Math Notes

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Contents

1 Statistics

1.1 Discrete Probability Distribution

Let X be a random variable with a finite number of finite outcomes $x_1, x_2, \ldots, x_n$ occurring with probabilities $p_1, p_2, \ldots, p_n$ respectively. The expectation of X is defined as

$$\mu = E[X] = \sum_{i=1}^{n} x_i p_i = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n$$

The variance of X is

$$\sigma^2 = Var(X) = \sum_{i=1}^{n} p_i \cdot (x_i - \mu)^2$$

For a collection of n equally likely values:

$$\mu = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$$

1.2 Continuous Probability Distribution

If the random variable X has a probability density function $f(x)$, and $F(x)$ is the corresponding cumulative distribution function, then

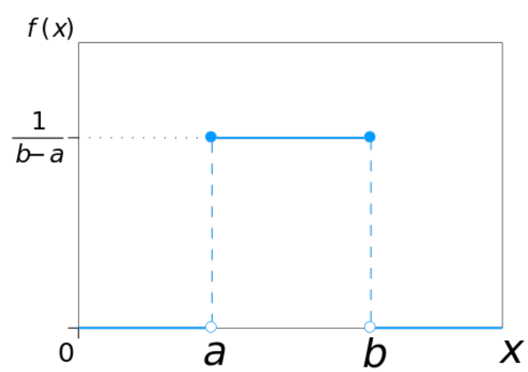
$$P(a \leq X \leq b) = \int_a^b f(x)dx$$

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)dt$$

$$\mu = \int_{\mathbb{R}} xf(x)dx = \int_{\mathbb{R}} xdF(x)$$

$$\sigma^2 = \int_{\mathbb{R}} (x - \mu)^2 f(x)dx$$

1.2.1 Uniform Distribution

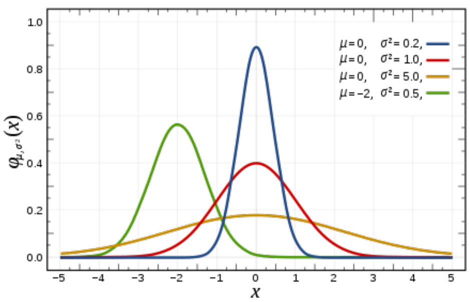


$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x \le b, \\ 0 & \text{for } x < a \text{ or } x > b \end{cases}$$

$$\mu = \tfrac{1}{2}(a+b)$$

$$\sigma^2 = \tfrac{1}{12}(b-a)^2$$

1.2.2 Normal Distribution



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$