

Iteratively Reweighted Algorithms for Compressive Sensing

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1 Introduction

Compressive Sensing concept says that sparse signals can be reconstructed exactly with few measurements. Since MRI acquisition is a time consuming process, this technique comes handy in reducing the time required to acquire as well as memory required to store the data. The principle behind this is that we measure few linear combinations of original signal vector in an manner that the measurements are incoherent, which is explained later. The incoherence ensures that info about entire signal is captured in few measurements.

Let ϕ be a measuring matrix of size $M * N$, x be a sparse vector which we seek to measure and $\phi x = b$ be the measured M -dimensional vector of N -dimensional signal x where $M < N$. The reconstructed signal x^* is given by

$$\min_u ||u||_1 \quad (1)$$

subject to $\phi u = b$.

We ideally want to minimize $L - 0$ norm rather than $L - 1$ norm but former being NP-hard, we perform a softer version called Basis Pursuit.

2 Theorem supporting recovery by basis pursuit

- Consider a signal vector f (having length n) with a sparse representation in basis ψ , i.e. $f = \psi\theta$ where $|\theta| \ll n$. ϕ be $M * N$ measurement matrix as mentioned before. If $m \geq C \log(n/\delta) K \mu^2(\psi, \phi)$ (C is some constant) then the solution to the Compressive Sensing problem is exact with probability $1 - \delta$

Here, K denoted $L - 0$ norm aka sparsity of x . C depends on δ but tends to 1 as N tends to ∞ . ϕ can be any of a random fourier submatrix, random Gaussian matrix with mean zero and columns normalized or random Bernoulli matrix with columns normalized etc. All these constructions are incoherent with any orthonormal basis.

3 Iteratively Reweighted Least Squares(IRLS) Algorithm

The approach is to replace the l^p objective function in (1) by a weighted l^2 norm

$$\min_u \sum_i w_i u_i^2 \quad (2)$$

subject to $\phi u = b$.

The weights are iteratively computed so as to approximate in first order sense the l^p objective, given by $w_i = |u_i^{(n-1)}|^{p-2}$. The solution to (2) is then explicitly given by

$$u^{(n)} = Q_n \phi^T (\phi Q_n \phi^T)^{-1} b \quad (3)$$

where Q_n is the diagonal matrix with entries $1/w_i$. To overcome the case where w_i shoots up due to u_i being zero, we use damping approach as follows

$$w_i = ((u_i^{(n-1)})^2 + \epsilon)^{p/2-1} \quad (4)$$

4 IRL1 variant of previous algorithm

The entire approach remains same except for the objective function being

$$\min_u \sum_i w_i |u_i| \quad (5)$$

subject to $\phi u = b$. The weights are given by

$$w_i = (|u_i^{(n-1)}| + \epsilon)^{p-1} \quad (6)$$

5 Dataset

We used the Cardiac MRI dataset from York University to test our implementation. The dataset consisted of 7098(of size 256*256 each) images of 33 persons in various orientations. We randomly chose one image per person (total 33 images). Since the data was given as images, we represented the data in sparse basis like Fourier and DCT for our experiments.

We are ideally supposed to reshape the images to 65526*1 images and acquire combinations of sparse data. But here, size of ϕ and Q_n is so huge and not supported by MATLAB. Hence, we preferred to reconstruct each 256*1 column separately to yield a whole image.

6 Implementation

- For each image, we first calculated the sparse representation in DCT basis.
- We chose ϕ to be random bernouli matrix with $p=0.5$ (symmetric) and normalized the columns.
- Then, we calculated $b = \phi u$.
- We then initialized $u^{(0)}$ to minimum l^2 norm solution, i.e . psuedo inverse solution.
- Performed the iterations varying p among 0,1 and algorithm among IRLS and IRL1

7 Results

For 2/3 compression, Figure 1 and 2 show some of the good and bad reconstructions. For 1/2 compression, Figure 3 and 4 show some of the good and bad reconstructions.

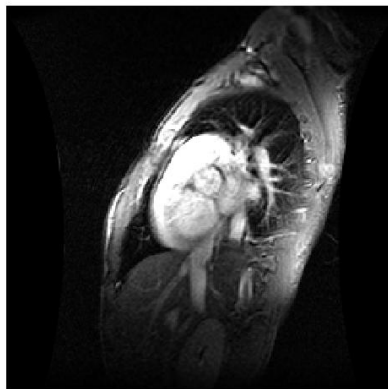
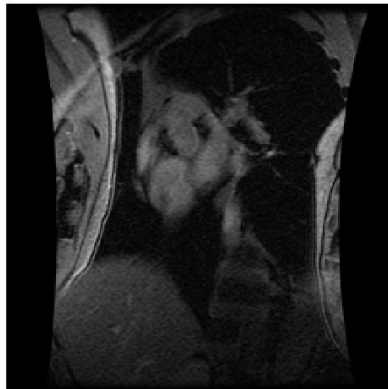


Figure 1: Good Reconstructions for 2/3 Compression

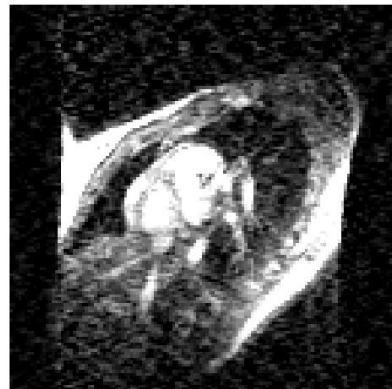
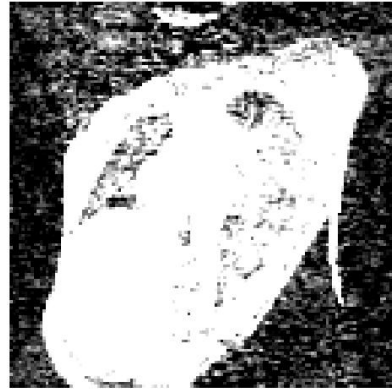


Figure 2: Bad Reconstructions for 2/3 Compression

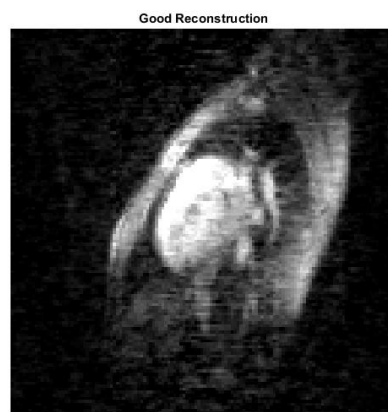
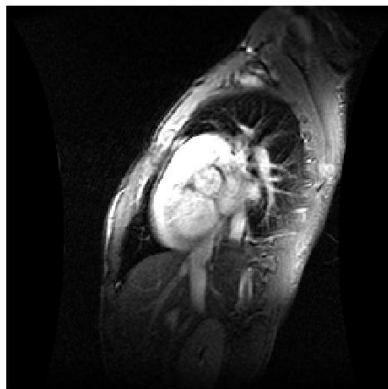
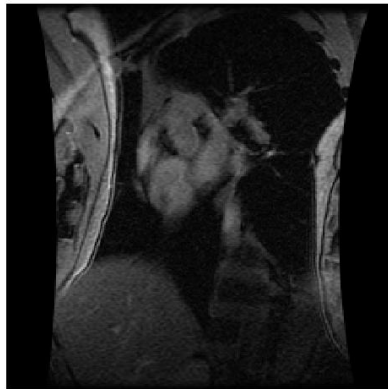


Figure 3: Good Reconstructions for 1/2 Compression



Figure 4: Bad Reconstructions for 1/2 Compression

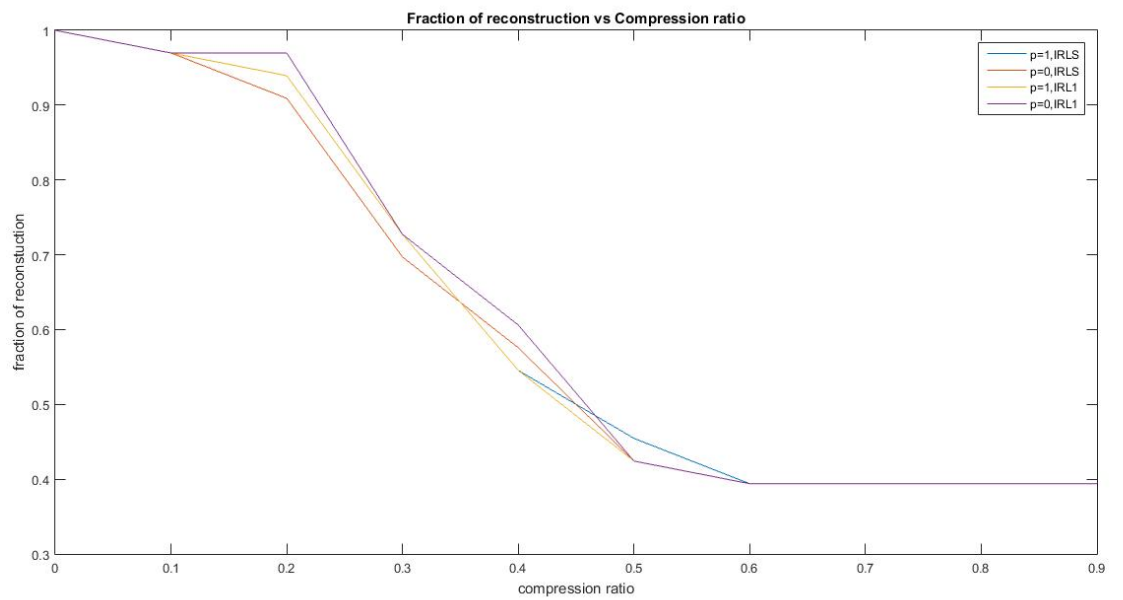


Figure 5: Reconstruction vs Compression