Assignment 8

1. Known that $L_{SSM}(\theta) = E_{x\sim p(x)} \| S(x;\theta)\|^2 + E_{x\sim p(x)} E_{v\sim p(v)} [2v^T \nabla_x (v^T S(x;\theta))]$,

and $v \sim N(0, I)$, then $E[vv^T] = I$ from lecture note.

$\|v^T S(x;\theta)\|^2 = S(x;\theta)^T (v v^T) S(x;\theta)$

$\Rightarrow E_{x\sim p(x)} E_{v\sim p(v)} [\|v^T S(x;\theta)\|^2]$

$= E_{x\sim p(x)} [S(x;\theta)^T E_{v\sim p(v)} (vv^T) S(x;\theta)]$

$= E_{x\sim p(x)} [S(x;\theta)^T S(x;\theta)]$ since $E_{v\sim p(v)} [vv^T] = I$

$= E_{x\sim p(x)} \|S(x;\theta)\|^2$.

Thus, $L_{SSM}(\theta) = E_{x\sim p(x)} E_{v\sim p(v)} [\|v^T S(x;\theta)\|^2 + 2v^T \nabla_x (v^T S(x;\theta))]$. $\square$

2. An SDE is a generalization from ODE which describes a dynamical system of the form:

$$d x_t = f(x_t, t)\, dt + G(x_t, t)\, d W_t \text{, where}$$

$f(x_t, t)$ is called a drift term, $G(x_t, t)$ is called a diffusion term and W_t is Wiener process

i.e. a differential equation of stochastic process function.

It describes how a random variable change w.r.t. time with the effect of the drift term and the diffusion term.

The drift term $f(x_t, t)$ defines the direction of change and the diffusion term defines the random noise.

And to solve the eq. above, we have Ito's integral form:

$$x_t = x_0 + \int_0^t f(x_s, s)\, ds + \int_0^t G(x_s, s)\, d W_s .$$

3. If the drift and diffusion don't satisfy certain regularity condition, then what will happen, why we need this condition?