# Market-based Coordination for Integrated Electricity & Natural Gas Systems Under Uncertain Supply

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This document serves as an electronic companion (EC) for the paper "Market-based Coordination for Integrated Electricity & Natural Gas Systems under Uncertain Supply". It contains four sections that present the MPEC formulation of the bilevel model *V-B*, provide proofs regarding the positive profits of flexible producers, a nomenclature and additional results.

# 1. MPEC formulation of volume-based coupled electricity and natural gas model (V-B)

In this section the bilevel V-B model is reformulated as a Mathematical Program with Equilibrium Constraints (MPEC) by replacing the linear, and thus convex, lower level problems by their Karush-Kuhn-Tucker (KKT) conditions. Then, the resulting MPEC is transformed into a Mixed-Integer Linear Program (MILP) in order to deal with the bilinear terms that arise from the complementarity conditions. We introduce a mapping  $M_a^{ig}$  of the natural gas-fired power plants  $i_g$  at area a (entries are equal to 1 if NGFPP is connected to an area and 0 otherwise). The model writes as follows,

$$\underset{\Theta^{\text{MUL}}}{\text{Min.}} \sum_{t \in T} \left[ \sum_{i_c \in I_c} C_{i_c} p_{i_c,t} + \sum_{k \in K} C_k g_{k,t} + \sum_{\omega \in \Omega} \pi_{\omega} \left( \sum_{k \in K} (C_k^+ g_{k,\omega,t}^+ - C_k^- g_{k,\omega,t}^-) + \sum_{i_c \in I_c} (C_{i_c}^+ p_{i_c,\omega,t}^+ - C_{i_c}^- p_{i_c,\omega,t}^-) \right) \right] + \sum_{r_e \in R_e} C^{\text{sh,E}} l_{r_e,\omega,t}^{\text{sh,E}} + \sum_{r_g \in R_g} C^{\text{sh,G}} l_{r_g,\omega,t}^{\text{sh,G}} \right) \right]$$
(1a)

subject to

$$-p_{i,t} \le \Delta p_{i,\omega',t} \le P_i^{\max} - p_{i,t}, \ \forall i,t,$$

$$\tag{1b}$$

$$-P_i^- \le \Delta p_{i,\omega',t} \le P_i^+, \ \forall i,t, \tag{1c}$$

$$0 \le w_{j,\omega',t}^{\mathrm{sp}} \le W_{j,\omega',t}, \ \forall j,t, \tag{1d}$$

$$0 \le l_{r_e,\omega',t}^{\text{sh,E}} \le D_{r_e,t}^{\text{E}}, \ \forall t, \tag{1e}$$

$$\sum_{i \in I} \Delta p_{i,\omega',t} + \sum_{r_e \in R_e} l_{r_e,\omega',t}^{\text{sh,E}} + \sum_{j \in J} (W_{j,\omega',t} - w_{j,\omega',t}^{\text{sp}} - w_{j,t}) = 0 : \tilde{\lambda}_{\omega',t}^{\text{E}}, \ \forall t,$$
 (1f)

$$-g_{k,t} \le \Delta g_{k,\omega',t} \le G_k^{\max} - g_{k,t}, \quad \forall k, t, \tag{1g}$$

$$-G_k^- \le \Delta g_{k,\omega',t} \le G_k^+, \quad \forall k, t, \tag{1h}$$

$$0 \le l_{r_a,\omega',t}^{\mathrm{sh,G}} \le D_{r_a,t}^{\mathrm{G}}, \ \forall t, \tag{1i}$$

$$\sum_{k \in K} \Delta g_{k,\omega',t} + \sum_{r_g \in R_g} l_{r_g,\omega',t}^{\text{sh,G}} - \sum_{i_g \in I_g} \phi_{i_g} \Delta p_{i_g,\omega',t} = 0 : \tilde{\lambda}_{\omega',t}^{\text{G}}, \quad \forall t,$$

$$(1j)$$

$$0 \leq \sum_{t \in T_{i_{\sigma}} \in A_{z}^{I_{g}}} \phi_{i_{g}}(p_{i_{g},t} + \Delta p_{i_{g},\omega',t}) \leq F_{z}^{A}, \quad \forall z,$$

$$(1k)$$

$$0 \leq \sum_{i_g \in A_z^{I_g}} \phi_{i_g}(p_{i_g,t} + \Delta p_{i_g,\omega',t}) \leq F_{z,t}^{\mathcal{M}}, \quad \forall z, t,$$
(11)

$$0 \le \chi_{\psi}^{v} \le |T| \sum_{k \in K} G_k^{\text{max}} - \sum_{t \in T} \sum_{r_g \in R_g} D_{r_g, t}^{G}, \quad \forall \psi,$$

$$\tag{1m}$$

$$0 \le \chi_{\psi,t}^v \le F_{\psi,t}^{\text{max}} - \sum_{r_g \in A_{\psi}^{R_g}} D_{r_g,t}^{G}, \ \forall \psi, t, \tag{1n}$$

$$0 \le p_{i,t} \le P_i^{\text{max}} : \mu_{i,t}^{\text{P}}, \overline{\mu}_{i,t}^{\text{P}}, \ \forall i, t, \tag{10}$$

$$0 \le w_{j,t} \le \widehat{W}_{j,t} : \underline{\mu}_{j,t}^{\widehat{W}}, \overline{\mu}_{j,t}^{\widehat{W}}, \ \forall j,t,$$

$$\tag{1p}$$

$$\sum_{i \in I} p_{i,t} + \sum_{j \in J} w_{j,t} - \sum_{r_e \in R_e} D_{r_e,t}^{E} = 0 : \hat{\lambda}_t^{E}, \ \forall t,$$
(1q)

$$0 \le g_{k,t} \le G_k^{\text{max}} : \underline{\mu}_{k,t}^{\text{G}}, \overline{\mu}_{k,t}^{\text{G}}, \quad \forall k, t, \tag{1r}$$

$$\sum_{k \in K} g_{k,t} - \sum_{r_g \in R_g} D_{r_g,t}^{G} - \sum_{i_g \in I_g} \phi_{i_g} p_{i_g,t} = 0 : \hat{\lambda}_t^{G}, \ \forall t,$$
(1s)

$$0 \le \sum_{t \in T} \sum_{i_g \in A_{\psi}^{I_g}} \phi_{i_g} p_{i_g, t} \le x_{\psi}^v : \underline{\nu}_{\psi}^v, \overline{\nu}_{\psi}^v, \quad \forall \psi,$$

$$\tag{1t}$$

$$0 \le \sum_{i_g \in A_{\psi}^{I_g}} \phi_{i_g} p_{i_g,t} \le x_{\psi,t}^v : \underline{\mu}_{\psi,t}^v, \overline{\mu}_{\psi,t}^v, \quad \forall \psi, t$$

$$\tag{1u}$$

$$C_{i_c} - \hat{\lambda}_t^{\mathrm{E}} - \underline{\mu}_{i_c,t}^{\mathrm{P}} + \overline{\mu}_{i_c,t}^{\mathrm{P}} = 0, \ \forall i_c, t,$$
 (1v)

$$\phi_{i_g} \hat{\lambda}_t^{G} - \hat{\lambda}_t^{E} - \underline{\mu}_{i_g,t}^{P} + \overline{\mu}_{i_g,t}^{P} + \sum_{\psi \in \Psi} M_{\psi}^{i_g} (\phi_{i_g} \overline{\mu}_{\psi,t}^{v} + \phi_{i_g} \overline{\nu}_{\psi}^{v} - \phi_{i_g} \underline{\mu}_{\psi,t}^{v} - \phi_{i_g} \underline{\nu}_{\psi}^{v}) = 0, \quad \forall i_g, t,$$
(1w)

$$-\hat{\lambda}_t^{\mathrm{E}} - \underline{\mu}_{j,t}^{\widehat{\mathrm{W}}} + \overline{\mu}_{j,t}^{\widehat{\mathrm{W}}} = 0, \ \forall j, t, \tag{1x}$$

$$C_k - \hat{\lambda}_t^{G} - \underline{\mu}_{k,t}^{G} + \overline{\mu}_{k,t}^{G} = 0, \ \forall k, t,$$
 (1y)

$$0 \le \overline{\mu}_{i,t}^{P} \perp P_{i}^{\text{max}} - p_{i,t} \ge 0, \quad \forall i, t,$$

$$(1z)$$

$$0 \le \underline{\mu}_{i,t}^{\mathrm{P}} \perp p_{i,t} \ge 0, \quad \forall i, t, \tag{1aa}$$

$$0 \le \overline{\mu}_{j,t}^{\widehat{W}} \perp \widehat{W}_j - w_{j,t} \ge 0, \ \forall j, t,$$
 (1ab)

$$0 \le \underline{\mu}_{j,t}^{\widehat{\mathbf{W}}} \perp w_{j,t} \ge 0, \ \forall j, t, \tag{1ac}$$

$$0 \le \overline{\mu}_{\psi,t}^{v} \perp x_{\psi,t}^{v} - \sum_{i_g \in A_{\psi}^{I_g}} \phi_{i_g} p_{i_g,t} \ge 0, \ \forall \psi, t,$$
(1ad)

$$0 \le \overline{\nu}_{\psi}^{v} \perp x_{\psi}^{v} - \sum_{t \in T} \sum_{i_{g} \in A_{\psi}^{I_{g}}} \phi_{i_{g}} p_{i_{g}, t} \ge 0, \ \forall \psi,$$
(1ae)

$$0 \le \underline{\mu}_{\psi,t}^{v} \perp \sum_{i_g \in A_{\psi}^{I_g}} \phi_{i_g} p_{i_g,t} \ge 0, \quad \forall \psi, t, \tag{1af}$$

$$0 \le \underline{\nu}_{\psi}^{v} \perp \sum_{t \in T} \sum_{i_g \in A_{\psi}^{I_g}} \phi_{i_g} p_{i_g, t} \ge 0, \quad \forall \psi, \tag{1ag}$$

$$0 \le \overline{\mu}_{k,t}^{G} \perp G_k^{\max} - g_{k,t} \ge 0, \quad \forall k, t, \tag{1ah}$$

$$0 \le \underline{\mu}_{k,t}^{G} \perp g_{k,t} \ge 0, \ \forall k, t, \tag{1ai}$$

The nonlinearities that arise from complementarity conditions are linearized via the Fortuny-Amat transformation [1]. We introduce the set of dual variables  $(\lambda, \mu \text{ and } \nu) \Theta^{\text{dual}}$ , thus  $\Theta^{\text{MUL}} = \{\Theta^{\text{VUL}}, \Theta^{\text{dual}}\}$ . For the network constrained balancing market, we use the set of constraints  $\{(2b)-(2e),(2g)-(2i),(3a)-(3c),(6a)-(6p)\}$  (numbered from the original manuscript) instead of (1b)-(11).

## 2. Proofs for the non-negative profits of flexible producers

In this section, we provide the proofs for the non-negativity of profits of flexible producers, when considering a sequential clearing of day-ahead and balancing markets. We would initially like to notice that model Seq in the original manuscript is an optimization problem. Let us then introduce model Seq-Eq, which is a two-settlement equilibrium model. Considering a different optimization problem for each market participant and the market-clearing conditions for each trading floor (i.e. day-ahead and balancing), we can formulate Seq-Eq by writing the KKT conditions for each individual optimization model. This set of KKT conditions is identical to those conditions associated with the Seq model, which proves that Seq and Seq-Eq are equivalent. Thus, any solution of one model is a solution of the other model too. The aforementioned statement holds if the problems are convex. We refer the reader to [2] and [3] for an extensive discussion on this topic and presentation of the approach to equivalently formulate the equilibrium and optimization models.

First, we focus on the thermal power plants that are not consuming natural gas. Focusing to the equilibrium model Seq-Eq, the profit maximization problem of each power producer for the day-ahead stage writes as follows,

$$\left\{ \underset{p_{i_c,t}}{\text{Max.}} \sum_{t \in T} \left[ p_{i_c,t} (\hat{\lambda}_t^{\text{E}} - C_{i_c}) \right] \right. \tag{2a}$$

subject to

$$0 \le p_{i_c,t} \le P_i^{\text{max}} : \underline{\mu}_{i_c,t}^{\text{P}}, \overline{\mu}_{i_c,t}^{\text{P}}$$
,  $\forall i_c, t.$  (2b)

Since program (2) is linear and thus convex, the strong duality theorem holds for the optimal solution and,

$$\sum_{t \in T} \left[ p_{i_c, t} (\hat{\lambda}_t^{\mathcal{E}} - C_{i_c}) \right] = \sum_{t \in T} \overline{\mu}_{i_c, t}^{\mathcal{P}} P_{i_c}^{\max}, \tag{3}$$

where  $\overline{\mu}_{i_c,t}^P \ge 0$  and  $P_{i_c}^{\max} \ge 0$  which shows that the profits in the day-ahead market are positive. Similarly, we can write the profit maximization problem for the balancing market by having the day-ahead decision  $p_{i_c,t}^*$  fixed,

$$\left\{ \underset{\Delta p_{i,\omega',t}}{\text{Max.}} \sum_{t \in T} \left[ \Delta p_{i,\omega',t} (\tilde{\lambda}_{\omega',t}^{\text{E}} - C_{i_c}) \right] \right\}$$
(4a)

subject to

$$-p_{i_c,t}^* \le \Delta p_{i_c,\omega',t} \le P_{i_c}^{\max} - p_{i_c,t}^* : \underline{\mu}_{i_c,\omega',t}^{\mathrm{R}}, \overline{\mu}_{i_c,\omega',t}^{\mathrm{R}}, \tag{4b}$$

$$-P_{i}^{-} \leq \Delta p_{i_{g},\omega',t} \leq P_{i}^{+} : \underline{\mu}_{i_{c},\omega',t}^{RR}, \overline{\mu}_{i_{c},\omega',t}^{RR} \Big\}, \quad \forall i_{c},\omega',t.$$

$$(4c)$$

We can also write the strong duality theorem for the optimal solution for program (4),

$$\sum_{t \in T} \left[ \Delta p_{i_c,\omega',t} (\tilde{\lambda}_{\omega',t}^{\mathrm{E}} - C_{i_c}) \right] = \sum_{t \in T} \left( \overline{\mu}_{i_c,\omega',t}^{\mathrm{R}} (P_{i_c}^{\mathrm{max}} - p_{i_c,t}^*) + \underline{\mu}_{i_c,\omega',t}^{\mathrm{R}} p_{i_c,t}^* + \overline{\mu}_{i_c,\omega',t}^{\mathrm{RR}} P_{i_c}^+ + \underline{\mu}_{i_c,\omega',t}^{\mathrm{RR}} P_{i_c}^- \right), \quad (5)$$

where  $\overline{\mu}_{i_c,\omega',t}^{\mathrm{R}}, \underline{\mu}_{i_c,\omega',t}^{\mathrm{R}}, \overline{\mu}_{i_c,\omega',t}^{\mathrm{RR}}, \underline{\mu}_{i_c,\omega',t}^{\mathrm{RR}} \geq 0$ . Moreover, the quantities  $(P_{i_c}^{\mathrm{max}} - p_{i_c,t}^*), p_{i_c,t}^*, P_{i_c}^+, P_{i_c}^- \geq 0$ . Thus, the profits in balancing market are also positive and this completes the proof. Since cost-recovery holds for model Seq-Eq, then it means that it is also ensured in optimization model Seq due to their equivalence.

The similar proof can be written for the GFPPs, where the marginal cost  $(C_{i_c})$  is replaced by  $\phi_{i_g}\hat{\lambda}_t^{G}$  and  $\phi_{i_g}\tilde{\lambda}_{\omega',t}^{G}$  for the day-ahead and balancing markets, respectively. The relative proof regarding cost recovery for flexible producers in stochastic dispatch model only in expectation and not per scenario realization of stochastic power production is presented in [4], while authors in [5] provide a detailed discussion on the topic.

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Finally, a stochastic market-clearing model that ensures cost-recovery and revenue adequacy per scenario is proposed in [6].

# 3. Nomenclature

In Tables 1-3, we present the symbols used in the original paper and the description for each one of them.

Table 1. Nomenclature for the variables

Symbol	Description
$p_{i,t}$	Day-ahead dispatch of power plants $i$ in period $t$ [MW]
$w_{j,t}$	Day-ahead dispatch of stochastic power producers $j$ in period $t$ [MW]
$p_{i,\omega,t}^+$	Up regulation provided by dispatchable power plant i in scenario $\omega$ , period t [MW]
$p_{i,\omega,t}^-$	Down regulation provided by dispatchable power plant $i$ in scenario $\omega$ , period $t$ [MW]
$w_{j,\omega,t}^{\mathrm{sp}}$	Power spilled by stochastic power plant $j$ in scenario $\omega$ , period $t$ [MW]
$l_{r_e,\omega,t}^{\mathrm{sh,E}}$	Electric power load shedding at node $n$ in scenario $\omega$ , period $t$ [MW]
$\begin{array}{c} r_{i,\omega,t}^{\text{w}}\\ p_{i,\omega,t}^{\text{sp}}\\ w_{j,\omega,t}^{\text{sp}}\\ l_{r_{e},\omega,t}^{\text{sh,E}}\\ l_{r_{g},\omega,t}^{\text{sh,G}}\\ \tilde{\delta}_{n,\omega,t} \end{array}$	Natural gas load shedding at node $m$ in scenario $\omega$ , period $t$ [kcf/h]
$ ilde{\delta}_{n,\omega,t}$	Voltage angle at node $n$ in scenario $\omega$ , period $t$ [rad]
$g_{k,t}$	Day-ahead dispatch of natural gas producer $k$ in period $t$ [kcf/h]
$g_{k,\omega,t}^+$	Up regulation provided by natural gas producer $k$ in scenario $\omega$ , period $t$ [kcf/h]
$g_{k,\omega,t}^-$	Down regulation provided by natural gas producer $k$ in scenario $\omega$ , period $t$ [kcf/h]
$pr_{m,\omega,t}^{\mathrm{r}}$	Pressure at node $m$ in scenario $\omega$ , period $t$ [psig]
$h_{m,u,\omega,t}^{\mathrm{r}}$	Average mass of natural gas (linepack) in pipeline (m,u), scenario $\omega$ , period t [kcf]
$q_{m,u,\omega,t}^{\mathrm{r}}$ $q_{m,u,\omega,t}^{\mathrm{in,r}}$	Inflow natural gas rates of pipeline (m,u) in scenario $\omega$ , period t [kcf/h]
$q_{m,u,\omega,t}^{\mathrm{out,r}}$	Outflow natural gas rates of pipeline (m,u) in scenario $\omega$ , period $t$ [kcf/h]
$q_{m,u}$	Natural gas flow in pipeline (m,u) [kcf/h]
$y_{m,u,\omega,t}^{\mathrm{r}}$	Binary variable defining the direction of the natural gas flow in pipeline (m,u), scenario $\omega$ , period $t$ {0,1}
$\chi_t^p$	Natural gas price adjustment in period $t$ [\$/kcf]
$\overline{\chi}_{\psi}^{v}$	Daily natural gas volume availability for GFPPs' group in specific area $\psi$ [kcf]
$\chi_{\psi,t}^{v'}$	Hourly natural gas volume availability for GFPPs' group in specific area $\psi$ in period $t$ [kcf]

Table 2. Nomenclature for the parameters

	Table 2. Nomenclature for the parameters
Symbol	Description
$ \begin{array}{c} D_{r_e,t}^{\mathrm{E}} \\ D_{r_g,t}^{\mathrm{G}} \\ C_i \\ C_i^+ \\ C_i^- \\ C^{\mathrm{sh,E}} \end{array} $	Electricity demand $r_e$ and in period $t$ [MW]
$D_{r_q,t}^{\mathrm{G}}$	Natural gas demand $r_g$ and in period $t$ [kcf/h]
$C_i$	Day-ahead offer price of dispatchable power plant $i$ [\$/MWh]
$C_i^+$	Up regulation offer price of dispatchable power plant $i$ [\$/MWh]
$C_i^-$	Down regulation offer price of dispatchable power plant $i$ [\$/MWh]
$C^{ m sh,E}$	Cost of electricity load shedding [\$/MWh]
$C_k$	Day-ahead offer price of natural gas producer $k$ [\$/kcf]
$C_k^+$	Up regulation offer price of natural gas producer $k$ [\$/kcf]
$C_k^-$	Down regulation offer price of natural gas producer $k$ [\$/kcf]
$C_k^+$ $C_k^-$ $C^{\text{sh,G}}$	Cost of natural gas load shedding [\$/kcf]
$P_i^{\max}$	Capacity of dispatchable power plant $i$ [MW]
$P_i^{\text{max}}$ $P_i^+$ $P_i^-$ $\phi_{i_g}$	Maximum up regulation capability of dispatchable power plant $i  [MW]$
$P_i^-$	Maximum down regulation capability of dispatchable power plant $i \ [MW]$
$\phi_{i_g}$	Power conversion factor of natural gas-fired power plant $i_g$ [kcf/MWh]
$W_{j,\omega,t}$	Power production by stochastic power plant $j$ in scenario $\omega$ , period $t$ [MW]
$W_{j,\omega,t}$ $\widehat{W}_{j,t}$ $\overline{W}_{j}$ $\overline{W}_{j}$ $G_{k}^{\max}$ $G_{k}^{+}$	Expected power production by stochastic power plant $j$ in period $t$ [MW]
$\overline{W}_j$	Capacity of stochastic power plant $j$ [MW]
$G_k^{\max}$	Capacity of natural gas producer $k \text{ [kcf/h]}$
$G_k^+$	Maximum up regulation capability of natural gas producer $k \text{ [kcf/h]}$
$G_k^-$	Maximum down regulation capability of natural gas producer $k  [kcf/h]$
$B_{n,r}$	Absolute value of the susceptance of line (n,r) [per unit]
$F_{n,r}^{\max}$	Transmission capacity of line (n,r) [MW]
$F_{n,r}^{\max}$ $K_{m,u}^{\mathrm{h}}$ $F_{m,u}^{\mathrm{min}}$	Natural gas flow constant of pipeline (m,u) [kcf/psig]
$K_{m,u}^{\mathrm{f}}$	Linepack constant of pipeline $(m,u)$ $[kcf/(psig \cdot h)]$
$PR_m^{\min}$	Minimum pressure at node $m$ [psig]
$PR_m^{\max}$	Maximum pressure at node $m$ [psig]
$\Gamma_z$	Compressor factor located at natural gas network branch $z$ [-]
$\pi_{\omega}$	Probability of scenario $\omega$ [-]
$ ilde{M}$	Sufficiently large constant [-]
$F_{z,t}^{ m M}$	Capacity of natural gas pipeline $z$ in period $t$ [kcf/h]
$F_{z,t}^{ m M} \ F_z^{ m A}$	Daily contract limit of natural gas pipeline $z$ [kcf]
$F_{\psi,t}^{\max}$	Maximum natural gas availability for a specific area $\psi$ containing the group of GFPPs [kcf/h]

	Table 3. Nomenclature for the sets
Symbol	Description
I	Set of dispatchable power plants $i$
$I_c$	Set of thermal power plants $i_c$ $(I_c \subset I)$
$I_g$	Set of natural gas-fired power plants $i_g$ $(I_g \subset I)$
J	Set of stochastic power plants $j$
L	Set of electricity transmission lines $l$
N	Set of electricity network nodes $n$
K	Set of natural gas producers $k$
Z	Set of natural gas network branches $z$
M	Set of natural gas network nodes $m$
$R_e$	Set of electricity demands $r_e$
$R_g$	Set of natural gas demands $r_g$
V	Set of fixed pressure points $v$ for the linearization of Weymouth equation
$\Omega$	Set of stochastic power production scenarios $\omega$
T	Set of time periods $t$
$A_n^I$	Set of dispatchable power plants $i$ located at electricity network node $n$
$A_n^J \ A_m^{I_g}$	Set of stochastic power plants $j$ located at electricity network node $n$
$A_m^{I_g}$	Set of natural gas-fired power plants $i_g$ located at natural gas network node $m$
$A_{\psi}^{I_g} \ A_m^K$	Set of natural gas-fired power plants $i_g$ located in a specific area $\psi$
$A_m^K$	Set of natural gas producers $k$ located at natural gas network node $m$
$A_n^{R_e}$	Set of electricity demands $r_e$ located at electricity network node $n$
$A_n^{R_e} \ A_m^{R_g}$	Set of natural gas demands $r_g$ located at natural gas network node $m$
$\Psi$	Set of groups of natural gas-fired power plants located in a specific area $\psi$
Θ	Set of primal optimization variables defined for each optimization model

## 4. Additional Results

Fig. 1 shows the natural gas price adjustment  $(\chi_t^p)$  and the day-ahead payment/charge in order to generate this signal. Moreover, Fig. 2 illustrate the natural gas price adjustment  $(\chi_t^p)$  in relation to the difference in the hourly GFPPs share of the total power production between P-B and Seq. A detailed analysis of these results is presented in [7].

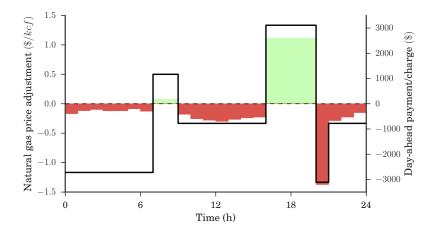


FIGURE 1. Hourly natural gas price adjustment (black line: left y-axis) and day-ahead financial settlement of the system operator to adjust the natural gas price (colored areas: right y-axis). Wind power penetration 50%.

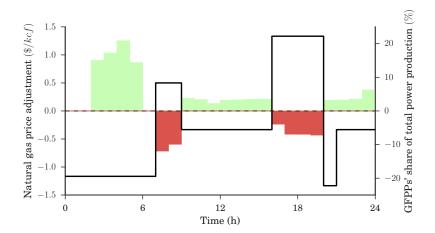


FIGURE 2. Hourly natural gas price adjustment (black line: left y-axis) and difference in GFPPs share of total power production between P-B and Seq (colored areas: right y-axis). Wind power penetration 50%.

Fig. 3 shows the natural gas volume  $(\chi_{a,t}^v)$  in relation to the natural gas volume consumed in Seq and the the difference in the hourly GFPPs share of the total power production between V-B and Seq. Moreover, Fig. 4 illustrate the natural gas volume  $(\chi_{a,t}^v)$  change compared to the natural gas volume consumed in Seq in relation to the difference in the hourly GFPPs share of the total power production between V-B gen and Seq. Thus, the left y-axis illustrates the quantity  $\frac{F(V-B)-F(Seq)}{F(Seq)}$  or  $\frac{F(V-B\ gen)-F(Seq)}{F(Seq)}$ , where F is the natural gas volume made available at the day-ahead stage for each dispatch model.

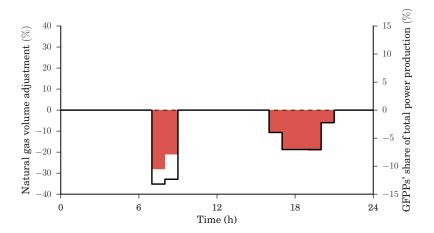


FIGURE 3. Hourly natural gas volume adjustment (black line: left y-axis) and difference in GFPPs share of total power production between V-B and Seq (colored areas: right y-axis). Wind power penetration 50%.

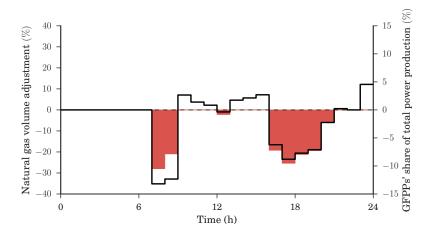


FIGURE 4. Hourly natural gas volume adjustment (black line: left y-axis) and difference in GFPPs share of total power production between *V-B gen* and *Seq* (colored areas: right y-axis). Wind power penetration 50%.

It can be noticed in Fig. 3 that a decrease in the natural gas volume results in a decrease of the power production share of GFPPs compared to the scheduling provided from model Seq. A similar effect is also observed in Fig. 4, except for two periods in the middle of the day. During these hours, the total natural gas consumed by GFFPs in V-B gen is higher than in Seq, while GFPPs' share of total power production is not affected. This happens because there is a sift of "X MW" between the two GFPPs, resulting in the one with the higher power conversion factor to produce more than in Seq; hence increase the total gas consumption for power production.

Table 4 illustrates the performance of the dispatch models in terms of expected cost when the electricity demand is equal to 344 MW in order to explore an alternative case of natural gas price adjustment and volume availability. Models Stoch and Seq provide the two extreme solutions in terms of expected cost. We highlight though that P-B, V-B and V-B gen manage to return the same expected cost as Stoch. This fact illustrates that it is possible in specific cases to have an efficient sequential system dispatch if the future balancing costs are communicated into the day-ahead market through the operator-defined parameters  $\chi$ .

TABLE 4. Expected system cost and its breakdown in \$ when total power load is 344 MW

	Total	Day-ahead	Balancing	Up regulation	Down regulation
Seq	8,932.8	$8,\!566.8$	366.0	825.0	-459.0
Stoch	8,859.6	8,206.8	652.8	917.4	-264.6
$P ext{-}B \ / \ V ext{-}B \ / \ V ext{-}B \ gen$	8,859.6	8,638.8	220.8	679.8	-459.0

The schedule of power plants is given in Table 5 for  $D^{\rm E}$ =344 MW. In P-B, the natural gas price adjustment is  $\chi^p_{t_2} = +\$0.5/{\rm kcf}$  which increases the marginal cost of GFPP  $I_3$  to  $\$30/{\rm MWh}$ . Thus, an improved dayahead schedule is achieved by sifting 12 MW from GFPP  $I_3$  to unit  $I_1$ , resulting in a lower expected cost. Models V-B and V-B gen reduce the total natural gas availability from 600 kcf to 456 kcf and return the same improved dispatch as P-B.

TABLE 5. Power system schedule in MW when total power load is 344 MW (variation from Seq day-ahead (DA) schedule in bold)

	Seq			P-B		V-B			V-B gen			
Unit	DA	$\omega_1$	$\omega_2$									
$\overline{I_1}$	58	-10	+10	70	-10	+10	70	-10	+10	70	-10	+10
$I_2$	110	0	0	110	0	0	110	0	0	110	0	0
$I_3$	50	-30	0	38	-30	+12	38	-30	+12	38	-30	+12
$I_4$	0	0	+25	0	0	+18	0	0	+18	0	0	+18
$I_5$	0	0	+5	0	0	0	0	0	0	0	0	0
WP	126	+40	-40	126	+40	-40	126	+40	-40	126	+40	-40

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