2021

Final-term for Engineering Mathematics 06/21/~~2020~~

Dept. of CSIE, NTUST, Taiwan

Instructor: Prof. Binayak Kar

[Number your answer sheets sequentially and upload them in order before the deadline]

1. [25] Answer the following questions.

[3] (a) Find the Laplace transform of $f(t) = sinh\ t\ cos\ t$. Apply s-shifting theorem.

[3] (b) Solve initial value problem by the LT: $y'' - 4y' + 3y = 6t - 8,\ y(0) = 0,\ y'(0) = 0$.

[3] (c) Evaluate $L^{-1}\left\{\frac{s^2+6s+9}{(s-1)(s-2)(s+4)}\right\}$.

[3] (d) Find the Laplace transform of $f(t) = sin\ t\ (2\pi < t < 4\pi)$. Apply t-shifting theorem.

[5] (e) Using Laplace transform, solve the IVP. $y'' + 9y = 8\sin t$, if $0 < t < \pi$ and 0 if $t > \pi$; $y(0) = 0, y'(0) = 4$.

[3] (f) Find the Laplace transform by differentiation of $f(t) = te^{-t}\cos t$.

[2] (g) Find the inverse transform by convolution, if $L(f) = \frac{\omega}{s^2(s^2-\omega^2)}$.

[3] (h) $L\{f(t)\} = \frac{2s^3}{s^4-81}$, Find $f(t)$.

2. [20] Answer the following questions

[2] (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & -2 & 3 & -4 \\ 2 & -3 & 4 & -1 \\ 3 & -4 & 1 & -2 \\ 4 & -1 & 2 & -3 \end{bmatrix}$.

[2] (b) Find the determinant of the matrix $A = \begin{bmatrix} 0 & 2 & 2 & 2 \\ 2 & 1 & 3 & 2 \\ 1 & 3 & 3 & 3 \\ 1 & 2 & 3 & 2 \end{bmatrix}$.

[3+5] (c) Find the inverse of the matrices: $A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ by determinants and $B = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 3 & 1 & 1 \\ -1 & -1 & 2 & 3 \\ 1 & 3 & 3 & 4 \end{bmatrix}$ by Gauss-Jordan elimination method.

[3+5] (d) Find the eigen-values and eigen-vectors of matrices: $A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$, and $B = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix}$.

3. [15] Answer the following questions

[3] (a) Find the angles of the triangle ABC whose vertices are A: (1,1,0), B: (5,3,0), and C: (2,8,0).

[2] (b) Let $\vec{a} = [1,2,0], \vec{b} = [-3,2,0], \vec{c} = [2,3,4], \vec{d} = [6,-7,2]$. Find $(a \times b) \cdot (c \times d)$.

[3] (c) Find the gradient of the function $f = \sin(x + z)$ at $(\frac{\pi}{8}, 1, \frac{\pi}{8})$.

[3] (d) Find the Curl of v, where $v = \frac{x\hat{\imath}+y\hat{\jmath}+z\hat{k}}{(x^2+y^2+z^2)^{3/2}}$.

[4] (e) Show that $\int_{0,2,3}^{1,1,1}(yz\sinh xy\ dx + \cosh xy\ dy + \text{xy}\sinh xz\ dz)$ is independent on path and exact. Evaluate the integral.