THE UNIVERSITY OF MELBOURNE

# COMP90038 Algorithms and Complexity

## Tutorial 1 Introduction to algorithms and data structures

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**Tutorial Slides can be accessed at** ***https://github.com/chuangw46/COMP90038_Algorithms***

# Prerequisite

- **Chapter 1.** *Levitin, A. (2012). Introduction to the design & analysis of algorithms. (3rd Ed.)*
- ***Week 1 Lectures***

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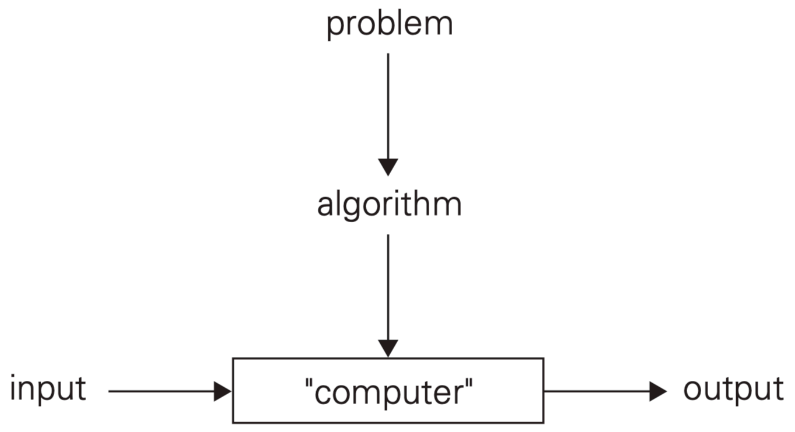
# 1. Review

# Algorithms & Data Structures

## Algorithm:

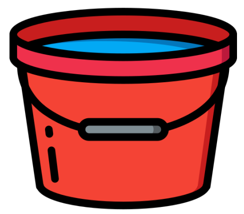
- A sequence of well-defined, computer-implementable instructions for **solving a problem**
- Algorithms operate on data.




*Sourced from Levitin's book*

## Data Structure:

- Storing data
- Organizing data so that it can be used efficiently.



Think of a data structure as a bucket storing something

# Bird's eye view of what we will study

## Problem Types
- Searching
- Sorting
- String processing
- Graph problems
- Numerical problems
- Geometric problems
- Combinatorial problems

## Data Structures
- Linear
  - Array
  - List
    - Singly Linked List
    - Doubly linked list
  - Stack
  - Queue
    - Simple Queue
    - Priority Queue(Heaps)
- Non-Linear
  - Tree
    - Binary Tree
    - Binary Search Tree
    - AVL Tree
    - 2-3 Tree
    - Red-Black Tree
    - B Tree
  - Graph
    - Directed
    - Undirected
    - Unweighted
    - Weighted
  - Hashtable
  - Sets and Dictionaries

## Algorithms Techniques
- Brute Force
- Decrease and Conquer
- Divide and Conquer
- Transform-and-Conquer
- Dynamic Programming
- Greedy Algorithms
- Backtracking

## Algorithm Efficiency
- Time Efficiency
- Space Efficiency
- Time/space tradeoffs

*It's okay! We will get through these together!!*

# Array

## Array

### What

1. A simple data structure for holding a sequence of cells(data) in memory.
2. Array elements are stored in adjacent memory cells.
3. The index is used to locate the desired element in the array by calculating an offset.
4. The first index of an array is 0.

### Example

| | | | | | | |
|---|---|---|---|---|---|---|
| 6 | 9 | 2 | 3 | 7 | 5 | 8 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |

data = [6, 9, 2, 3, 7, 5, 8]

data[4] = 7

Memory representation

| | |
|---|---|
| 42148 | 6 |
| 42150 | 9 |
| 42152 | 2 |
| 42154 | 3 |
| 42156 | 7 |
| 42158 | 5 |
| 42160 | 8 |

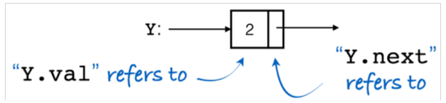
### Tradeoffs

- Pros
  - Locating, storing(append), or retrieving a cell -> very fast (constant time)
- Cons
  - insert or delete an element -> slow as we have to shift subsequent elements
  - Maintaining a contiguous bank of cells with information can be difficult and time-consuming.

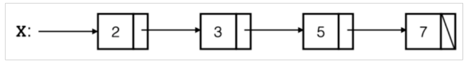
# (Singly) Linked List

## Linked List

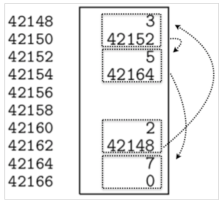
- **What**
  - 1. A data structure in which the elements are not stored at contiguous/adjacent memory locations.
  - 2. To accommodate this feature, each node(cell) has two things:
    Value(Data) + Pointer(also called reference in Java) that indicates where the next data is in the memory.



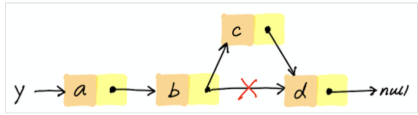

- **Example**



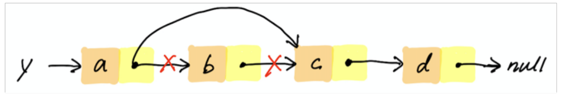

  - x is (a pointer to) the head node of the linked list.
  - x represents the linked list.
  - Memory representation




- **Operations**
  - Insert




  - Delete




  - Traverse — Use loop(while loop or for loop)
- **Pros&Cons**
  - Pros
    - Inserting and deleting data -> fast as it does not require moving and shifting subsequent data elements
  - Cons
    - insert or delete an element -> slow as  we have to shift subsequent elements
    - access a specific element -> slow as we need to traverse the list from the head node to find it
      Note: Generally, we can not randomly access any element as we do in array by index.(Depends on programming language)
    - Require more memory storage ->  as each node contains a pointer and it requires extra memory for itself.

# 2. Tutorial Questions

# Question 1

1. Consider the usual (unsigned) binary representation of integers. For example, `10110010` represents 178, and `000011` represents 3.

   (a) If we call the bits in an $n$-bit word $x_{n-1}, x_{n-2}, \ldots, x_2, x_1, x_0$ (so $x_0$ is the *least significant* bit), which natural number is denoted by $x_{n-1}x_{n-2}\cdots x_2x_1x_0$?

   (b) Describe, in English, an algorithm for converting from binary to decimal notation.

   (c) Write the algorithm in (pseudo-) code.

   (d) Describe, in English, how to convert the decimal representation to binary.

## Answers:

$$10110010: 1 * 2^7 + 0 * 2^6 + 1 * 2^5 + 1 * 2^4 + 0 * 2^3 + 0 * 2^2 + 1 * 2^1 + 0 * 2^0 = 178$$

(a) $x_{n-1}x_{n-2} \ldots x_2x_1x_0$ represents the integers from **0 to $2^n - 1$ (inclusive)** because

it denotes $x_{n-1} \times 2^{n-1} + x_{n-2} \times 2^{n-2} + \cdots + x_2 \times 2^2 + x_1 \times 2^1 + x_0 \times 2^0 = \boldsymbol{\sum_{i=0}^{n-1} x_i \times 2^i}$

# Question 1

1. Consider the usual (unsigned) binary representation of integers. For example, 10110010 represents 178, and 000011 represents 3.

   (a) If we call the bits in an $n$-bit word $x_{n-1}, x_{n-2}, \ldots, x_2, x_1, x_0$ (so $x_0$ is the *least significant* bit), which natural number is denoted by $x_{n-1}x_{n-2}\cdots x_2x_1x_0$?

   (b) Describe, in English, an algorithm for converting from binary to decimal notation.

   (c) Write the algorithm in (pseudo-) code.

   (d) Describe, in English, how to convert the decimal representation to binary.

## Answers:

1011: $1 * 2^3 + 0 * 2^2 + 1 * 2^1 + 1 * 2^0$

$= 2 * (2 * (2 * (x_3) + x_2) + x_1) + x_0$

$= 2 * (2 * (2 * (1) + 0) + 1) + 1$

(b) To convert from binary to decimal notation,

1. visit the binary digits from left to right
2. construct the result in an "accumulator", the accumulator begins with 0
3. as long as there is a next bit to process,

   double the value of the accumulator and add the value of that next bit.
4. get the result

(c)

```
function BINTODEC(x_{n-1}x_{n-2} ... x_2x_1x_0)
    a ← 0
    for i ← 0 to n - 1 do
        a ← 2a + x_{n-1-i}
    return a
```

## Question 1

1. Consider the usual (unsigned) binary representation of integers. For example, `10110010` represents 178, and `000011` represents 3.

   (a) If we call the bits in an $n$-bit word $x_{n-1}, x_{n-2}, \ldots, x_2, x_1, x_0$ (so $x_0$ is the *least significant* bit), which natural number is denoted by $x_{n-1}x_{n-2}\cdots x_2x_1x_0$?

   (b) Describe, in English, an algorithm for converting from binary to decimal notation.

   (c) Write the algorithm in (pseudo-) code.

   (d) Describe, in English, how to convert the decimal representation to binary.

## Answers:

(d) 1001011 $\quad 1*2^6+0*2^5+0*2^4+1*2^3+0*2^2+1*2^1+1*2^0=75$

Successive division:

$75 \div 2 = 37\ R\ 1$ <- lease significant bit

$37 \div 2 = 18\ R\ 1$

$18 \div 2 = 9\ R\ 0$

$9 \div 2 = 4\ R\ 1$

$4 \div 2 = 2\ R\ 0$

$2 \div 2 = 1\ R\ 0$

$1 \div 2 = 0\ R\ 1$ <- most significant bit

```
function DECTOBIN(d)
        n ← 0
        while d ≠ 0 do
                x_n ← d mod 2
                d ← ⌊d / 2⌋
                n ← n + 1
        return x_{n-1} x_{n-2} ... x_2 x_1 x_0
```

# Question 2

2. Below are three (correct) formulas for calculating the area of a triangle with sides $A$, $B$, and $C$, of length $a$, $b$, and $c$, respectively.

   (a) $S = \sqrt{p(p-a)(p-b)(p-c)}$, where $p = (a+b+c)/2$

   (b) $S = \frac{1}{2}ab\sin\theta$, where $\theta$ is the angle between sides $A$ and $B$

   (c) $S = \frac{1}{2}ah_A$, where $h_A$ is the height to base $A$

   Which of these would you accept as an *algorithm*?

## Answers:

(a) Heron's formula is a fine algorithm as long as the square root operation is a primitive operation available on our computing device, or we know how to calculate square roots. For example, we can implement it as follows:

```
function SQRT(n)
        root ← 1
        while root * root ≤ n  do
                root ← root + 1
        return x_{n−1}x_{n−2} … x_2x_1x_0
```

**Based on $\sum_{i=1}^{k}(2i-1) = k^2$**

```
function SQRT(n)
        square ← 1
        i ← 1
        while square ≤ n  do
                square ← square + 2i + 1
                i ← i + 1
        return i − 1
```

# Question 2

2. Below are three (correct) formulas for calculating the area of a triangle with sides $A$, $B$, and $C$, of length $a$, $b$, and $c$, respectively.

   (a) $S = \sqrt{p(p-a)(p-b)(p-c)}$, where $p = (a+b+c)/2$
   (b) $S = \frac{1}{2}ab\sin\theta$, where $\theta$ is the angle between sides $A$ and $B$
   (c) $S = \frac{1}{2}ah_A$, where $h_A$ is the height to base $A$

   Which of these would you accept as an *algorithm*?

**Answers:**

(b) This is a problematic formulation, because, even if the sine function is considered a primitive, there is no indication of how to compute the angle.

(c) Again, the formula says, “the height to base A”, without any indication of how to find that. So we can’t really call that an algorithm.

# Question 3

3. Consider the following problem: You are to design an algorithm to determine the best route for a subway passenger to take from one station to another in a city such as Kolkata or Tokyo.

   (a) Discuss ways of making the problem statement less vague. In particular, what is "best" supposed to mean?

   (b) How would you model this problem by a graph?

**Answers:**

(a) "best" can mean many things. We might want to minimize

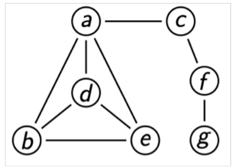
- The travel time
- The number of train stops
- The number of train changes
- Some combination of these

# Graphs

## What

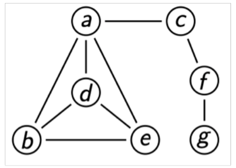
1. A data structure that consists of a finite set of nodes (or vertices) and a set of edges connecting them.
2. A pair (x,y) is referred to as an edge, which communicates that the x vertex connects to the y vertex.
3. Mathematic notation:

G = ⟨V , E⟩
V : Set of nodes or vertices
E: Set of edges (a binary relation on V )



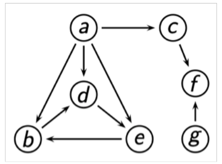

## Types

### Undirected



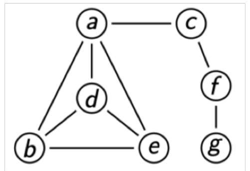

V = { a, b, c, d, e, f , g }
E = symmetric closure of { (a, b), (a, c), (a, d), (a, e), (b, d), (b, e), (c, f ), (d, e), (f , g) }

### Directed

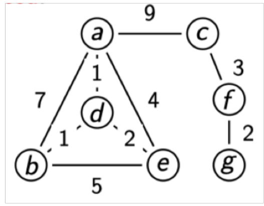



V = { a, b, c, d, e, f , g }
E = { (a, b), (a, c), (a, d),(a, e), (b, d), (c, f ), (d, e), (e, b), (g, f ) }

### Unweighted




### Weighted




## Common Graph Representation

### Adjacency Matrix

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 1 | | | 1 |
| 1 | | | 1 | |
| 2 | | 1 | | |
| 3 | 1 | | | 1 |

An Adjacency Matrix is a 2D array of size V x V where V is the number of nodes in a graph.

A slot matrix[i][j] = 1 indicates that there is an edge from node i to node j.

# Question 3

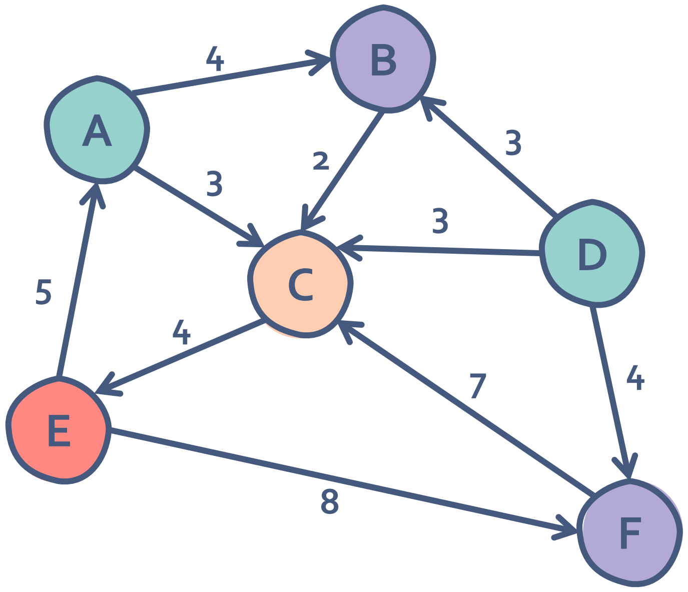
3. Consider the following problem: You are to design an algorithm to determine the best route for a subway passenger to take from one station to another in a city such as Kolkata or Tokyo.

   (a) Discuss ways of making the problem statement less vague. In particular, what is "best" supposed to mean?

   (b) How would you model this problem by a graph?

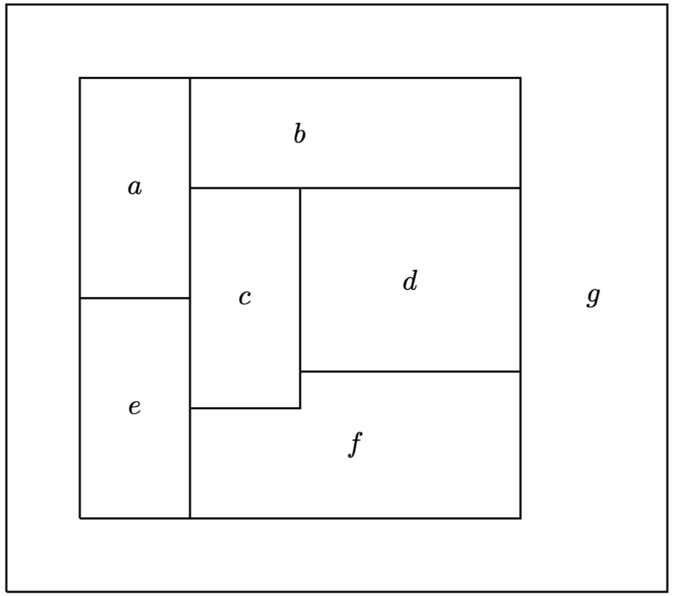
(b)

- The nodes A,B,C,D,E,F represent 6 stations in Tokyo.
- There is an edge between two nodes iff the stations that correspond to the incident nodes are directly connected by a train line.
- The direction(e.g. A->B) indicates that the route only goes in one direction.
- The weight between two stations denotes the travel time.



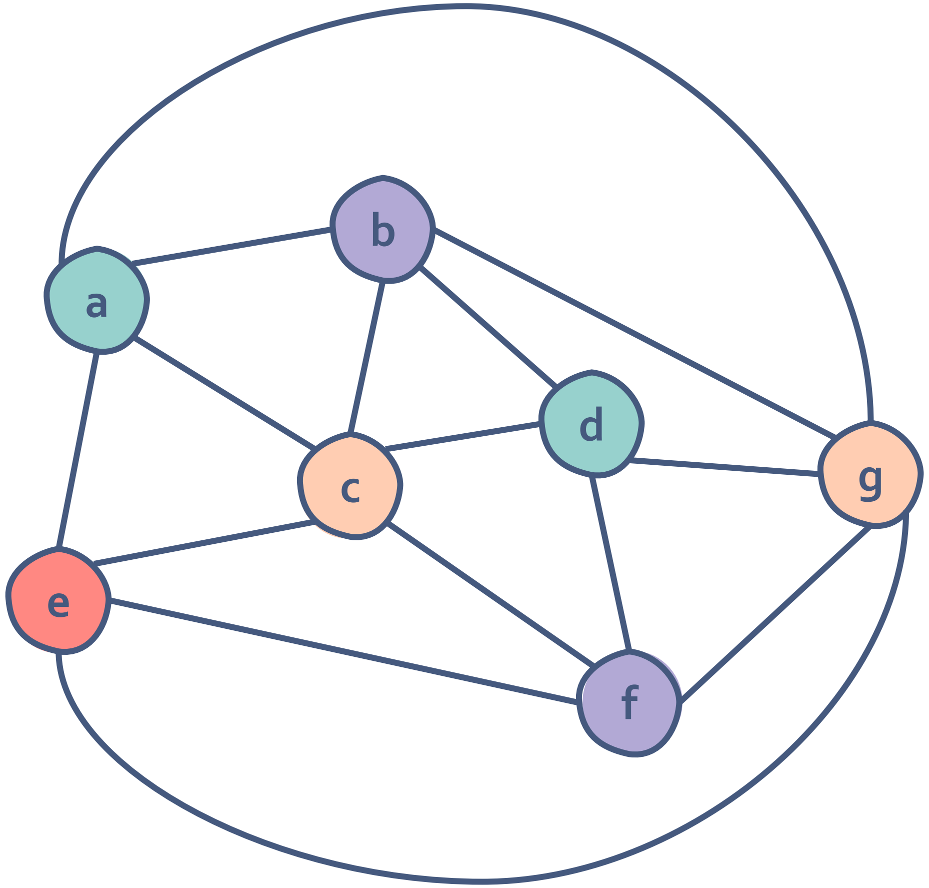
# Question 4

4. Consider the following map:




(a) A cartographer wants to colour the map so that no two neighbouring countries have the same colour. How few colours can she get away with?

(b) Show how to reduce the problem to a graph-colouring problem.




Algorithm:

1. Color first vertex with first color.
2. Do following for remaining V-1 vertices.

- Consider the currently picked vertex and color it with the lowest numbered color that has not been used on any previously colored vertices adjacent to it.
- If all previously used colors appear on vertices adjacent to v, assign a new color to it.

# Question 5

5. You have to search for a given number $n$ in a *sorted* list of numbers.

   (a) How can you take advantage of knowing that the list is represented as a linked (and sorted) list?

   (b) How can you take advantage of knowing the list is represented as an array?

**Answers:**

(a) We can stop searching as soon as we find (n or) a number greater than n.

*In a linked list, we have to traverse the whole list from the head node until we find the target because we cannot access a specific item.*

(b) With an array we can use binary search (we will cover this algorithm later in this course).

*In an array, we can access a specific element by its index, which makes binary search possible.*

# Question 6

6. In the first lecture we discussed different ways of calculating the greatest common divisor of two positive integers. A mathematician might simply write

$$gcd(x, y) = max\{z \mid \exists u, v : x = uz \land y = vz\}$$

and suggest we develop a functional programming language that allows us to write this, leaving it to the language implementation to translate this definition to an efficient algorithm. Do you imagine a time when we may be able to do this? If we restrict our attention to functions like *gcd* which takes a pair of integers and returns an integer, do you think we may some day be able to automatically turn any function definition into a working algorithm?

**Answers:**

The point of asking this question is to call attention to that fact that there are functions that are ***not computable.***

Recall the first slide in this tutorial:

***An algorithm is a sequence of well-defined, computer-implementable instructions for solving a problem***

# Next Week:

*Algorithm Analysis*

*- how to reason about an algorithm’s resource consumption(time and space).*

# *Thank you !*