Jumps and Diffusive Variance: A Granular Analysis of Individual Stock Returns

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Abstract

Jumps and diffusive changes in stock prices are different ways in which information is reflected in the prices. We use nonparametric methods to decompose returns on individual stocks into jumps and diffusive components. Contrary to the conventional assumption that jump intensity is positively related to diffusive variance, we find abundant evidence that realized jump intensity and diffusive variance are uncorrelated or negatively related for a majority of stocks. The jump-diffusion beta is found to positively contribute to the implied volatility smile of options on individual stocks. We also document a counter-cyclical pattern of realized jump sizes, which challenges the i.i.d. jump size assumption commonly seen in the literature. The findings provide useful guidance on modeling option prices.

1. Introduction

Stock prices change as new information about their future payoffs or discount rates arrives and investors react to such information. At different times for different firms, however, price changes take different forms. At most of the time for most firms, the returns are small in magnitude and frequency, but occasionally, they can be large and abrupt. Based on limited evidence, financial economists have proposed to capture these different patterns with the so-called diffusive variance and jumps respectively in options pricing models, and shown that having jumps is important for pricing of options written on stock indexes and on some large stocks. For broader applications, however, many questions have left unanswered.

In this paper, we address a few issues related to jumps and diffusive variance. First, as the evidence on the phenomenon is mainly obtained on stock indexes and just a handful of large stocks, whether such division between jumps and diffusive variance is necessary for all stocks is unclear. Whether individual stocks exhibit the same pattern as stock indexes is not clear either. As such, our first step is to modify methodologies groomed for indexes to be applied to all individual stocks, in order to measure jumps and diffusive variance. Upon completing that, we gauge the extent to which jump intensity, jump size and diffusive volatility are stochastic.

Second, we examine the assumption made in the options pricing literature that jump intensity and the diffusive variance are positively related over time for a given underlying asset. In the development of options pricing models, stochastic diffusive variance represents a major breakthrough after the Black-Scholes model of options pricing with a constant variance and has been treated as the most important state variable in determining options prices, beside the price of the underlying asset. When jumps are added later to the model and when the jump intensity is found necessarily to be time-varying, it is very natural to specify the conditional jump intensity as an increasing function of the diffusive variance of the underlying asset. In popular affine jump diffusion models, for example, the conditional jump intensity is specified as an affine function of the diffusive variance of the underlying asset with a positive slope coefficient (in non-trivial cases). It seems natural to think that the probability of a jump is high when the diffusive variance is also high. After all, both diffusive variance and jump intensity are measures of the magnitude of possible future price changes.[1] The empirical evidence in Li and Zhang (2016) based on the the S&P 500 index options and a handful of large stocks, however, casts doubt on the assumption of positive correlation between the two characteristics of the stock returns. We extend the inquiry to all stocks traded in the major US exchanges.

Third, based on the findings of the first two issues, we address the issue of how options pricing depends on the covariation between the conditional jump intensity and diffusive variance, controlling for the diffusive variance, jump size, and volatility of volatility. This line of empirical examination has not been conducted explicitly in the literature, even for index options. We collect evidence from a large panel of options on individual stocks. The result can provide guidance to the work on modeling option prices.

We conduct our analyses on two data sets. One is for all stocks listed on major US stock exchanges from 1962 to 2019. The other is for stocks on which options are traded during the period from 1996 to 2019, for which options data are available. The latter is a subset of the former in terms of both time-series observations and cross-sectional coverage. We dub the former as the stocks sample and the latter as the options sample. With the stocks sample, we apply semi-nonparametric approach to daily stock returns for each firm, estimate the conditional diffusive variance, and detect jumps. Intuitively, a daily return whose magnitude is too large compared to the then conditional diffusive variance is identified as a jump. We then construct a firm-quarter panel data sample of realized jump intensity, realized average jump size, and realized diffusive variance for a firm during a year between 1962 and 2019. Using this panel, we document the statistical properties of jump

[1] In affine models encompassed in Duffie, Pan and Singleton (2000), the conditional jump intensity λ_t is an affine function of conditional diffusive variance D_t of the return: $\lambda_t = \alpha + \beta D_t$, which may be extended to include other state variables. Since D_t is unbounded from above and λ_t is positive, $\beta \geq 0$ is the only choice. As such, the assumption of positive relation between jump intensity and diffusive variance arises more from a technical viewpoint, rather than from economic rationale.

intensity, jump size, and diffusive variance. We also examine how jump intensity, jump size, and diffusive variance vary with macroeconomic conditions and find that both jump size and diffusive variance increase during recessions, but jump intensity does not vary with macroeconomic conditions. The principal component analysis suggests that jump intensity and jump size of negative jumps exhibit more commonality than those of positive jumps. We also find that the semi-deviations of negative diffusive returns comove more than those of the positive diffusive returns. We then investigate the relationship among jump intensity, jump size, and diffusive variance. In the regressions of realized jump intensity on diffusive variance firm by firm, the slope coefficients (i.e., the jump-diffusion beta) tends to be more negative than positive, with a large amount of them being insignificantly different from zero. This is consistent with Li and Zhang (2016) for the case of S&P index returns and a handful large stocks.

Using the options sample, we examine how the jump-diffusion beta affects options pricing. We choose implied volatility smile as the main characteristics of options prices. We view the jump-diffusion beta at the end of each quarter estimated with past twelve quarters as a state variable/firm specific characteristic and examine how implied volatility smile depends on jump-diffusion beta among others. The result shows that implied volatility smile positively depends on jump-diffusion beta controlling for the diffusive variance, jump size, and volatility of volatility. We also show that the restriction of a non-negative relation between jump intensity and diffusive volatility imposed in the popular affine jump-diffusion option pricing models introduces biases to options pricing. If the jump intensity-diffusive volatility relation is negative, the models with affine jump intensity under (over) estimate the curvature of the implied volatility function when the diffusive volatility is low (high). These results provide valuable guidance on how options pricing model should be designed. In particular, models that do not allow negative jump-diffusion beta should be reconsidered and models that do not imply a positive relationship between implied volatility smile and jump-diffusion beta should also be reconsidered.

The analysis in the paper contributes to the options pricing literature in the following sense. First, how jumps are useful in options pricing for individual stocks has never been thoroughly investigated. Our paper is the first to conduct the analysis for all stocks during 1962-2019 on jump properties and for all stocks with options during 1996-2019 on the usefulness of jumps for options pricing. Second, the sign of jump-diffusion beta has been an issue for options pricing modeling. The limited evidence on the S&P500 index and a handful large stocks casts doubt on the assumption of positivity of jump-diffusion beta. The results in the paper provide further evidence against such an assumption, based on all individual stocks. Third, the effect of jump-diffusion beta on options pricing has never been examined in the literature, not even for options written on the S&P500 index. The results of this paper on all stocks with options fill a gap and provide a starting point for further analysis.

The rest of the paper is organized as follows. Section 2 discusses the nonparametric jump detection methods. Section 3 describes the data, conducts jump detection, constructs the firm-quarter observations of realized jump intensity, jump size and diffusive variance, and provide descriptive analysis of these variables in the stocks sample, options sample, and the top 50 stocks ranked by market capitalization in the options sample. Section 4 conduct empirical analysis of the relation between realized jump intensity and realized diffusive variance in the stocks sample and the options sample. Section 5 explores how jump-diffusion beta affects implied volatility smile in the options sample. Section 6 concludes the paper.

2. The Jump Detection Method

In examining affine options pricing models, the relation between the conditional jump intensity and the diffusive volatility is typically estimated as part of a parametric model. For example, Pan (2002) and Eraker (2004) find a significant increasing relation between the conditional jump intensity and the diffusive volatility of the underlying asset, using options data.[2] Bates (2000) explicitly tests the difference in jump intensity-diffusive variance relation under the actual and risk-neutral probabilities and finds no relation under the former, but a positive relation under the latter. Using S&P 500 index return data only without options prices, Andersen, Benzoni and Lund (2002) find an insignificant relation between the conditional jump intensity and the diffusive volatility. In short, the issues of whether there is a relation between jump intensity of the underlying asset price and its diffusive volatility, whether the relation is positive, and whether the relation is monotonic are unsettled even for S&P 500 index, let alone individual stocks.[3]

The results of parametric analysis of jump intensity can be sensitive to the model assumptions. Recent studies have shown that many standard options pricing models are misspecified. Jones (2003) finds that the square-root stochastic volatility model is incapable of generating realistic return behavior and the data are better represented by a stochastic volatility model in the constant-elasticity-of-variance class or a model with a time-varying leverage effect. Christoffersen, Jacobs and Mimouni (2010) find that a stochastic volatility model with a linear diffusion term is more consistent with the data on the underlying asset and options than a stochastic volatility model with a square-root diffusion term is. Li and Zhang (2013) show that the affine drift of the diffusive volatility model is mis-specified because the mean reversion is particularly strong at the high end of volatility. These results suggest that the standard options pricing model with the square-root volatility process falls short of generating sharp increases and decreases in volatility when the level of volatil-

[2] Two issues involved with using options data make the results difficult to interpret. First, jumps in options prices can result from jumps in state variables, rather than from those in the underlying asset price. In potentially misspecified models, especially those that force the underlying asset price and other state variables to jump together, a relation between the jump intensity of the underlying asset price and diffusive volatility can be found spuriously. Second, since options pricing is conducted under the risk-neutral probability which involves risk premium, for models that implicitly assume no risk premium associated with jump intensity, a relation between the jump intensity and diffusive volatility under the risk-neutral probability may be attributed to that under the actual probability spuriously.

[3] Eraker (2004) also has estimation results without using options data in his Table 4, in which he does not report the relation between jump intensity and diffusive volatility and does not explain why not. Andersen et al. (2002) attribute the insignificance result to the approximation used to calculate standard errors, and a multicollinearity type problem. As shown by Andersen et al. (2002), when the conditional jump intensity is specified as an affine function of diffusive variance, instead of a constant, the standard errors of the parameters related to jump intensity are about 20 times larger.

ity is high. As a result, when there are in fact spikes in the volatility, the model with a square-root volatility process attributes large changes in asset price to jumps and produces a spurious positive relation between the conditional jump intensity and diffusive volatility.

The jumps referred to in this paper are infrequent and have large magnitudes from the viewpoint of long-term investors, rather than the smaller, infinite activity jumps that are used to model high-frequency returns. We adopt a semi-nonparametric jump detection method which in turn is based on many studies in the literature.[4] The method is described as follows.

Suppose S_t is the logarithm of a dividend included stock index at the end of day t and $r_{t+1} = S_{t+1} - S_t$ is the daily log return over day $t+1$. As in the jump-diffusion models in the continuous-time framework and the GARCH models in the discrete-time framework, the daily return r_{t+1} consists of two components, $r_{t+1} = \tilde{r}_{t+1} + z_{t+1}$, where $\tilde{r}_{t+1}$ is the diffusive component and is approximately normally distributed with a conditional variance $\tilde{D}_t$ and a negligible conditional mean, and z_{t+1} is the size of a jump with a conditional jump intensity λ_t. On most of the days, a jump does not occur and z_{t+1} is zero. While we do not make assumptions on the distribution of z_{t+1}, it is practically impossible to identify a jump if the realized jump size is small relative to $\sqrt{\tilde{D}_t}$. To detect a jump based on observed daily returns, a test statistic of the form is used which is based on the estimated diffusive variance $\hat{D}_t$:

$$L_{t+1} = \frac{r_{t+1}}{\sqrt{\hat{D}_t}}. \tag{1}$$

Asymptotically, L_{t+1} follows the standard normal distribution under the null hypothesis of there being no jumps on day $t+1$. A jump is detected if L_{t+1} exceeds a critical value

[4] There are several nonparametric jump detection tests in the literature. Barndorff-Nielsen and Shephard (2004, 2006) propose a bipower variation measure to separate the diffusive variance from the jump variance. Jiang and Oomen (2008) propose a jump detection test based on variance swap prices. Lee and Mykland (2008) develop a rolling jump detection test based on large increments relative to the instantaneous volatility. The test proposed by Aït-Sahalia and Jacod (2009) is based on power variations sampled at different frequencies. Except for the Lee and Mykland (2008) test, other tests are applied to a block of return observations, so the number (except zero) and exact timing of the jumps in the block are not known. The Lee and Mykland (2008) test is applied to individual return observations so that the exact timing and sign of jumps can be identified. The methodology in this paper follows closely those of Barndorff-Nielsen and Shephard (2004, 2006) and Lee and Mykland (2008).

determined by the standard normal distribution. A crucial matter here is how $\hat{D}_t$ depends the daily returns up to day t, as these returns may contain jumps, while $\tilde{D}_t$ is the diffusive variance only.

The estimator $\hat{D}_t$ is calculated recursively. At jth round,

$$\hat{D}_t^{(j)} = \frac{\sum_{k=0}^{K-1} \psi^k r_{t-k}^2 1_{\{c_{L,t-k} \leq |r_{t-k}| \leq c_{U,t-k}\}}}{\sum_{k=0}^{K-1} \psi^k 1_{\{c_{L,t-k} \leq |r_{t-k}| \leq c_{U,t-k}\}}}, \tag{2}$$

where $1_{\{.\}}$ is an indicator function, K is a window size, $\psi \in (0,1)$ is a smoothing constant chosen such that the log-likelihood function

$$l(\psi) = \sum_{t=K}^{T-1} (-\log \hat{D}_t^{(j-1)} - r_{t+1}^2 / \hat{D}_t^{(j-1)}) 1_{\{|r_{t+1}| \leq c_{U,t}\}} \tag{3}$$

is maximized, and T is the number of return observations of the stock in the sample. The estimator is recursive because the threshold used to exclude jumps, $c_{U,t} = 3\sqrt{\hat{D}_t^{(j-1)}}$, depending on the previous round estimator of the diffusive variance and $c_{L,t}$ is determined to ensure that the diffusive variance with the truncation is the same as that without truncations. The first round estimator is chosen as the daily bipower variation estimated based on the returns from $t-K+1$ to t which places equal weights on all past observations in the window $[t-K+1, t]$.

$$\hat{D}_t^{(1)} = \frac{\pi}{2(K-1)} \sum_{i=1}^{K-1} |r_{t-i}||r_{t-i+1}| \tag{4}$$

Starting from the second round, the estimated diffusive variance depends less on remote past because $\psi < 1$. If ψ is not too close to one, the weights of past return observations will decay quickly and the window size K will not be crucial, as an experiment with data indicates. We thus simply set $K = 63$ trading days, i.e., past return observations of the recent three months. This estimator is similar to the commonly used exponentially weighted moving average estimator of return variance.[5] The iteration stops if the average proportional changes $\sum_{t=K}^{T-1} (\hat{D}_t^{(j)} - \hat{D}_t^{(j-1)}) / \hat{D}_t^{(j-1)}$ is smaller than a tolerance level 0.01%.

[5] The jump-robust return variance estimators with truncations are proposed by Jacod (2008), Mancini (2009) and Corsi, Pirino and Renò (2010), among others. For the estimated diffusive variance to converge to the true value asymptotically, the upper truncation is required to take the form of $c_{U,t} = c_t K^{-\omega}$ for any

3. Data and Sample Description

3.1. Return Properties

We adopt the iterative jump detection method using daily returns on all common stocks traded in NYSE, AMEX and NASDAQ obtained from the Center for Research in Security Prices (CRSP) for January 1, 1962 (1962Q1) to June 30, 2019 (2019Q2). For the jump detection method to produce reliable results, we drop those stocks with a time series less than 250 trading days or with more than half of daily returns equal to zero. For each firm, we calculate the time-series mean, standard deviation, skewness, kurtosis, the 5th, 25th, 50th, 75th and 95th percentiles of the daily log returns (in percentage), and the number of business days with available observations, and report their cross-sectional averages across the firms in Table 1.

Table 1 here

Panel A of Table 1 shows that an average individual stock has a daily mean log return of -0.06% and a daily volatility of 4.59%. The skewness of the time-series distribution of returns is slightly negative, -0.12 for an average stock. The cross-sectional mean of time-series excess kurtosis for individual stocks is as large as 29.42, highlighting the importance of jumps in modeling the fat tail distribution. The 5th, 25th, 50th, 75th, and 95th percentiles of daily returns on an average stock are -6.5%, -1.96%, -0.03%, 1.68%, and 6.65%, respectively, and the average length of the time series of returns is 3,040.1 trading days. In total, there are 19,132 individual stocks in our sample (stocks sample) from 1962 to 2019.

Panel B shows that an average stock with options (options sample) has a daily mean log return of -0.07%, comparable with that of the stocks sample. The average daily volatility is

$c_t > 0$ and $\omega \in (0, 0.5)$. However, there is little theoretical guidance on how c_t and ω should be chosen in finite sample problems. In practice, ω is chosen to be close to and less than 0.5, and c_t is chosen to be a few times the size of the estimated diffusive volatility from $t - K$ to t. Our approach follows this practice. The lower truncation is set to a value such that $E(\xi^2) = E(\xi^2 | c^2_{L,t} \leq \xi^2 \leq c^2_{U,t})$, where ξ follows the normal distribution with mean zero and variance BV_t.

3.89%, smaller than that of the stocks sample. This is expected as stocks with options tend to be larger and more liquid. The cross-sectional mean of time-series skewness of individual stocks with options is -0.37, more negative than that from the stocks sample. The excess kurtosis of an average stock with options is 26.58, indicating that jumps also play a crucial role in modeling these return distributions. The 5th, 25th, 50th, 75th, and 95th percentiles of the daily returns on an average stock with options are -5.38%, -1.78%, -0.08%, 1.58%, and 5.47%, respectively, and the average length of the time series is 1,917.4 trading days.

Panel C shows that for the top 50 stocks ranked by market capitalization (big 50 sample) at the end of year 1996 in the options sample, the mean daily log return is 0.03%. The cross-sectional mean daily volatility of the big 50 stocks is 2.11%, even smaller than that of the stocks with options. The mean of time-series skewness is -0.67, comparable to that of the options sample. The excess kurtosis of an average stock in the big 50 sample is 29.23, comparable to those from the stocks sample and options sample. The 5th, 25th, 50th, 75th, and 95th percentiles of the daily returns of the average of the 50 big stocks are -2.98%, -0.91%, 0.03%, 0.99%, and 3.03%, respectively, and the average length of the time series is 4,757.3 trading days.

We apply the recursive method discussed in Section 2 to daily returns on each stock in the sample and classify each daily return as a diffusive change or a jump. In Table 2, we report the distribution of the smoothing constant ψ and the number of iterations before convergence across firms. For an average stock in the stocks sample, the approach takes 5.9 iterations to converge and the smoothing constant $\psi = 0.82$. The smoothing constant becomes closer to 1 for the big 50 stocks and stocks with options. The average smoothing constant is 0.89 for the options sample and 0.94 for the big 50 sample. The recursive method converges faster when applied to these two samples than to the stocks sample. The average number of iterations is 5.4 for the options sample and 5.1 for the big 50 sample. Compared with previous studies using similar method on market indices, the smoothing constant is smaller for stocks, implying that the volatilities of individual stocks, especially small stocks, are more volatile and less persistent over time than market indices are.

Table 2 here

3.2. Variation Decomposition

For a stock i at the end of calendar quarter q, we define its realized total variance in the past year as $V_{iq} = \sum_{t \in W_q} r_{it}^2$ where W_q is the set of business days in the one-year window before the end of q. If a jump is detected for stock i on a trading day t, the diffusive return on that day is set to be zero and r_{it} is treated as the signed jump size. Let W_{iq}^J (W_{iq}^{J+}, W_{iq}^{J-}) be the set of all jump days (positive jump days, negative jump days) and λ_{iq} (λ_{iq}^+, λ_{iq}^-) be the number of realized jumps (positive jumps, negative jumps) for stock i in the window W_q. The realized quarterly diffusive variance is defined as $D_{iq} = \sum_{t \in W_q, t \notin W_{iq}^J} r_{it}^2$. The realized quarterly jump variance is defined as $J_{iq} = \sum_{t \in W_{iq}^J} r_{it}^2$, and similarly, we define $J_{iq}^+ = \sum_{t \in W_{iq}^{J+}} r_{it}^2$ and $J_{iq}^- = \sum_{t \in W_{iq}^{J-}} r_{it}^2$. The average squared jump size is $Z_{iq} = \frac{J_{iq}}{\lambda_{iq}}$ for $\lambda_{iq} > 0$, and similarly, we define $Z_{iq}^+ = \frac{J_{iq}^+}{\lambda_{iq}^+}$ for $\lambda_{iq}^+ > 0$, and $Z_{iq}^- = \frac{J_{iq}^-}{\lambda_{iq}^-}$ for $\lambda_{iq}^- > 0$. The annualized total variance is decomposed as

$$V_{iq} = D_{iq} + J_{iq} = D_{iq} + J_{iq}^+ + J_{iq}^- = D_{iq} + \lambda_{iq}^+ Z_{iq}^+ + \lambda_{iq}^- Z_{iq}^-. \tag{5}$$

Panel A of Table 3 reports the cross-sectional average of the time-series descriptive statistics of the firm-quarter quantities $\sqrt{V_{iq}}$, $\sqrt{D_{iq}}$, λ_{iq}, λ_{iq}^+, λ_{iq}^-, $\sqrt{Z_{iq}}$, $\sqrt{Z_{iq}^+}$, $\sqrt{Z_{iq}^-}$, and jump variance ratios, J_{iq}/V_{iq}, J_{iq}^+/V_{iq}, and J_{iq}^-/V_{iq} for the stocks sample. Panel B and C are for the options sample and the big 50 samples, respectively. The stocks sample and the options sample share similar characteristics, but the 50 big stocks exhibit some differences from the rest of the stocks. The return volatility of the big 50 sample is much smaller than those of the two large samples. The time-series average of diffusive (total) volatility is 0.66 (0.81) for a representative stock in the stocks sample, 0.47 (0.59) in the options sample, but only 0.26 (0.30) in the big 50 sample. The realized jump intensity is comparable between the stocks sample (on average, 9.07) and the options sample (on average, 6.99), but the big 50 sample has a lower realized jump intensity (on average 4.54). The typical stocks in the two large samples have more positive jumps than negative jumps, and the representative stocks in the big 50 sample have slightly more negative jumps than positive jumps. The jump size in the two large samples is much larger than that in the big 50 sample. The average jump size is 0.15 for the stocks sample, 0.13 for the options sample, and 0.06 for the big 50 sample. The two large samples have a larger jump variance ratio than the big 50 sample. The average J_{iq}/V_{iq} is 0.30 for the stocks sample, 0.32 for the options sample, and 0.21 for the big 50 sample. From the above results, we find the two large samples of individual stocks are quite different from the big 50 sample in features of jump and diffusive returns.

Table 3 here

3.3. Variation over Business Cycles

To investigate whether realized diffusive volatility, jump size, and jump intensity tend to distribute differently in expansions and recessions, we run the time-series regressions

$$X_{iq} = \overline{X}_i + \overline{\Delta X}_i \times Recession_q + \varepsilon_{iq}, \tag{6}$$

for each stock i, where X_{iq} is variable of interest, the coefficient $\overline{X}_i$ captures the average level of X_{iq} for stock i in expansion periods, and the coefficient $\overline{\Delta X}_i$ captures the average change of X_{iq} from expansion to recession periods for stock i, $Recession_q = 1$ if the estimation window W_q contains at least two recession quarters defined by NBER, and zero otherwise, and ε_{iq} is a generic term for regression errors. Since the error terms are auto-correlated due to the overlapped estimation window for X_{iq}, we adjust the standard errors and t-statistics using Newey and West (1987) method with 3 lags. In order to obtain more reliable estimates, we require the stock-quarter time series longer than 10 quarters, and at least two observations in and out of recession, respectively.

Table 4 reports the cross-sectional distributions of coefficients estimates for the regressions where $X_{iq} = \sqrt{D}_{iq}$. Panel A is for the stocks sample, Panel B is for the options sample, and Panel C is for the big 50 sample. It is shown that the realized diffusive volatility exhibits a counter-cyclical pattern, i.e., diffusive volatility increases in recessions. The average increase in diffusive volatility is 14% for the stocks sample, 19% for the options sample, and 14% for the big 50 sample. For most of the stocks, especially for the stocks with options, the increase is statistically significant. The average Newey-West adjusted t-statistic is 1.72 for the stocks sample, 2.91 for the options sample, and 2.29 for the big 50 sample.

Table 4 here

Then, we examine whether the average realized jump size varies with business cycles. Table 5 summarizes the firm-level regression results for $X_{iq} = \sqrt{Z}_{iq}$, $X_{iq} = \sqrt{Z^{+}}_{iq}$, or $X_{iq} = \sqrt{Z^{-}}_{iq}$. The average realized jump size generally increases in recessions, and the increase is statistically significant for at least one quarter of the firms. The average change of jump size is 3% for the stocks sample, 4% for the options sample, and 3% for the big 50 sample. Given the mean (standard deviation) of jump size is 15% (6%) for the stocks sample, 13% (4%) for the options sample, and 6% (3%) for the big 50 sample, as shown in Table 3, the increase in jump size during the recession is economically large, especially for the options sample and the big 50 sample. Similar patterns are observed for both size of positive jump and size of negative jump. The above results suggest that jump size may not follow an i.i.d. distribution, as assumed in the typical models of stock prices.

Table 5 here

Table 6 reports the cross-sectional distributions of coefficients estimates for the regressions where $X_{iq} = \lambda_{iq}$, $X_{iq} = \lambda^{+}_{iq}$, or $X_{iq} = \lambda^{-}_{iq}$. The realized jump intensity does not change with business cycles as significantly as the realized diffusive volatility and realized jump size do. The patterns across samples and positive and negative jumps are interesting, however. The realized jump intensity on average tends to increase in recessions for the stocks sample, but tends to decrease for the options sample and the big 50 sample. The realized intensity of positive jumps tends to decrease in recessions for all the samples, and the realized intensity of negative jumps tends to increase in recessions.

$\boxed{\text{Table 6 here}}$

To sum up, we find a strong counter-cyclical pattern for realized jump size and even stronger counter-cyclical behaviors for realized diffusive volatility, but a weak relation between realized jump intensity and business cycles.

3.4. Commonality of Jumps and Diffusive Variance

In this subsection, we investigate which of the jump-related variables are relatively more systematic (idiosyncratic). To this end, we further define total and diffusive variation of positive and negative returns separately. Let W_{iq}^{+} (W_{iq}^{-}) be the set of all days with positive (negative) returns for the stock i in the window W_q. We define the semi-deviation of positive returns, $\sqrt{V_{iq}^{+}} = \sqrt{\sum_{t \in W_{iq}^{+}} r_{it}^2}$, the semi-deviation of negative returns, $\sqrt{V_{iq}^{-}} = \sqrt{\sum_{t \in W_{iq}^{-}} r_{it}^2}$, semi-deviation of diffusive positive returns, $\sqrt{D_{iq}^{+}} = \sqrt{\sum_{t \in W_{iq}^{+}, t \notin W_{iq}^{J+}} r_{it}^2}$, and semi-deviation of diffusive negative returns, $\sqrt{D_{iq}^{-}} = \sqrt{\sum_{t \in W_{iq}^{-}, t \notin W_{iq}^{J-}} r_{it}^2}$.

To form a balanced panel for principal component analysis (PCA), we construct 10×10 portfolios (indexed by j) based on the NYSE 10th, 20th, up to 90th percentiles of the market value of equity (ME) and book-to-market ratio of equity (BM). We do not use the Fama-French breakpoints directly because we use a smaller universe of stocks than they do, i.e., excluding those stocks with a time series less than 250 trading days or having more than half of daily returns equal to zero. For the early years in the 1960s, there are some empty ME×BM portfolios, so we drop those quarters. Then, we get a balanced panel of equally-weighted portfolios with 191 quarters for λ_{jq}, λ_{jq}^{+}, λ_{jq}^{-}, $\sqrt{Z_{jq}}$, $\sqrt{V_{jq}}$, $\sqrt{V_{jq}^{+}}$, $\sqrt{V_{jq}^{-}}$, $\sqrt{D_{jq}}$, $\sqrt{D_{jq}^{+}}$, and $\sqrt{D_{jq}^{-}}$, 182 for $\sqrt{Z_{jq}^{+}}$, and 172 for $\sqrt{Z_{jq}^{-}}$.

Table 7 reports the summary statistics of the 10,000 pair-wise correlations among the 100 portfolios and the fraction of variance explained by the first and the second principal components. The average correlations of λ_{jq}, λ_{jq}^{+} and λ_{jq}^{+}, are 0.44, 0.49, and 0.51, respectively. Realized intensity of negative jump generally exhibits more commonality than that of positive jump. The first principal component, PC1, explains only 34% variance of realized intensity of positive jump but 68% variance of realized intensity of negative jump. For overall jump intensity, PC1 explains 52% variance. The jump intensities are the least systematic among the variables we considers. For jump size measures, we also observe a larger systematic component in negative jumps than in positive jumps, although the difference in variation explained by PC1 is not as drastic as that in jump intensities. We also find that the semi-deviations of negative diffusive returns comove more than those of positive diffusive returns, and it is the same case for the total returns as well. The correlations of semi-deviations of diffusive and total returns are of similar magnitudes, but correlations of semi-deviations of positive total returns are slightly lower than those of positive diffusive returns. The overall results suggest that negative jumps are relatively more systematic, whereas positive jumps are relatively more idiosyncratic.

Table 7 here

4. The Relationship between Jumps and Diffusive Variance

4.1. The Relationship between Realized Jumps Intensity and Realized Diffusive Variance

To investigate the relationship between realized jump intensity and diffusive variance, we run time-series regressions

$$\lambda_{i,q-k} = \alpha_{iq} + \beta_{iq} D_{i,q-k} + \varepsilon_{i,q-k}, \tag{7}$$

$$\lambda^{+}_{i,q-k} = \alpha^{+}_{iq} + \beta^{+}_{iq} D_{i,q-k} + \varepsilon_{i,q-k}, \tag{8}$$

$$\lambda^{-}_{i,q-k} = \alpha^{-}_{iq} + \beta^{-}_{iq} D_{i,q-k} + \varepsilon_{i,q-k}, \tag{9}$$

in a rolling 3-year window (k = 0, 1, 2, ..., 11) for each stock-quarter iq, where ε_{iq} is a generic term for regression errors.

Table 8 reports the time-series average of cross-sectional descriptive statistics of least-square estimates of β_{iq}, β_{iq}^{+}, β_{iq}^{-}, $t(\beta_{iq})$, $t(\beta_{iq}^{+})$ and $t(\beta_{iq}^{-})$, where $t(\cdot)$ is the Newey-West adjusted (3 lags) t-statistic. The estimates of β_{iq} are on average negative for the stocks sample, and even more negative for the options sample. The mean (median) of jump-diffusion beta, β_{iq}, is -12.41 (-5.41) for the stocks sample, and -11.37 (-6.98) for the options sample. The mean (median) of the t-statistic of the jump-diffusion beta, $t(\beta_{iq})$, is -0.78 (-0.64) for the stocks sample, and -0.77 (-0.68) for the options sample. The jump-diffusion beta of positive jumps, β_{iq}^{+}, is also negative on average, and is less dispersed compared with β_{iq}. The mean (median) of β_{iq}^{+}, is -10.32 (-4.21) for the stocks sample, and -10.63 (-6.01) for the options sample. The 25th percentile of β_{iq}^{+} is -23.32 for the stocks sample, three times in absolute value as the 75th percentile value 7.27. For the options sample, the 25th percentile is -23.24, whose absolute value more than three times larger than the 75th percentile value 5.29. In contrast, for the jump-diffusion beta of negative jumps, β_{iq}^{-}, the average is much closer to 0 for both the stocks sample and the options sample. The 25th and 75th percentiles of β_{iq}^{-} are similar in magnitude, -14.54 and 11.01 for the stocks sample, and -14.08 and 11.90 for the options sample, respectively. Overall, we find limited evidence that realized jump intensity and diffusive variance are positively correlated, but abundant evidence that the two variables are not correlated or even negatively correlated.

Table 8 here

Since jumps are identified based on an estimate of diffusive variance, the estimation error in the diffusive variance may cause negative biases in the slope coefficient estimates in the regressions (**??**), (**??**) and (**??**). To mitigate these error-in-variable biases, we perform the estimation using the one-quarter lagged diffusive variance $D_{i,q-1}$ as the instrumental variable (IV). Table 9 reports the results. The medians of β_{iq}, β_{iq}^{+}, and β_{iq}^{-} are all below 0, and the 5th and 25th percentiles are considerably larger in absolute value than the 95th and 75th percentiles, suggesting that the negative relations between realized jump intensity and diffusive variance dominate among the stocks. The distributions of coefficients estimates are slightly more dispersed than those reported in Table 8, and the t-statistics are more or less the same. Overall, results in Table 9 are qualitatively consistent with those in Table 8, and our findings that realized jump intensity and diffusive variance are not correlated or even negatively correlated cannot be due to the error-in-variable biases.

Table 9 here

4.2. The Relationship between Realized Jumps Size and Realized Diffusive Variance

To investigate the relationship between jump size and diffusive variance, we run time-series regressions

$$Z_{i,q-k} = \gamma_{iq} + \theta_{iq} D_{i,q-k} + \varepsilon_{i,q-k}, \tag{10}$$

$$Z^{+}_{i,q-k} = \gamma^{+}_{iq} + \theta^{+}_{iq} D_{i,q-k} + \varepsilon_{i,q-k}, \tag{11}$$

$$Z^{-}_{i,q-k} = \gamma^{-}_{iq} + \theta^{-}_{iq} D_{i,q-k} + \varepsilon_{i,q-k}, \tag{12}$$

in a rolling window ($k = 0$, 1, 2, ..., 11) for each stock-quarter iq.

Table 10 reports the time-series averages of cross-sectional descriptive statistics of least-square estimates of θ_{iq}, θ^{+}_{iq}, θ^{-}_{iq}, $t(\theta_{iq})$, $t(\theta^{+}_{iq})$, and $t(\theta^{-}_{iq})$. The t-statistics are adjusted using the Newey-West method with 3 lags. There is a significantly positive relationship between jump size and diffusive variance for more than 50% of individual stocks. The average cross-sectional mean (median) θ_{iq} is 0.05 (0.03) for the stocks sample and 0.06 (0.04) for the options sample. The average cross-sectional statistics are quite similar for positive jumps and negative jumps. The cross-sectional dispersion of θ^{-}_{iq} is slightly larger than that of θ^{+}_{iq}. Consistent with the pro-cyclical patterns for both jump size and diffusive volatility shown in Table 4 and Table 5, the positive relationship shown in Table 10 can be attributed to a common underlying state variable governing both jump size and diffusive variance through business cycles.

Table 10 here

We also estimate (**??**), (**??**), and (**??**) using the lagged diffusive variance $D_{i,q-1}$ as the IV for $D_{i,q}$. The results in Table 11 show that the distributions of regression estimates are more negatively skewed and more dispersed than those in Table 10, and the statistical significance is slightly lower. Overall, the jump size-diffusive variance relations are positive for most of the stocks, and results are consistent with those reported in Table 10.

Table 11 here

5. The Jump-diffusion Beta and Options Pricing

5.1. Empirical Analysis of Jump-diffusion Beta and Implied Volatility Smile

In this subsection, we examine the implications of the jump-diffusion beta on options pricing. When there are no jumps in the underlying asset price and the diffusive volatility is constant, the underlying asset returns follow the log-normal distribution and the classical Black-Scholes option pricing model holds. In this case, the Black-Scholes implied volatilities of options for a given maturity is constant across strike prices. When the underlying asset price jumps or the diffusive volatility is time-varying, the tails of the underlying asset return distribution are fatter than those of the log-normal distribution. In this case, the Black-Scholes implied volatility is no longer flat, but typically a U-shaped function of strike prices, which is referred to as the implied volatility smile. The implied volatility smile is a robust fact for equity options, and many recent options pricing models are developed to capture the effect. Jumps in underlying asset prices and the stochastic volatility affect the implied volatility smile in different ways. Since the impact of jumps relative to the stochastic volatility is more important for distributions of underlying asset returns in short horizons than those in long horizons, jumps contribute more to the implied volatility smile of short-maturity options than the stochastic volatility does. We examine the cross-sectional relationship between the jump-diffusion beta and the implied volatility smile of short-maturity options.

The implied volatility data is retrieved from the implied volatility surface of OptionMetrics. OptionMetrics uses the binomial tree approach to calculate the implied volatilities from option prices so that the early exercise premiums in the individual stock options are adjusted for. OptionMetrics then uses a kernel smoothing method to interpolate the implied volatility surface at the fixed maturities and deltas. We use the interpolated implied volatility instead of the actual implied volatility to minimize the errors in the implied volatility smile measure arising from the variations of maturities and deltas of the actual implied volatility across time.

We measure the implied volatility smile at each week w for each stock i as

$$\mathrm{SM}_{iw} = \mathrm{IV}^{p}_{iw}(-0.3) - \mathrm{IV}^{p}_{iw}(-0.5) + \mathrm{IV}^{c}_{iw}(0.3) - \mathrm{IV}^{c}_{iw}(0.5), \tag{13}$$

where $\mathrm{IV}^{j}_{iw}(x)$ is the week-w implied volatility with a 30-day to maturity on stock i with delta x, and $j = c$ for call and $j = p$ for put. The 30-day maturity is the shortest on the implied volatility surface in the OptionMetrics database. $\mathrm{IV}^{p}_{iw}(-0.5)$ and $\mathrm{IV}^{c}_{iw}(0.5)$ are the at-the-money implied volatilities, and $\mathrm{IV}^{p}_{iw}(-0.3)$ ($\mathrm{IV}^{c}_{iw}(0.3)$) is the out-of-the-money implied volatility at a low (high) strike price. We use out-of-the-money implied volatilities rather than in-the-money implied volatilities because of the better liquidity and less contaminated by early exercise premiums for out-of-the-money options.[6] There are choices of deltas for the out-of-the-money implied volatilities. Using implied volatilities of deeper out-of-the-money options helps better isolate the information related to jumps from that related to diffusive volatility, however, these options are less liquid and their prices may contain large errors. In some cases, deep out-of-the-money options do not exist at all and extrapolating the existing implied volatilities may be required so that the estimated implied volatilities are not reliable. Given these considerations, we choose the moderately deep out-of-the-money implied volatilities, i.e., absolute delta of 0.3.[7] If the implied volatility as a

[6]For European type options, the implied volatilities from call and put options of the same strike price are expected to be the same because of the put-call parity. However, in reality, due to informed trading in options market and microstructure issues, such as bid-ask spread and short selling constraint, the implied volatilities from call and put options of the same strike price usually are not the same. The early exercise premiums in the American type options of individual stocks also contribute to the differences.

[7]The empirical results are quantitatively similar for absolute deltas between 0.2 to 0.4.

function of strike prices is U-shaped, SM_{iw} is positive, and SM_{iw} increases with the degree of the curvature of the function.

We examine the effect of the jump-diffusion beta on the implied volatility smile using the following Fama-Macbeth cross-sectional regression,

$$\overline{\mathrm{SM}}_{iq} = b_0 + b_1\beta_{iq} + b_2\alpha_{iq} + b_3\overline{D}_{iq} + b_4\sqrt{\bar{Z}_{iq}} + b_5\sigma(D_{iq}) + \varepsilon_{iq}, \tag{14}$$

where $\overline{\mathrm{SM}}_{iq}$ is the average of weekly SM_{iw} observed every Wednesday in the past 3 years until quarter q.[8] $\overline{D}_{iq}$ and $\sigma(D_{iq})$ are the average and standard deviation of diffusive variance, D_{iq}, in the past 3 years, respectively, and $\sqrt{\overline{Z}_{iq}}$ and $\overline{\lambda}_{iq}$ are the averages of jump size, $\sqrt{Z_{iq}}$, respectively. Since the underlying return variance due to jumps can be written as $(\alpha_{iq} + \beta_{iq}D_{iq})Z_{iq}$ in this paper, we include these variables or their rolling averages in the regression specification. As argued earlier, since in addition to jumps, the stochastic volatility can also lead to the implied volatility smile, $\sigma(D_{iq})$ is included in the regression to control for the effect of the stochastic volatility on the implied volatility smile.

Table 12 reports the descriptive statistics of the variables used in the regression analysis in additional to those reported in Panel B of Table 8. The sample of stocks with options traded covers the period from 1996Q1 to 2019Q2. Almost all SM_{iq}s are positive, suggesting that the implied volatility smile is a robust fact for individual stock options. The average SM_{iq} is about 3.4% in annual percentage volatility terms, and there is a large cross-sectional variation in SM_{iq}, ranging from 0.2% at the 5th percentile to 11% at the 95 percentile. The estimated α_{iq}s are mostly positive, and for the stocks with negative α_{iq}s, β_{iq}s are positive with large magnitudes shown in Panel B of Table 8. Since stocks with options traded tend to be large stocks, the average diffusive variance and jump size are smaller for this sample than for the whole sample. The standard deviation of diffusive variance varies in the cross section, and it is positively correlated with the average diffusive variance.

Table 12 here

[8] We require at least 50 weekly SM_{iw} observations for a stock to be included in the sample.

The regression results are shown in Table 13. β_{iq} is positive and significant, and the result is consistent with the explanation that a higher β_{iq} leads to a higher jump intensity so that the jump variance contributes more to the total variance of the underlying asset returns. For other components of the jump variance, α_{iq} is also positive and significant, but $\overline{D}_{iq}$ and $\sqrt{\overline{Z}_{iq}}$ are insignificant. The explanatory power of β_{iq} does not change when $\sigma(D_{iq})$ is included in the regression which controls for the effect of the stochastic volatility on the implied volatility smile. $\sigma(D_{iq})$ is also insignificant. All these variables are regarded as the determinants of the implied volatility smile in option pricing models, however, we find that only are β_{iq} and α_{iq} important for options pricing empirically.

Table 13 here

5.2. The Restriction in the Affine Jump Intensity

In the popular affine jump-diffusion option pricing models, the jump intensity is specified as an affine function of instantaneous volatility, $\lambda_t = \alpha + \beta D_t$. Since the jump intensity is non-negative, the specification does not allow a negative jump intensity-diffusive volatility relation. In this subsection, we examine the effects of such a restriction in the affine model on options pricing.

To quantify the effect, we assume that the log stock price S_t evolves according to the following process under the risk-neutral probability,

$$dS_t = (r_t - 0.5D_t)dt + \sqrt{D_t}dW_t^S + Y_t dJ_t - E_t(e_t^Y - 1)\lambda_t dt \quad (15)$$

$$dD_t = \kappa(\theta - D_t)dt + \sigma\sqrt{D_t}dW_t^D, \quad (16)$$

where W_t^S and W_t^D are the standard Brownian motion with correlation ρ, J_t is a counting process with jump intensity λ_t, independent of W_t^S and W_t^D, Y_t is a random jump size of S_t, and D_t is the instantaneous variance with unconditional mean θ, mean reverting speed κ, and volatility σ.[9] Y_t follows the normal distribution with constant mean μ_j and variance σ_j^2

[9] Modelling stock prices under the risk-neutral probability is sufficient for determining options prices.

as commonly assumed in the literature. Since it is impossible to exhaust all specifications of non-affine jump intensity-diffusive volatility relation, we consider a simple exponential affine function, $\lambda_t = \exp(l_0 + l_1 D_t)$, which allows negative jump intensity-diffusive volatility relations, i.e., $l_1 < 0$.

We calibrate the parameters to match the characteristics of the sample of stocks with options in the period of from 1996Q1 to 2019Q2 as much as possible. $r_t = 2.2\%$ is the average of risk-free during the sample period. $\mu_j = 0.1\%$ and $\sigma_j = 16\%$ are mean and standard deviation of the daily jump size for this sample of stocks. $\theta = 0.2$ matches with the mean of diffusive volatility $\sqrt{D_{iq}}$ in Panel B of Table 3. κ is calibrated to the annualized autoregressive coefficient of an AR(1) model of the quarterly observations $\sqrt{D_{iq}}$, and σ and ρ are estimated from the residuals of the model. The values averaged across the stocks are used, $\kappa = 0.84$, $\sigma = 0.081$ and $\rho = -0.12$. The values of l_0 and l_1 are determined according to the following conditions. First, the unconditional mean of the jump intensity $E(\lambda_t) = 6.4$, the mean jump intensity λ_{iq} reported in Panel B of Table 3. Second, the jump-diffusion beta is -25, the median of the estimated β_{iq} for the sample of stocks with options and with negative jump-diffusion betas. Since the jump-diffusion beta varies with D_t for the exponential affine jump intensity specification, we calculate jump-diffusion beta, $\beta_t = \frac{d\lambda_t}{dD_t}$ at the value $D_t = \theta$ in this case.

As explicit solutions of option prices do not exist for the non-affine models, we use the Monte Carlo simulations to calculate option prices. We simulate a discretized version of the above model for 10000 sample paths, and calculate call option prices with the maturity of one month and moneyness $K/\exp(S_0)$ between 0.5 to 1.5, where K is the strike price of an option. We use a number of numerical techniques to increase numerical efficiency, namely, the empirical martingale method of Duan and Simonato (1998), stratified random numbers, and antithetic variates. Then, we invert the Black-Scholes implied volatility from the option prices.

The dash lines in Figure 1 show the implied volatility as a function of moneyness

$K/\exp(S_0)$. The three panels are for the cases of different levels of instantaneous volatility D_0. These figures show clear smile patterns which are mostly due to jumps in the underlying stock prices. The smile pattern is rather symmetrical because the average jump size is close to zero. The smile pattern is more pronounced for low D_0 case. This is so because when $l_1 < 0$, the jump variance, i.e., $\lambda_0 \sigma_j^2$, contributes more to the total variance of underlying stock returns, $D_0 + \lambda_0 \sigma_j^2$, for low D_0 than for high D_0. The figure also shows that the level of implied volatility increases with D_0 as expected.

Figure 1 here

When the relation between the jump intensity and diffusive volatility is negative, the option pricing models with affine jump intensity, $\lambda_t = \alpha + \beta D_t$, would restrict $\beta = 0$, the lowest possible value allowed by the specification. In this case, λ_t is equal to a constant α which is set to be 6.4, the unconditional mean of the jump intensity for the exponential jump intensity. To illustrative the impact of the affine restriction, we plot the implied volatility functions generated from the model with the affine jump intensity by the solid lines in Figure 5. To ensure that the levels of the implied volatility are comparable to those from the model with exponential affine jump intensity, the instantaneous total variance, $D_0 + \lambda_0 \sigma_j^2$, are kept the same for the two models. The figure shows that the implied volatility function exhibits similar simile patterns across different values of instantaneous volatility. It also shows that when D_0 is low (high), the smile shape of the implied volatility curve is less (more) pronounced than that for the case of exponential affine jump intensity. The middle panel shows that when $D_0 = \theta$, the smile patterns of the two specifications are very close because the conditional jump intensities of the two cases are approximately the same. Since the jump intensity in the affine case is constant, the contribution of jump variance to the total variance solely depends on the level of instantaneous volatility. For the exponential affine jump intensity case, jump intensity is negatively related to diffusive volatility which results in a large variation in the contribution of jump variance to the total variance across different values of instantaneous volatility. Overall, the results suggest that the restriction that the affine jump intensity specification has a quantitatively important implication for options pricing. If the jump-diffusion beta is negative, the model with affine jump intensity restricts the jump-diffusion beta to zero, and under (over) estimates the conditional jump intensity and the curvature of the implied volatility function when the diffusive volatility is low (high).

6. Conclusion

We use nonparametric methods to decompose returns on a large cross-section of individual stocks into jumps and diffusive components. The descriptive analysis on the firm-quarter panel of jump-related variables highlights the importance of the division between jumps and diffusive variances for individual stocks. The average jump-variance ratio for the large samples of individual stocks are around 30%, considerably larger than that of the big 50 stocks, 21%. Individual stocks generate more frequent jumps with larger magnitudes than stock index and the big 50 stocks. The relative propensity of positive jumps to negative jumps are also different between the large sample of individual stocks and the handful of big stocks. We also observe pro-cyclical patterns for jump size and diffusive volatility. These patterns challenge the commonly used assumption that jump size is i.i.d. and indicate that jump size and diffusive volatility are correlated.

We examine the relationship between realized jump intensity and diffusive variance for all individual stocks from 1962 to 2019. We find limited evidence of positive relationship but plenty of evidence for no significant relationship or negative relationship. This finding casts doubt on the assumption of positive correlation between jump intensity and diffusive volatility. We also investigate the relationship between realized (unsigned) jump size and diffusive variance. We find that more than 50% of the individual stocks exhibit a significantly positive relationship.

We then investigate how options pricing depends on the covariation between jump intensity and diffusive variance. The jump-diffusion beta is found to positively contribute to the implied volatility smile of options on individual stocks. The results suggest that the relationship between jump intensity and diffusive variance matters for option pricing. Mis-specification will distort options pricing outcome. We find that the jump intensity specification imposed in the popular affine jump-diffusion option pricing models hinders the option pricing performance because they do not allow the negative relationship between the jump intensity and diffusive variance found in many individual stocks. For these stocks, the models with affine jump intensity under (over) estimate the curvature of the implied volatility function when the diffusive volatility is low (high). The findings on the jump-diffusion relationship and implied volatility smile of individual stocks provide guidance for modeling option prices.

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Table 1
Cross-sectional averages of time-series statistics of daily returns
This table reports the cross-sectional average of the time-series mean, standard deviation, skewness, excess kurtosis, the 5th, 25th, 50th, 75th, 95th percentiles of the daily log returns on individual stocks, and the average number of trading days in a stock ($\bar{T}$). Panel A summarizes the sample during 1962Q1-2019Q2 (stocks sample) with a total number of 19,132 stocks. Panel B summarizes the sample of 6,863 stocks that have options traded on them during 1996Q1-2019Q2 (options sample). Panel C summarizes the top 50 stocks ranked by market capitalization at the end of year 1996 in the options sample (big 50 sample). All returns are in percentages.

	Mean	StdDev	Skew	Kurt	P5	P25	P50	P75	P95	$\bar{T}$
Panel A: Stocks sample										
r_{it}	-0.06	4.59	-0.12	29.42	-6.50	-1.96	-0.03	1.68	6.65	3,040.1
Panel B: Options sample										
r_{it}	-0.07	3.89	-0.37	26.58	-5.38	-1.78	-0.08	1.58	5.47	1,917.4
Panel C: Big 50 sample										
r_{it}	0.03	2.11	-0.67	29.23	-2.98	-0.91	0.03	0.99	3.03	4,757.3

Table 2
Summary statistics for iterative jump detection procedures
This table reports the distribution of the final smoothing constant (ψ_i) and the number of iterations ($ITER_i$) before convergence across stocks i. Panel A is for the sample of all stocks during 1962Q1-2019Q2 (stocks sample), Panel B is for the sample of stocks that have options traded on them during 1996Q1-2019Q2 (options sample), and Panel C is for the top 50 stocks ranked by market capitalization at the end of year 1996 in the options sample (big 50 sample). N is the number of stocks.

	Mean	StdDev	P5	P25	P50	P75	P95	N
Panel A: Stocks sample								
ψ_i	0.82	0.15	0.54	0.77	0.86	0.92	0.96	19,132
$ITER_i$	5.86	2.68	3	4	5	7	10	19,132
Panel B: Options sample								
ψ_i	0.89	0.07	0.78	0.87	0.91	0.93	0.96	6,863
$ITER_i$	5.43	2.35	3	4	5	6	8	6,863
Panel C: Big 50 sample								
ψ_i	0.94	0.01	0.91	0.93	0.94	0.94	0.96	50
$ITER_i$	5.1	1.22	4	5	5	6	7	50

Table 3
Summary statistics for firm-quarter jump and volatility variables

This table reports the cross-sectional average of the time-series mean, standard deviation, skewness, kurtosis, the 5th, 25th, 50th, 75th, 95th percentiles of the quarterly quantities for individual stocks, including total volatility ($\sqrt{V_{iq}}$), diffusive volatility ($\sqrt{D_{iq}}$), realized jump intensity (λ_{iq}), realized intensity of positive jump (λ_{iq}^{+}), realized intensity of negative jump (λ_{iq}^{-}), jump size ($\sqrt{Z_{iq}}$), size of positive jump ($\sqrt{Z_{iq}^{+}}$), size of negative jump ($\sqrt{Z_{iq}^{-}}$), jump variance ratio (J_{iq}/V_{iq}), variance ratio of positive jump (J_{iq}^{+}/V_{iq}), and variance ratio of negative jump (J_{iq}^{-}/V_{iq}), and the average number of calendar quarters with available observations in a stock ($\bar{T}$). Panel A is for the sample of all stocks during 1962Q1-2019Q2 (stocks sample), Panel B is for the sample of stocks that have options traded on them during 1996Q1-2019Q2 (options sample), and Panel C is for the top 50 stocks ranked by market capitalization at the end of year 1996 in the options sample (big 50 sample).

	Mean	StdDev	Skew	Kurt	P5	P25	P50	P75	P95	$\bar{T}$
Panel A: Stocks sample										
$\sqrt{V_{iq}}$	0.81	0.27	0.79	1.03	0.51	0.62	0.75	0.96	1.32	45.7
$\sqrt{D_{iq}}$	0.66	0.21	0.66	0.59	0.42	0.51	0.62	0.78	1.07	45.7
λ_{iq}	9.07	3.96	0.39	0.21	3.61	6.24	8.72	11.54	15.85	45.7
λ_{iq}^{+}	4.87	2.71	0.46	0.19	1.29	2.87	4.59	6.56	9.54	45.7
λ_{iq}^{-}	4.21	2.57	0.55	0.34	1.00	2.28	3.87	5.77	8.71	45.7
$\sqrt{Z_{iq}}$	0.15	0.06	0.95	1.68	0.09	0.11	0.13	0.17	0.25	43.8
$\sqrt{Z_{iq}^{+}}$	0.14	0.06	0.90	1.43	0.08	0.10	0.13	0.17	0.25	40.5
$\sqrt{Z_{iq}^{-}}$	0.14	0.06	0.95	1.70	0.08	0.10	0.13	0.17	0.26	37.9
$V(J_{iq})/V_{iq}$	0.30	0.14	0.39	0.00	0.10	0.19	0.28	0.39	0.54	45.7
$V(J_{iq}^{+})/V_{iq}$	0.16	0.11	0.72	0.59	0.03	0.08	0.15	0.23	0.36	45.7
$V(J_{iq}^{-})/V_{iq}$	0.13	0.10	0.87	0.84	0.02	0.06	0.11	0.19	0.32	45.7

Table 3 (cont'd)

	Mean	StdDev	Skew	Kurt	P5	P25	P50	P75	P95	$\bar{T}$
Panel B: Options sample										
$\sqrt{V_{iq}}$	0.59	0.15	0.70	0.45	0.42	0.48	0.56	0.68	0.89	90.3
$\sqrt{D_{iq}}$	0.47	0.11	0.62	0.27	0.34	0.39	0.45	0.54	0.69	90.3
λ_{iq}	6.99	2.35	0.24	-0.34	3.57	5.24	6.85	8.61	10.98	90.3
λ_{iq}^{+}	3.66	1.64	0.35	-0.28	1.39	2.42	3.51	4.77	6.51	90.3
λ_{iq}^{-}	3.33	1.62	0.33	-0.32	1.09	2.09	3.18	4.43	6.11	90.3
$\sqrt{Z_{iq}}$	0.13	0.04	0.76	0.72	0.08	0.10	0.12	0.15	0.21	89.3
$\sqrt{Z_{iq}^{+}}$	0.12	0.04	0.76	0.71	0.07	0.09	0.11	0.14	0.19	83.6
$\sqrt{Z_{iq}^{-}}$	0.13	0.05	0.75	0.79	0.07	0.09	0.12	0.16	0.23	82.5
$V(J_{iq})/V_{iq}$	0.32	0.13	0.28	-0.33	0.14	0.22	0.31	0.41	0.53	90.3
$V(J_{iq}^{+})/V_{iq}$	0.16	0.09	0.68	0.32	0.04	0.09	0.14	0.21	0.33	90.3
$V(J_{iq}^{-})/V_{iq}$	0.16	0.11	0.58	-0.01	0.04	0.08	0.14	0.23	0.35	90.3
Panel C: Big 50 sample										
$\sqrt{V_{iq}}$	0.30	0.13	1.15	1.33	0.17	0.21	0.26	0.36	0.59	230.8
$\sqrt{D_{iq}}$	0.26	0.11	1.04	0.97	0.14	0.18	0.23	0.32	0.50	230.8
λ_{iq}	4.54	2.50	0.51	0.04	1.05	2.61	4.25	6.10	9.20	230.8
λ_{iq}^{+}	2.22	1.56	0.59	0.16	0.08	1.03	1.98	3.21	5.18	230.8
λ_{iq}^{-}	2.31	1.72	0.71	0.15	0.08	1.01	1.98	3.35	5.60	230.8
$\sqrt{Z_{iq}}$	0.06	0.03	1.39	2.30	0.03	0.04	0.05	0.08	0.14	225.0
$\sqrt{Z_{iq}^{+}}$	0.06	0.03	1.22	1.66	0.03	0.04	0.05	0.07	0.12	199.2
$\sqrt{Z_{iq}^{-}}$	0.07	0.04	1.31	2.19	0.03	0.04	0.06	0.08	0.15	199.2
$V(J_{iq})/V_{iq}$	0.21	0.12	0.51	-0.16	0.04	0.12	0.20	0.29	0.44	230.8
$V(J_{iq}^{+})/V_{iq}$	0.10	0.08	0.82	0.35	0.00	0.04	0.08	0.14	0.26	230.8
$V(J_{iq}^{-})/V_{iq}$	0.11	0.10	0.95	0.61	0.00	0.04	0.09	0.17	0.31	230.8

Table 4
Diffusive volatility and recession
This table reports the cross-sectional distribution of coefficient estimates and t-statistics of the NBER recession dummy in the univariate regressions of diffusive volatility. Panel A is for the sample of all stocks during 1962Q1-2019Q2 (stocks sample), Panel B is for the sample of stocks that have options traded on them during 1996Q1-2019Q2 (options sample), and Panel C is for the top 50 stocks ranked by market capitalization at the end of year 1996 in the options sample (big 50 sample). t-statistics are adjusted using the Newey-West method with 3 lags. N is the number of stocks.

	Mean	StdDev	Skew	P5	P25	P50	P75	P95	N
Panel A: Stocks sample									
$\overline{\Delta\sqrt{\mathcal{D}}}_i$	0.14	0.32	3.37	-0.21	0.02	0.11	0.22	0.57	11,964
$t(\overline{\Delta\sqrt{\mathcal{D}}}_i)$	1.72	2.50	0.80	-2.07	0.35	1.68	2.86	5.74	11,964
Panel B: Options sample									
$\overline{\Delta\sqrt{\mathcal{D}}}_i$	0.19	0.15	1.39	-0.01	0.09	0.17	0.26	0.45	2,460
$t(\overline{\Delta\sqrt{\mathcal{D}}}_i)$	2.91	3.67	15.77	-0.18	1.66	2.44	3.46	7.72	2,460
Panel C: Big 50 sample									
$\overline{\Delta\sqrt{\mathcal{D}}}_i$	0.14	0.10	0.64	0.00	0.07	0.12	0.20	0.36	50
$t(\overline{\Delta\sqrt{\mathcal{D}}}_i)$	2.29	1.23	-1.30	0.04	1.82	2.47	3.04	3.78	50

Table 5
Realized jump size and recession
This table reports the cross-sectional distribution of coefficient estimates and t-statistics of the NBER recession dummy in the univariate regressions of realized jump size. Panel A is for the sample of all stocks during 1962Q1-2019Q2 (stocks sample), Panel B is for the sample of stocks that have options traded on them during 1996Q1-2019Q2 (options sample), and Panel C is for the top 50 stocks ranked by market capitalization at the end of year 1996 in the options sample (big 50 sample). t-statistics are adjusted using the Newey-West method with 3 lags. N is the number of stocks.

	Mean	StdDev	Skew	P5	P25	P50	P75	P95	N
Panel A: Stocks sample									
$\overline{\Delta\sqrt{Z}_i}$	0.03	0.06	2.33	-0.05	0.00	0.02	0.04	0.13	11,766
$\overline{\Delta\sqrt{Z^+}_i}$	0.02	0.06	1.88	-0.05	0.00	0.02	0.04	0.12	11,292
$\overline{\Delta\sqrt{Z^-}_i}$	0.03	0.07	2.37	-0.05	0.00	0.02	0.05	0.14	11,273
$t(\overline{\Delta\sqrt{Z}_i})$	1.22	2.35	1.90	-2.14	-0.08	1.19	2.25	4.70	11,766
$t(\overline{\Delta\sqrt{Z^+}_i})$	1.13	2.35	2.04	-2.16	-0.15	1.10	2.15	4.50	11,292
$t(\overline{\Delta\sqrt{Z^-}_i})$	1.09	2.27	1.66	-2.15	-0.20	1.05	2.12	4.56	11,273
Panel B: Options sample									
$\overline{\Delta\sqrt{Z}_i}$	0.04	0.05	1.49	-0.02	0.01	0.03	0.06	0.12	2,439
$\overline{\Delta\sqrt{Z^+}_i}$	0.03	0.04	2.01	-0.02	0.01	0.03	0.05	0.10	2,345
$\overline{\Delta\sqrt{Z^-}_i}$	0.04	0.06	1.24	-0.03	0.01	0.03	0.06	0.14	2,356
$t(\overline{\Delta\sqrt{Z}_i})$	2.16	2.89	2.64	-1.21	0.87	1.82	3.00	6.39	2,439
$t(\overline{\Delta\sqrt{Z^+}_i})$	1.92	2.49	1.92	-1.39	0.60	1.70	2.79	5.96	2,345
$t(\overline{\Delta\sqrt{Z^-}_i})$	1.82	2.63	2.56	-1.54	0.47	1.56	2.66	5.98	2,356
Panel C: Big 50 sample									
$\overline{\Delta\sqrt{Z}_i}$	0.03	0.03	1.01	-0.01	0.01	0.03	0.05	0.11	50
$\overline{\Delta\sqrt{Z^+}_i}$	0.03	0.03	1.05	-0.01	0.01	0.03	0.05	0.09	50
$\overline{\Delta\sqrt{Z^-}_i}$	0.04	0.04	0.99	-0.01	0.01	0.03	0.05	0.12	50
$t(\overline{\Delta\sqrt{Z}_i})$	1.80	1.53	-1.18	-1.94	1.16	1.81	2.77	3.93	50
$t(\overline{\Delta\sqrt{Z^+}_i})$	1.66	1.42	-0.87	-1.10	1.08	1.86	2.25	4.27	50
$t(\overline{\Delta\sqrt{Z^-}_i})$	1.78	1.80	0.67	-0.79	0.71	1.69	2.52	4.49	50

Table 6
Realized jump intensity and recession
This table reports the cross-sectional distribution of coefficient estimates and t-statistics of the NBER recession dummy in the univariate regressions of realized jump intensity. Panel A is for the sample of all stocks during 1962Q1-2019Q2 (stocks sample), Panel B is for the sample of stocks that have options traded on them during 1996Q1-2019Q2 (options sample), and Panel C is for the top 50 stocks ranked by market capitalization at the end of year 1996 in the options sample (big 50 sample). t-statistics are adjusted using the Newey-West method with 3 lags. N is the number of stocks.

	Mean	StdDev	Skew	P5	P25	P50	P75	P95	N
Panel A: Stocks sample									
$\overline{\Delta\lambda_i}$	0.61	3.67	1.36	-4.33	-1.34	0.25	2.06	6.75	11,927
$\overline{\Delta\lambda^+{}_i}$	-0.13	2.30	0.78	-3.49	-1.35	-0.26	0.90	3.63	11,795
$\overline{\Delta\lambda^-{}_i}$	0.80	2.44	1.79	-2.27	-0.49	0.51	1.72	4.86	11,825
$t(\overline{\Delta\lambda_i})$	0.19	2.08	0.36	-3.05	-1.03	0.18	1.33	3.43	11,927
$t(\overline{\Delta\lambda^+{}_i})$	-0.29	2.02	0.27	-3.50	-1.47	-0.28	0.86	2.81	11,795
$t(\overline{\Delta\lambda^-{}_i})$	0.61	2.05	0.57	-2.51	-0.59	0.59	1.68	3.91	11,825
Panel B: Options sample									
$\overline{\Delta\lambda_i}$	-0.30	1.98	0.21	-3.36	-1.50	-0.33	0.88	3.00	2,443
$\overline{\Delta\lambda^+{}_i}$	-0.44	1.27	-0.15	-2.51	-1.22	-0.44	0.30	1.59	2,420
$\overline{\Delta\lambda^-{}_i}$	0.18	1.36	0.53	-1.79	-0.72	0.08	0.97	2.53	2,427
$t(\overline{\Delta\lambda_i})$	-0.45	2.20	-0.28	-4.01	-1.62	-0.31	0.83	2.84	2,443
$t(\overline{\Delta\lambda^+{}_i})$	-0.76	2.09	-0.31	-4.21	-1.96	-0.68	0.40	2.37	2,420
$t(\overline{\Delta\lambda^-{}_i})$	0.12	2.13	0.44	-3.23	-1.11	0.13	1.28	3.43	2,427
Panel C: Big 50 sample									
$\overline{\Delta\lambda_i}$	-0.20	1.43	0.61	-2.06	-1.36	-0.28	0.49	2.33	50
$\overline{\Delta\lambda^+{}_i}$	-0.38	0.77	0.01	-1.53	-0.80	-0.50	0.16	0.87	50
$\overline{\Delta\lambda^-{}_i}$	0.18	1.18	1.00	-1.36	-0.64	0.12	0.69	2.59	50
$t(\overline{\Delta\lambda_i})$	-0.34	1.47	0.16	-2.49	-1.38	-0.28	0.62	2.62	50
$t(\overline{\Delta\lambda^+{}_i})$	-0.70	1.33	0.07	-2.72	-1.80	-0.82	0.28	1.25	50
$t(\overline{\Delta\lambda^-{}_i})$	0.11	1.76	0.41	-2.78	-0.98	0.14	0.88	3.16	50

Table 7
Correlations and variances explained by principal components
This table reports the mean, standard deviation, and the 5th, 25th, 50th, 75th, and 95th percentiles of the 10,000 pair-wise quarterly time-series correlations and the fraction of variance explained by the first and the second principal components of jump and volatility variables from the 10×10 equally-weighted portfolios of all stocks based on the NYSE 10th, 20th, up to 90th percentiles of the market value of equity (ME) and book-to-market ratio of equity (BM).

	Mean	Std	P5	P25	P50	P75	P95	PC1	PC2
λ_{jq}	0.44	0.20	0.12	0.30	0.44	0.59	0.73	0.52	0.08
λ_{jq}^{+}	0.49	0.17	0.20	0.36	0.48	0.61	0.75	0.34	0.11
λ_{jq}^{-}	0.51	0.14	0.26	0.42	0.52	0.61	0.71	0.68	0.05
$\sqrt{Z_{jq}}$	0.69	0.11	0.47	0.62	0.71	0.77	0.84	0.69	0.07
$\sqrt{Z_{jq}^{+}}$	0.64	0.11	0.44	0.57	0.66	0.72	0.79	0.64	0.07
$\sqrt{Z_{jq}^{-}}$	0.67	0.10	0.49	0.61	0.69	0.74	0.81	0.68	0.06
$\sqrt{V_{jq}}$	0.73	0.15	0.42	0.66	0.77	0.83	0.89	0.74	0.10
$\sqrt{V_{jq}^{+}}$	0.71	0.14	0.42	0.64	0.75	0.81	0.87	0.72	0.10
$\sqrt{V_{jq}^{-}}$	0.74	0.15	0.43	0.68	0.79	0.85	0.90	0.75	0.10
$\sqrt{D_{jq}}$	0.73	0.16	0.38	0.66	0.78	0.84	0.90	0.77	0.08
$\sqrt{D_{jq}^{+}}$	0.72	0.15	0.39	0.65	0.77	0.83	0.89	0.73	0.11
$\sqrt{D_{jq}^{-}}$	0.74	0.16	0.36	0.67	0.79	0.85	0.90	0.75	0.12

Table 8

Relationship between realized jump intensity and realized diffusive variance

This table reports the time-series average of cross-sectional descriptive statistics of the regression coefficients. The following time-series regressions

$$\begin{aligned}
\lambda_{i,q-k} &= \alpha_{iq} + \beta_{iq} D_{i,q-k} + \varepsilon_{i,q-k}, \\
\lambda^{+}_{i,q-k} &= \alpha^{+}_{iq} + \beta^{+}_{iq} D_{i,q-k} + \varepsilon_{i,q-k}, \\
\lambda^{-}_{i,q-k} &= \alpha^{-}_{iq} + \beta^{-}_{iq} D_{i,q-k} + \varepsilon_{i,q-k},
\end{aligned}$$

are estimated for each stock-quarter iq, in a 3-year rolling window (k = 0, 1, 2, ..., 11), where $\lambda_{i,q-k}$, $\lambda^{+}_{i,q-k}$, and $\lambda^{-}_{i,q-k}$ are jump intensity, intensity of positive jump, and intensity of negative jump, respectively, and $D_{i,q-k}$ is diffusive variance. The t-statistics, $t(\beta_{iq})$, $t(\beta^{+}_{iq})$ and $t(\beta^{-}_{iq})$, are adjusted using the Newey-West method with 3 lags. Panel A reports the results for all the stocks in the sample period 1962Q1-2019Q2 (stocks sample). Panel B reports the results for the stocks with options in the sample period 1996Q1-2019Q2 (options sample). N is the average number of stocks in the cross section.

	Mean	Std	Skew	P5	P25	P50	P75	P95	N
Panel A: Stocks sample									
β_{iq}	-12.41	101.30	-3.22	-127.57	-30.91	-5.41	11.39	83.76	3608.5
β^{+}_{iq}	-10.32	74.54	-3.12	-94.69	-23.32	-4.21	7.27	57.91	3608.2
β^{-}_{iq}	-2.09	62.11	-1.24	-70.46	-14.54	-0.93	11.01	64.67	3608.1
$t(\beta_{iq})$	-0.78	4.07	-1.23	-6.69	-2.53	-0.64	1.14	4.72	3608.5
$t(\beta^{+}_{iq})$	-0.86	4.04	-0.47	-6.74	-2.60	-0.75	1.03	4.62	3608.2
$t(\beta^{-}_{iq})$	-0.15	4.04	-0.22	-5.63	-1.95	-0.21	1.61	5.49	3608.1
Panel B: Options sample									
β_{iq}	-11.37	59.45	0.17	-101.81	-30.22	-6.98	9.59	66.97	1758.9
β^{+}_{iq}	-10.63	42.44	-0.32	-77.10	-23.24	-6.01	5.29	42.11	1758.9
β^{-}_{iq}	-0.74	41.04	0.71	-59.03	-14.08	-0.98	11.90	58.94	1758.9
$t(\beta_{iq})$	-0.77	3.38	-0.23	-6.20	-2.49	-0.68	1.05	4.34	1758.9
$t(\beta^{+}_{iq})$	-0.96	3.41	-0.19	-6.40	-2.69	-0.88	0.86	4.18	1758.9
$t(\beta^{-}_{iq})$	-0.05	3.35	0.41	-5.08	-1.84	-0.12	1.63	5.24	1758.9

Table 9
Relationship between realized jump intensity and realized diffusive variance (instrumental variable approach)
This table reports the time-series average of cross-sectional descriptive statistics of the regression coefficients. The following time-series regressions

$$\begin{aligned}\lambda_{i,q-k} &= \alpha_{iq} + \beta_{iq} D_{i,q-k} + \varepsilon_{i,q-k},\\ \lambda^{+}_{i,q-k} &= \alpha^{+}_{iq} + \beta^{+}_{iq} D_{i,q-k} + \varepsilon_{i,q-k},\\ \lambda^{-}_{i,q-k} &= \alpha^{-}_{iq} + \beta^{-}_{iq} D_{i,q-k} + \varepsilon_{i,q-k},\end{aligned}$$

are estimated for each stock-quarter iq, in a 3-year rolling window ($k = 0$, 1, 2, ..., 11), with $D_{i,q-k-1}$ as the instrumental variable, where $\lambda_{i,q-k}$, $\lambda^{+}_{i,q-k}$, and $\lambda^{-}_{i,q-k}$ are jump intensity, intensity of positive jump, and intensity of negative jump, respectively, and $D_{i,q-k}$ is diffusive variance. The t-statistics, $t(\beta_{iq})$, $t(\beta^{+}_{iq})$ and $t(\beta^{-}_{iq})$, are adjusted using the Newey-West method with 3 lags. Panel A reports the results for all the stocks in the sample period 1962Q1-2019Q2 (stocks sample). Panel B reports the results for the stocks with options in the sample period 1996Q1-2019Q2 (options sample). N is the average number of stocks in the cross section.

	Mean	Std	Skew	P5	P25	P50	P75	P95	N
Panel A: Stocks sample									
β_{iq}	-13.00	3387	4.19	-215.39	-47.25	-9.54	10.79	122.21	3540.8
β^{+}_{iq}	-6.73	2175	1.33	-150.17	-30.89	-4.90	10.59	97.26	3540.6
β^{-}_{iq}	-6.27	1932	-0.04	-127.47	-24.93	-3.76	9.24	85.80	3540.3
$t(\beta_{iq})$	-1.07	3.84	-0.78	-6.94	-2.76	-0.88	0.85	4.16	3540.8
$t(\beta^{+}_{iq})$	-0.74	3.93	-0.15	-6.45	-2.49	-0.66	1.11	4.69	3540.5
$t(\beta^{-}_{iq})$	-0.73	3.76	-0.62	-6.27	-2.37	-0.64	1.07	4.39	3540.3
Panel B: Options sample									
β_{iq}	-78.52	4701	-0.30	-166.25	-45.21	-12.50	7.24	78.40	1758.3
β^{+}_{iq}	-33.21	2447	-1.03	-113.36	-30.43	-7.38	6.50	60.89	1758.3
β^{-}_{iq}	-45.31	2551	-1.43	-102.58	-23.81	-4.51	9.69	67.91	1758.3
$t(\beta_{iq})$	-1.27	3.27	-0.67	-6.67	-2.86	-1.08	0.55	3.45	1758.3
$t(\beta^{+}_{iq})$	-1.04	3.34	-0.51	-6.41	-2.69	-0.93	0.74	3.90	1758.3
$t(\beta^{-}_{iq})$	-0.68	3.13	-0.19	-5.67	-2.25	-0.62	0.98	4.02	1758.3

Table 10
Relationship between realized jump size and realized diffusive variance
This table reports the time-series average of cross-sectional descriptive statistics of the regression coefficients. The following time-series regressions

$$
\begin{aligned}
Z_{i,q-k} &= \gamma_{iq} + \theta_{iq} D_{i,q-k} + \varepsilon_{i,q-k}, \\
Z^{+}_{i,q-k} &= \gamma^{+}_{iq} + \theta^{+}_{iq} D_{i,q-k} + \varepsilon_{i,q-k}, \\
Z^{-}_{i,q-k} &= \gamma^{-}_{iq} + \theta^{-}_{iq} D_{i,q-k} + \varepsilon_{i,q-k},
\end{aligned}
$$

are estimated for each stock-quarter iq, in a 3-year rolling window ($k = 0$, 1, 2, ..., 11), where $Z_{i,q-k}$, $Z^{+}_{i,q-k}$, and $Z^{-}_{i,q-k}$ are jump size, size of positive jump, and size of negative jump, respectively, and $D_{i,q-k}$ is diffusive variance. The t-statistics, $t(\theta_{iq})$, $t(\theta^{+}_{iq})$ and $t(\theta^{-}_{iq})$, are adjusted using the Newey-West method with 3 lags. Panel A reports the results for all the stocks in the sample period 1962Q1-2019Q2 (stocks sample). Panel B reports the results for the stocks with options in the sample period 1996Q1-2019Q2 (options sample). N is the average number of stocks in the cross section.

	Mean	Std	Skew	P5	P25	P50	P75	P95	N
Panel A: Stocks sample									
θ_{iq}	0.05	0.10	3.16	-0.03	0.01	0.03	0.07	0.16	3401.5
θ^{+}_{iq}	0.04	0.10	4.77	-0.04	0.01	0.03	0.07	0.16	3261.4
θ^{-}_{iq}	0.05	0.17	5.58	-0.06	0.00	0.03	0.07	0.22	3103.5
$t(\theta_{iq})$	4.55	5.73	3.17	-1.64	1.20	3.41	6.60	14.45	3401.5
$t(\theta^{+}_{iq})$	3.92	5.80	3.90	-2.27	0.77	2.91	5.91	13.32	3261.4
$t(\theta^{-}_{iq})$	3.72	6.05	3.03	-2.90	0.46	2.73	5.84	13.56	3103.4
Panel B: Options sample									
θ_{iq}	0.06	0.12	3.63	-0.04	0.02	0.04	0.08	0.22	1756.9
θ^{+}_{iq}	0.05	0.11	3.64	-0.05	0.01	0.04	0.08	0.19	1731.2
θ^{-}_{iq}	0.07	0.24	4.57	-0.10	0.01	0.04	0.09	0.33	1722.5
$t(\theta_{iq})$	4.91	5.66	2.64	-1.23	1.46	3.72	7.01	14.83	1756.9
$t(\theta^{+}_{iq})$	4.24	5.85	3.29	-1.83	0.96	3.14	6.19	13.78	1731.2
$t(\theta^{-}_{iq})$	3.90	5.90	3.30	-2.30	0.63	2.88	5.91	13.33	1722.5

Table 11
Relationship between realized jump size and realized diffusive variance (instrumental variable approach)
This table reports the time-series average of cross-sectional descriptive statistics of the regression coefficients. The following time-series regressions

$$
\begin{aligned}
Z_{i,q-k} &= \gamma_{iq} + \theta_{iq} D_{i,q-k} + \varepsilon_{i,q-k}, \\
Z^{+}_{i,q-k} &= \gamma^{+}_{iq} + \theta^{+}_{iq} D_{i,q-k} + \varepsilon_{i,q-k}, \\
Z^{-}_{i,q-k} &= \gamma^{-}_{iq} + \theta^{-}_{iq} D_{i,q-k} + \varepsilon_{i,q-k},
\end{aligned}
$$

are estimated for each stock-quarter iq, in a 3-year rolling window ($k = 0$, 1, 2, ..., 11), with $D_{i,q-k-1}$ as the instrumental variable, where $Z_{i,q-k}$, $Z^{+}_{i,q-k}$, and $Z^{-}_{i,q-k}$ are jump size, size of positive jump, and size of negative jump, respectively, and $D_{i,q-k}$ is diffusive variance. The t-statistics, $t(\theta_{iq})$, $t(\theta^{+}_{iq})$ and $t(\theta^{-}_{iq})$, are adjusted using the Newey-West method with 3 lags. Panel A reports the results for all the stocks in the sample period 1962Q1-2019Q2 (stocks sample). Panel B reports the results for the stocks with options in the sample period 1996Q1-2019Q2 (options sample). N is the average number of stocks in the cross section.

	Mean	Std	Skew	P5	P25	P50	P75	P95	N
Panel A: Stocks sample									
θ_{iq}	0.04	2.15	-1.96	-0.07	0.01	0.04	0.07	0.21	3337.4
θ^{+}_{iq}	0.02	3.42	-3.11	-0.09	0.01	0.03	0.08	0.22	3201.2
θ^{-}_{iq}	0.04	2.68	-2.51	-0.13	0.00	0.03	0.08	0.29	3043.5
$t(\theta_{iq})$	3.44	4.73	2.61	-2.05	0.72	2.65	5.25	11.57	3337.4
$t(\theta^{+}_{iq})$	3.22	5.20	3.19	-2.55	0.43	2.41	5.05	11.68	3201.2
$t(\theta^{-}_{iq})$	2.79	5.26	2.62	-3.32	0.03	2.06	4.69	11.32	3043.5
Panel B: Options sample									
θ_{iq}	0.04	1.97	1.67	-0.08	0.02	0.05	0.09	0.27	1756.0
θ^{+}_{iq}	0.02	2.15	2.90	-0.09	0.01	0.04	0.09	0.24	1730.1
θ^{-}_{iq}	0.09	1.72	5.76	-0.17	0.00	0.04	0.11	0.42	1721.5
$t(\theta_{iq})$	3.77	4.62	2.44	-1.55	1.01	2.93	5.56	11.85	1756.0
$t(\theta^{+}_{iq})$	3.49	5.09	3.07	-2.08	0.65	2.61	5.27	11.89	1730.1
$t(\theta^{-}_{iq})$	3.07	4.89	2.42	-2.53	0.32	2.27	4.84	11.33	1721.5

Table 12
Summary statistics for the sample of stocks with options traded
This table reports the time-series averages of the mean, standard deviation, skewness, kurtosis, and the 5th, 25th, 50th, 75th, 95th percentiles of the cross-sectional distributions of the following variables. $\overline{\text{SM}}_{iq}$ is the average of weekly implied volatility smile measure in the past 3 years until quarter q for stock i, in annual percentage terms. β_{iq} and α_{iq} are the slope coefficient and intercept of the regression of jump intensity, λ_{iq}, on diffusive variance, D_{iq}, in the past 3 years, respectively. $\overline{D}_{iq}$ and $\sigma(D_{iq})$ are the average and standard deviation of D_{iq} in the past 3 years, respectively, and $\sqrt{\bar{Z}_{iq}}$ is the average of jump size, $\sqrt{Z_{iq}}$. The sample include the stocks with options in the period 1996Q1-2019Q2.

	Mean	Std	Skew	P5	P25	P50	P75	P95
$\overline{\text{SM}}_{iq}$	3.44	3.90	2.42	0.24	0.96	2.17	4.64	11.03
α_{iq}	6.28	4.32	0.52	-0.17	3.57	6.06	8.75	13.43
$\overline{D}_{iq}$	0.18	0.15	2.34	0.04	0.08	0.14	0.25	0.47
$\sigma(D_{iq})$	0.08	0.09	6.17	0.01	0.03	0.05	0.10	0.24
$\sqrt{\bar{Z}_{iq}}$	0.11	0.05	1.58	0.05	0.07	0.09	0.13	0.20

Table 13
Jump-diffusion beta and options pricing
This table reports the coefficient estimates of the following Fama-Macbeth cross-sectional regression,

$$\overline{\mathrm{SM}}_{iq} = b_0 + b_1\beta_{iq} + b_2\alpha_{iq} + b_3\overline{D}_{iq} + b_4\sqrt{\bar{Z}_{iq}} + b_5\sigma(D_{iq}) + \varepsilon_{iq},$$

where $\overline{\mathrm{SM}}_{iq}$ is the average of weekly implied volatility smile measure in the past 3 years until quarter q for stock i, β_{iq} and α_{iq} are the slope coefficient and intercept of the regression of jump intensity, λ_{iq}, on diffusive variance, D_{iq}, in the past 3 years, respectively, $\overline{D}_{iq}$ and $\sigma(D_{iq})$ are the average and standard deviation of D_{iq} in the past 3 years, respectively, and $\sqrt{\bar{Z}_{iq}}$ is the average of jump size, $\sqrt{Z_{iq}}$, in the past 3 years. The coefficient of β_{iq} is multiplied by 100. t-statistics adjusted using the Newey-West method with 11 lags are reported in parentheses. $\bar{R}^2$ is the average R^2 of the cross-sectional regressions. The sample include the stocks with options in the period 1996Q1-2019Q2.

β_{iq}	α_{iq}	$\overline{D}_{iq}$	$\sqrt{\bar{Z}_{iq}}$	$\sigma(D_{iq})$	$\bar{R}^2$
0.4588	0.0284	-1.2173			0.0112
(2.97)	(2.61)	(-1.38)			
0.4699	0.0301	-0.8392	-0.8334		0.0129
(2.96)	(2.52)	(-0.97)	(-0.36)		
0.4650	0.0297	-0.9096	-0.7091	0.1423	0.0148
(2.95)	(2.40)	(-1.37)	(-0.33)	(0.08)	

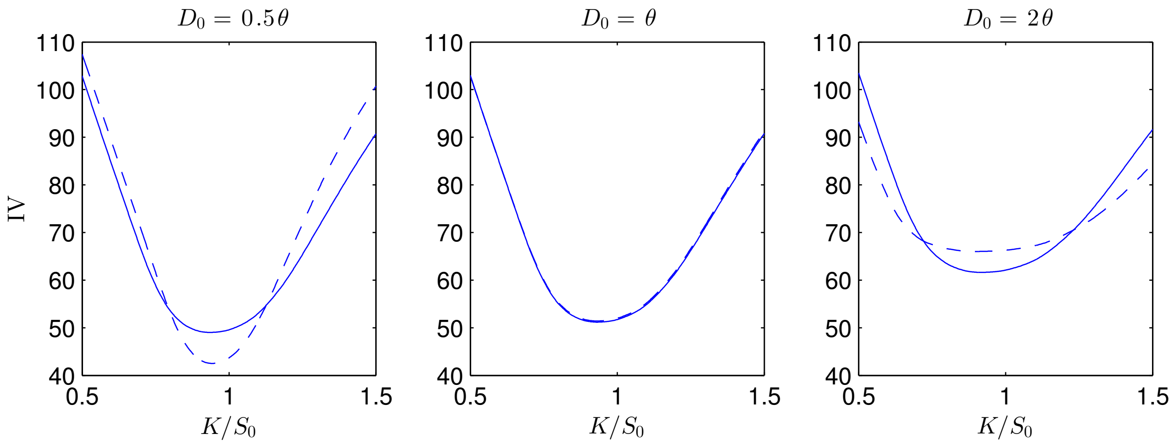

Figure 1. Implied volatility curve

This figure shows the one-month maturity implied volatlity in percetage and annual terms as a function of moneyness $K/\exp(S_0)$ from the simulated data, where K is the strike price and S_0 is the current log underlying stock price. The three panels show the cases of different values of diffusive volatility, D_0, relative to its unconditional mean, θ. The dash line is for the model with the exponential affine jump intensity and the solid line is for the model with the affine jump intensity.