Final-term for Engineering Mathematics 06/19/2020

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[Number your answer sheets sequentially and upload them in order before deadline]

1. **[20]** Answer the following questions.

   [3]   (a) Find the Laplace transform of $f(t) = sinh\ t\ cos\ t$. Apply s-shifting theorem.

   [3]   (b) Solve initial value problem by the Laplace transform $y' + 3y = 10 \sin t$, $y(0) = 0$.

   [3]   (c) Find the inverse transform by integration. $L(f) = \frac{4}{s^3+2s^2}$, find $f(t)$.

   [3]   (d) Find the Laplace transform of $f(t) = \sin t\ (2\pi < t < 4\pi)$. Apply t-shifting theorem.

   [5]   (e) Using Laplace transform solve the initial value problem. $y'' + 9y = 8 \sin t$, if $0 < t < \pi$ and 0 if $t > \pi$; $y(0) = 0, y'(0) = 4$.

   [3]   (f) Find the Laplace transform by differentiation of $f(t) = te^{-t} \cos t$.

2. **[15]** Answer the following questions

   [2]   (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & -2 & 3 & -4 \\ 2 & -3 & 4 & -1 \\ 3 & -4 & 1 & -2 \\ 4 & -1 & 2 & -3 \end{bmatrix}$.

   [4]   (b) Find the inverse of the matrix $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ by Determinants.

   [4]   (c) Find the inverse of the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ by Gauss-Jordan elimination method.

   [5]   (d) Find the eigen values and eigen vectors of matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

3. **[15]** Answer the following questions

   [3]   (a) Find the angles of the triangle ABC whose vertices are A: (1,1,0), B: (5,3,0), and C: (2,8,0).

   [2]   (b) Let $a = [1,2,0], b = [-3,2,0], c = [2,3,4], d = [6,-7,2]$. Find $(a \times b) \cdot (c \times d)$.

   [3]   (c) Find the gradient of the function $f = z/(x^2 + y^2)$ at (0,1,2).

   [3]   (d) Find the Curl of v, where $v = xyz[x, y, z]$.

   [4]   (e) Show that $\int_{0,\pi}^{3,\pi/2} e^x(\cos y\ dx - \sin y\ dy)$ is independent on path and exact. Evaluate the integral.

4. **[10]** Answer the following questions.

   [4]   (a) Find the Fourier series of the function $f(x) = x^2$, $(-\pi < x < \pi)$.

   [3]   (b) Find the Fourier series of the function $f(x) = \begin{cases} 0, & (-2 < x < 0) \\ 2, & (0 < x < 2) \end{cases}$

   [3]   (b) $f(x) = e^{|x|}, -\pi < x < \pi$. Test whether $f(x)$ is even or odd.