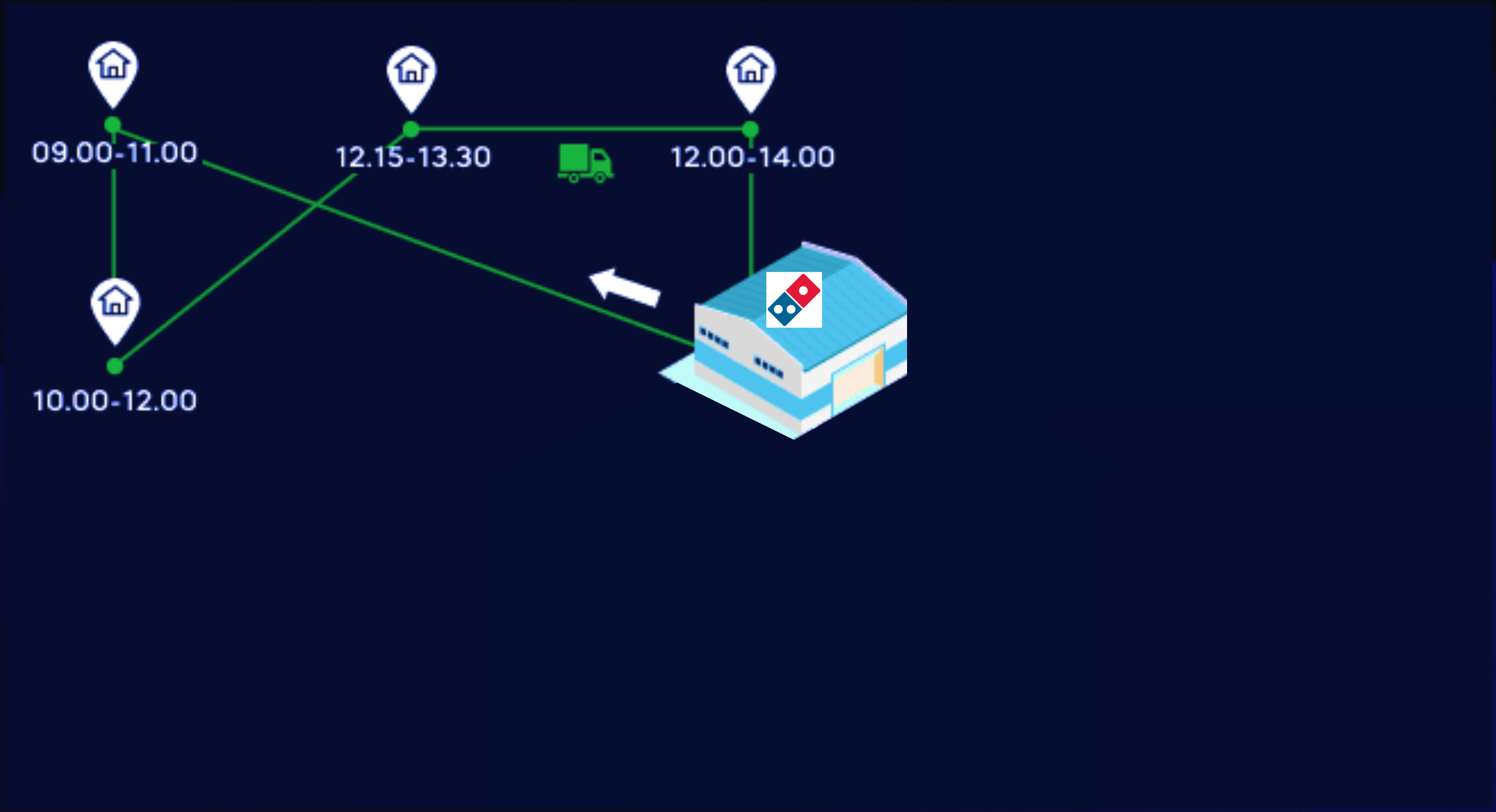
Hardware-accelerated QAOA for graph cutting

Better graph cuts than last time

Caffeinated Qrackheads

The Vehicle Routing Problem (VRP)

Why is this interesting?!



Example of the TSP & VRP

- TSP: optimal route to visit all customers
- VRP: optimal set of routes to visit all customers with fleet

The Vehicle Routing Problem (VRP)

Where can we use it?

- Delivery services
- Public transport
- Waste collection
- Emergency services
- Agriculture

Can we translate this?

And why is the math so hard?!

$$H_{VRP} = H_A + H_B + H_C + H_D + H_E$$

$$H_A = \sum_{i \to j} w_{ij} x_{ij}$$

$$H_B = A \sum_{i \in 1,\dots,n-1} \Big(1 - \sum_{j \in source[i]} x_{ij}\Big)^2$$

$$H_C = A \sum_{i \in 1,\dots,n-1} \Big(1 - \sum_{j \in target[i]} x_{ji}\Big)^2$$

$$H_D = A\Big(k - \sum_{j \in source[0]} x_{0j}\Big)^2$$

$$H_E = A\Big(k - \sum_{j \in target[0]} x_{j0}\Big)^2$$

VRP math

$$f(x)_{QUBO} = min_{x \in \{0,1\}^{(N \times V)}} \ x^T Q x + g^T x + c$$

QUBO

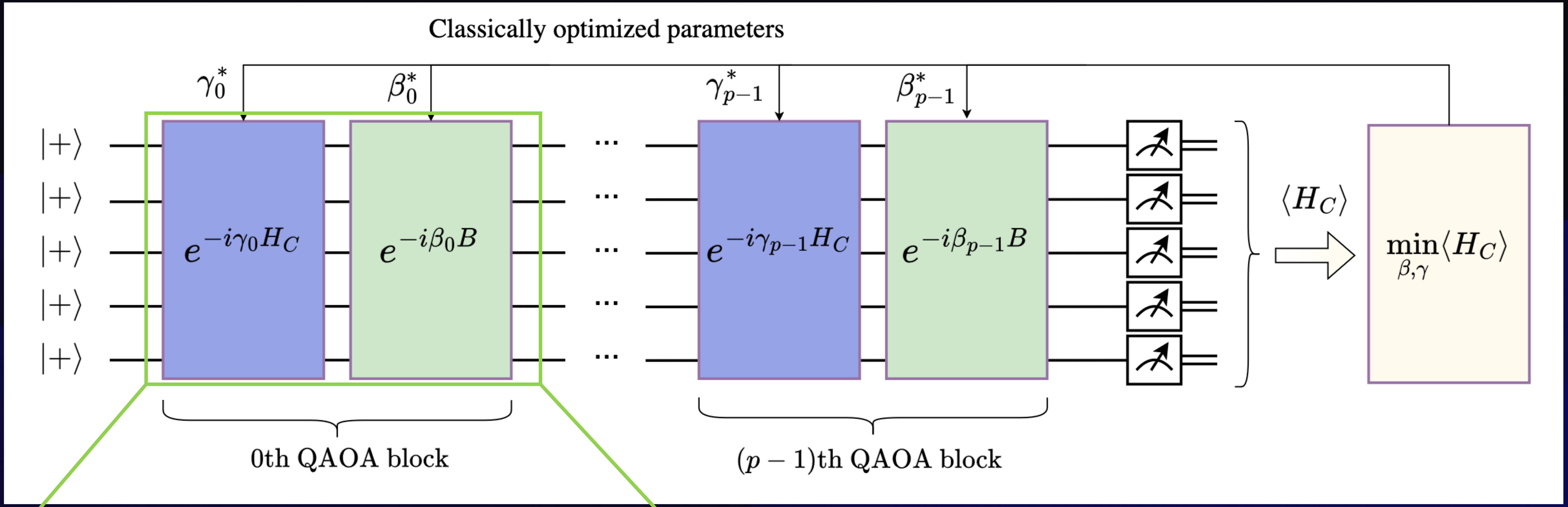
$$H_{ISING} = -\sum_i \sum_{j<i} J_{ij} s_i s_j + \sum_i h_i s_i + d$$

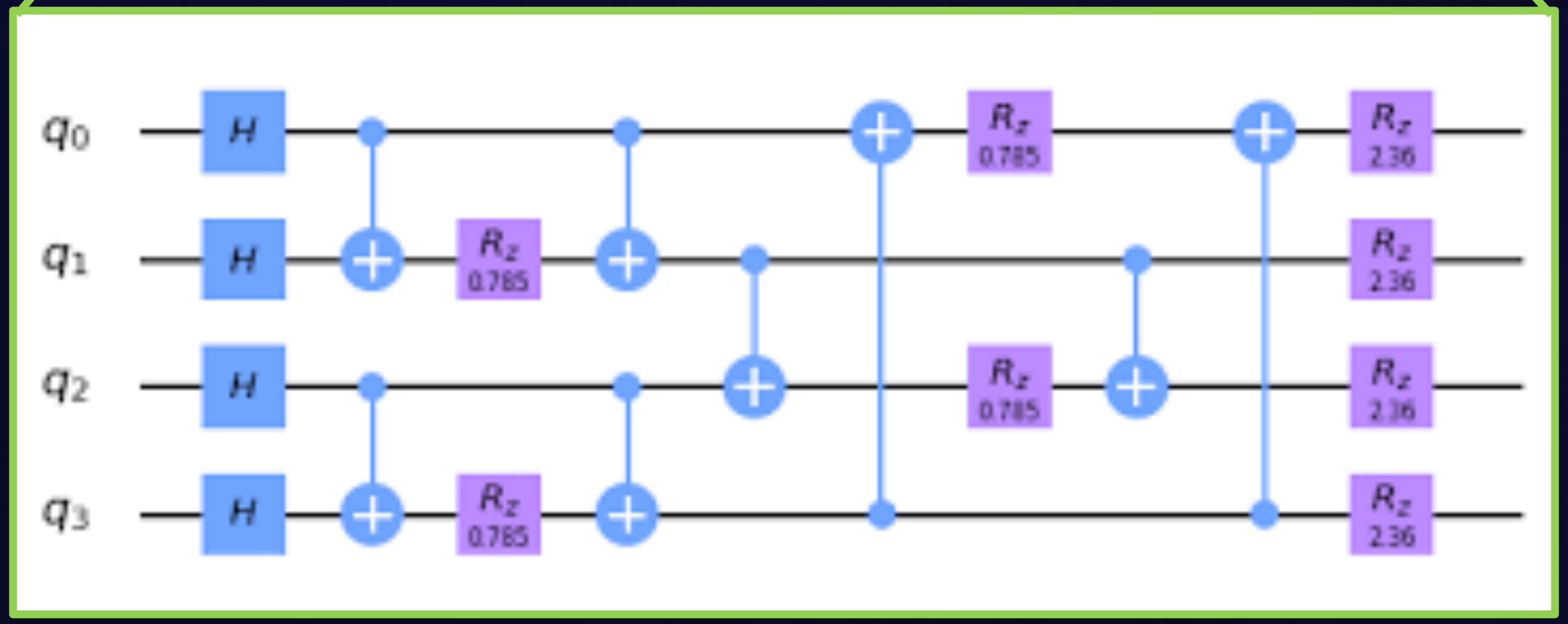
Ising

Key idea: Map each node in a graph to a qubit!

What is QAOA?

And what is the circuit?





- p layers
- 2 parameters per layer

What to analyze with QAOA?

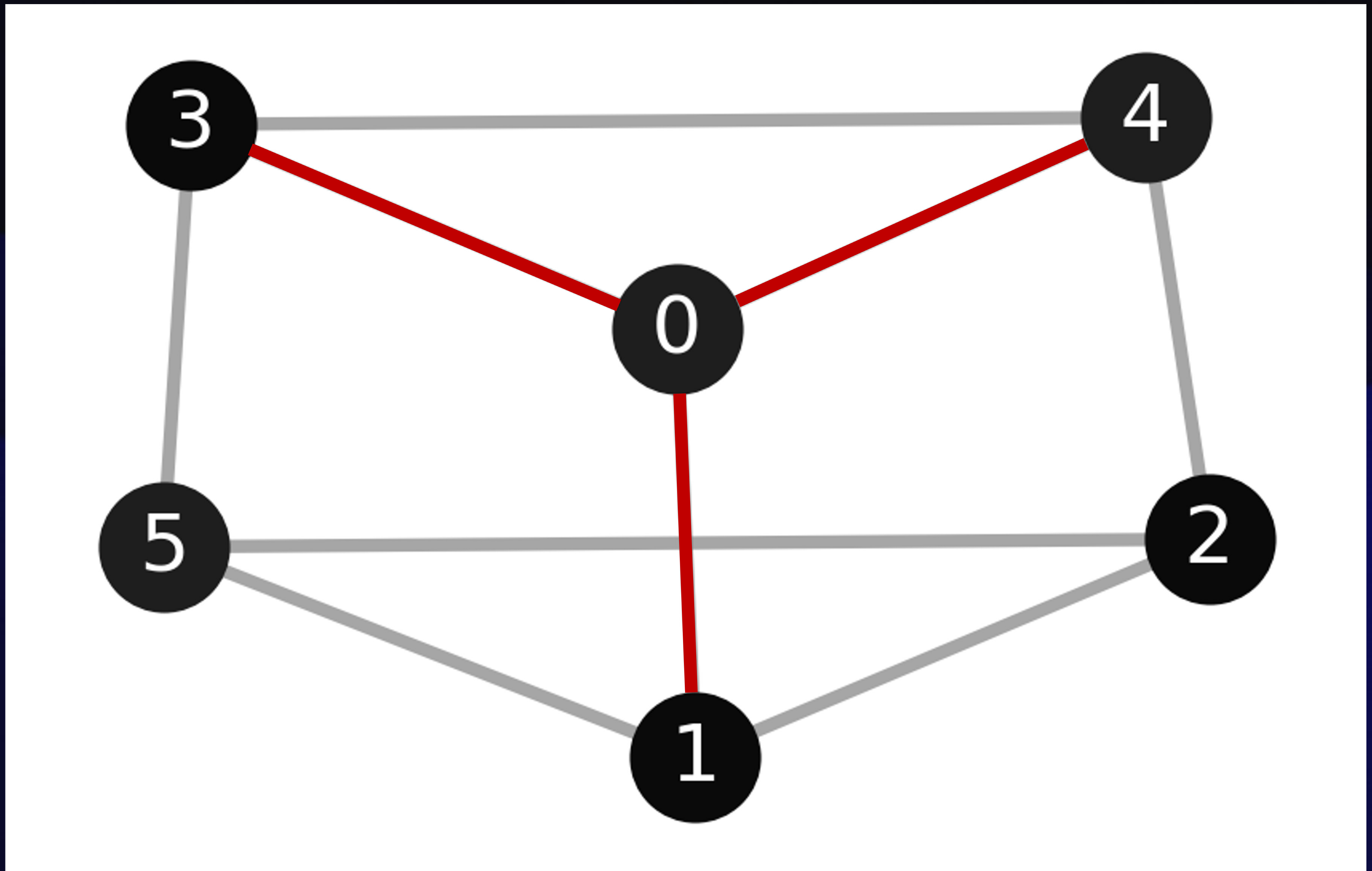
Problem	Graph/Hypergraph	Algorithm	Performance
MaxCut	2-regular	QAOA_p [12, 138, 159]	$\geq \frac{2p+1}{2p+2}$
	3-regular	QAOA_1 [12]	≥ 0.6924
		QAOA_2 [160]	≥ 0.7559
		QAOA_3 [160]	≥ 0.7924
	D-regular with girth > 3	QAOA_1 [138]	$\geq \frac{1}{2} + \frac{0.303}{\sqrt{D}}$
		1-step threshold algorithm [95]	$\geq \frac{1}{2} + \frac{0.335}{\sqrt{D}}$
	D-regular with girth > 5	QAOA_2 [161]	$\geq \frac{1}{2} + \frac{0.407}{\sqrt{D}}$
		2-step threshold algorithm [162]	$\geq \frac{1}{2} + \frac{0.417}{\sqrt{D}}$
		Any 1-local algorithm [161]	$\leq \frac{1}{2} + \frac{1/\sqrt{2}}{\sqrt{D}} \approx \frac{1}{2} + \frac{0.707}{\sqrt{D}}$
	D-regular with girth $> 2p+1$	p-local ALR algorithm [162]	$\geq \frac{1}{2} + \frac{2/\pi}{\sqrt{D}} \approx \frac{1}{2} + \frac{0.6366}{\sqrt{D}}$
		QAOA_p [163]	$\geq \frac{1}{2} + \frac{0.6408}{\sqrt{D}}$ at $p = 11$
		Gaussian wave process [164]	$\geq \frac{1}{2} + \frac{c}{\sqrt{D}}$ with $c > 2/\pi$ not optimized

2-layer QAOA for 3-regular graphs!

Reference???

But what is a 3-regular graph?

And why do we need it?

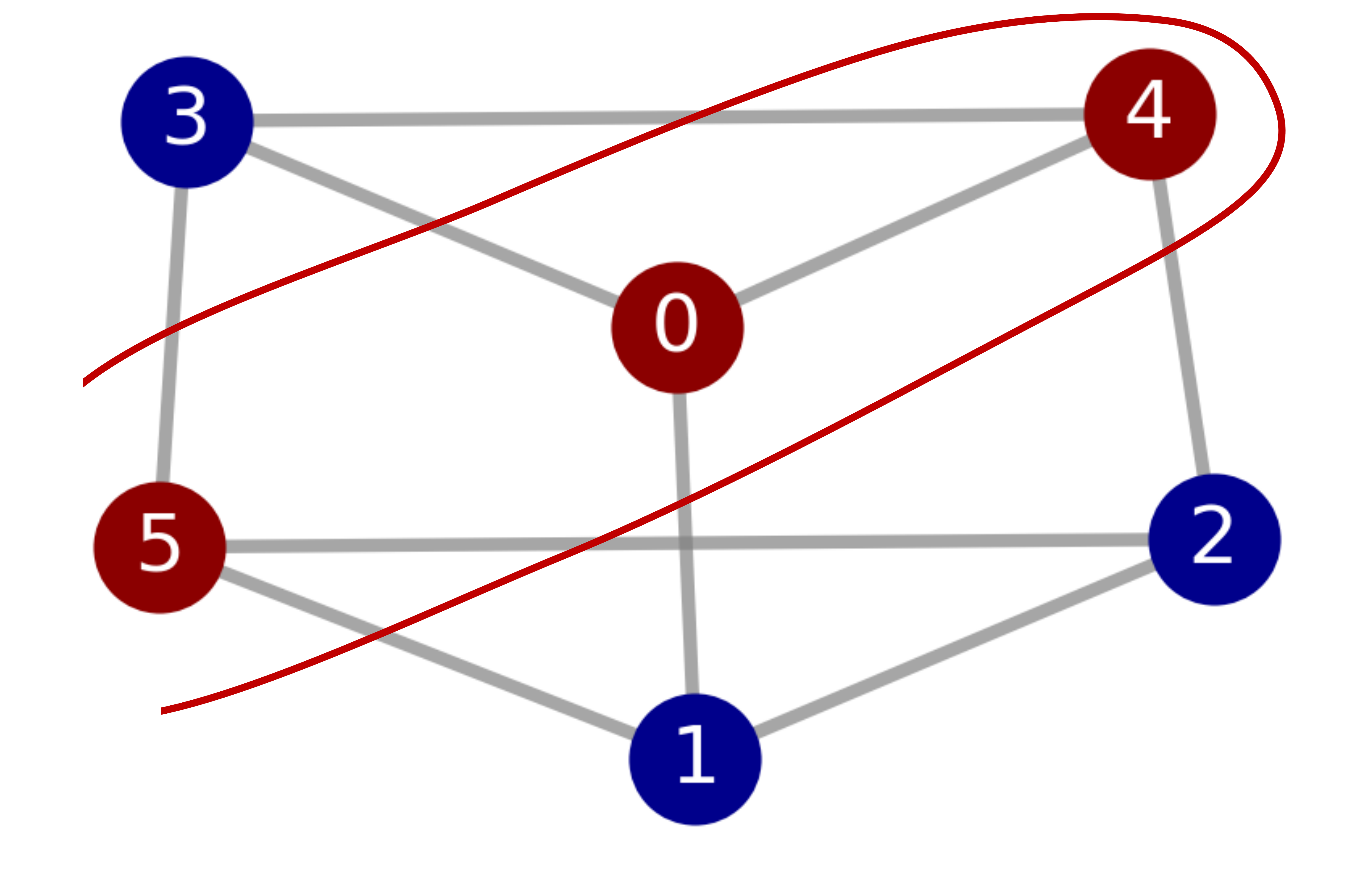


3-regular graph with 6 nodes

- Each node has 3 connections
- Cubic graph

Can we cut a graph?

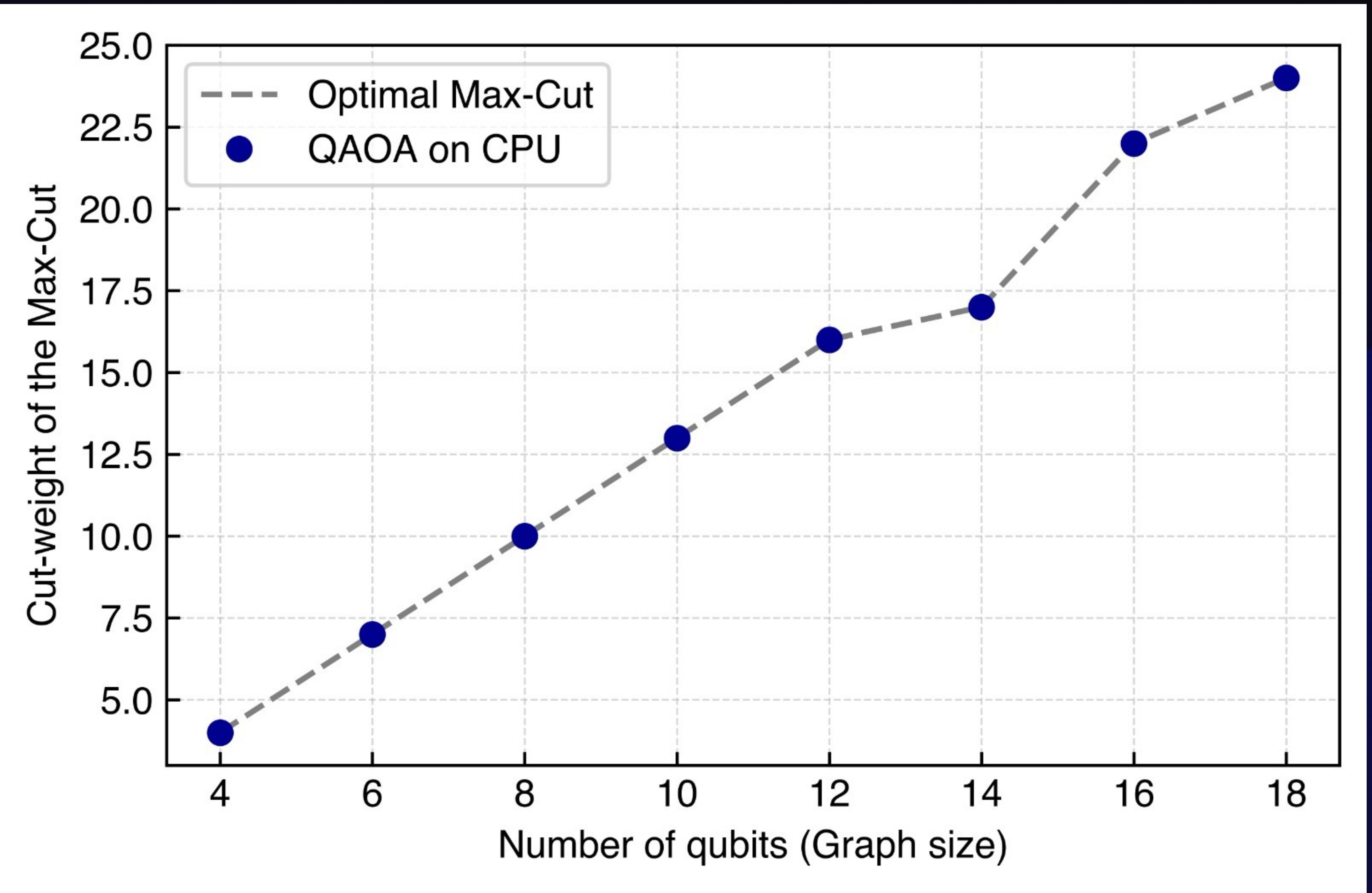
Knifes out



3-regular graph with 6 nodes and highlighted maxcut

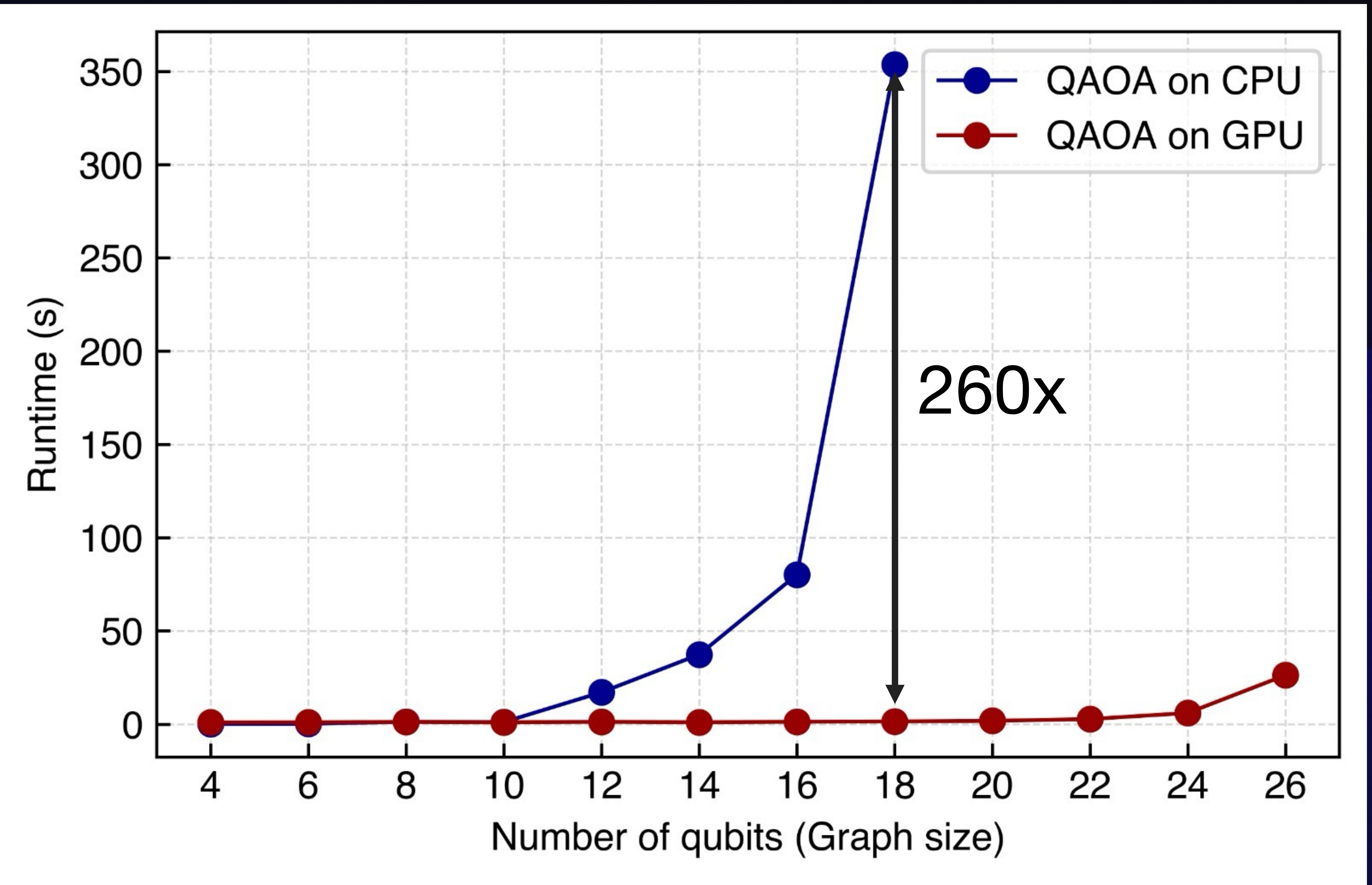
- Define cut weight $\mathrm{CW} = \#edges\ \ cut$
- Here: $\mathrm{CW} = 7$

Is QAOA any good?



- Compare CW_{QAOA} with $CW_{optimal}$
- 100 % accuracy!

How long does it take?



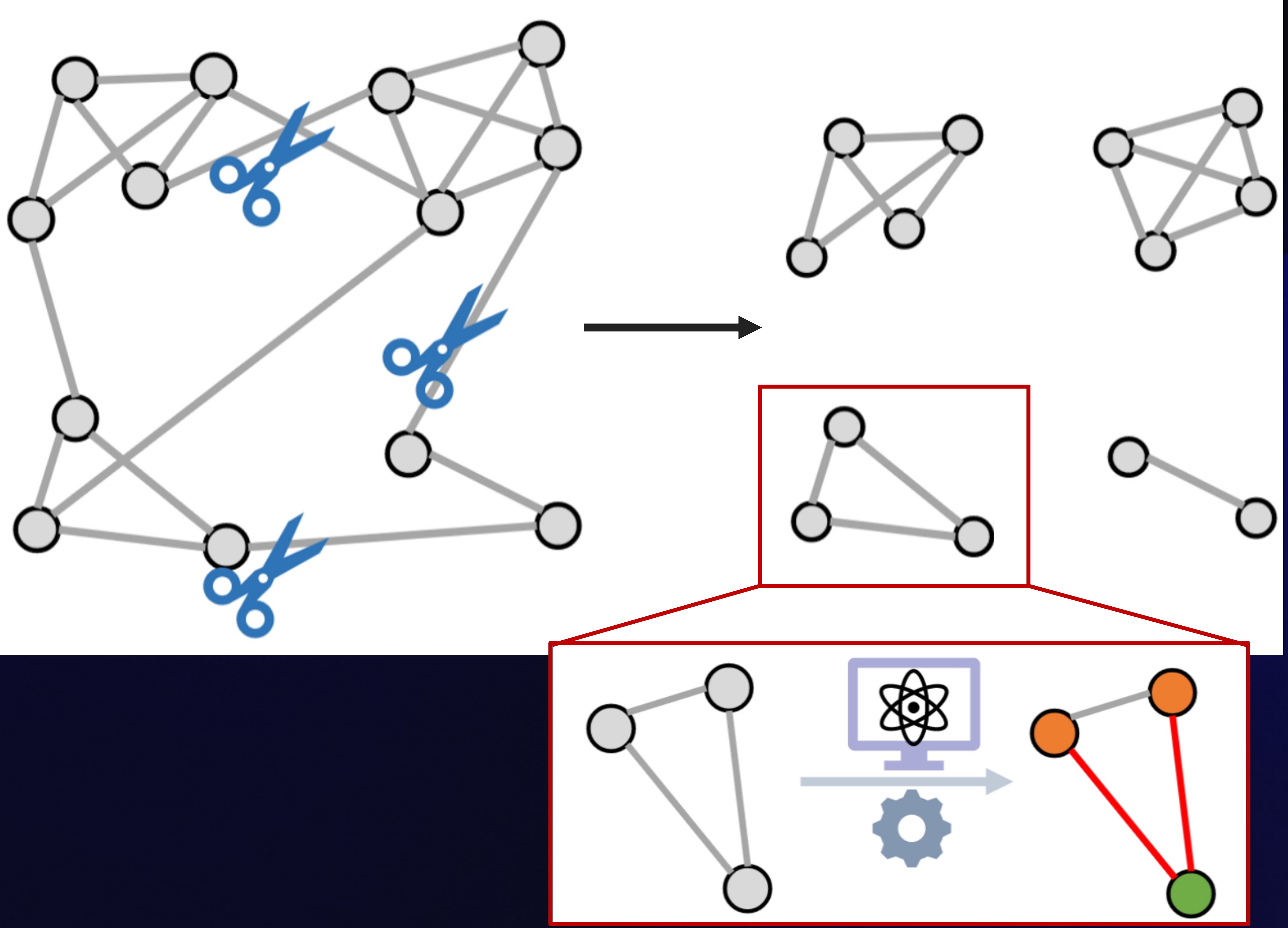
Runtime for increasing qubit number

- CPU breaks down for >18 qubits
- Enterprise GPU (A100) doesn’t break a sweat

Let’s divide and conquer

What do we divide?

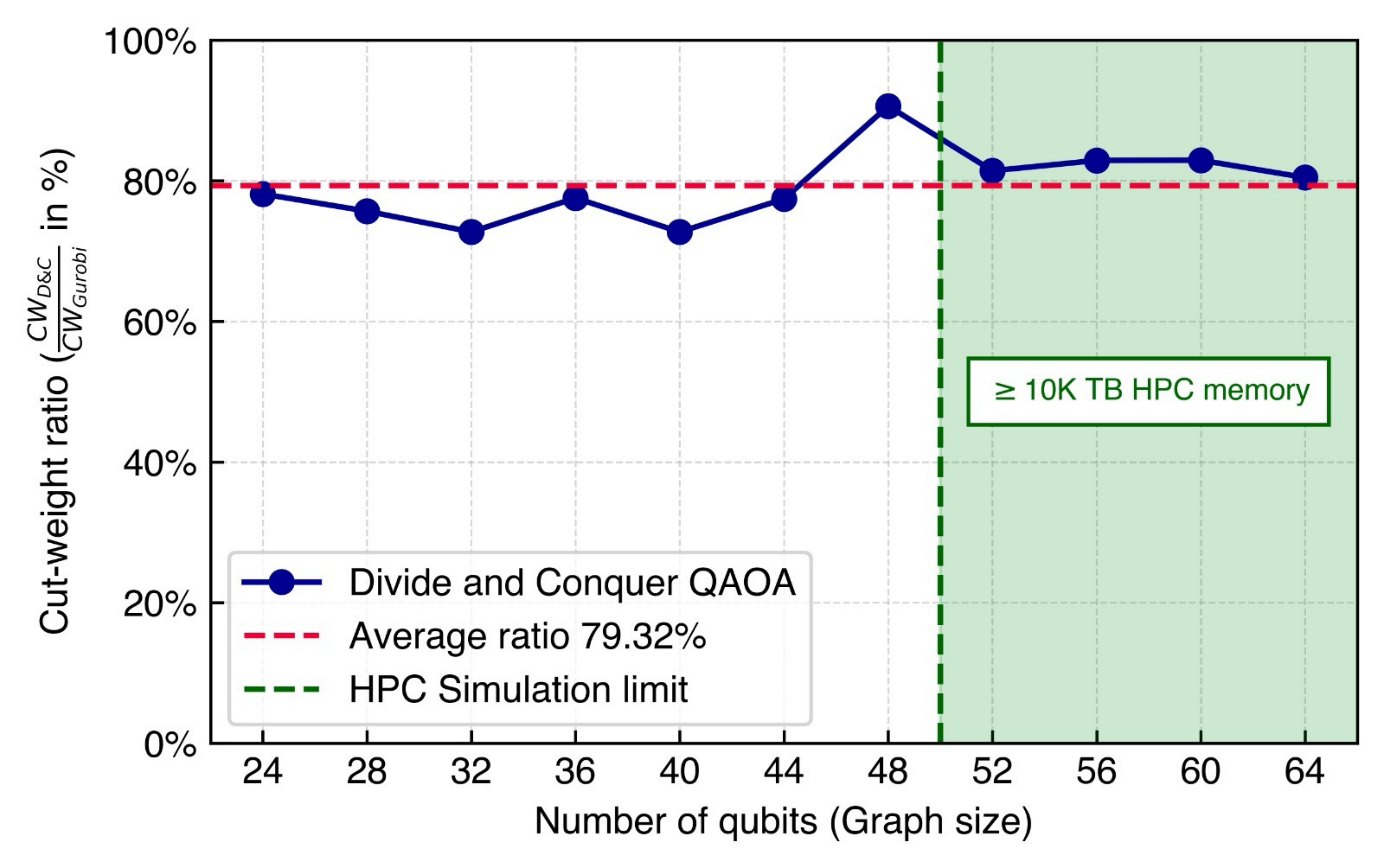
And what do we conquer?



- Divide large graph
- Conquer subgraphs individually

Performance of DAC

Is this any good?



- DAC > 80% of optimal cut
- HPC memory to store qubits

BACKUP Slides

Comparison of different partitions of graphs