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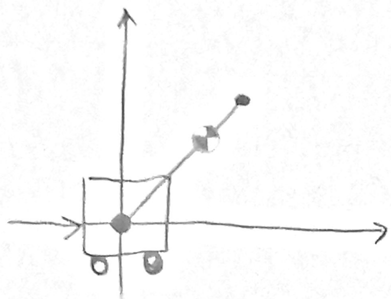

$$\begin{cases} f = (m_\theta + m_x)\ddot{x} + m_\theta l_g \ddot{\theta}\cos\theta - m_\theta l_g \dot{\theta}^2 \sin\theta \\ 0 = I\ddot{\theta} + m_\theta l_g \ddot{x}\cos\theta + m_\theta l_g^2 \ddot{\theta} - m_\theta g l_g \sin\theta \end{cases}$$

$\theta \sim 0, \ \dot{\theta} \sim 0$

1. Linearizing

$$\begin{cases} f = (m_\theta + m_x)\ddot{x} + m_\theta l_g \ddot{\theta} \\ 0 = I\ddot{\theta} + m_\theta l_g \ddot{x} + m_\theta l_g^2 \ddot{\theta} - m_\theta g l_g \theta \end{cases}$$

2. Laplace Transform

$$\begin{cases} F(s) = (m_\theta + m_x) s^2 X(s) + m_\theta l_g s^2 \Theta(s) \\ 0 = m_\theta l_g s^2 X(s) + (I + m_\theta l_g^2) s^2 \Theta(s) - m_\theta g l_g \Theta(s) \end{cases}$$

$$X(s) = \frac{-(I + m_\theta l_g^2) s^2 + m_\theta g l_g}{m_\theta l_g s^2} \Theta(s)$$

$$F(s) = \left[-\frac{(m_\theta + m_x)}{m_\theta l_g}(I + m_\theta l_g^2) s^2 + m_\theta l_g s^2 + (m_\theta + m_x) g \right] \Theta(s)$$

$$F(s) = \left\{ -\left[\frac{I(m_\theta + m_x)}{m_\theta l_g} + m_x l_g \right] s^2 + (m_\theta + m_x) g \right\} \Theta(s)$$

3. Transfer function

$$G(s) = \frac{\Theta(s)}{F(s)} = \frac{1}{-\left[\frac{I(m_\theta + m_x)}{m_\theta l g} + m_x l g\right] s^2 + (m_\theta + m_x) g}$$

4. Stability Analysis

Characteristic equation,

$$-\left[\frac{I(m_\theta + m_x)}{m_\theta l g} + m_x l g\right] s^2 + (m_\theta + m_x) g = 0$$

$$\because A = \left[\frac{I(m_\theta + m_x)}{m_\theta l g} + m_x l g\right] > 0$$

$$B = (m_\theta + m_x) g > 0$$

$\therefore$ The system has positive real roots

$\therefore$ The system is unstable.

Hurwitz stability criterion

$$[H] = \begin{bmatrix} 0 & 0 & 0 \\ -A & B & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \left.\begin{array}{l} \Delta_1 = 0 \\ \Delta_2 = 0 \\ \Delta_3 = 0 \end{array}\right\} \Rightarrow \text{Unstable system.}$$