

Exam 1 - 10/13

In class

No calculator.

No class Monday we meet Tuesday & Wednesday.

Format of exam.

MIC
T/F
a few-

Show
work.

3 or 4

~ still in the
works.

14.1 Functions of Several Variables.

(10) $F(x, y) = 1 + \sqrt{4 - y^2}$

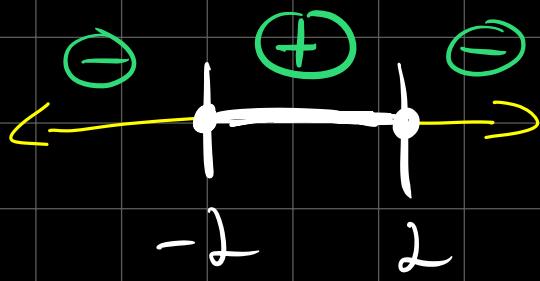
(a) $F(3, 1) = 1 + \sqrt{4 - (1)^2} = 1 + \sqrt{3}$

(b) Find & sketch domain of $F(x, y)$.

$x \in \mathbb{R}$.

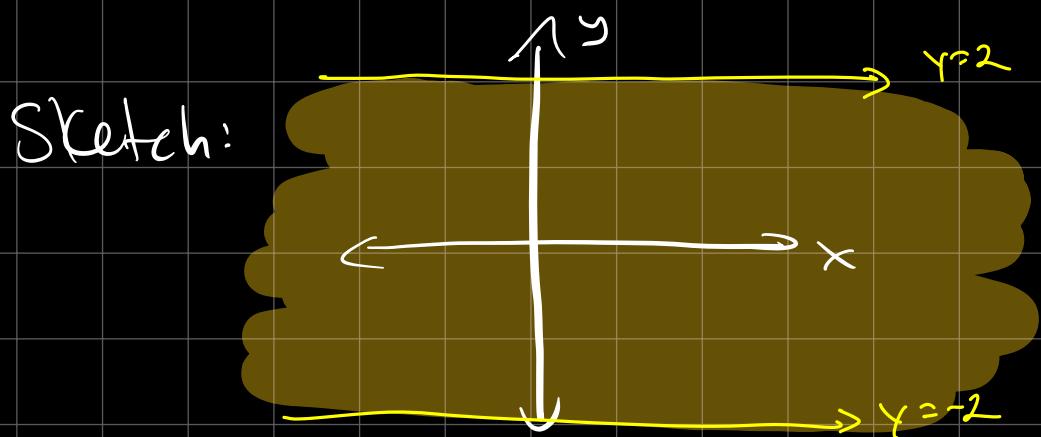
Need $4 - y^2 \geq 0$

$$(2+y)(2-y) \geq 0$$



$$-2 \leq y \leq 2$$

$$D = \{(x, y) \mid x \in \mathbb{R}, -2 \leq y \leq 2\}$$



$$\text{Range of } f(x, y) = 1 + \sqrt{4 - y^2}$$

Smallest = 0

Smallest value for $f(x, \pm 2) = 1 + 0 = 1$
when $y = \pm 2$

Biggest occurs when $y = 0$
 $f = 3$.

Look at

$$z = 1 + \sqrt{4 - y^2}$$

$$(z-1)^2 = (\sqrt{4-y^2})^2$$

$$(z-1)^2 = 4 - y^2 \Rightarrow y^2 + (z-1)^2 = 4$$

Circle centred at
 $(0, 1)$ in yz -plane.

$$-2 \leq y \leq 2$$

$$1 \leq z \leq 3$$

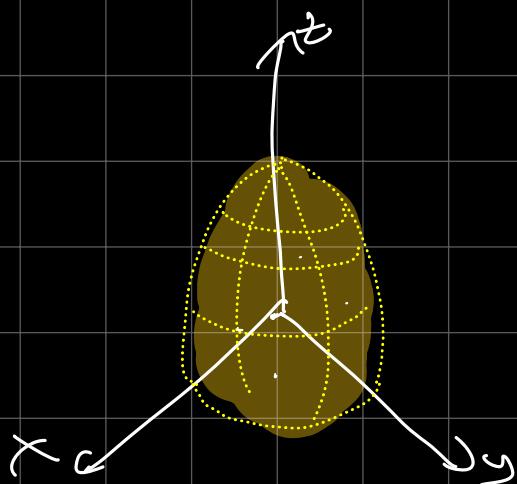
11) $f(x, y, z) = \sqrt{x} + \sqrt{y} + \sqrt{z} + \ln(4-x^2-y^2-z^2)$

find $\{$ describe domain.

$$x \geq 0, y \geq 0, z \geq 0$$

$$4-x^2-y^2-z^2 > 0 \Rightarrow x^2+y^2+z^2 < 4$$

Open ball.



Range of $f(x,y,z)$?

$$f(x,y,z) = \sqrt{x} + \sqrt{y} + \sqrt{z} + \ln(4-x^2-y^2-z^2)$$

upper bound may be a bit less clear.

$-\infty < f \leq \sqrt{4x}$? need more calculus.
try a few things.

① look at $\sqrt{x} + \ln(4-x^2)$

$$\underbrace{f'(x)}_{=0}$$

appears that x, y, z

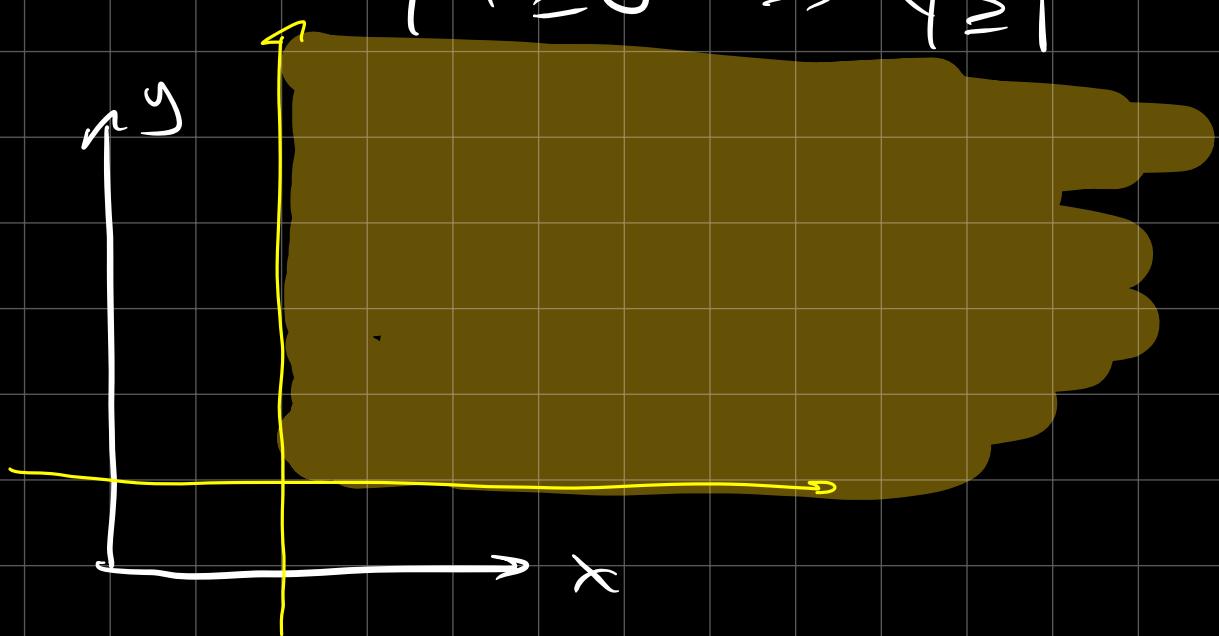
should
be just
smaller
than 1.

⑬ $f(x,y) = \sqrt{x-2} + \sqrt{y-1}$

find & sketch domain

need $x-2 \geq 0 \Rightarrow x \geq 2$

$y-1 \geq 0 \Rightarrow y \geq 1$

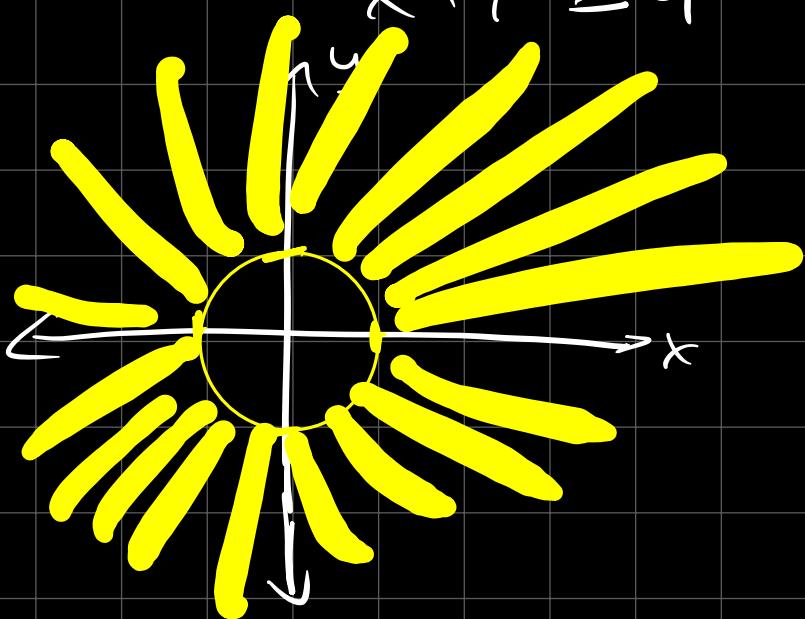


⑯ $f(x,y) = \sqrt{x^2 + y^2 - 4}$

Sketch domain.

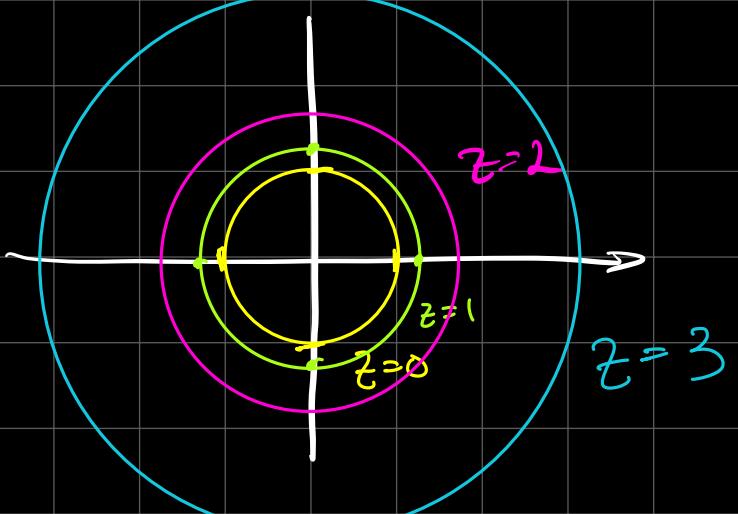
Need $x^2 + y^2 - 4 \geq 0$

$$x^2 + y^2 \geq 4$$

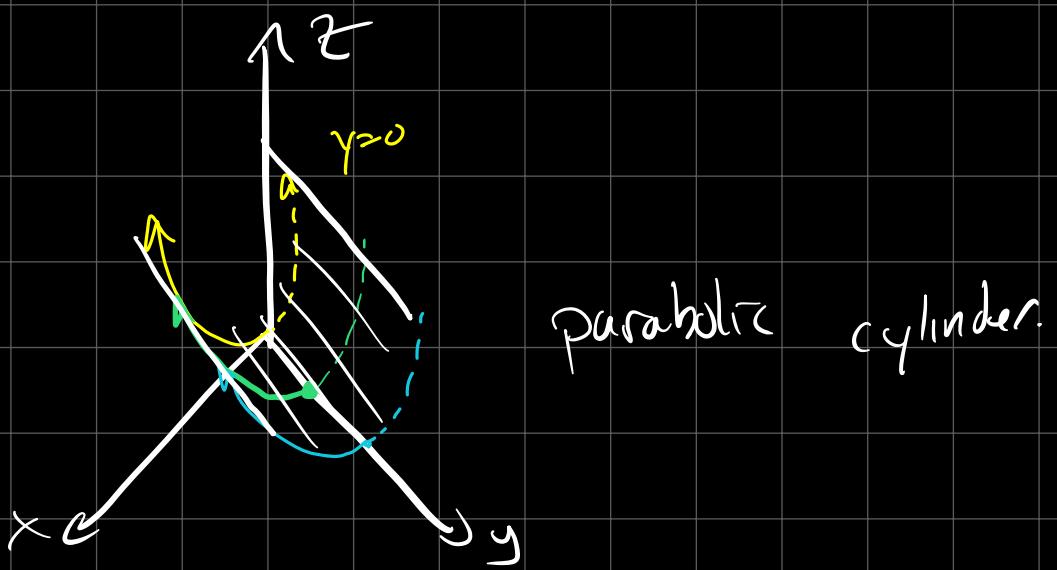


Level curves of $z = \sqrt{x^2 + y^2 - 4}$
(contour)

$\frac{z}{0}$	$z = \sqrt{x^2 + y^2 - 4}$ $0 = \sqrt{x^2 + y^2 - 4} \Rightarrow x^2 + y^2 = 4$ •
1	$1 = \sqrt{x^2 + y^2 - 4} \Rightarrow x^2 + y^2 = 5$ •
2	$2 = \sqrt{x^2 + y^2 - 4} \Rightarrow x^2 + y^2 = 8$ •

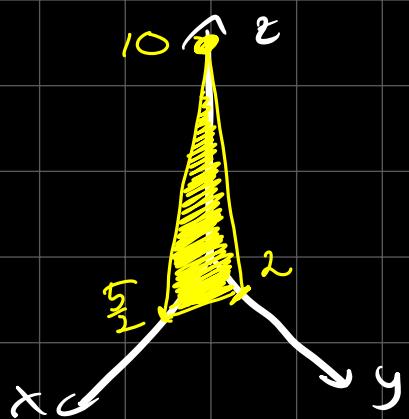


(24) Sketch graph of $f(x,y)=x^2$



(25) $f(x,y)=10-4x-5y$.

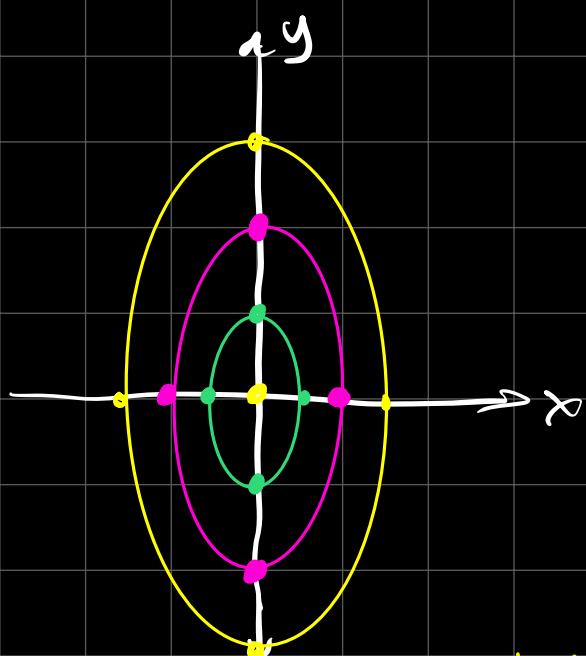
$$z = 10 - 4x - 5y \rightarrow 4x + 5y + z = 10$$

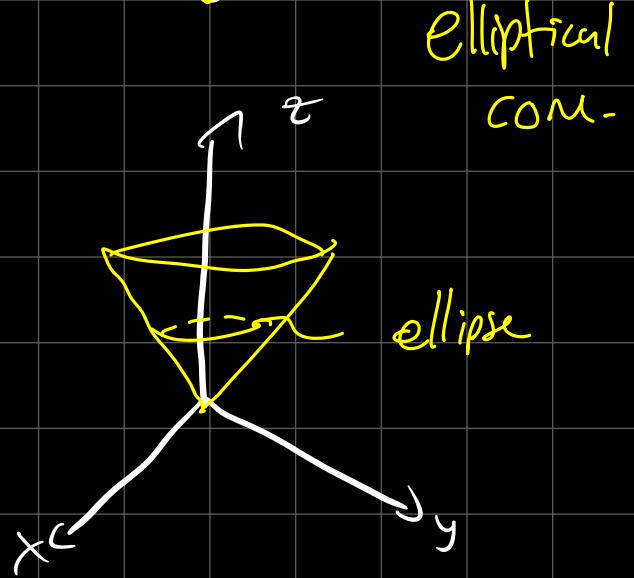


30 $f(x,y) = \sqrt{4x^2 + y^2}$ Sketch graph

Domain: $4x^2 + y^2 \geq 0$ $x \in \mathbb{R}$ or \mathbb{R}^2
 $y \in \mathbb{R}$

\underline{z} 0 1 2 3	$\frac{z = \sqrt{4x^2 + y^2}}{0 = \sqrt{4x^2 + y^2}}$ $x = y = 0$ • $1 = \sqrt{4x^2 + y^2} \Rightarrow 4x^2 + y^2 = 1$ • $2 = \sqrt{4x^2 + y^2} \Rightarrow 4x^2 + y^2 = 4$ • $4x^2 + y^2 = 9$ •
---	--





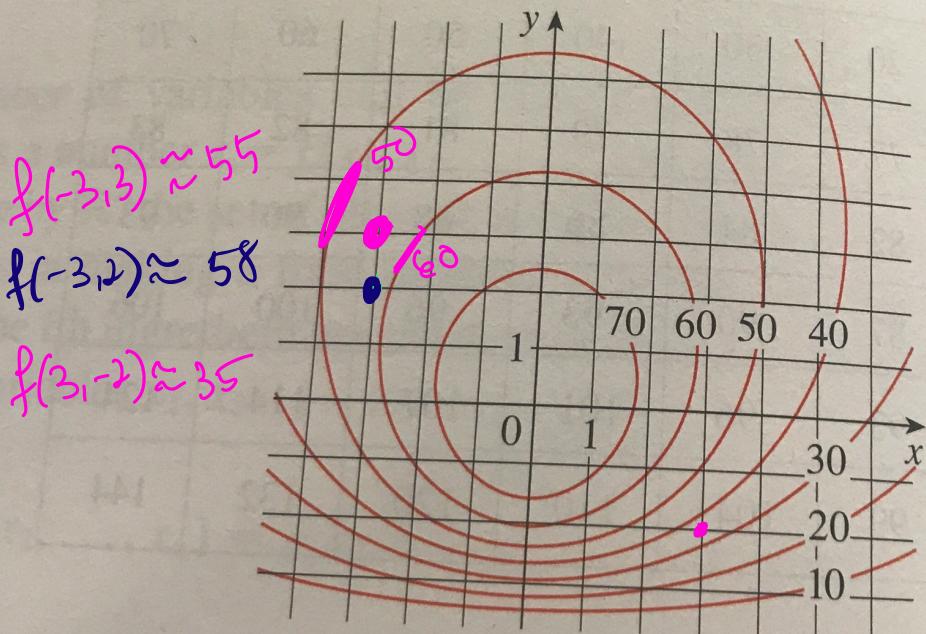
elliptical

com-

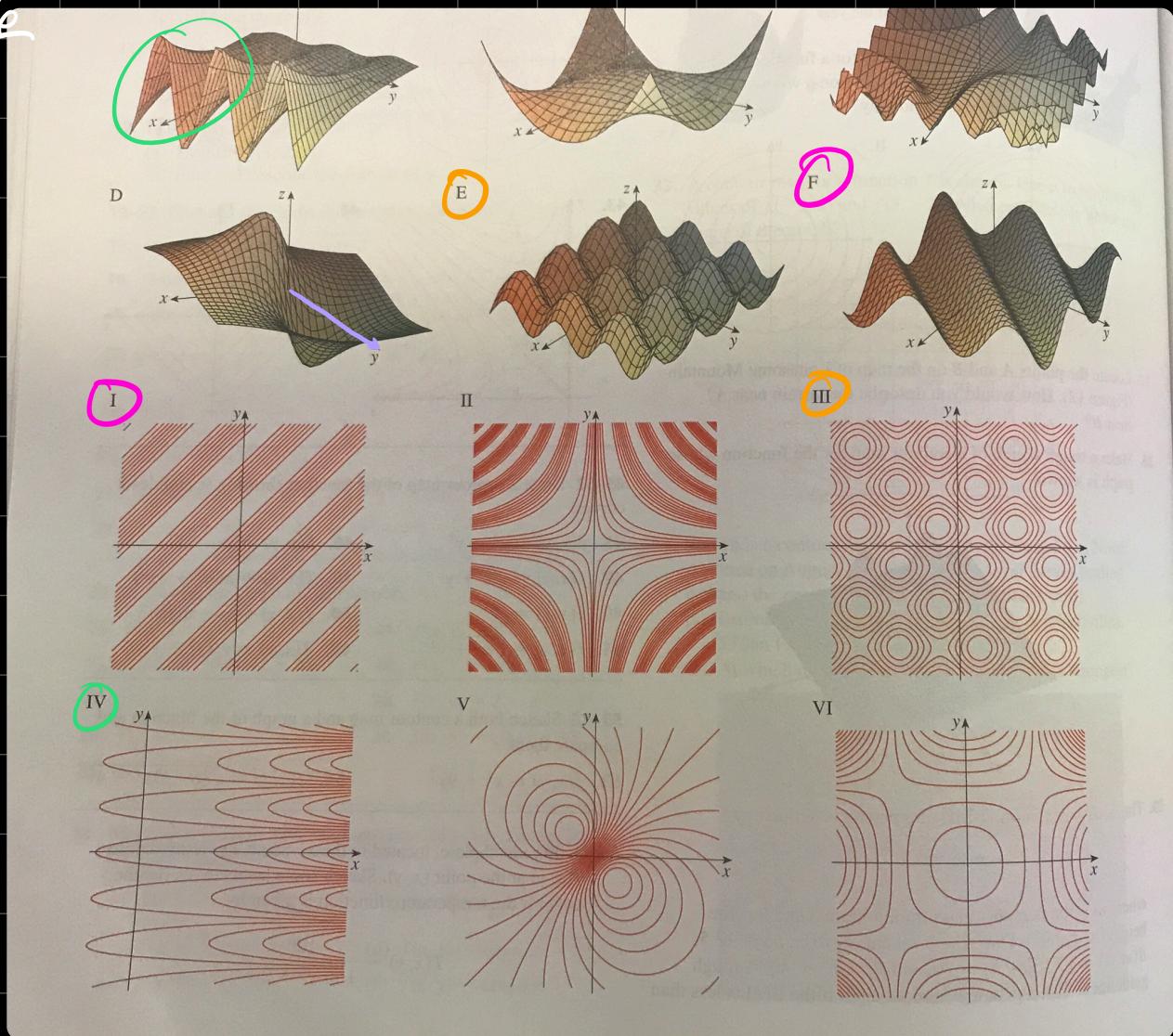
ellipse

33

33. A contour map for a function f is shown. Use it to estimate the values of $f(-3, 3)$ and $f(3, -2)$. What can you say about the shape of the graph?



61-66



$$z = \frac{x-y}{1+x^2+y^2}$$