EE4C10 Analog Circuit Design Fundamentals

Homework Assignment IV

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October 17, 2021

Contents

Simulation Files

Each question with simulation files will have their respective subfolder.

Running the simulation files should be able to directly plot the graphs used (configured in the *.plt file). The folders for each question are arranged as follows after extracting:

spice
q4

Problem 1

(a) Small-signal gain, A_v

$$G_m = g_m$$
$$R_{out} = R_D$$

$$A_V = -g_m R_D$$

Thermal noise of:

1. Resistor, R_D
   Output noise current and voltage,

$$S_{i_{out},th} = \frac{4kT}{R_D}$$

$$S_{v_{out},th} = S_{i,output} R_{out}^2 = 4kTR_D$$

2. Transistor, M
   Output noise current and voltage,

$$S_{i_{out},th} = 4kT\gamma g_m$$

$$S_{v_{out},th} = S_{i,output} R_{out}^2 = 4kT\gamma g_m R_D^2$$

Total output thermal noise,

$$
\begin{aligned}
S_{v_{out},th} &= 4kTR_D + 4kT\gamma g_m R_D^2 \\
&= 4kTR_D^2(\gamma g_m + \frac{1}{R_D})
\end{aligned}
$$

Total input thermal noise,

$$
\begin{aligned}
S_{v_{in},th} &= \frac{S_{v_{out},th}}{A_V^2} \\
&= \frac{4kT}{g_m}(\gamma + \frac{1}{g_m R_D})
\end{aligned}
$$

(b) Flicker noise PSD at:

1. Input

$$
\begin{aligned}
S_{in,\frac{1}{f}} &= \frac{K}{C_{OX}WL} \cdot \frac{1}{f} \\
&= \frac{1.11 \times 10^{-12}}{f} \frac{V^2}{Hz}
\end{aligned}
$$

2. Output

$$
\begin{aligned}
S_{in,\frac{1}{f}} &= S_{in,\frac{1}{f}} A_V^2 \\
&= \frac{K(g_m R_D)^2}{C_{OX}WL} \cdot \frac{1}{f} \\
&= \frac{4.0 \times 10^{-11}}{f} \frac{V^2}{Hz}
\end{aligned}
$$

3. Sketch of PSD:

img/q1/flicker-noise.png

Figure 1: Input and output flicker noise

(c) $\frac{1}{f}$ noise corner frequency

$$
\begin{aligned}
S_{in,\frac{1}{f}}(f_c) &= S_{v_{in},th}(f_c) \\
\frac{K}{C_{OX}WL} \cdot \frac{1}{f_c} &= \frac{4kT}{g_m}(\gamma + \frac{1}{g_m R_D}) \\
f_c &= \frac{K}{C_{OX}WL} \cdot \frac{1}{\frac{4kT}{g_m}(\gamma + \frac{1}{g_m R_D})} \\
&= 48.3kHz
\end{aligned}
$$

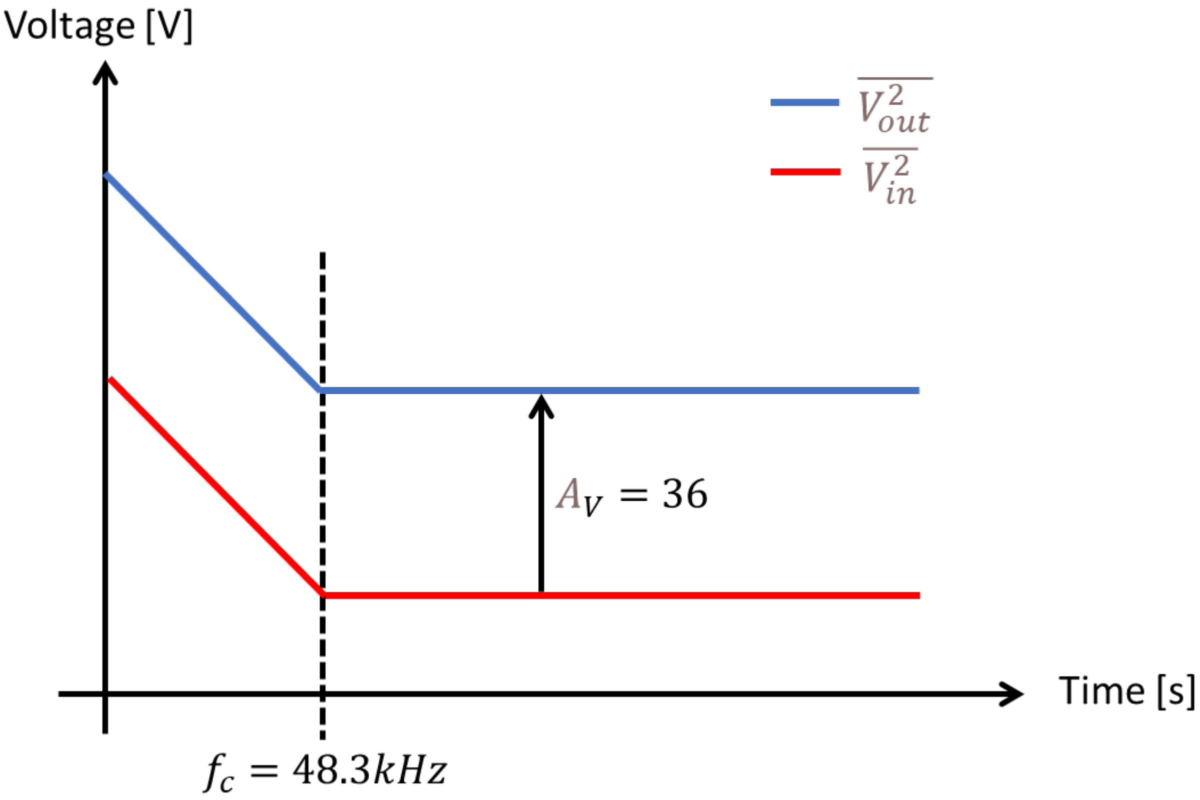

Figure 2: Sketch

(d) RMS integrated output noise voltage,

$$
\begin{aligned}
v_{rms,noise,out} &= \sqrt{\int_{10^3}^{10\times10^6} \frac{K(g_m R_D)^2}{C_{OX}WL} \cdot \frac{1}{f}\,df + 4kTR_D^2(\gamma g_m + \frac{1}{R_D})\Delta f} \\
&= \sqrt{[\frac{K(g_m R_D)^2}{C_{OX}WL} ln(f) + 4kTR_D^2(\gamma g_m + \frac{1}{R_D})]_{10^3}^{10\times10^6}} \\
&= 9.30 \times 10^{-5} \frac{V}{\sqrt{Hz}}
\end{aligned}
$$

(e) RMS integrated output noise voltage,

$$
\begin{aligned}
v_{rms,noise,out} &= \sqrt{\int_{10^3}^{10\times10^6} \frac{K(g_m R_D)^2}{C_{OX}WL} \cdot \frac{1}{f}\,df + 4kTR_D^2(\gamma g_m + \frac{1}{R_D})\Delta f} \\
&= \sqrt{[\frac{K(g_m R_D)^2}{C_{OX}WL} ln(f) + 4kTR_D^2(\gamma g_m + \frac{1}{R_D})]_{10^3}^{10\times10^3}} \\
&= 9.98 \times 10^{-6} \frac{V}{\sqrt{Hz}}
\end{aligned}
$$

(f) SNR,

$$
\begin{aligned}
SNR &= 20log(\frac{V_{rms,signal}}{V_{rms,noise}}) \\
&= 80dB
\end{aligned}
$$

Problem 2

Output resistance, transconductance and gain of amplifier,

$$
\begin{aligned}
R_{out} &= (r_o + R_s) + g_m R_s r_o \\
G_m &= \frac{g_m}{(1+\frac{R_s}{r_o}) + g_m R_s} \\
A_V &= \frac{g_m[(r_o + R_s) + g_m R_s r_o]}{(1+\frac{R_s}{r_o}) + g_m R_s}
\end{aligned}
$$

(a) Thermal noise from:

Transistor, M

$$
\begin{aligned}
\overline{v_{M,out}^2} &= 4kT\gamma g_m R_{out}^2 \\
&= 4kT\gamma g_m[(r_o + R_s) + g_m R_s r_o]^2
\end{aligned}
$$

Resistor, R_S

$$
\begin{aligned}
\frac{V_S - V_o}{r_o} &= -g_m V_s \\
V_o = (1 + g_m r_o)V_s
\end{aligned}
$$

$$\overline{v_{R_s,out}^2} = 4kTR_s(1 + g_m r_o)^2$$

Total output noise voltage,

$$\overline{v_{n,out}^2} = 4kT\gamma g_m[(r_o + R_s) + g_m R_s r_o]^2 + 4kTR_s(1 + g_m r_o)^2$$

(b) Ratio between thermal noise,

$$
\begin{aligned}
\frac{\overline{v_{M,out}^2}}{\overline{v_{R_s,out}^2}} &= \frac{4kT\gamma g_m[(r_o + R_s) + g_m R_s r_o]^2}{4kTR_s(1 + g_m r_o)^2} \\
&= \frac{\gamma g_m[(r_o + R_s) + g_m R_s r_o]^2}{R_s(1 + g_m r_o)^2} \\
&\approx \gamma g_m R_s
\end{aligned}
$$

(c) Input referred thermal noise,

$$
\begin{aligned}
\overline{v_{n,in}^2} &= \frac{\overline{v_{n,out}^2}}{A_v^2} \\
&= \{4kT\gamma g_m[(r_o + R_s) + g_m R_s r_o]^2 + 4kTR_s(1 + g_m r_o)^2\}[\frac{(1+\frac{R_s}{r_o}) + g_m R_s}{g_m[(r_o + R_s) + g_m R_s r_o]}]^2 \\
&= 6.4653 \times 10^{-16}\frac{V^2}{Hz}
\end{aligned}
$$

Problem 3

(a) Since the circuit is symmetrical the differential noise due to the current mirrors, M_{N3} and M_{N4} is 0.

$$\overline{v_{n,diff,out}^2} = 0\frac{V^2}{Hz}$$

(b) Common mode noise at output.

Noise current spectrum density at source of M_{N_4},

$$
\begin{aligned}
\overline{i_{n,M_{N3}}^2} &= \frac{4kT\gamma g_{m4}^2}{g_{m3}} \\
\overline{i_{n,M_{N4}}^2} &= 4kT\gamma g_{m4} \\
\overline{i_n^2} &= \frac{4kT\gamma g_{m4}^2}{g_{m3}} + 4kT\gamma g_{m4} \\
&= 4kT\gamma g_{m4}(1 + \frac{g_{m4}}{g_{m3}})
\end{aligned}
$$

Input referred noise voltage,

$$\begin{aligned}
\overline{v_{n,in}^2} &= \frac{\overline{i_n^2}}{4g_{m1}^2} \\
&= \frac{g_{m4}kT\gamma}{g_{m1}^2}(1 + \frac{g_{m4}}{g_{m3}})
\end{aligned}$$

Output referred common-mode noise,

$$A_{CM-CM}(s) = \frac{-g_{m1}}{g_{m2} + sC_L}$$

$$\begin{aligned}
\overline{v_{n,out}^2} &= \overline{v_{n,in}^2} A_{CM-CM}(s)^2 \\
&= \frac{g_{m4}kT\gamma}{g_{m1}^2}(1 + \frac{g_{m4}}{g_{m3}})[\frac{g_{m1}}{g_{m2} + sC_L}]^2 \\
&= \frac{g_{m4}kT\gamma}{(g_{m2} + sC_L)^2}(1 + \frac{g_{m4}}{g_{m3}}) \\
&= \frac{2g_{m4}kT\gamma}{(g_{m2} + sC_L)^2}
\end{aligned}$$

Doubling I_B and W_3, will not increase the current at the amplifier or g_{m3}. Therefore

$$\begin{aligned}
\overline{v_{n,out}^2} &= \frac{g_{m4}kT\gamma}{(g_{m2} + sC_L)^2}(1 + \frac{g_{m4}}{g_{m3}}) \\
&= \frac{2g_{m4}kT\gamma}{(g_{m2} + sC_L)^2}
\end{aligned}$$

(c) Total differential input and output noise PSD,

$$\begin{aligned}
Z_{out}(s) &= \frac{1}{g_{m2} + sC_L} \\
A_{DM-DM} &= \frac{g_{m1}}{g_{m2} + sC_L}
\end{aligned}$$

$$\begin{aligned}
\overline{v_{n,MN1\&MN2}(s)^2} &= (\frac{1}{g_{m2} + sC_L})^2(4kT\gamma g_{m1} + 4kT\gamma g_{m1}) \\
&= \frac{8kT\gamma g_{m1}}{(g_{m2} + sC_L)^2}
\end{aligned}$$

$$\begin{aligned}
\overline{v_{n,MP1\&MP2}(s)^2} &= (\frac{1}{g_{m2} + sC_L})^2(4kT\gamma g_{m2} + 4kT\gamma g_{m2}) \\
&= \frac{8kT\gamma g_{m2}}{(g_{m2} + sC_L)^2}
\end{aligned}$$

$$\overline{v_{n,out}^2}(s) = \frac{8kT\gamma}{(g_{m2} + sC_L)^2}(g_{m1} + g_{m2})$$

$$\overline{v_{n,in}^2} = \frac{8kT\gamma}{g_{m1}^2}(g_{m1} + g_{m2})$$

(d) Integrated differential output thermal noise,

$$
\begin{aligned}
\overline{v_{n,out}^2}(s) &= \frac{8kT\gamma}{(g_{m2}+sC_L)^2}(g_{m1}+g_{m2}) \\
&= \frac{8kT\gamma(g_{m1}+g_{m2})}{g_{m2}^2(1+\frac{sC_L}{g_{m2}})^2} \\
&= \frac{8kT\gamma(g_{m1}+g_{m2})}{g_{m2}^2}\frac{1}{(1+\frac{sC_L}{g_{m2}})^2}
\end{aligned}
$$

$$
\begin{aligned}
ENBW &= \frac{\pi}{2}\frac{g_{m2}}{2\pi C_L} \\
&= \frac{g_{m2}}{4C_L}
\end{aligned}
$$

$$
\begin{aligned}
v_{rms}^2 &= \frac{8kT\gamma(g_{m1}+g_{m2})}{g_{m2}^2} \times ENBW \\
&= \frac{2kT\gamma(g_{m1}+g_{m2})}{g_{m2}C_L}
\end{aligned}
$$

(e) Integrated differential output thermal noise,

$$
\begin{aligned}
v_{rms}^2 &= \frac{8kT\gamma(g_{m1}+g_{m2})}{g_{m2}^2} \times ENBW \\
&= \frac{2kT\gamma(g_{m1}+g_{m2})}{g_{m2}C_L} \\
&= 2.76 \times 10^{-08}\frac{V^2}{Hz}
\end{aligned}
$$

Problem 4

Gain, cutoff frequency, BW, and ENBW of amplifier

$$
\begin{aligned}
A_{DM-DM} &= \frac{g_{m1}}{g_{m2}+sC_L} \\
f_c &= \frac{g_{m2}}{2\pi C_L} \\
ENBW &= \frac{g_{m2}}{4C_L}
\end{aligned}
$$

Steps taken for determining the dimensions of transistors.

1. Select values so that the gain, $|A_v \geq 5|$. The tail current is increased when the dimensions exceeds the allowed limit in the model, by doing so the transconductance is increased without increasing the dimensions.
2. The capacitance of the output capacitance is selected to give a cutoff of $f_c \geq 2MHz$.
3. The 1/f noise corner is decreased below, $f_c \leq 100kHz$, by changing the dimensions of the transistors but keeping the ratio of W/L constant.
4. The total integrated noise is from 1Hz to 100MHz is $52.675\mu V$.

Final dimensions of transistors and bias current

I_B	$0.05mA$
W_{N1}	$80um$
L_{N1}	$2um$
W_{N2}	$80um$
L_{N2}	$2um$
W_{N3}	$20um$
L_{N3}	$2um$
W_{N4}	$20um$
L_{N4}	$2um$
W_{P1}	$20um$
L_{P1}	$2um$
W_{P2}	$20um$
L_{P2}	$2um$

Testbench of amplifier

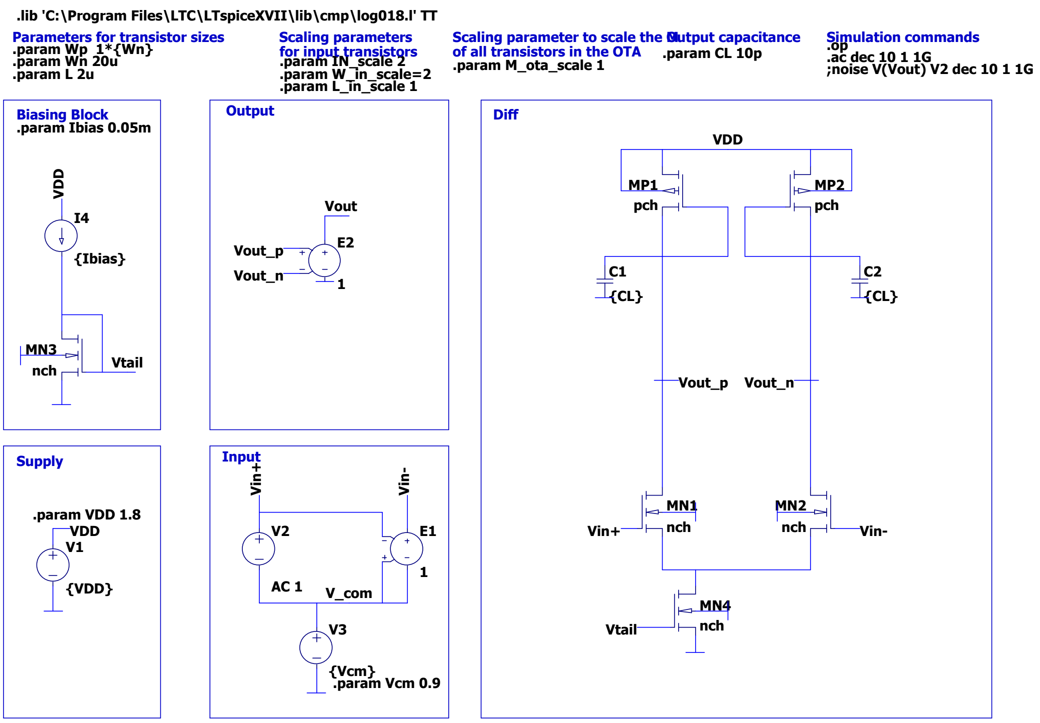

Figure 3: Testbench

Gain of amplifier

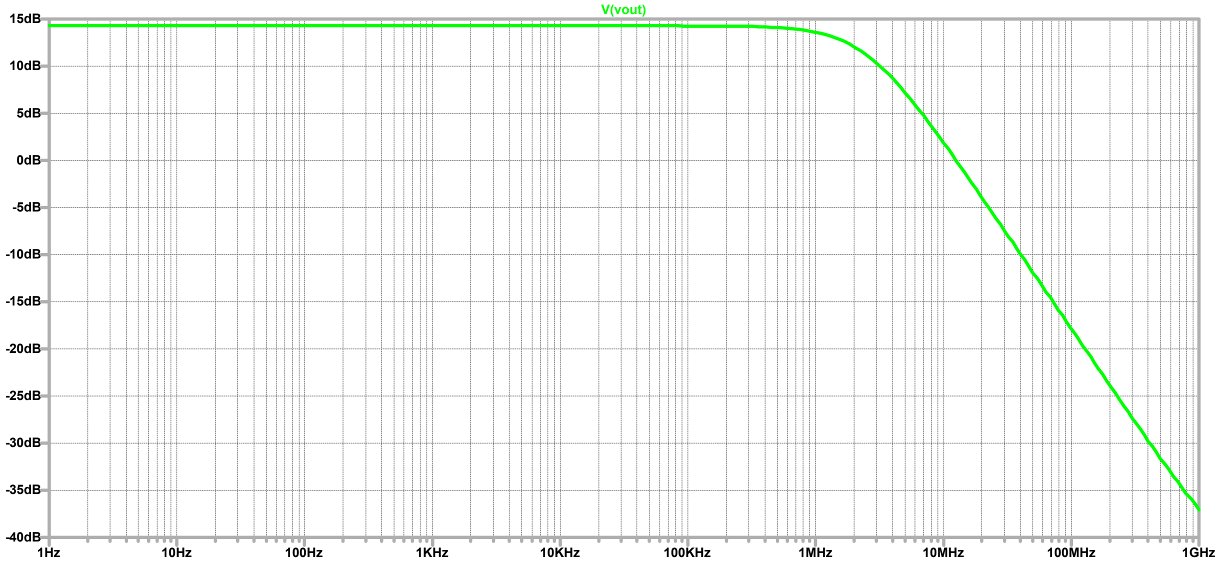

Figure 4: Gain

Noise analysis

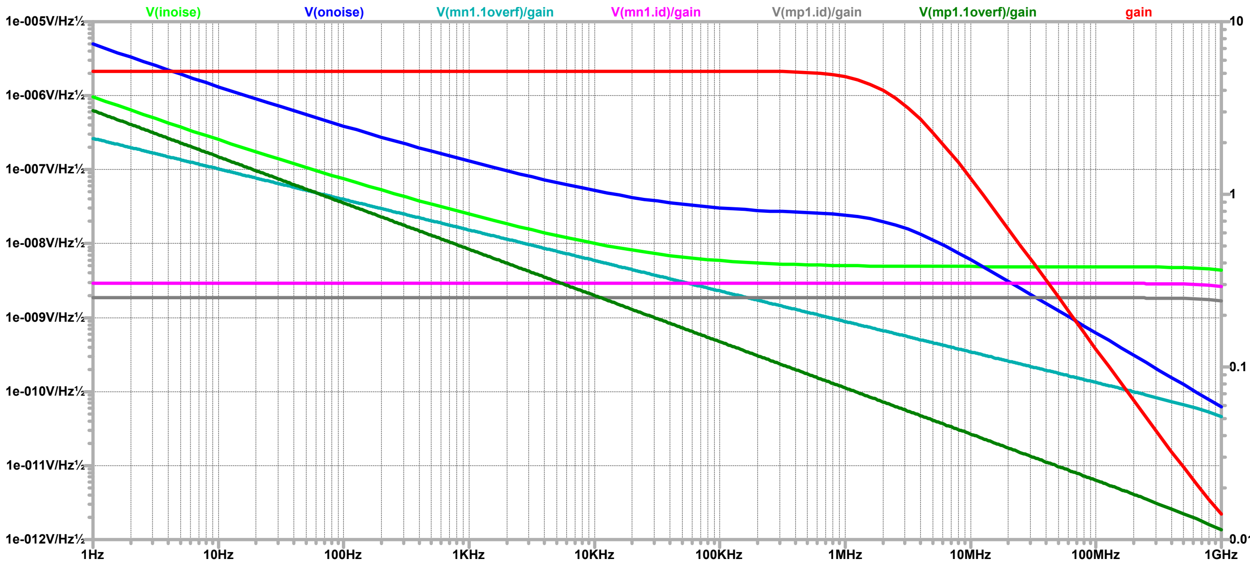

Figure 5: Noise analysis

Transient analysis

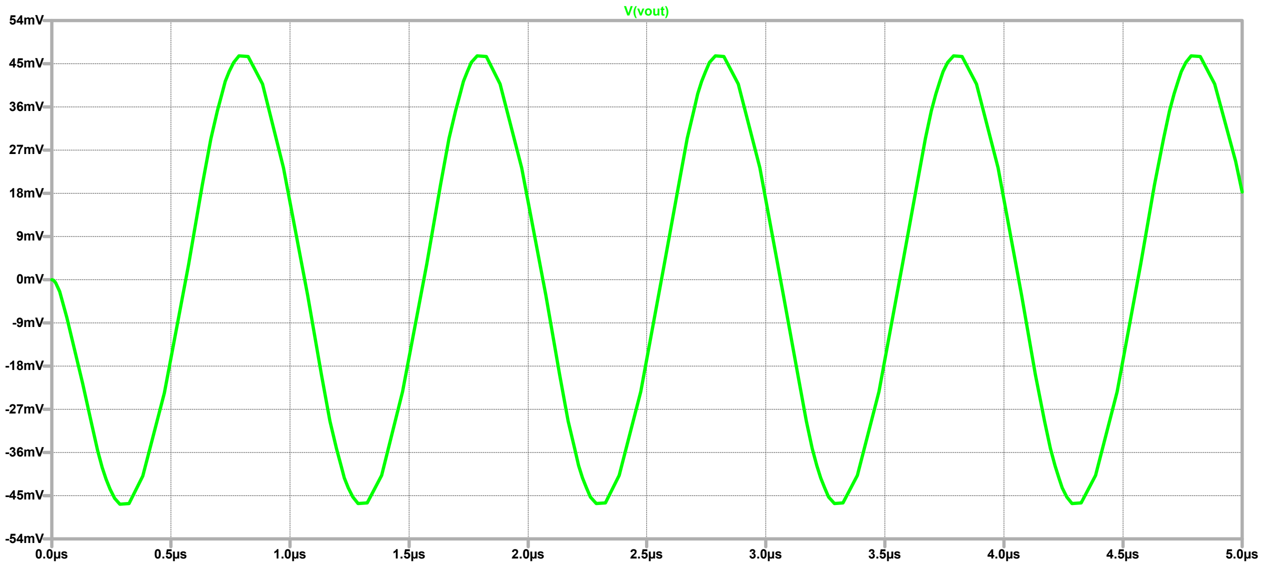

Figure 6: Transient analysis

Final results of amplifier,

Small-signal Gain, A_V	5.17
3dB bandwidth, f_{-3dB}	2.4 MHz
Integrated output noise from 1Hz to 100MHz	$52.675\mu V$
1/f noise corner	50.7 KHz