FEEDBACK OF ASSIGNMENTS 12 AND 13

1. Assignment 12

Exercise 1. *For* (a)*, the matrix is diagonalizable iff* $\dim E_1 = 2$*, then you should try to solve the linear system* $\mathrm{Ker}(A - I_3)$ *with parameters* a*,* b *and* c *to find an equivalent condition on them. For* (b)*, we work with* ***real*** *numbers.*

Exercise 2. *Please read the statement carefully,* A *must have nonzero* (A)*, integer entries* (B) *and eigenvalues* 1*,* 2 *and* 3*. So you may either try your luck and conjugate* $\mathrm{diag}(1, 2, 3)$ *with random elements in* $\mathrm{SL}_3(\mathbb{Z})$ *(*3×3 *matrix with entry in* $\mathbb{Z}$ *and determinant equals* 1*) or follow the clever solution given in the recitation. (Does AI or google have a good answer for this one ?)*

Exercise 3. *For the space* $\mathrm{Span}(\begin{bmatrix} 2 \\ 1 \end{bmatrix})$ *to be the only eigenspace, you need to make sure that*

(1) There is no other eigenvalues.

(2) The dimension of E_1 *is* 1*.*

2. Assignment 13

Exercise. *3. (a) Please note that for questions like "find/give* ***a*** *... ", you don't need to find equivalent conditions/find all possible cases. It is better but won't give you more scores in the exam.*

Exercise. *4. Geometrically, the image of the unit sphere under* A *is the ellipsoid with semi-axes 2, 3, 4.*

Exercise. *5. Showing only something like* $\langle Ax, x - Ax \rangle = 0$ *is not enough. Please follow Definition 2.2.1 of the text book: show that you have space decomposition* $\mathbb{R}^n = E_1 \oplus E_0$ $(x^{||} \in E_1$ *and* $x^{\perp} \in E_0)$ *and* $E_0 \perp E_1$*.*

Exercise. *7. Use what you learned in the linear algebra course to solve it if you haven't. Write down the associated symmetric matrix, and find its eigenvalues, then find the equation with respect to the principal axis (base change):* $4c_1^2 - 6c_2^2 + c_3^2 = 1$*. Write down the distance (using the eigenvectors as an orthogonal basis)* $d^2 = c_1^2 + c_2^2 + c_3^2 = 1 - 3c_1^2 + 7c_2^2$*, one can easily see that when* $c_2 = c_3 = 0$ *the distance is minimum...*