PHYS 64 Final Project: Simulating Quantum Errors

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1 Abstract

One of the biggest hurdles facing quantum computing is the current high error rates in quantum computers. There are two main types of errors in quantum computing: (1) quantum gates being imperfect (infidelity); (2) qubits may develop entanglement with its environment (decoherence). In this project, I will introduce errors in quantum computing, build composite noise models, then apply noise models to quantum circuits to test their impact.

2 Introduction

Quantum computing is one of the key technologies that is developing at the intersection of computer science and physics. The concept of quantum computing revolves around leveraging the principles of quantum mechanics, such as superposition and entanglement, to perform computations in a fundamentally different manner than classical computers.

In 1982, Richard Feynman daringly predicted that the very foundation of physics - quanta - could be used to build computers [1]. In 1994, Peter Shor developed a quantum algorithm for factoring large numbers exponentially faster than the best known classical algorithms, posing a significant threat to widely used cryptographic systems [2]. This breakthrough demonstrated the potential superiority of quantum computers over classical computers for certain tasks.

In the following years, several quantum algorithms, such as Grover's search algorithm and quantum error correction codes, were developed, further fueling the interest in quantum computing [3, 4]. Experimental efforts also began to build actual quantum computers, with the first few-qubit quantum computers emerging in the late 1990s and early 2000s. Since then, research in quantum computing has accelerated, with academic institutions, technology companies, and government agencies investing heavily in the field. In 2019, Google first achieved quantum supremacy (the ability of quantum computers to solve problems significantly faster than classical computers) with their superconducting quantum computer [5]. Google's team demonstrated that Sycamore could perform a specific computation in 200 seconds that would take the world's most powerful classical supercomputer, IBM's Summit, approximately 10,000 years to complete.

However, we are far from reaching complete quantum supremacy. One of the biggest hurdles facing quantum computing today is quantum errors. Qubits are highly sensitive to various types of noise and environmental interactions, which can cause them to lose their quantum states and introduce errors in computations [4]. These errors can quickly accumulate and corrupt the output of quantum algorithms, limiting the potential of quantum computers. Addressing quantum errors is crucial for building reliable and scalable quantum computers capable of solving practical problems.

Quantum error correction techniques have been developed to detect and correct errors in quantum computations, analogous to classical error correction codes used in digital communications and data storage. These techniques involve encoding qubits into logical qubits using additional ancilla qubits, allowing for the detection and correction of errors during computation. However, implementing robust quantum error correction remains a significant challenge, as it requires a large number of physical qubits and introduces additional complexity and overhead.

3 Quantum Computing Basics

Quantum computers are built from quantum bits – qubits, that have states $|0\rangle$ and $|1\rangle$ as their binary bits. Unlike classical computers where states are either 0 or 1, a generic single qubit is in a superposition of the two states:

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle ,$$

where $\alpha, \beta \in \mathbb{C}$. This quantum parallelism makes quantum computers much more powerful than normal computers. The probability of measuring the qubit in the states $|0\rangle$ and $|1\rangle$ are $|\alpha|^2$ and $|\beta|^2$ respectively. We can write $|\psi\rangle$ as a column vector $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$. In general, an arbitrary number of qubits can be entangled together and have a state

$$|\psi\rangle_n = c_0 |0\rangle + c_1 |1\rangle + \cdots + c_{2^n-1} |2^n - 1\rangle + c_{2^n} |2^n\rangle ,$$

where $|p\rangle$ means the state of the binary string representing the number p. This can be written as a column vector $\begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_{2^n-1} \\ c_{2^n} \end{pmatrix}$. From this representation, we can see that n qubits can store 2^n pieces of information - exponentially more than classical computers.

A more universal way of representing quantum states is through the density matrix representation. A density matrix is the weighted sum (by probability) of the outer products of possible quantum states. For a single qubit in one possible state, it looks like

$$\rho = \begin{pmatrix} \alpha^* & \beta^* \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix} .$$

To alter a quantum state, we apply quantum gates onto the qubits. A quantum gate is represented by a unitary matrix (a matrix equal to its conjugate transpose) that acts on the column vector representation of quantum states.The two most important one-qubit states are (1) the X gate, which switches the coefficients of $|0\rangle$ and $|1\rangle$: $\hat{X} |\psi\rangle = \beta |0\rangle + \alpha |1\rangle$; (2) the Z gate, which negates the coefficient in front of $|1\rangle$: $\hat{Z} |\psi\rangle = \alpha |0\rangle - \beta |1\rangle$. In general, any single-qubit gate can be composed as a linear combination of the identity gate, X gate, Z gate, and XZ gates (i.e. acting a Z gate then an X gate). Another way of constructing an arbitrary single-qubit gate is through the universal rotation matrices $\hat{U}_3(\theta, \phi, \lambda) = \begin{pmatrix} \cos\theta/2 & -e^{i\lambda} \sin\theta/2 \\ e^{i\phi} \sin\theta/2 & e^{i(\phi+\lambda)} \cos\theta/2 \end{pmatrix}$.

4 Quantum Errors

Quantum errors can occur everywhere - from manipulation to measurement to thermal fluctuations to even just the pure existence of quanta themselves. I will introduce the two basic types of errors: the bit-flip error and the phase-flip error. The bit-flip error is an accidental X gate on a qubit and the phase-flip error is an accidental Z gate on a qubit. The bit-flip and phase-flip errors are important because they form a complete basis for all possible single-qubit errors, since an arbitrary quantum gate can be expressed as

$$\hat{U} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{a+d}{2}\hat{I} + \frac{a-d}{2}\hat{Z} + \frac{b+c}{2}\hat{X} + \frac{c-b}{2}\hat{X}\hat{Z}.$$

Ideally, a qubit would pass through a noisy channel, be transformed by some noise operator $\hat{U}_{\text{noise}}$.

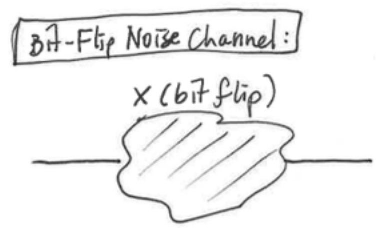

Figure 1: Quantum Noise Channel (taken from Prof. Lynn's textbook)

One of the most realistic models for quantum errors is the T1/T2 relaxation model [6]. T1 and T2 relaxations are generalizations of bit-flip and phase-flip errors explored earlier − T1 relaxation flips a qubit while T2 relaxation dephases it. Specifically, T1 relaxation refers to energy relaxation, or the spontaneous decay of a qubit from its excited state to the ground state. Energy relaxation is caused by the interaction of a qubit with its environment and lead to energy dissipation and decoherence. A lot of qubit systems such as superconducting qubits and trapped ion qubits use the ground and excited states as their $|0\rangle$ and $|1\rangle$, meaning that energy relaxation is a nontrivial problem to solve for quantum computing. Under T1 relaxation, the density matrix of a qubit initially at $\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{01}^* & \rho_{11} \end{pmatrix}$ changes as

$$\rho(t) = \begin{pmatrix} \rho_{00} + (1 - e^{\Gamma_1 t})\rho_{11} & e^{-\Gamma_1 t/2}\rho_{01} \\ e^{-\Gamma_1 t/2}\rho_{01}^* & e^{-\Gamma_1 t}\rho_{11} \end{pmatrix} \rightarrow_{t\to\infty} |0\rangle\langle 0|\,.$$

As the equation above suggests, the terms not on the upper left corner decay exponentially. The term on the upper left corner of the density matrix represents the density of states in the ground state, and the terms on the other parts represent excited states or superpositions. Thus T1 relaxation causes a qubit to exponentially decay to ground state.

T2 relaxation refers to dephasing, or the loss of phase coherence between the qubit and its environment. Dephasing is caused by interactions with other qubits or with environmental fluctuations such as magnetic or electric field noises. Under T2 relaxation, the density matrix of a qubit evolves

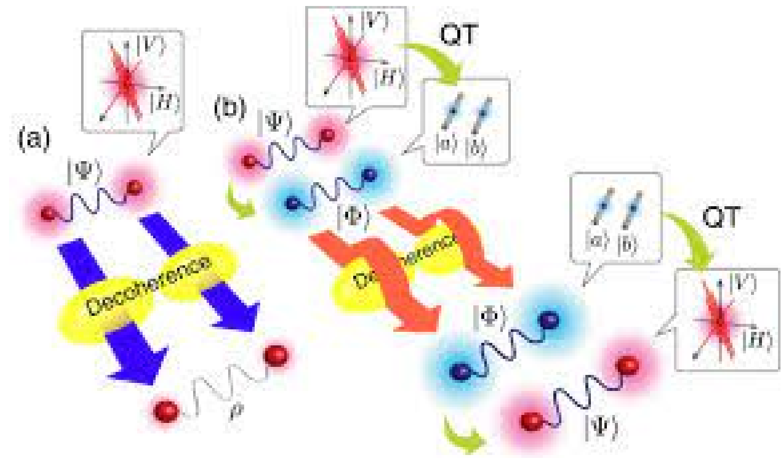

Figure 2: Quantum Decoherence

as

$$\rho(t) = \begin{pmatrix} \rho_{00} & e^{-\Gamma_2 t}\rho_{01} \\ e^{-\Gamma_2 t}\rho_{01}^* & \rho_{11} \end{pmatrix} \rightarrow_{t\rightarrow\infty} \begin{pmatrix} \rho_{00} & 0 \\ 0 & \rho_{11} \end{pmatrix}.$$

Longer T1 and T2 times are generally desirable. Longer T1 times mean that the qubit can maintain its coherence (excited state) for a longer period of time, allowing for more operations to be performed before information is lost in relaxation. Longer T2 times indicate that the qubit can maintain its phase coherence. Phase coherence is important because (1) it allows for entanglement between different qubits and thus allows for further exploitation of the exponential advantage; (2) a lot of quantum algorithms depend on interference between different states, in which case phase is important.

References

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