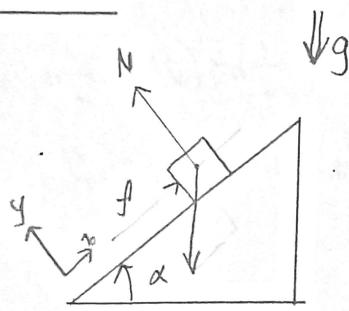


Robotics II

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Exercise 1



Newton-Euler method

$$\sum F = ma$$

$$f - mgs \sin \alpha = m\ddot{x} \quad //$$

Lagrangian

$$q_1 = x, \dot{q}_1 = \dot{x}, Q_1 = f$$

$$K = \frac{1}{2}m\dot{x}^2, V = mgx \sin \alpha$$

$$L = \frac{1}{2}m\dot{x}^2 - mgx \sin \alpha$$

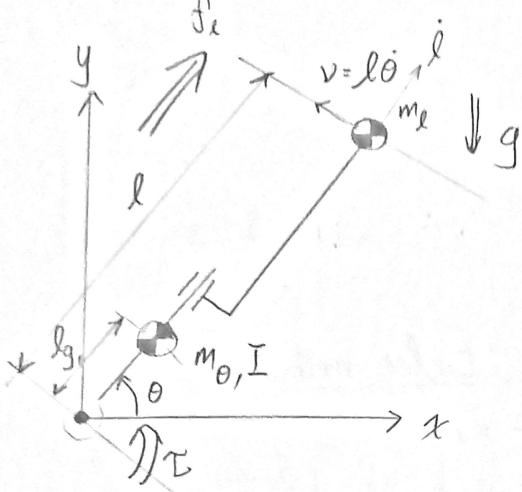
$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i$$

$$m\ddot{x} - (-mgs \sin \alpha) = f$$

$$\text{EOM, } f - mgs \sin \alpha = m\ddot{x}$$

$$f = m\ddot{x} + mgs \sin \alpha \quad //$$

Exercise 2



Lagrangian

$$\begin{cases} q_1 = \theta, q_2 = l, Q_1 = \tau \\ \dot{q}_1 = \ddot{\theta}, \dot{q}_2 = \dot{l}, Q_2 = f_x \end{cases}$$

$$T_{rot.} = \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} m_e (l \dot{\theta})^2$$

$$T_{trans.} = \frac{1}{2} m_e \dot{l}^2$$

$$T = \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} m_e (l \dot{\theta})^2 + \frac{1}{2} m_e \dot{l}^2$$

$$V = (m_e l g_s + m_e l) g \sin \theta$$

$$L = T - V$$

$$\tau - 2m_e l \dot{\theta} - (m_e l g_s + m_e l) g \cos \theta = (I + m_e l^2) \ddot{\theta}$$

$$f - m_e g \sin \theta + m_e l \ddot{\theta} = m_e \ddot{l}$$

$$\begin{cases} \tau = (I + m_e l^2) \ddot{\theta} + 2m_e l \dot{\theta} + (m_e l g_s + m_e l) g \cos \theta \\ f = m_e \ddot{l} - m_e l \dot{\theta}^2 + m_e g \sin \theta \end{cases}$$

$$q_1, \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = \tau$$

$$\frac{\partial L}{\partial \dot{\theta}} = I \ddot{\theta} + m_e l^2 \dot{\theta}$$

$$\begin{aligned} \frac{d}{dt} (\partial_{\dot{\theta}} L) &= I \ddot{\theta} + 2m_e l \dot{\theta} + m_e l^2 \dot{\theta} \\ &= (I + m_e l^2) \ddot{\theta} + 2m_e l \dot{\theta} \end{aligned}$$

$$-\partial_{\theta} L = (m_e l g_s + m_e l) g \cos \theta$$

$$(I + m_e l^2) \ddot{\theta} + 2m_e l \dot{\theta} + (m_e l g_s + m_e l) g \cos \theta = \tau //$$

$$q_2, \frac{d}{dt} (\partial_l L) - \partial_l L = f_x$$

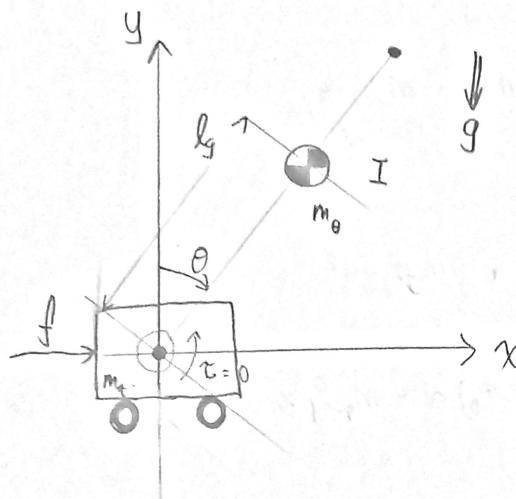
$$\partial_l L = m_e \ddot{l}$$

$$\frac{d}{dt} (\partial_l L) = m_e \ddot{l}$$

$$-\partial_l L = m_e g \sin \theta - m_e \dot{\theta}^2 l$$

$$-m_e l \dot{\theta}^2 m_e \ddot{l} + m_e g \sin \theta = f //$$

Exercise 3



Lagrangian

$$\begin{cases} q_1 = \theta, \dot{q}_1 = \dot{\theta}, Q_1 = -\tau \\ q_2 = x, \dot{q}_2 = \dot{x}, Q_2 = f \end{cases}$$

$$T_{\text{trans}} = \frac{1}{2} m_x \dot{x}^2 + \frac{1}{2} m_\theta [(\dot{x} + l_g \dot{\theta} \cos \theta)^2 + (-l_g \dot{\theta} \sin \theta)^2]$$

$$T_{\text{rot}} = \frac{1}{2} I \dot{\theta}^2$$

$$T = \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} (m_x + m_\theta) \dot{x}^2 + \frac{1}{2} m_\theta (l_g \dot{\theta})^2 + m_\theta l_g \dot{\theta} \cos \theta \dot{x}$$

$$V = m g l_g \cos \theta$$

$$q_1, \quad \partial_{\dot{\theta}} L = I \ddot{\theta} + m_\theta l_g^2 \ddot{\theta} + m_\theta l_g \dot{x} \cos \theta$$

$$\frac{d}{dt} (\partial_{\dot{\theta}} L) = I \ddot{\theta} + m_\theta l_g^2 \ddot{\theta} + m_\theta l_g \dot{x} \cos \theta - m_\theta l_g \dot{x} \dot{\theta} \sin \theta$$

$$\partial_{\theta} L = -m_\theta l_g \dot{x} \dot{\theta} \sin \theta + m g l_g \sin \theta$$

$$I \ddot{\theta} + m_\theta l_g \ddot{\theta} + m_\theta l_g \dot{x} \cos \theta - m_\theta l_g \dot{x} \dot{\theta} \sin \theta + m_\theta l_g \dot{x} \dot{\theta} \sin \theta - m g l_g \sin \theta = \tau$$

$$-\tau + m g l_g \sin \theta = (I + m_\theta l_g^2) \ddot{\theta} + m_\theta l_g \cos \theta \dot{x}$$

$$m g l_g \sin \theta = (I + m_\theta l_g^2) \ddot{\theta} + m_\theta l_g \cos \theta \dot{x} //$$

$$q_2, \quad \partial_{\dot{x}} L = (m_x + m_\theta) \ddot{x} + m_\theta l_g \cos \theta \ddot{\theta}$$

$$\frac{d}{dt} (\partial_{\dot{x}} L) = (m_x + m_\theta) \ddot{x} + m_\theta l_g \cos \theta \ddot{\theta} - m_\theta l_g \sin \theta \dot{\theta}^2$$

$$\partial_x L = 0$$

$$f + m_\theta l_g \sin \theta \dot{\theta}^2 = (m_x + m_\theta) \ddot{x} + m_\theta l_g \cos \theta \ddot{\theta} //$$

EOM

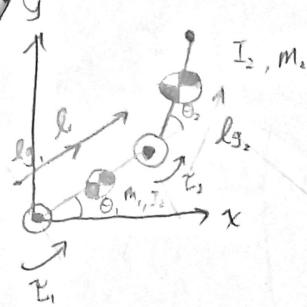
$$m_0 l g \sin \theta = (I + m_0 l^2) \ddot{\theta} + m_0 l g \cos \theta \dot{x}$$

$$f - m_0 l g \sin \theta \dot{\theta}^2 = m_0 l g \cos \theta \ddot{\theta} + (m_x + m_0) \ddot{x}$$

$$\left\{ \begin{array}{l} 0 = (I + m_0 l^2) \ddot{\theta} + m_0 l g \cos \theta \dot{x} - m_0 l g \sin \theta \\ f = m_0 l g \cos \theta \ddot{\theta} + (m_x + m_0) \ddot{x} - m_0 l g \sin \theta \dot{\theta}^2 \end{array} \right.$$

//

case 4



$$\begin{cases} q_1 = \theta_1, \dot{q}_1 = \dot{\theta}_1, Q_1 = \Gamma_1 \\ q_2 = \theta_2, \dot{q}_2 = \dot{\theta}_2, Q_2 = \Gamma_2 \end{cases}$$

$$T_1 = \frac{1}{2} (I_1 + m_1 l_{g_1}^2) \dot{\theta}_1^2$$

$$T_2 = \frac{1}{2} (I_2 + m_2 l_{g_2}^2) \dot{\theta}_2^2$$

$$r_1 = l_1 e^{i\theta_1} + l_{g_1} e^{i(\theta_1 + \theta_2)}$$

$$+ \frac{1}{2} m_1 \left[(l_1^2 + l_{g_1}^2 + 2l_1 l_{g_1} \cos \theta_2) \dot{\theta}_1^2 \right]$$

$$+ m_2 \left[(l_{g_2}^2 + l_1 l_{g_2} \cos \theta_2) \dot{\theta}_1 \dot{\theta}_2 \right]$$

$$\dot{r}_1 = i\dot{\theta}_1 l_1 e^{i\theta_1} + i(\dot{\theta}_1 + \dot{\theta}_2) l_{g_1} e^{i(\theta_1 + \theta_2)}$$

$$V_1 = m_1 g l_{g_1} \sin \theta_1$$

$$= [\dot{\theta}_1 l_1 + (\dot{\theta}_1 + \dot{\theta}_2) l_{g_1} e^{i\theta_2}] e^{i\theta_1}$$

$$|\dot{r}_1|^2 = [\dot{\theta}_1 l_1 + (\dot{\theta}_1 + \dot{\theta}_2) l_{g_1} e^{i\theta_2}]$$

$$\cdot [\dot{\theta}_1 l_1 + (\dot{\theta}_1 + \dot{\theta}_2) l_{g_1} e^{-i\theta_2}]$$

$$= (\dot{\theta}_1 l_1)^2 + (\dot{\theta}_1 + \dot{\theta}_2)^2 l_{g_1}^2$$

$$+ \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2) l_1 l_{g_1} [e^{i\theta_2} + e^{-i\theta_2}]$$

$$= (\dot{\theta}_1 l_1)^2 + (\dot{\theta}_1 + \dot{\theta}_2)^2 l_{g_1}^2$$

$$+ 2\dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2) l_1 l_{g_1} \cos \theta_2$$

$$= \dot{\theta}_1^2 (l_1^2 + l_{g_1}^2 + 2l_1 l_{g_1} \cos \theta_2)$$

$$+ \dot{\theta}_2^2 l_{g_1}^2$$

$$+ 2\dot{\theta}_1 \dot{\theta}_2 (l_{g_1}^2 + l_1 l_{g_1} \cos \theta_2)$$

$$\partial_{\dot{\theta}_1} L = (I_1 + m_1 l_{g_1}^2) \ddot{\theta}_1$$

$$+ m_2 [(l_1^2 + l_{g_1}^2 + 2l_1 l_{g_1} \cos \theta_2)] \ddot{\theta}_1$$

$$+ m_2 [(l_{g_2}^2 + l_1 l_{g_2} \cos \theta_2)] \ddot{\theta}_2$$

$$\frac{d}{dt} (\partial_{\dot{\theta}_1} L) = (I_1 + m_1 l_{g_1}^2) \ddot{\theta}_1 + m_2 [(l_1^2 + l_{g_1}^2 + 2l_1 l_{g_1} \cos \theta_2)] \ddot{\theta}_1$$

$$+ m_2 [-2l_1 l_{g_1} \sin \theta_2] \dot{\theta}_1 \dot{\theta}_2$$

$$+ m_2 [(l_{g_2}^2 + l_1 l_{g_2} \cos \theta_2)] \ddot{\theta}_2$$

$$+ m_2 [-l_1 l_{g_2} \sin \theta_2] \dot{\theta}_2^2$$

$$\partial_{\dot{\theta}_2} L = -m_1 g l_{g_1} \cos \theta_1 - m_1 g l_1 \cos \theta_1 - m_2 g l_{g_2} \cos (\theta_1 + \theta_2)$$

$$\partial_{\dot{\theta}_2} L = (I_2 + m_2 l_{g_2}^2) \ddot{\theta}_2$$

$$+ m_2 (l_{g_2}^2 + l_1 l_{g_2} \cos \theta_2) \ddot{\theta}_1$$

$$\frac{d}{dt} (\partial_{\dot{\theta}_2} L) = (I_2 + m_2 l_{g_2}^2) \ddot{\theta}_2$$

$$+ m_2 (l_{g_2}^2 + l_1 l_{g_2} \cos \theta_2) \ddot{\theta}_1$$

$$+ m_2 (-l_1 l_{g_2} \sin \theta_2) \dot{\theta}_1 \dot{\theta}_2$$

$$\partial_{\dot{\theta}_3} L = -m_1 l_1 l_{g_2} \sin \theta_1 \dot{\theta}_2^2$$

$$- m_1 l_1 l_{g_2} \sin \theta_1 \dot{\theta}_1 \dot{\theta}_2$$

$$- m_2 g l_{g_2} \cos (\theta_1 + \theta_2)$$

EOM

$$\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} = \begin{bmatrix} (I_1 + m_1 l g_1^2 + m_2 (l_1^2 + l_2^2 + 2l_1 l_2 \cos \theta_2)) & m_2 (l g_1^2 + l_1 l_2 \cos \theta_2) \\ m_2 (l g_1^2 + l_1 l_2 \cos \theta_2) & I_2 + m_2 l g_2^2 \end{bmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}$$

$$+ m_2 l_1 l_2 g_2 \sin \theta_2 \begin{pmatrix} -\dot{\theta}_2^2 \\ \dot{\theta}_1 \end{pmatrix} + 2m_2 l_1 l_2 g_2 \begin{pmatrix} -\dot{\theta}_1 \dot{\theta}_2 \\ 0 \end{pmatrix}$$

$$+ \begin{pmatrix} m_1 g l g_1 \cos \theta_1 + m_2 g l \cos \theta_1 + m_2 g l g_2 \cos(\theta_1 + \theta_2) \\ m_2 g l g_2 \cos(\theta_1 + \theta_2) \end{pmatrix}$$