

$$7. S = U^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & \frac{1}{2} & -1 \\ 1 & -\frac{1}{2} & 0 \end{pmatrix}$$

$$8. A \Rightarrow \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$(a) \{ \{1, 0, 2\}^T, \{0, 1, 0\}^T \}$$

$$(b) \{ \{1, 2, 4\}^T, \{3, 1, 1\}^T \}$$

$$(c) \text{ set } x_3 = \alpha, \alpha \in \mathbb{R}.$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -2\alpha \\ 0 \\ \alpha \end{pmatrix} \Rightarrow \left\{ \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \right\}$$

9.

$$A \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 1 \\ -\frac{14}{5} & \frac{6}{5} \end{pmatrix}$$

$$L\left(\begin{pmatrix} 8 \\ 7 \end{pmatrix}\right) = \begin{pmatrix} 0 & 1 \\ -\frac{14}{5} & \frac{6}{5} \end{pmatrix} \begin{pmatrix} 8 \\ 7 \end{pmatrix} = \begin{pmatrix} 7 \\ -14 \end{pmatrix}$$

10.

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$$

11. Let,  $P, Q$  is nonsingular matrix

$$B = P^{-1}AP, \quad C = Q^{-1}BQ \quad \because \det(PQ) \neq 0$$

$$C = \underbrace{Q^{-1}(P^{-1}AP)Q}_{= (PQ)^{-1}A(PQ)} \quad \rightarrow \quad \therefore C \sim A$$

12.

a.  $U = \begin{bmatrix} 2 & 7 \\ 1 & 4 \end{bmatrix}$

$$V = \begin{bmatrix} 7 & 4 \\ 5 & 3 \end{bmatrix}$$

$$U^{-1} = \begin{bmatrix} 4 & -7 \\ -1 & 2 \end{bmatrix}$$

$$S = U^{-1}V = \begin{bmatrix} -7 & -5 \\ 3 & 2 \end{bmatrix} \quad \text{✗}$$

b.  $B = S^{-1}AS$

$$= \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} \quad \text{✗}$$