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Content

1\. A is a nonsingular matrix, so $\det(A) \neq 0$, and there exist A^{-1} such that $A^{-1}A = I$

since $\det(I) = 1$, $\det(A^{-1}A) = 1$

$\det(A^{-1}A) = \det(A^{-1})\det(A)$

$\Rightarrow \det(A^{-1})\det(A) = 1 \Rightarrow \det(A^{-1}) = \frac{1}{\det(A)}$ ※

2\.

a. $$\det(\operatorname{adj}(A)) = \begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 4 & 3 & 2 \\ 0 & -2 & -1 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 & 0 \\ 4 & 3 & 2 \\ -2 & -1 & 2 \end{vmatrix}$$

$$= 2 \cdot \left(2\begin{vmatrix} 3 & 2 \\ -1 & 2 \end{vmatrix} - 1\begin{vmatrix} 4 & 2 \\ -2 & 2 \end{vmatrix}\right)$$

$$= 2 \cdot [2 \cdot 8 - 12] = \underline{8}_{\#}$$

$(\det(A))^3 = \det(\operatorname{adj}(A)) = 8 \Rightarrow \det(A) = \underline{2}_{\#}$

b. $$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & -1 & 1 \\ 0 & -6 & 2 & -2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$ #

3\. $\forall$ nonsingular matrix A, $\det(A^{-1}) = \frac{1}{\det(A)}$

$$\det(B) = \det(S^{-1}AS)$$
$$= \det(S^{-1})\det(A)\det(S)$$
$$= \frac{1}{\det(S)}\det(A)\det(S)$$
$$= \det(A)$$

4.

$$\left[\begin{array}{ccc|c} 2 & 3 & 6 & 0 \\ 1 & 2 & -1 & 0 \\ -2 & -3 & -6 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 15 & 0 \\ 0 & 1 & -8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x_1 + 15x_3 = 0 \\ x_2 - 8x_3 = 0 \\ x_3 = x_3 \end{cases} \qquad N(A) = \left\{ \begin{bmatrix} -15x_3 \\ 8x_3 \\ x_3 \end{bmatrix} \middle| \, x_3 \in \mathbb{R} \right\}$$

5. a. no b. yes

6. a. $c_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} c_1 & c_2 \\ c_2 & c_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad c_1 = c_2 = 0$$

the only linear combination of two matrix is equal to $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

yes.#

b.

$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 2 & 1 & 3 & 0 \\ 4 & 2 & 6 & 0 \end{bmatrix} \begin{matrix} \\ -2 \\ -4 \end{matrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 3 & -1 & 0 \\ 0 & 6 & -2 & 0 \end{bmatrix} -2 \rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

No#