \underline{\mu}_{j,t}^{\widehat{\text{W}}}, \overline{\mu}_{j,t}^{\widehat{\text{W}}}, \ \ \forall j,t, \tag{1e}$$

$$0 \le \sum_{i_g\in A_l^{I_g}} \phi_{i_g} p_{i_g,t} \le F_{l,t}^{\text{M}} : \underline{\mu}_{l,t}^{\text{M}}, \overline{\mu}_{l,t}^{\text{M}}, \ \ \forall l,t, \tag{1f}$$

$$0 \le \sum_{t\in T}\sum_{i_g\in A_l^{I_g}} \phi_{i_g} p_{i_g,t} \le F_l^{\text{A}} : \underline{\mu}_l^{\text{A}}, \overline{\mu}_l^{\text{A}}, \ \ \forall l, \tag{1g}$$

$$\sum_{i\in I} p_{i,t} + \sum_{j\in J} w_{j,t} - D_t^{\text{E}} = 0 : \hat{\lambda}_t^{\text{E}}, \ \ \forall t, \tag{1h}$$

$$C_{i_c} - \hat{\lambda}_t^{\text{E}} - \underline{\mu}_{i_c,t}^{\text{P}} + \overline{\mu}_{i_c,t}^{\text{P}} = 0, \ \ \forall i_c, t, \tag{1i}$$

$$\phi_{i_g}(\hat{\lambda}_t^{\text{G}} + x_t) - \hat{\lambda}_t^{\text{E}} - \underline{\mu}_{i_g,t}^{\text{P}} + \overline{\mu}_{i_g,t}^{\text{P}} + \sum_{l\in L} M_l^{i_g}(\phi_{i_g}\overline{\mu}_{l,t}^{\text{M}} + \phi_{i_g}\overline{\mu}_l^{\text{A}} - \phi_{i_g}\underline{\mu}_{l,t}^{\text{M}} - \phi_{i_g}\underline{\mu}_l^{\text{A}}) = 0, \ \ \forall i_g, t, \tag{1j}$$

$$-\hat{\lambda}_t^{\text{E}} - \underline{\mu}_{j,t}^{\widehat{\text{W}}} + \overline{\mu}_{j,t}^{\widehat{\text{W}}} = 0, \ \ \forall j,t, \tag{1k}$$

$$0 \le g_{k,t} \le G_k^{\max} : \underline{\mu}_{k,t}^{\text{G}}, \overline{\mu}_{k,t}^{\text{G}}, \ \ \forall k,t, \tag{1l}$$

$$\sum_{k\in K} g_{k,t} - D^{\mathrm{G}}_t - \sum_{i_g\in I_g} \phi_{i_g} p_{i_g,t} = 0 : \hat{\lambda}^{\mathrm{G}}_t, \ \ \forall t, \tag{1m}$$

$$C_k - \hat{\lambda}^{\mathrm{G}}_t - \underline{\mu}^{\mathrm{G}}_{k,t} + \overline{\mu}^{\mathrm{G}}_{k,t} = 0, \ \ \forall k,t, \tag{1n}$$

$$0 \le p^{+}_{i,\omega,t} \le P^{\max}_i - p_{i,t} : \underline{\mu}^{\text{PR+}}_{i,\omega,t}, \overline{\mu}^{\text{PR+}}_{i,\omega,t}, \ \ \forall i,\omega,t, \tag{1o}$$

$$0 \le p^{-}_{i,\omega,t} \le p_{i,t} : \underline{\mu}^{\text{PR-}}_{i,\omega,t}, \overline{\mu}^{\text{PR-}}_{i,\omega,t}, \ \ \forall i,\omega,t, \tag{1p}$$

$$0 \le p^{+}_{i,\omega,t} \le P^{+}_i : \underline{\mu}^{\text{P+}}_{i,\omega,t}, \overline{\mu}^{\text{P+}}_{i,\omega,t}, \ \ \forall i,\omega,t, \tag{1q}$$

$$0 \le p^{-}_{i,\omega,t} \le P^{-}_i : \underline{\mu}^{\text{P-}}_{i,\omega,t}, \overline{\mu}^{\text{P-}}_{i,\omega,t}, \ \ \forall i,\omega,t, \tag{1r}$$

$$0 \le w^{\mathrm{sp}}_{j,\omega,t} \le W_{j,\omega,t} : \underline{\mu}^{\mathrm{sp}}_{j,\omega,t}, \overline{\mu}^{\mathrm{sp}}_{j,\omega,t}, \ \ \forall j,\omega,t, \tag{1s}$$

$$0 \le l^{\mathrm{sh,E}}_{\omega,t} \le D^{\mathrm{E}}_t : \underline{\mu}^{\mathrm{sh,E}}_{\omega,t}, \overline{\mu}^{\mathrm{sh,E}}_{\omega,t}, \ \ \forall \omega,t, \tag{1t}$$

$$0 \le \sum_{i_g\in A^{I_g}_l} \phi_{i_g} p^{+}_{i_g,\omega,t} \le F^{\mathrm{M}}_{l,t} - \sum_{i_g\in A^{I_g}_l} \phi_{i_g} p_{i_g,t} : \underline{\mu}^{\text{MR+}}_{l,\omega,t}, \overline{\mu}^{\text{MR+}}_{l,\omega,t}, \ \forall l,\omega,t, \tag{1u}$$

$$0 \le \sum_{i_g\in A^{I_g}_l} \phi_{i_g} p^{-}_{i_g,\omega,t} \le \sum_{i_g\in A^{I_g}_l} \phi_{i_g} p_{i_g,t} : \underline{\mu}^{\text{MR-}}_{l,\omega,t}, \overline{\mu}^{\text{MR-}}_{l,\omega,t}, \ \forall l,\omega,t, \tag{1v}$$

$$0 \le \sum_{t\in T}\sum_{i_g\in A^{I_g}_l} \phi_{i_g} p^{+}_{i_g,\omega,t} \le F^{\mathrm{A}}_l - \sum_{t\in T}\sum_{i_g\in A^{I_g}_l} \phi_{i_g} p_{i_g,t} : \underline{\mu}^{\text{AR+}}_{l,\omega}, \overline{\mu}^{\text{AR+}}_{l,\omega}, \ \forall l,\omega, \tag{1w}$$

$$0 \le \sum_{t\in T}\sum_{i_g\in A^{I_g}_l} \phi_{i_g} p^{-}_{i_g,\omega,t} \le \sum_{t\in T}\sum_{i_g\in A^{I_g}_l} \phi_{i_g} p_{i_g,t} : \underline{\mu}^{\text{AR-}}_{l,\omega}, \overline{\mu}^{\text{AR-}}_{l,\omega}, \ \forall l,\omega, \tag{1x}$$

$$\sum_{i\in I}(p^{+}_{i,\omega,t} - p^{-}_{i,\omega,t}) + l^{\mathrm{sh,E}}_{\omega,t} + \sum_{j\in J}(W_{j,\omega,t} - w^{\mathrm{sp}}_{j,\omega,t} - w_{j,t}) = 0 : \tilde{\lambda}^{\mathrm{E}}_{\omega,t}, \ \ \forall \omega,t, \tag{1y}$$

$$C^{+}_{i_c} - \tilde{\lambda}^{\mathrm{E}}_{\omega,t} + \overline{\mu}^{\text{PR+}}_{i_c,\omega,t} + \overline{\mu}^{\text{P+}}_{i_c,\omega,t} - \underline{\mu}^{\text{P+}}_{i_c,\omega,t} = 0, \ \ \forall i_c,\omega,t, \tag{1z}$$

$$-C^{-}_{i_c} + \tilde{\lambda}^{\mathrm{E}}_{\omega,t} + \overline{\mu}^{\text{PR-}}_{i_c,\omega,t} + \overline{\mu}^{\text{P-}}_{i_c,\omega,t} - \underline{\mu}^{\text{P-}}_{i_c,\omega,t} = 0, \ \ \forall i_c,\omega,t, \tag{1aa}$$

$$\begin{aligned}&\phi_{i_g}(\tilde{\lambda}^{\mathrm{G}}_{\omega,t} + x_t) - \tilde{\lambda}^{\mathrm{E}}_{\omega,t} + \overline{\mu}^{\text{PR+}}_{i_g,\omega,t} + \overline{\mu}^{\text{P+}}_{i_g,\omega,t} - \underline{\mu}^{\text{P+}}_{i_g,\omega,t}\\ &+ \sum_{l\in L} M^{i_g}_l (\phi_{i_g}\overline{\mu}^{\text{MR+}}_{l,\omega,t} + \phi_{i_g}\overline{\mu}^{\text{AR+}}_{l,\omega} - \phi_{i_g}\underline{\mu}^{\text{MR+}}_{l,\omega,t} - \phi_{i_g}\underline{\mu}^{\text{AR+}}_{l,\omega}) = 0, \ \ \forall i_g,\omega,t,\end{aligned} \tag{1ab}$$

$$\begin{aligned}&-\phi_{i_g}(\tilde{\lambda}^{\mathrm{G}}_{\omega,t} + x_t) + \tilde{\lambda}^{\mathrm{E}}_{\omega,t} + \overline{\mu}^{\text{PR-}}_{i_g,\omega,t} + \overline{\mu}^{\text{P-}}_{i_g,\omega,t} - \underline{\mu}^{\text{P-}}_{i_g,\omega,t}\\ &+ \sum_{l\in L} M^{i_g}_l (\phi_{i_g}\overline{\mu}^{\text{MR-}}_{l,\omega,t} + \phi_{i_g}\overline{\mu}^{\text{AR-}}_{l,\omega} - \phi_{i_g}\underline{\mu}^{\text{MR-}}_{l,\omega,t} - \phi_{i_g}\underline{\mu}^{\text{AR-}}_{l,\omega}) = 0, \ \ \forall i_g,\omega,t,\end{aligned} \tag{1ac}$$

$$C^{\mathrm{sh,E}} - \tilde{\lambda}^{\mathrm{E}}_{\omega,t} + \overline{\mu}^{\mathrm{sh,E}}_{\omega,t} - \underline{\mu}^{\mathrm{sh,E}}_{\omega,t} = 0, \ \ \forall \omega,t, \tag{1ad}$$

$$C^{\mathrm{sp}} + \tilde{\lambda}^{\mathrm{E}}_{\omega,t} + \overline{\mu}^{\mathrm{sp}}_{j,\omega,t} - \underline{\mu}^{\mathrm{sp}}_{j,\omega,t} = 0, \ \ \forall j,\omega,t, \tag{1ae}$$

$$g^{+}_{k,\omega,t} \le G^{\max}_k - g_{k,t} : \underline{\mu}^{\text{GR+}}_{k,\omega,t}, \overline{\mu}^{\text{GR+}}_{k,\omega,t}, \ \ \forall k,\omega,t, \tag{1af}$$

$$0 \le g^{-}_{k,\omega,t} \le g_{k,t} : \underline{\mu}^{\text{GR-}}_{k,\omega,t}, \overline{\mu}^{\text{GR-}}_{k,\omega,t}, \ \ \forall k,\omega,t, \tag{1ag}$$

$$0 \le g^{+}_{k,\omega,t} \le G^{+}_k : \underline{\mu}^{\text{G+}}_{k,\omega,t}, \overline{\mu}^{\text{G+}}_{k,\omega,t}, \ \ \forall k,\omega,t, \tag{1ah}$$

$$0 \le g^{-}_{k,\omega,t} \le G^{-}_k : \underline{\mu}^{\text{G-}}_{k,\omega,t}, \overline{\mu}^{\text{G-}}_{k,\omega,t}, \ \ \forall k,\omega,t, \tag{1ai}$$

$$0 \le l^{\mathrm{sh,G}}_{\omega,t} \le D^{\mathrm{G}}_t : \underline{\mu}^{\mathrm{sh,G}}_{\omega,t}, \overline{\mu}^{\mathrm{sh,G}}_{\omega,t}, \ \ \forall \omega,t, \tag{1aj}$$

$$\sum_{k\in K}(g^{+}_{k,\omega,t} - g^{-}_{k,\omega,t}) + l^{\mathrm{sh,G}}_{\omega,t} - \sum_{i_g\in I_g}\phi_{i_g}(p^{+}_{i_g,\omega,t} - p^{-}_{i_g,\omega,t}) = 0 : \tilde{\lambda}^{\mathrm{G}}_{\omega,t}, \ \ \forall \omega,t, \tag{1ak}$$

$$C^{+}_k - \tilde{\lambda}^{\mathrm{G}}_{\omega,t} + \overline{\mu}^{\text{GR+}}_{k,\omega,t} + \overline{\mu}^{\text{G+}}_{k,\omega,t} - \underline{\mu}^{\text{G+}}_{k,\omega,t} = 0, \ \ \forall k,\omega,t, \tag{1al}$$

$$-C^{-}_k + \tilde{\lambda}^{\mathrm{G}}_{\omega,t} + \overline{\mu}^{\text{GR-}}_{k,\omega,t} + \overline{\mu}^{\text{G-}}_{k,\omega,t} - \underline{\mu}^{\text{G-}}_{k,\omega,t} = 0, \ \ \forall k,\omega,t, \tag{1am}$$

$$C^{\mathrm{sh,G}} - \tilde{\lambda}^{\mathrm{G}}_{\omega,t} + \overline{\mu}^{\mathrm{sh,G}}_{\omega,t} - \underline{\mu}^{\mathrm{sh,G}}_{\omega,t} = 0, \ \ \forall \omega,t, \tag{1an}$$

$$0 \le \overline{\mu}^{\mathrm{P}}_{i,t} \perp P^{\max}_i - p_{i,t} \ge 0, \ \ \forall i,t, \tag{1ao}$$

$$0 \le \underline{\mu}^{\mathrm{P}}_{i,t} \perp p_{i,t} \ge 0, \ \ \forall i,t, \tag{1ap}$$

$$0 \le \overline{\mu}^{\widehat{\mathrm{W}}}_{j,t} \perp \widehat{W}_j - w_{j,t} \ge 0, \ \ \forall j,t, \tag{1aq}$$

$$0 \le \underline{\mu}^{\widehat{\mathrm{W}}}_{j,t} \perp w_{j,t} \ge 0, \ \ \forall j,t, \tag{1ar}$$

$$0 \leq \overline{\mu}^{\mathrm{M}}_{l,t} \perp F^{\mathrm{M}}_{l,t} - \sum_{i_g \in A^{I_g}_l} \phi_{i_g} p_{i_g,t} \geq 0, \ \ \forall l, t, \tag{1as}$$

$$0 \leq \overline{\mu}^{\mathrm{A}}_{l} \perp F^{\mathrm{A}}_{l} - \sum_{t \in T} \sum_{i_g \in A^{I_g}_l} \phi_{i_g} p_{i_g,t} \geq 0, \ \ \forall l, \tag{1at}$$

$$0 \leq \underline{\mu}^{\mathrm{M}}_{l,t} \perp \sum_{i_g \in A^{I_g}_l} \phi_{i_g} p_{i_g,t} \geq 0, \ \ \forall l, t, \tag{1au}$$

$$0 \leq \underline{\mu}^{\mathrm{A}}_{l} \perp \sum_{t \in T} \sum_{i_g \in A^{I_g}_l} \phi_{i_g} p_{i_g,t} \geq 0, \ \ \forall l, \tag{1av}$$

$$0 \leq \overline{\mu}^{\mathrm{G}}_{k,t} \perp G^{\max}_k - g_{k,t} \geq 0, \ \ \forall k, t, \tag{1aw}$$

$$0 \leq \underline{\mu}^{\mathrm{G}}_{k,t} \perp g_{k,t} \geq 0, \ \ \forall k, t, \tag{1ax}$$

$$0 \leq \overline{\mu}^{\mathrm{PR+}}_{i,\omega,t} \perp P^{\max}_i - p_{i,t} - p^{+}_{i,\omega,t} \geq 0, \ \ \forall i, \omega, t, \tag{1ay}$$

$$0 \leq \underline{\mu}^{\mathrm{PR\text{-}}}_{i,\omega,t} \perp p_{i,t} - p^{-}_{i,\omega,t} \geq 0, \ \ \forall i, \omega, t, \tag{1az}$$

$$0 \leq \overline{\mu}^{\mathrm{P+}}_{i,\omega,t} \perp P^{+}_i - p^{+}_{i,\omega,t} \geq 0, \ \ \forall i, \omega, t, \tag{1ba}$$

$$0 \leq \underline{\mu}^{\mathrm{P+}}_{i,\omega,t} \perp p^{+}_{i,\omega,t} \geq 0, \ \ \forall i, \omega, t, \tag{1bb}$$

$$0 \leq \overline{\mu}^{\mathrm{P\text{-}}}_{i,\omega,t} \perp P^{\text{-}}_i - p^{-}_{i,\omega,t} \geq 0, \ \ \forall i, \omega, t, \tag{1bc}$$

$$0 \leq \underline{\mu}^{\mathrm{P\text{-}}}_{i,\omega,t} \perp p^{-}_{i,\omega,t} \geq 0, \ \ \forall i, \omega, t, \tag{1bd}$$

$$0 \leq \overline{\mu}^{\mathrm{sp}}_{j,\omega,t} \perp W_{j,\omega,t} - w^{\mathrm{sp}}_{j,\omega,t} \geq 0, \ \ \forall j, \omega, t, \tag{1be}$$

$$0 \leq \underline{\mu}^{\mathrm{sp}}_{j,\omega,t} \perp w^{\mathrm{sp}}_{j,\omega,t} \geq 0, \ \ \forall j, \omega, t, \tag{1bf}$$

$$0 \leq \overline{\mu}^{\mathrm{sh,E}}_{\omega,t} \perp D^{\mathrm{E}}_t - l^{\mathrm{sh,E}}_{\omega,t} \geq 0, \ \ \forall \omega, t, \tag{1bg}$$

$$0 \leq \underline{\mu}^{\mathrm{sh,E}}_{\omega,t} \perp l^{\mathrm{sh,E}}_{\omega,t} \geq 0, \ \ \forall \omega, t, \tag{1bh}$$

$$0 \leq \overline{\mu}^{\mathrm{MR+}}_{l,\omega,t} \perp F^{\mathrm{M}}_{l,t} - \sum_{i_g \in A^{I_g}_l} \phi_{i_g} p_{i_g,t} - \sum_{i_g \in A^{I_g}_l} \phi_{i_g} p^{+}_{i_g,\omega,t} \geq 0, \forall l, \omega, t, \tag{1bi}$$

$$0 \leq \underline{\mu}^{\mathrm{MR+}}_{l,\omega,t} \perp \sum_{i_g \in A^{I_g}_l} \phi_{i_g} p^{+}_{i_g,\omega,t} \geq 0, \forall l, \omega, t, \tag{1bj}$$

$$0 \leq \overline{\mu}^{\mathrm{MR\text{-}}}_{l,\omega,t} \perp \sum_{i_g \in A^{I_g}_l} \phi_{i_g} p_{i_g,t} - \sum_{i_g \in A^{I_g}_l} \phi_{i_g} p^{-}_{i_g,\omega,t} \geq 0, \forall l, \omega, t, \tag{1bk}$$

$$0 \leq \underline{\mu}^{\mathrm{MR\text{-}}}_{l,\omega,t} \perp \sum_{i_g \in A^{I_g}_l} \phi_{i_g} p^{-}_{i_g,\omega,t} \geq 0, \forall l, \omega, t, \tag{1bl}$$

$$0 \leq \overline{\mu}^{\mathrm{AR+}}_{l,\omega} \perp F^{\mathrm{A}}_{l} - \sum_{t \in T} \sum_{i_g \in A^{I_g}_l} \phi_{i_g} p_{i_g,t} - \sum_{t \in T} \sum_{i_g \in A^{I_g}_l} \phi_{i_g} p^{+}_{i_g,\omega,t} \geq 0, \forall l, \omega, \tag{1bm}$$

$$0 \leq \underline{\mu}^{\mathrm{AR+}}_{l,\omega} \perp \sum_{t \in T} \sum_{i_g \in A^{I_g}_l} \phi_{i_g} p^{+}_{i_g,\omega,t} \geq 0, \forall l, \omega, \tag{1bn}$$

$$0 \leq \overline{\mu}^{\mathrm{AR\text{-}}}_{l,\omega} \perp \sum_{t \in T} \sum_{i_g \in A^{I_g}_l} \phi_{i_g} p_{i_g,t} - \sum_{t \in T} \sum_{i_g \in A^{I_g}_l} \phi_{i_g} p^{-}_{i_g,\omega,t} \geq 0, \forall l, \omega, \tag{1bo}$$

$$0 \leq \underline{\mu}^{\mathrm{AR\text{-}}}_{l,\omega} \perp \sum_{t \in T} \sum_{i_g \in A^{I_g}_l} \phi_{i_g} p^{-}_{i_g,\omega,t} \geq 0, \forall l, \omega, \tag{1bp}$$

$$0 \leq \overline{\mu}^{\mathrm{GR+}}_{k,\omega,t} \perp G^{\max}_k - g_{k,t} - g^{+}_{k,\omega,t} \geq 0, \ \ \forall k, \omega, t, \tag{1bq}$$

$$0 \leq \underline{\mu}^{\mathrm{GR\text{-}}}_{k,\omega,t} \perp g_{k,t} - g^{-}_{k,\omega,t} \geq 0, \ \ \forall k, \omega, t, \tag{1br}$$

$$0 \leq \overline{\mu}^{\mathrm{G+}}_{k,\omega,t} \perp G^{+}_k - g^{+}_{k,\omega,t} \geq 0, \ \ \forall k, \omega, t, \tag{1bs}$$

$$0 \leq \underline{\mu}^{\mathrm{G+}}_{k,\omega,t} \perp g^{+}_{k,\omega,t} \geq 0, \ \ \forall k, \omega, t, \tag{1bt}$$

$$0 \leq \overline{\mu}^{\mathrm{G\text{-}}}_{k,\omega,t} \perp G^{-}_k - g^{-}_{k,\omega,t} \geq 0, \ \ \forall k, \omega, t, \tag{1bu}$$

$$0 \leq \underline{\mu}^{\mathrm{G\text{-}}}_{k,\omega,t} \perp g^{-}_{k,\omega,t} \geq 0, \ \ \forall k, \omega, t, \tag{1bv}$$

$$0 \leq \overline{\mu}^{\mathrm{sh,G}}_{\omega,t} \perp D^{\mathrm{G}}_t - l^{\mathrm{sh,G}}_{\omega,t} \geq 0, \ \ \forall \omega, t, \tag{1bw}$$

$$0 \leq \underline{\mu}^{\mathrm{sh,G}}_{\omega,t} \perp l^{\mathrm{sh,G}}_{\omega,t} \geq 0, \ \ \forall \omega, t. \tag{1bx}$$

The nonlinearities that arise from complementarity conditions are linearized via the Fortuny-Amat transformation [1]. We introduce the set of dual variables (λ and μ) Θ^{dual}, thus $\Theta^{\text{MUL}} = \{\Theta^{\text{UL}}, \Theta^{\text{dual}}\}$.

2 Linearization of cost-neutrality constraint

In this section, the aim is to linearize the fairness constraint (1c). First, we write the strong duality of problem (4d)-(4f) as presented in the original paper,

$$\sum_{t\in T}\Big(\sum_{i_c\in I_c} C_{i_c}p_{i_c,t} + \sum_{i_g\in I_g}(\hat{\lambda}^{\text{G}}_t + x_t)p_{i_g,t}\phi_{i_g}\Big) = \sum_{t\in T}\Big(-\sum_{i\in I}\overline{\mu}^{\text{P}}_{i,t}P^{\max}_i - \sum_{j\in J}\overline{\mu}^{\widehat{\text{W}}}_{j,t}\widehat{W}_{j,t} - \sum_{l\in L}\overline{\mu}^{\text{M}}_{l,t}F^{\text{M}}_{l,t} + \hat{\lambda}^{\text{E}}_t D^{\text{E}}_t\Big) - \sum_{l\in L}\overline{\mu}^{\text{A}}_l F^{\text{A}}_l, \tag{2a}$$

We solve for $\sum_{t\in T}\sum_{i_g\in I_g} x_t p_{i_g,t}\phi_{i_g}$,

$$\sum_{t\in T}\sum_{i_g\in I_g} x_t p_{i_g,t}\phi_{i_g} = \sum_{t\in T}\Big(-\sum_{i_c\in I_c} C_{i_c}p_{i_c,t} - \sum_{i\in I}\overline{\mu}^{\text{P}}_{i,t}P^{\max}_i - \sum_{j\in J}\overline{\mu}^{\widehat{\text{W}}}_{j,t}\widehat{W}_{j,t} - \sum_{l\in L}\overline{\mu}^{\text{M}}_{l,t}F^{\text{M}}_{l,t} + \hat{\lambda}^{\text{E}}_t D^{\text{E}}_t - \sum_{i_g\in I_g}\hat{\lambda}^{\text{G}}_t p_{i_g,t}\phi_{i_g}\Big) - \sum_{l\in L}\overline{\mu}^{\text{A}}_l F^{\text{A}}_l. \tag{3a}$$

In the case that $\hat{\lambda}^{\text{G}}_t = 0$, the nonlinear term $\sum_{t\in T}\hat{\lambda}^{\text{G}}_t\sum_{i_g\in I_g}\phi_{i_g}p_{i_g,t}$ vanishes. When $\hat{\lambda}^{\text{G}}_t \neq 0$, we multiply equation (1m) with $\hat{\lambda}^{\text{G}}_t$ and we sum over time periods t,

$$\begin{aligned}
&\sum_{t\in T}\Big(\hat{\lambda}^{\text{G}}_t\sum_{k\in K} g_{k,t} - \hat{\lambda}^{\text{G}}_t D^{\text{G}}_t - \hat{\lambda}^{\text{G}}_t\sum_{i_g\in I_g}\phi_{i_g}p_{i_g,t}\Big) = 0\\
&\Leftrightarrow \sum_{t\in T}\hat{\lambda}^{\text{G}}_t\sum_{i_g\in I_g}\phi_{i_g}p_{i_g,t} = \sum_{t\in T}\Big(\hat{\lambda}^{\text{G}}_t\sum_{k\in K} g_{k,t} - \hat{\lambda}^{\text{G}}_t D^{\text{G}}_t\Big).
\end{aligned} \tag{4a}$$

Morevover, we multiply equality (1n) with $g_{k,t}\neq 0$,

$$C_k g_{k,t} - \hat{\lambda}^{\text{G}}_t g_{k,t} - \underline{\mu}^{\text{G}}_{k,t}g_{k,t} + \overline{\mu}^{\text{G}}_{k,t}g_{k,t} = 0 \ \ \forall k, t. \tag{5a}$$

Then, we sum over the gas wells k and time periods t,

$$\sum_{t\in T}\Big(\sum_{k\in K} C_k g_{k,t} - \hat{\lambda}^{\text{G}}_t\sum_{k\in K} g_{k,t} - \sum_{k\in K}\underline{\mu}^{\text{G}}_{k,t}g_{k,t} + \sum_{k\in K}\overline{\mu}^{\text{G}}_{k,t}g_{k,t}\Big) = 0 \tag{6a}$$

$$\Leftrightarrow \sum_{t\in T}\hat{\lambda}^{\text{G}}_t\sum_{k\in K} g_{k,t} = \sum_{t\in T}\Big(\sum_{k\in K} C_k g_{k,t} - \sum_{k\in K}\underline{\mu}^{\text{G}}_{k,t}g_{k,t} + \sum_{k\in K}\overline{\mu}^{\text{G}}_{k,t}g_{k,t}\Big). \tag{6b}$$

In equation (4a) the term $\sum_{t\in T}\hat{\lambda}^{\text{G}}_t\sum_{k\in K} g_{k,t}$ is nonlinear and thus substituted by the equivalent expression given in (6b),

$$\sum_{t\in T}\hat{\lambda}^{\text{G}}_t\sum_{i_g\in I_g}\phi_{i_g}p_{i_g,t} = \sum_{t\in T}\Big(\sum_{k\in K} C_k g_{k,t} - \sum_{k\in K}\underline{\mu}^{\text{G}}_{k,t}g_{k,t} + \sum_{k\in K}\overline{\mu}^{\text{G}}_{k,t}g_{k,t} - \hat{\lambda}^{\text{G}}_t D^{\text{G}}_t\Big). \tag{7a}$$

Finally, we substitute the nonlinear term $\sum_{t\in T}\hat{\lambda}^{\text{G}}_t\sum_{i_g\in I_g}\phi_{i_g}p_{i_g,t}$ in the strong duality equation (3a) by (7a),

$$\sum_{t\in T}\sum_{i_g\in I_g} x_t p_{i_g,t}\phi_{i_g} = \sum_{t\in T}\Big(-\sum_{i_c\in I_c} C_{i_c}p_{i_c,t} - \sum_{i\in I}\overline{\mu}^{\text{P}}_{i,t}P^{\max}_i - \sum_{j\in J}\overline{\mu}^{\widehat{\text{W}}}_{j,t}\widehat{W}_{j,t} - \sum_{l\in L}\overline{\mu}^{\text{M}}_{l,t}F^{\text{M}}_{l,t} + \hat{\lambda}^{\text{E}}_t D^{\text{E}}_t\Big) - \sum_{l\in L}\overline{\mu}^{\text{A}}_l F^{\text{A}}_l - \sum_{t\in T}\Big(\sum_{k\in K} C_k g_{k,t} - \sum_{k\in K}\underline{\mu}^{\text{G}}_{k,t}g_{k,t} + \sum_{k\in K}\overline{\mu}^{\text{G}}_{k,t}g_{k,t} - \hat{\lambda}^{\text{G}}_t D^{\text{G}}_t\Big). \tag{8a}$$

From the complementarity condition (1aw) we have,

$$\overline{\mu}^{\text{G}}_{k,t}(G^{\max}_k - g_{k,t}) = 0 \Leftrightarrow \overline{\mu}^{\text{G}}_{k,t}G^{\max}_k = \overline{\mu}^{\text{G}}_{k,t}g_{k,t}, \ \ \forall k, t. \tag{9a}$$

Additionally, from the complementarity condition (1ax) we have,

$$\underline{\mu}^{\text{G}}_{k,t}g_{k,t} = 0, \ \ \forall k, t. \tag{10a}$$

By substituting (9a) and (10a) in (8a) we have the following linear representation of $\sum_{t\in T}\sum_{i_g\in I_g} xp_{i_g,t}\phi_{i_g}$,

$$\sum_{t\in T}\sum_{i_g\in I_g} x_t p_{i_g,t}\phi_{i_g} = \sum_{t\in T}\Big(-\sum_{i_c\in I_c} C_{i_c} p_{i_c,t} - \sum_{i\in I}\overline{\mu}^{\mathrm{P}}_{i,t} P^{\max}_i - \sum_{j\in J}\overline{\mu}^{\widehat{\mathrm{W}}}_{j,t}\widehat{W}_{j,t} - \sum_{l\in L}\overline{\mu}^{\mathrm{NGA}}_{l,t} F^{\mathrm{NGA}}_{l,t} + \hat{\lambda}^{\mathrm{E}}_t D^{\mathrm{E}}_t\Big) - \sum_{l\in L}\overline{\mu}^{\mathrm{NGA}}_{l} F^{\mathrm{NGA}}_{l} - \sum_{t\in T}\Big(\sum_{k\in K} C_k g_{k,t} + \sum_{k\in K}\overline{\mu}^{\mathrm{G}}_{k,t} G^{\max}_k - \hat{\lambda}^{\mathrm{G}}_t D^{\mathrm{G}}_t\Big). \tag{11a}$$

3 Supplementary results

In this section, we provide some additional graphs related to the results of the paper. We illustrate the demand and expected wind power profiles, as well as the day-ahead dispatch of the conventional units under the three different dispatch models for reader's convenience. This way the effect of changing the price of natural gas for power production is demonstrated by comparing the dispatch between *Seq* and *P-B*.

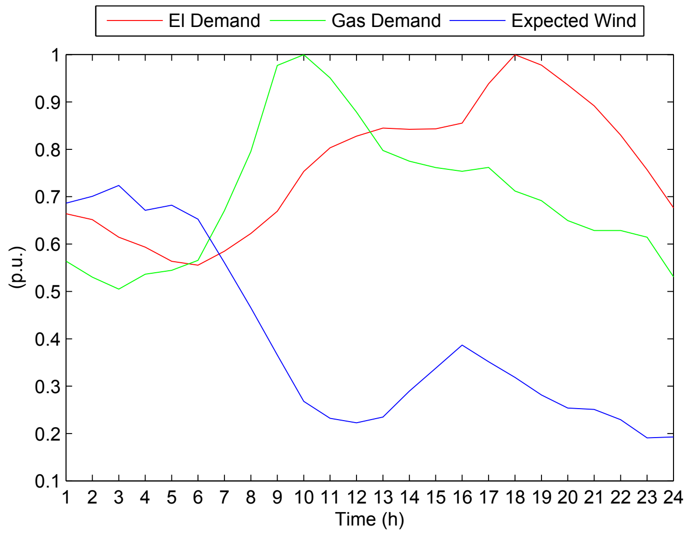

Figure 1: Electricity demand, natural gas demand and expected wind power production profiles.

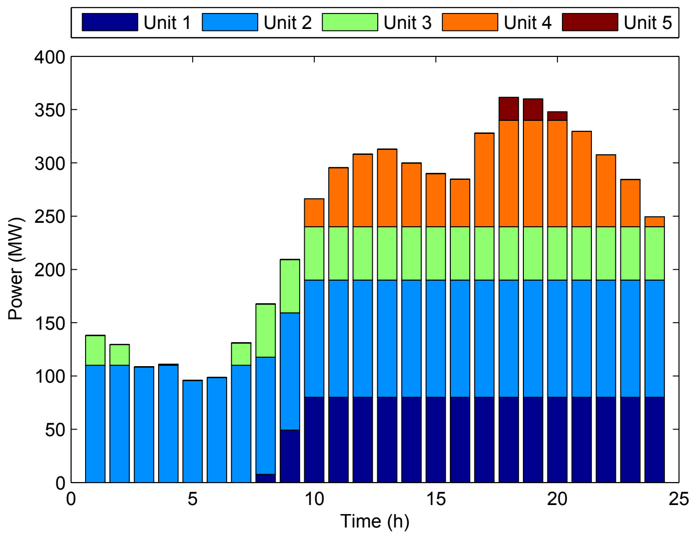

Figure 2: Hourly day-ahead schedule of power plants (Seq).

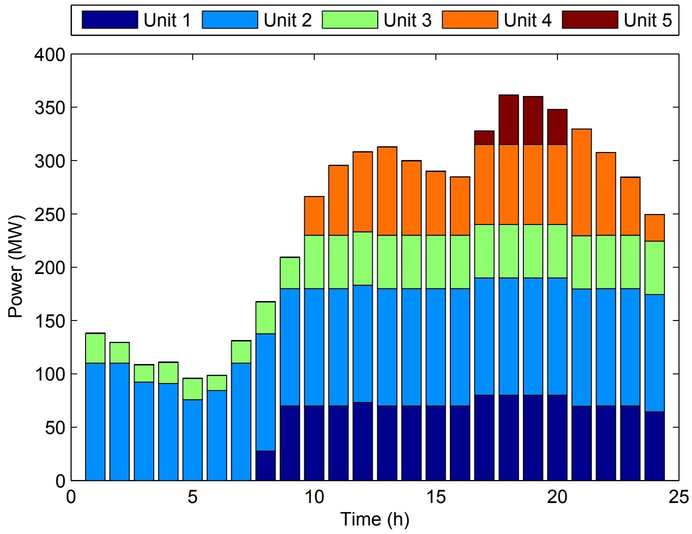

Figure 3: Hourly day-ahead schedule of power plants (P-B).

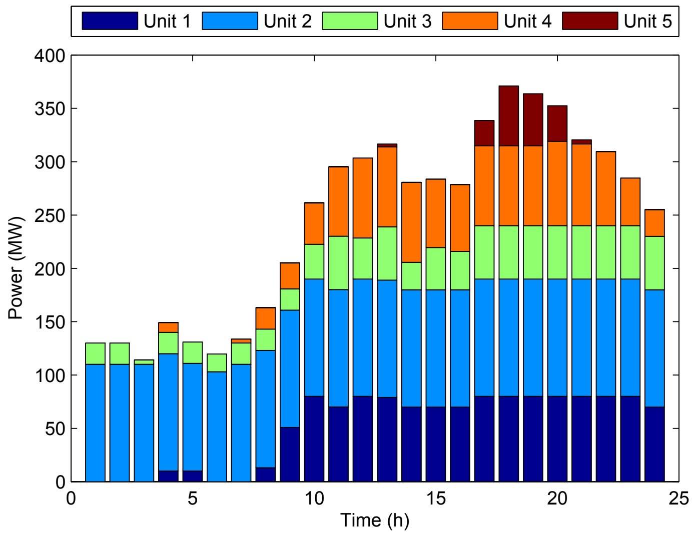

Figure 4: Hourly day-ahead schedule of power plants (Stoch).

Table 1: Profits. Wind power penetration 50%.

		I_1	I_3	I_4	I_5
Seq	Exp. profit ($)	18 725.35	24 634.10	46 411.69	55 360.59
Stoch	Exp. profit ($)	16 989.08	16 376.21	14 532.13	2 805.79
	Aver. losses ($)	-104.57	-106.09	-69.03	-28.37
	Prob. profit<0 (%)	1.2	7.6	9.4	0.3
P-B	Exp. profit ($)	18 452.14	22 044.05	24 001.30	19 771.83

The expected profits of flexible producers are shown in Table 1. The higher expected profits occur under *Seq* due to the high balancing prices that appear when costly balancing actions (e.g., electricity load shedding) take place. In this case, wind power producers have to bear this cost and this may result in negative profits in expectation. Model *P-B* attains to alleviate this effect by significantly reducing the cost of balancing actions. For *Stoch*, the average losses for the hours and scenarios that a negative profit realizes are provided. Moreover, we calculate the probability of having a negative profit for each hour and scenario. Although the average losses are relatively small, a considerable probability of having a negative profit per hour and scenario emerges. In this example, profits per scenario are positive under *Stoch* but this can not be guaranteed and generalized for every case. Positive profits for each scenario and hour are guaranteed under models *Seq* and *P-B*.

References

[1] J. Fortuny-Amat and B. McCarl, "A representation and economic interpretation of a two-level programming problem," *J. Oper. Res. Soc.*, vol. 32, no. 9, pp. 783-792, 1981.