EE4C10 Analog Circuit Design Fundamentals

Homework Assignment IV

Tzong Lin Chua

October 15, 2021

Contents

Simulation Files

Each question with simulation files will have their respective subfolder.

Running the simulation files should be able to directly plot the graphs used (configured in the *.plt file). The folders for each question are arranged as follows after extracting:

spice	
	q4

Problem 1

(a) Small-signal gain, A_v

$$G_m = g_m$$
$$R_{out} = R_D$$

$$A_V = -g_m R_D$$

Thermal noise of:

1. Resistor, R_D
   Output noise current and voltage,

$$S_{i_{out},th} = \frac{4kT}{R_D}$$

$$\begin{aligned} S_{v_{out},th} &= S_{i,output} R_{out}^2 \\ &= 4kTR_D \end{aligned}$$

2. Transistor, M
   Output noise current and voltage,

$$S_{i_{out},th} = 4kT\gamma g_m$$

$$\begin{aligned} S_{v_{out},th} &= S_{i,output} R_{out}^2 \\ &= 4kT\gamma g_m R_D^2 \end{aligned}$$

Total output thermal noise,

$$S_{v_{out},th} = 4kTR_D + 4kT\gamma g_m R_D^2$$
$$= 4kTR_D^2(\gamma g_m + \frac{1}{R_D})$$

Total input thermal noise,

$$S_{v_{in},th} = \frac{S_{v_{out},th}}{A_V^2}$$
$$= \frac{4kT}{g_m}(\gamma + \frac{1}{g_m R_D})$$

(b) Flicker noise PSD at:

1. Input

$$S_{in,\frac{1}{f}} = \frac{K}{C_{OX}WL} \cdot \frac{1}{f}$$
$$= \frac{1.11 \times 10^{-12}}{f} \frac{V^2}{Hz}$$

2. Output

$$S_{in,\frac{1}{f}} = S_{in,\frac{1}{f}} A_V^2$$
$$= \frac{K(g_m R_D)^2}{C_{OX}WL} \cdot \frac{1}{f}$$
$$= \frac{4.0 \times 10^{-11}}{f} \frac{V^2}{Hz}$$

(c) $\frac{1}{f}$ noise corner frequency

$$S_{in,\frac{1}{f}}(f_c) = S_{v_{in},th}(f_c)$$
$$\frac{K}{C_{OX}WL} \cdot \frac{1}{f_c} = \frac{4kT}{g_m}(\gamma + \frac{1}{g_m R_D})$$
$$f_c = \frac{K}{C_{OX}WL} \cdot \frac{1}{\frac{4kT}{g_m}(\gamma + \frac{1}{g_m R_D})}$$
$$= 48.3kHz$$

(d) RMS integrated output noise voltage,

$$v_{rms,noise,out} = \sqrt{\int_{10^3}^{10\times 10^6} \frac{K(g_m R_D)^2}{C_{OX}WL} \cdot \frac{1}{f}\, df + 4kTR_D^2(\gamma g_m + \frac{1}{R_D})\Delta f}$$
$$= \sqrt{[\frac{K(g_m R_D)^2}{C_{OX}WL} ln(f) + 4kTR_D^2(\gamma g_m + \frac{1}{R_D})]_{10^3}^{10\times 10^6}}$$
$$= 9.30 \times 10^{-5} \frac{V}{\sqrt{Hz}}$$

(e) RMS integrated output noise voltage,

$$v_{rms,noise,out} = \sqrt{\int_{10^3}^{10\times 10^6} \frac{K(g_m R_D)^2}{C_{OX}WL} \cdot \frac{1}{f}\, df + 4kTR_D^2(\gamma g_m + \frac{1}{R_D})\Delta f}$$
$$= \sqrt{[\frac{K(g_m R_D)^2}{C_{OX}WL} ln(f) + 4kTR_D^2(\gamma g_m + \frac{1}{R_D})]_{10^3}^{10\times 10^3}}$$
$$= 9.98 \times 10^{-6} \frac{V}{\sqrt{Hz}}$$

(f) SNR,

$$SNR = 20log(\frac{V_{rms,signal}}{V_{rms,noise}})$$
$$= 80dB$$

Problem 2

Output resistance, transconductance and gain of amplifier,

$$R_{out} = (r_o + R_s) + g_m R_s r_o$$
$$G_m = \frac{g_m}{(1 + \frac{R_s}{r_o}) + g_m R_s}$$
$$A_V = \frac{g_m[(r_o + R_s) + g_m R_s r_o]}{(1 + \frac{R_s}{r_o}) + g_m R_s}$$

(a) Thermal noise from:

Transistor, M

$$\begin{aligned}\overline{v_{M,out}^2} &= 4kT\gamma g_m R_{out}^2 \\ &= 4kT\gamma g_m[(r_o + R_s) + g_m R_s r_o]^2\end{aligned}$$

Resistor, R_S

$$\frac{V_S - V_o}{r_o} = -g_m V_s$$
$$V_o = (1 + g_m r_o)V_s$$

$$\overline{v_{R_s,out}^2} = 4kTR_s(1 + g_m r_o)^2$$

Total output noise voltage,

$$\overline{v_{n,out}^2} = 4kT\gamma g_m[(r_o + R_s) + g_m R_s r_o]^2 + 4kTR_s(1 + g_m r_o)^2$$

(b) Ratio between thermal noise,

$$\begin{aligned}\frac{\overline{v_{M,out}^2}}{\overline{v_{R_s,out}^2}} &= \frac{4kT\gamma g_m[(r_o + R_s) + g_m R_s r_o]^2}{4kTR_s(1 + g_m r_o)^2} \\ &= \frac{\gamma g_m[(r_o + R_s) + g_m R_s r_o]^2}{R_s(1 + g_m r_o)^2} \\ &\approx \gamma g_m R_s\end{aligned}$$

(c) Input referred thermal noise,

$$\begin{aligned}\overline{v_{n,in}^2} &= \frac{\overline{v_{n,out}^2}}{A_v^2} \\ &= \{4kT\gamma g_m[(r_o + R_s) + g_m R_s r_o]^2 + 4kTR_s(1 + g_m r_o)^2\}[\frac{(1 + \frac{R_s}{r_o}) + g_m R_s}{g_m[(r_o + R_s) + g_m R_s r_o]}]^2 \\ &= 6.4653 \times 10^{-16} \frac{V^2}{Hz}\end{aligned}$$

Problem 3

(a) Since the circuit is symmetrical the differential noise due to the current mirrors, M_{N3} and M_{N4} is 0.

$$\overline{v_{n,diff,out}^2} = 0 \frac{V^2}{Hz}$$

(b) Common mode noise at output.

Noise current spectrum density at source of M_{N_4},

$$\begin{aligned}\overline{i_{n,M_{N3}}^2} &= \frac{4kT\gamma g_{m4}^2}{g_{m3}} \\ \overline{i_{n,M_{N4}}^2} &= 4kT\gamma g_{m4} \\ \overline{i_n^2} &= \frac{4kT\gamma g_{m4}^2}{g_{m3}} + 4kT\gamma g_{m4} \\ &= 4kT\gamma g_{m4}(1 + \frac{g_{m4}}{g_{m3}})\end{aligned}$$

Input referred noise voltage,

$$\begin{aligned}\overline{v_{n,in}^2} &= \frac{\overline{i_n^2}}{4g_{m1}^2} \\ &= \frac{g_{m4}kT\gamma}{g_{m1}^2}(1+\frac{g_{m4}}{g_{m3}})\end{aligned}$$

Output referred common-mode noise,

$$A_{CM-CM}(s) = \frac{-g_{m1}}{g_{m2}+sC_L}$$

$$\begin{aligned}\overline{v_{n,out}^2} &= \overline{v_{n,in}^2}A_{CM-CM}(s)^2 \\ &= \frac{g_{m4}kT\gamma}{g_{m1}^2}(1+\frac{g_{m4}}{g_{m3}})[\frac{g_{m1}}{g_{m2}+sC_L}]^2 \\ &= \frac{g_{m4}kT\gamma}{(g_{m2}+sC_L)^2}(1+\frac{g_{m4}}{g_{m3}}) \\ &= \frac{2g_{m4}kT\gamma}{(g_{m2}+sC_L)^2}\end{aligned}$$

Doubling I_B and W_3, will not increase the current at the amplifier or g_{m3}. Therefore

$$\begin{aligned}\overline{v_{n,out}^2} &= \frac{g_{m4}kT\gamma}{(g_{m2}+sC_L)^2}(1+\frac{g_{m4}}{g_{m3}}) \\ &= \frac{2g_{m4}kT\gamma}{(g_{m2}+sC_L)^2}\end{aligned}$$

1. Total differential input and output noise PSD,

$$\begin{aligned}Z_{out}(s) &= \frac{1}{g_{m2}+sC_L} \\ A_{DM-DM} &= \frac{g_{m1}}{g_{m2}+sC_L}\end{aligned}$$

$$\begin{aligned}\overline{v_{n,MN1\&MN2}(s)^2} &= (\frac{1}{g_{m2}+sC_L})^2(4kT\gamma g_{m1}+4kT\gamma g_{m1}) \\ &= \frac{8kT\gamma g_{m1}}{(g_{m2}+sC_L)^2}\end{aligned}$$

$$\begin{aligned}\overline{v_{n,MP1\&MP2}(s)^2} &= (\frac{1}{g_{m2}+sC_L})^2(4kT\gamma g_{m2}+4kT\gamma g_{m2}) \\ &= \frac{8kT\gamma g_{m2}}{(g_{m2}+sC_L)^2}\end{aligned}$$

$$\overline{v_{n,out}^2}(s) = \frac{8kT\gamma}{(g_{m2}+sC_L)^2}(g_{m1}+g_{m2})$$

$$\overline{v_{n,in}^2} = \frac{8kT\gamma}{g_{m1}^2}(g_{m1}+g_{m2})$$

2. Integrated differential output thermal noise,

$$
\begin{aligned}
\overline{v_{n,out}^2}(s) &= \frac{8kT\gamma}{(g_{m2}+sC_L)^2}(g_{m1}+g_{m2}) \\
&= \frac{8kT\gamma(g_{m1}+g_{m2})}{g_{m2}^2(1+\frac{sC_L}{g_{m2}})^2} \\
&= \frac{8kT\gamma(g_{m1}+g_{m2})}{g_{m2}^2}\frac{1}{(1+\frac{sC_L}{g_{m2}})^2}
\end{aligned}
$$

$$
\begin{aligned}
ENBW &= \frac{\pi}{2}\frac{g_{m2}}{2\pi C_L} \\
&= \frac{g_{m2}}{4C_L}
\end{aligned}
$$

$$
\begin{aligned}
v_{rms}^2 &= \frac{8kT\gamma(g_{m1}+g_{m2})}{g_{m2}^2} \times ENBW \\
&= \frac{2kT\gamma(g_{m1}+g_{m2})}{g_{m2}C_L}
\end{aligned}
$$

3. Integrated differential output thermal noise,

$$
\begin{aligned}
v_{rms}^2 &= \frac{8kT\gamma(g_{m1}+g_{m2})}{g_{m2}^2} \times ENBW \\
&= \frac{2kT\gamma(g_{m1}+g_{m2})}{g_{m2}C_L} \\
&= 2.76 \times 10^{-08}\frac{V^2}{Hz}
\end{aligned}
$$

Problem 4

Step:

1. Gain and BW of amplifier

$$
\begin{aligned}
A_{DM-DM} &= \frac{g_{m1}}{g_{m2}+sC_L} \\
f_c &= \frac{g_2}{2\pi C_L}
\end{aligned}
$$

2.