Campaigning for Congress After Redistricting

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Introduction

- United States Representatives are elected by voters in subsets of each state called **districts**.
- Congressional districts for states are decided in the year following the United States Census in a process called **redistricting**.
- The choice of districts can have a positive or negative effect on each political party during congressional elections.
- There are a number of factors that can have a positive or negative effect during campaigning.

Redistricting

- Parties responsible for redistricting:
  - State Legislature
    - 37 states rely on state legislature for redistricting
  - Single District States
    - 7 states only have enough residents to qualify for one congressional district
  - Independent Commissions
    - These commissions have regulations limiting participation by elected officials

Regulations on Redistricting

- Requirements set for **all** states:
  - Equal Population
    - Populations across all districts in each state should be equal "as nearly as is practicable"
    - Result of *Wesberry v. Sanders in 1964*
  - Race and Ethnicity
    - These may not be used as predominant factors in the redistricting process
    - Result of *Voting Rights Act* passed by Congress in 1965

Regulations on Redistricting

- Regulations determined by each state:
  - Contiguity
    - A district is **contiguous** if travel between any two points can be made without crossing the boundary
  - Political Boundaries
    - Examples are county, city, or town lines
  - Communities of Interest
  - Political Outcomes

Problem Statement

- Our objective:
  - Maximize the expected number of votes that go to our political party in the state across all districts after redistricting
- Subject to the following constraints:
  - Our party has a limited campaign budget for each candidate in each district
  - We can influence the redistricting process provided that the new districts:
    - Have nearly equal population
    - Have race populations near the average race population of the state
    - Are contiguous

Problem Statement

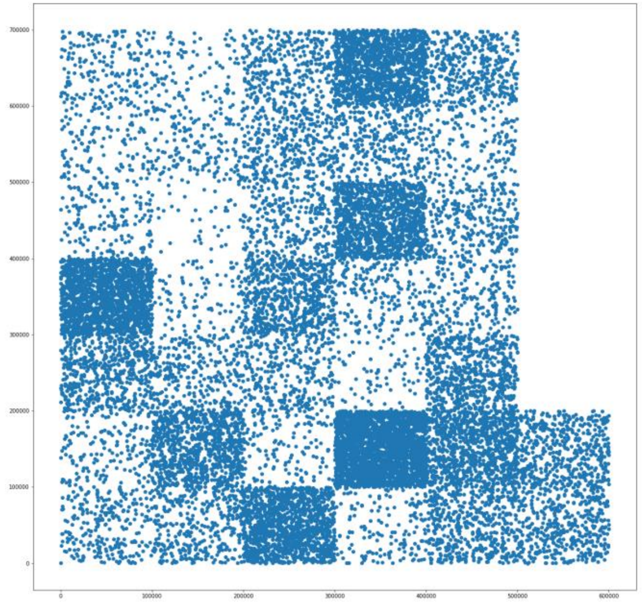
- Our assumptions:
  - Work from the perspective of a single political party in a single state
  - The opposing party spends the entirety of their budget on campaign expenditures
  - The counties in our state have the same shape and area

Model Formulation

- Stage one: **Redistricting**
  - Determine new districts for the state such that:
    - The population in each district is within five percent of the population in all other districts
    - The percentage of each race in each district is within ten percent of the mean race population for the entire state
    - The districts are contiguous

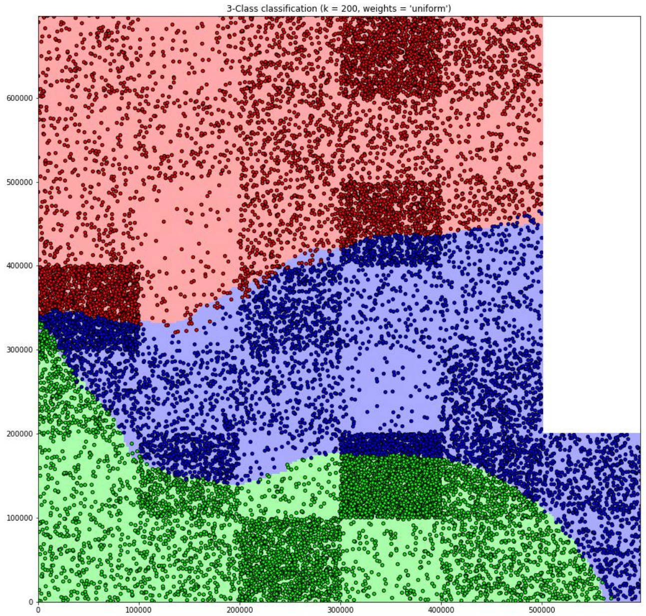
Model Formulation

- Stage one: **Redistricting**
  - First, we generate a geographical model of the state using population data from 37 counties in Florida available from the US Census Bureau.
    - Each point represents 1000 citizens with population statistics equal to the mean of the population statistics for the county in which they are located



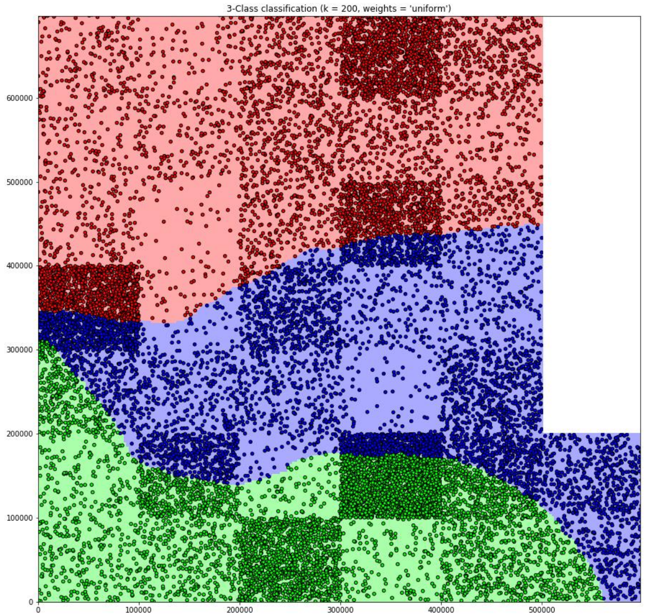
Model Formulation

- Stage one: **Redistricting**
  - Next, one initial point is chosen for each district
  - Each district then takes turns choosing points nearest to their initial point to keep the race percentages close to the mean race percentages for the entire state



Model Formulation

- Stage one: **Redistricting**
  - Districts are finalized and outliers are eliminated by using k-nearest neighbors classification
  - If this process results in districts that do not satisfy the constraints then the process is restarted with new initial points



Model Formulation

- Stage two: **Campaign Expenditures**
  - For each district we consider two different campaign budget and incumbency scenarios with some given probability
  - We aim to maximize the expected number of votes that go to our political party in the state across all districts by adjusting the amount of money we spend in each district
  - We used a batch stochastic gradient projection algorithm to determine a local maximum based on a gradient projection algorithm by Hager and Zhang.

Model Formulation

District 1				
Scenario	Probability	Our Budget	Opponent Budget	Incumbent Party
1	0.7	1,500,000	1,000,000	Opponent
2	0.3	4,000,000	2,000,000	Opponent

District 2				
Scenario	Probability	Our Budget	Opponent Budget	Incumbent Party
1	0.4	2,000,000	1,000,000	Our Party
2	0.6	4,000,000	3,000,000	Our Party

District 3				
Scenario	Probability	Our Budget	Opponent Budget	Incumbent Party
1	0.8	3,000,000	2,000,000	Opponent
2	0.2	2,000,000	3,000,000	Our Party

Model Formulation

- We modeled a function to predict the percentage of votes earned by a candidate in a district as a neural network using 173 parameters.
  - 165 from the US Census Fact Finder for population, age, gender, race, education, and income statistics
  - 5 from the Federal Election Commission and Ballotpedia for campaign expenditures, incumbency statistics, and voting statistics
  - 3 from the Florida Department of State and Iowa Secretary of State on voter registration information

Model Formulation

Data and Parameters

Given Data.

m	:	Number of districts in state (varies by state, we will take $m = 3$)
n	:	Number of counties in state (varies by state, we will take $n = 37$)
D	:	Index set $D = \{1, 2, \ldots, m\}$
C	:	Set of all counties in our state
R	:	Set of all races in our state
P	:	Set of all parties running in our state
s	:	Number of campaign scenarios (in our case $s = 24$)
L	:	Index set $L = \{1, 2, \ldots, s\}$

Parameters.

q	:	Total population of our state
q_{race}	:	Total population of a given race in our state, for all $race \in R$
p_{county}	:	Total population of given county, for all $county \in C$
$p_{race,county}$	:	Total population of given race in a given county, for all $race \in R$, $county \in C$
X_{county}	:	Population, age, gender, education, income, and voter registration parameters for a given county, for all $county \in C$
$\omega^{(\ell)}$	:	Probability of campaign scenario ℓ occurring, for all $\ell \in L$
$f_i^{(\ell)}$	:	Maximum amount of funds available to our party in campaign scenario, for $i \in D$ and $\ell \in L$
$g_i^{(\ell)}$	:	Amount of funds spent by opposing party in campaign scenario for for $i \in D$ and $\ell \in L$

Model Formulation

Stage One

Decision Variables. We have the following first-stage decision variable:

S_i	:	Set of population points used in District i, $i \in D$
$x_{c,i}$	:	Number of population points from County c used in District i, $c \in C$, $i \in D$
X_i	:	Population, age, gender, education, income, voter registration, incumbency, and opposing party budget parameters for District i, for all $i \in D$

Constraints. In addition to contiguity requirements, we have the following first-stage constraints:

$$\left| \sum_{c \in C} 1000 x_{c,i} \frac{p_c}{q} - \frac{1}{m} \right| \leq 0.05, \quad i \in D$$

: Population across all districts should be nearly identical (in particular, population in each district should be within five percent of one third of the population of our state)

$$\left| \sum_{c \in C} 1000 x_{c,i} \frac{p_{r,c}}{p_c} - \frac{q_r}{q} \right| \leq 0.10, \quad i \in D, \; r \in R$$

: The percentage of each race concentrated in each district should be nearly identical across all districts (in this case, the percentage of each race in each district should be within ten percent of the percentage of each race in the entire state)

$$X_i = \sum_{c \in C} \frac{1000 x_{c,i}}{p_c} X_c, \quad i \in D$$

: The parameters for each district are determined using the percentage of the population from each county in that district.

Model Formulation

Stage Two

Decision Variables. We have the following second-stage decision variables:

$y_i^{(\ell)}$: Amount of money to be spent by our party's candidate in district i in campaign scenario ℓ, for all $i \in D$ and $\ell \in L$

IVRs. We have the following basic constraint on the second-stage decision variables:

$y_i^{(\ell)} \geq 0, \quad i \in D$: The amount of money spent on the campaign must be nonnegative

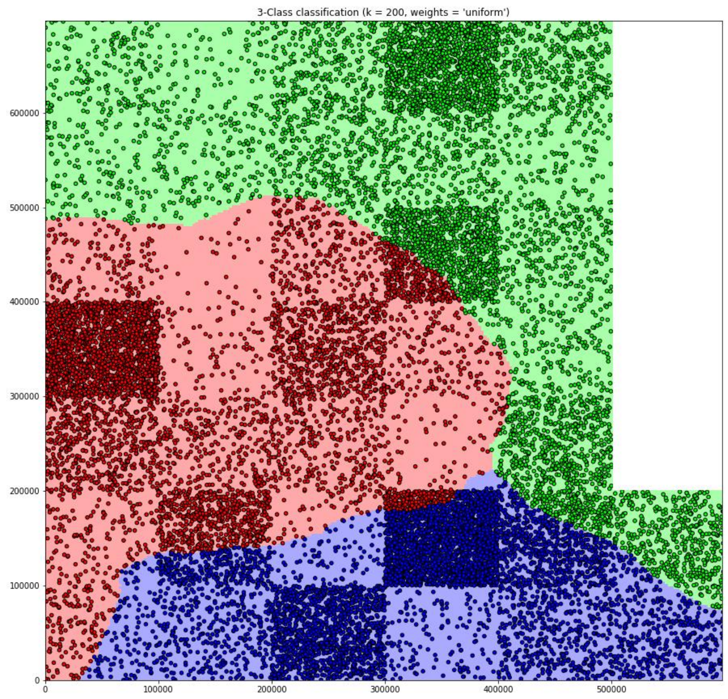
Constraints. We have the following constraints on the second-stage decision variables:

$y_i^{(\ell)} \leq f_i^{(\ell)}, \quad \ell \in L$: The amount of money spent on the campaign in each district must not exceed the amount of money available to the campaign

Objective

$$\max \sum_{i \in D} \left[\sum_{\ell \in L} \omega_\ell \, \Phi_{X_i}\left(y_i^{(\ell)}\right) \right]$$

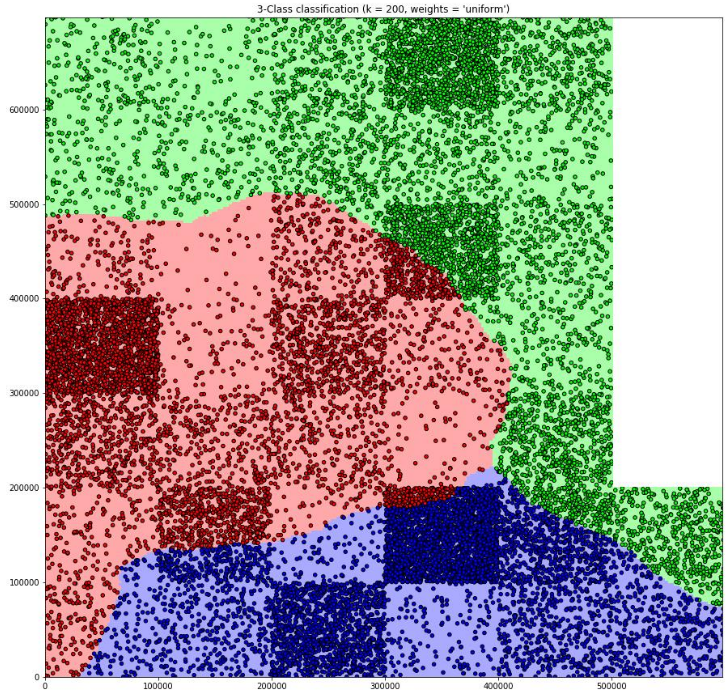
Some Results



Deviation from One-Third of Total Population		
District 1	District 2	District 3
1.385 %	0.740 %	0.645 %

	Deviation from Mean of Each Race Population		
Race	District 1	District 2	District 3
0	4.829 %	6.365 %	1.842 %
1	4.500 %	5.163 %	0.946 %

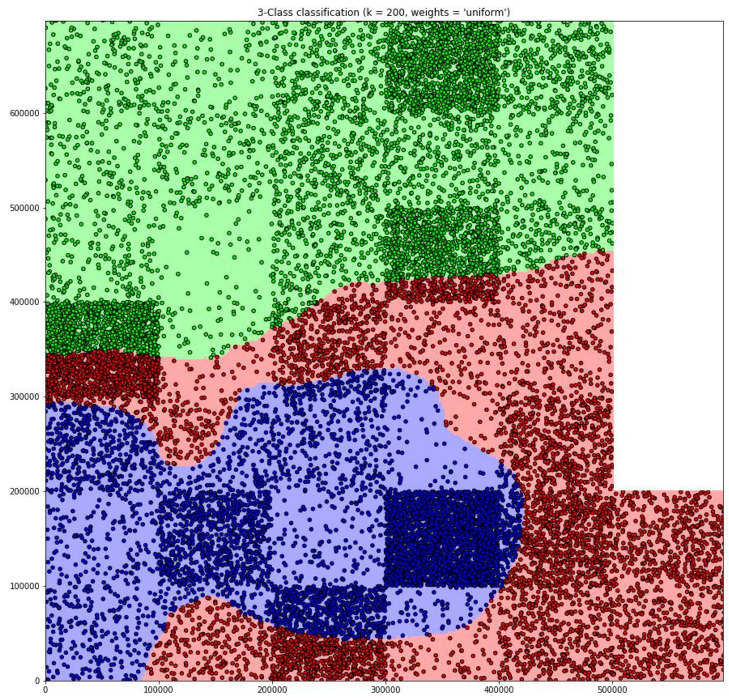
Some Results



	Campaign Expenditures (in Dollars)		
Scenario	District 1	District 2	District 3
1,1,1	0.00	0.00	1,215,183.00
2,1,1	795,441.30	1,327,494.84	1,285,359.83
1,2,1	0.00	1,846,470.85	0.00
1,1,2	203,391.60	3,387,493.00	943,895.74
2,2,1	3,003,245.75	752,255.16	2,376,795.19
2,1,2	3,763,949.55	1,697,554.59	1,728,911.95
1,2,2	2,552,034.91	3,060,235.66	1,987,804.61
2,2,2	3,648,446.16	3,726,293.13	1,601,049.73

Expected Percentage of Votes Across All Districts
41.49

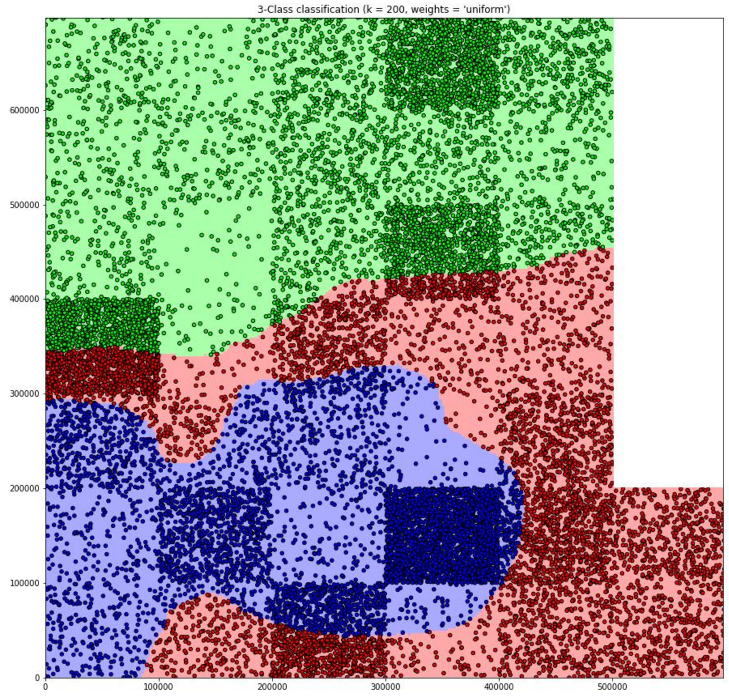
Some Results



Deviation from One-Third of Total Population		
District 1	District 2	District 3
0.238 %	0.381 %	0.143 %

	Deviation from Mean of Each Race Population		
Race	District 1	District 2	District 3
0	5.246 %	4.156 %	1.171 %
1	4.268 %	3.175 %	1.155 %

Some Results



	Campaign Expenditures (in Dollars)		
Scenario	District 1	District 2	District 3
1,1,1	0.00	0.00	436539.79
2,1,1	752,908.78	1,299,112.88	1,319,356.70
1,2,1	0.00	0.00	0.00
1,1,2	233,450.22	2,222,531.41	871,216.45
2,2,1	1,706,859.04	869,676.79	1,977,478.63
2,1,2	2,424,552.34	1,700,415.17	1,737,906.64
1,2,2	1,240,332.17	1,679,516.80	1,561,693.18
2,2,2	2,321,334.14	2,474,906.48	1,579,844.78

Expected Percentage of Votes Across All Districts
41.56

Future Work

- The optimization model should be updated to allow the districts determined from stage one to be modified by the gradients of the objective function.
  - To do this, treat Φ as a function of y and X_i then compute gradient of Φ with respect to each component of X_i
  - By modifying the algorithm used in redistricting, these gradients can be used to make decisions during the redistricting phase to increase the objective function value.

Future Work

- The neural network model should be trained on more data to obtain more accurate prediction results.
  - The primary difficulty faced when collecting congressional election and expenditure data to train the neural network with was that this data was not typically available in an easily modifiable file format.
  - The gradients of the current model often indicate that our party should decrease campaign expenditures to increase the percentage of votes obtained.

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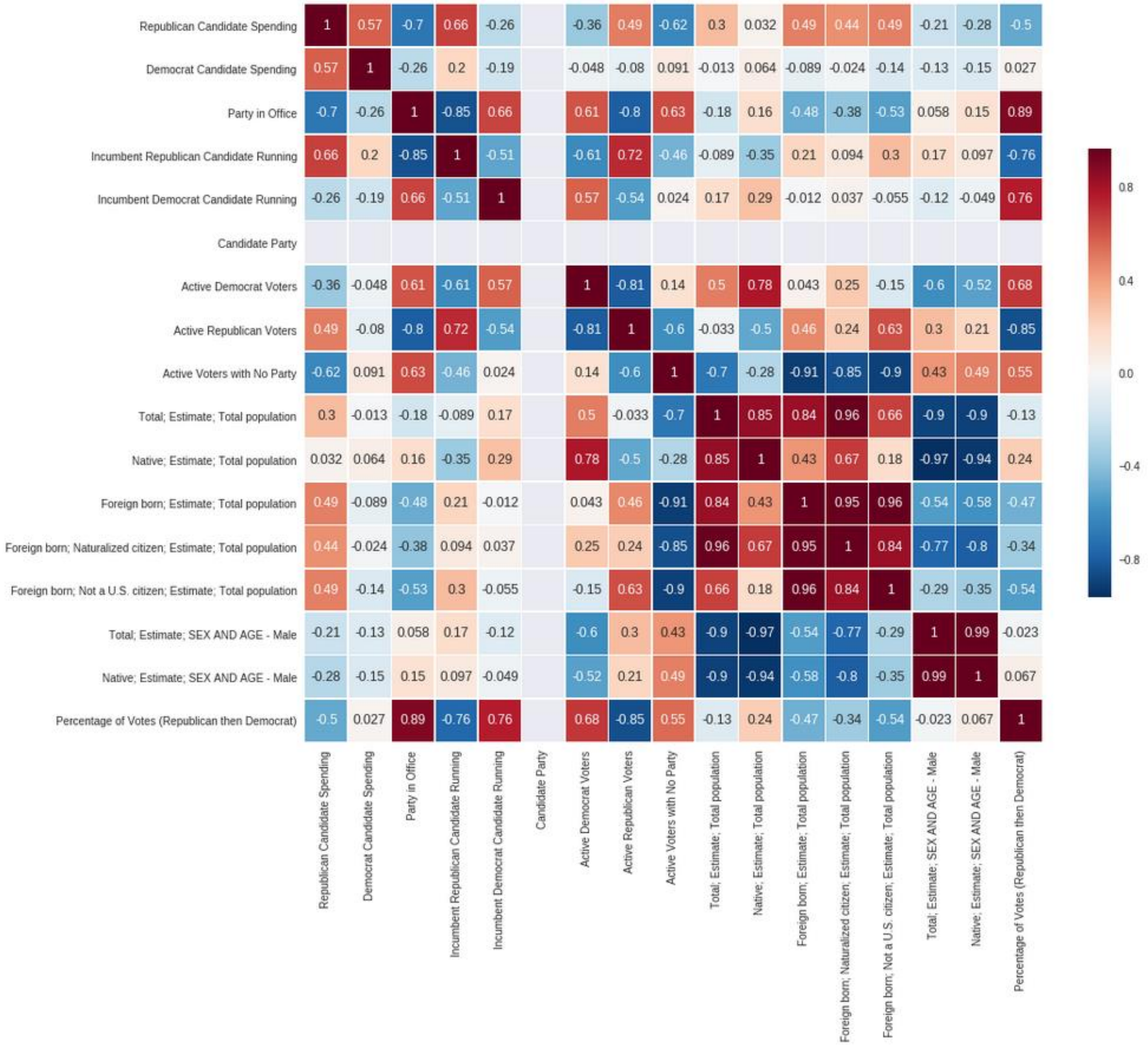
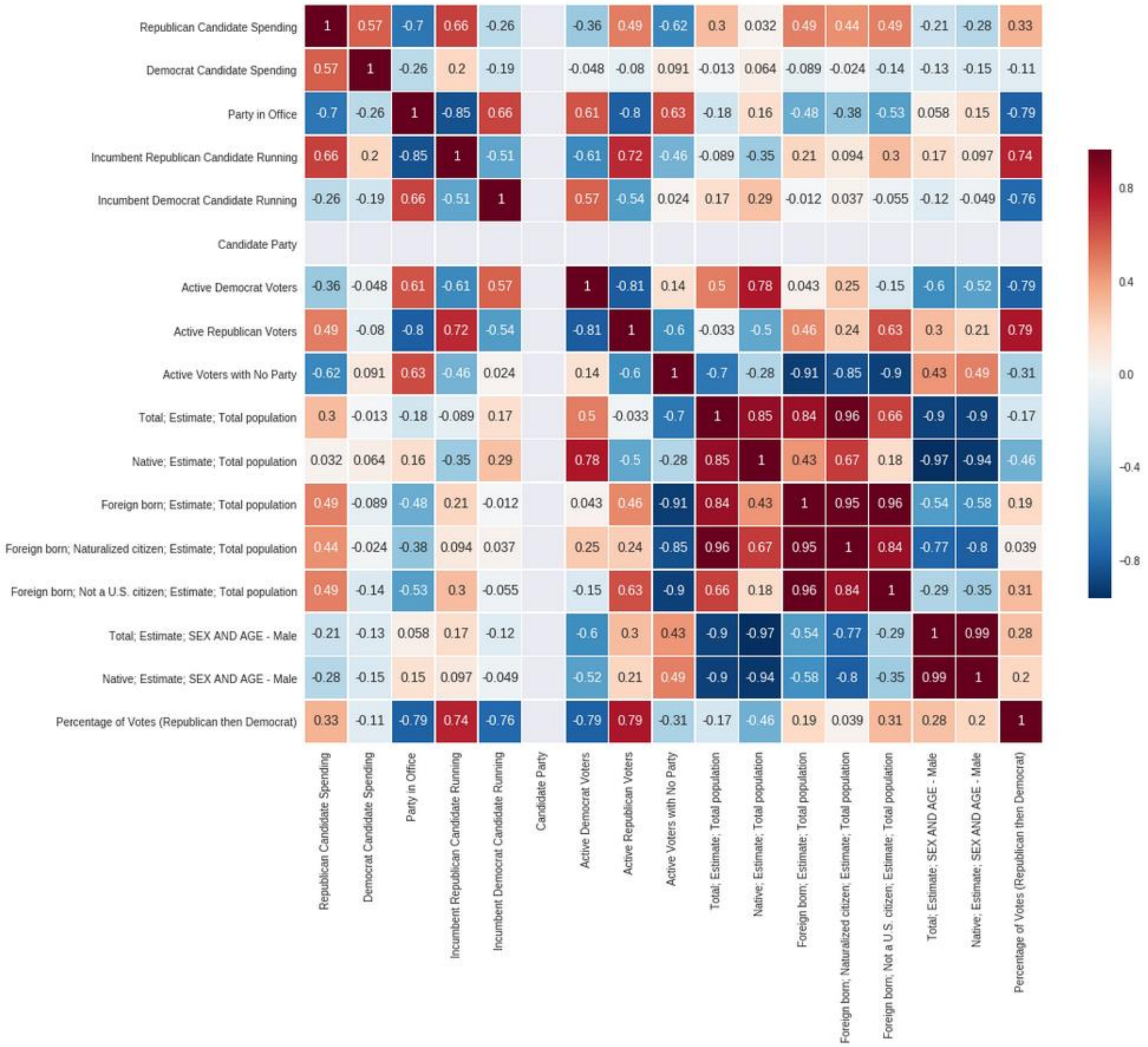
Additional Materials



Democrat Party Correlation Matrix

Additional Materials



Republican Party Correlation Matrix