given $f(x,y)$

$$z_x = \frac{\partial z}{\partial x} = f_x(x,y) = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$z_y = \frac{\partial z}{\partial y} = f_y(x,y) = \lim_{h \to 0} \frac{f(x,y+h) - f(x,y)}{h}$$

if f is continuous, f_x, f_y continuous. $f_{xy} = f_{yx}$ Clairaut's Thm.

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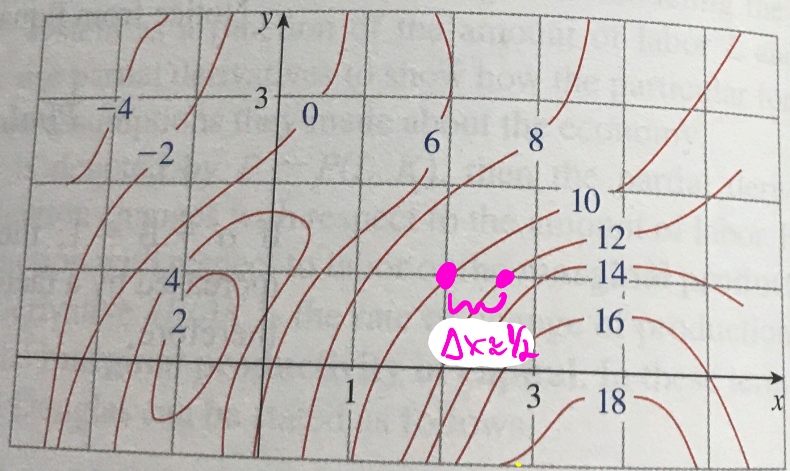
(10) **10.** A contour map is given for a function f. Use it to estimate $f_x(2, 1)$ and $f_y(2, 1)$.



$f_x \approx \frac{\Delta z}{\Delta x}$ as y is fixed.

at $(2,1)$

$f(2,1) = 10$

$f(2+\Delta x, 1) \approx 12$

$\Delta x \approx \frac{1}{2}$

$f_x(2,1) \approx \frac{12-10}{\frac{1}{2}} = 4.$

(15) $f(x,y) = x^4 + 5xy^3$

find 1st partial derivatives f_x f_y

$$f_x(x,y) = \frac{\partial}{\partial x}(x^4 + 5xy^3) = \frac{\partial}{\partial x}(x^4) + \frac{\partial}{\partial x}(5xy^3)$$

$$= 4x^3 + 5y^3 \frac{\partial}{\partial x}(x)$$

$$= 4x^3 + 5y^3$$

$$f_y(x,y) = \frac{\partial}{\partial y}(x^4 + 5xy^3) = \underbrace{\frac{\partial}{\partial y}(x^4)}_{0} + 5x\underbrace{\frac{\partial}{\partial y}(y^3)}_{3y^2}$$

$$= 15xy^2$$

(20) $z = x\sin(xy)$

$$z_x = \frac{\partial}{\partial x}(x\sin(xy)) = \underbrace{\left(\frac{\partial}{\partial x}x\right)}_{1}\sin(xy) + x\underbrace{\frac{\partial}{\partial x}\sin(xy)}_{y\cos(xy)}$$

$$= \sin(xy) + xy\cos(xy)$$

(28) $f(x,y) = \dfrac{x}{(x+y)^2}$

$$f_x(x,y) = \frac{\partial}{\partial x}\left(\frac{x}{(x+y)^2}\right) = \frac{(x+y)^2\overbrace{\left(\frac{\partial}{\partial x}x\right)}^{1} - x\overbrace{\frac{\partial}{\partial x}\left[(x+y)^2\right]}^{2(x+y)}}{(x+y)^4}$$

leave f_y for you

$$= \frac{(x+y)^{\cancel{2}} - 2x\cancel{(x+y)}}{(x+y)^{\cancel{4}\,3}}$$

$$= \frac{x+y-2x}{(x+y)^3} = \frac{y-x}{(x+y)^3}$$

(32) $f(x,y,z) = xy^2e^{-xz}$

find f_z

$$\frac{\partial}{\partial z}(xy^2e^{-xz}) = xy^2\frac{\partial}{\partial z}(e^{-xz})$$

$$= xy^2 e^{-xz} \underbrace{\frac{\partial}{\partial z}(-xz)}_{-x}$$

$$= -x^2 y^2 e^{-xz}$$

find f_{zz}

$$\frac{\partial^2}{\partial z^2}\left(xy^2 e^{-xz}\right)$$

$$= \frac{\partial}{\partial z}\left(-x^2 y^2 e^{-xz}\right)$$

$$= -x^2 y^2 \frac{\partial}{\partial z}\left(e^{-xz}\right)$$

$$= x^3 y^2 e^{-xz}$$

(40) $u = \sin(x_1 + 2x_2 + \cdots + nx_n) = \sin\left(\sum_{i=1}^{n} ix_i\right)$

find u_{x_i} $\quad i = 1, \ldots, n$

$$\frac{\partial}{\partial x_i} \sin(x_1 + 2x_2 + \cdots + nx_n) \quad f(x_1, x_2, \ldots, x_n)$$

$$= \cos(x_1 + 2x_2 + \cdots + nx_n) \frac{\partial}{\partial x_i} \underbrace{(x_1 + 2x_2 + \cdots + ix_i + \cdots + nx_n)}$$

$\frac{\partial}{\partial x_i} x_j = 0$ when $i \neq j$

$\frac{\partial}{\partial x_i}(ix_i) = i$

$\frac{\partial}{\partial x} y = 0$

$\frac{\partial}{\partial x} z = 0$

$\frac{\partial}{\partial x_1} x_2 = 0$

$$\boxed{= \cos(x_1 + 2x_2 + \cdots + nx_n)\, i \quad \text{for } i = 1, \ldots, n}$$

$$\frac{\partial f}{\partial x_n} = n\cos(x_1 + \cdots + nx_n)$$

$$\frac{\partial f}{\partial x_9} = 9\cos(x_1 + \cdots + nx_n)$$

note
$g'(x, y)$ makes no sense.

(51a) $z = f(x) + g(y)$ find $\frac{\partial z}{\partial x} = \frac{\partial}{\partial x} f(x) + \underbrace{\frac{\partial}{\partial x} g(y)}_{0} = f'(x)$

(52b) find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of $z = f(xy)$

$\frac{\partial}{\partial x} f(xy) = f' \frac{\partial}{\partial x}(xy) = y f'$, $\frac{\partial}{\partial y} f(xy) = x f'$

on WA
$y f'(xy)$
$x f'(xy)$

(52a) $z = f(x) g(y)$ find $z_x = f'(x) g(y)$

$= \frac{\partial}{\partial x}[f(x) g(y)]$

$= g(y) \underbrace{\frac{\partial}{\partial x}[f(x)]}_{f'(x)}$

$\frac{\partial}{\partial x} \overset{f}{\sin}(xy)$
$= \underset{f'}{\cos}(xy)\, y$

difference b/w

$f(xy)$	$f(x,y)$
↓	↓
1 variable.	2 variables.

(53) $f(x,y) = x^4 y - 2x^3 y^2$

find all 2nd p.d.

$f_{xx}, f_{yy}, f_{xy}, f_{yx}$

$f_x = 4x^3 y - 6x^2 y^2$ $f_y = x^4 - 4x^3 y$

$f_{xx} = 12x^2 y - 12xy^2$ $f_{yy} = -4x^3$

$$f_{xy} = \frac{\partial}{\partial y}(f_x) = \frac{\partial}{\partial y}(4x^3y - 6x^2y^2) = 4x^3 - 12x^2y$$

$$f_{yx} = \frac{\partial}{\partial x}(f_y) = \frac{\partial}{\partial x}(x^4 - 4x^3y) = 4x^3 - 12x^2y$$

$$z_{xy} = (z_x)_y = \boxed{\frac{\partial z}{\partial y \partial x}} = \frac{\partial}{\partial y}(z_x)$$

Question: $w = f(x, y, z)$

how many 2^{nd} order p.d. can we find?

w_{xx} $\quad w_{xy}$ $\quad w_{yx}$ $\quad w_{zx}$

w_{yy} $\quad w_{xz}$ $\quad w_{yz}$ $\quad w_{zy}$

w_{zz}

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(54) $f(x,y) = \ln(ax+by)$ Find all 2^{nd} p.d.

$$f_x(x,y) = \frac{a}{ax+by} \qquad f_y(x,y) = \frac{b}{ax+by}$$

$$f_{xx}(x,y) = \frac{\partial}{\partial x}\left[a(ax+by)^{-1}\right]$$

$$= a(-1)(ax+by)^{-2}(a)$$

$$= \frac{-a^2}{(ax+by)^2}$$

$$f_{yy}(x,y) = \frac{-b^2}{(ax+by)^2}$$

$$f_{xy}(x,y) = \frac{-ab}{(ax+by)^2}$$

(68) $V=\ln(r+s^2+t^3)$ find $\dfrac{\partial^3 V}{\partial r\,\partial s\,\partial t} = V_{tsr}$

$$V_t = \frac{1}{r+s^2+t^3}(3t^2) = \frac{3t^2}{r+s^2+t^3} = 3t^2(r+s^2+t^3)^{-1}$$

$$(V_t)_s = \frac{\partial}{\partial s}\left(3t^2(r+s^2+t^3)^{-1}\right) = 3t^2\frac{\partial}{\partial s}(r+s^2+t^3)^{-1}$$
$$= -3t^2(r+s^2+t^3)^{-2}(2s)$$
$$= -6st^2(r+s^2+t^3)^{-2}$$

$$(V_{ts})_r = \frac{\partial}{\partial r}\left(-6st^2(r+s^2+t^3)^{-2}\right)$$
$$= -6st^2\frac{\partial}{\partial r}(r+s^2+t^3)^{-2}$$
$$= 12st^2(r+s^2+t^3)^{-3}(1)$$
$$= \boxed{\frac{12st^2}{(r+s^2+t^3)^3}}$$

(76) $u_{xx}+u_{yy}=0$ Laplace's Equation.

$\Delta u=0$

verify if $u=x^2+y^2$, $u=x^2-y^2$,

$u=\sin x\cosh y+\cos x\sinh y$

are solutions to Laplace's equation.

$u = x^2 + y^2$ $\quad u_y = 2y \quad u_{yy} = 2 \quad u_{xx} + u_{yy} = 4 \neq 0$

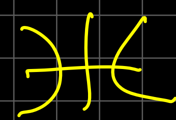
$u_x = 2x \quad u_{xx} = 2$

$u = x^2 - y^2 \quad u_{xx} = 2 \quad u_{yy} = -2 \quad u_{xx} + u_{yy} = 0$

$u = \sin x \cosh y + \cos x \sinh y$

$x^2 + y^2 = 1$

$x^2 - y^2 = 1$



$$\cosh y = \frac{e^y + e^{-y}}{2}$$

$$\sinh y = \frac{e^y - e^{-y}}{2}$$

$(\cosh y)' = \sinh y \quad , (\sinh y)' = \cosh y$

$u_x = \cos x \cosh y + (-\sin x)(\sinh y)$

$u_{xx} = -\sin x \cosh y - \cos x \sinh y$

$u_y = \sin x \sinh y + \cos x \cosh y$

$u_{yy} = \sin x \cosh y + \cos x \sinh y$

$+ = 0.$

Heat: $u_t = \alpha^2 u_{xx} \qquad u = e^{-\alpha^2 k^2 t} \sin(kx)$

(80) if $u = e^{a_1 x_1 + a_2 x_2 + \cdots + a_n x_n}$ $\quad \langle a_1, \ldots, a_n \rangle \cdot \langle x_1, \ldots, x_n \rangle$

$\|\vec{a}\| = 1$

where $a_1^2 + a_2^2 + \cdots + a_n^2 = 1$

show $u_{x_1 x_1} + u_{x_2 x_2} + \cdots + u_{x_n x_n} = u.$ $\iff L(u) = u$

$$u_{x_i} = \frac{\partial}{\partial x_i} e^{a_1x_1+\cdots+a_nx_n} = a_i e^{a_1x_1+\cdots+a_nx_n}$$

$$u_{x_ix_i} = \frac{\partial}{\partial x_i}\left(a_i e^{a_1x_1+\cdots+a_nx_n}\right) = a_i^2 e^{a_1x_1+\cdots+a_nx_n}$$

$$\underbrace{u_{x_1x_1} + u_{x_2x_2} + \cdots + u_{x_nx_n}} = u.$$

$$\sum_{i=1}^{n} u_{x_ix_i} = \sum_{i=1}^{n} a_i^2 \underbrace{e^{a_1x_1\cdots+a_nx_n}}_{u} = \sum_{i=1}^{n} a_i^2 u$$

$$= u \underbrace{\sum_{i=1}^{n} a_i^2}_{1}$$

$$= u.$$