

## Bitwise operators

& AND

| OR

~ COMPLEMENT

^ XOR

These operators do not modify their operands

int x = 2; // 10 binary

int y = 1; // 1 binary

x | y; // doesn't modify x or y

x + y; // also doesn't modify x or y

x = x | y; // now x is 11 binary = 3 decimal

## Shift operators

- compute a value resulting from shifting a number left or right.

>> - right shift

<< - left shift

$x \ll 3$  - shifts the value of  $x$  left by three bits, filling in zeros in the rightmost 3 bits.

int  $x = 0xf;$  F hex = 1111 binary  
int  $y = x \ll 5;$  //  $y$  is 00..01110000

Since each 1-bit shift left doubles the value of the number, shifting left by 5 multiplies the number by  $2^5 = 32$ .  
- shifting  $n$  bits left multiplies a number by  $2^n$ .

What happens if the number is negative, such that it looks like:

10...01

↑ negative

and we shift left by 1 bit:

0...010

↑ positive!

Overflow situation:

- number is too big to be contained in the bits we have.

Right shift:

$x \gg 4$

Computes the result of shifting the value of  $x$  to the right by 4.

- if  $x$  is unsigned (positive or zero)  
then zeros are written into the leftmost 4 bits.

- if  $x$  is signed, then the right shift has to preserve the sign of  $x$ .
  - if  $x$  is positive or zero, fill in the leftmost bits with zeros.
  - if  $x$  is negative, fill in the leftmost bits with 1.

This causes a right shift by  $n$  bits to compute  $x/2^n$ .

unsigned int  $x = 0xFA;$

$\begin{array}{r} 110\ldots01111010 \\ \text{F A} \end{array}$

unsigned int  $y = x >> 3;$

$\begin{array}{r} 110\ldots01111 \\ \text{y} \end{array}$

int  $w = -2; // 11\ldots10$   
 int  $z = w >> 1; // 11\ldots11$

$\uparrow$  new bit = 1       $\uparrow$  shifted off

We use bitwise and shift operators to read and set individual bits of a variable.

Example: how do we determine if the rightmost bit of a number is 0 or 1?

- use the `&` operator with a "mask" that contains all zeros and a 1 in the rightmost bit position.

unsigned int n = 37;

unsigned int result = n & 0x1;  
                            ^  
                            mask = 00..01

n            00..100101  
0x1              00 000001  
                  0000...1

So, result will be 1 since n had a 1 in the rightmost position.

unsigned int  $x = 36$ ;  
unsigned int result =  $x \& 0x1$

$$\begin{array}{r} 0..000100100 \\ \& \underline{0..000000001} \\ 000 - ..00 \end{array}$$

result is  $\emptyset$ , telling us that  $x$  had a zero in the rightmost position.

What if I wanted to know if the  $n^{th}$  bit of  $x$  is 1 or 0?

Solution 1: Perform an  $\&$  on  $x$  and a mask with  $\& 1$  in the  $n^{th}$  position and zeros everywhere else.

$$\text{unsigned int result} = x \& (\underbrace{0x1}_{\substack{\text{bit } n \\ \downarrow}} \ll n)$$

mask w/ 1  
in the  $n^{th}$  position

$$\begin{array}{r} x \quad 0001010101101 \\ \text{mask} \quad 0000001000000 \\ \hline \text{result} \quad 0000000000000 \end{array}$$

Result will be 0 if the  $n^{th}$  bit of  $x$  is 0.

What if the  $n^{th}$  bit of  $x$  is 1?

Result is  $\neq \phi$ , but it's not 1 either.

- in fact, it's  $2^n$  (which is not important)

```
if ( $x \& (0x1 << n)$ )
    printf("yes");
else
    printf("no");
```

Solution 2:

Shift  $x$  right by  $n$  bits and & with  $0x1$ ;

unsigned int result =  $(x >> n) \& 0x1;$

$x$  0..0010110101  
 $(x >> 4)$  00...001011  
mask & 00-----01  
————— 00...01

Here result will either be 0 or 1;

How do we determine the value of  
the lowest 4 bits of  $X$ .

$$\begin{array}{r} X \quad 00..101101(0) \\ \text{mask } \& 000000001111 \leftarrow F_{\text{hex}} \\ \hline 0000..01101 \end{array}$$

all zeros      lowest 4 bits of  $X$ .

unsigned int result =  $X \& 0xf;$

↓ instead

#define MASK 0xf

unsigned int result =  $X \& \text{MASK};$

In general, if I want to know the value of the  $M$  bits starting at position  $n$ :

- shift  $x$  to the right by  $n$  bits
  - & it with a mask whose lower  $M$  bits are  $1$ .

$x$

M bits

00101010100100

↑  
position 0

$$\begin{array}{r}
 x \gg n \\
 \begin{array}{c}
 000\ldots 00101010 \\
 \hline
 8 \\
 \begin{array}{r}
 000000111111 \\
 \hline
 000\ldots 0101010
 \end{array}
 \end{array}
 \end{array}$$

A mask with  $M$  1's in the lowest position would be

$$(1 \ll M) - 1$$

This value is  $2^M - 1$ , which is  $M$  1's in binary  $(111\ldots)$

<sup>unsigned</sup>  
int result = ( $x >> n$ ) &  $((1 \ll M) - 1);$

How do we set the  $n^{th}$  bit of  $x$  to 1?

- OR, "1" the number of a mask that has a 1 in the  $n^{th}$  position and a zero everywhere else.