

Robotics II

Day 9: Dynamics

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1. Derive the equation of motion for the following systems.

The PDF of handwritten working is included together inside the attached folder under the name [RoboticsIIHW_Workings.pdf](#).

Following are the final results of the calculation.

- 1.1. Equation of motion for the 1st System

$$f = m\ddot{x} + mg\sin(\alpha)$$

The python script is implemented under the filename [Exercise 1.py](#).

- 1.2. Equation of motion for the 2nd System

$$\begin{cases} \tau = (I + m_1 l^2) \ddot{\theta} + 2m_1 l \dot{\theta} + (m_\theta l_{g_1} + m_1 l) g \cos(\theta) \\ f = m_1 \ddot{l} - m_1 l \dot{\theta}^2 + m_1 g \sin(\theta) \end{cases}$$

The python script is implemented under the filename [Exercise 2.py](#).

- 1.3. Equation of motion for the 3rd System

$$\begin{pmatrix} 0 \\ f \end{pmatrix} = \begin{pmatrix} I + m_\theta l_g^2 & m_\theta l_g \cos(\theta) \\ m_\theta l_g \cos(\theta) & m_x + m_\theta \end{pmatrix} \begin{pmatrix} \ddot{\theta} \\ \ddot{x} \end{pmatrix} - \begin{pmatrix} 0 \\ m_\theta l_g \sin(\theta) \theta^2 \end{pmatrix} - \begin{pmatrix} m_\theta g l_g \sin(\theta) \\ 0 \end{pmatrix}$$

The python script is implemented under the filename [Exercise 3.py](#).

- 1.4. Equation of motion for the 4th System

$$\begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} = \begin{pmatrix} I_1 + m_1 l_{g_1}^2 + m_2 (l_1^2 + l_2^2 + 2l_1 l_2 \cos(\theta_2)) & m_2 (l_{g_2}^2 + l_1 l_2 \cos(\theta_2)) \\ m_2 (l_{g_2}^2 + l_1 l_2 \cos(\theta_2)) & I_2 + m_2 l_{g_2}^2 \end{pmatrix} \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} \\ + m_2 l_1 l_{g_2} \sin(\theta_2) \begin{pmatrix} -\dot{\theta}_2^2 \\ \dot{\theta}_1^2 \end{pmatrix} + 2m_2 l_1 l_{g_2} \begin{pmatrix} -\dot{\theta}_1 \dot{\theta}_2 \\ 0 \end{pmatrix} \\ - g \begin{pmatrix} m_1 l_{g_1} \cos(\theta_1) + m_2 l_1 \cos(\theta_1) + m_2 l_{g_2} \cos(\theta_1 + \theta_2) \\ m_2 l_{g_2} \cos(\theta_1 + \theta_2) \end{pmatrix}$$

The python script is implemented under the filename [Exercise 4.py](#).

In this case the of the python script, only forces can be applied to the system, whereas torques have to be resolved into forces to be applied. Hence in this case, in the python script, the non-restorative force on the 2nd link is assumed to be zero.