Problem 1: for function $f(x) = e^{x^2}$, we know some values as

X_i	0	1/6	2/6	3/6	4/6	5/6	1
$f(x_i)$	1.0	1.0281672	1.1175191	1.2840254	1.5596235	2.0025962	2.7182818

Using the composite trapezoidal rule and composite Simpson rule (no additional middle points) on above values to calculate the integral $\int_0^1 e^{x^2} dx = 1.462651745907182...$ and find the error.

Problem 2: for above problem, using the Romberg method based on composite trapezoidal rule results T_1, T_2, T_4, T_8 , we can listed all extrapolation values.

Problem 3: Using composite Gaussian quadrature to calculate $\int_0^1 e^{x^2} dx$ with $h = \frac{1}{6}$, here two Gauss points are selected on every subinterval. (hint: Coordinate transformation should be used)

说明:由于下周起无法使用实验大楼进行集中上机,请大家将上面三道题加大计算区间数模后编程实践一下。

即复化梯形, Romberg 算法和复化两点高斯积分方法。