

# Probability Introduction



Week 02 - Day 04

# Probability

Probability = chance of an event (coin flip, result of a match, etc.)

$$P(\text{head}) = 0.5 = 50\%$$

$$P(\text{tail}) = 0.5 = 50\%$$

# Axioms

## Axiom 1: non negativity

The probability of an event is greater or equal to zero

$$P(A) \geq 0$$

$$P(\text{Head}) = -0.7 \text{ ???}$$

## **Axiom 2: unit measure**

The probability of all the possible events is 1

$$P(\text{head}) + P(\text{tail}) = 1$$

## **Axiom 2: additivity**

$$P(\text{Head or Tail}) = P(\text{head}) + P(\text{Tails})$$

(disjoint events = “non overlapping” = mutually exclusive)

# Properties



# **1 - Probability of No Event**

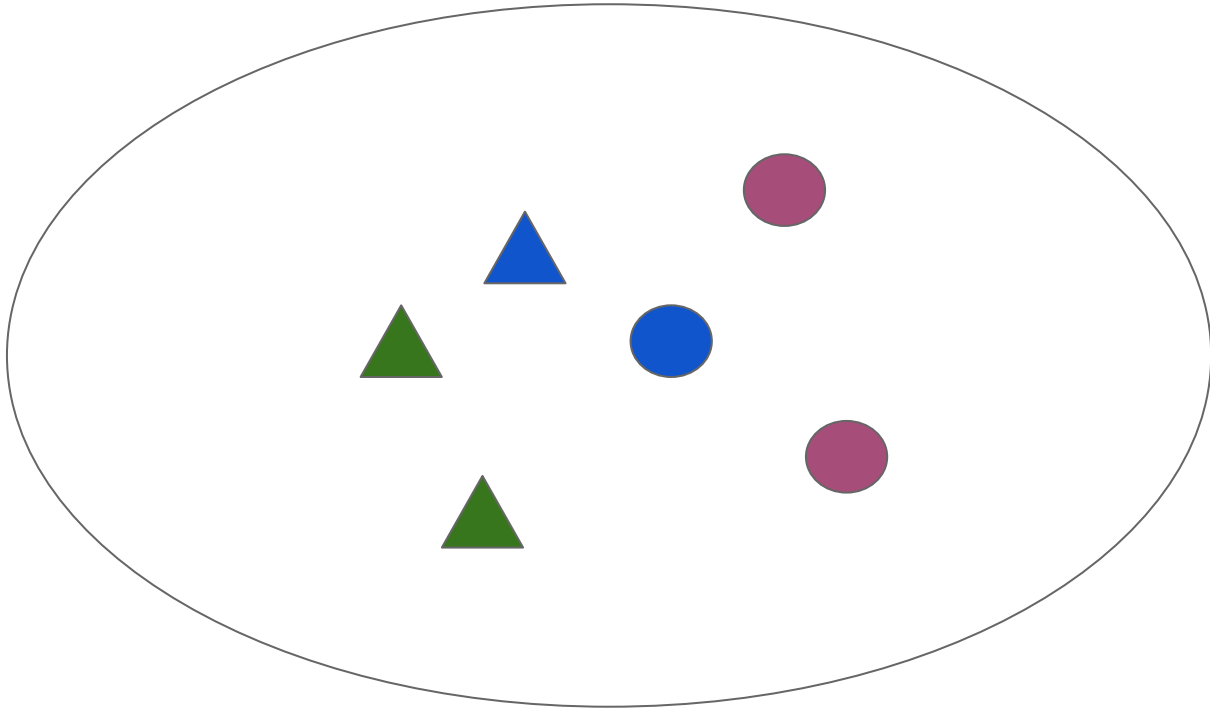
is zero!... (it's either tail or head)

## 2 - Probability of A or B (union)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

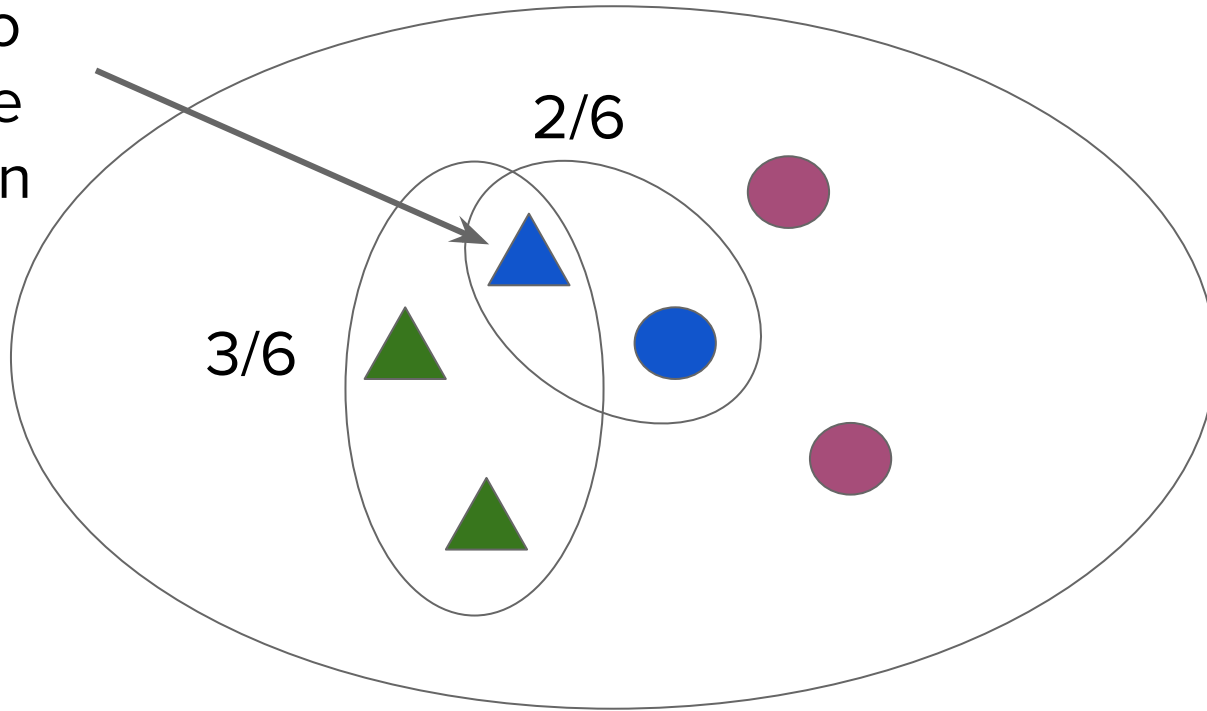
If no overlapping  $\rightarrow$  just  $P(A) + P(B)$

$$P(\text{triangle} \cup \text{blue}) = ?$$



$$P(\text{triangle} \cup \text{blue}) = ?$$

We have to  
remove the  
intersection



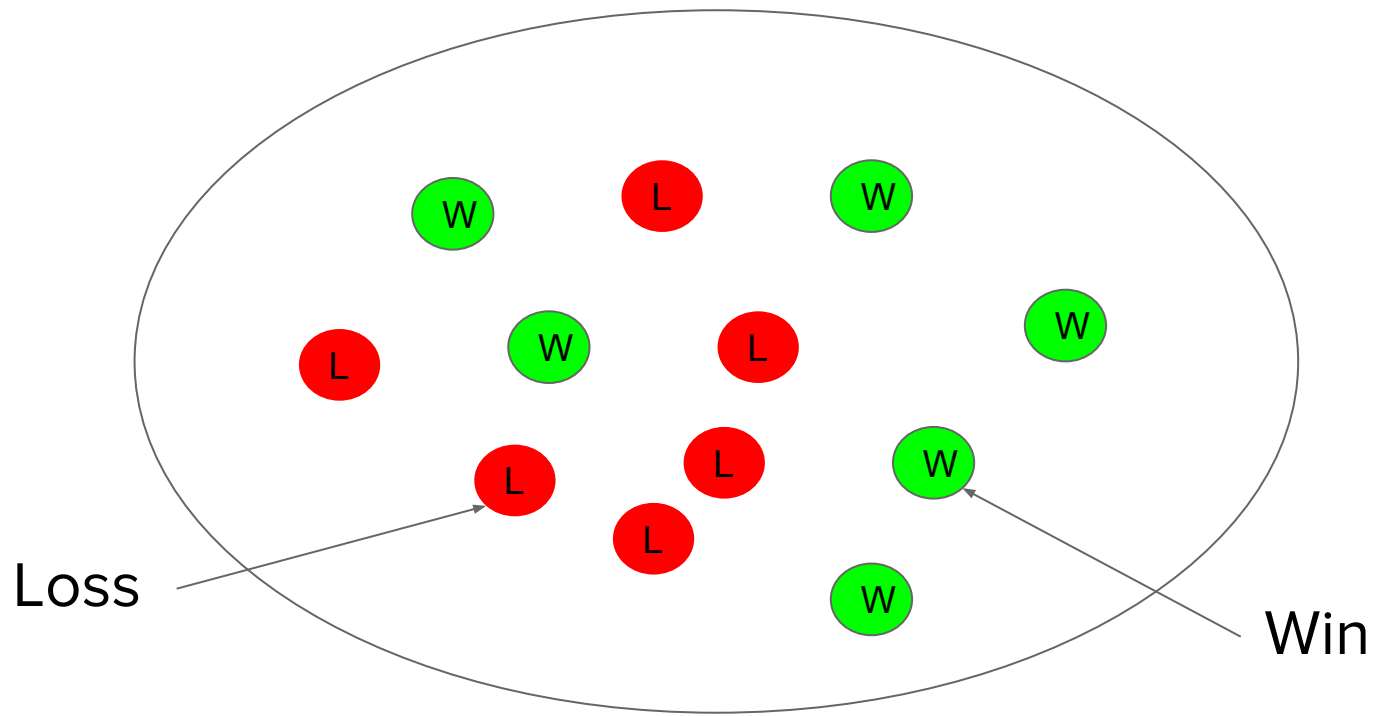
## 3 - Conditional Probability

$$P(A|B) = P(A \cap B) / P(B)$$

- 1) Useful for ML
- 2) Example:  $P(\text{good movie} \mid \text{category}=\text{romantic})$

## Example

$P(\text{win} \mid \text{play against juventus})$

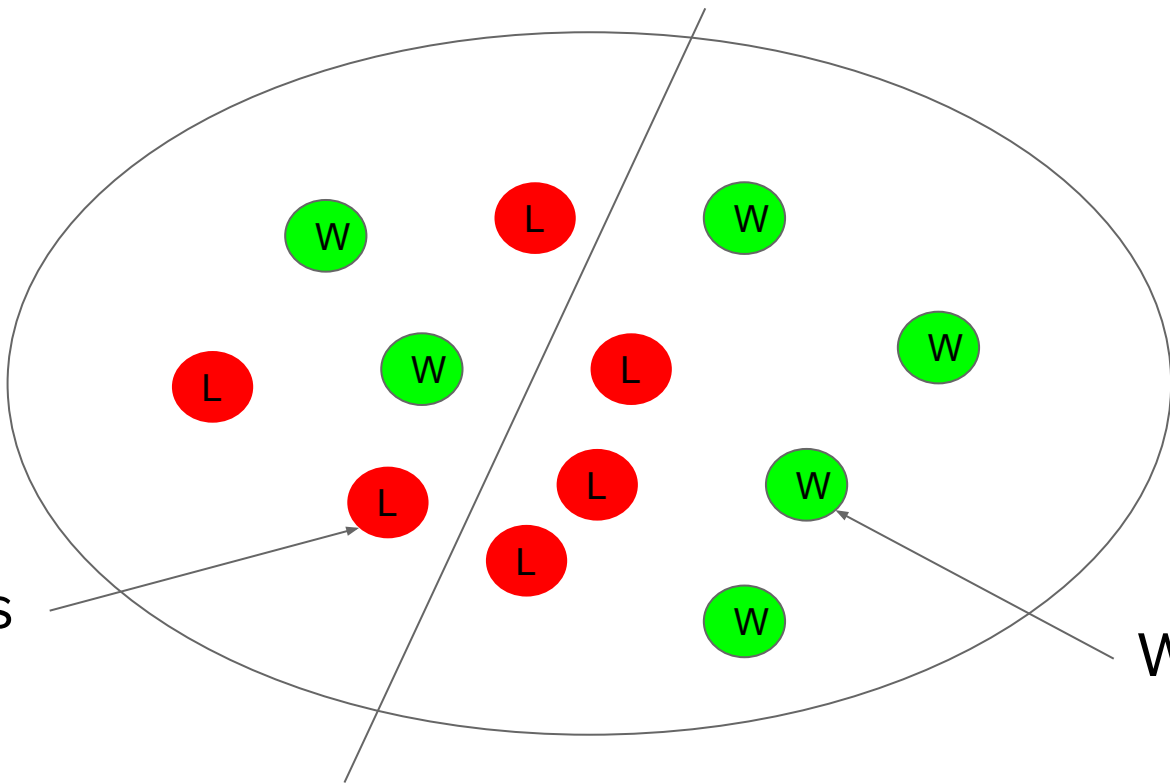


Matches  
against  
other teams

Matches  
against  
juve

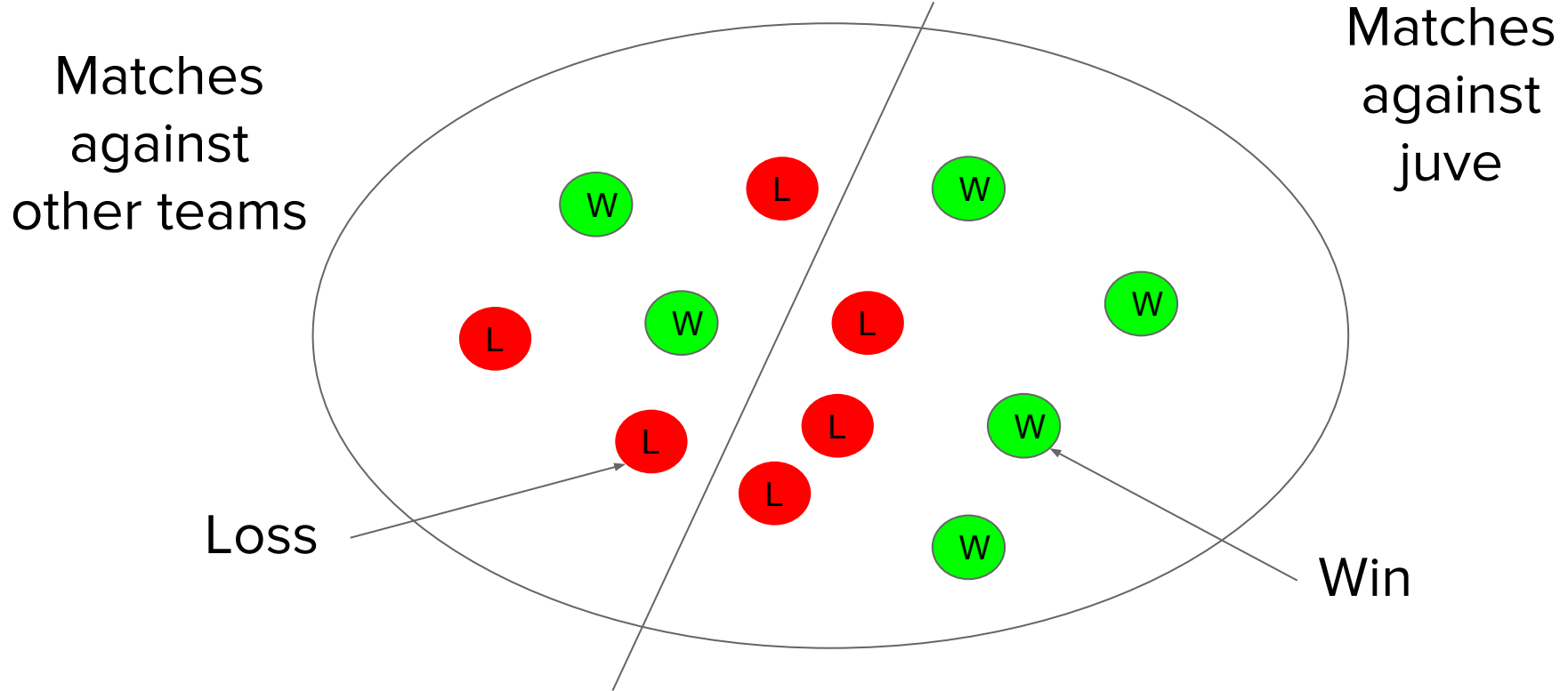
Loss

Win

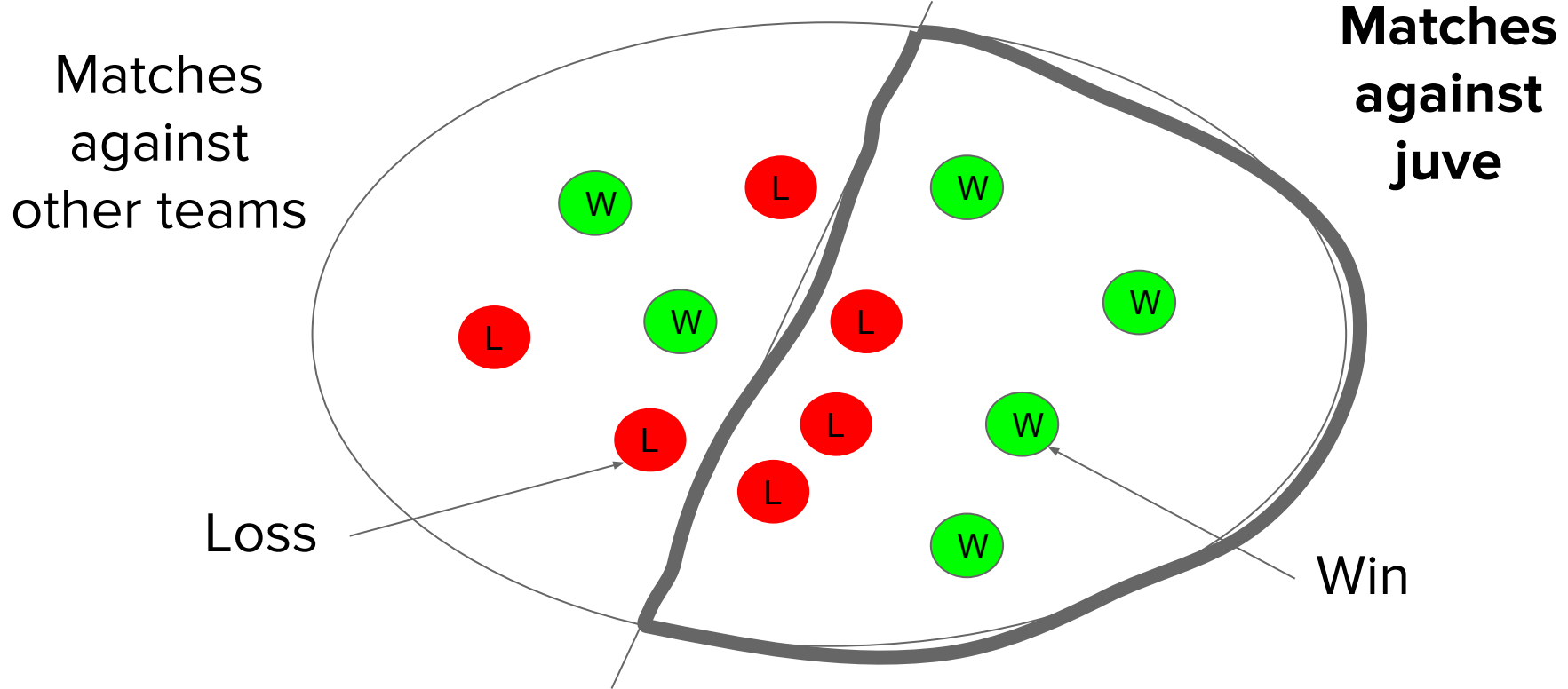




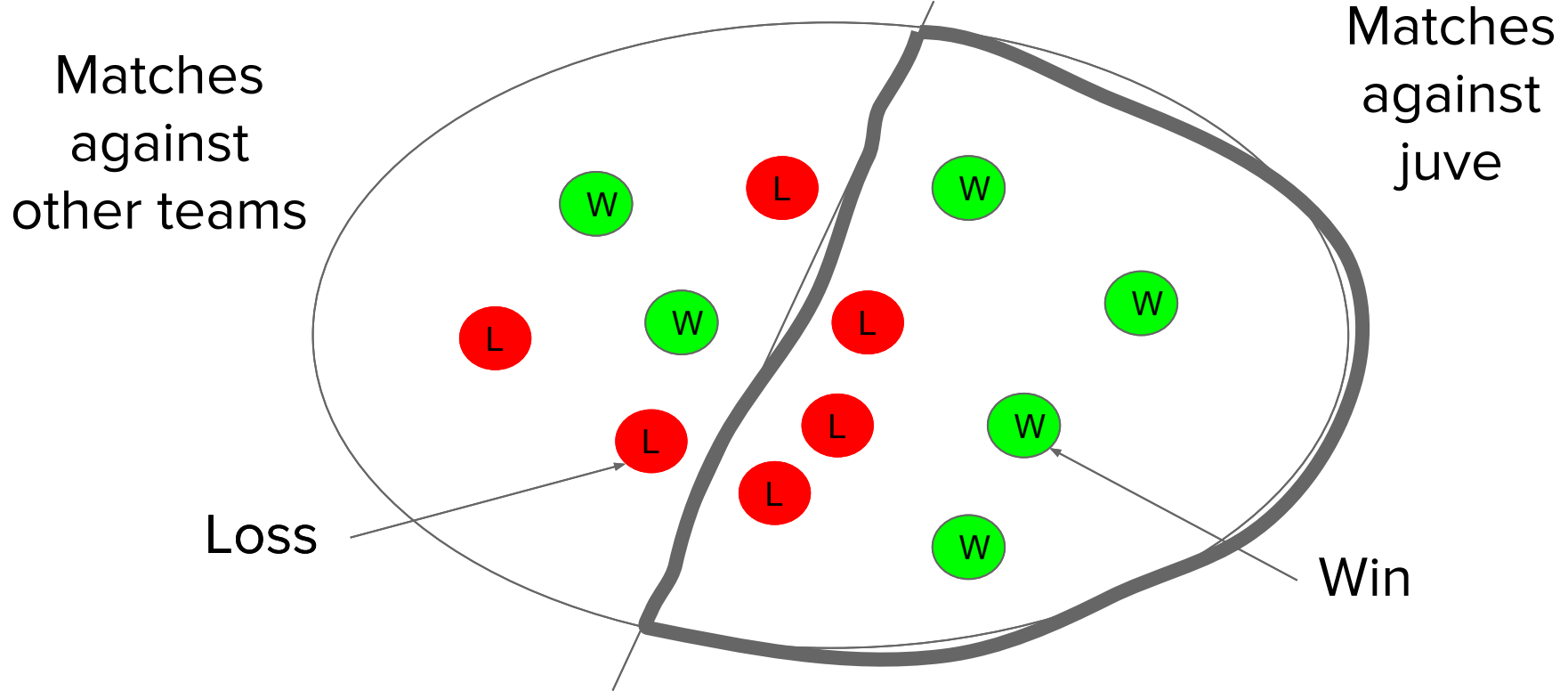
$$P(\text{win} \mid \text{playing against juve}) = ?$$



$P(\text{win} \mid \text{playing against juve}) = ?$



$$P(\text{win} \mid \text{playing against juve}) = \underline{\underline{4/7}}$$



## 4 - Joint probability

$$P(A \cap B) = P(A|B) * P(B)$$

Just play with the previous formula!

# 5 - Total probability

[video](#)

# Combinatorics

# Basic definition

Combinatorics is the study of how sets can be enumerated

“How many strings of 5 letters can I build using only [a,b,e,n,o].”

# Counting principle

3 girls = (a,b,c)

4 boys = (1,2,3,4)

How many possible couples like (girl,boy) can you have?



# Counting principle

$a_1, a_2, a_3, a_4$

$b_1, b_2, b_3, b_4$

$c_1, c_2, c_3, c_4$

$4 \times 3 = 12 \rightarrow$  double for loop! :)

# Permutations

[Video](#)

# Combinations

[Video](#)

## **With replacement (sampling)**

The same element can be chosen more than one time

# **Birthday paradox**

[https://en.wikipedia.org/wiki/Birthday\\_problem](https://en.wikipedia.org/wiki/Birthday_problem)

# Lesson material

Skip:

Distinguishable vs. Indistinguishable (*"balls and urns"*)

**What to do**

## **If you are new to probability**

Get the basics:  $P(a \text{ and } b)$ ,  $P(a \text{ or } b)$ ,  $P(a|b)$ , etc.

## **If you are OK to probability**

Play with combinatorics