EE4C10 Analog Circuit Design Fundamentals

Homework Assignment II

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Contents

Problem 1

(a) Overdrive voltage, V_{gt}, for:

1. M1

$$I_{D1} = \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_1(V_{GS_1} - V_{TH_1})^2(1 + \lambda_1 V_{DS_1})$$

$$I_{D1} \approx \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_1(V_{gt_1})^2$$

$$V_{gt_1} \approx \sqrt{\frac{2I_{D_1}}{\mu_n C_{OX}}(\frac{L}{W})_1}$$

$$V_{gt_1} \approx 109.11mV$$

2. M2

$$V_{gt_2} \approx \sqrt{\frac{2I_{D_2}}{\mu_p C_{OX}}(\frac{L}{W})_2}$$

$$V_{gt_2} \approx 377.96mV$$

(b) Small-signal gain

$$g_{m1}V_{in} = \frac{-V_{out}}{r_{o1}//r_{o2}}$$
$$\frac{V_{out}}{V_{in}} = -g_{m1}(r_{o1}//r_{o2})$$

$$g_{m1} = \mu_n C_{OX}(\frac{W}{L})_1 V_{gt_1}$$
$$= 4.582mS$$

$$r_{o1} = \frac{1}{I_{D1}\lambda_n}$$
$$= 20k\Omega$$

$$r_{o2} = \frac{1}{I_{D2}\lambda_p}$$
$$= 40k\Omega$$

$$\frac{V_{out}}{V_{in}} \approx -61.09$$

(c) V_{out} output swing

For M_1 to be in saturation,

$$V_{DS1} \geq V_{gt1}$$
$$V_{out} \geq 0.109V$$

For M_2 to be in saturation,

$$V_{DS2} \geq V_{gt2}$$
$$V_{DD} - V_{out} \geq 0.377V$$
$$V_{out} \leq 3.3V - 0.377V$$
$$V_{out} \leq 2.923V$$

Swing of V_{out},

$$0.109V < V_{out} < 2.923V$$

$$V_{out,pp} = 2.923V - 0.109V$$
$$= 2.814V$$

(d) The up-swing is limited by the PMOS leaving saturation, while the down swing is limited by the NMOS leaving saturation. The up-swing can be increased by connecting the base of the PMOS to the drain to form a diode connected structure, therefore V_{SD2} will always be larger than V_{gt2} by V_{TH2}. As a result, V_{out} can be increased to V_{DD} - V_{TH2} without PMOS leaving saturation.

Problem 2

(a) For M1 to be 100mV from triode,

$$V_{DS1} = V_{GS1} - V_{TH,N} + 100mV$$
$$X = V_{in} - V_{TH,N} + 100mV$$

V_{in} for M1 to be in saturation with I_{D1} of 0.35 mA,

$$
\begin{aligned}
I_{D1} &= \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_1(V_{GS1} - V_{TH,N})^2 \\
I_{D1} &= \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_1(V_{in} - V_{TH,N})^2 \\
V_{in} &= \sqrt{\frac{2I_{D1}}{\mu_n C_{OX}}(\frac{L}{W})_1} + V_{TH,N} \\
&= 0.653V
\end{aligned}
$$

$$
\begin{aligned}
X &= \sqrt{\frac{2I_{D1}}{\mu_n C_{OX}}(\frac{L}{W})_1} + 100mV \\
&\approx 0.253V
\end{aligned}
$$

V_b for M2 to be in saturation with I_{D2} of 0.35 mA,

$$
\begin{aligned}
I_{D2} &= \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_2(V_{GS2} - V_{TH,N})^2 \\
I_{D2} &= \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_2(V_b - X - V_{TH,N})^2 \\
V_b &= \sqrt{\frac{2I_{D2}}{\mu_n C_{OX}}(\frac{L}{W})_2} + X + V_{TH,N} \\
&\approx 0.906V
\end{aligned}
$$

(b) Small-signal gain

$$
\begin{aligned}
G_m &= \frac{g_{m1}(g_{m2}r_{o1}r_{o2} + r_{o1})}{g_{m2}r_{o1}r_{o2} + r_{o1} + r_{o2}} \\
&\approx g_{m1} \\
R_{out} &= (g_{m2}r_{o1}r_{o2} + r_{o1} + r_{o2})//R_d
\end{aligned}
$$

Small-signal gain,

$$\begin{aligned}\frac{V_{out}}{V_{in}} &= -G_m R_{out} \\ &= -g_{m1}[(g_{m2} r_{o1} r_{o2} + r_{o1} + r_{o2})//R_d]\end{aligned}$$

$$\begin{aligned}g_{m1} &= \mu_n C_{OX} (\frac{W}{L})_1 (V_{GS1} - V_{TH,N}) \\ &= \mu_n C_{OX} (\frac{W}{L})_1 (V_{in} - V_{TH,N}) \\ &= 4.583 mS\end{aligned}$$

$$\begin{aligned}g_{m2} &= \mu_n C_{OX} (\frac{W}{L})_2 (V_{GS2} - V_{TH,N}) \\ &\approx \mu_n C_{OX} (\frac{W}{L})_2 (V_b - X - V_{TH,N}) \\ &= 4.583 mS\end{aligned}$$

$$\begin{aligned}r_{o1} &= \frac{1}{I_{D1} \lambda_n} \\ &= 28.571 k\Omega\end{aligned}$$

$$\begin{aligned}r_{o2} &= \frac{1}{I_{D2} \lambda_p} \\ &= 28.571 k\Omega\end{aligned}$$

$$\frac{V_{out}}{V_{in}} \approx -22.88$$

(c) Assume V_b to be 1.65V, For M2 to be in saturation,

$$\begin{aligned}V_{out} - X &\geq V_b - X - V_{TH,N} \\ V_{out} &\geq 1.15V\end{aligned}$$

When $I_D \geq 0$,

$$\begin{aligned}V_{out} &\leq V_{DD} \\ 1.15V \leq V_{out} &\leq 3.3V\end{aligned}$$

$$V_{out,pp} = 2.15V$$

(d) X_{pp}

Gain of X,

$$\begin{aligned}\frac{X}{V_{in}} &= \frac{-g_{m1}}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}} \\ &\approx \frac{-g_{m1}}{g_{m2}} \\ &\approx -1\end{aligned}$$

X_{pp}

$$\begin{aligned}\frac{X}{V_{out}} &= \frac{X}{V_{in}} \frac{V_{in}}{V_{out}} \\ &= \frac{1}{22.88}\end{aligned}$$

$$X_{pp} = 54.63 mV$$

Problem 3

(a) Sketch of:

1. Output voltage [[

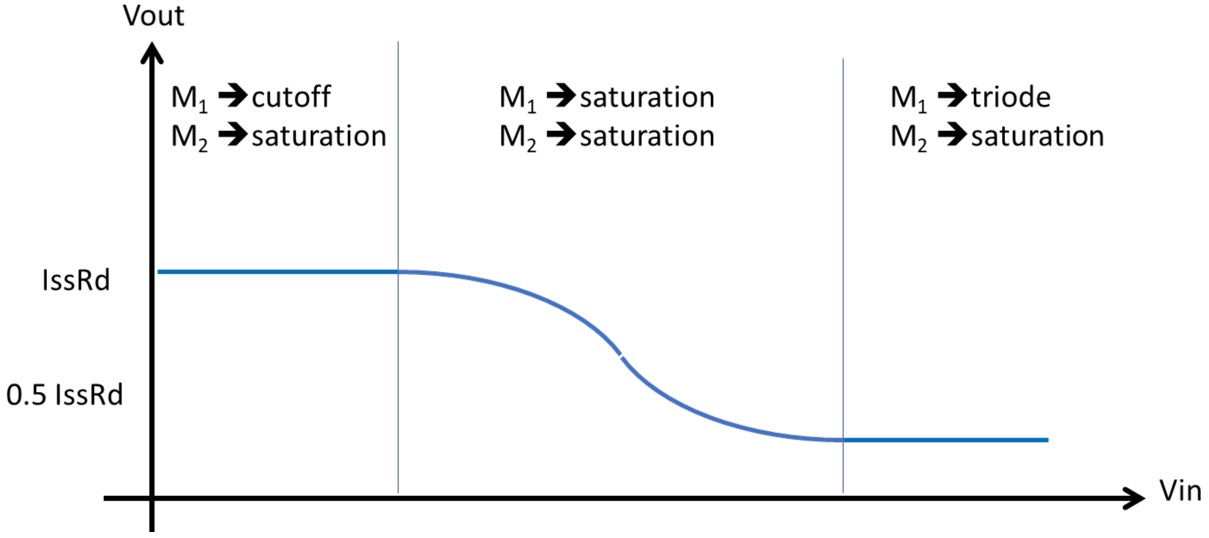

Figure 1: Output voltage sketch

2. Small-signal gain

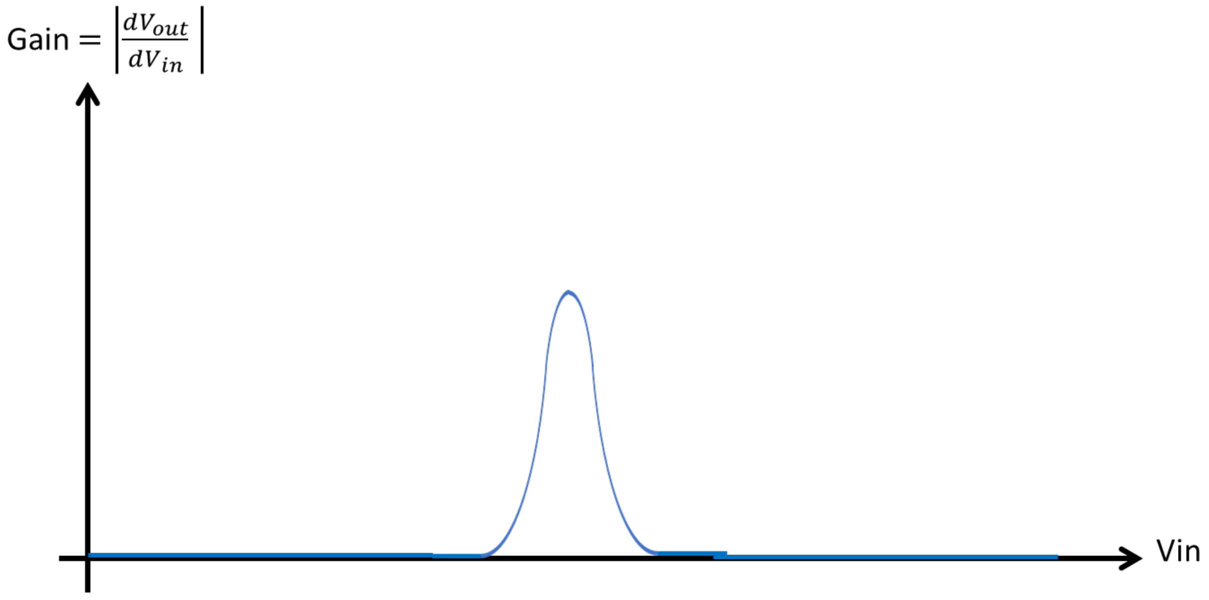

Figure 2: Small-signal gain

(b) Small-signal model

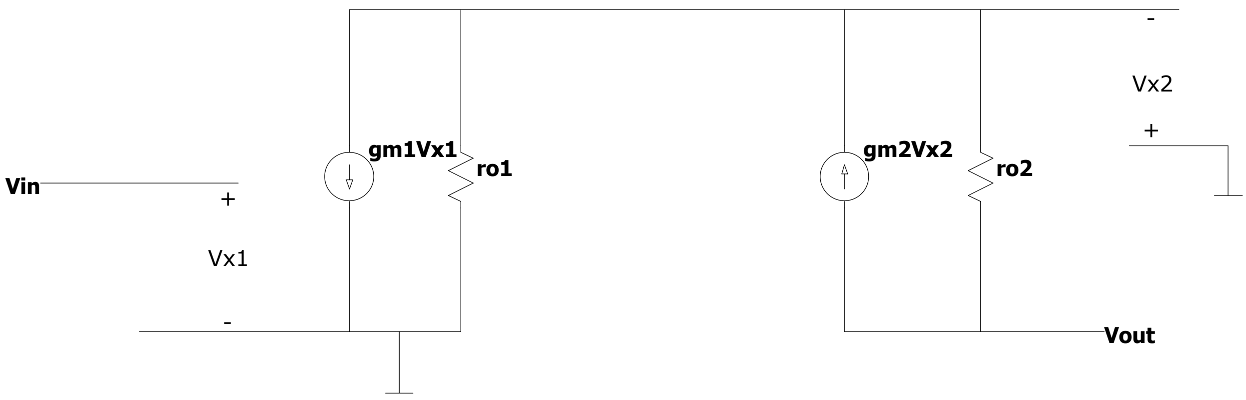

Figure 3: Small-signal model of folded-cascode stage

$$
\begin{aligned}
R_{out} &= g_{m2} r_{o1} r_{o2} + r_{o1} + r_{o2} \\
&\approx g_{m2} r_{o1} r_{o2} \\
G_m &= \frac{-g_{m1}(g_{m2} + \frac{1}{r_{o1}})}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}} \\
&\approx -g_{m1}
\end{aligned}
$$

$$
\frac{V_{out}}{V_{in}} = g_{m1} g_{m2} r_{o1} r_{o2}
$$

Problem 4

(a) Testbench

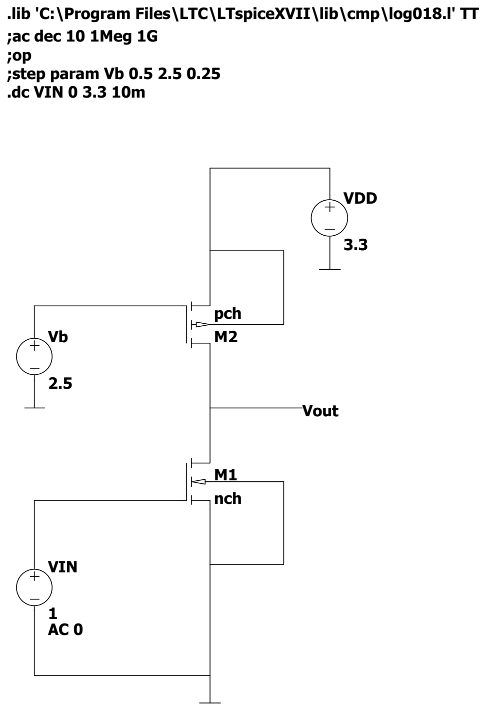

Figure 4: Testbench

V_{out} - V_{in} characteristics

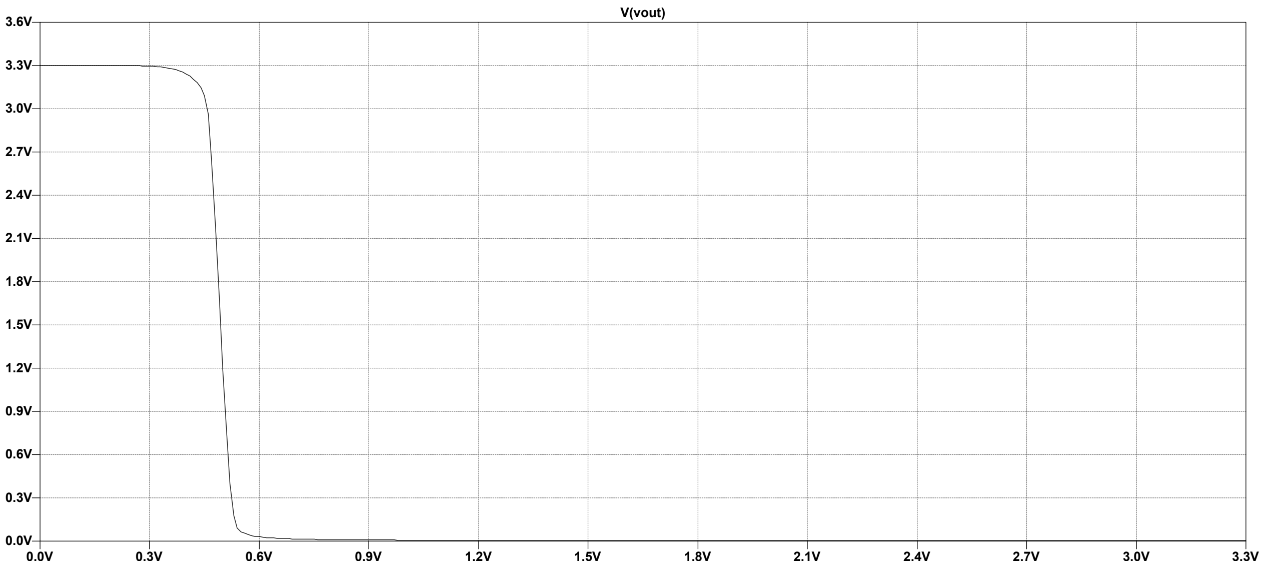

Figure 5: V_{out} - V_{in} characteristics

(b) Small-signal gain, $\frac{dV_{out}}{dV_{in}}$

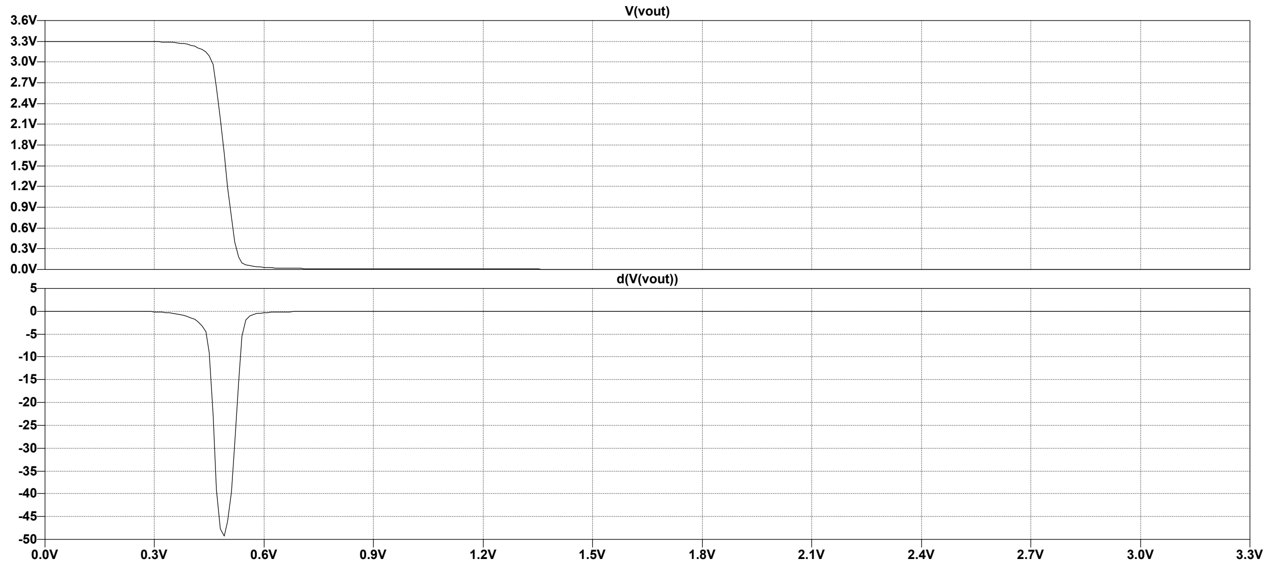

Figure 6: Small-signal gain, $\frac{dV_{out}}{dV_{in}}$

From figure 6 the gain when:

1. $V_{out} = 0.6$ V
   $\frac{dV_{out}}{dV_{in}} = -35.59$
2. $V_{out} = 2.8$ V
   $\frac{dV_{out}}{dV_{in}} = -32.77$

(c) From figure 6, the input voltage, V_{in}, for maximum gain, $max(|\frac{dV_{out}}{dV_{in}}|)$ is given to be:

$$max(|\frac{dV_{out}}{dV_{in}}|) = 50.07$$
$$V_{in} = 489mV$$

(d) Output voltage swing for gain of 1,

$$V_{out,max} = 3.24V$$
$$V_{out,min} = 56mV$$
$$V_{out,pp} = 3.184V$$

Output peak to peak voltage

$$V_{out,pp} = 3.184V$$

(e) Small-signal voltage gain when:

1. $V_{out} = 0.6$ V, $V_{in} = 0.514$ V

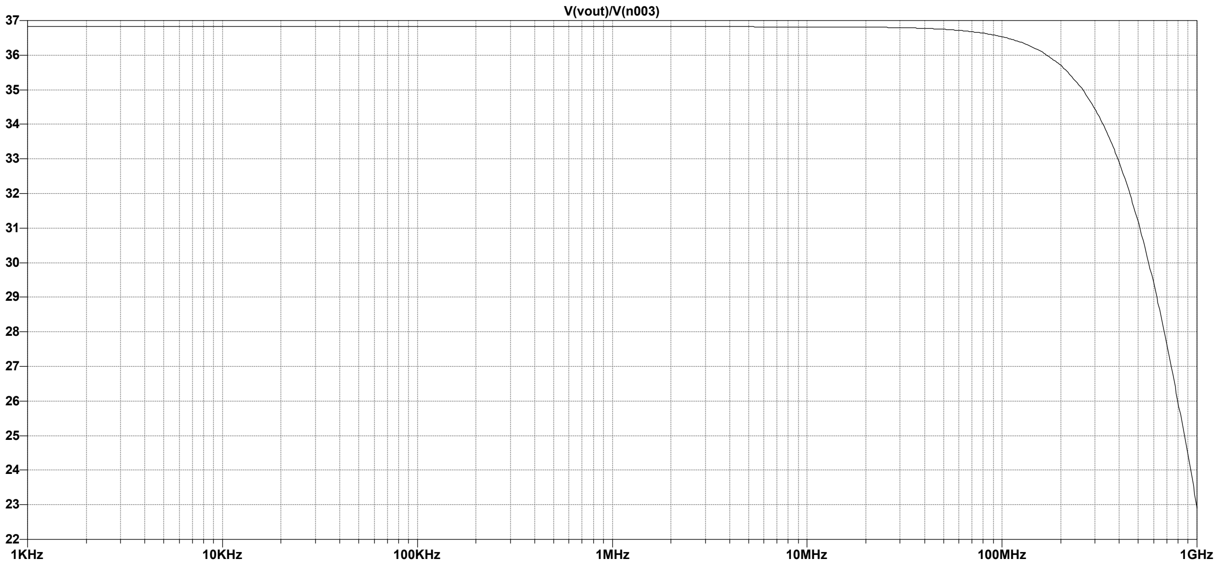

Figure 7: Small-signal gain, $|\frac{V_{out}}{V_{in}}|$, $\mathrm{V_{out}} = 0.6$ V, $\mathrm{V_{in}} = 0.514$ V

Gain $= 36.82$

2. $\mathrm{V_{out}} = 2.8$ V, $\mathrm{V_{in}} = 0.464$ V

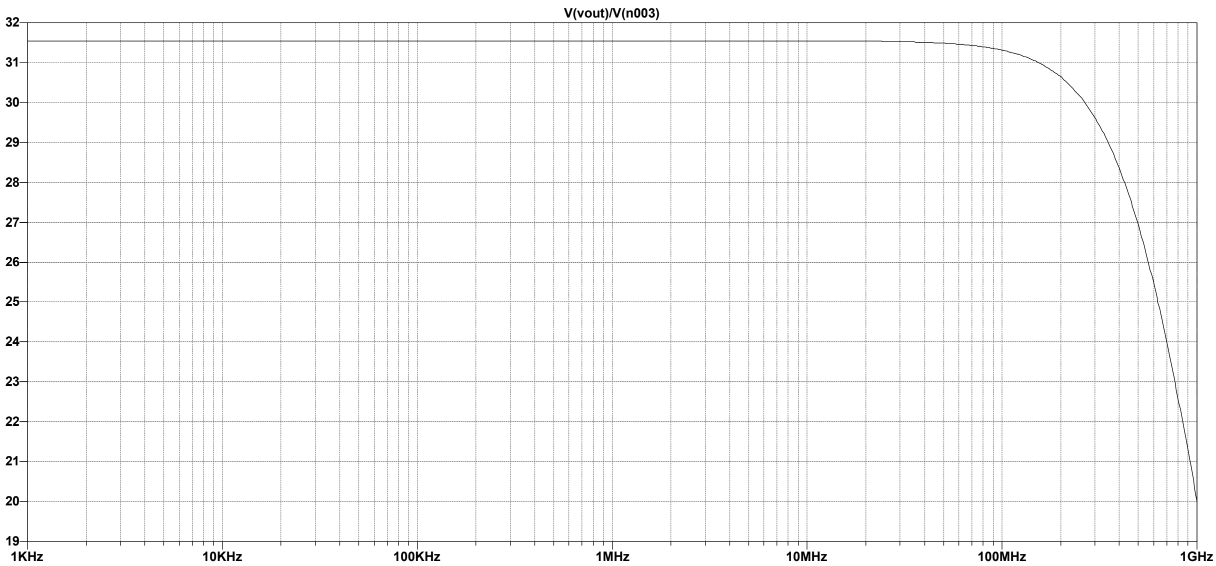

Figure 8: Small-signal gain, $|\frac{V_{out}}{V_{in}}|$, $\mathrm{V_{out}} = 2.8$ V, $\mathrm{V_{in}} = 0.464$ V

Gain $= 31.54$

Problem 5

(a) Procedure for designing $\mathrm{V_b}$ for M1 to be 100mV away from triode, $\mathrm{V_b} = 0.825$V, is determined by the following:

1. Select an initial value $\mathrm{V_b}$ for M1 and M2 to be in saturation when $\mathrm{V_{in}} > \mathrm{V_{TH1}}$. In figure 9, $\mathrm{V_b} = 1.5$ V. From the error logfile 10, $\mathrm{V_{DS1}} > \mathrm{V_{GS1}}$ - $\mathrm{V_{TH1}}$ and $\mathrm{V_{DS2}} > \mathrm{V_{GS2}}$ - $\mathrm{V_{TH2}}$.

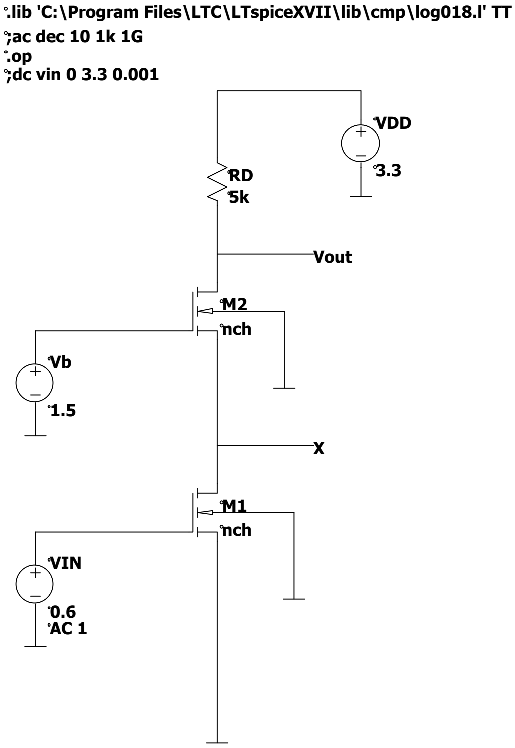

Figure 9: Testbench for Q5

Semiconductor Device Operating Points:

--- BSIM3 MOSFETS ---

Name:	m2	m1
Model:	nch.3	nch.3
Id:	3.74e-04	3.74e-04
Vgs:	7.73e-01	6.00e-01
Vds:	7.05e-01	7.27e-01
Vbs:	-7.27e-01	0.00e+00
Vth:	6.78e-01	4.97e-01
Vdsat:	1.18e-01	1.35e-01
Gm:	4.21e-03	4.42e-03
Gds:	1.29e-04	4.49e-05
Gmb	7.98e-04	1.27e-03
Cbd:	0.00e+00	0.00e+00
Cbs:	0.00e+00	0.00e+00
Cgsov:	6.59e-15	1.32e-14
Cgdov:	6.59e-15	1.32e-14
Cgbov:	3.18e-17	6.47e-17
dQgdVgb:	3.21e-14	1.00e-13
dQgdVdb:	-6.47e-15	-1.32e-14
dQgdVsb:	-2.27e-14	-7.78e-14
dQddVgb:	-6.61e-15	-1.33e-14
dQddVdb:	6.60e-15	1.32e-14
dQddVsb:	2.09e-17	9.01e-17
dQbdVgb:	-3.63e-15	-1.24e-14
dQbdVdb:	1.50e-17	2.88e-18
dQbdVsb:	-2.22e-15	-1.44e-14

Figure 10: Semiconductor Device Operating Points

2. Doing a DC-sweep for $\mathrm{I_{D1}}$-$\mathrm{V_{in}}$, $V_{in} \approx 0.6V$ when $\mathrm{I_{D1}} = 0.35\mathrm{mA}$.

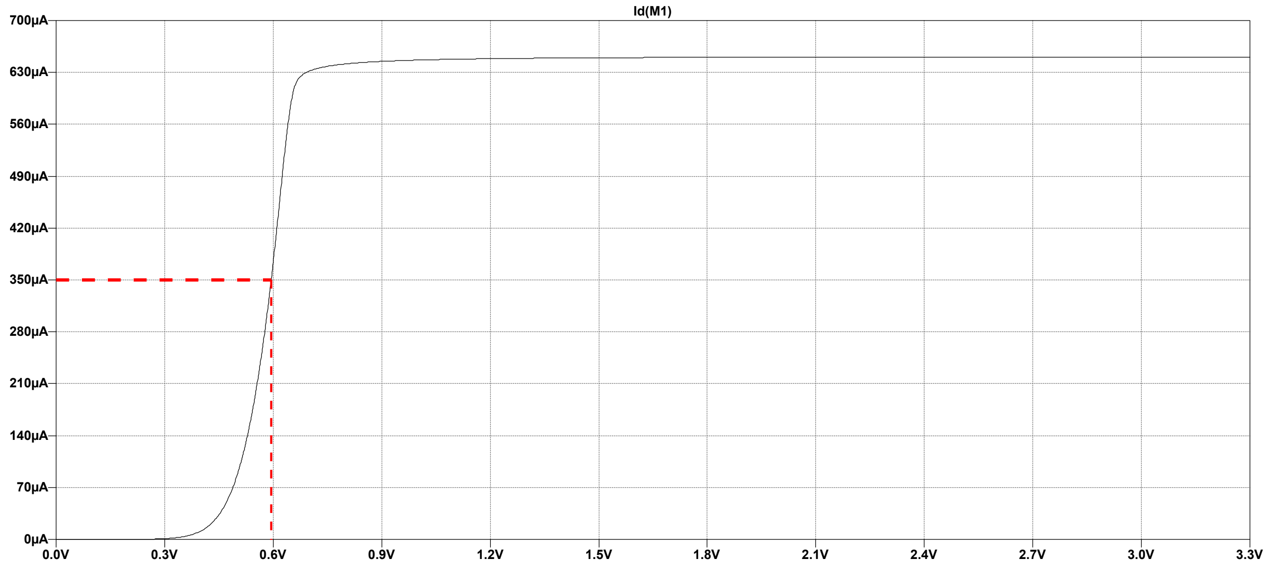

Figure 11: I_{D1}-V_{in}

3. From the operation point simulation int figure 10, V_{TH1} is determined to be 0.497V. For M1 to be 100mV from triode region,

$$V_x = V_{in} - V_{TH1} + 100mV$$
$$\approx 0.2V$$

4. DC-sweeping V_x-V_b and determining V_b for $V_x = 0.2$V. From the figure 12 , $V_b = 0.825$V.

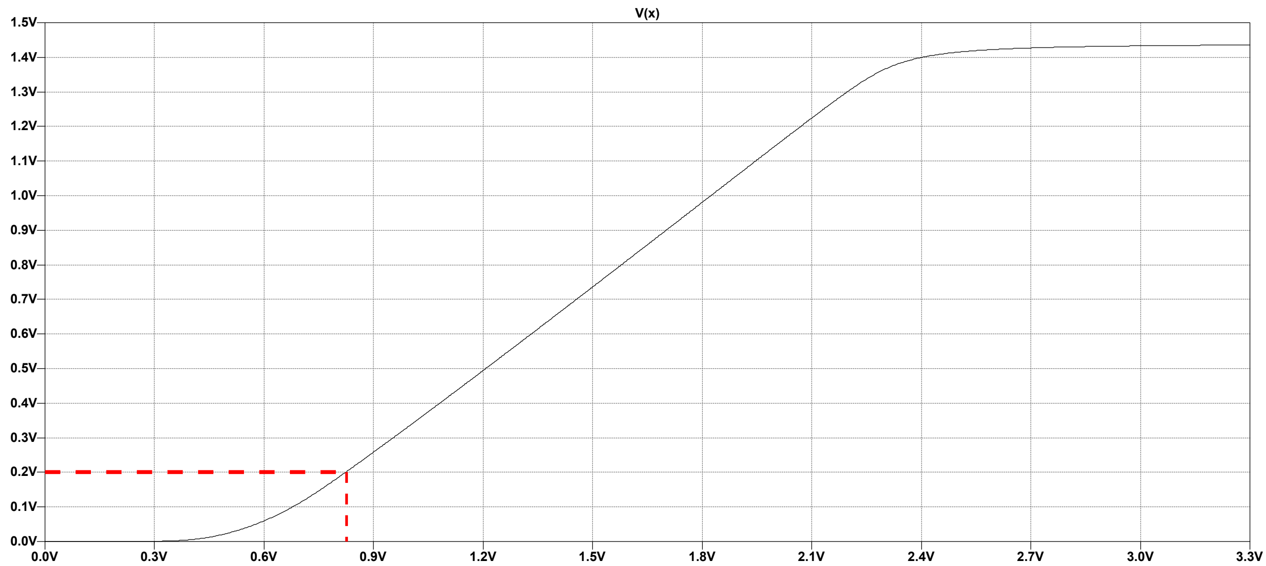

Figure 12: V_x-V_b

(b) I_{OUT}-V_{OUT} when V_{IN} is fixed for $I_{D1} = 0.35$ mA

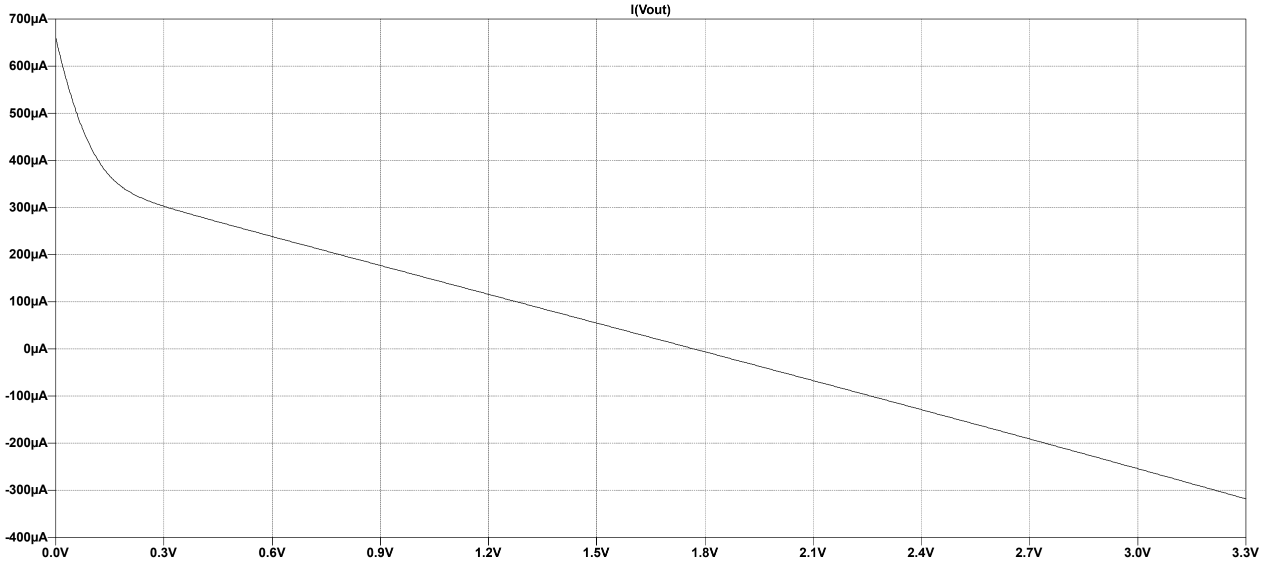

Figure 13: I_{OUT}-V_{OUT}

Problem 6

The output resistance, R_{out} is the reciprocal of the slope of I_{OUT}-V_{OUT},

$$R_{out} = (\frac{I_{OUT}}{V_{OUT}})^{-1}$$

The results are shown in figure 14,

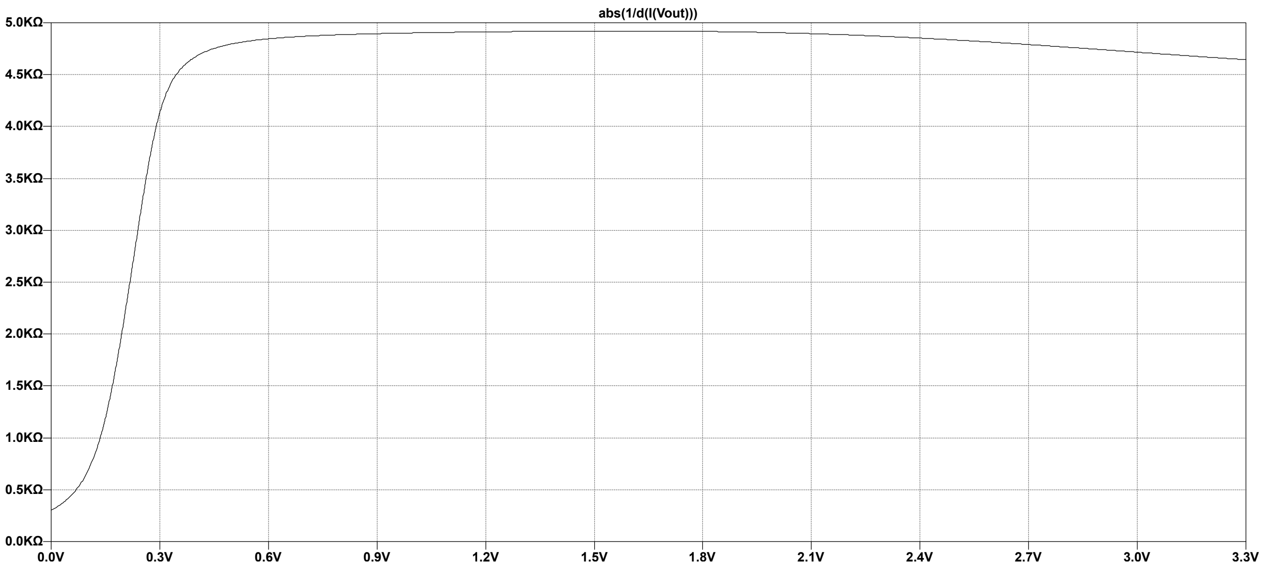

Figure 14: R_{OUT}-V_{OUT}

From figure 15,
$\max(R_{out}) = 4920\ \Omega$
when,
$V_{out} = 1.53$ V

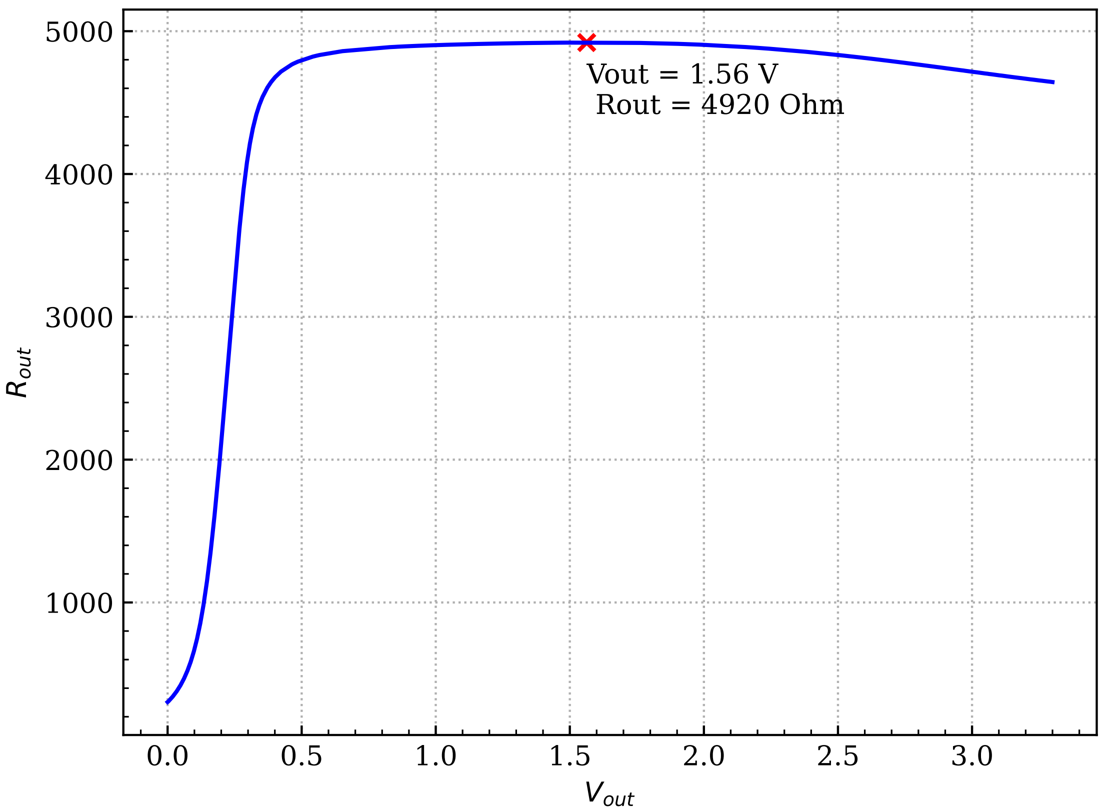

Figure 15: Maximum R_{out}