Project4 - Car

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1 Warmup

1.1

$P(C_2|D_2 = 0) = \frac{P(C_2, D_2=0)}{P(D_2=0)}$

So we can get:

$P(C_2|D_2 = 0) \propto P(C_2, D_2 = 0)$
$P(C_2|D_2 = 0) \propto P(D_2 = 0|C_2)P(C_2)$
$P(C_2|D_2 = 0) \propto P(D_2 = 0|C_2) \sum\limits_{C_1} P(C_2, C_1)$

Then I compute:

$\sum\limits_{C_1} P(C_2 = 0, C_1) = \frac{1}{2} * \epsilon + \frac{1}{2} * (1 - \epsilon)$
$\sum\limits_{C_1} P(C_2 = 1, C_1) = \frac{1}{2} * \epsilon + \frac{1}{2} * (1 - \epsilon)$

So we can simplify:

$P(C_2|D_2 = 0) \propto P(D_2 = 0|C_2)$

Normalized:

$P(C_2 = 1|D_2 = 0) = \frac{P(D_2=0|C_2=1)}{P(D_2=0|C_2=1)+P(D_2=0|C_2=0)}$
$P(C_2 = 1|D_2 = 0) = \frac{\eta}{\eta+1-\eta} = \eta$

1.2

$P(C_2 = 1|D_2 = 0, D_3 = 1) = \frac{C_2=1,D_2=0,D_3=1}{D_2=0,D_3=1}$
$P(C_2 = 1|D_2 = 0, D_3 = 1) = \frac{\sum\limits_{C_3} P(C_2=1,C_3,D_2=0,D_3=1)}{\sum\limits_{C_3}\sum\limits_{C_2} P(C_2,C_3,D_2=0,D_3=1)}$
$P(C_2 = 1|D_2 = 0, D_3 = 1) = \frac{\eta(1-\epsilon)(1-\eta)+\eta\eta\epsilon}{\eta(1-\epsilon)(1-\eta)+\eta\eta\epsilon+\eta(1-\epsilon)(1-\eta)+(1-\eta)(1-\eta)\epsilon}$
$P(C_2 = 1|D_2 = 0, D_3 = 1) = \frac{\eta[\eta\epsilon+(1-\eta)(1-\epsilon)]}{4\eta_2\epsilon-2\eta_2-4\eta\epsilon+2\eta+\epsilon}$

1.3

1.3.1 i

Use what I have computed before, I can directly get:

$P(C_2 = 1|D_2 = 0) = 0.2000$
$P(C_2 = 1|D_2 = 0, D_3 = 1) = 0.4157$

1.3.2 ii

Because $\eta = 0.1$ and $\epsilon = 0.2$, so C_3 has a higher probability of being 1 like D_3 and C_2 has a higher probability of being 1 like C_2. So based original evidence $D_2 = 0$, $D_3 = 1$ gives more possibility of being 1 for C_2.

1.3.3 iii

$\frac{0.2[0.2\epsilon+(1-0.2)(1-\epsilon)]}{0.16\epsilon-0.08-0.8\epsilon+0.4+\epsilon} = 0.2$
$\epsilon = 0.5$
The intuition is although C_3 has a higher probability of being 1 like D_3, C_3 should not provide more evidence for $C_2 = 1$. When $\epsilon = 0.5$,$C_2 = 1$ and $C_2 = 0$ have the same probability of turning out $C_3 = 1$.

2 Other parts

Other parts please refer to my submitted codes.