Assignment E

Common Notations

We use the following notation:

S_t	:	Stock price at time t
σ	:	Volatility of the Stock (assumed constant)
r	:	Interest rate
T	:	Time to option expiration (in years)
K	:	Strike price
W_t	:	Brownian motion process
$N(0,1)$	:	Standard normal distribution
$(A-B)^+$	:	Max(A-B, 0)

Assignment E (C/C++)

- Due: July 1 by 6 PM.
- Write a function to price European Call options using Monte Carlo.
- Use the function you wrote to price 1 million options.
- You may use the function shown below to generate input values randomly:
- Measure time taken to price 1 million options.
- Vectorizing/parallelizing code is NOT required.

Monte Carlo Technique: Background

- The (time 0) value of an option is the discounted *expectation* of its payoff under the risk neutral measure[1].
- That means: to price an option we need to compute an *expected value*, i.e. an integral.
- Monte Carlo is a *numerical technique* used to **estimate** an integral.

[1] `http://en.wikipedia.org/wiki/Risk-neutral_measure`

Stock Price SDE

- We assume a stock price process follows a *Geometric Brownian Motion*[2].
- Under the *risk neutral measure* the process of the stock price is given by the SDE:

$$dS_t = rS_t dt + \sigma S_t dW_t \tag{1}$$

where,

$$dW_t = N(0, dt) \tag{2}$$

$$W_t = \sqrt{t} N(0,1) \quad \because W_0 = 0 \tag{3}$$

I.e. The Brownian motion has a normal distribution with mean zero (0) and variance t.

- Using the above SDE we get:

$$S_t = S_0 \exp^{((r - \sigma^2/2)t + \sigma\sqrt{t} N(0,1))} \tag{4}$$

[2] http://en.wikipedia.org/wiki/Geometric_Brownian_motion

- Using equation (4), we can simulate the stock price at any time t, in the future.
- $N(0,1)$ is a random observation (random number) drawn from the standard normal distribution.
- Let's say, $z =$ a random number drawn from N(0, 1).
- The stock price at the expiration (time T) is given by:

$$S_T = S_0 \exp^{((r-\sigma^2/2)T+\sigma z\sqrt{T}} \tag{5}$$

- That means, we can simulate the terminal stock price S_T (i.e. price at time T).
- What do we need to find the price of a European style option?

Monte Carlo Option Pricer: Step by Step

- Draw a random number (z_i) from the standard normal distribution, N(0, 1). Here, the subscript i denotes the i^{th} draw.
- Simulate the stock price at the option expiration (S_T). The stock price at the expiration is given by:

$$S_{T,i} = S_0 \exp^{((r-\sigma^2/2)T+\sigma z_i \sqrt{T}} \tag{6}$$

- For a given price path (i^{th}realization), the European call payoff at time zero is given by:

$$C_i = \exp^{-rT}(S_{T,i} - K)^+ \tag{7}$$

- Repeat above steps for (M) trials, and calculate the option payoff for each path.
- Find the mean value of the option payoffs:

$$\hat{C} = \frac{\exp^{-rT}}{M} \sum_{i=1}^{M} (S_{T,i} - K)^{+} \tag{8}$$

- This is an **estimate** of the option price.
- *The law of large numbers* states that such an estimate converges to the correct value as the number of trials (M) increases.