Introduction to Robot Intelligence [Spring 2023]

Gradient Descent

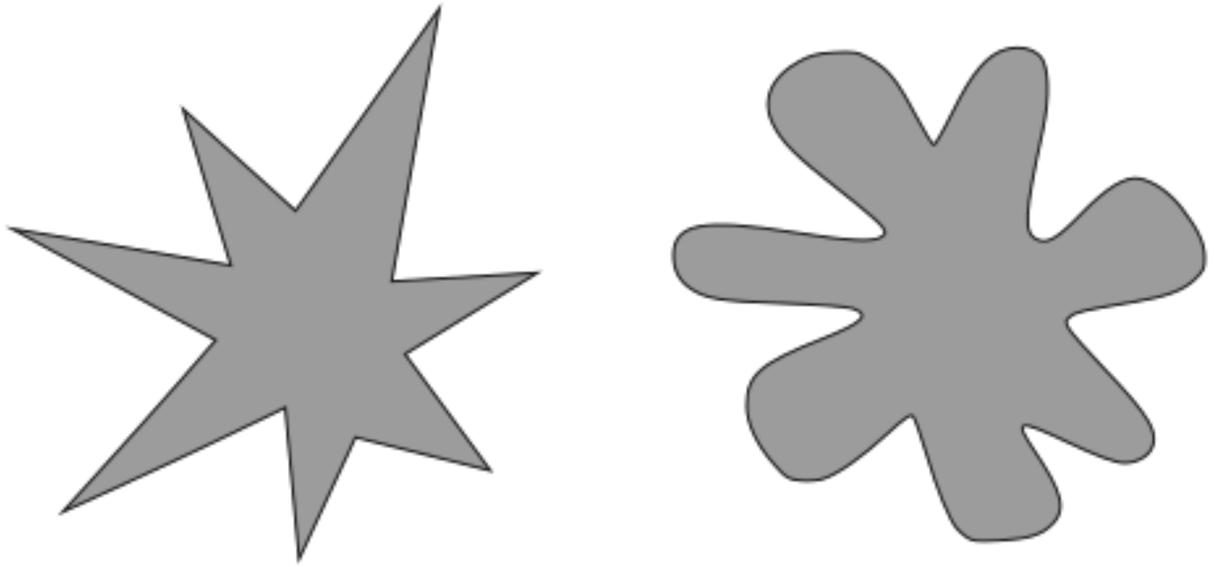
February 2, 2023

Lerrel Pinto

Some class logistics

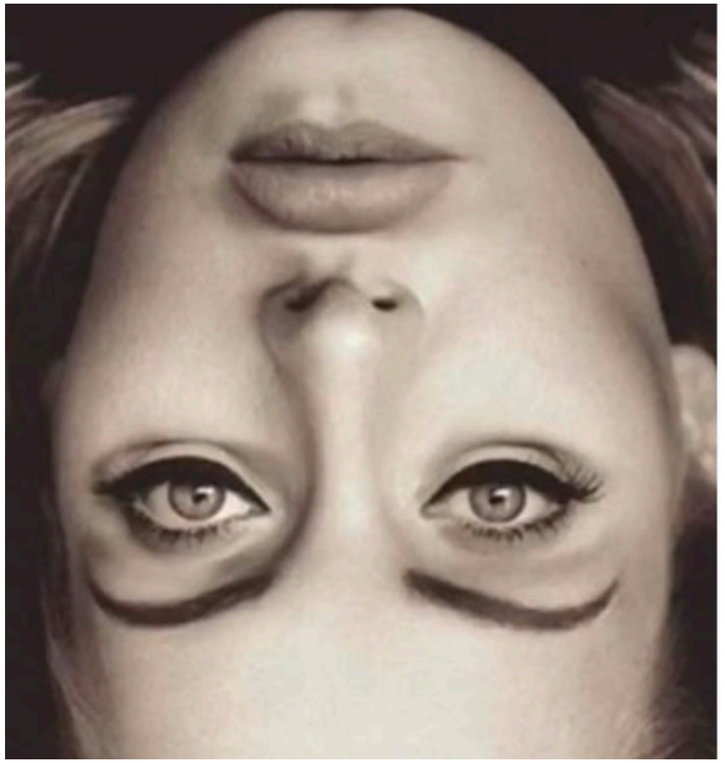
- HW1 is due today!

Musings



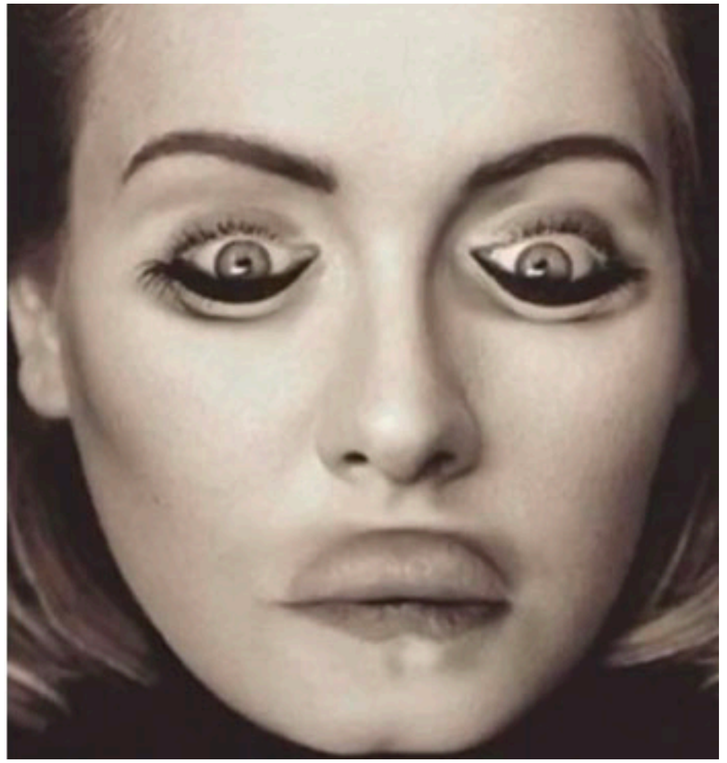
https://en.wikipedia.org/wiki/Bouba/kiki_effect

Musings



https://en.wikipedia.org/wiki/Thatcher_effect

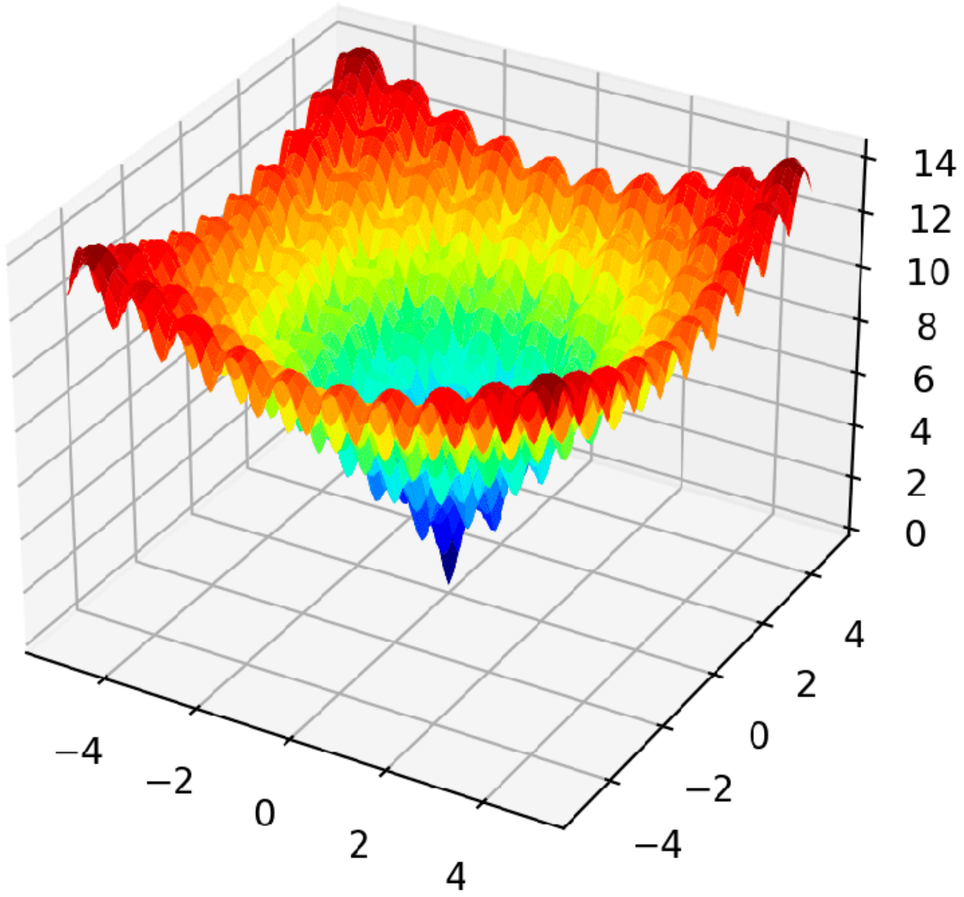
Musings



https://en.wikipedia.org/wiki/Thatcher_effect

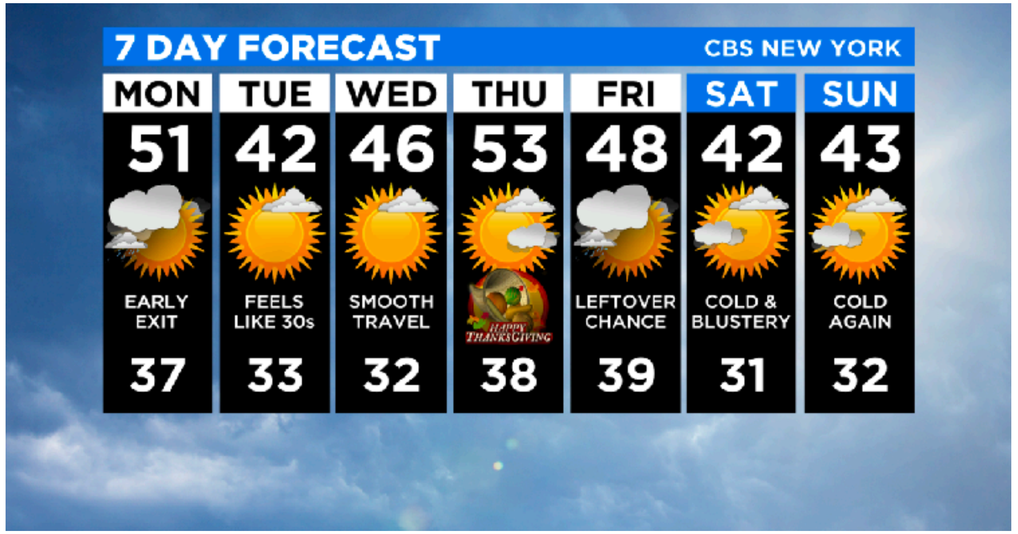
Goal for today: A soft introduction to gradient descent

Solve this:



What if you do not have the analytic equation?

Financial markets



Weather models

Image recognition

Cross Entropy Method

Initialize $\mu \in \mathbb{R}^d, \sigma \in \mathbb{R}^d$

for iteration $= 1, 2, \ldots$ **do**

Collect n samples of $\theta_i \sim N(\mu, \text{diag}(\sigma))$

Perform a noisy evaluation $R_i \sim \theta_i$

Select the top $p\%$ of samples (e.g. $p = 20$), which we'll call the **elite set**

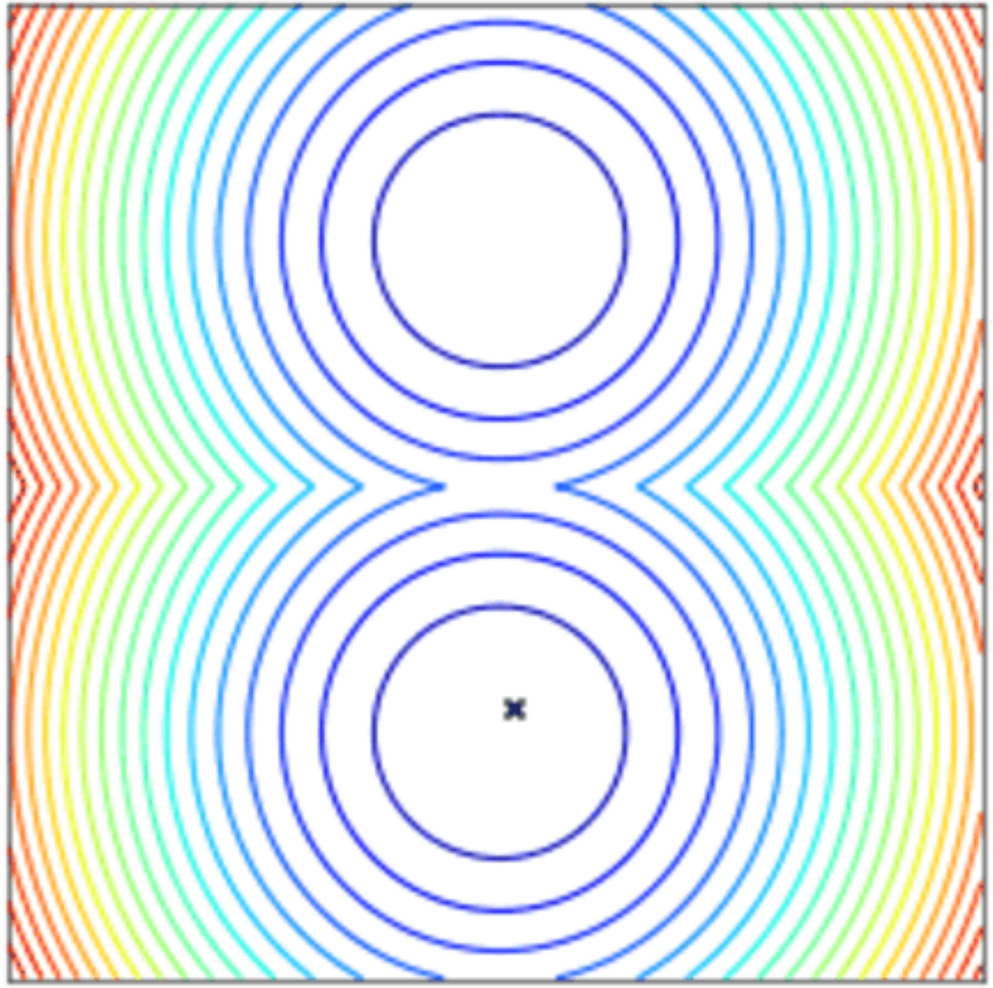
Fit a Gaussian distribution, with diagonal covariance, to the elite set, obtaining a new μ, σ.

end for

Return the final μ.

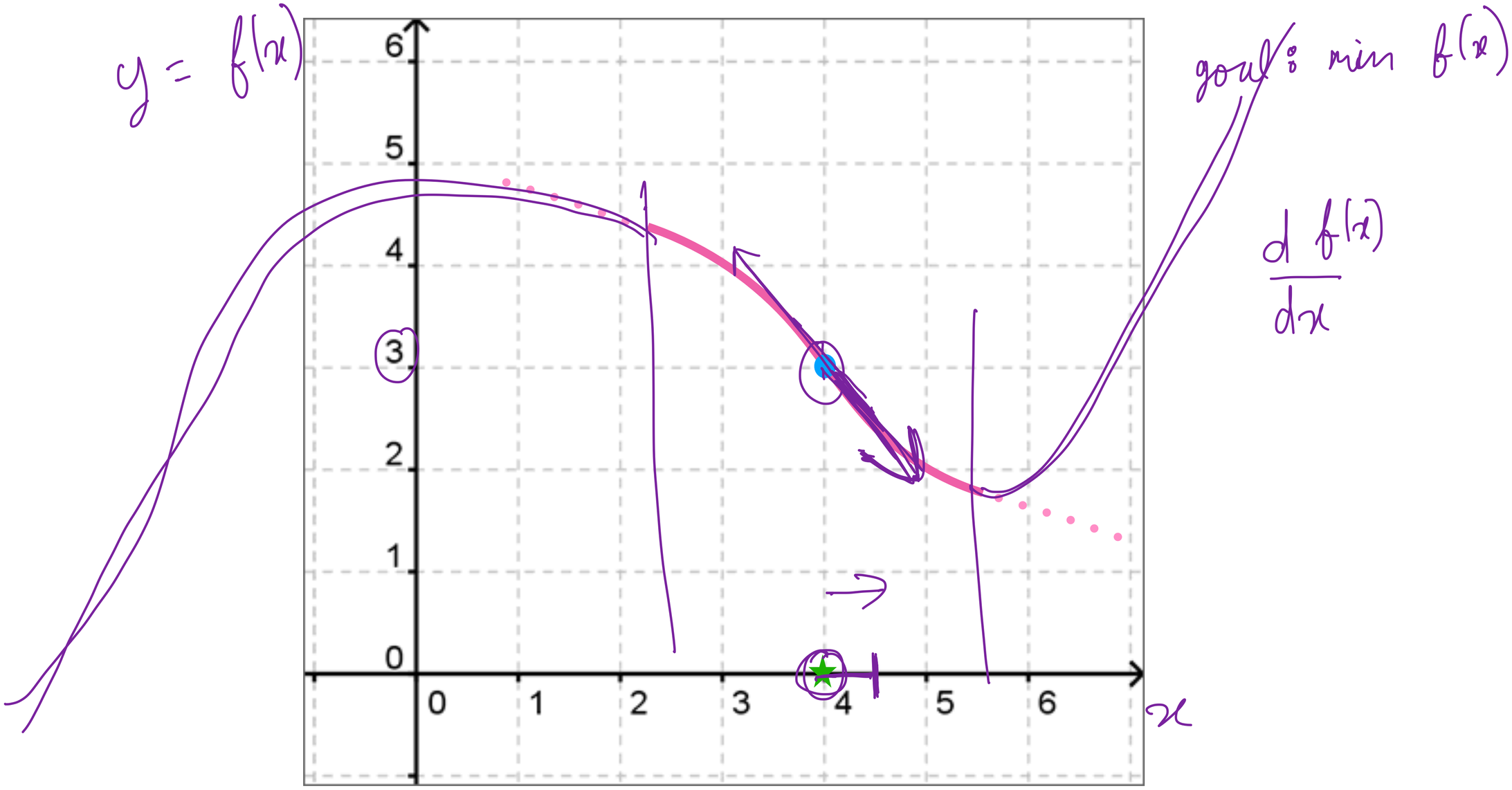
Slide credits: MLSS 2016 on Deep Reinforcement Learning by John Schulman

Visualizing DFO with two variables



Lets go back to having analytic information

Where to search next?



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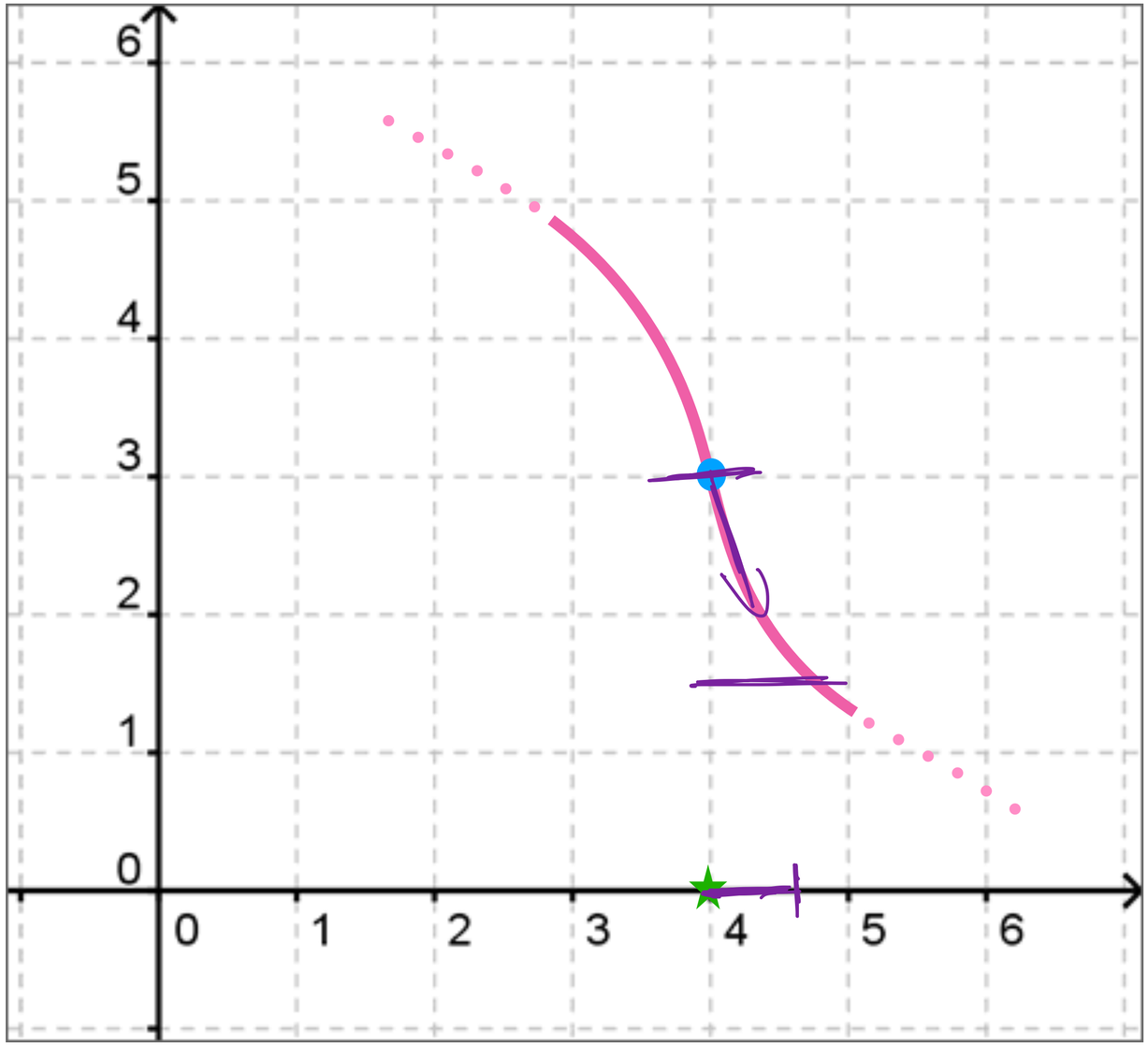

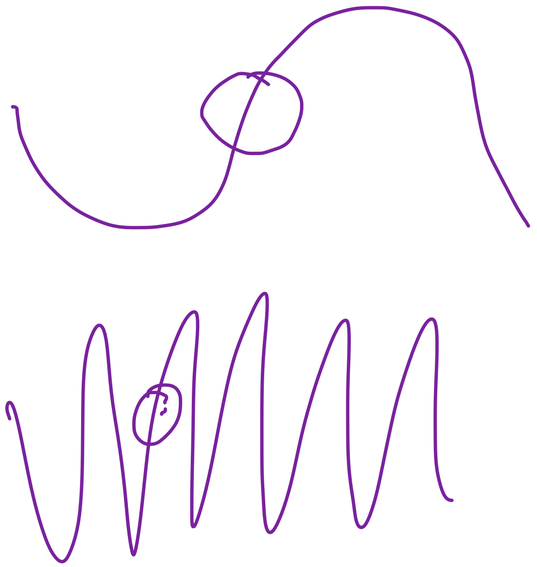

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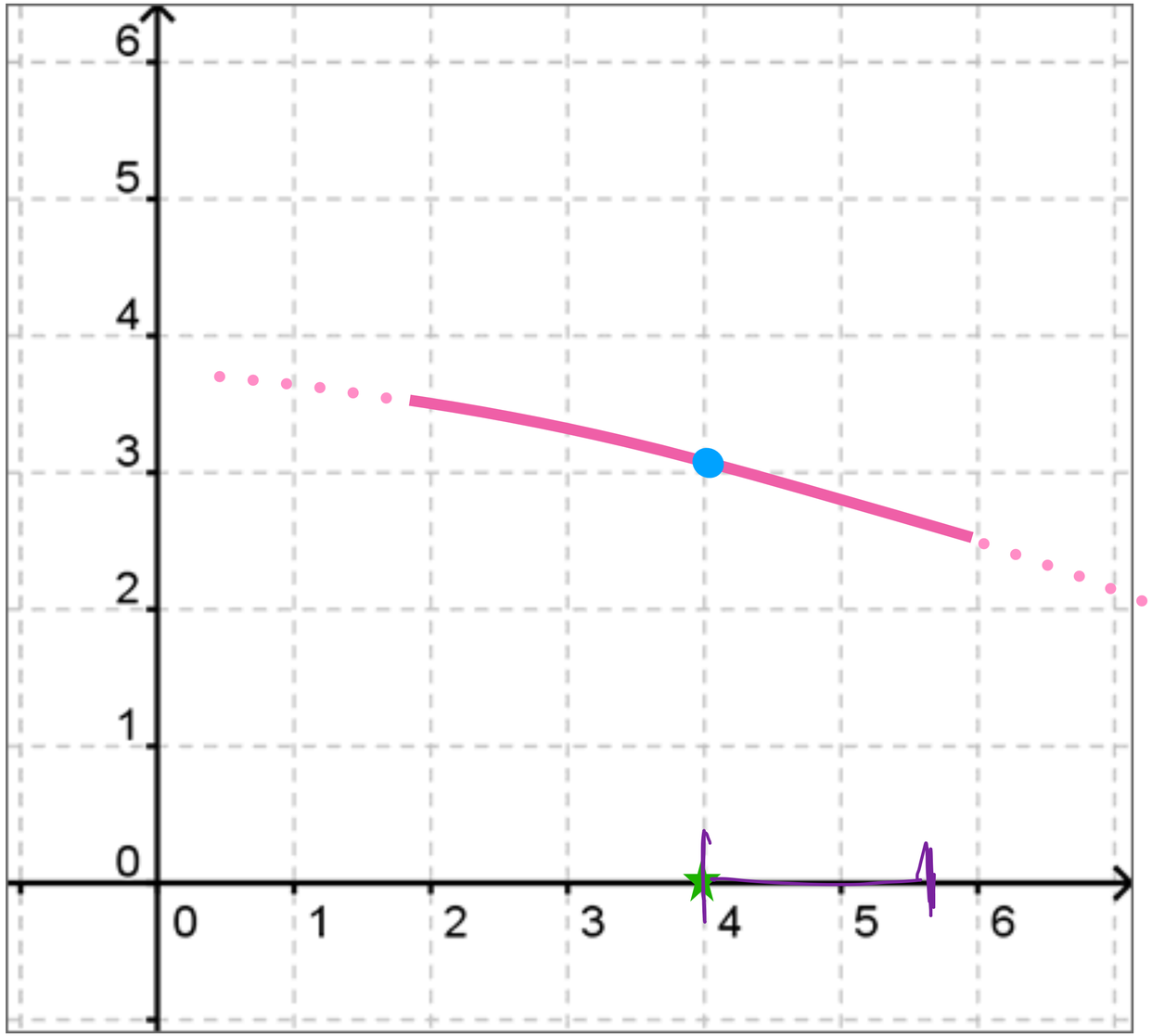

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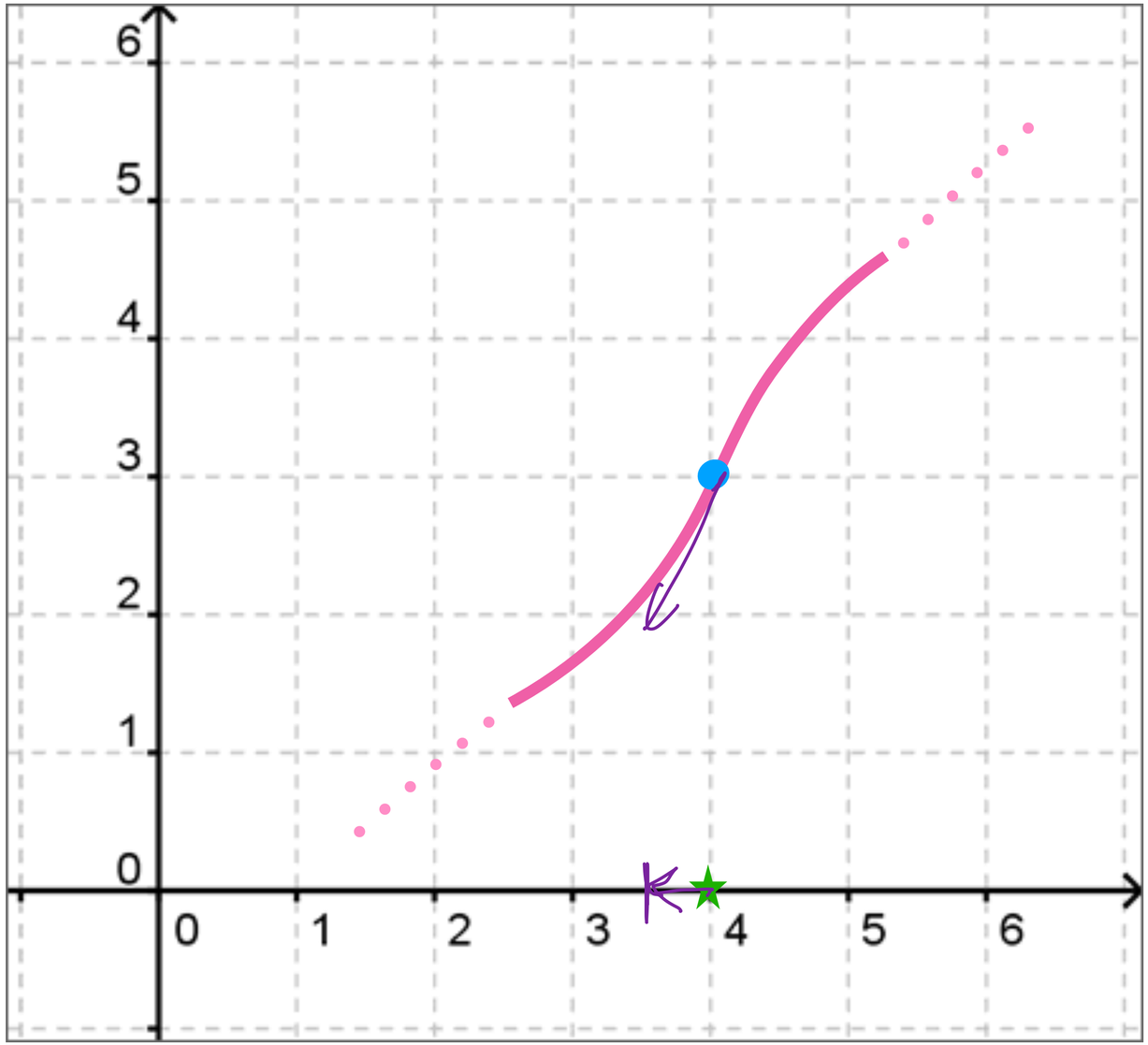
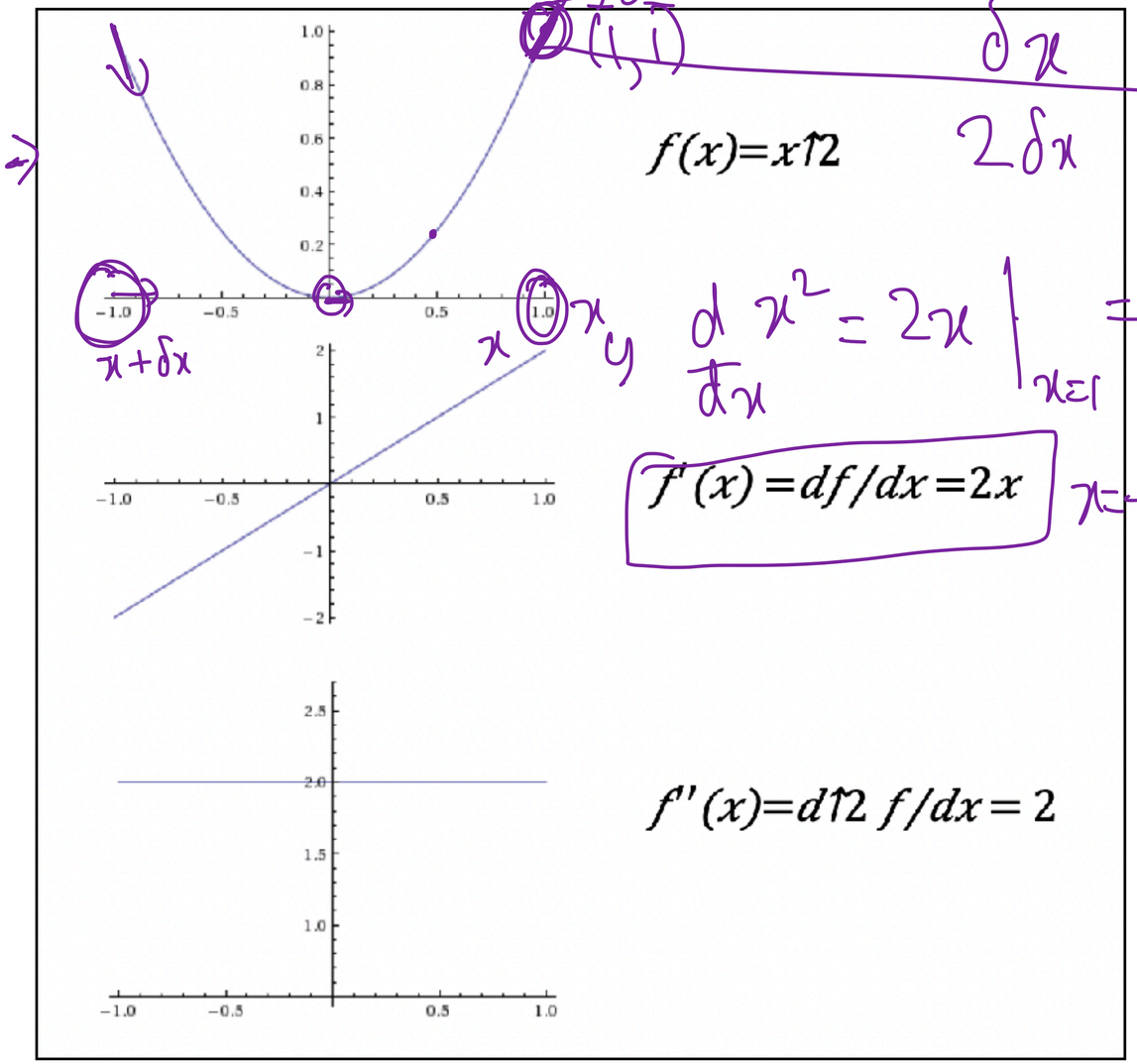
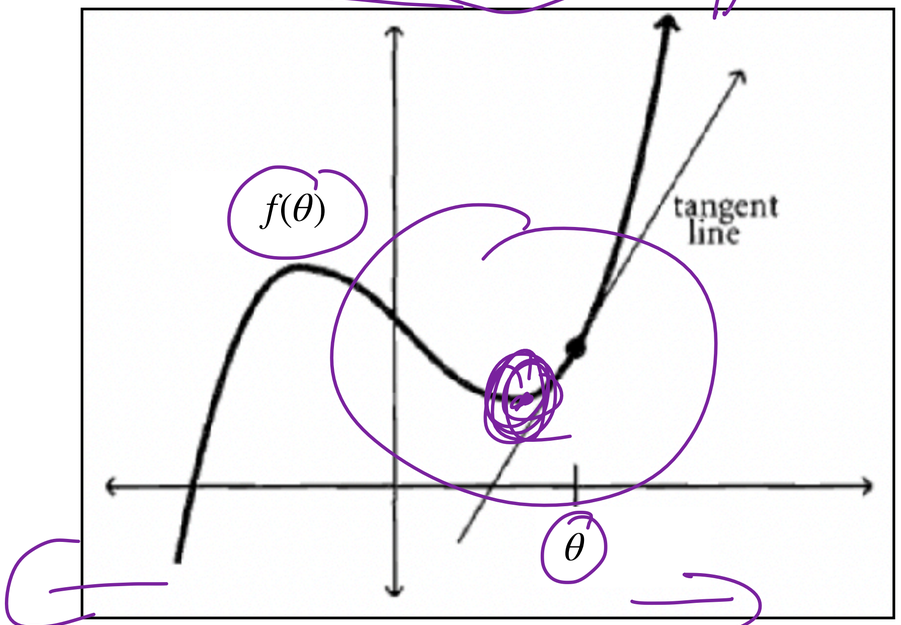


Recap: Derivatives





Gradient Descent Algorithm

- Given: cost / loss/ objective function $f(\theta)$.
- Goal: find θ^* such that $f(\theta^*) = \min_{\theta} f(\theta)$.

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- Gradient descent solution:
  - Start from initial guess θ^0 and learning rate α
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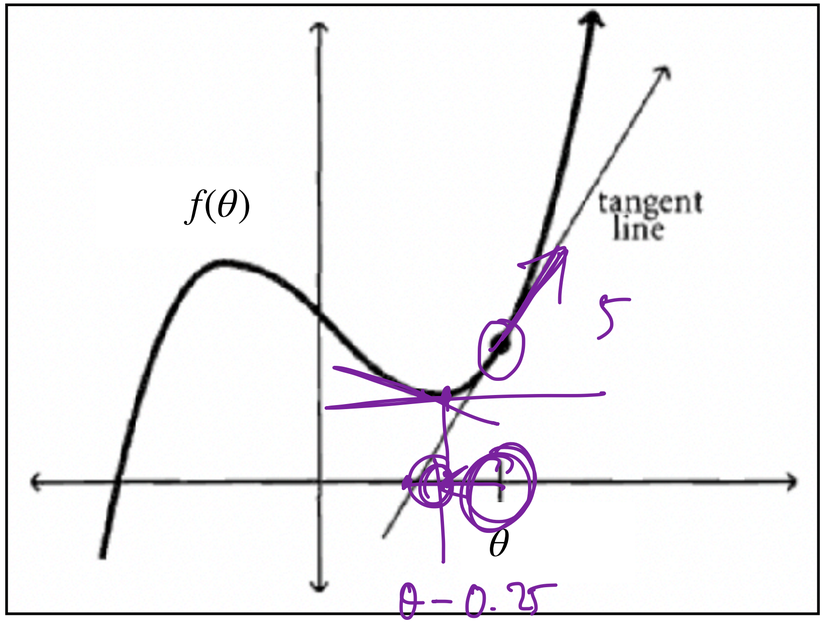
    - Repeat until change in θ is small, or maximum number of steps reached.



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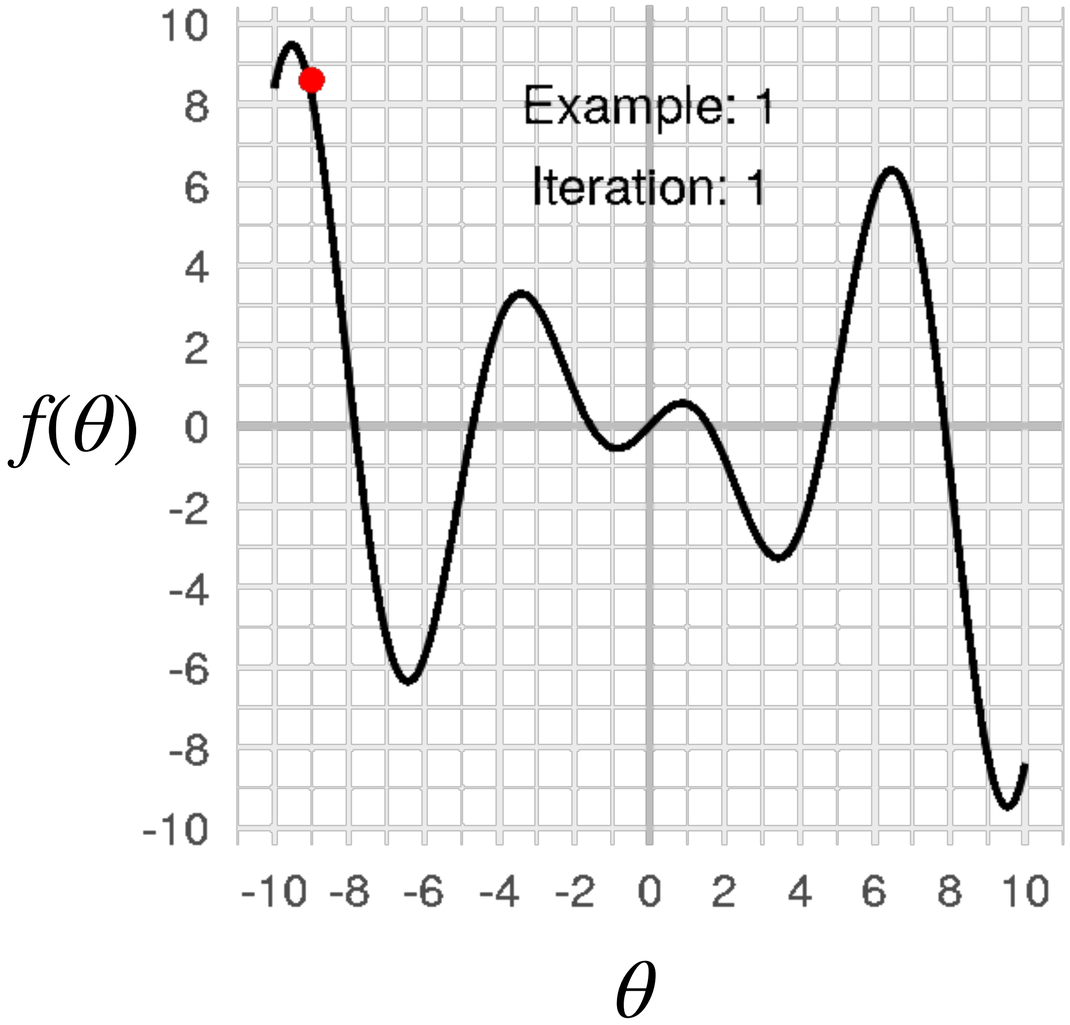
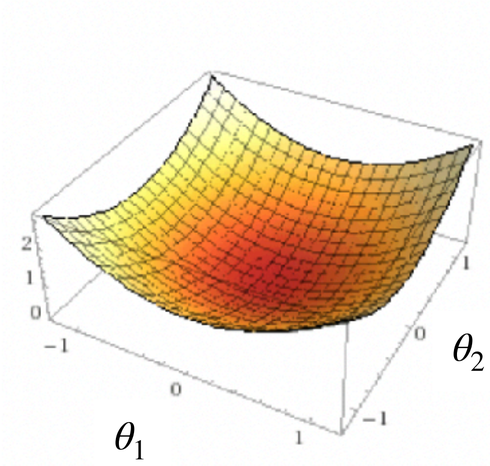


Credits: Charles Bordet

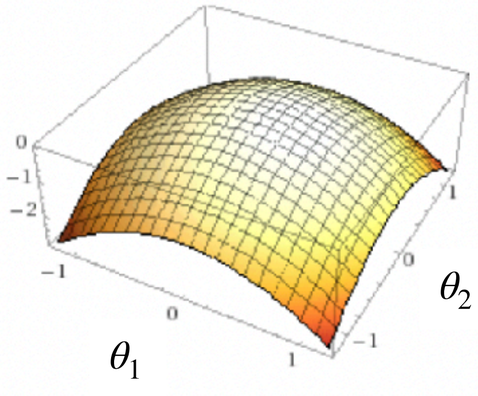
Interactive demo:

https://uclaacm.github.io/gradient-descent-visualiser/

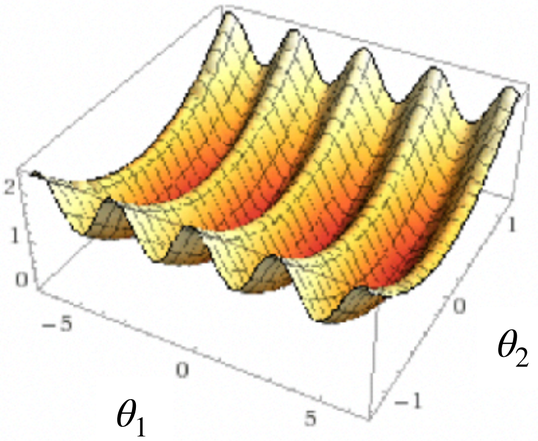
What happens when number of dimensions is high?



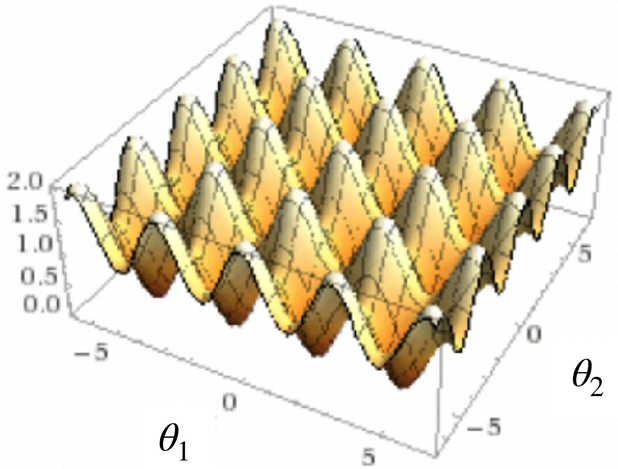
$f(\theta_1, \theta_2) = \theta_1^2 + \theta_2^2$



$f(\theta_1, \theta_2) = -\theta_1^2 - \theta_2^2$



$f(\theta_1, \theta_2) = cos(\theta_1)^2 + \theta_2^2$



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Credits: Michail Michailidis & Patrick Maiden

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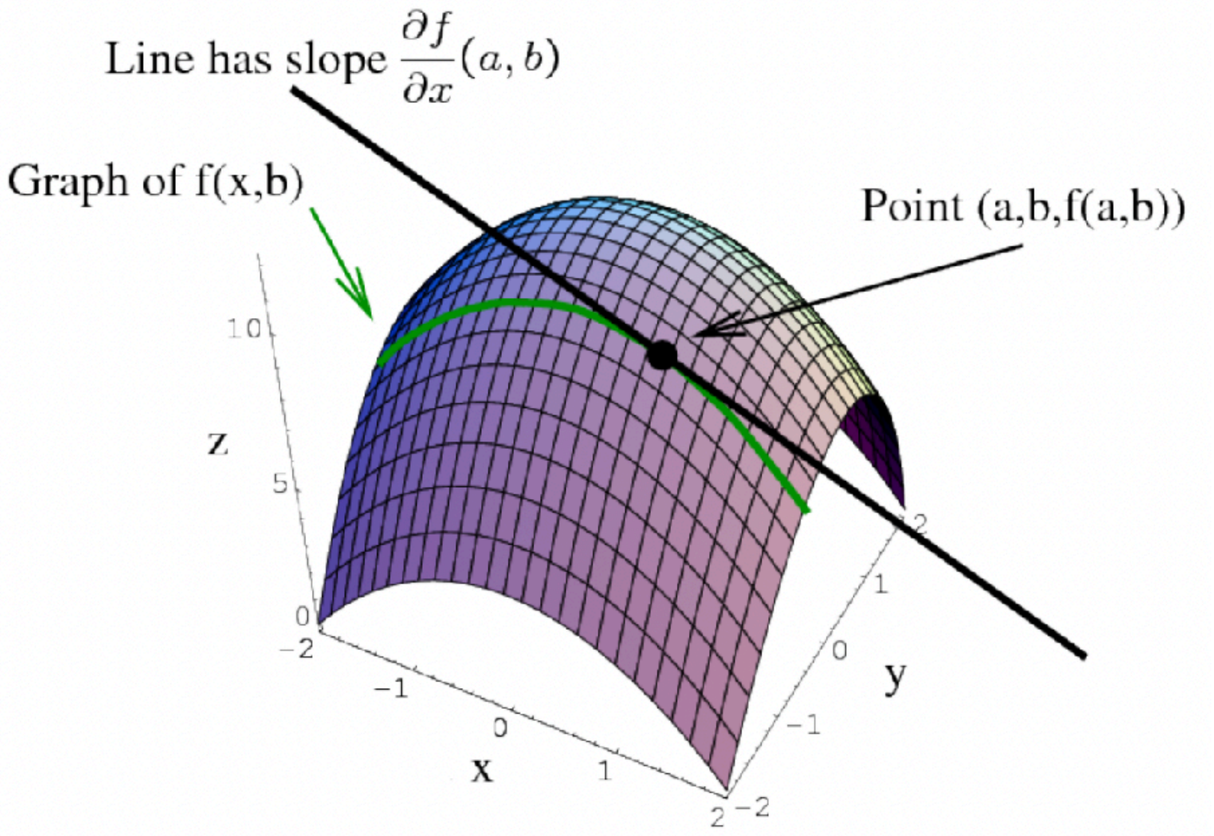

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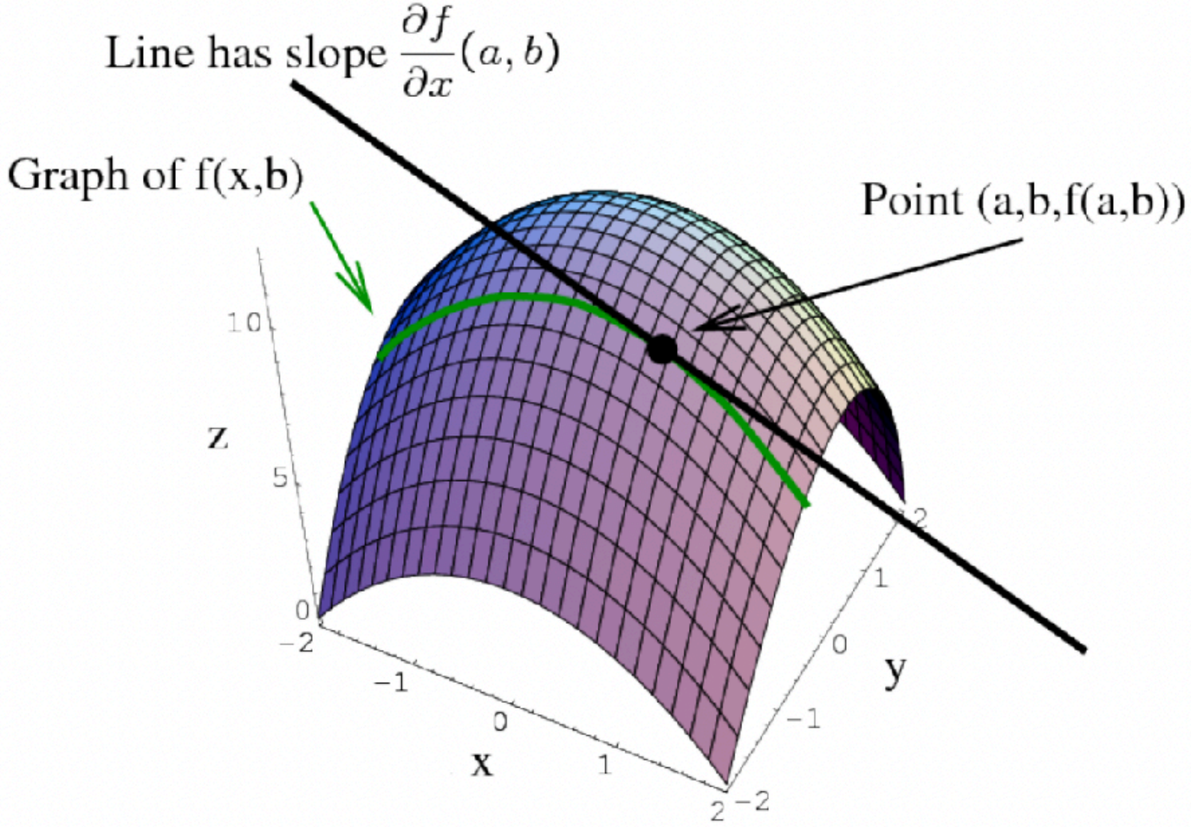

$$f(x,y) = 9 - x^2 - y^2$$

$$f(x,y) = 9 - x^2 - c^2 \qquad f(x,y) = 9 - c^2 - y^2$$

$$\frac{\partial f(x,y)}{\partial x} = -2x \qquad \frac{\partial f(x,y)}{\partial y} = -2y$$

Gradient vector:

$$\nabla f(x,y) = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial x})$$

$$\nabla f(x,y) = (-2x, -2y)$$

Credits: Michail Michailidis & Patrick Maiden

Gradient Descent Algorithm with multiple params

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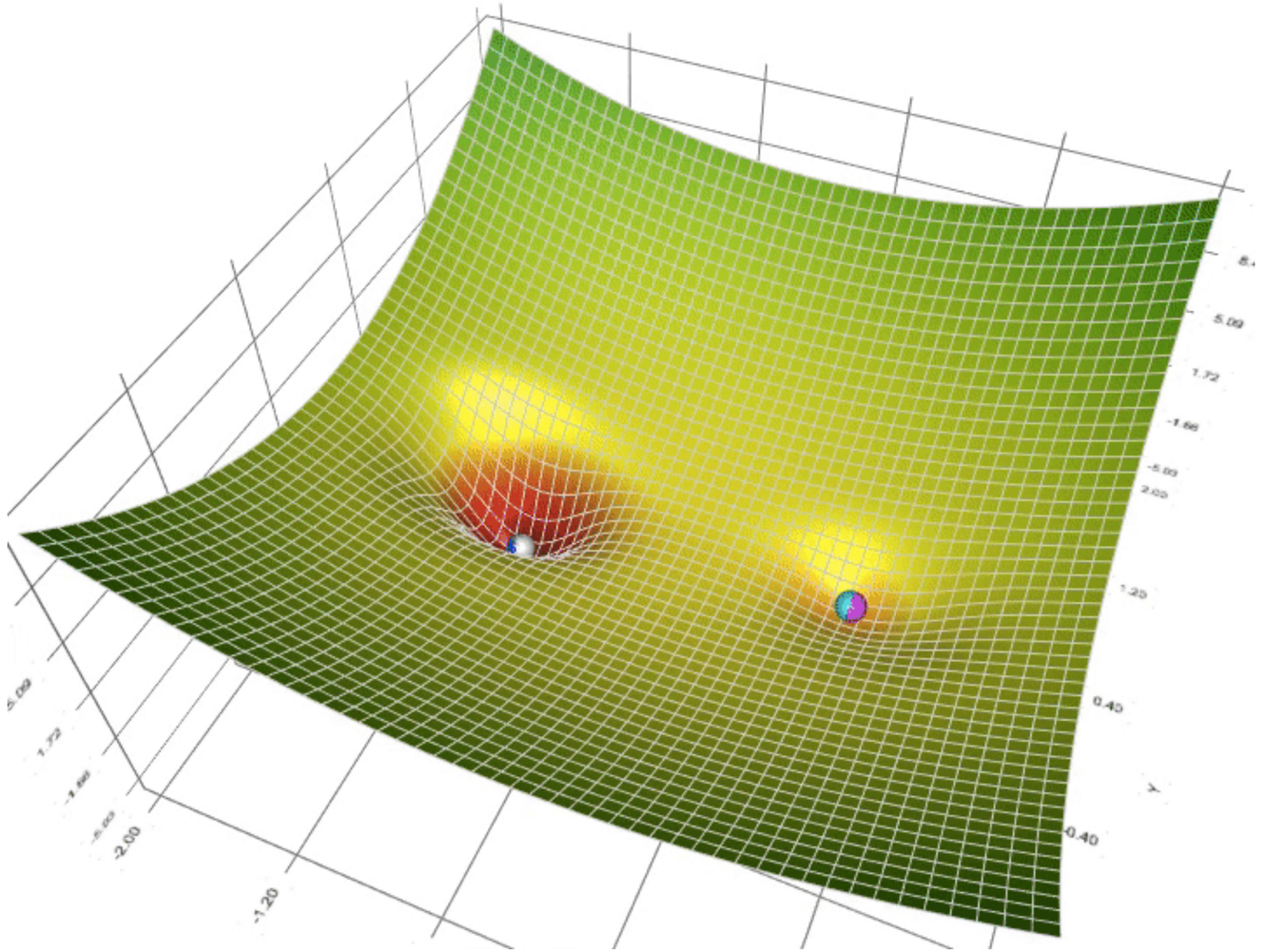
Gradient Descent Algorithm Visualization



Credits: Lili Jiang

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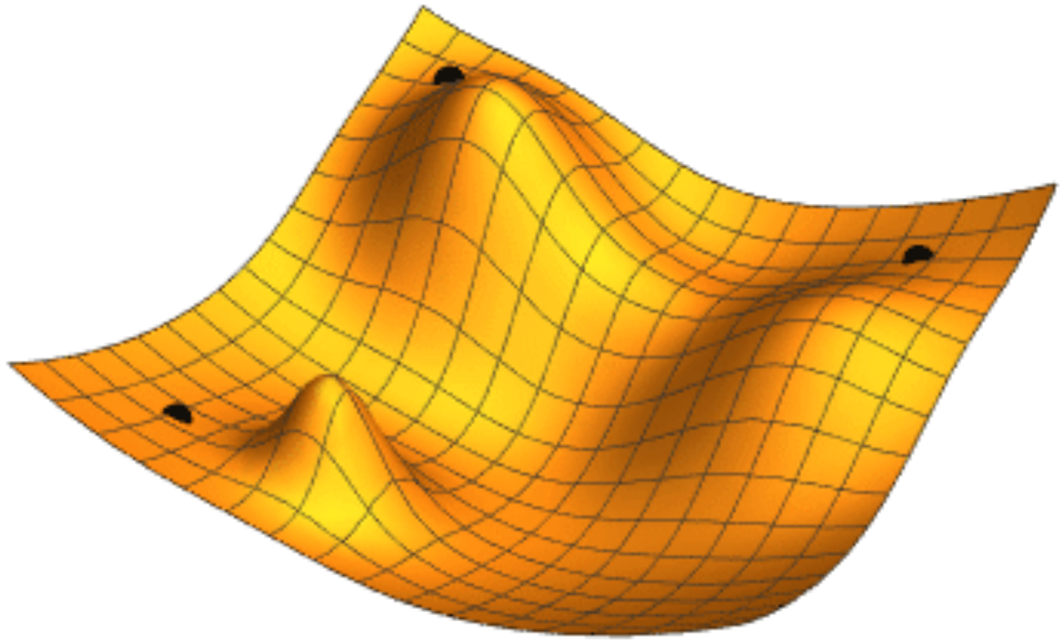

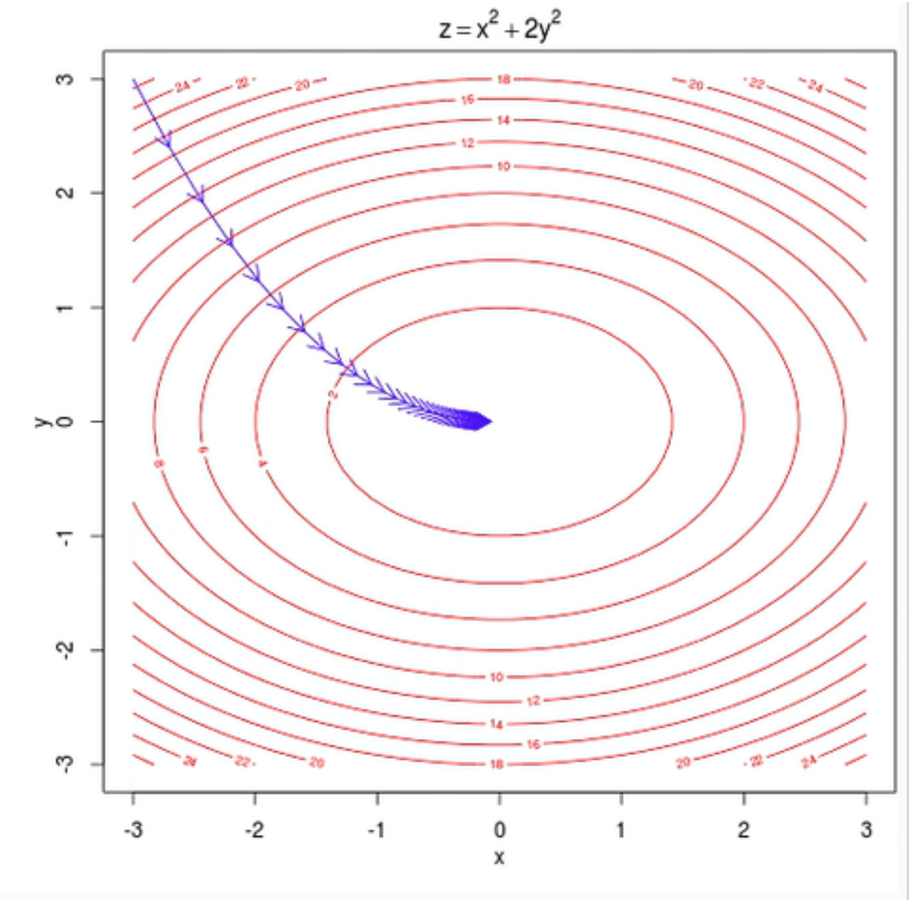

Credits: Wikimedia, Hoang Duong

Additional Reading

- Cross entropy method: https://people.smp.uq.edu.au/DirkKroese/ps/aortut.pdf
- Intro to Optimization: http://www.lewissoft.com/pdf/INTRO_OPT.pdf
- Linear Algebra Blog from Gregory Gundersen (https://gregorygundersen.com/blog/tags/la/)
- Mathematics for Machine Learning e-book by Deisenroth, Faisal, Ong (https://mml-book.github.io/book/mml-book.pdf)

Additional Reading

- Interactive tutorial: https://uclaacm.github.io/gradient-descent-visualiser/
- Book chapter: https://www.cs.utah.edu/~jeffp/IDABook/T6-GD.pdf
- SGD + variants: https://ruder.io/optimizing-gradient-descent

Questions?