

EE4C10 Analog Circuit Design Fundamentals

Homework Assignment I

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Simulation Files

Each question with simulation files will have their respective subfolder. q7 subfolder for problem 7 will have subfolders for the subquestions.

The graphs used for the sub problems in problem 3 to 6 are plotted together. Running the simulation files should be able to directly plot the graphs used (configured in the *.plt file). The folders for each question are arranged as follows after extracting:

spice
q3
q4
q5
q7
b
c
d
e

Problem 1

For $I_D = 40\mu A$:

$$I_D = \frac{1.8V - V_D}{R}$$

$$V_D = 1.8V - I_D R$$

$$V_D = 1.0V$$

Saturation region:

$$V_{GS} = 1.0V > V_{TH}$$

$$V_{GS} - V_{TH} = 0.4V < V_{DS}$$

(a) $\lambda = 0V^{-1}$

$$I_D = \frac{\mu_n C_{OX}}{2} \frac{W}{L} (V_{GS} - V_{TH})^2$$

$$L = \frac{\mu_n C_{OX}}{2} \frac{W}{I_D} (V_{GS} - V_{TH})^2$$

$$L = 0.39\mu m$$

(b) $\lambda = 0.06V^{-1}$

$$I_D = \frac{\mu_n C_{OX}}{2} \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$$

$$L = \frac{\mu_n C_{OX}}{2} \frac{W}{I_D} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$$

$$L = 0.41\mu m$$

Problem 2

(a) Bulk of the transistors are connected to the source, $V_B = V_S$

$$V_{TH} = V_{TH0} + \gamma(\sqrt{|2\varphi_F| + V_{BS}} - \sqrt{|2\varphi_F|})$$

$$V_{TH} = V_{TH0} = 0.33V$$

1. Transistor M_1

$$V_{SG} = 2.5V - 1.7V = 0.8V$$

$$I_D = \frac{\mu_p C_{OX}}{2} \frac{W}{L} (V_{SG} - V_{TH})^2$$

$$W = \frac{2LI_D}{\mu_p C_{OX}} \frac{1}{(V_{SG} - V_{TH})^2}$$

$$W_1 = 2.72\mu m$$

2. Transistor M_2

$$V_{SG} = 1.7V - 1V = 0.7V$$

$$W = \frac{2LI_D}{\mu_p C_{OX}} \frac{1}{(V_{SG} - V_{TH})^2}$$

$$W_2 = 4.38\mu m$$

3. Transistor M_3

$$V_{SG} = 1V$$

$$W = \frac{2LI_D}{\mu_p C_{OX}} \frac{1}{(V_{SG} - V_{TH})^2}$$

$$W_3 = 1.37\mu m$$

(b) Bulk terminals are attached to the V_{DD} , $V_B = V_{DD}$.

1. Transistor M_1

$$V_{BS} = 2.5V - 2.5V = 0V$$

$$V_{TH} + \gamma(\sqrt{|2\varphi_F| + V_{BS}} - \sqrt{|2\varphi_F|})$$

$$V_{TH} = 0.33V$$

$$W = \frac{2LI_D}{\mu_p C_{OX}} \frac{1}{(V_{SG} - V_{TH})^2}$$

$$W_1 = 2.72\mu m$$

2. Transistor M₂

$$V_{BS} = 2.5V - 1.7V = 0.8V$$

$$V_{TH} + \gamma(\sqrt{|2\varphi_F| + V_{BS}} - \sqrt{|2\varphi_F|})$$

$$V_{TH} = 0.43V$$

$$W = \frac{2LI_D}{\mu_p C_{OX}} \frac{1}{(V_{SG} - V_{TH})^2}$$

$$W_2 = 8.23\mu m$$

3. Transistor M₃

$$V_{BS} = 2.5V - 1.0V = 1.5V$$

$$V_{TH} + \gamma(\sqrt{|2\varphi_F| + V_{BS}} - \sqrt{|2\varphi_F|})$$

$$V_{TH} = 0.49V$$

$$W = \frac{2LI_D}{\mu_p C_{OX}} \frac{1}{(V_{SG} - V_{TH})^2}$$

$$W_3 = 2.31\mu m$$

Problem 3

(a) Testbench and I_D-V_{GS} characteristics of NMOS and PMOS

1. NMOS

i. Testbench

```
.lib 'C:\Program Files\LTC\LTspiceXVII\lib\cmp\log018.l' TT
.dc VGS 0 1.8 0.001
```

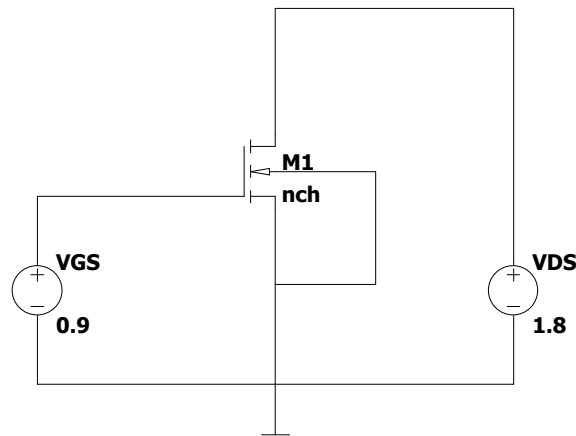


Figure 1: NMOS Testbench

ii. I_D-V_{GS}

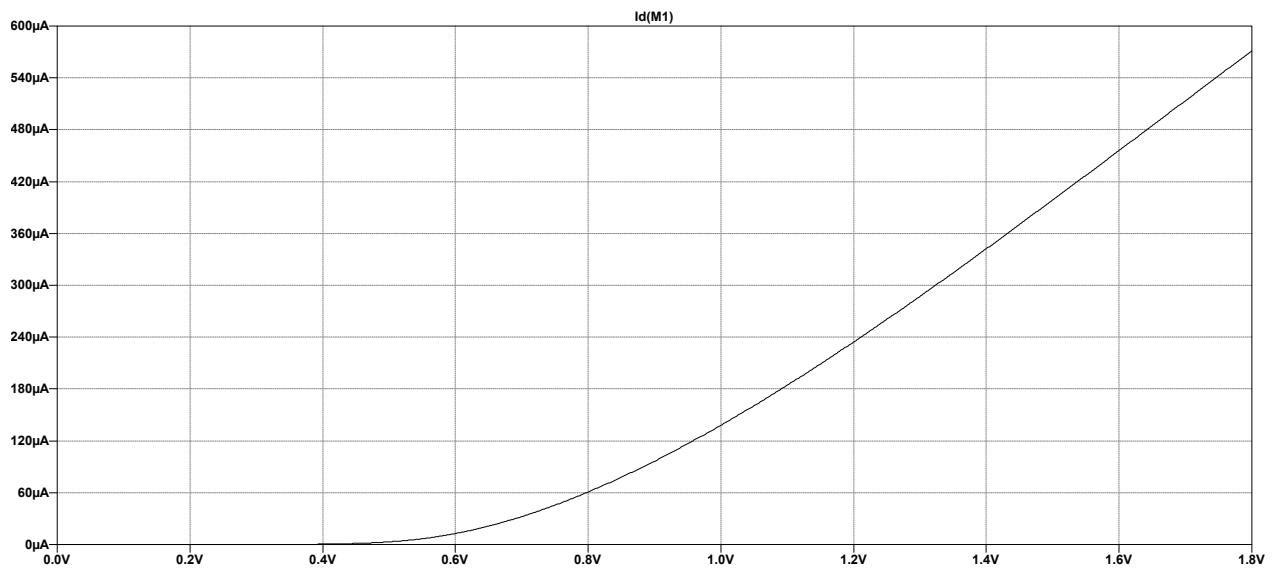


Figure 2: NMOS I_D - V_{GS}

2. PMOS

i. Testbench

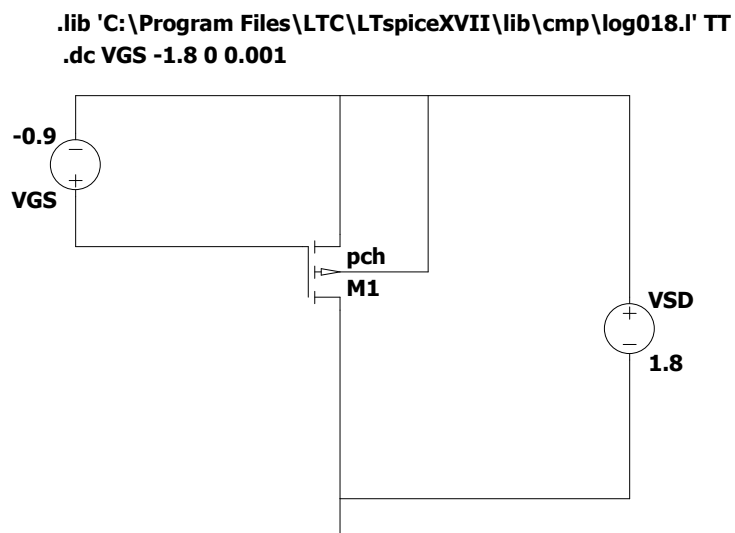


Figure 3: PMOS Testbench

ii. I_S - V_{GS}

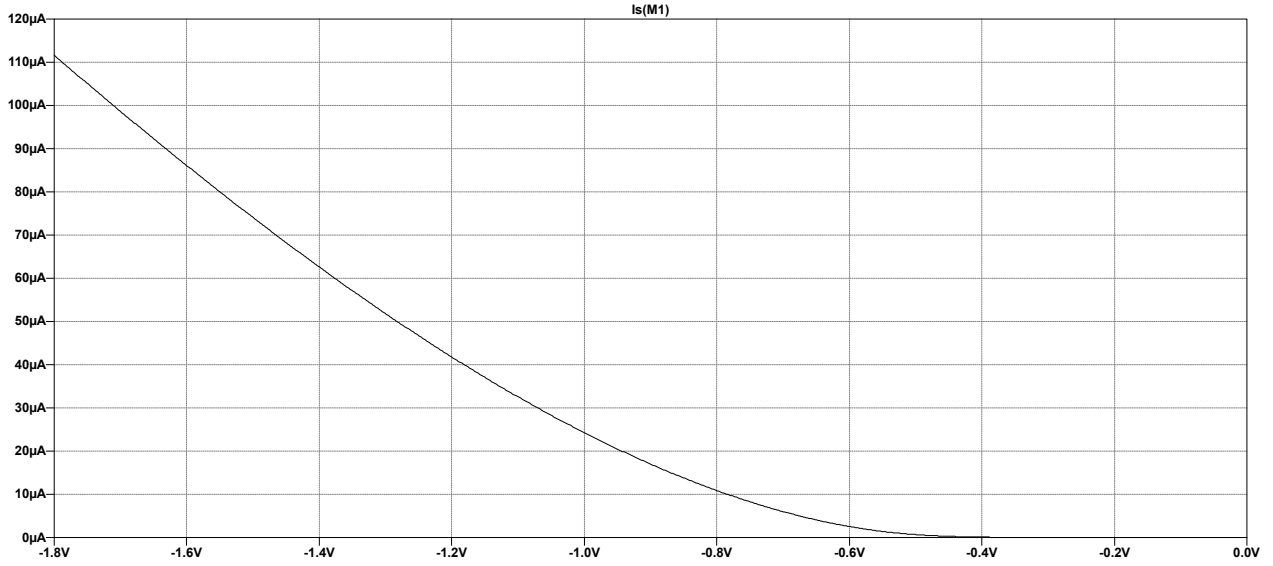


Figure 4: PMOS I_S - V_{GS}

(b) $\mu_{n(p)}C_{OX}$ and $V_{THn(p)}$

Assuming that channel length modulation is negligible, V_{THn} for NMOS can be derived from the following relation:

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_{THn})^2$$

$$\frac{2I_D}{\mu_n C_{ox}} \frac{L}{W} = (V_{GS} - V_{THn})^2$$

$$\sqrt{\frac{2I_D}{\mu_n C_{ox}} \frac{L}{W}} = V_{GS} - V_{THn}$$

V_{THn} is the x-axis intercept when the saturation region is extrapolated. In the case of PMOS, the relation becomes:

$$\sqrt{\frac{2I_S}{\mu_p C_{ox}} \frac{L}{W}} = V_{SG} + V_{THp}$$

For deriving $\mu_n C_{OX}$, since $V_{THn(p)}$ is constant at specific temperatures. Differentiating both sides with respect to $V_{GS(SG)}$ will give:

$$\frac{d}{dV_{GS}} \sqrt{\frac{2I_D}{\mu_n C_{ox}} \frac{L}{W}} = \frac{d}{dV_{GS}} (V_{GS} - V_{THn})$$

$$\frac{1}{2} \frac{dI_D}{dV_{GS}} \sqrt{\frac{2}{I_D \mu_n C_{ox}} \frac{L}{W}} = 1$$

$$\sqrt{\mu_n C_{ox}} = \frac{1}{2} \frac{dI_D}{dV_{GS}} \sqrt{\frac{2}{I_D} \frac{L}{W}}$$

$$\mu_n C_{ox} = \frac{1}{2} \frac{L}{W} \frac{1}{I_D} \left(\frac{dI_D}{dV_{GS}} \right)^2$$

$$\mu_n C_{ox} = \frac{1}{6I_D} \left(\frac{dI_D}{dV_{GS}} \right)^2$$

In the case for PMOS, the relation becomes:

$$\mu_p C_{ox} = \frac{1}{6I_S} \left(\frac{dI_S}{dV_{GS}} \right)^2$$

1. NMOS

i. $\mu_n C_{OX}$

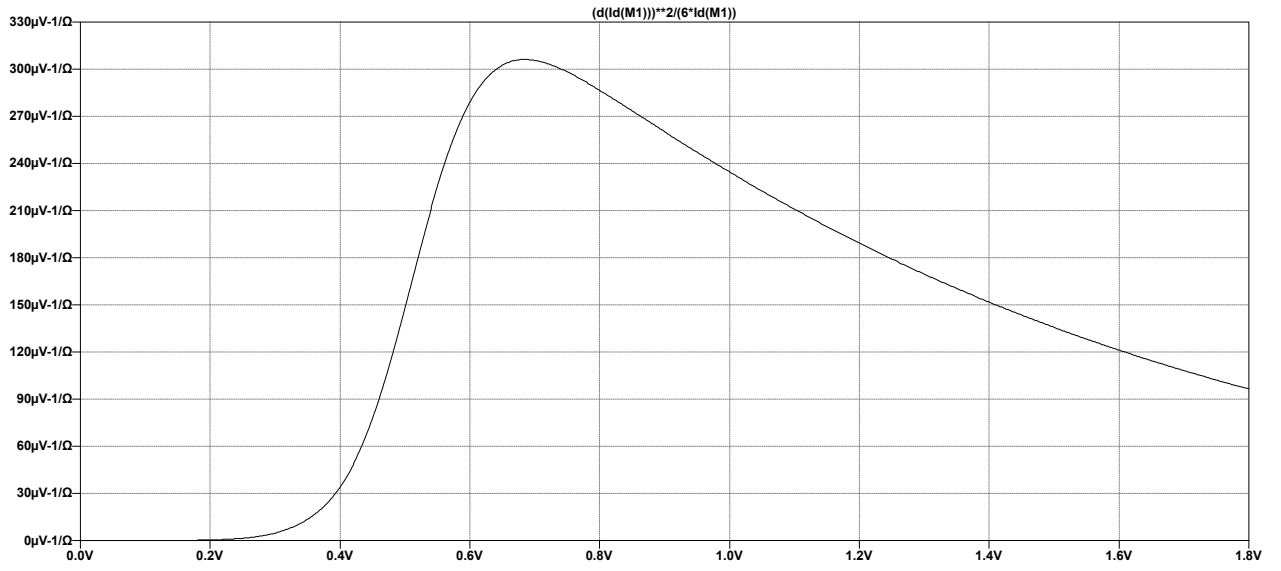


Figure 5: NMOS $\mu_n C_{OX}$ - V_{GS}

ii. $V_{THn} = 0.44V$

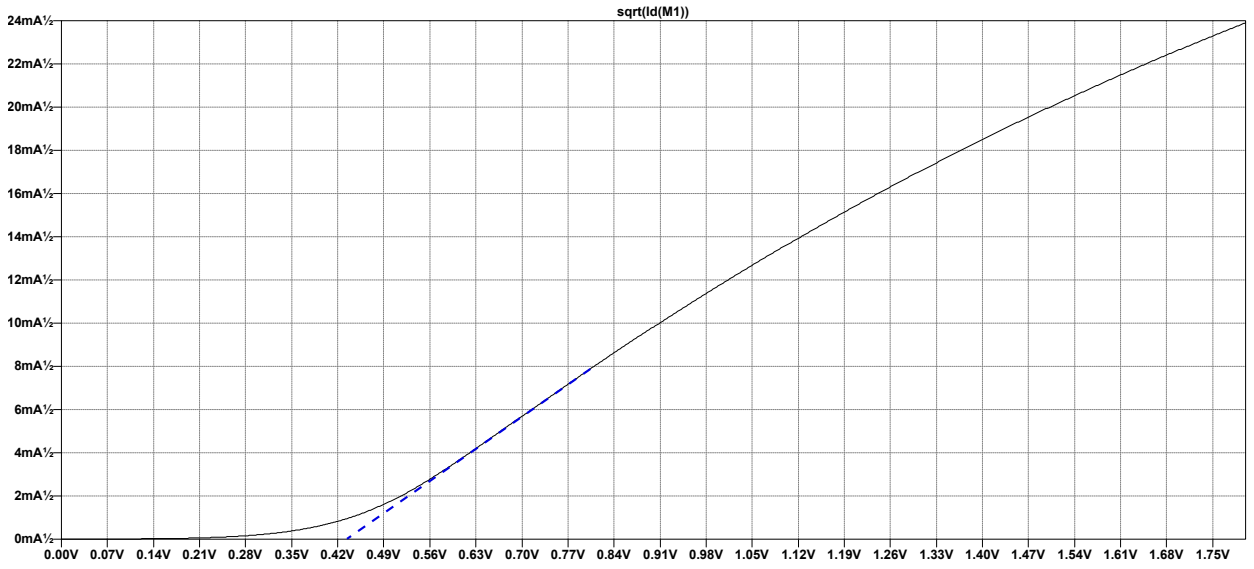


Figure 6: NMOS $\sqrt{I_D} - V_{GS}$

2. PMOS

i. $\mu_p C_{OX}$

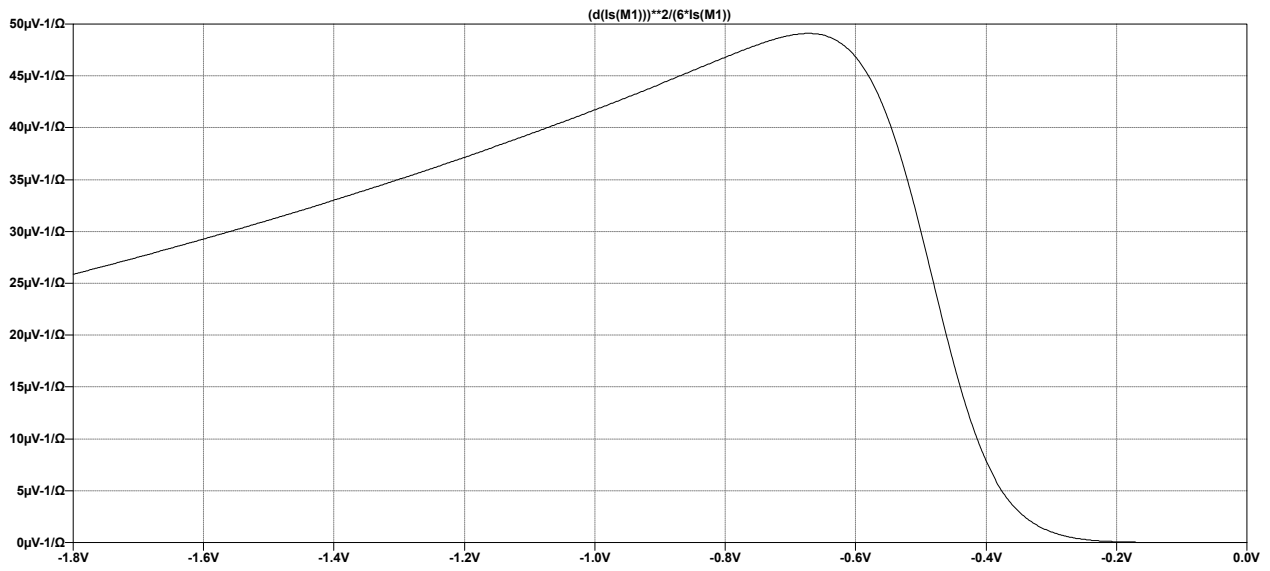


Figure 7: PMOS $\mu_p C_{OX}$ - V_{GS}

ii. $V_{THp} = -0.42V$

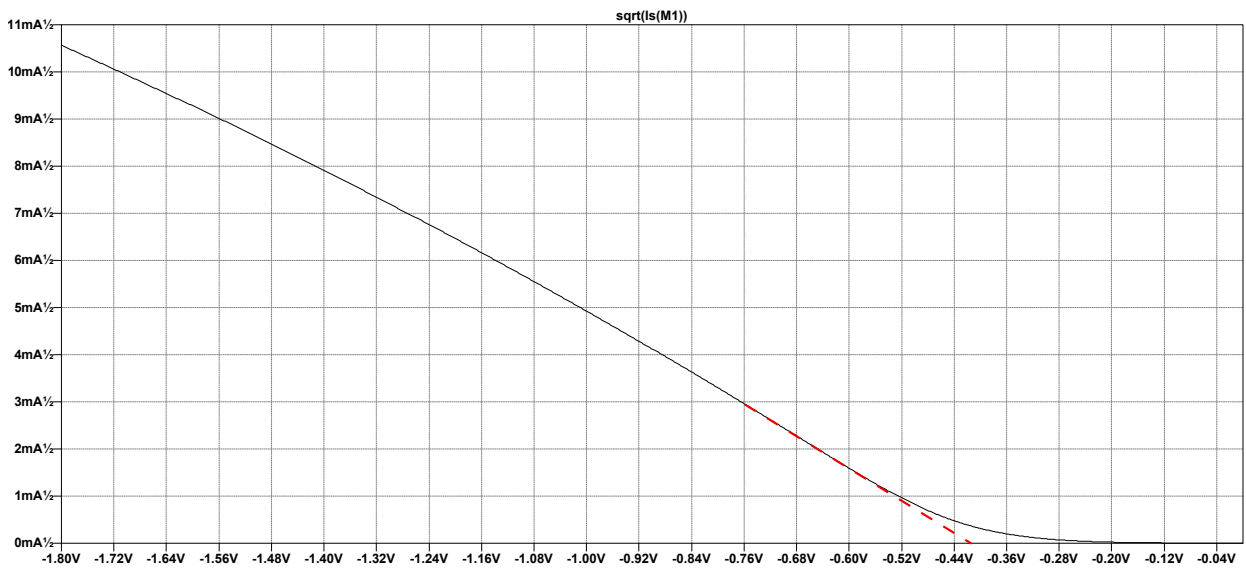


Figure 8: PMOS $\sqrt{I_S} - V_{GS}$

Problem 4

(a) Testbench and I_D - V_{DS} characteristics of NMOS and PMOS

1. NMOS

i. Testbench

```
.lib 'C:\Program Files\LTC\LTspiceXVII\lib\cmp\log018.l' TT
.dc VDS 0 1.8 0.001
```

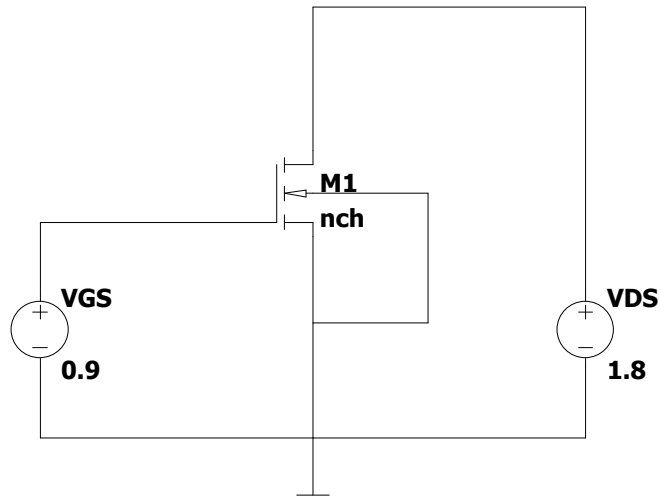


Figure 9: NMOS Testbench

ii. I_D - V_{DS} characteristics

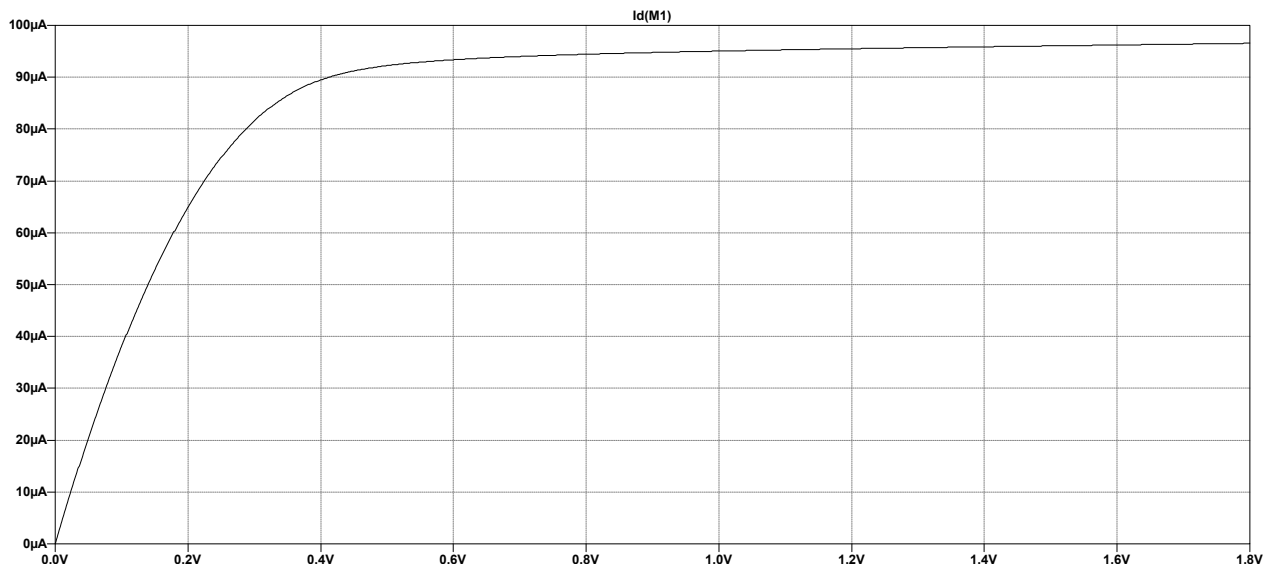


Figure 10: NMOS I_D - V_{DS}

2. PMOS

i. Testbench

.lib 'C:\Program Files\LTC\LTspiceXVII\lib\cmp\log018.l' TT
 .dc VDS -1.8 0 0.001

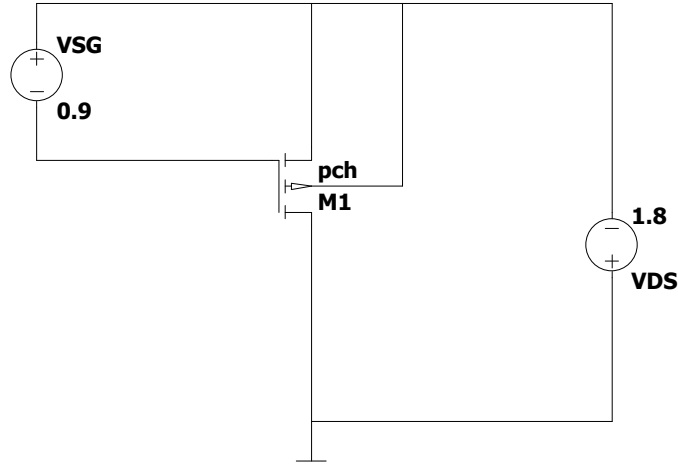


Figure 11: PMOS Testbench

ii. I_S - V_{DS} characteristics

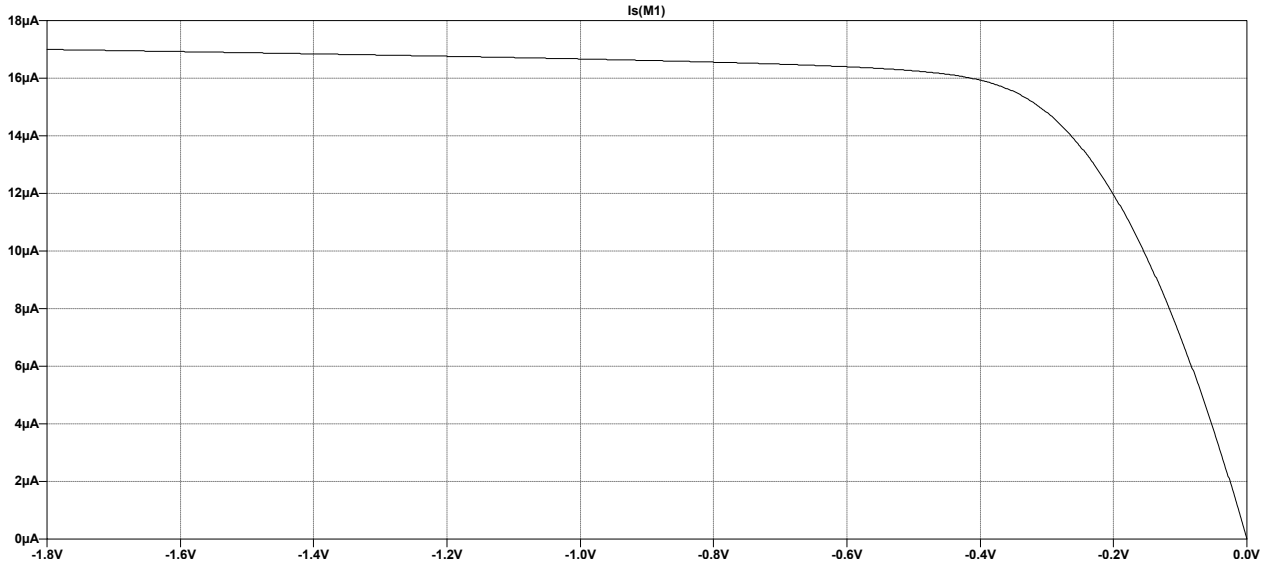


Figure 12: PMOS I_S - V_{DS}

(b) $\lambda_{n(p)}$

Drain current characteristics for NMOS under saturation conditions:

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda_n V_{DS})$$

Differentiating both side with respect to V_{DS} .

$$\begin{aligned} \frac{dI_D}{dV_{DS}} &= \frac{d}{dV_{DS}} \left(\frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda_n V_{DS}) \right) \\ \frac{dI_D}{dV_{DS}} &= \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_{TH})^2 \lambda_n \end{aligned}$$

Assuming that the body-effect is small:

$$I_D \approx \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_{TH})^2$$

$$\frac{dI_D}{dV_{DS}} \approx I_D \lambda_n$$

$$\lambda_n \approx \frac{1}{I_D} \frac{dI_D}{dV_{DS}}$$

In the case of PMOS:

$$\lambda_p \approx \frac{1}{I_S} \frac{dI_S}{dV_{DS}}$$

1. NMOS, $\lambda_n = 0.20V^{-1}$

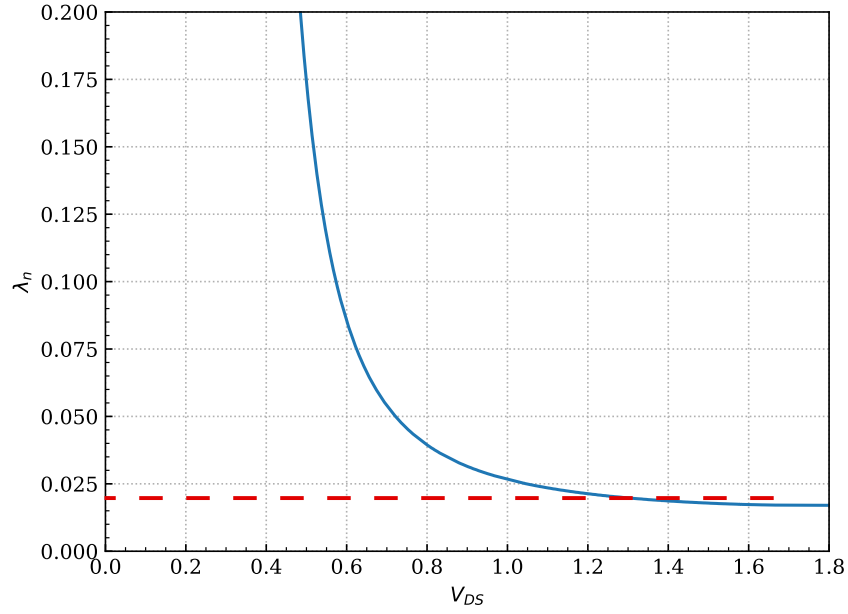


Figure 13: NMOS $\lambda_n - V_{DS}$

2. PMOS, $\lambda_p = -0.025V^{-1}$

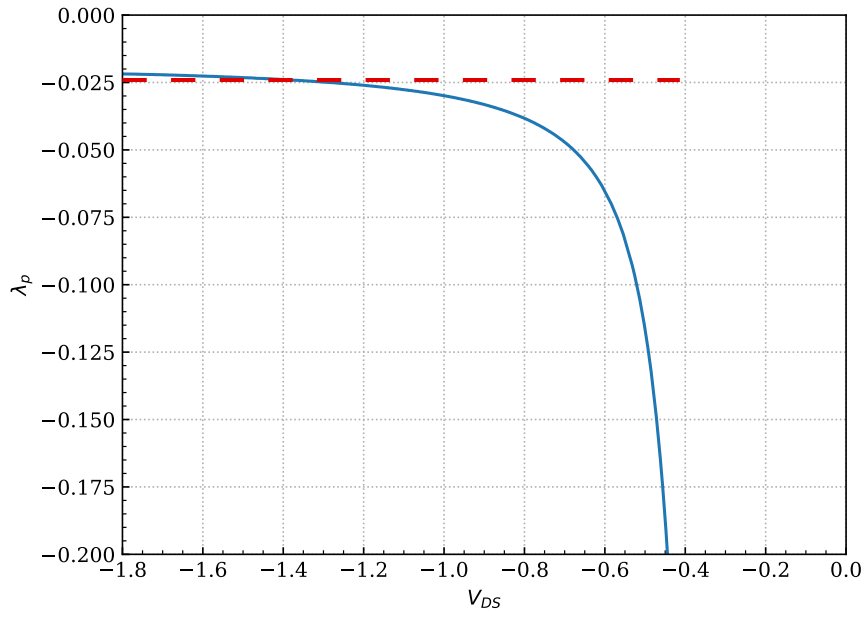


Figure 14: PMOS $\lambda_p - V_{DS}$

Problem 5

g_m for NMOS is approximately:

$$g_m \approx \frac{\partial I_D}{\partial V_{GS}}$$

For PMOS:

$$g_m \approx \frac{\partial I_S}{\partial V_{GS}}$$

(a) $\frac{g_m}{I_D} - V_{GS}$

1. NMOS

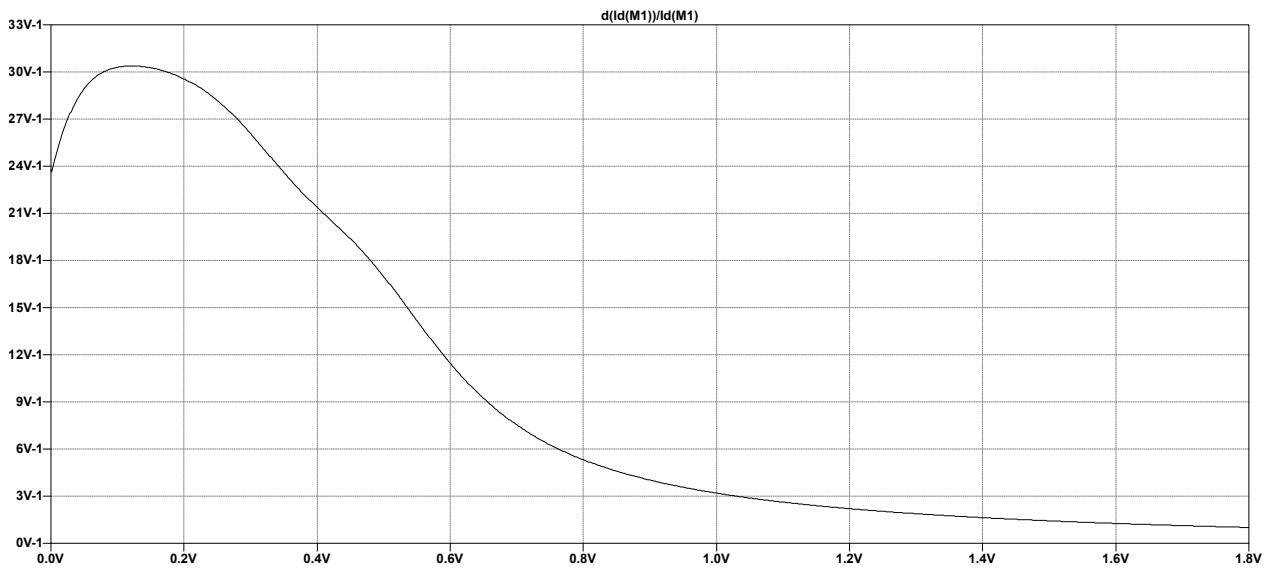


Figure 15: NMOS $\frac{g_m}{I_D} - V_{GS}$

2. PMOS

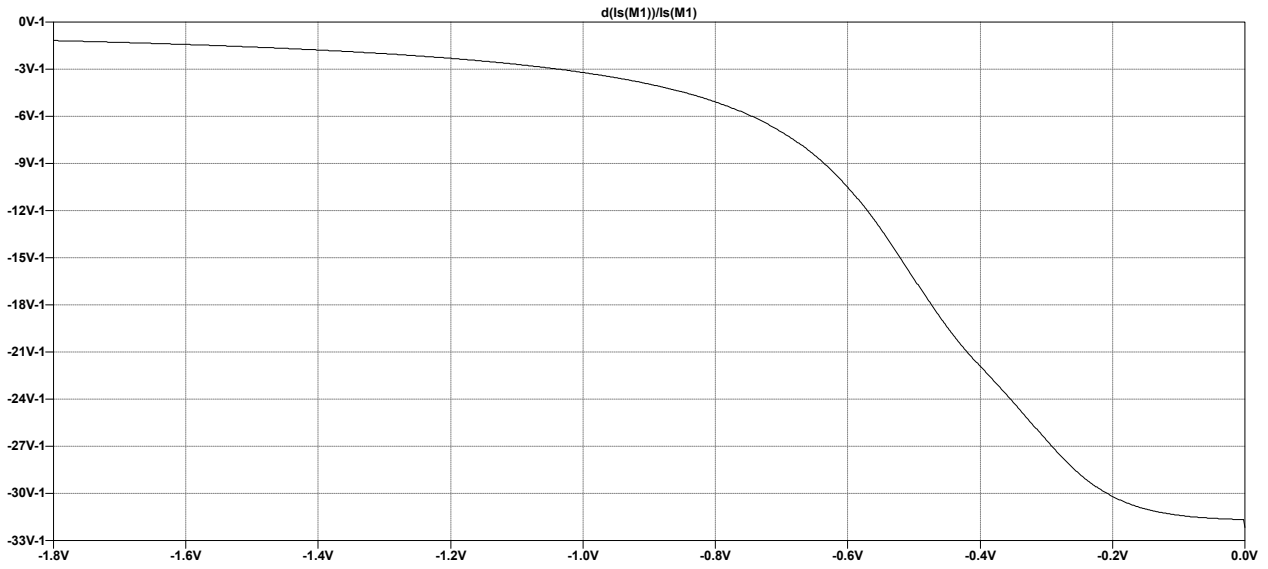


Figure 16: PMOS $\frac{g_m}{I_S} - V_{GS}$

(b) $\max(|\frac{g_m}{I_{D(S)}}|)$

1. NMOS

$$\max(|\frac{g_m}{I_D}|) = 30.4V^{-1}$$

2. PMOS

$$\max(|\frac{g_m}{I_S}|) = 31.7V^{-1}$$

(c) Slope factor, n

1. NMOS

$$\max(|\frac{g_m}{I_D}|) = 30.4V^{-1}$$

$$\begin{aligned} \frac{1}{nV_t} &= 30.4V^{-1} \\ n &= \frac{1}{0.026 \times 30.4} \\ n &= 1.27 \end{aligned}$$

2. PMOS

$$\max(|\frac{g_m}{I_S}|) = 31.7V^{-1}$$

$$\begin{aligned} \frac{1}{nV_t} &= 31.7V^{-1} \\ n &= \frac{1}{0.026 \times 31.7} \\ n &= 1.21 \end{aligned}$$

Problem 6

(a) Small-signal Model

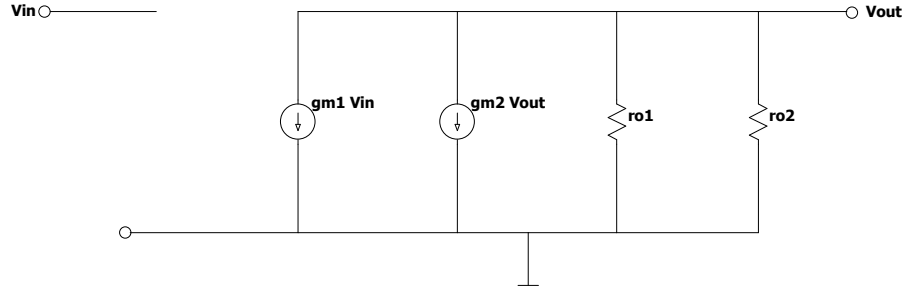


Figure 17: Small signal model

(b) $\lambda = 0V^{-1}$

1. $A_V = \frac{v_{out}}{v_{in}}$

$$(g_{m1}v_{in} + g_{m2}v_{out}) = 0$$

$$A_V = \frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{m2}}$$

2. R_{out}

$$R_{out} = \frac{1}{g_{m2}}$$

(c) $\lambda \neq 0V^{-1}$

1. $A_V = \frac{v_{out}}{v_{in}}$

$$-v_{out} = (g_{m1}v_{in} + g_{m2}v_{out})(r_{o1} // r_{o2})$$

$$-v_{in}g_{m1}(r_{o1} // r_{o2}) = (1 + g_{m2}(r_{o1} // r_{o2}))v_{out}$$

$$A_V = \frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$

2. R_{out}

$$R_{out} = \frac{1}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$

Problem 7

(a) V_{out} - V_{in} relation when:

1. M_1 and M_2 under subthreshold conditions

$$V_{TH_n} = 0.44V$$

$$V_{TH_p} = -0.42V$$

$$\mu_n C_{OX_n} = 306 \mu A V^{-2}$$

$$\mu_p C_{OX_p} = 49 \mu A V^{-2}$$

$$n_n = 1.27$$

$$n_p = 1.21$$

$$I_{D_1} = I_{D_2}$$

$$(\mu_n C_{OX_n} (n-1) \frac{W_n}{L_n} V_T^2) e^{\frac{V_{in} - V_{TH_n}}{n_n V_T}} = (\mu_p C_{OX_p} (n-1) \frac{W_p}{L_p} V_T^2) e^{\frac{V_{DD} - V_{out} + V_{TH_p}}{n_p V_T}}$$

$$247 e^{\frac{V_{in} - 0.44}{0.033}} = 154 e^{\frac{1.8 - V_{out} - 0.42}{0.031}}$$

$$\ln(1.6) + \frac{V_{in} - 0.44}{0.033} = \frac{1.38 - V_{out}}{0.031}$$

$$0.015 + V_{in} - 0.44 \approx 1.38 - V_{out}$$

$$V_{out} \approx 1.8V - V_{in}$$

$$V_{in} < 0.44V$$

2. M_1 and M_2 at saturation

$$I_{D_1} = I_{D_2}$$

$$\frac{\mu_n C_{OX_n}}{2} \frac{W_n}{L_n} (V_{GS_1} - V_{TH_n})^2 = \frac{\mu_p C_{OX_p}}{2} \frac{W_p}{L_p} (V_{SG_2} + V_{TH_p})^2$$

$$918(V_{in} - 0.44)^2 = 735(1.8 - V_{out} - 0.42)^2$$

$$1.12(V_{in} - 0.44) = 1.38 - V_{out}$$

$$V_{out} = 1.87V - 1.12V_{in}$$

Saturation conditions for M_1 :

$$V_{GS_1} - V_{TH_1} < V_{DS}$$

$$V_{in} - 0.44V < V_{out}$$

$$V_{in} - 0.44V < 1.87V - 1.12V_{in}$$

$$2.12V_{in} < 2.31V$$

$$V_{in} < 1.09V$$

Condition for M_1 and M_2 at saturation:

$$0.44V < V_{in} < 1.09V$$

3. M_1 at triode and M_2 at saturation

$$I_{D_1} = I_{D_2}$$

$$\mu_n C_{OX_n} \frac{W_n}{L_n} [(V_{GS_1} - V_{TH_n}) V_{DS} - \frac{V_{DS}^2}{2}] = \frac{\mu_p C_{OX_p}}{2} \frac{W_p}{L_p} (V_{SG_2} + V_{TH_p})^2$$

$$2.50[(V_{in} - 0.44V) V_{out} - \frac{V_{out}^2}{2}] = (V_{DD} - V_{out} - 0.42V)^2$$

$$V_{out} = \frac{(2.5V_{in} + 1.66) - \sqrt{(2.5V_{in} + 1.66)^2 - 17.1}}{4.5}$$

Condition:

$$V_{in} > 1.09V$$

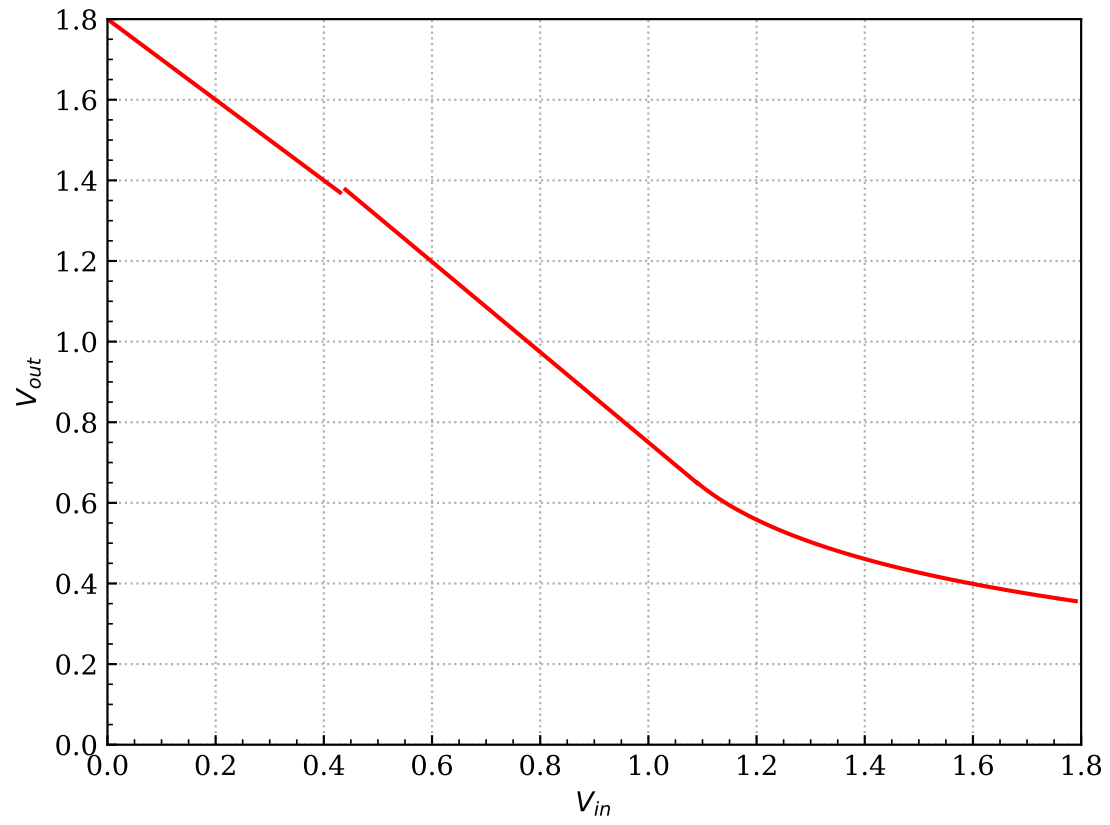


Figure 18: Calculated $V_{out} - V_{in}$

(b) Simulated V_{out} - V_{in} relation using LTSpice

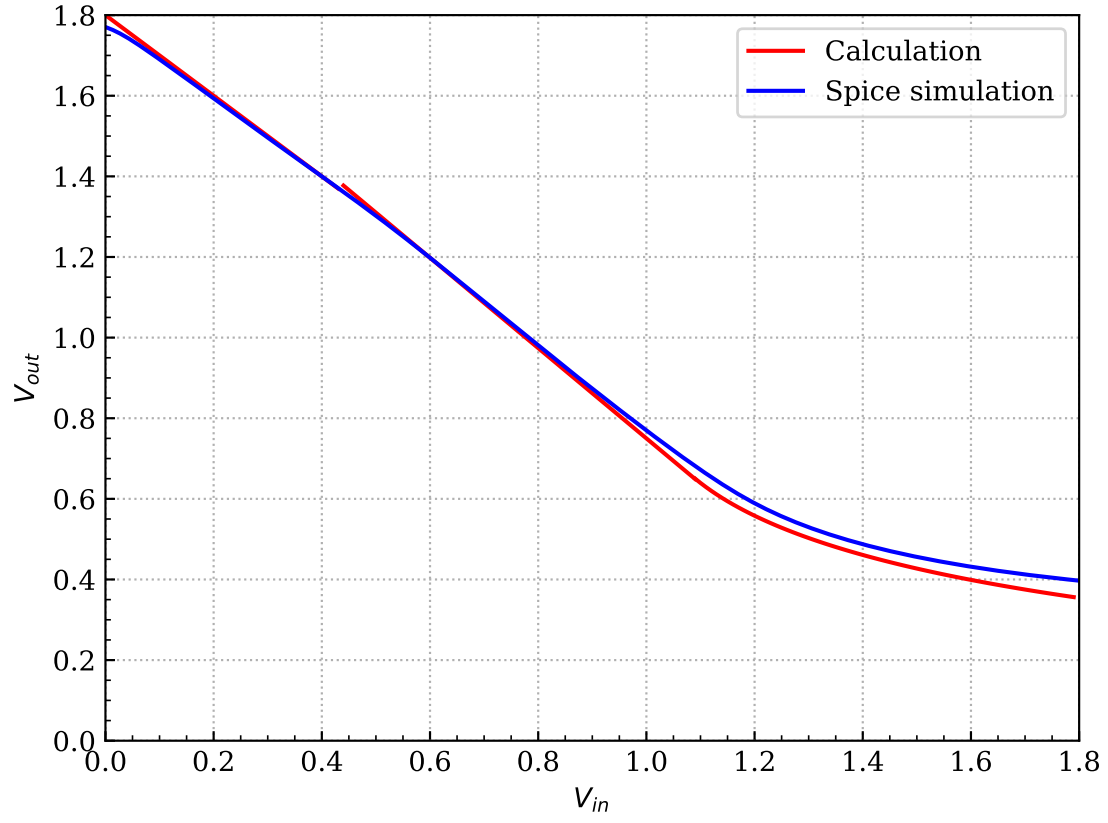


Figure 19: Simulated and calculated $V_{out} - V_{in}$

(c) Maximum small-signal gain For maximum small-signal gain:

$$\max(|A_V|) = \max\left(\left|\frac{\partial V_{out}}{\partial V_{in}}\right|\right)$$

$$V_{in} \approx 0.69V$$

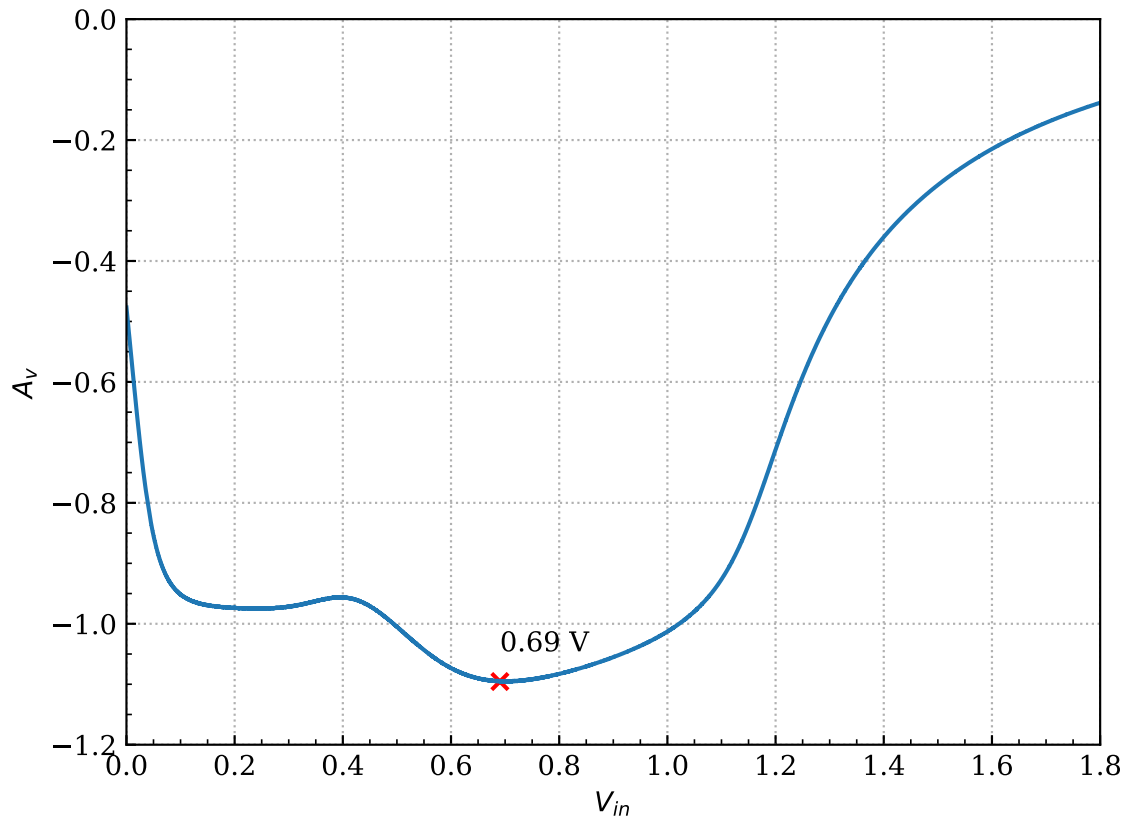


Figure 20: $A_v - V_{in}$

(d) Small signal parameters:

1. $|A_v| = 787.83 \text{ mdB} = 1.09$

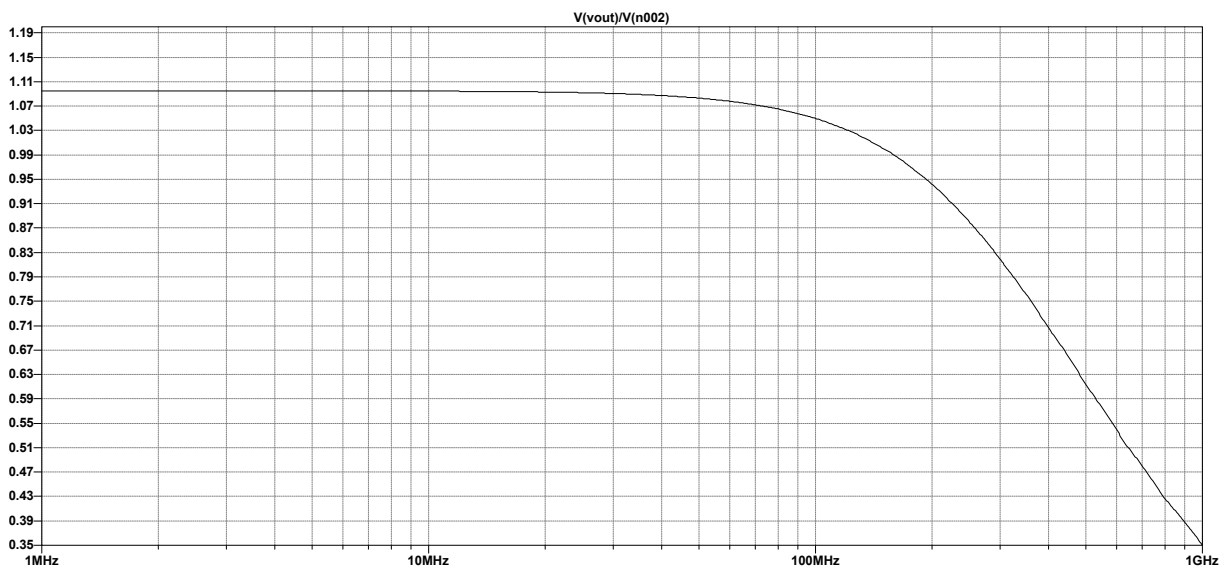


Figure 21: A_v at $V_{in} = 0.69V$

2. $R_{out} = 4.73k\Omega$

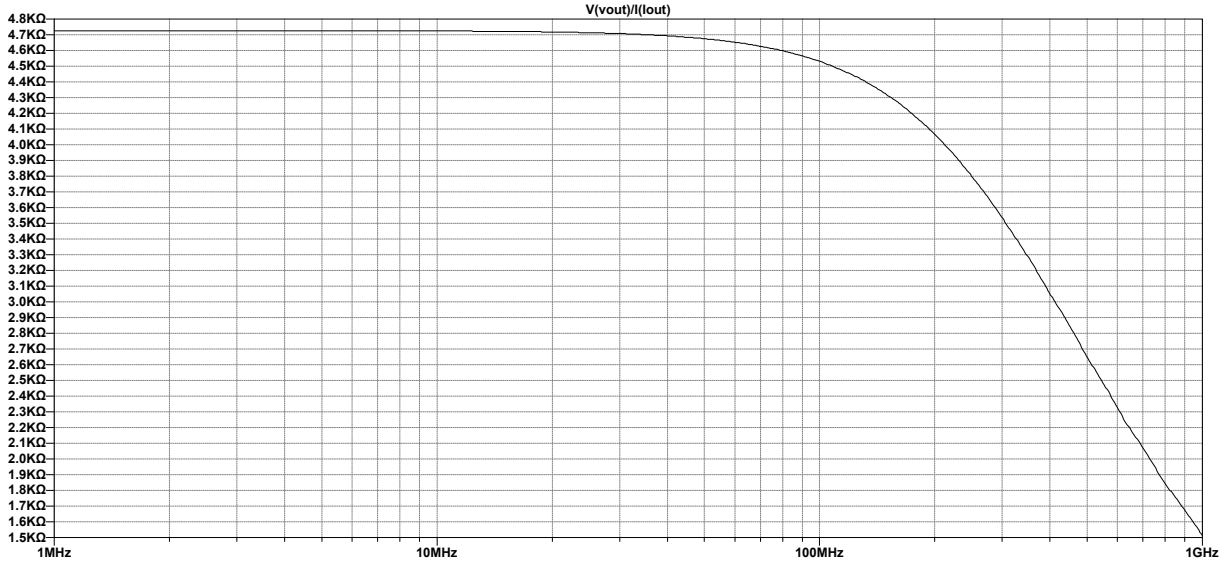


Figure 22: R_{out} at $V_{in} = 0.69V$

(e) g_m and g_{ds}

From 7(d),

$$A_V = 1.09$$

$$R_{out} = 4.73k\Omega$$

From 6(c),

$$g_{m_1} = 2.32e-04\Omega^{-1}$$

$$g_{m_2} = 2.09e-04\Omega^{-1}$$

$$g_{DS_1} = \frac{1}{r_{o_1}} = 8.66e-07\Omega^{-1}$$

$$g_{DS_2} = \frac{1}{r_{o_2}} = 1.28e-06\Omega^{-1}$$

$$A_V = \frac{v_{out}}{v_{in}} = -\frac{g_{m_1}}{g_{m_2} + \frac{1}{r_{o_1}} + \frac{1}{r_{o_2}}}$$

$$\approx 1.10$$

$$R_{out} = \frac{1}{g_{m_2} + \frac{1}{r_{o_1}} + \frac{1}{r_{o_2}}}$$

$$R_{out} = 4.74k\Omega$$