41080 Theory of Computing Science
Week 2 Tutorial Class

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Outline

- **Review**: languages and operations on them
- **Keynote**: DFAs and their relation with languages
- **Tutorial**: how to do the product construction of two DFAs

What is a language?

- Σ: an alphabet set;
- Σ^n: the set of all length-n strings over Σ;
- Σ^*: the set of ALL strings over Σ.

Definition (Language)

L is a language if $L \subseteq \Sigma^*$ for some Σ.

Example (Language)

Let $\Sigma = \{0, 1\}$ and $L = \{w \in \{0, 1\}^* \mid w \text{ starts with 0 and ends with 1}\}$.

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Basic operations on languages

Given two languages $L_1, L_2 \subseteq \Sigma^*$, we can make the following operations:

- Union: $L_1 \cup L_2 = \{w \in \Sigma^* : w \in L_1 \text{ or } w \in L_2\}$.
- Intersection: $L_1 \cap L_2 = \{w \in \Sigma^* : w \in L_1 \text{ and } w \in L_2\}$.
- Complement: $\neg L_1 = \{w \in \Sigma^* : w \notin L_1\}$.
- Reverse: $L_1^R = \{a_k \dots a_1 \in \Sigma^* : a_1 \dots a_k \in L_1 \text{ for each } a_i \in \Sigma\}$.
- Concatenation: $L_1 \circ L_2 = \{w_1 w_2 \in \Sigma^* : w_1 \in L_1 \text{ and } w_2 \in L_2\}$.
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What is a deterministic finite automaton?

Definition (DFA)

A deterministic finite automaton (DFA) is a five tuple $(Q, \Sigma, q_0, F, \delta)$:

1. Q: a set of states;
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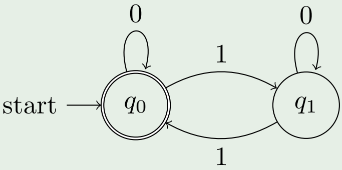
From DFA to language

Example (DFA)



Exercise

Write down the language that the above DFA recognises.

Solution: $L = \{w \in \{0,1\}^* \mid w \text{ contains even number of 1s}\}$.

From DFA to language

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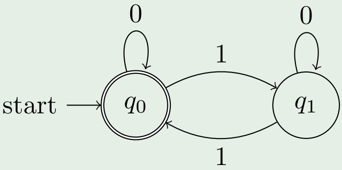

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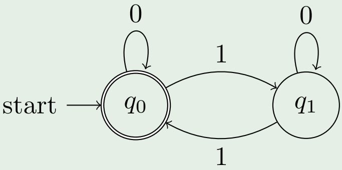

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Design a DFA that recognises the above language.

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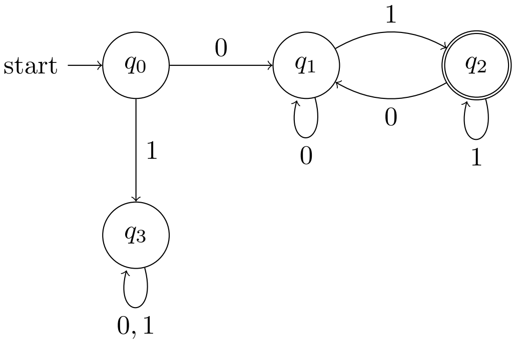

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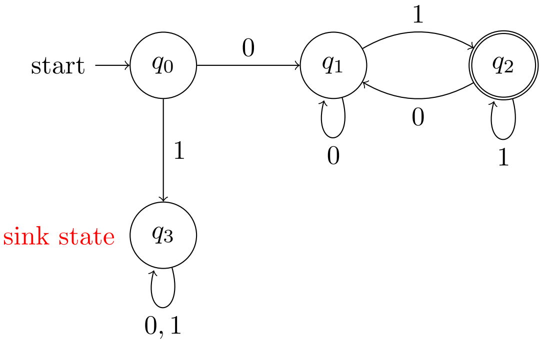

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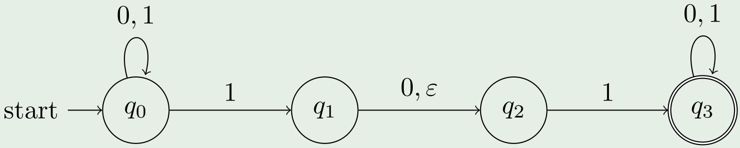
Note that 2^Q refers to the set consisting of all subsets of Q.

From NFA to language

Example (NFA)



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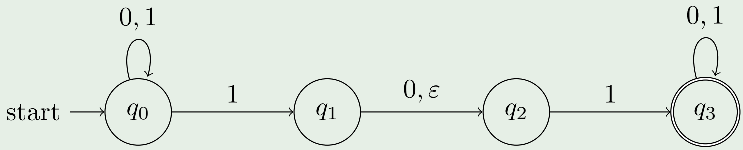

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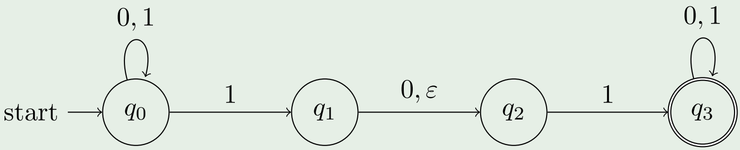

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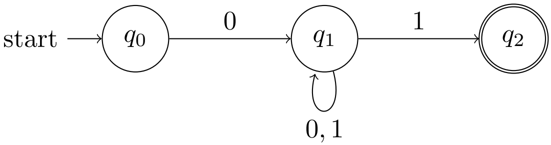

What is a product construction?

Definition (Product construction of two DFAs)

Let $M = (P, \Sigma, p_0, E, \alpha)$ and $N = (Q, \Sigma, q_0, F, \beta)$ be two DFAs. The product construction for recognising $L(M) \cup L(N)$ is to construct $O = (R, \Sigma, r_0, G, \gamma)$ where

1. the state set $R = P \times Q$;
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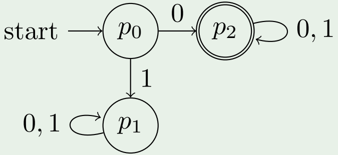
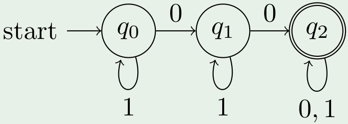
2. the start state $r_0 = (p_0, q_0)$;
3. the accept state set $G = \{(p, q) \mid p \in E \text{ and } q \in F\}$.
4. the transition function $\gamma : (P \times Q) \times \Sigma \to (P \times Q)$ given by $\gamma((p, q), a)) = (\alpha(p, a), \beta(q, a))$.

Tutorial: product construction

Construct a DFA for $L_1 \cup L_2$ (recognised by the given two DFAs, respectively).





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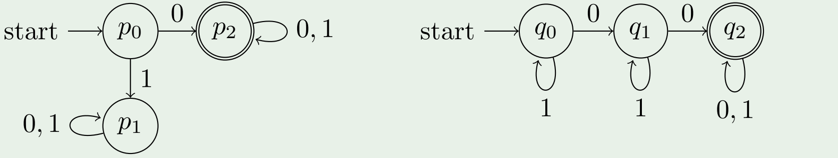

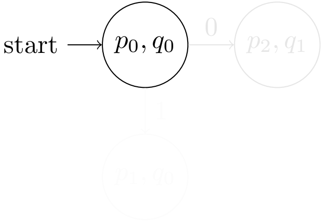

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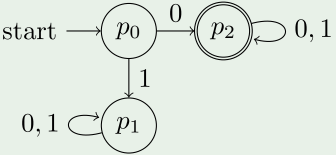

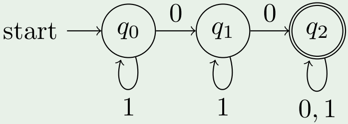

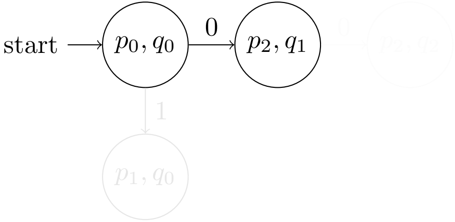

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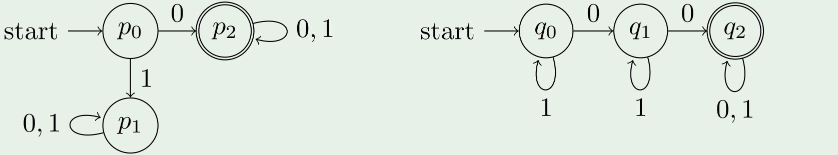

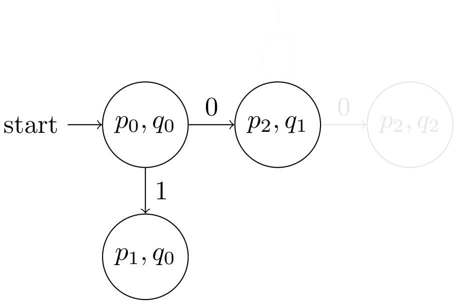

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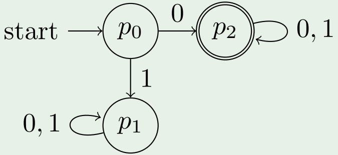

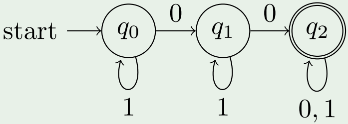

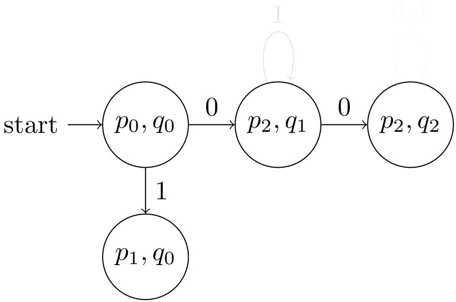

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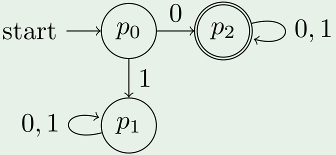

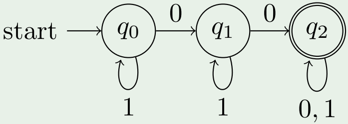

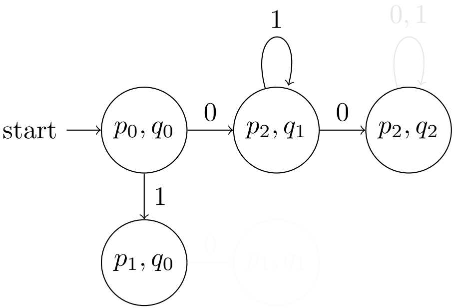

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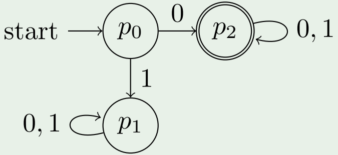

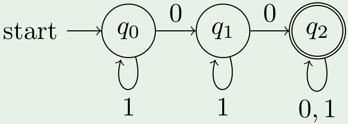

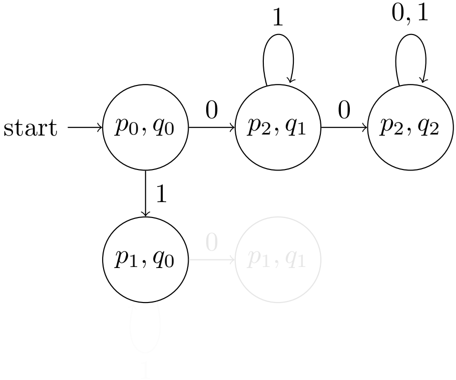

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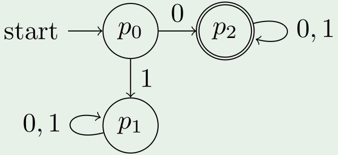

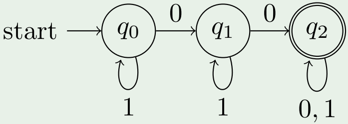

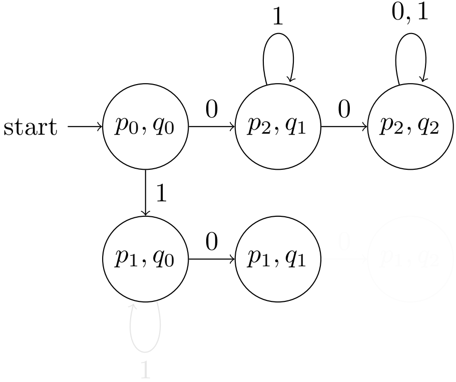

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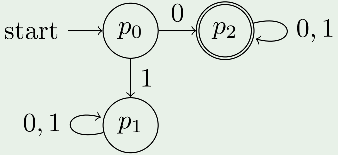

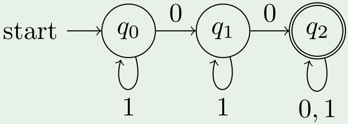

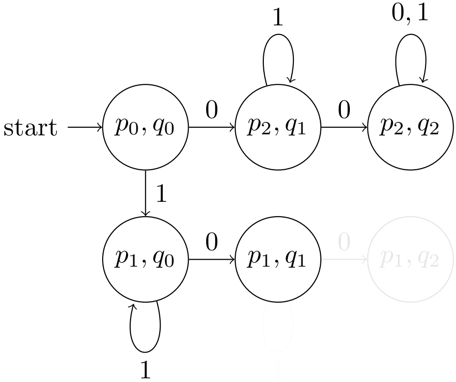

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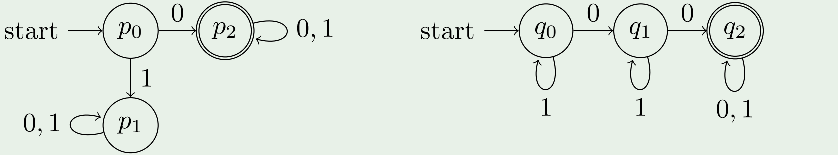

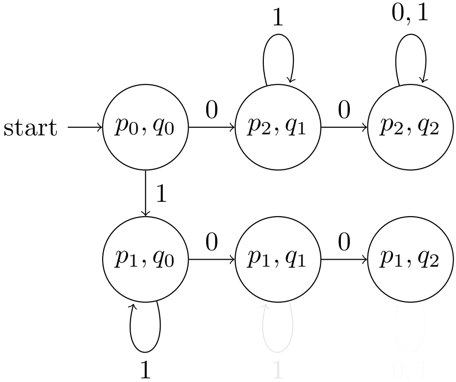

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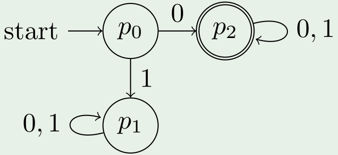

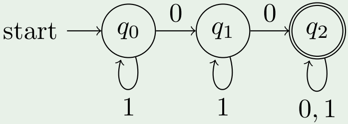

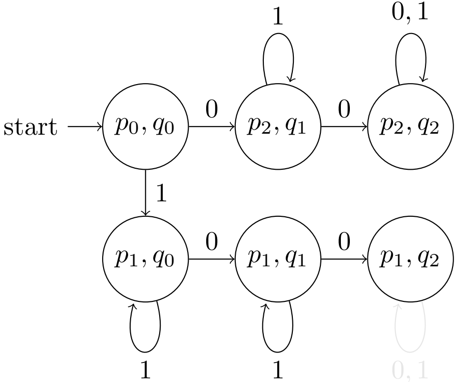

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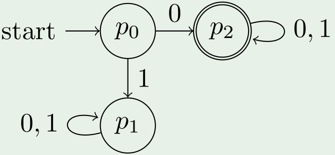

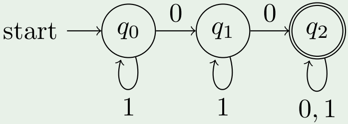

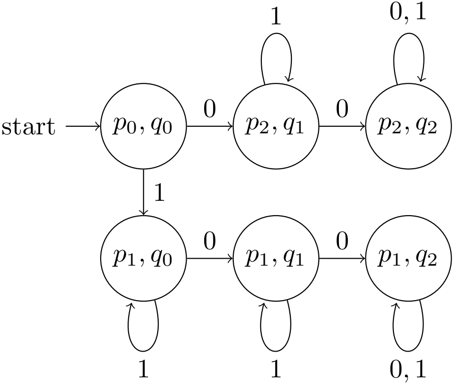

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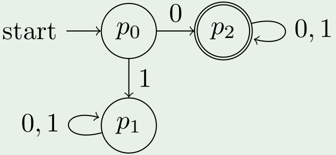

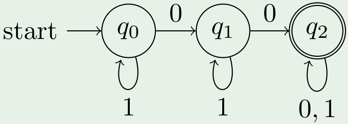

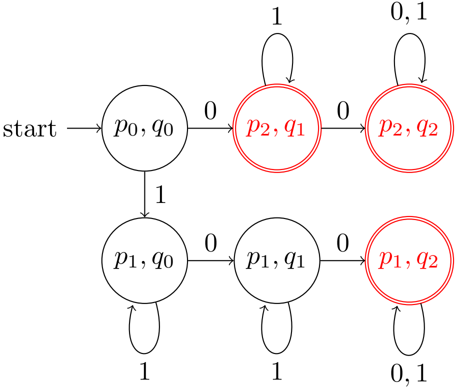

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