Transformers can learn pairwise—but not three-wise—functions

Clayton Sanford

Joint work with Daniel Hsu and Matus Telgarsky

Transformer architecture

What is it?

Transformer architecture

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- **Self-attention unit:**
  $f(X) = \mathrm{softmax}(XQK^T X^T)XV$ for input $X \in \mathbb{R}^{N \times d}$, model parameters $Q, K, V \in \mathbb{R}^{d \times m}$.

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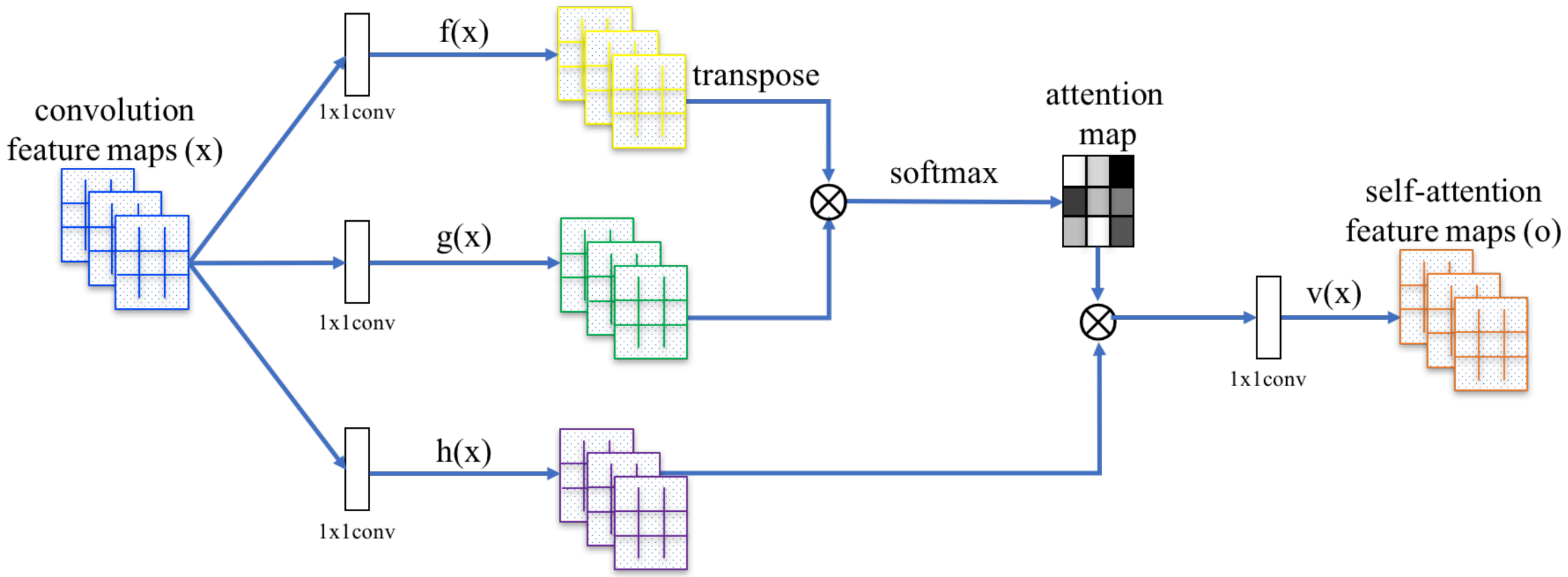

Source: https://lilianweng.github.io/posts/2018-06-24-attention/

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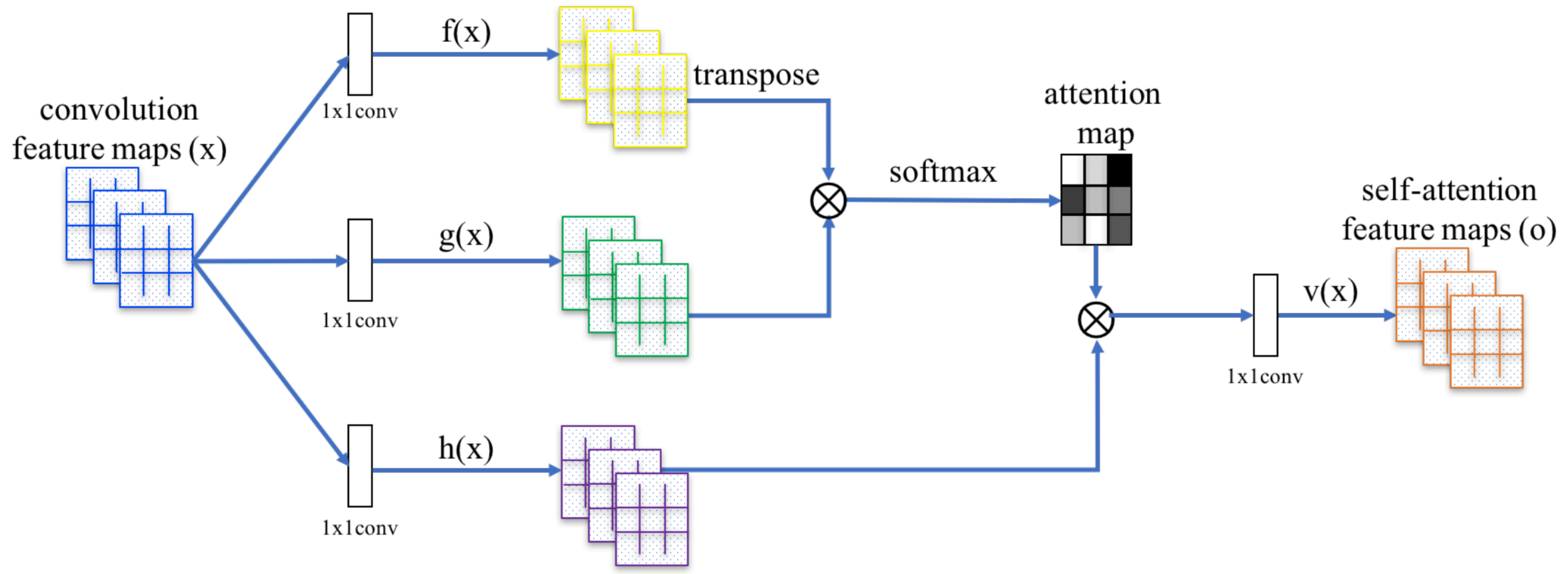

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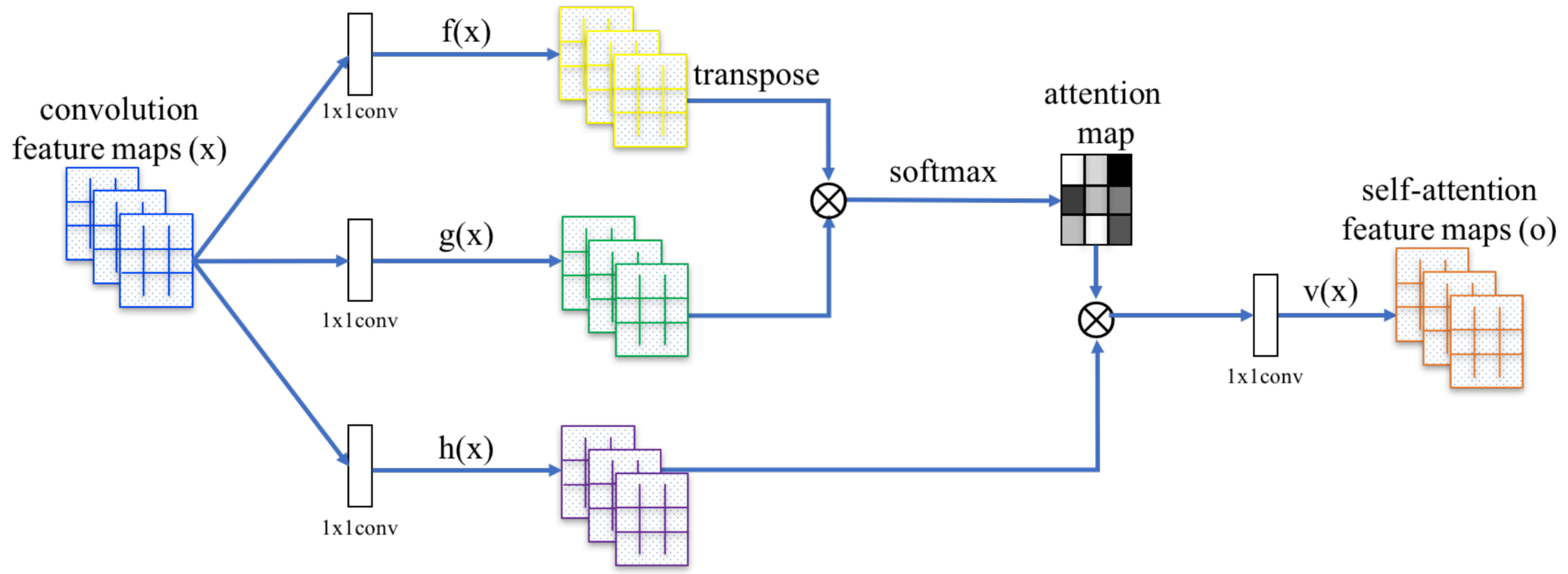

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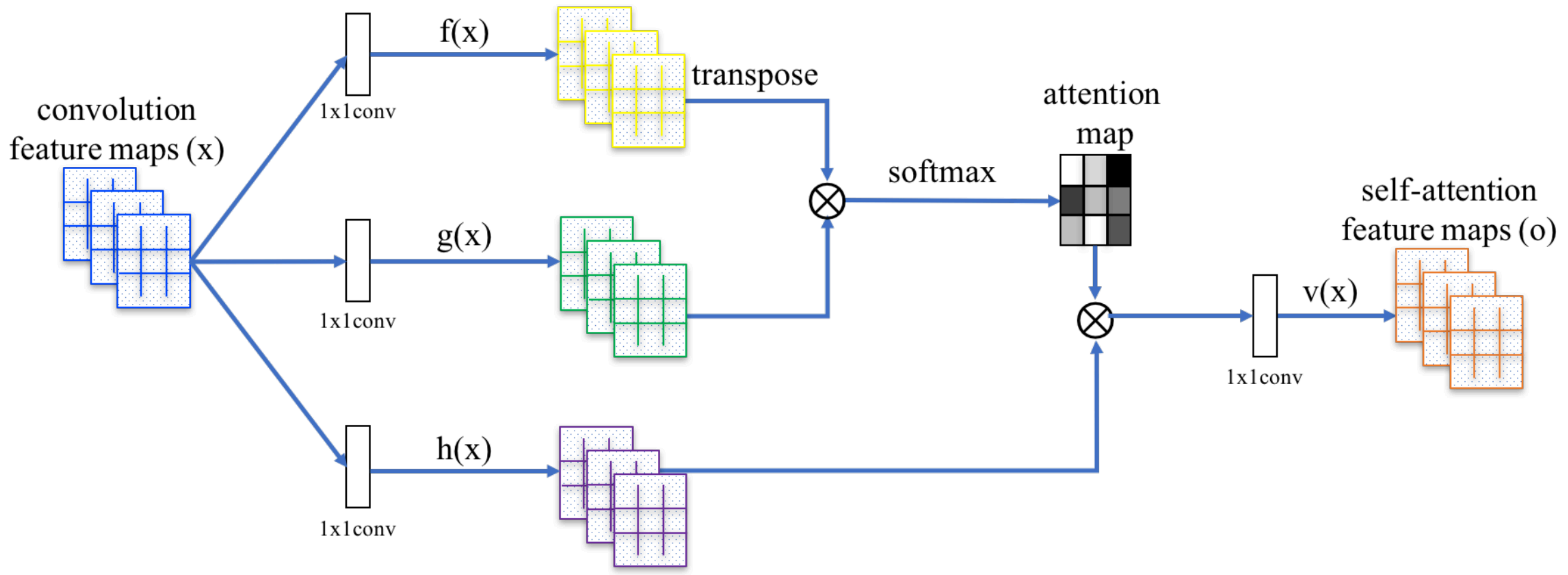

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- **Attuned to pairwise linguistic structure**: self-attention encodes syntactic and semantic linkages between words*

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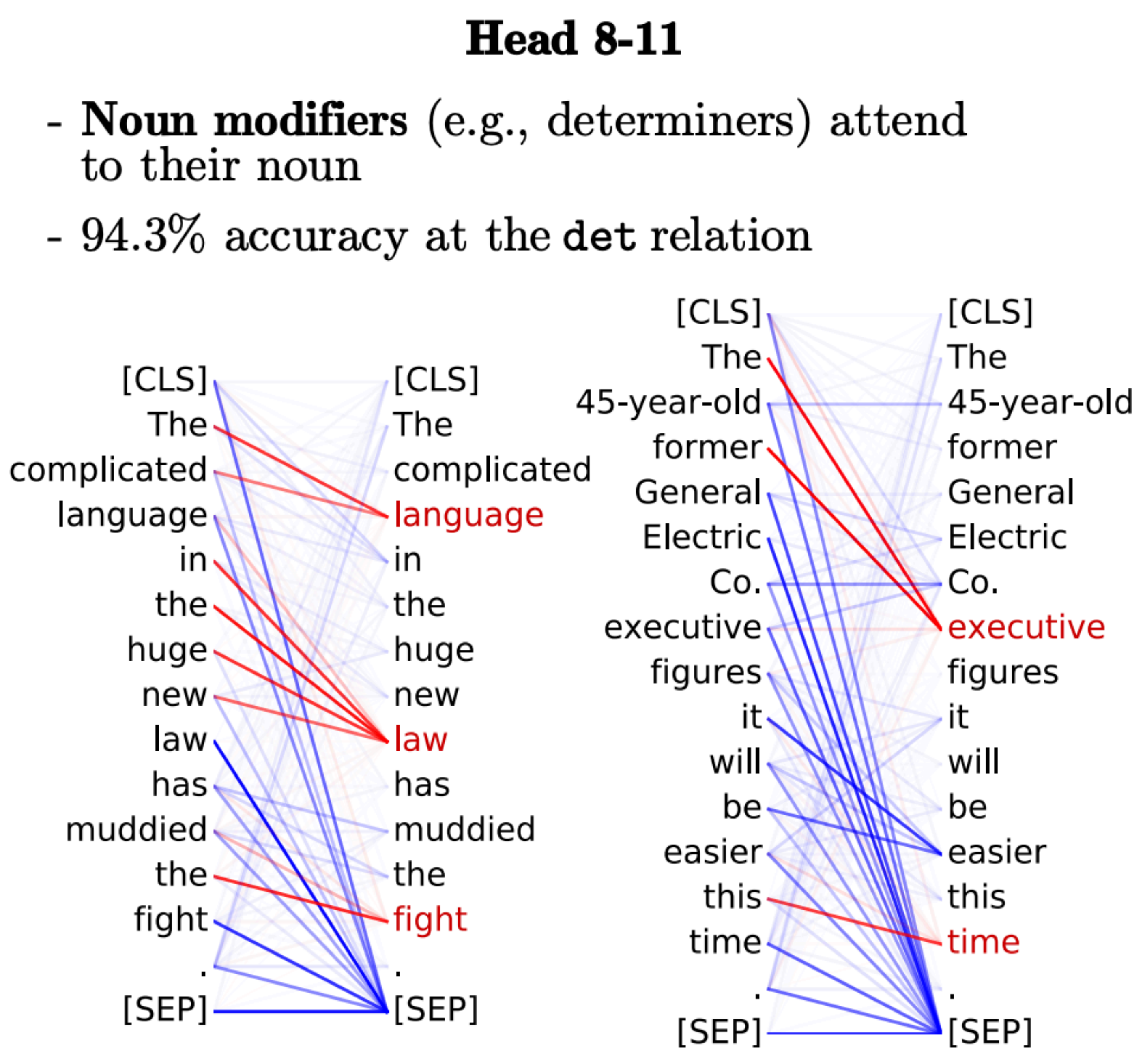

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Head 8-11

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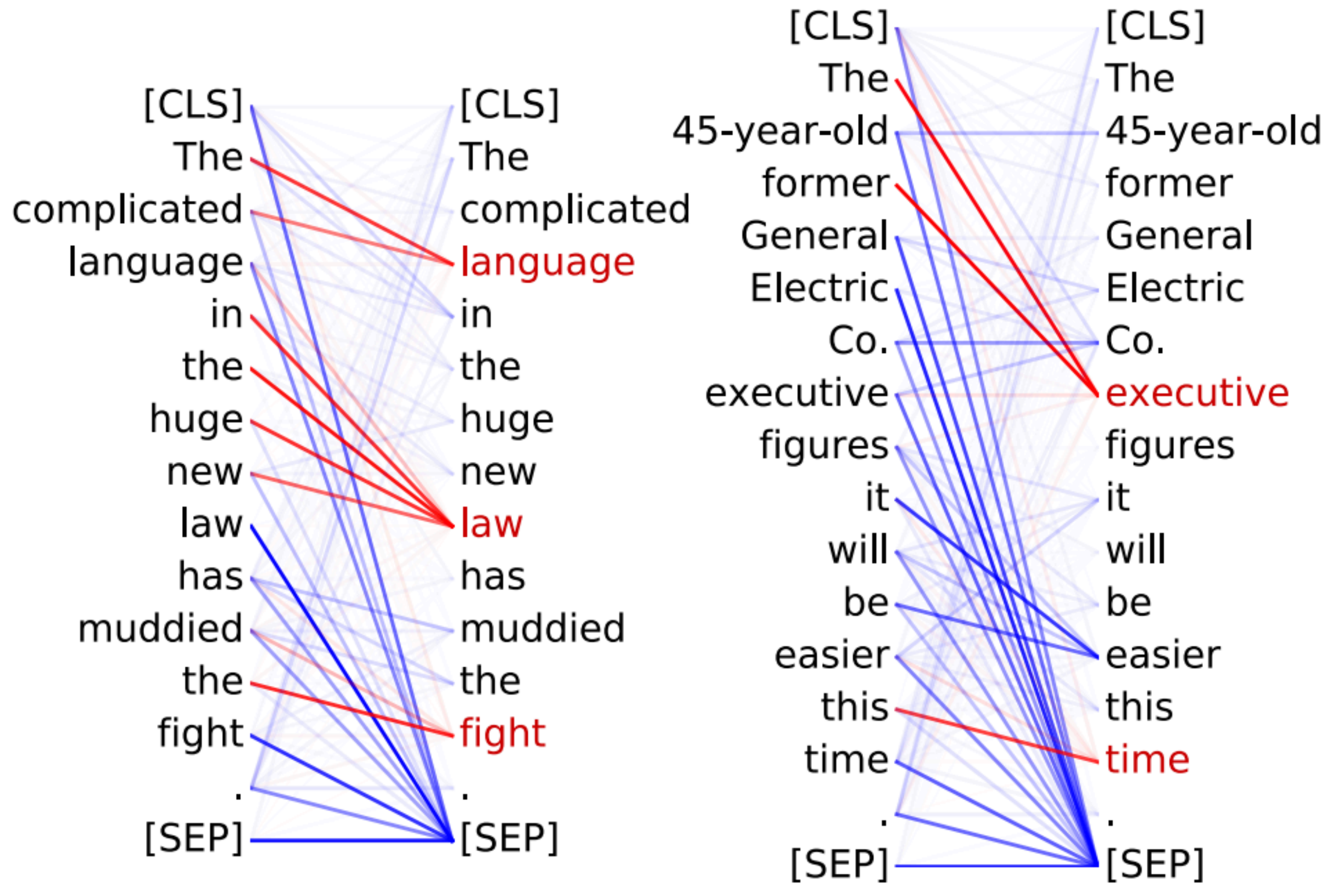

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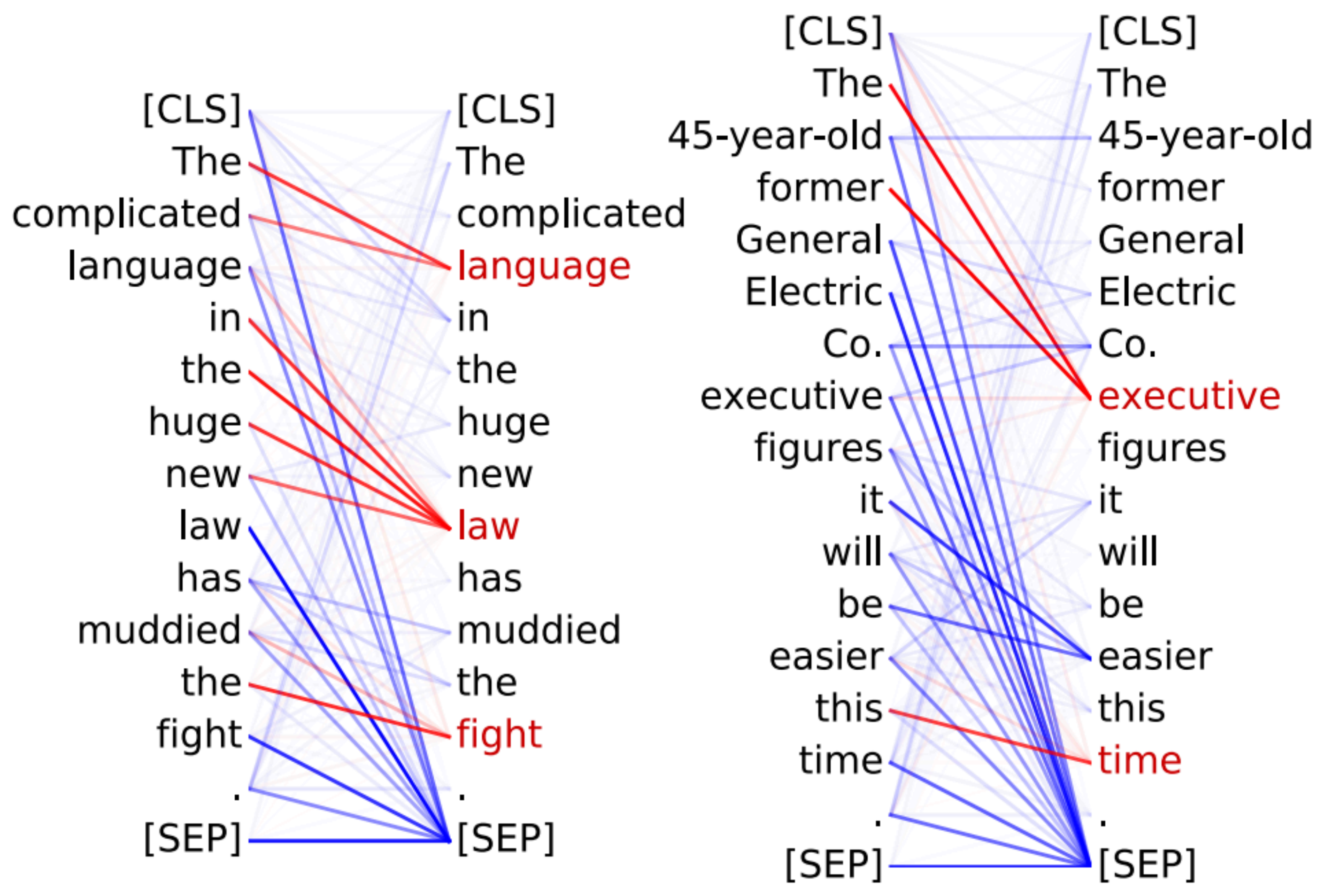

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Model	Context length (N)	#layers (D)	#heads (H)	*#param self-attn* (m)	*#param MLP* (k)
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  $\Longrightarrow$ **unlimited element-wise computational power**

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- #MLP params $\gg$ #self-attention params $\Longrightarrow$ key representational bottleneck as limitations on pairwise communication

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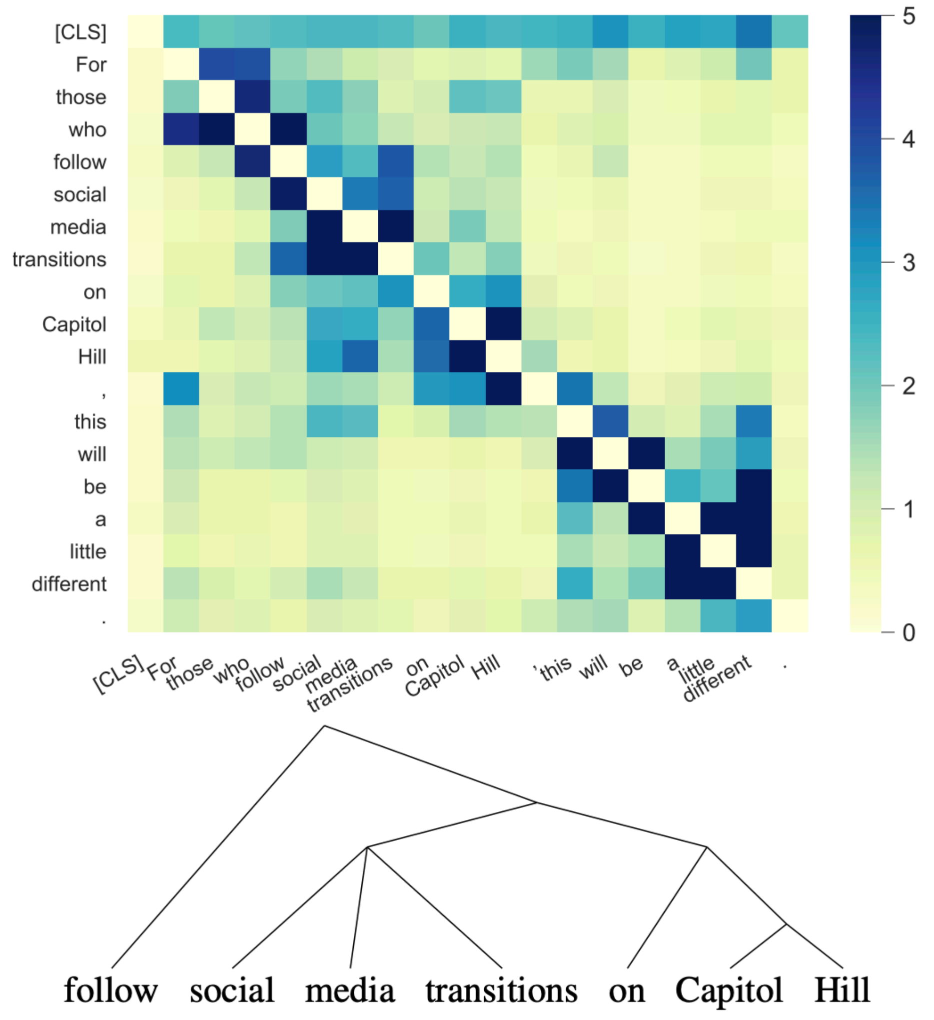

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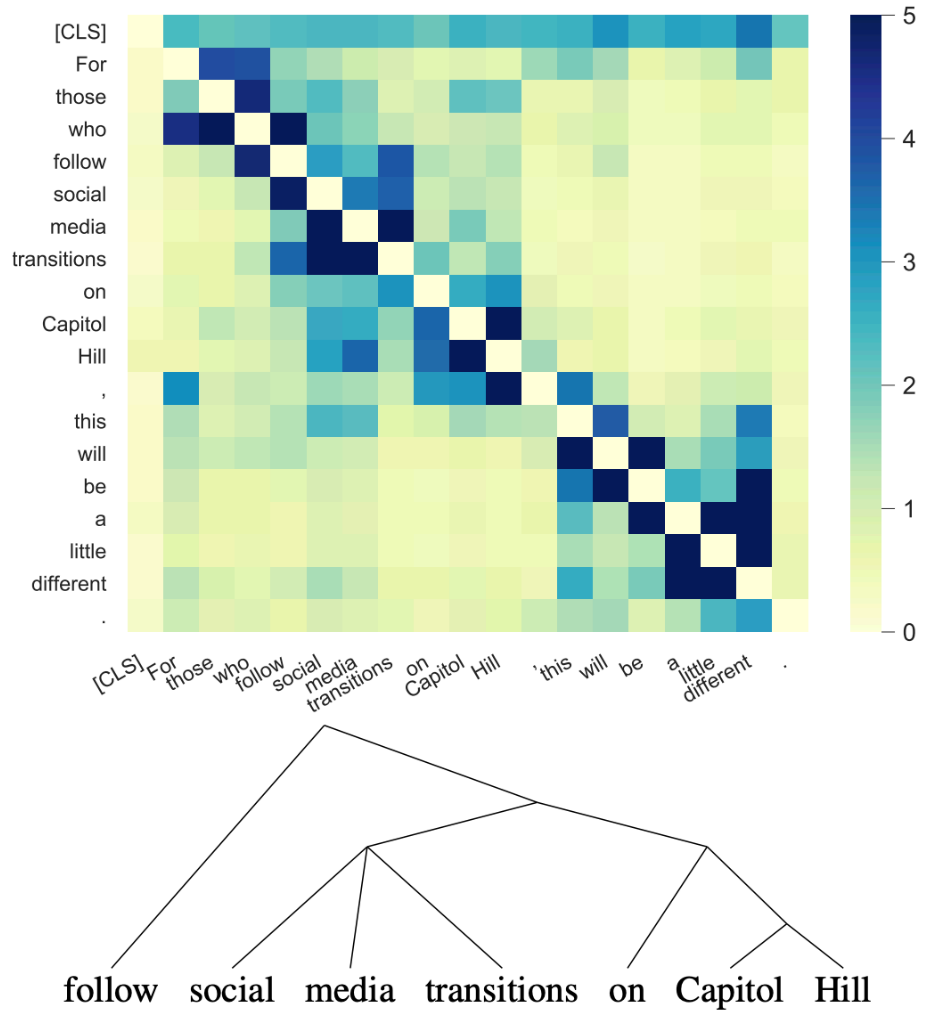

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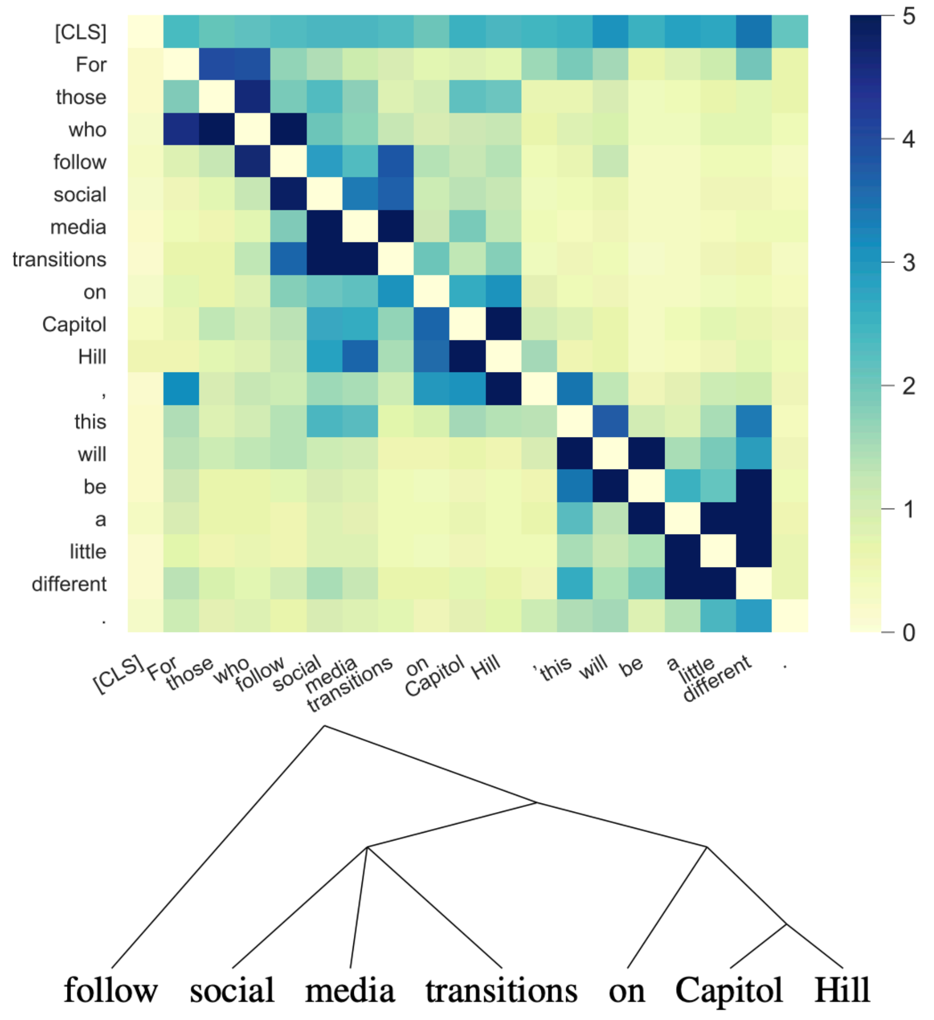
- Apply communication complexity to obtain matching bounds
- How apt is the “sparse pairwise connectedness” framework for understanding language?



- Are there practical “intrinsically three-wise” learning tasks where modern transformers fail?

Our Results

Formulation & bounds

- $\mathrm{PairID}(X) = 1\{\exists j: \ x_i + x_j = 0\}_{i \in [N]}$
- $\mathrm{TriID}(X) = 1\{\exists j_1, j_2: \ x_i + x_{j_1} + x_{j_2} = 0\}_{i \in [N]}$

Result	Target	Architecture	Bound
[+]	PairID	Self-attention unit, MLP input	$m = O(1)$
[-]	TriID	Multi-headed attention, MLP input	$\max(H, m) \geq N^{\Omega(1)}$
[-]	Modified TriID	Full transformer	$\max(D, H, m) \geq N^{\Omega(1)}$
[+]	TriID	"Three-wise tensor self-attention unit"	$m = O(1)$

Thank you!

Want to discuss or learn more?
Check out the poster at 2pm.