Robotics II
Day 9: Dynamics

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1. Derive the equation of motion for the following systems.

   The PDF of handwritten working is included together inside the attached folder under the name RoboticsIIHW_Workings.pdf.
   Following are the final results of the calculation.

   1.1. Equation of motion for the 1st System

   $$f = m\ddot{x} + mgsin(\alpha)$$

   The python script is implemented under the filename Exercise_1.py.

   1.2. Equation of motion for the 2nd System

   $$\begin{cases} \tau = (\mathrm{I} + \mathrm{m_l l^2})\ddot{\theta} + 2\mathrm{m_l l}\dot{\mathrm{l}}\dot{\theta} + (\mathrm{m_\theta} l_{g_1} + \mathrm{m_l l})gcos(\theta) \\ \mathrm{f} = \mathrm{m_l}\ddot{\mathrm{l}} - \mathrm{m_l l}\dot{\theta}^2 + \mathrm{m_l g sin}(\theta) \end{cases}$$

   The python script is implemented under the filename Exercise_2.py.

   1.3. Equation of motion for the 3rd System

   $$\begin{pmatrix} 0 \\ \mathrm{f} \end{pmatrix} = \begin{pmatrix} \mathrm{I} + \mathrm{m_\theta} {l_g}^2 & \mathrm{m_\theta} l_g cos(\theta) \\ \mathrm{m_\theta} l_g cos(\theta) & \mathrm{m_x} + \mathrm{m_\theta} \end{pmatrix} \begin{pmatrix} \ddot{\theta} \\ \ddot{x} \end{pmatrix} - \begin{pmatrix} 0 \\ \mathrm{m_\theta} l_g sin(\theta)\, \dot{\theta}^2 \end{pmatrix} - \begin{pmatrix} \mathrm{m_\theta} g l_g sin(\theta) \\ 0 \end{pmatrix}$$

   The python script is implemented under the filename Exercise_3.py.

   1.4. Equation of motion for the 4th System

   $$\begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} = \begin{pmatrix} \mathrm{I_1} + \mathrm{m_1} {l_{g_1}}^2 + \mathrm{m_2}({l_1}^2 + {l_2}^2 + 2 l_1 l_2 cos(\theta_2)) & \mathrm{m_2}({l_{g_2}}^2 + l_1 l_2 cos(\theta_2)) \\ \mathrm{m_2}({l_{g_2}}^2 + l_1 l_2 cos(\theta_2)) & \mathrm{I_2} + \mathrm{m_2} {l_{g_2}}^2 \end{pmatrix} \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} + \mathrm{m_2} l_1 l_{g_2} sin(\theta_2) \begin{pmatrix} -\dot{\theta}_2^{\,2} \\ \dot{\theta}_1^{\,2} \end{pmatrix} + 2\mathrm{m_2} l_1 l_{g_2} \begin{pmatrix} -\dot{\theta}_1 \dot{\theta}_2 \\ 0 \end{pmatrix} - g \begin{pmatrix} \mathrm{m_1} l_{g_1} cos(\theta_1) + \mathrm{m_2} l_1 cos(\theta_1) + \mathrm{m_2} l_{g_2} cos(\theta_1 + \theta_2) \\ \mathrm{m_2} l_{g_2} cos(\theta_1 + \theta_2) \end{pmatrix}$$

   The python script is implemented under the filename Exercise_4.py.

   In this case the of the python script, only forces can be applied to the system, whereas torques have to be resolved into forces to be applied. Hence in this case, in the python script, the non-restorative force on the 2nd link is assumed to be zero.