400 Lab Assignment 1

Chuan Du (Sophie)

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Problem 1

```
#read in table and make matrix
markov = read.table("~/Desktop/400_Lab1/markov100.txt", header = FALSE)
M = as.matrix(markov)
```

Problem 1 (a)

```
library(expm)
```

```
## Loading required package: Matrix
```

```
##
## Attaching package: 'expm'
```

```
## The following object is masked from 'package:Matrix':
##
##     expm
```

```
#generate initial vector (1, 0, ... , 0)
a1 = c(1, rep(0, 99))
#n-step transition, p(10)
prob10 = a1 %*% (M %^% 10)
p_15_10 = prob10[1, 5]; p_15_10
```

```
##       V5
## 0.045091
```

Answer: The probability of being in State 5 after 10 transitions $P_{1,5}(10) = 0.045091$.

Problem 1 (b)

```
#initial states for State 1, 2, 3 with 1/3 prob
a_123 = c(rep(1/3, 3), rep(0, 97))
#10-step transitions, get P^n
prob123_10 = a_123 %*% (M %^% 10)
#get P^10[i, j]
prob_123_10 = prob123_10[1, 10]; prob_123_10
```

```
##        V10
## 0.08268901
```

Answer: The probability of being in State 10 after 10 transitions is **0.08268901**.

Problem 1 (c)

```
#calculate pi
Q1 = t(M) - diag(100)
Q1[100, ] = c(rep(1, 100))
rhs = c(rep(0, 99), 1)
Pi = solve(Q1) %*% rhs
Pi[1]
```

```
## [1] 0.01256589
```

Answer: The steady state probability of being in State 1 is **0.01256589**.

Problem 1 (d)

$$m = (I - B)^{-1}e$$

```
#submatrix of M obtained by deleting r and c corresponding state 100
B2 = M[1:99, 1:99]
Q2 = diag(99) - B2
e1 = c(rep(1, 99))
m = solve(Q2) %*% e1; m[1]
```

```
## [1] 254.9395
```

Answer: The mean first passage time from State 1 to State 100 is **254.9395**.

Problem 2

```
#read in table and make matrix
page = read.table("~/Desktop/400_Lab1/webtraffic.txt", header = TRUE)
P = as.matrix(page) #1000*81
```

Problem 2 (a)

The traffic matrix is shown below.

```
sums = colSums(P)
m_99 = matrix(sums, 9, 9)
#note: t_9i = 0 for all i
Traffic = t(m_99); Traffic
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##  [1,]    0  447  553    0    0    0    0    0    0
##  [2,]    0   23  230  321    0    0    0    0   63
##  [3,]    0  167   43  520    0    0    0    0   96
##  [4,]    0    0    0   44  158  312  247    0  124
##  [5,]    0    0    0    0   22   52   90  127  218
##  [6,]    0    0    0    0   67   21    0  294   97
##  [7,]    0    0    0    0    0   94    7  185   58
##  [8,]    0    0    0    0  262    0    0   30  344
##  [9,]    0    0    0    0    0    0    0    0    0
```

Problem 2 (b)

The transition probability matrix is shown below.

```
Traffic[9, 1] = 1000
P = Traffic / rowSums(Traffic); P
```

```
##       [,1]       [,2]       [,3]       [,4]      [,5]       [,6]
##  [1,]    0 0.44700000 0.55300000 0.00000000 0.0000000 0.00000000
##  [2,]    0 0.03610675 0.36106750 0.50392465 0.0000000 0.00000000
##  [3,]    0 0.20217918 0.05205811 0.62953995 0.0000000 0.00000000
##  [4,]    0 0.00000000 0.00000000 0.04971751 0.1785311 0.35254237
##  [5,]    0 0.00000000 0.00000000 0.00000000 0.0432220 0.10216110
##  [6,]    0 0.00000000 0.00000000 0.00000000 0.1398747 0.04384134
##  [7,]    0 0.00000000 0.00000000 0.00000000 0.0000000 0.27325581
##  [8,]    0 0.00000000 0.00000000 0.00000000 0.4119497 0.00000000
##  [9,]    1 0.00000000 0.00000000 0.00000000 0.0000000 0.00000000
##             [,7]       [,8]      [,9]
##  [1,] 0.00000000 0.00000000 0.0000000
##  [2,] 0.00000000 0.00000000 0.0989011
##  [3,] 0.00000000 0.00000000 0.1162228
##  [4,] 0.27909605 0.00000000 0.1401130
##  [5,] 0.17681729 0.24950884 0.4282908
##  [6,] 0.00000000 0.61377871 0.2025052
##  [7,] 0.02034884 0.53779070 0.1686047
##  [8,] 0.00000000 0.04716981 0.5408805
##  [9,] 0.00000000 0.00000000 0.0000000
```

Problem 2 (c)

```
Q3 = t(P) - diag(9)
Q3[9, ] = c(rep(1, 9))
rhs2 = c(rep(0, 8), 1)
Pi1 = solve(Q3) %*% rhs2; Pi1
```

```
##             [,1]
##  [1,] 0.15832806
##  [2,] 0.10085497
##  [3,] 0.13077897
##  [4,] 0.14012033
##  [5,] 0.08058898
##  [6,] 0.07583914
##  [7,] 0.05446485
##  [8,] 0.10069664
##  [9,] 0.15832806
```

Problem 2 (d)

```
B3 = P[1:8, 1:8]
Q4 = diag(8) - B3
e8 = c(rep(1, 8))
m8 = solve(Q4) %*% e8; m8
```

```
##          [,1]
## [1,] 5.316000
## [2,] 4.401776
## [3,] 4.246666
## [4,] 3.392390
## [5,] 2.429794
## [6,] 2.749343
## [7,] 2.940475
## [8,] 2.100010
```

```
t2 = c(0.1, 2, 3, 5, 5, 3, 3, 2, 0)
t2 %*% Pi1 * m8[1] #stationary avg time
```

```
##          [,1]
## [1,] 12.25727
```

Answer: The average time a visitor spend on the website is **12.25727**.

Problem 2 (e)

```
Traffic_new = Traffic
Traffic_new[2, 6] = 0.3 * Traffic_new[2, 3]
Traffic_new[2, 3] = 0.7 * Traffic_new[2, 3]
Traffic_new[2, 7] = 0.2 * Traffic_new[2, 4]
Traffic_new[2, 4] = 0.8 * Traffic_new[2, 4]

P_newtraffic = Traffic_new / rowSums(Traffic_new)
Qe = t(P_newtraffic) - diag(9)
Qe[9, ] = c(rep(1, 9))
rhse = c(rep(0, 8), 1)
Pi2 = solve(Qe) %*% rhse; Pi2
```

```
##             [,1]
##  [1,] 0.16162840
##  [2,] 0.10034341
##  [3,] 0.12104331
##  [4,] 0.12275720
##  [5,] 0.08164613
##  [6,] 0.08250884
##  [7,] 0.06003218
##  [8,] 0.10841213
##  [9,] 0.16162840
```

```
Pi2 - Pi1
```

```
##               [,1]
##  [1,]  0.0033003475
##  [2,] -0.0005115633
##  [3,] -0.0097356600
##  [4,] -0.0173631313
##  [5,]  0.0010571466
##  [6,]  0.0066696974
##  [7,]  0.0055673326
##  [8,]  0.0077154832
##  [9,]  0.0033003475
```

Answer: By comparing Pi2 to Pi, we could see that the probability of visiting Page 2 and Page 3 decreases, so that the link works.