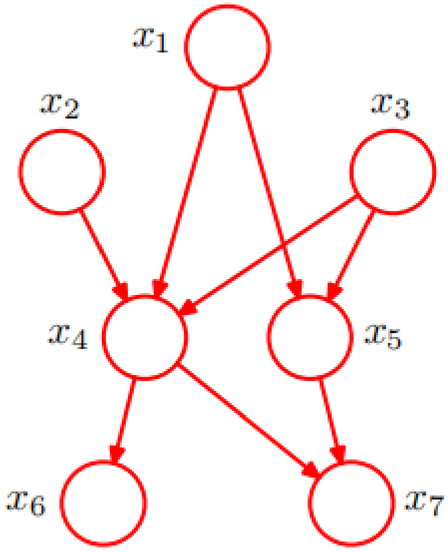
Probabilistic Graphical Models

8.1. Bayesian Networks — directed graphical models

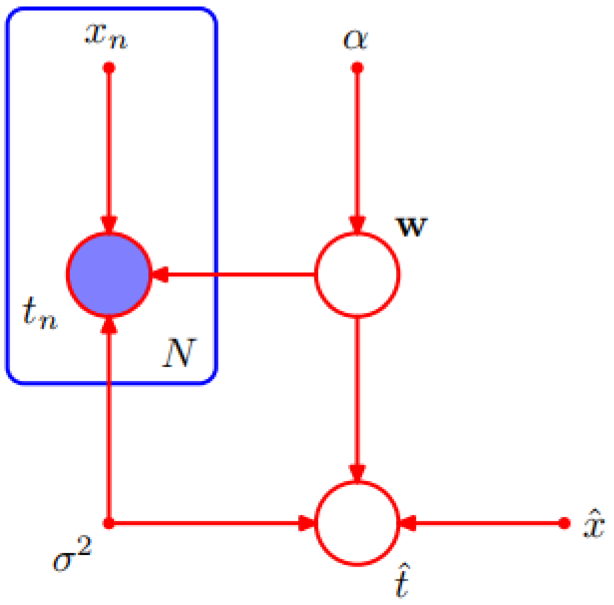


$$p(x_1)p(x_2)p(x_3)p(x_4|x_1, x_2, x_3)p(x_5|x_1, x_3)p(x_6|x_4)p(x_7|x_4, x_5).$$

$$p(\mathbf{x}) = \prod_{k=1}^{K} p(x_k|\mathrm{pa}_k)$$

where pa_k denotes the set of parents of x_k, and $\mathbf{x} = \{x_1, \ldots, x_K\}$.

There must be no directed cycles → directed acyclic graphs, or DAGs



new input value $\hat{x}$

plate (the box labelled *N*) — *N* nodes

shaded — t_n are observed variables (training set)

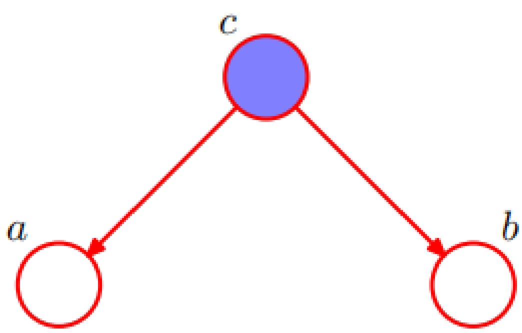
8.2. Conditional Independence

a is conditionally independent of *b* given *c*, $a \perp\!\!\!\perp b \mid c$

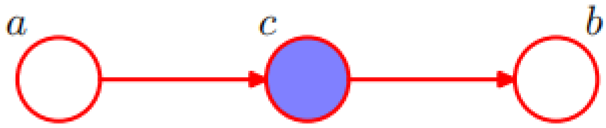
$p(a|b,c) = p(a|c)$

$p(a,b|c) = p(a|b,c)p(b|c) = p(a|c)p(b|c)$

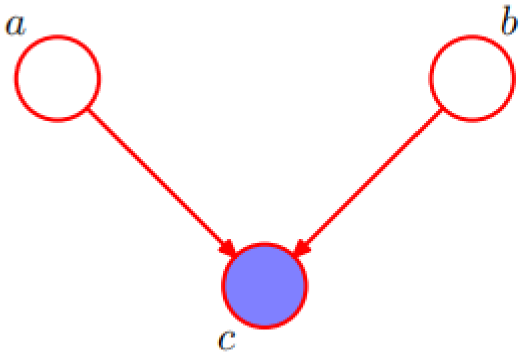
Three examples:



$a \not\!\perp\!\!\!\perp b \mid \emptyset, a \perp\!\!\!\perp b \mid c$ “tail to tail” blocked



$a \not\perp\!\!\!\perp b \mid \emptyset$, $a \perp\!\!\!\perp b \mid c$ “head to tail” blocked



$a \not\perp\!\!\!\perp b \mid c$, $a \perp\!\!\!\perp b \mid \emptyset$ “head to head” unblocked

D-separation

A path is blocked if it includes a node such that either

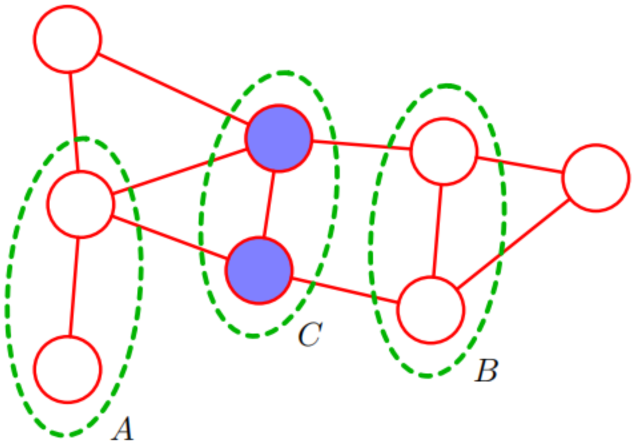
1. the arrows on the path meet either head-to-tail or tail-to-tail at the node, and the node is in the set C, or

2. the arrows meet head-to-head at the node, and neither the node, nor any of its descendants, is in the set C.

If all paths are blocked, then A is said to be d-separated from B by C, and the joint distribution over all of the variables in the graph will satisfy $a \perp\!\!\!\perp b \mid c$

8.3. Markov Random Fields

conditional independence is determined by simple graph separation



An example of an undirected graph in which every path from any node in set A to any node in set B passes through at least one node in set C. Consequently the conditional independence property $A \perp\!\!\!\perp B \mid C$ holds for any probability distribution described by this graph.

clique: a subset of the nodes in a graph such that there exists a link between all pairs of nodes in the subset; fully connected.