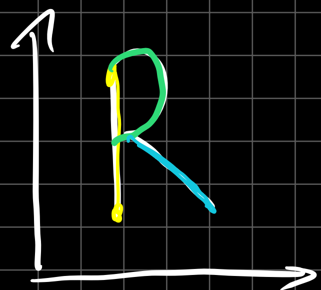
Comment about project.



$$\vec{r}(t) = \begin{cases} \vec{r}_1(t), & a \le t < b \\ \vec{r}_2(t) & b \le t < c \\ \vec{r}_3(t) & c \le t \le d \end{cases}$$

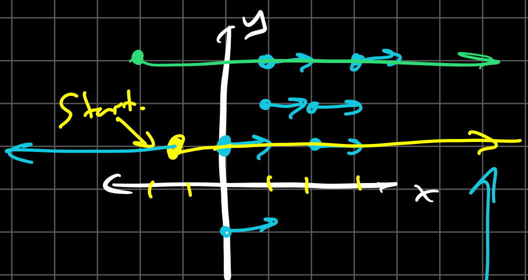
$$\vec{v}(t) = \vec{r}\,'(t) = \begin{cases} \vec{r}_1\,' & a < t < b \\ \vec{r}_2\,' & b < t < c \\ r_3\,' & c < t < d \end{cases}$$

Can $\underline{\underline{\text{not}}}$ include a, b, c, d.

<u>Make sure you label</u>

16.1 Vector fields

① $\vec{F}(x,y) = \hat{i}$



path of a particle
flow line.
or
stream line.

find a param. of the curve.

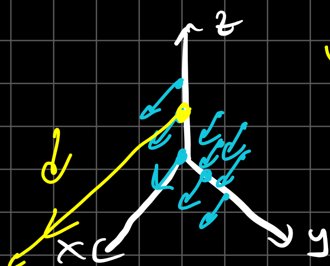
$x = t$, $t \ge -1$
$y = 1$

or

$x = t-1$, $t \ge 0$
$y = 1$

$\vec{r}(t) = \langle t-1, 1 \rangle$
$t \ge 0$.

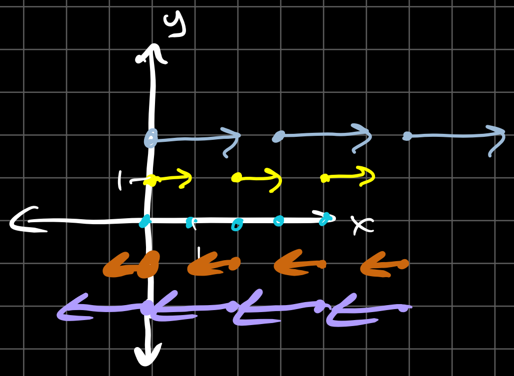
$\vec{F}(x,y,z) = \hat{i}$



param. this flow line.

$y = 0$
$z = 1$
$x = t$, $t \ge 0$

⑧ $\vec{F}(x,y) = y\hat{\imath}$ — observation: $\|\vec{F}\| = \sqrt{y^2+0^2} = \sqrt{y^2} = |y|$

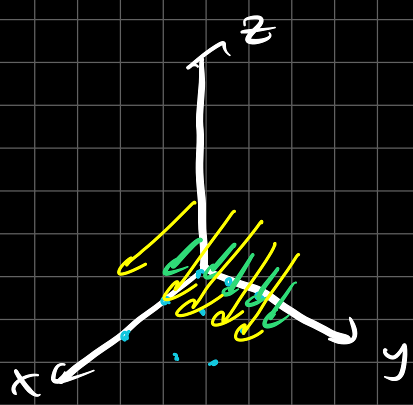


$\vec{F}(0,0) = \vec{0}$

$\vec{F}(0,1) = \hat{\imath}$

$\vec{F}(1,0) = \vec{0}$

$\vec{F}(x,y,z) = z\hat{\imath}$



use - mathematica
wolfram alpha
desmos
geogebra
to help visualize.

(25) $f(x,y) = \frac{1}{2}(x-y)^2$

find $\nabla f = \langle x-y, -(x-y)\rangle = \langle x-y, y-x\rangle$

gradient is a vector field.

See video

as vector field magnitude gets larger
contour lines get closer

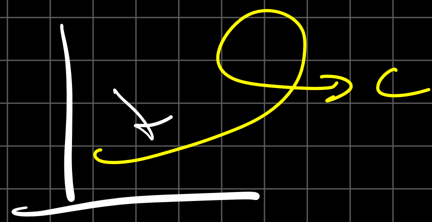
along the contour curves V.F. is same.

vector field $\perp$ to contour lines.

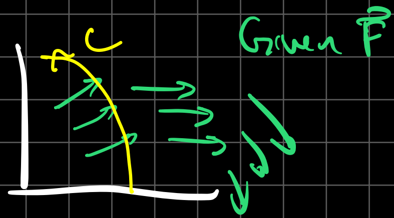
flow line (stream line) $\perp$ contour curve.

16.2 Line Integrals

Basic idea



$$\int_C f(x,y)\,ds$$



evaluate $\int_C \vec{F}\cdot d\vec{r}$

Definition

$$\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

if C is param by $x(t), y(t)$, $\vec{r}(t) = \langle x(t), y(t)\rangle$

$$= \int_a^b f(\vec{r}(t))\,\|\vec{r}\,'(t)\|\,dt$$

$$\int_C f(x,y)\,dx = \int_a^b f(x(t),y(t))\,x'(t)\,dt \qquad \text{same for } dy.$$

also

$$\int_C \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt = \int_C \vec{F}\cdot\vec{T}\,ds$$

unit tangent vector

16.2

② $\boxed{\int_C \left(\frac{x}{y}\right) ds}$ $\quad C: x = t^3 \quad y = t^4 \quad 1 \le t \le 2$

$\frac{dx}{dt} = 3t^2 \qquad \frac{dy}{dt} = 4t^3$

$$= \int_1^2 \left(\frac{t^3}{t^4}\right)\sqrt{(3t^2)^2+(4t^3)^2}\;dt$$

$$= \int_1^2 \left(\frac{1}{t}\right)\sqrt{9t^4+16t^6}\,dt$$

factor out t^4
take square root.

$$= \int_1^2 \frac{t^2}{t}\sqrt{9+16t^2}\,dt$$

$$= \int_1^2 t\sqrt{9+16t^2}\,dt$$

$u = 9+16t^2$
$du = 32t\,dt$

$$= \frac{1}{32}\int_{25}^{73} u^{\frac{1}{2}}\,du$$

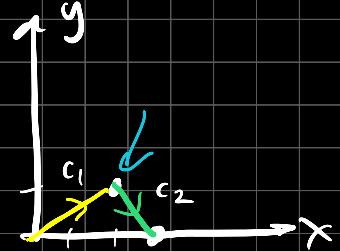
$$= \frac{1}{32}\left(\frac{2}{3}\right)u^{3/2}\Big|_{u=25}^{u=73}$$

think as: $\int_C x+2y\,dx + \int_C x^2\,dy$

7 $\int_C (x+2y)\,dx + x^2\,dy$

– looks weird.

C line segments from $(0,0)$ to $(2,1)$ and $(2,1)$ to $(3,0)$.



on C_1: need a param. Line segment $\vec{r}(t) = (1-t)\vec{r}_0 + t\vec{r}_1$

$0 \le t \le 1$

$\vec{r}(t) = (1-t)\langle 0,0\rangle + t\langle 2,1\rangle$

$\vec{r}_0 = \langle 0,0\rangle$

$= \langle 2t, t\rangle$, $0\le t\le 1$

$\vec{r}_1 = \langle 2,1\rangle$

$\frac{dx}{dt} = 2$

$x(t) = 2t$ $\quad y(t) = t$ $\rightarrow$ $dx = 2\,dt$ $\quad dy = dt$

$$\int_{C_1} (x+2y)\,dx + x^2\,dy = \int_{C_1}(x+2y)\,dx + \int_{C_1} x^2\,dy$$

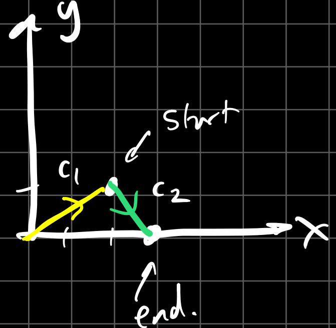
$$= \int_0^1 (\underbrace{2t+2(t)}_{4t})\,2\,dt + \int_0^1 (2t)^2\,dt$$

$$= \int_0^1 \left((4t)2 + (2t)^2\right)dt$$

$$= \int_0^1 8t + 4t^2\,dt = \left(4t^2 + \frac{4}{3}t^3\right)\Big|_{t=0}^{t=1} = 4 + \frac{4}{3}$$

$$= \frac{16}{3}.$$

now evaluate $\int_{C_2} (x+2y)\,dx + x^2\,dy = \int_{C_2} x+2y\,dx + \int_{C_2} x^2\,dy$



$\vec{r}_0 = \langle 2,1\rangle$, $\vec{r}_1 = \langle 3,0\rangle$

$\vec{r}(t) = (1-t)\,\vec{r}_0 + t\,\vec{r}_1 = (1-t)\langle 2,1\rangle + t\langle 3,0\rangle$

$0 \le t \le 1$

$= \langle 2-2t+3t,\ 1-t\rangle$

$= \langle \underbrace{2+t}_{x},\ \underbrace{1-t}_{y}\rangle$

$dx = dt$ $\quad dy = -dt$

$$\int_{C_2} (x+2y)\,dx + x^2\,dy = \int_0^1 (2+t) + 2(1-t)\,dt + \int_0^1 (2+t)^2(-dt)$$

$$= \int_0^1 (2+t) + 2 - 2t - (2+t)^2\,dt = \boxed{}$$

then $\int_C x+2y\,dx + x^2\,dy = \frac{16}{3} + \boxed{}$

(21) $\vec{F}(x,y,z) = \sin x\,\hat{i} + \cos y\,\hat{j} + xz\,\hat{k}$

$\vec{r}(t) = t^3\,\hat{i} - t^2\,\hat{j} + t\,\hat{k} \qquad 0 \le t \le 2$

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt$$

$$\vec{r}\,'(t) = 3t^2\hat{i} - 2t\hat{j} + \hat{k}$$

$$\vec{F}(\vec{r}(t)) = \sin(t^3)\hat{i} + \cos(-t^2)\hat{j} + t^3(t)\hat{k}$$

$$\vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t) = 3t^2\sin(t^3) - 2t\cos(-t^2) + t^4$$

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^2 \left(3t^2\sin(t^3) - 2t\cos(-t^2) + t^4\right)dt$$

$$= \int_0^2 3t^2\sin t^3\,dt + \int_0^2 -2t\cos(-t^2)\,dt + \int_0^2 t^4\,dt$$

$u = t^3$ $\qquad$ $v = -t^2$

$du = 3t^2\,dt$ $\qquad$ $dv = -2t\,dt$

$$= \int_0^8 \sin u\,du + \int_0^{-4}\cos v\,dv + \frac{t^5}{5}\Big|_{t=0}^{t=2}$$ becomes easy.

output = how much vector field and path work together.

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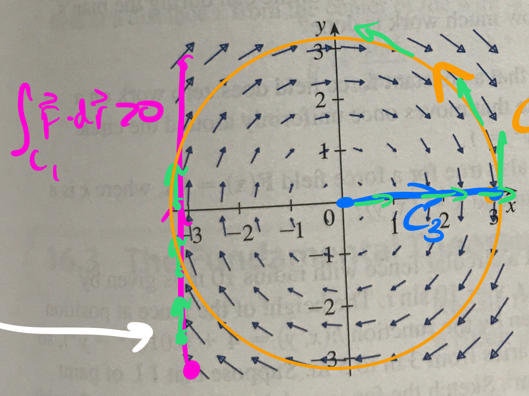
17. Let **F** be the vector field shown in the figure.
(a) If C_1 is the vertical line segment from $(-3, -3)$ to $(-3, 3)$, determine whether $\int_{C_1} \mathbf{F}\cdot d\mathbf{r}$ is positive, negative, or zero.
(b) If C_2 is the counterclockwise-oriented circle with radius 3 and center the origin, determine whether $\int_{C_2} \mathbf{F}\cdot d\mathbf{r}$ is positive, negative, or zero.



dot product will be >0

$$\int_{C_2} \vec{F}\cdot d\vec{r} < 0$$

$$\int_{C_3} \vec{F}\cdot d\vec{r} = 0$$