# Market-based Coordination of Integrated Electricity & Natural Gas Systems Under Uncertain Supply

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This document serves as an electronic companion (EC) for the paper "Market-based Coordination of Integrated Electricity and Natural Gas Systems under Uncertain Supply". It contains two sections that present the MPEC formulation of the bilevel model *V-B* and additional results.

## 1. MPEC formulation of volume-based coupled electricity and natural gas model (V-B)

In this section the bilevel *V-B* model is reformulated as a Mathematical Program with Equilibrium Constraints (MPEC) by replacing the linear, and thus convex, lower level problems by their Karush-Kuhn-Tucker (KKT) conditions. Then, the resulting MPEC is transformed into a Mixed-Integer Linear Program (MILP) in order to deal with the bilinear terms that arise from the complementarity conditions. We introduce a mapping $M_a^{i_g}$ of the natural gas-fired power plants $i_g$ at area $a$ (entries are equal to 1 if a GFPP is connected to an area and 0 otherwise). The model writes as follows,

$$\begin{aligned}
\underset{\Theta^{\mathrm{MUL}}}{\text{Min.}} \; & \sum_{t\in T}\Big[\sum_{i_c\in I_c} C_{i_c} p_{i_c,t} + \sum_{k\in K} C_k g_{k,t} + \sum_{\omega\in\Omega}\pi_\omega\Big(\sum_{k\in K}(C_k^+ g_{k,\omega,t}^+ - C_k^- g_{k,\omega,t}^-) + \sum_{i_c\in I_c}(C_{i_c}^+ p_{i_c,\omega,t}^+ - C_{i_c}^- p_{i_c,\omega,t}^-) \\
& + \sum_{r_e\in R_e} C^{\mathrm{sh,E}} l_{r_e,\omega,t}^{\mathrm{sh,E}} + \sum_{r_g\in R_g} C^{\mathrm{sh,G}} l_{r_g,\omega,t}^{\mathrm{sh,G}}\Big)\Big] && \text{(1a)}\\
\text{s.t} \; & -p_{i,t} \le \Delta p_{i,\omega',t} \le P_i^{\max} - p_{i,t}, \;\; \forall i,t, && \text{(1b)}\\
& -P_i^- \le \Delta p_{i,\omega',t} \le P_i^+, \;\; \forall i,t, && \text{(1c)}\\
& 0 \le w_{j,\omega',t}^{\mathrm{sp}} \le W_{j,\omega',t}, \;\; \forall j,t, && \text{(1d)}\\
& 0 \le l_{r_e,\omega',t}^{\mathrm{sh,E}} \le D_{r_e,t}^{\mathrm{E}}, \;\; \forall t, && \text{(1e)}\\
& \sum_{i\in I}\Delta p_{i,\omega',t} + \sum_{r_e\in R_e} l_{r_e,\omega',t}^{\mathrm{sh,E}} + \sum_{j\in J}(W_{j,\omega',t} - w_{j,\omega',t}^{\mathrm{sp}} - w_{j,t}) = 0 : \tilde{\lambda}_{\omega',t}^{\mathrm{E}}, \;\; \forall t, && \text{(1f)}\\
& -g_{k,t} \le \Delta g_{k,\omega',t} \le G_k^{\max} - g_{k,t}, \;\; \forall k,t, && \text{(1g)}\\
& -G_k^- \le \Delta g_{k,\omega',t} \le G_k^+, \;\; \forall k,t, && \text{(1h)}\\
& 0 \le l_{r_g,\omega',t}^{\mathrm{sh,G}} \le D_{r_g,t}^{\mathrm{G}}, \;\; \forall t, && \text{(1i)}\\
& \sum_{k\in K}\Delta g_{k,\omega',t} + \sum_{r_g\in R_g} l_{r_g,\omega',t}^{\mathrm{sh,G}} - \sum_{i_g\in I_g}\phi_{i_g}\Delta p_{i_g,\omega',t} = 0 : \tilde{\lambda}_{\omega',t}^{\mathrm{G}}, \;\; \forall t, && \text{(1j)}\\
& 0 \le \sum_{t\in T}\sum_{i_g\in A_z^{I_g}}\phi_{i_g}(p_{i_g,t} + \Delta p_{i_g,\omega',t}) \le F_z^{\mathrm{A}}, \;\; \forall z, && \text{(1k)}\\
& 0 \le \sum_{i_g\in A_z^{I_g}}\phi_{i_g}(p_{i_g,t} + \Delta p_{i_g,\omega',t}) \le F_{z,t}^{\mathrm{M}}, \;\; \forall z,t, && \text{(1l)}\\
& 0 \le \chi_\psi^v \le |T|\sum_{k\in K} G_k^{\max} - \sum_{t\in T}\sum_{r_g\in R_g} D_{r_g,t}^{\mathrm{G}}, \;\; \forall \psi, && \text{(1m)}\\
& 0 \le \chi_{\psi,t}^v \le F_{\psi,t}^{\max} - \sum_{r_g\in A_\psi^{R_g}} D_{r_g,t}^{\mathrm{G}}, \;\; \forall \psi,t, && \text{(1n)}
\end{aligned}$$

$$0 \leq p_{i,t} \leq P_i^{\max} : \underline{\mu}_{i,t}^{\mathrm{P}}, \overline{\mu}_{i,t}^{\mathrm{P}}, \ \ \forall i, t, \tag{1o}$$

$$0 \leq w_{j,t} \leq \widehat{W}_{j,t} : \underline{\mu}_{j,t}^{\widehat{\mathrm{W}}}, \overline{\mu}_{j,t}^{\widehat{\mathrm{W}}}, \ \ \forall j, t, \tag{1p}$$

$$\sum_{i \in I} p_{i,t} + \sum_{j \in J} w_{j,t} - \sum_{r_e \in R_e} D_{r_e,t}^{\mathrm{E}} = 0 : \hat{\lambda}_t^{\mathrm{E}}, \ \ \forall t, \tag{1q}$$

$$0 \leq g_{k,t} \leq G_k^{\max} : \underline{\mu}_{k,t}^{\mathrm{G}}, \overline{\mu}_{k,t}^{\mathrm{G}}, \ \ \forall k, t, \tag{1r}$$

$$\sum_{k \in K} g_{k,t} - \sum_{r_g \in R_g} D_{r_g,t}^{\mathrm{G}} - \sum_{i_g \in I_g} \phi_{i_g} p_{i_g,t} = 0 : \hat{\lambda}_t^{\mathrm{G}}, \ \ \forall t, \tag{1s}$$

$$0 \leq \sum_{t \in T} \sum_{i_g \in A_\psi^{I_g}} \phi_{i_g} p_{i_g,t} \leq x_\psi^v : \underline{\nu}_\psi^v, \overline{\nu}_\psi^v, \ \ \forall \psi, \tag{1t}$$

$$0 \leq \sum_{i_g \in A_\psi^{I_g}} \phi_{i_g} p_{i_g,t} \leq x_{\psi,t}^v : \underline{\mu}_{\psi,t}^v, \overline{\mu}_{\psi,t}^v, \ \ \forall \psi, t \tag{1u}$$

$$C_{i_c} - \hat{\lambda}_t^{\mathrm{E}} - \underline{\mu}_{i_c,t}^{\mathrm{P}} + \overline{\mu}_{i_c,t}^{\mathrm{P}} = 0, \ \ \forall i_c, t, \tag{1v}$$

$$\phi_{i_g} \hat{\lambda}_t^{\mathrm{G}} - \hat{\lambda}_t^{\mathrm{E}} - \underline{\mu}_{i_g,t}^{\mathrm{P}} + \overline{\mu}_{i_g,t}^{\mathrm{P}} + \sum_{\psi \in \Psi} M_\psi^{i_g} (\phi_{i_g} \overline{\mu}_{\psi,t}^v + \phi_{i_g} \overline{\nu}_\psi^v - \phi_{i_g} \underline{\mu}_{\psi,t}^v - \phi_{i_g} \underline{\nu}_\psi^v) = 0, \ \ \forall i_g, t, \tag{1w}$$

$$-\hat{\lambda}_t^{\mathrm{E}} - \underline{\mu}_{j,t}^{\widehat{\mathrm{W}}} + \overline{\mu}_{j,t}^{\widehat{\mathrm{W}}} = 0, \ \ \forall j, t, \tag{1x}$$

$$C_k - \hat{\lambda}_t^{\mathrm{G}} - \underline{\mu}_{k,t}^{\mathrm{G}} + \overline{\mu}_{k,t}^{\mathrm{G}} = 0, \ \ \forall k, t, \tag{1y}$$

$$0 \leq \overline{\mu}_{i,t}^{\mathrm{P}} \perp P_i^{\max} - p_{i,t} \geq 0, \ \ \forall i, t, \tag{1z}$$

$$0 \leq \underline{\mu}_{i,t}^{\mathrm{P}} \perp p_{i,t} \geq 0, \ \ \forall i, t, \tag{1aa}$$

$$0 \leq \overline{\mu}_{j,t}^{\widehat{\mathrm{W}}} \perp \widehat{W}_j - w_{j,t} \geq 0, \ \ \forall j, t, \tag{1ab}$$

$$0 \leq \underline{\mu}_{j,t}^{\widehat{\mathrm{W}}} \perp w_{j,t} \geq 0, \ \ \forall j, t, \tag{1ac}$$

$$0 \leq \overline{\mu}_{\psi,t}^v \perp x_{\psi,t}^v - \sum_{i_g \in A_\psi^{I_g}} \phi_{i_g} p_{i_g,t} \geq 0, \ \ \forall \psi, t, \tag{1ad}$$

$$0 \leq \overline{\nu}_\psi^v \perp x_\psi^v - \sum_{t \in T} \sum_{i_g \in A_\psi^{I_g}} \phi_{i_g} p_{i_g,t} \geq 0, \ \ \forall \psi, \tag{1ae}$$

$$0 \leq \underline{\mu}_{\psi,t}^v \perp \sum_{i_g \in A_\psi^{I_g}} \phi_{i_g} p_{i_g,t} \geq 0, \ \ \forall \psi, t, \tag{1af}$$

$$0 \leq \underline{\nu}_\psi^v \perp \sum_{t \in T} \sum_{i_g \in A_\psi^{I_g}} \phi_{i_g} p_{i_g,t} \geq 0, \ \ \forall \psi, \tag{1ag}$$

$$0 \leq \overline{\mu}_{k,t}^{\mathrm{G}} \perp G_k^{\max} - g_{k,t} \geq 0, \ \ \forall k, t, \tag{1ah}$$

$$0 \leq \underline{\mu}_{k,t}^{\mathrm{G}} \perp g_{k,t} \geq 0, \ \ \forall k, t, \tag{1ai}$$

The nonlinearities that arise from complementarity conditions are linearized via the Fortuny-Amat transformation [1]. We introduce the set of dual variables ($\lambda$, $\mu$ and $\nu$) $\Theta^{\mathrm{dual}}$, thus $\Theta^{\mathrm{MUL}} = \{\Theta^{\mathrm{VUL}}, \Theta^{\mathrm{VLL}}, \Theta^{\mathrm{dual}}\}$. For the network constrained balancing market, we use the set of constraints {(2b)-(2e),(2g)-(2i),(3a)-(3c),(6a)-(6p)} (numbered from the original manuscript) instead of (1b)-(1l).

## 2. Additional Results

Fig. 1 shows the natural gas price adjustment ($\chi_t^p$) and the day-ahead payment/charge in order to generate this signal. Moreover, Fig. 2 illustrate the natural gas price adjustment ($\chi_t^p$) in relation to the difference in the hourly GFPPs share of the total power production between *P-B* and *Seq*. A detailed analysis of these results is presented in [2].

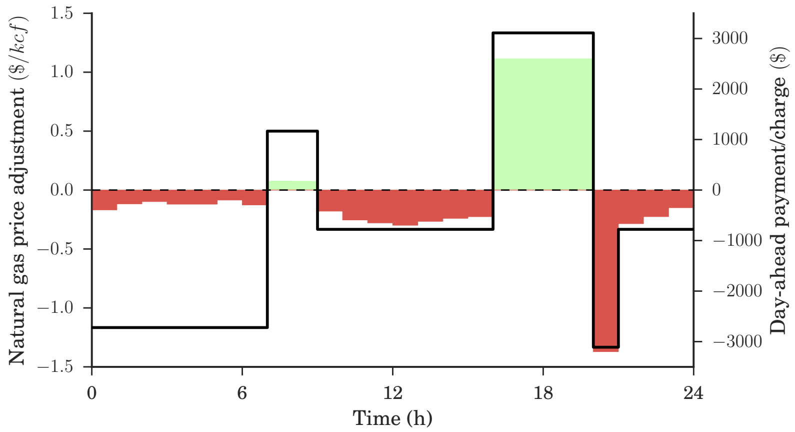


FIGURE 1. Hourly natural gas price adjustment (black line: left y-axis) and day-ahead financial settlement of the system operator to adjust the natural gas price (colored areas: right y-axis). Wind power penetration 50%.

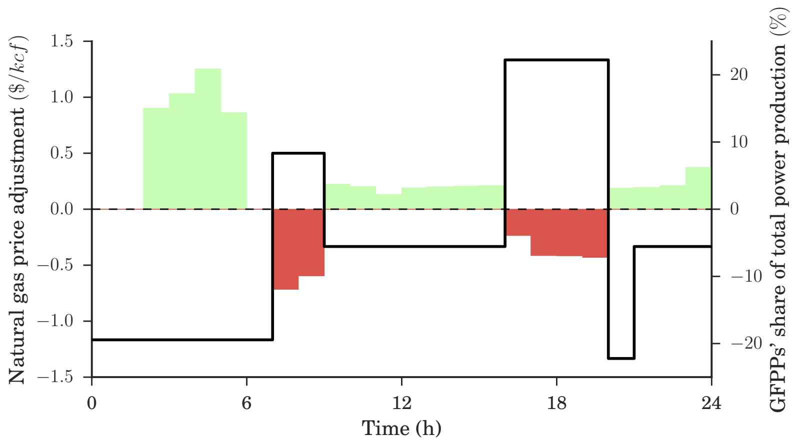


FIGURE 2. Hourly natural gas price adjustment (black line: left y-axis) and difference in GFPPs share of total power production between P-B and Seq (colored areas: right y-axis). Wind power penetration 50%.

Fig. 3 shows the natural gas volume ($\chi^{v}_{a,t}$) in relation to the natural gas volume consumed in *Seq* and the the difference in the hourly GFPPs share of the total power production between *V-B* and *Seq*. Moreover, Fig. 4 illustrate the natural gas volume ($\chi^{v}_{a,t}$) change compared to the natural gas volume consumed in *Seq* in relation to the difference in the hourly GFPPs share of the total power production between *V-B gen* and *Seq*. Thus, the left y-axis illustrates the quantity $\frac{F(V\text{-}B)-F(Seq)}{F(Seq)}$ or $\frac{F(V\text{-}B\ gen)-F(Seq)}{F(Seq)}$, where $F$ is the natural gas volume made available at the day-ahead stage for each dispatch model.

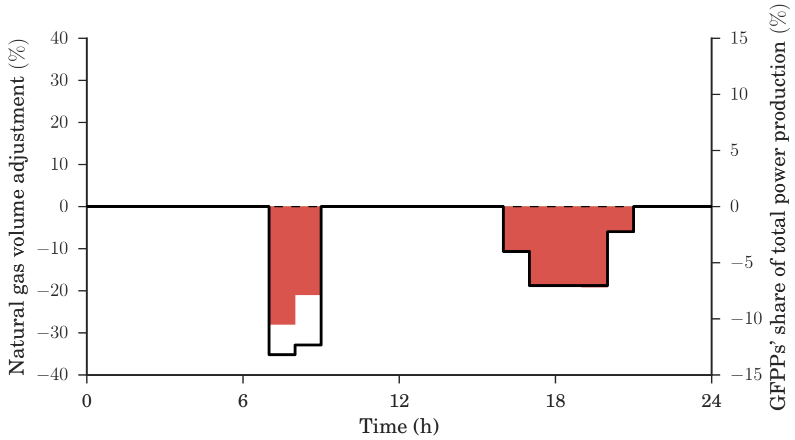


FIGURE 3. Hourly natural gas volume adjustment (black line: left y-axis) and difference in GFPPs share of total power production between *V-B* and *Seq* (colored areas: right y-axis). Wind power penetration 50%.

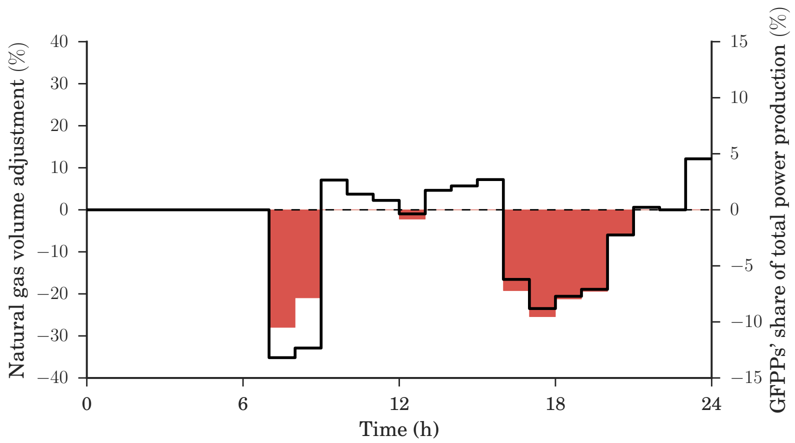


FIGURE 4. Hourly natural gas volume adjustment (black line: left y-axis) and difference in GFPPs share of total power production between *V-B gen* and *Seq* (colored areas: right y-axis). Wind power penetration 50%.

It can be noticed in Fig. 3 that a decrease in the natural gas volume results in a decrease of the power production share of GFPPs compared to the scheduling provided from model *Seq*. A similar effect is also observed in Fig. 4, except for two periods in the middle of the day. During these hours, the total natural gas consumed by GFFPs in *V-B gen* is higher than in *Seq*, while GFPPs' share of total power production is not affected. This happens because there is a sift of "X MW" between the two GFPPs, resulting in the one with the higher power conversion factor to produce more than in *Seq*; hence increase the total gas consumption for power production.

Table 1 illustrates the performance of the dispatch models in terms of expected cost when the electricity demand is equal to 344 MW in order to explore an alternative case of natural gas price adjustment and volume availability. Models *Stoch* and *Seq* provide the two extreme solutions in terms of expected cost. We highlight though that *P-B*, *V-B* and *V-B gen* manage to return the same expected cost as *Stoch*. This fact illustrates that it is possible in specific cases to have an efficient sequential system dispatch if the future balancing costs are communicated into the day-ahead market through the operator-defined parameters $\chi$.

TABLE 1. Expected system cost and its breakdown in $ when total power load is 344 MW

| | Total | Day-ahead | Balancing | Up regulation | Down regulation |
|---|---|---|---|---|---|
| *Seq* | 8,932.8 | 8,566.8 | 366.0 | 825.0 | -459.0 |
| *Stoch* | 8,859.6 | 8,206.8 | 652.8 | 917.4 | -264.6 |
| *P-B / V-B / V-B gen* | 8,859.6 | 8,638.8 | 220.8 | 679.8 | -459.0 |

The schedule of power plants is given in Table 2 for $D^{\mathrm{E}}$=344 MW. In *P-B*, the natural gas price adjustment is $\chi^{p}_{t_2} = +\$0.5$/kcf which increases the marginal cost of GFPP $I_3$ to \$30/MWh. Thus, an improved day-ahead schedule is achieved by sifting 12 MW from GFPP $I_3$ to unit $I_1$, resulting in a lower expected cost. Models *V-B* and *V-B gen* reduce the total natural gas availability from 600 kcf to 456 kcf and return the same improved dispatch as *P-B*.

TABLE 2. Power system schedule in MW when total power load is 344 MW (variation from *Seq* day-ahead (DA) schedule in bold)

| | *Seq* | | | *P-B* | | | *V-B* | | | *V-B gen* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Unit | DA | $\omega_1$ | $\omega_2$ | DA | $\omega_1$ | $\omega_2$ | DA | $\omega_1$ | $\omega_2$ | DA | $\omega_1$ | $\omega_2$ |
| $I_1$ | 58 | -10 | +10 | **70** | -10 | +10 | **70** | -10 | +10 | **70** | -10 | +10 |
| $I_2$ | 110 | 0 | 0 | 110 | 0 | 0 | 110 | 0 | 0 | 110 | 0 | 0 |
| $I_3$ | 50 | -30 | 0 | **38** | -30 | +12 | **38** | -30 | +12 | **38** | -30 | +12 |
| $I_4$ | 0 | 0 | +25 | 0 | 0 | +18 | 0 | 0 | +18 | 0 | 0 | +18 |
| $I_5$ | 0 | 0 | +5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| WP | 126 | +40 | -40 | 126 | +40 | -40 | 126 | +40 | -40 | 126 | +40 | -40 |

## References

[1] J. Fortuny-Amat and B. McCarl, "A representation and economic interpretation of a two-level programming problem," *The Journal of the Operational Research Society*, vol. 32, no. 9, pp. 783–792, 1981.

[2] C. Ordoudis, S. Delikaraoglou, P. Pinson, and J. Kazempour, "Exploiting flexibility in coupled electricity and natural gas markets: A price-based approach," in *2017 IEEE Manchester PowerTech*, pp. 1–6, June 2017.