Final-term for Engineering Mathematics 06/21/2020

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[Number your answer sheets sequentially and upload them in order before the deadline]

1. **[25]** Answer the following questions.

   [3]   (a) Find the Laplace transform of $f(t) = sinh\, t\, cos\, t$. Apply s-shifting theorem.

   [3]   (b) Solve initial value problem by the LT: $y'' - 4y' + 3y = 6t - 8$, $y(0) = 0$, $y'(0) = 0$.

   [3]   (c) Evaluate $L^{-1}\left\{\frac{s^2+6s+9}{(s-1)(s-2)(s+4)}\right\}$.

   [3]   (d) Find the Laplace transform of $f(t) = sin\, t\ (2\pi < t < 4\pi)$. Apply t-shifting theorem.

   [5]   (e) Using Laplace transform, solve the IVP. $y'' + 9y = 8\sin t$, if $0 < t < \pi$ and 0 if $t > \pi$; $y(0) = 0, y'(0) = 4$.

   [3]   (f) Find the Laplace transform by differentiation of $f(t) = te^{-t}\cos t$.

   [2]   (g) Find the inverse transform by convolution, if $L(f) = \frac{\omega}{s^2(s^2-\omega^2)}$.

   [3]   (h) $L\{f(t)\} = \frac{2s^3}{s^4-81}$, Find $f(t)$.

2. **[20]** Answer the following questions

   [2]   (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & -2 & 3 & -4 \\ 2 & -3 & 4 & -1 \\ 3 & -4 & 1 & -2 \\ 4 & -1 & 2 & -3 \end{bmatrix}$.

   [2]   (b) Find the determinant of the matrix $A = \begin{bmatrix} 0 & 2 & 2 & 2 \\ 2 & 1 & 3 & 2 \\ 1 & 3 & 3 & 3 \\ 1 & 2 & 3 & 2 \end{bmatrix}$.

   [3+5]   (c) Find the inverse of the matrices: $A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ by determinants and $B = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 3 & 1 & 1 \\ -1 & -1 & 2 & 3 \\ 1 & 3 & 3 & 4 \end{bmatrix}$ by

   Gauss-Jordan elimination method.

   [3+5]   (d) Find the eigen-values and eigen-vectors of matrices: $A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$, and $B = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix}$.

3. **[15]** Answer the following questions

   [3]   (a) Find the angles of the triangle ABC whose vertices are A: (1,1,0), B: (5,3,0), and C: (2,8,0).

   [2]   (b) Let $\vec{a} = [1,2,0], \vec{b} = [-3,2,0], \vec{c} = [2,3,4], \vec{d} = [6,-7,2]$. Find $(a \times b) \cdot (c \times d)$.

   [3]   (c) Find the gradient of the function $f = \sin(x + z)$ at $(\frac{\pi}{8}, 1, \frac{\pi}{8})$.

   [3]   (d) Find the Curl of v, where $v = \frac{x\hat{\imath}+y\hat{\jmath}+z\hat{k}}{(x^2+y^2+z^2)^{3/2}}$.

   [4]   (e) Show that $\int_{0,2,3}^{1,1,1}(yz \sinh xy\, dx + \cosh xy\, dy + xy \sinh xz\, dz)$ is independent on path and exact. Evaluate the integral.