EE4C10 Analog Circuit Design Fundamentals

Homework Assignment I

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Simulation Files

Each question with simulation files will have their respective subfolder. `q7` subfolder for problem 7 will have subfolders for the subquestions.

The graphs used for the sub problems in problem 3 to 6 are plotted together. Running the simulation files should be able to directly plot the graphs used (configured in the *.plt file). The folders for each question are arranged as follows after extracting:

```
spice
    q3
    q4
    q5
    q7
        b
        c
        d
        e
```

Problem 1

For $I_D = 40\mu A$:

$$I_D = \frac{1.8V - V_D}{R}$$
$$V_D = 1.8V - I_D R$$
$$V_D = 1.0V$$

Saturation region:

$$V_{GS} = 1.0V > V_{TH}$$
$$V_{GS} - V_{TH} = 0.4V < V_{DS}$$

(a) $\lambda = 0V^{-1}$

$$I_D = \frac{\mu_n C_{OX}}{2}\frac{W}{L}(V_{GS} - V_{TH})^2$$
$$L = \frac{\mu_n C_{OX}}{2}\frac{W}{I_D}(V_{GS} - V_{TH})^2$$
$$L = 0.39\mu m$$

(b) $\lambda = 0.06V^{-1}$

$$I_D = \frac{\mu_n C_{OX}}{2}\frac{W}{L}(V_{GS} - V_{TH})^2(1+\lambda V_{DS})$$
$$L = \frac{\mu_n C_{OX}}{2}\frac{W}{I_D}(V_{GS} - V_{TH})^2(1+\lambda V_{DS})$$
$$L = 0.41\mu m$$

Problem 2

(a) Bulk of the transistors are connected to the source, $V_B = V_S$

$$V_{TH} = V_{TH0} + \gamma(\sqrt{|2\varphi_F| + V_{BS}} - \sqrt{|2\varphi_F|})$$
$$V_{TH} = V_{TH0} = 0.33V$$

1. Transistor M_1

$$V_{SG} = 2.5V - 1.7V = 0.8V$$

$$I_D = \frac{\mu_p C_{OX}}{2}\frac{W}{L}(V_{SG} - V_{TH})^2$$
$$W = \frac{2LI_D}{\mu_p C_{OX}}\frac{1}{(V_{SG} - V_{TH})^2}$$
$$W_1 = 2.72\mu m$$

2. Transistor M_2

$$V_{SG} = 1.7V - 1V = 0.7V$$

$$W = \frac{2LI_D}{\mu_p C_{OX}}\frac{1}{(V_{SG} - V_{TH})^2}$$
$$W_2 = 4.38\mu m$$

3. Transistor M_3

$$V_{SG} = 1V$$

$$W = \frac{2LI_D}{\mu_p C_{OX}}\frac{1}{(V_{SG} - V_{TH})^2}$$
$$W_3 = 1.37\mu m$$

(b) Bulk terminals are attached to the V_{DD}, $V_B = V_{DD}$.

1. Transistor M_1

$$V_{BS} = 2.5V - 2.5V = 0V$$

$$V_{TH} + \gamma(\sqrt{|2\varphi_F| + V_{BS}} - \sqrt{|2\varphi_F|})$$
$$V_{TH} = 0.33V$$

$$W = \frac{2LI_D}{\mu_p C_{OX}}\frac{1}{(V_{SG} - V_{TH})^2}$$
$$W_1 = 2.72\mu m$$

2. Transistor M_2

$$V_{BS} = 2.5V - 1.7V = 0.8V$$

$$V_{TH} + \gamma(\sqrt{|2\varphi_F| + V_{BS}} - \sqrt{|2\varphi_F|})$$
$$V_{TH} = 0.43V$$

$$W = \frac{2LI_D}{\mu_p C_{OX}} \frac{1}{(V_{SG} - V_{TH})^2}$$
$$W_2 = 8.23\mu m$$

3. Transistor M_3

$$V_{BS} = 2.5V - 1.0V = 1.5V$$

$$V_{TH} + \gamma(\sqrt{|2\varphi_F| + V_{BS}} - \sqrt{|2\varphi_F|})$$
$$V_{TH} = 0.49V$$

$$W = \frac{2LI_D}{\mu_p C_{OX}} \frac{1}{(V_{SG} - V_{TH})^2}$$
$$W_3 = 2.31\mu m$$

Problem 3

(a) Testbench and I_D-V_{GS} characteristics of NMOS and PMOS

1. NMOS
   i. Testbench

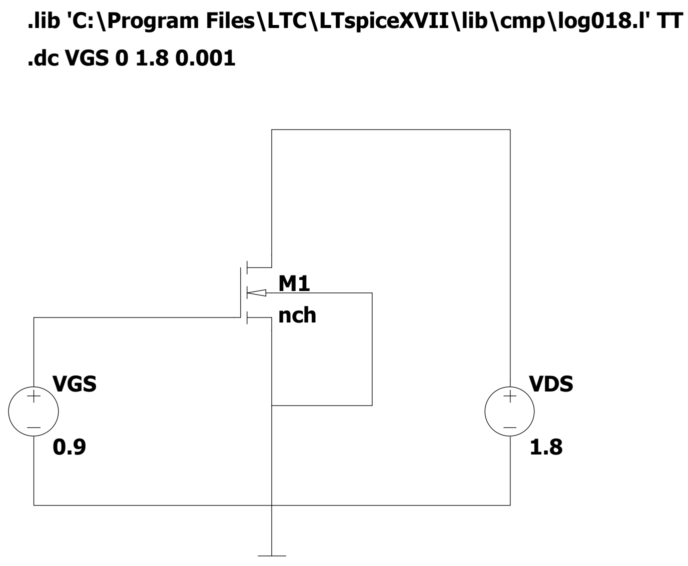

Figure 1: NMOS Testbench

   ii. I_D-V_{GS}

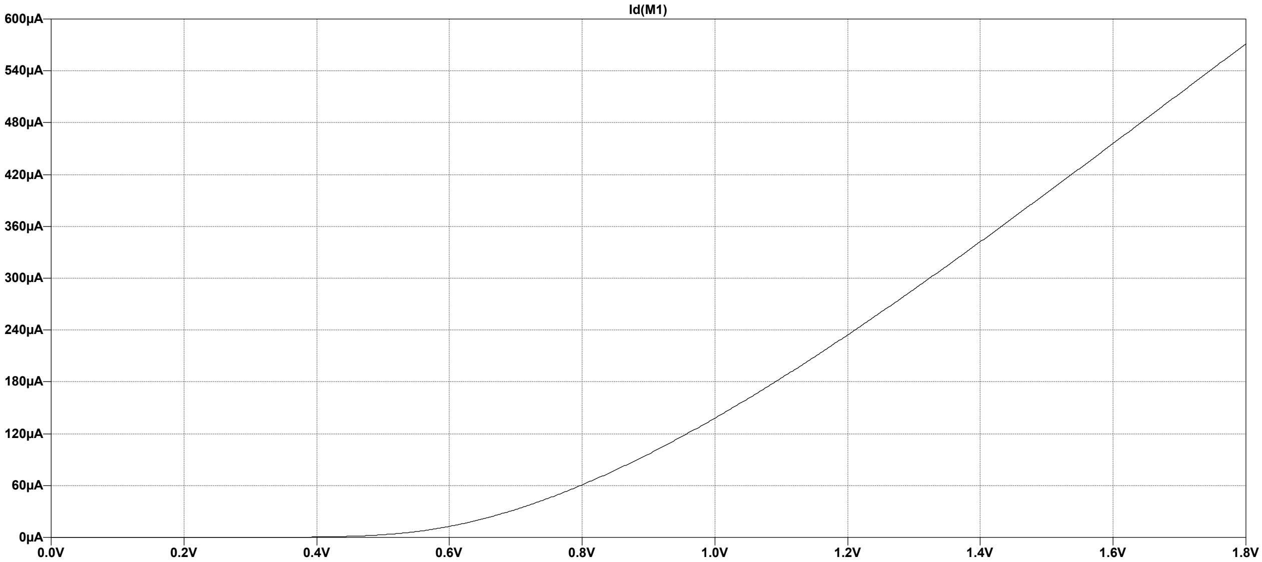

Figure 2: NMOS I_D-V_{GS}

2. PMOS
   i. Testbench

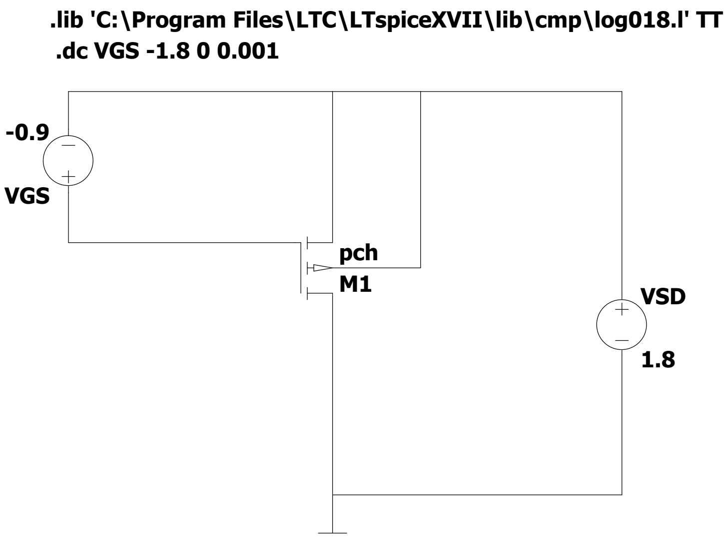

Figure 3: PMOS Testbench

   ii. I_S-V_{GS}

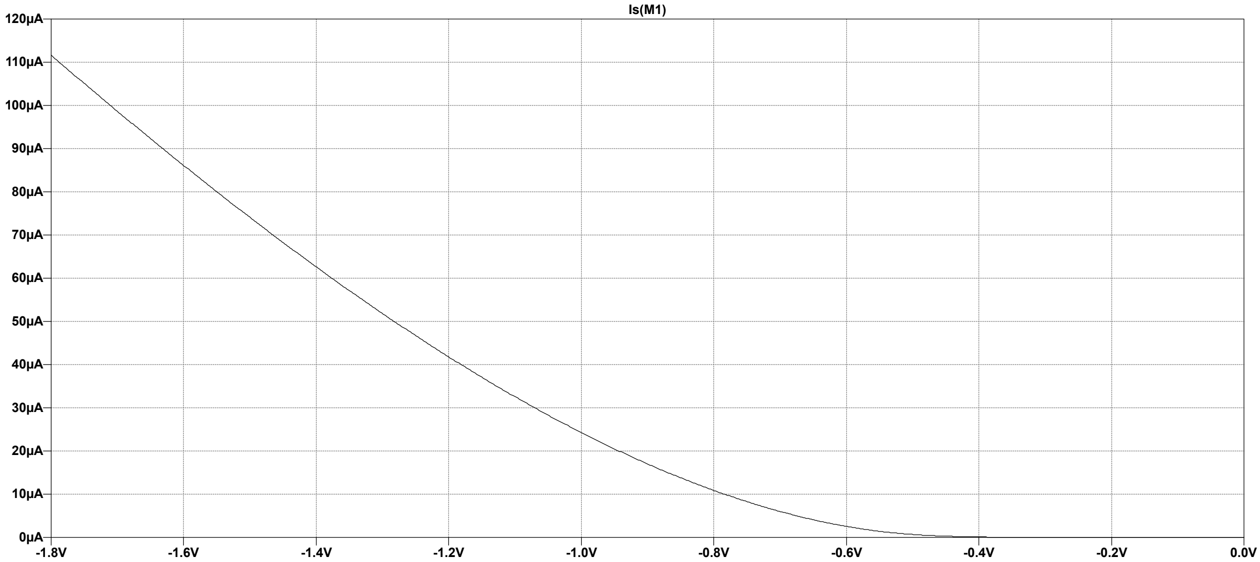

Figure 4: PMOS I_S-V_{GS}

(b) $\mu_{n(p)}C_{OX}$ and $V_{THn(p)}$

Assuming that channel length modulation is negligible, V_{THn} for NMOS can be derived from the following relation:

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_{THn})^2$$
$$\frac{2I_D}{\mu_n C_{ox}}\frac{L}{W} = (V_{GS} - V_{THn})^2$$
$$\sqrt{\frac{2I_D}{\mu_n C_{ox}}\frac{L}{W}} = V_{GS} - V_{THn}$$

V_{THn} is the x-axis intercept when the saturation region is extrapolated. In the case of PMOS, the relation becomes:

$$\sqrt{\frac{2I_S}{\mu_p C_{ox}}\frac{L}{W}} = V_{SG} + V_{THp}$$

For deriving $\mu_n C_{OX}$, since $V_{THn(p)}$ is constant at specific temperatures. Differentiating both sides with respect to $V_{GS(SG)}$ will give:

$$\frac{d}{dV_{GS}}\sqrt{\frac{2I_D}{\mu_n C_{ox}}\frac{L}{W}} = \frac{d}{dV_{GS}}(V_{GS} - V_{THn})$$
$$\frac{1}{2}\frac{dI_D}{dV_{GS}}\sqrt{\frac{2}{I_D\mu_n C_{ox}}\frac{L}{W}} = 1$$
$$\sqrt{\mu_n C_{ox}} = \frac{1}{2}\frac{dI_D}{dV_{GS}}\sqrt{\frac{2}{I_D}\frac{L}{W}}$$
$$\mu_n C_{ox} = \frac{1}{2}\frac{L}{W}\frac{1}{I_D}(\frac{dI_D}{dV_{GS}})^2$$
$$\mu_n C_{ox} = \frac{1}{6I_D}(\frac{dI_D}{dV_{GS}})^2$$

In the case for PMOS, the relation becomes:

$$\mu_p C_{ox} = \frac{1}{6I_S}(\frac{dI_S}{dV_{GS}})^2$$

1. NMOS
   i. $\mu_n C_{OX}$

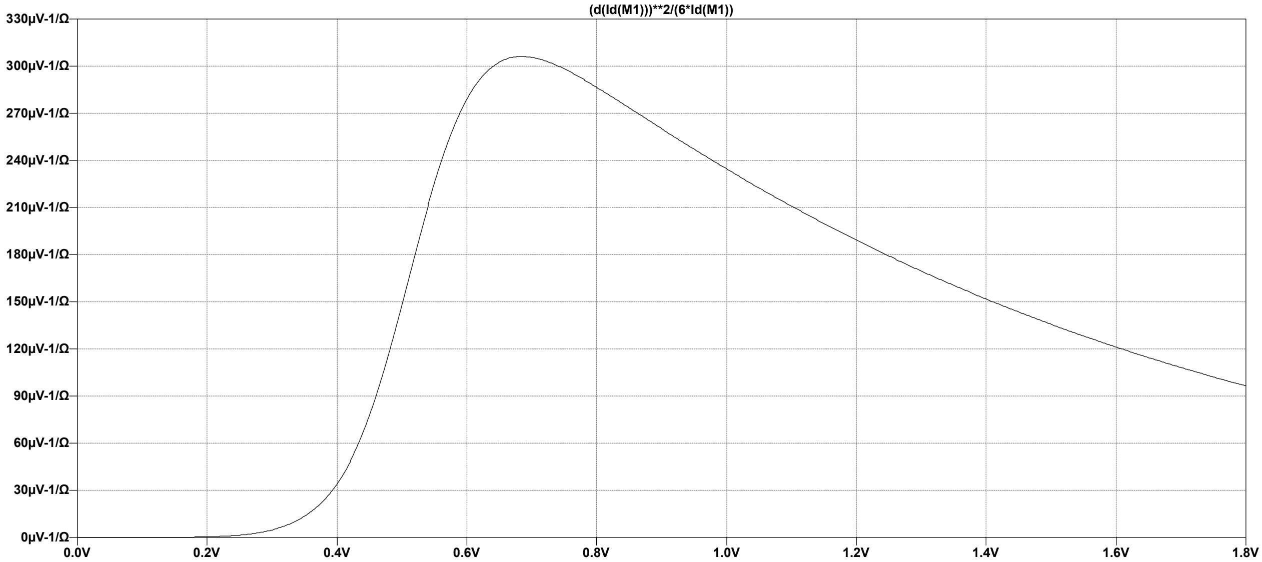

Figure 5: NMOS $\mu_{\mathrm{n}}\mathrm{C}_{\mathrm{OX}}$-$\mathrm{V}_{\mathrm{GS}}$

ii. $V_{THn} = 0.44V$

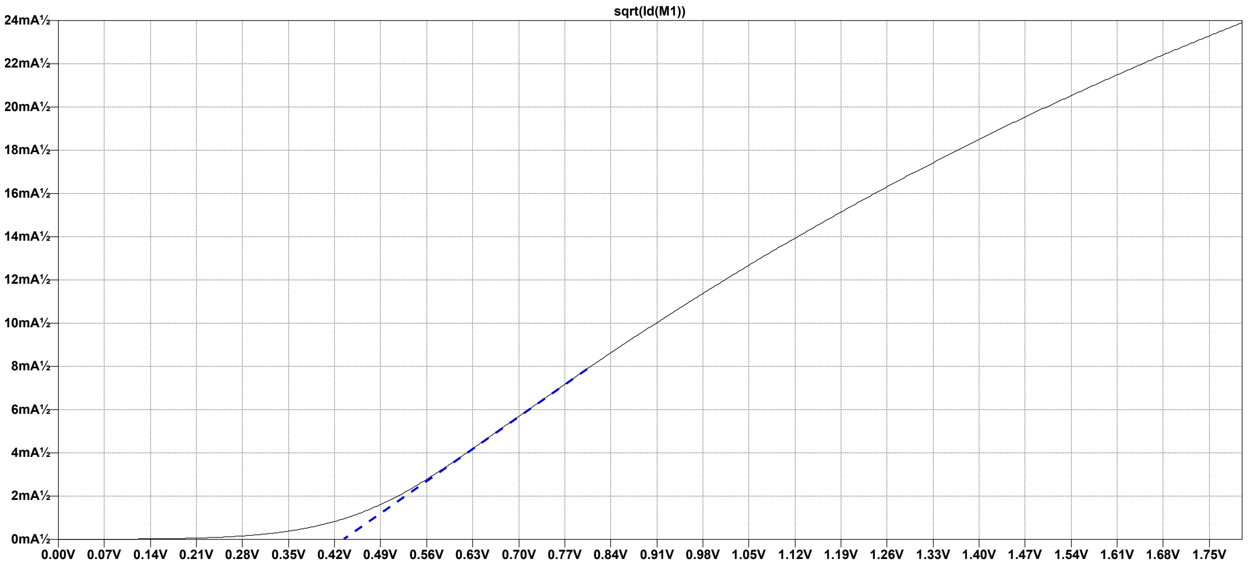

Figure 6: NMOS $\sqrt{I_D} - V_{GS}$

2. PMOS

   i. $\mu_p C_{OX}$

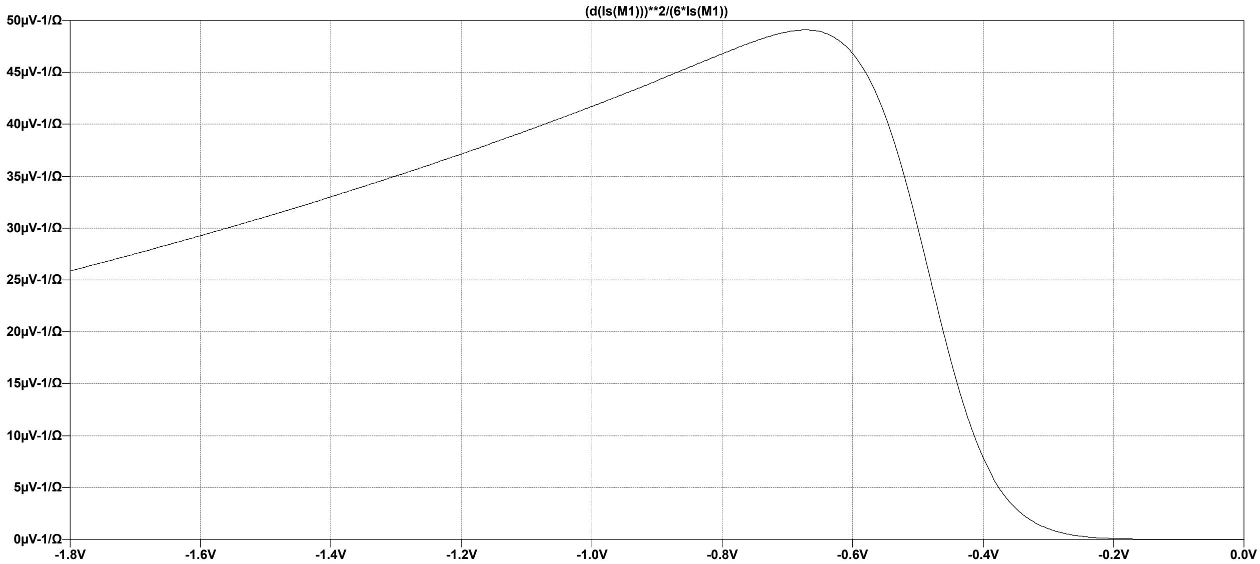

Figure 7: PMOS $\mu_p C_{OX}$-V_{GS}

ii. $V_{THp} = -0.42V$

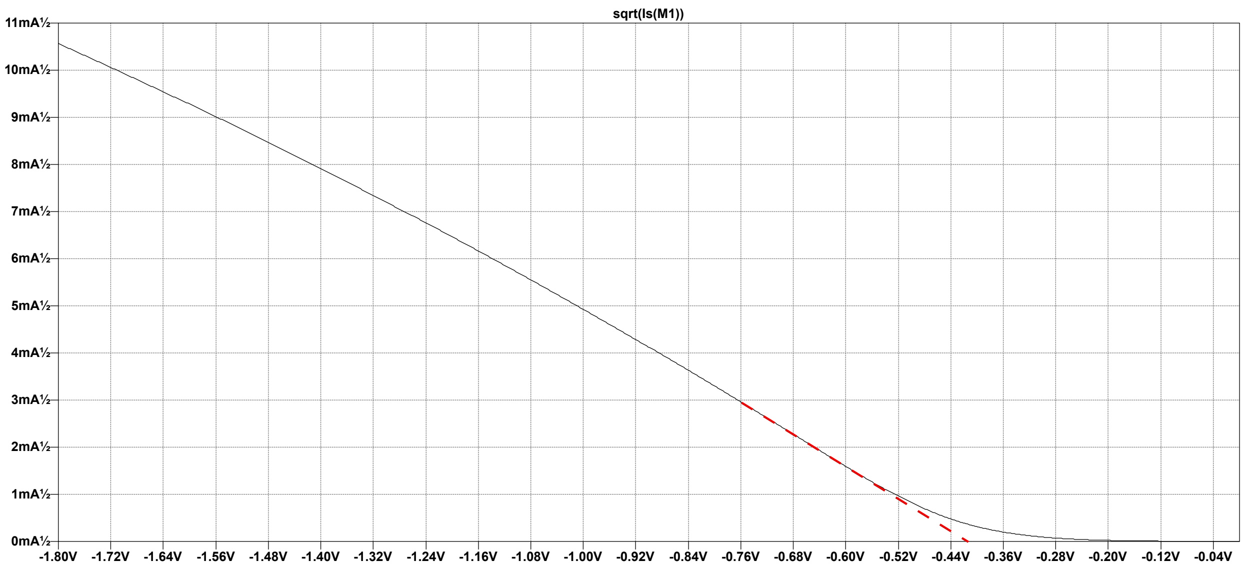

Figure 8: PMOS $\sqrt{I_S} - V_{GS}$

Problem 4

(a) Testbench and I_D-V_{DS} characteristics of NMOS and PMOS

1. NMOS

   i. Testbench

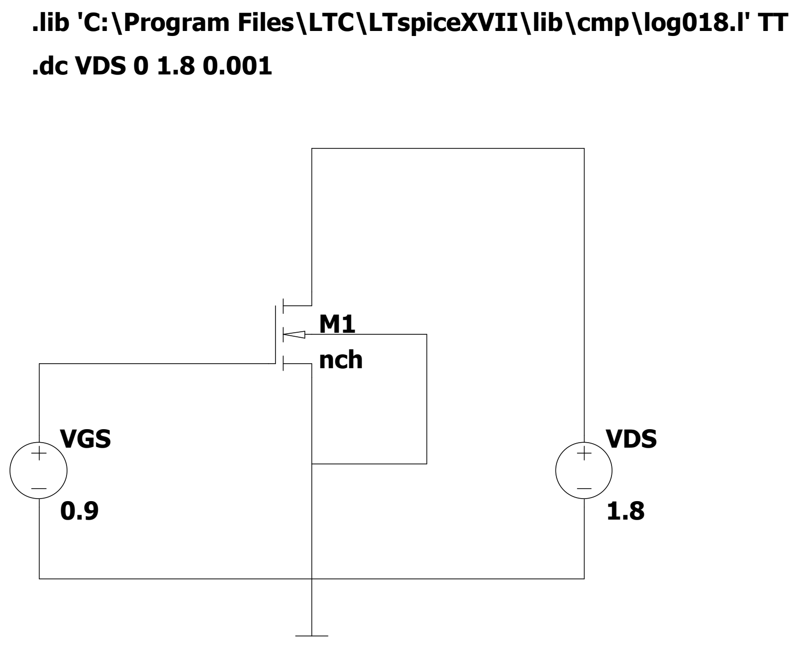

Figure 9: NMOS Testbench

ii. I_D-V_{DS} characteristics

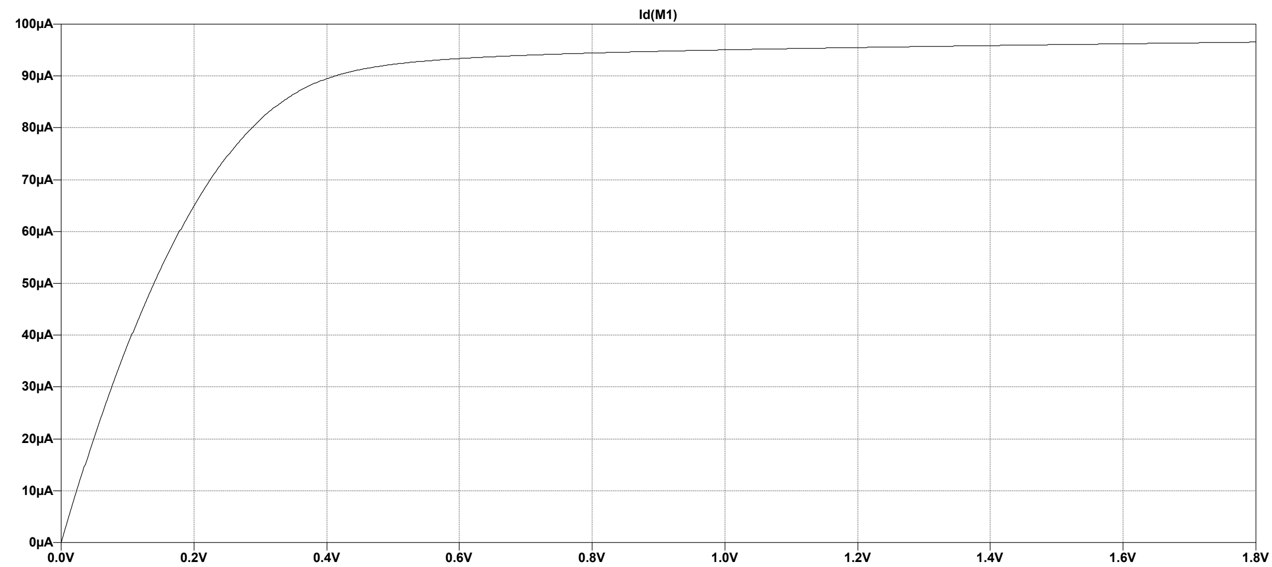

Figure 10: NMOS I_D-V_{DS}

2. PMOS
   i. Testbench

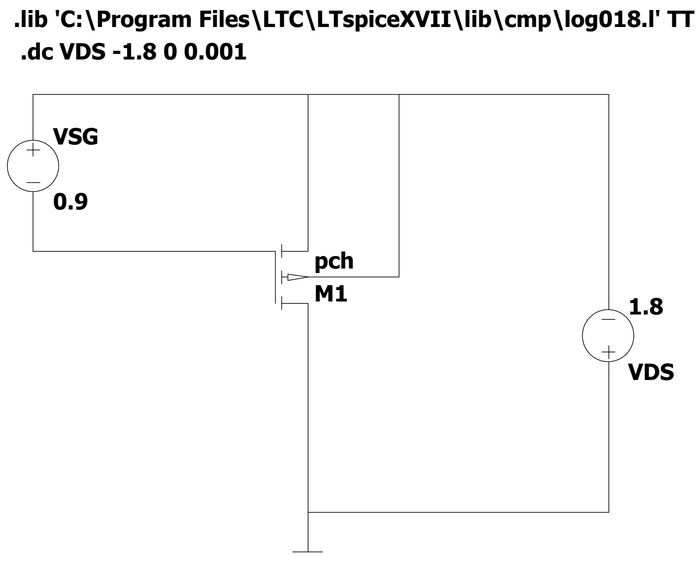

Figure 11: PMOS Testbench

ii. I_S-V_{DS} characteristics

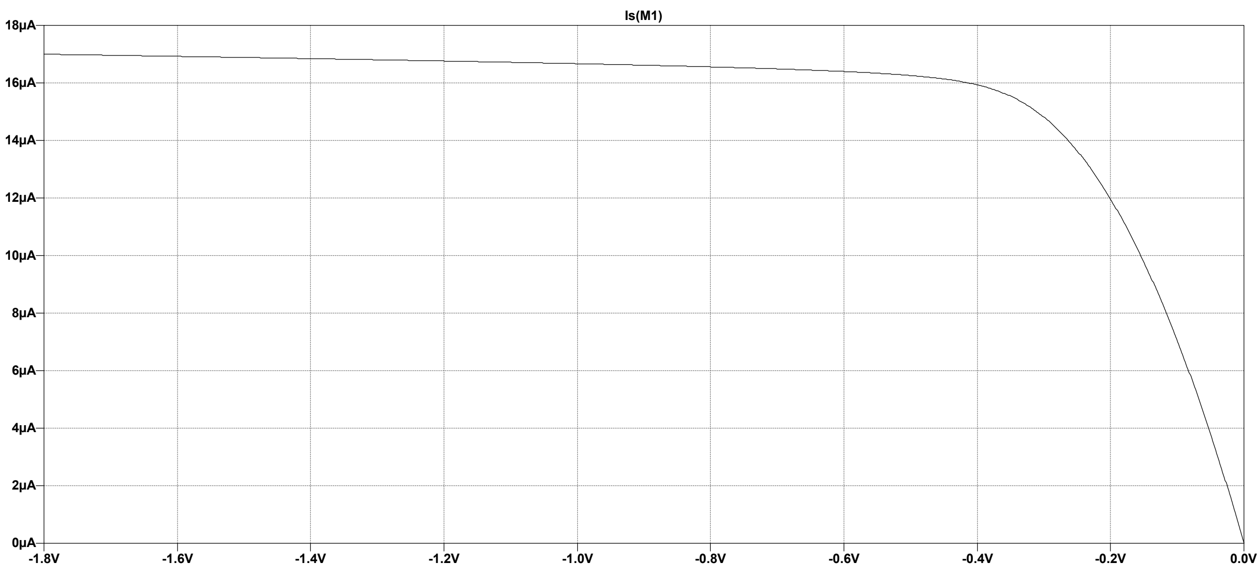

Figure 12: PMOS I_S-V_{DS}

(b) $\lambda_{n(p)}$

Drain current characteristics for NMOS under saturation conditions:

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda_n V_{DS})$$

Differentiating both side with respect to V_{DS}.

$$\frac{dI_D}{dV_{DS}} = \frac{d}{dV_{DS}} (\frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda_n V_{DS}))$$
$$\frac{dI_D}{dV_{DS}} = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_{TH})^2 \lambda_n$$

Assuming that the body-effect is small:

$$I_D \approx \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_{TH})^2$$

$$\frac{dI_D}{dV_{DS}} \approx I_D \lambda_n$$
$$\lambda_n \approx \frac{1}{I_D}\frac{dI_D}{dV_{DS}}$$

In the case of PMOS:

$$\lambda_p \approx \frac{1}{I_S}\frac{dI_S}{dV_{DS}}$$

1. NMOS, $\lambda_n = 0.20V^{-1}$

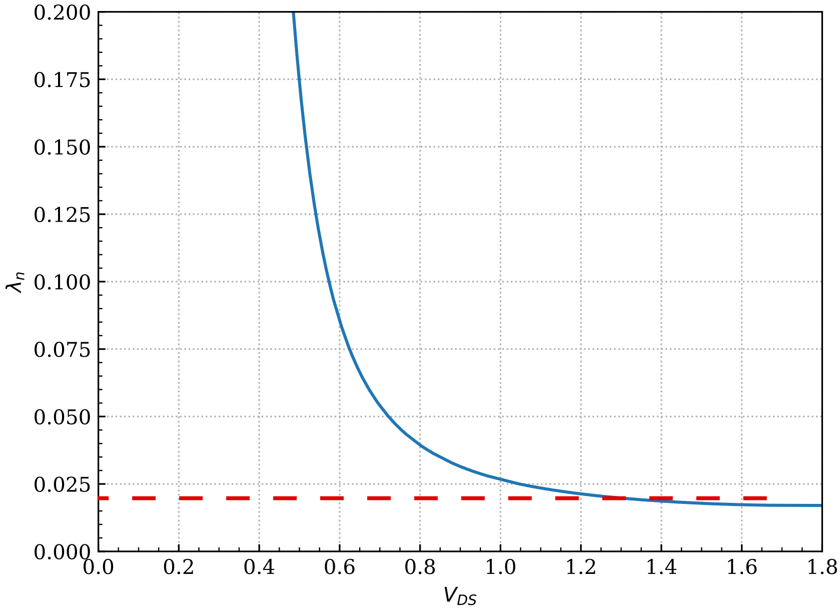

Figure 13: NMOS $\lambda_n - V_{DS}$

2. PMOS, $\lambda_p = -0.025V^{-1}$

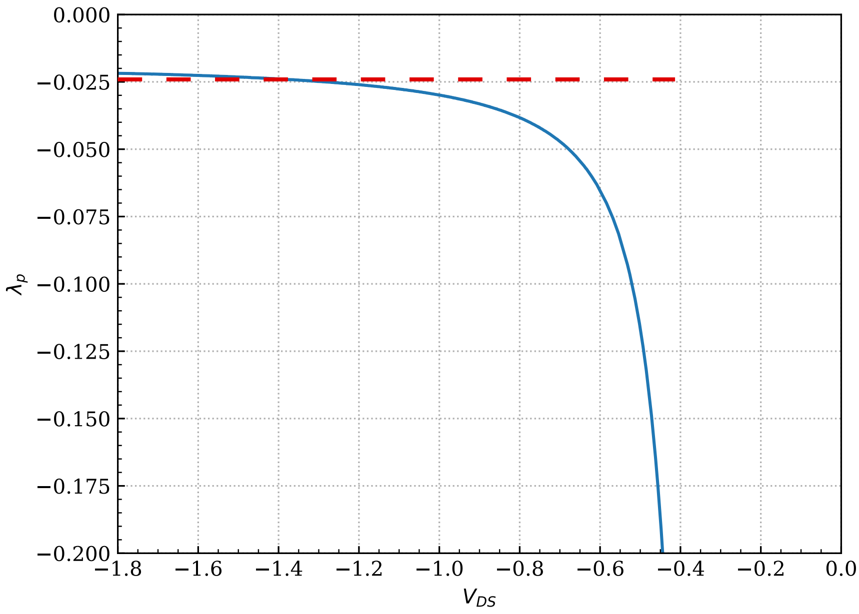

Figure 14: PMOS $\lambda_p - V_{DS}$

Problem 5

g_m for NMOS is approximately:

$$g_m \approx \frac{\partial I_D}{\partial V_{GS}}$$

For PMOS:

$$g_m \approx \frac{\partial I_S}{\partial V_{GS}}$$

(a) $\frac{g_m}{I_D} - V_{GS}$

1. NMOS

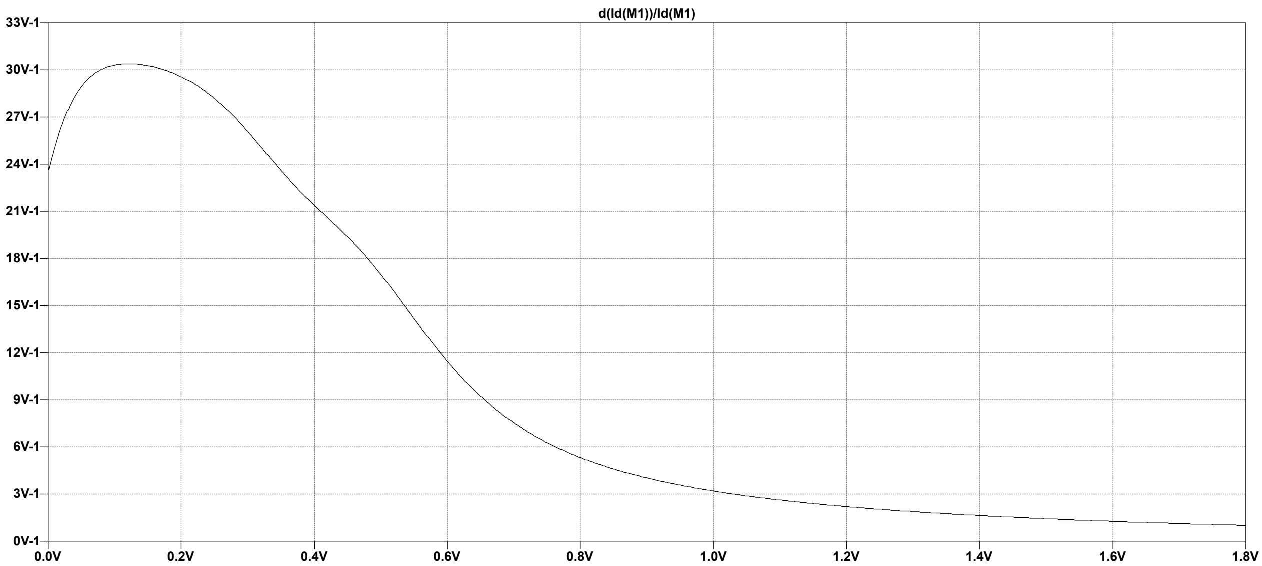

Figure 15: NMOS $\frac{g_m}{I_D} - V_{GS}$

2. PMOS

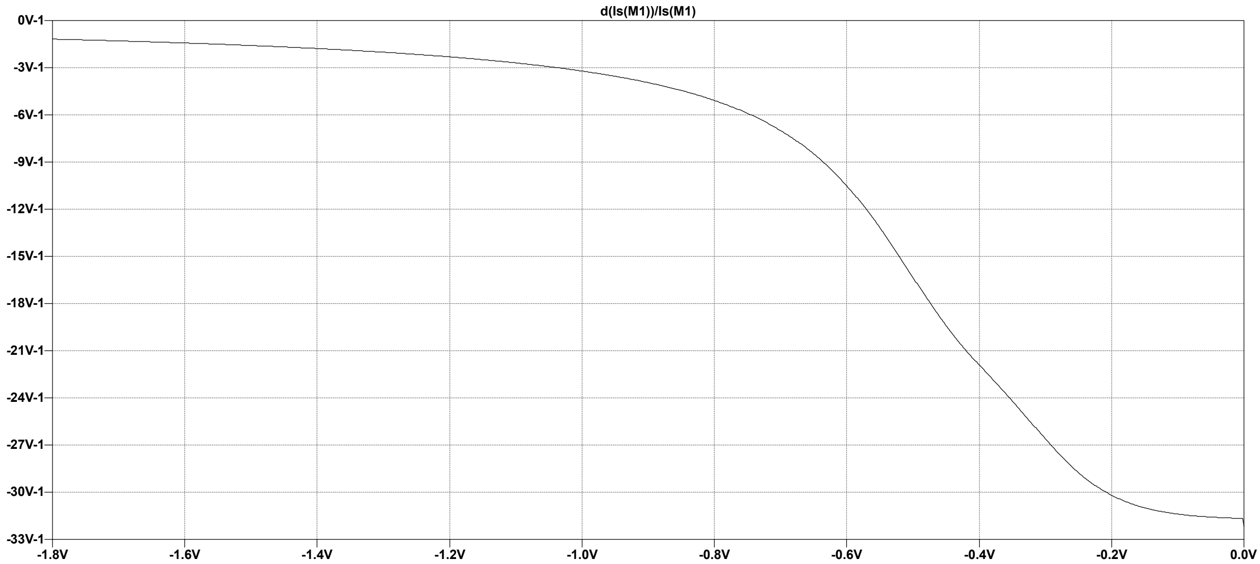

Figure 16: PMOS $\frac{g_m}{I_S} - V_{GS}$

(b) $max(|\frac{g_m}{I_{D(S)}}|)$

1. NMOS
   $max(|\frac{g_m}{I_D}|) = 30.4V^{-1}$
2. PMOS
   $max(|\frac{g_m}{I_S}|) = 31.7V^{-1}$

(c) Slope factor, n

1. NMOS

$$max(|\frac{g_m}{I_D}|) = 30.4V^{-1}$$

$$\begin{aligned} \frac{1}{nV_t} &= 30.4V^{-1} \\ n &= \frac{1}{0.026 \times 30.4} \\ n &= 1.27 \end{aligned}$$

2. PMOS

$$max(|\frac{g_m}{I_S}|) = 31.7V^{-1}$$

$$\begin{aligned} \frac{1}{nV_t} &= 31.7V^{-1} \\ n &= \frac{1}{0.026 \times 31.7} \\ n &= 1.21 \end{aligned}$$

Problem 6

(a) Small-signal Model

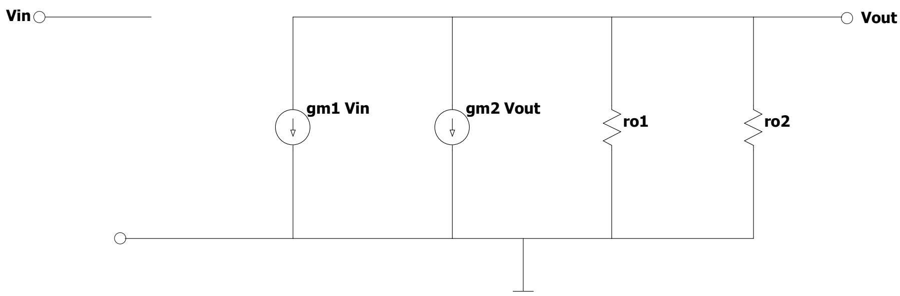

Figure 17: Small signal model

(b) $\lambda = 0V^{-1}$

1. $A_V = \frac{v_{out}}{v_{in}}$

$$(g_{m1}v_{in} + g_{m2}v_{out}) = 0$$
$$A_V = \frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{m2}}$$

2. R_{out}

$$R_{out} = \frac{1}{g_{m2}}$$

(c) $\lambda \neq 0V^{-1}$

1. $A_V = \frac{v_{out}}{v_{in}}$

$$-v_{out} = (g_{m1}v_{in} + g_{m2}v_{out})(r_{o1}//r_{o2})$$
$$-v_{in}g_{m1}(r_{o1}//r_{o2}) = (1 + g_{m2}(r_{o1}//r_{o2}))v_{out}$$
$$A_V = \frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$

2. R_{out}

$$R_{out} = \frac{1}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$

Problem 7

(a) V_{out}-V_{in} relation when:

1. M_1 and M_2 under subthreshold conditions

$$V_{TH_n} = 0.44V$$
$$V_{TH_p} = -0.42V$$

$$\mu_n C_{OX_n} = 306\mu A V^{-2}$$
$$\mu_p C_{OX_p} = 49\mu A V^{-2}$$

$$n_n = 1.27$$
$$n_p = 1.21$$

$$I_{D_1} = I_{D_2}$$
$$(\mu_n C_{OX_n}(n-1)\frac{W_n}{L_n}V_T^2)e^{\frac{V_{in}-V_{TH_n}}{n_n V_T}} = (\mu_p C_{OX_p}(n-1)\frac{W_p}{L_p}V_T^2)e^{\frac{V_{DD}-V_{out}+V_{TH_p}}{n_p V_T}}$$
$$247e^{\frac{V_{in}-0.44}{0.033}} = 154e^{\frac{1.8-V_{out}-0.42}{0.031}}$$
$$ln(1.6) + \frac{V_{in}-0.44}{0.033} = \frac{1.38 - V_{out}}{0.031}$$
$$0.015 + V_{in} - 0.44 \approx 1.38 - V_{out}$$
$$V_{out} \approx 1.8V - V_{in}$$

$$V_{in} < 0.44V$$

2. M_1 and M_2 at saturation

$$I_{D_1} = I_{D_2}$$
$$\frac{\mu_n C_{OX_n}}{2}\frac{W_n}{L_n}(V_{GS_1} - V_{TH_n})^2 = \frac{\mu_p C_{OX_p}}{2}\frac{W_p}{L_p}(V_{SG_2} + V_{TH_p})^2$$
$$918(V_{in} - 0.44)^2 = 735(1.8 - V_{out} - 0.42)^2$$
$$1.12(V_{in} - 0.44) = 1.38 - V_{out}$$
$$V_{out} = 1.87V - 1.12V_{in}$$

Saturation conditions for M_1:

$$V_{GS_1} - V_{TH_1} < V_{DS}$$
$$V_{in} - 0.44V < V_{out}$$
$$V_{in} - 0.44V < 1.87V - 1.12V_{in}$$
$$2.12V_{in} < 2.31V$$
$$V_{in} < 1.09V$$

Condition for M_1 and M_2 at saturation:

$$0.44V < V_{in} < 1.09V$$

3. M_1 at triode and M_2 at saturation

$$I_{D_1} = I_{D_2}$$
$$\mu_n C_{OX_n}\frac{W_n}{L_n}[(V_{GS_1} - V_{TH_n})V_{DS} - \frac{V_{DS}^2}{2}] = \frac{\mu_p C_{OX_p}}{2}\frac{W_p}{L_p}(V_{SG_2} + V_{TH_p})^2$$
$$2.50[(V_{in} - 0.44V)V_{out} - \frac{V_{out}^2}{2}] = (V_{DD} - V_{out} - 0.42V)^2$$
$$V_{out} = \frac{(2.5V_{in} + 1.66) - \sqrt{(2.5V_{in} + 1.66)^2 - 17.1}}{4.5}$$

Condition:

$$V_{in} > 1.09V$$

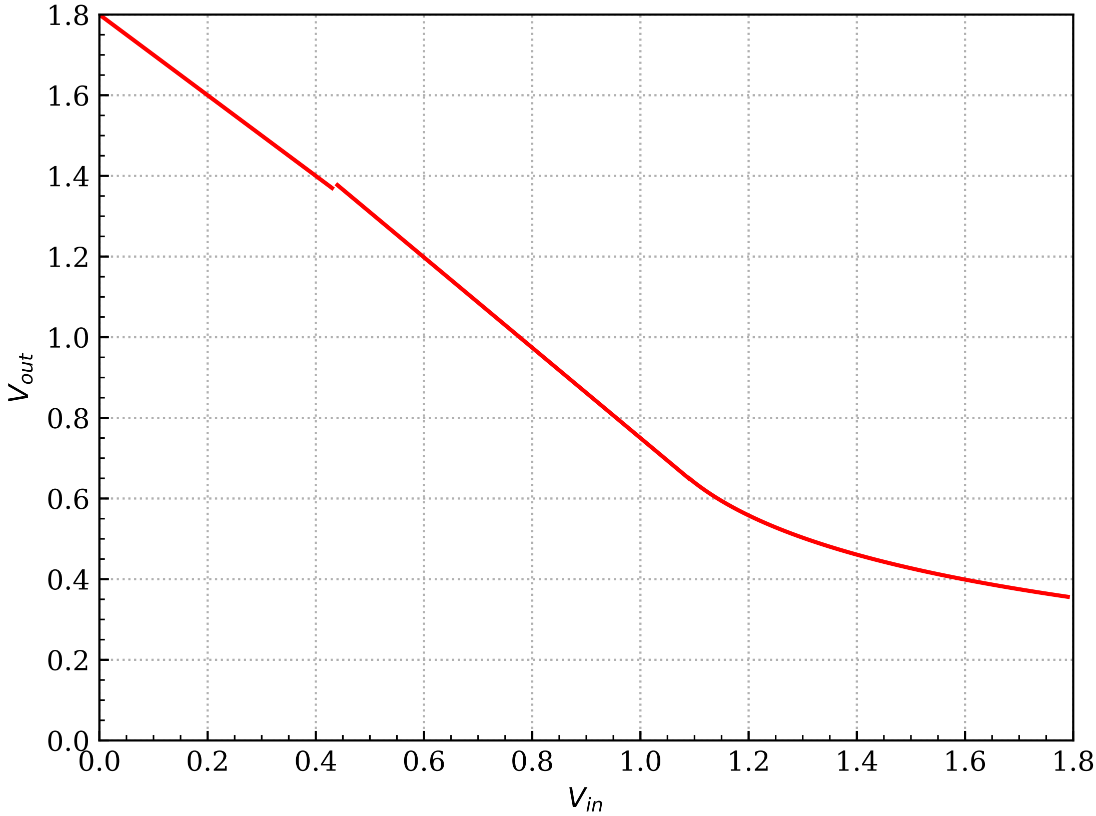

Figure 18: Calculated $V_{out} - V_{in}$

(b) Simulated $\mathrm{V_{out}}$-$\mathrm{V_{in}}$ relation using LTSpice

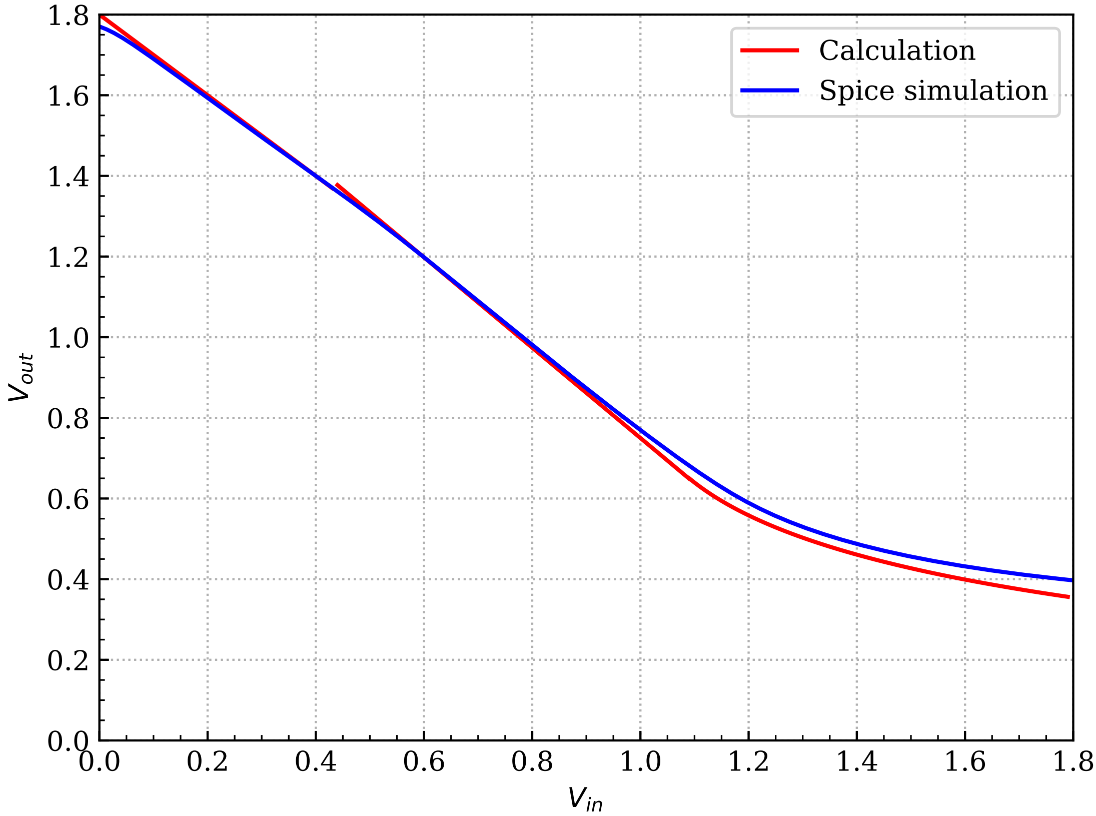

Figure 19: Simulated and calculated $V_{out} - V_{in}$

(c) Maximum small-signal gain For maximum small-signal gain:

$$max(|A_V|) = max(|\frac{\partial V_{out}}{\partial V_{in}}|)$$

$$V_{in} \approx 0.69V$$

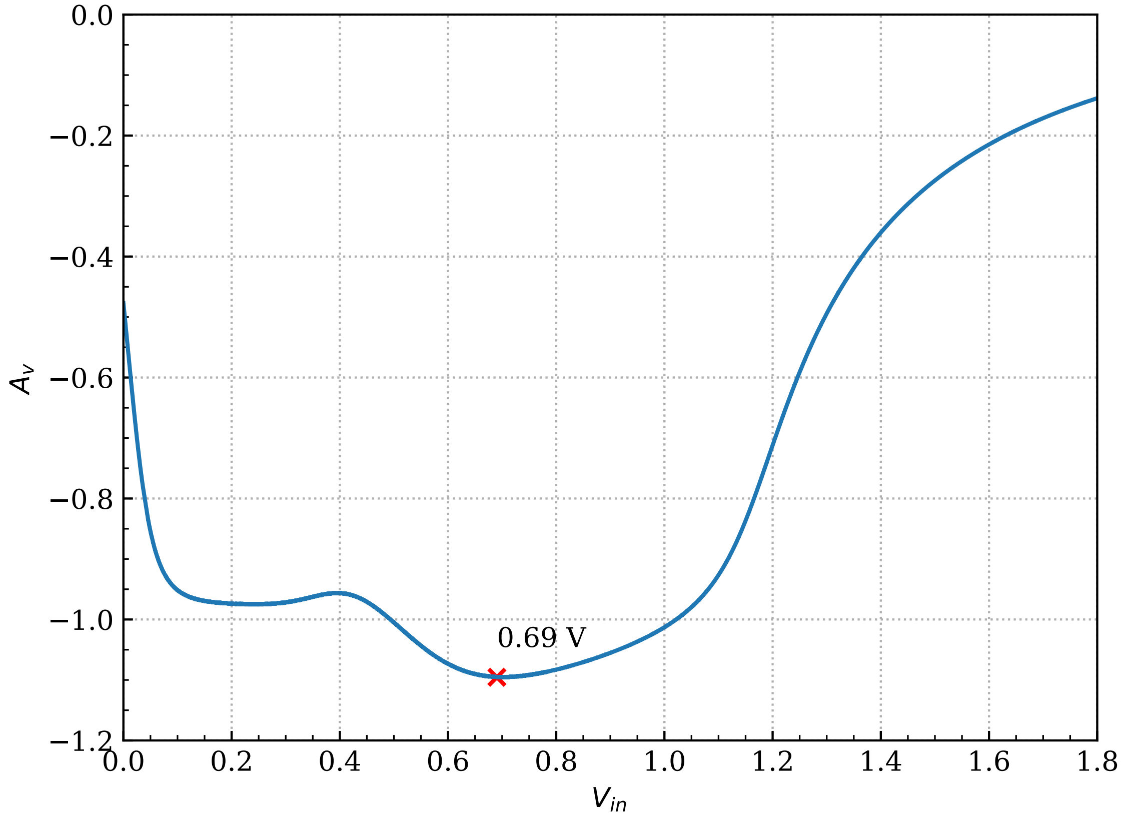

Figure 20: $A_v - V_{in}$

(d) Small signal parameters:

1. $|A_v| = 787.83mdB = 1.09$

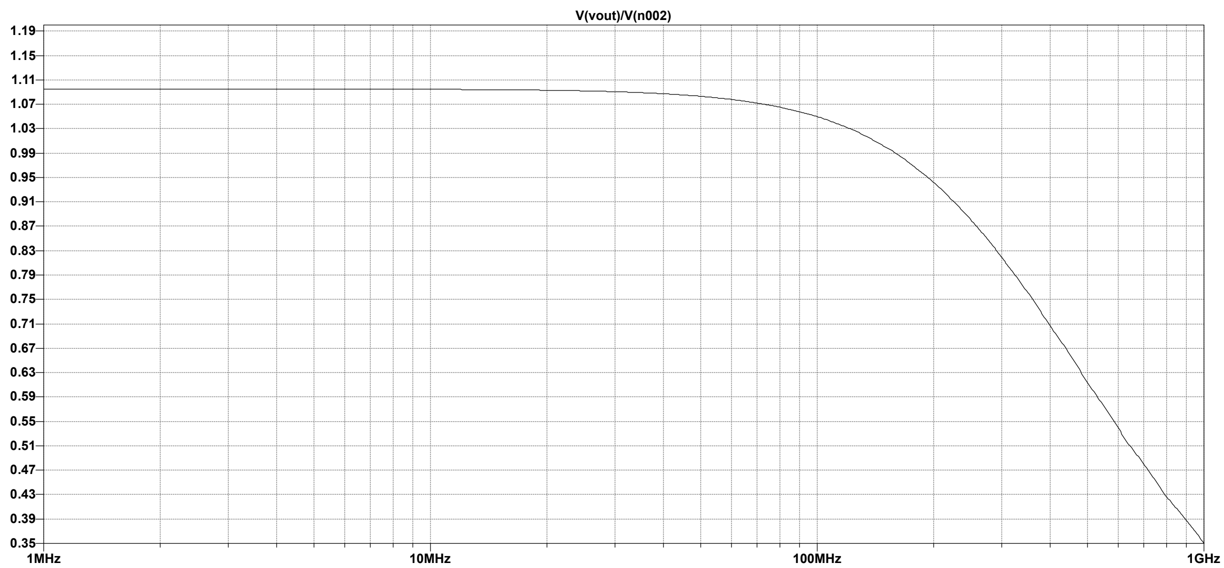

Figure 21: A_v at $V_{in} = 0.69V$

2. $R_{out} = 4.73k\Omega$

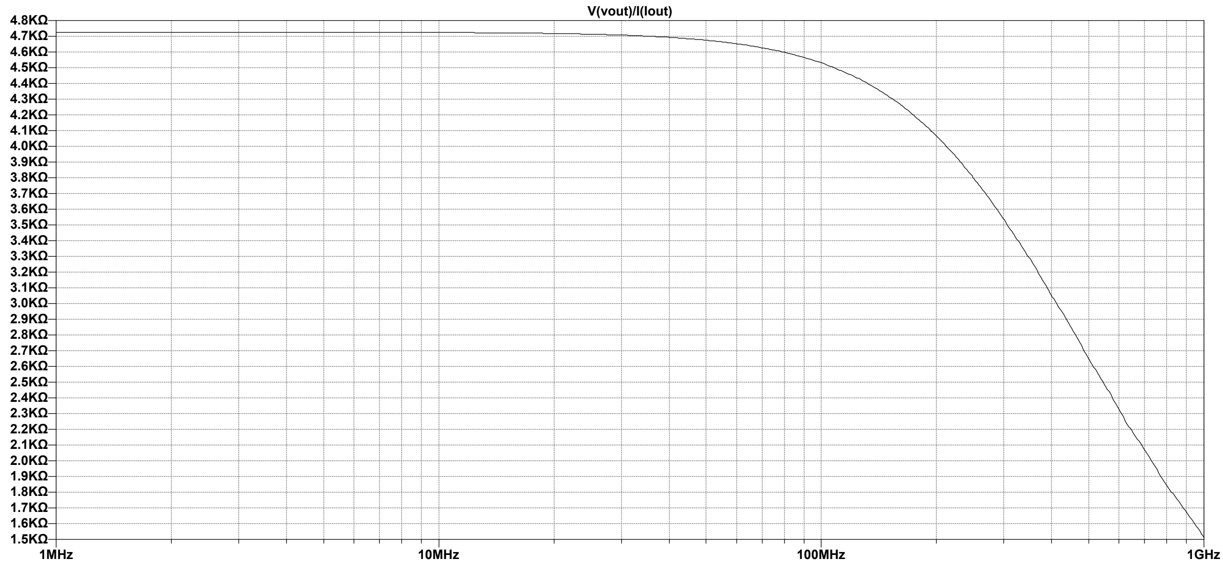

Figure 22: R_{out} at $V_{in} = 0.69V$

(e) $\mathrm{g_m}$ and $\mathrm{g_{ds}}$

From 7(d),

$$A_V = 1.09$$
$$R_{out} = 4.73k\Omega$$

From 6(c),

$$g_{m_1} = 2.32e - 04\Omega^{-1}$$
$$g_{m_2} = 2.09e - 04\Omega^{-1}$$
$$g_{DS_1} = \frac{1}{r_{o_1}} = 8.66e - 07\Omega^{-1}$$
$$g_{DS_2} = \frac{1}{r_{o_2}} = 1.28e - 06\Omega^{-1}$$

$$A_V = \frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$
$$\approx 1.10$$
$$R_{out} = \frac{1}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$
$$R_{out} = 4.74k\Omega$$