EE4C10 Analog Circuit Design Fundamentals

Homework Assignment I

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Contents

Problem 1

For $I_D = 40\mu A$:

$$\begin{aligned} I_D &= \frac{1.8V - V_D}{R} \\ V_D &= 1.8V - I_D R \\ V_D &= 1.0V \end{aligned}$$

Saturation region:

$$\begin{aligned} V_{GS} = 1.0V > V_{TH} \\ V_{GS} - V_{TH} = 0.4V < V_{DS} \end{aligned}$$

1. $\lambda = 0V^{-1}$

$$\begin{aligned} I_D &= \frac{\mu_n C_{OX}}{2}\frac{W}{L}(V_{GS} - V_{TH})^2 \\ L &= \frac{\mu_n C_{OX}}{2}\frac{W}{I_D}(V_{GS} - V_{TH})^2 \\ L &= 0.39\mu m \end{aligned}$$

2. $\lambda = 0.06V^{-1}$

$$\begin{aligned} I_D &= \frac{\mu_n C_{OX}}{2}\frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS}) \\ L &= \frac{\mu_n C_{OX}}{2}\frac{W}{I_D}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS}) \\ L &= 0.41\mu m \end{aligned}$$

Problem 2

1. Bulk of the transistors are connected to the source, $V_B = V_S$

$$\begin{aligned} V_{TH} &= V_{TH0} + \gamma(\sqrt{|2\varphi_F| + V_{BS}} - \sqrt{|2\varphi_F|}) \\ V_{TH} &= V_{TH0} = 0.33V \end{aligned}$$

(a) Transistor M_1

$$V_{SG} = 2.5V - 1.7V = 0.8V$$

$$I_D = \frac{\mu_p C_{OX}}{2}\frac{W}{L}(V_{SG} - V_{TH})^2$$
$$W = \frac{2LI_D}{\mu_p C_{OX}}\frac{1}{(V_{SG} - V_{TH})^2}$$
$$W_1 = 2.72\mu m$$

(b) Transistor M_2

$$V_{SG} = 1.7V - 1V = 0.7V$$

$$W = \frac{2LI_D}{\mu_p C_{OX}}\frac{1}{(V_{SG} - V_{TH})^2}$$
$$W_2 = 4.38\mu m$$

(c) Transistor M_3

$$V_{SG} = 1V$$

$$W = \frac{2LI_D}{\mu_p C_{OX}}\frac{1}{(V_{SG} - V_{TH})^2}$$
$$W_3 = 1.37\mu m$$

2. Bulk terminals are attached to the V_{DD}, $V_B = V_{DD}$.

(a) Transistor M_1

$$V_{BS} = 2.5V - 2.5V = 0V$$

$$V_{TH} = V_{TH0} + \gamma(\sqrt{|2\varphi_F| + V_{BS}} - \sqrt{|2\varphi_F|})$$
$$V_{TH} = V_{TH0} = 0.33V$$

$$W = \frac{2LI_D}{\mu_p C_{OX}}\frac{1}{(V_{SG} - V_{TH})^2}$$
$$W_1 = 2.72\mu m$$

(b) Transistor M_2

$$V_{BS} = 2.5V - 1.7V = 0.8V$$

$$V_{TH} = V_{TH0} + \gamma(\sqrt{|2\varphi_F| + V_{BS}} - \sqrt{|2\varphi_F|})$$
$$V_{TH} = V_{TH0} = 0.43V$$

$$W = \frac{2LI_D}{\mu_p C_{OX}}\frac{1}{(V_{SG} - V_{TH})^2}$$
$$W_2 = 8.23\mu m$$

(c) Transistor M_3

$$V_{BS} = 2.5V - 1.0V = 1.5V$$

$$V_{TH} = V_{TH0} + \gamma(\sqrt{|2\varphi_F| + V_{BS}} - \sqrt{|2\varphi_F|})$$
$$V_{TH} = V_{TH0} = 0.49V$$

$$W = \frac{2LI_D}{\mu_p C_{OX}}\frac{1}{(V_{SG} - V_{TH})^2}$$
$$W_3 = 2.31\mu m$$

Problem 3

1. Testbench and I_D-V_{GS} characteristics of NMOS and PMOS

   (a) NMOS

      i. Testbench

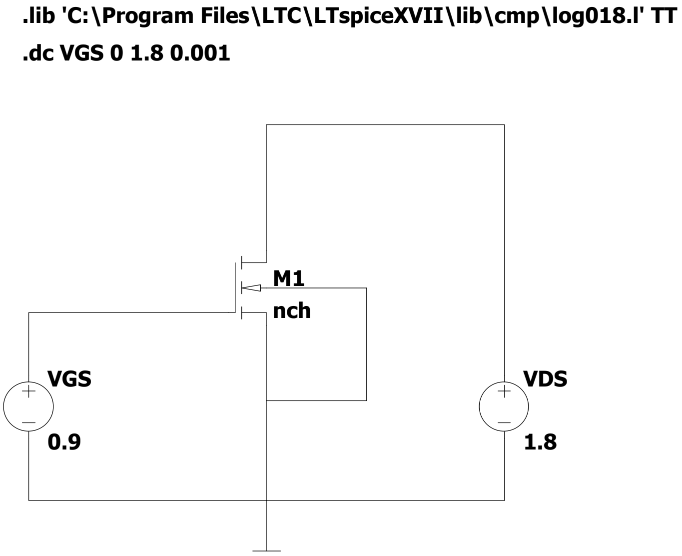

Figure 1: NMOS Testbench

      ii. I_D-V_{GS}

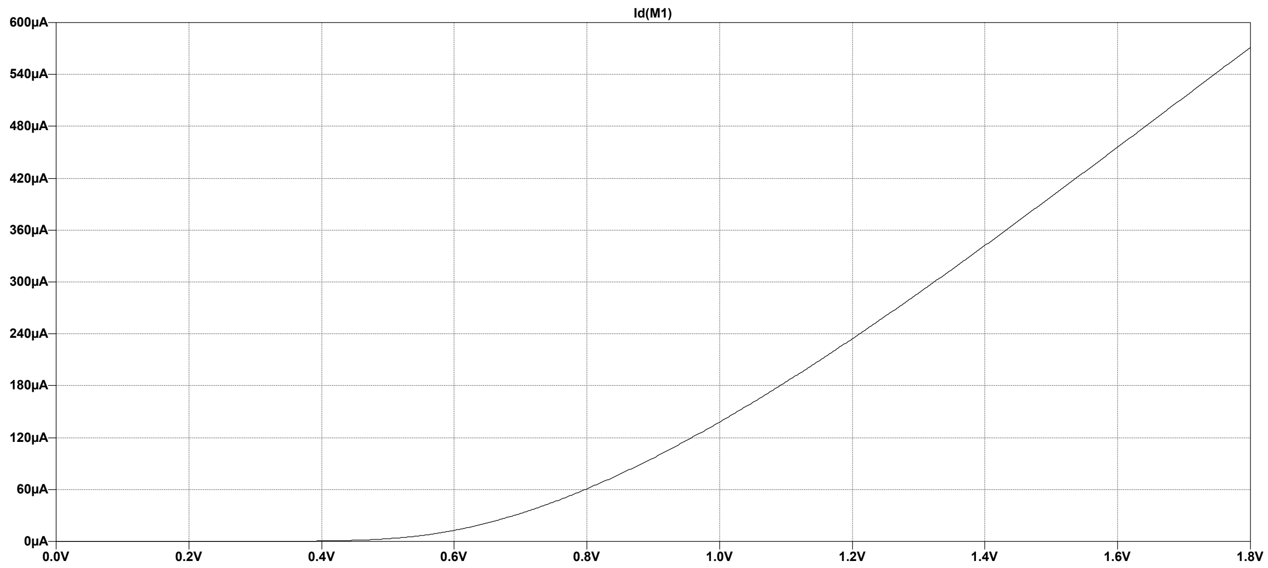

Figure 2: NMOS I_D-V_{GS}

   (b) PMOS

      i. Testbench

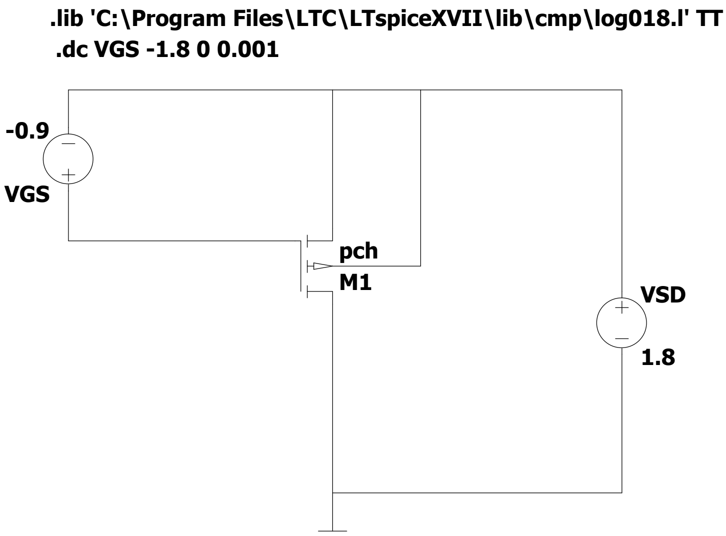

Figure 3: PMOS Testbench

ii. I_S-V_{GS}

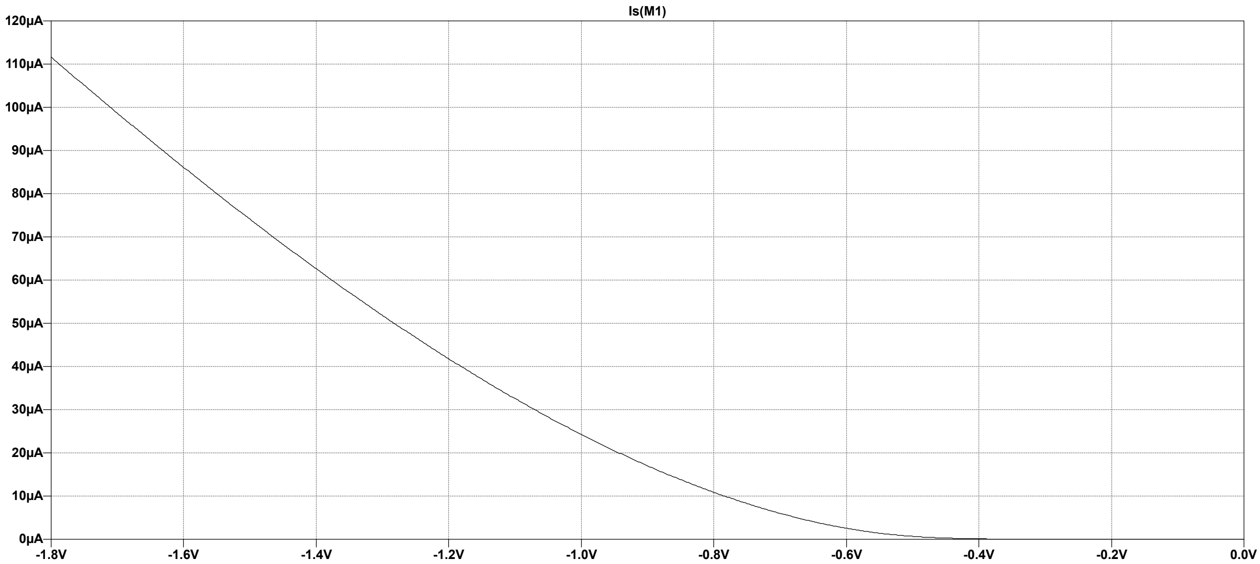

Figure 4: PMOS I_S-V_{GS}

2. $\mu_{n(p)}C_{OX}$ and $V_{THn(p)}$

Assuming that channel length modulation is negligible, V_{THn} for NMOS can be derived from the following relation:

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_{THn})^2$$
$$\frac{2I_D}{\mu_n C_{ox}}\frac{L}{W} = (V_{GS} - V_{THn})^2$$
$$\sqrt{\frac{2I_D}{\mu_n C_{ox}}\frac{L}{W}} = V_{GS} - V_{THn}$$

V_{THn} is the x-axis intercept when the saturation region is extrapolated. In the case of PMOS, the relation becomes:

$$\sqrt{\frac{2I_S}{\mu_p C_{ox}}\frac{L}{W}} = V_{SG} + V_{THp}$$

For deriving $\mu_n C_{OX}$, since $V_{THn(p)}$ is constant at specific temperatures. Differentiating both sides with respect to

$V_{GS(SG)}$ will give:

$$\frac{d}{dV_{GS}}\sqrt{\frac{2I_D}{\mu_n C_{ox}}\frac{L}{W}} = \frac{d}{dV_{GS}}(V_{GS} - V_{THn})$$

$$\frac{1}{2}\frac{dI_D}{dV_{GS}}\sqrt{\frac{2}{I_D\mu_n C_{ox}}\frac{L}{W}} = 1$$

$$\sqrt{\mu_n C_{ox}} = \frac{1}{2}\frac{dI_D}{dV_{GS}}\sqrt{\frac{2}{I_D}\frac{L}{W}}$$

$$\mu_n C_{ox} = \frac{1}{2}\frac{L}{W}\frac{1}{I_D}(\frac{dI_D}{dV_{GS}})^2$$

$$\mu_n C_{ox} = \frac{1}{6I_D}(\frac{dI_D}{dV_{GS}})^2$$

In the case for PMOS, the relation becomes:

$$\mu_p C_{ox} = \frac{1}{6I_S}(\frac{dI_S}{dV_{SG}})^2$$

(a) NMOS

i. $\mu_n C_{OX}$

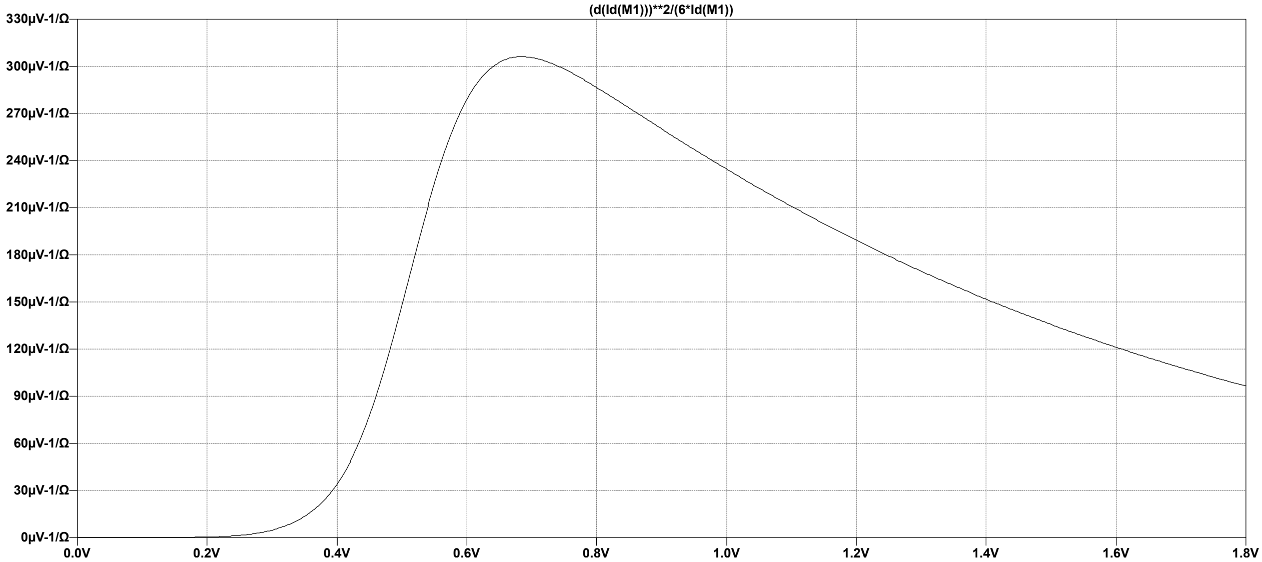

Figure 5: NMOS $\mu_n C_{OX}$-V_{GS}

ii. $V_{THn} = 0.44V$

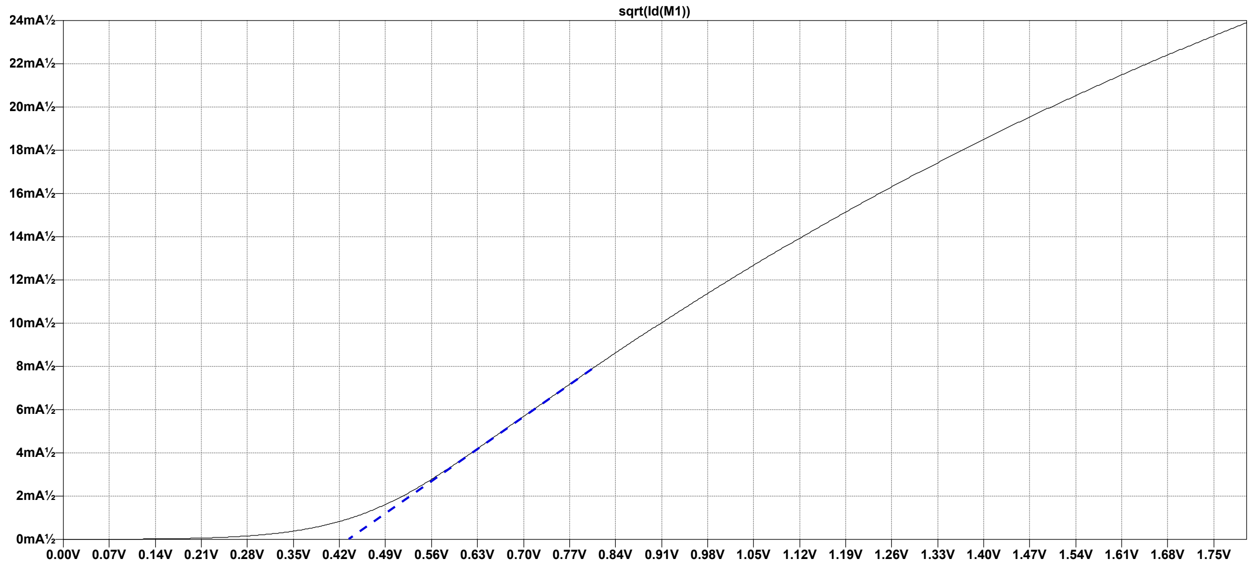

Figure 6: NMOS $\sqrt{I_D} - V_{GS}$

(b) PMOS

i. $\mu_p C_{OX}$

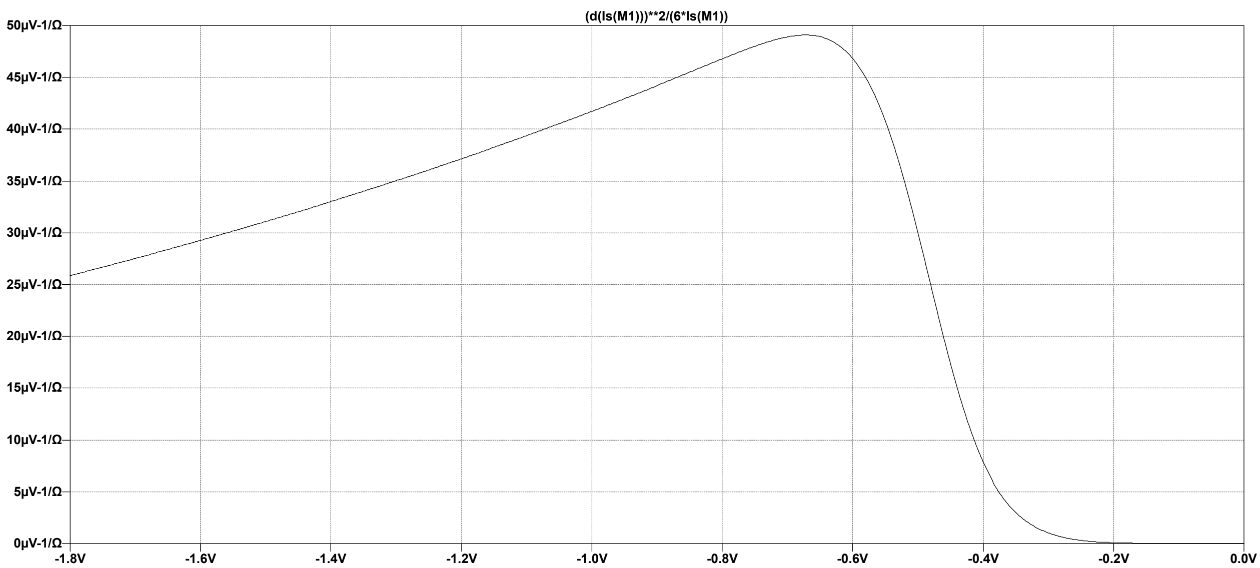

Figure 7: PMOS $\mathrm{\mu_p C_{OX}}$-$\mathrm{V_{GS}}$

ii. $V_{THp} = -0.42V$

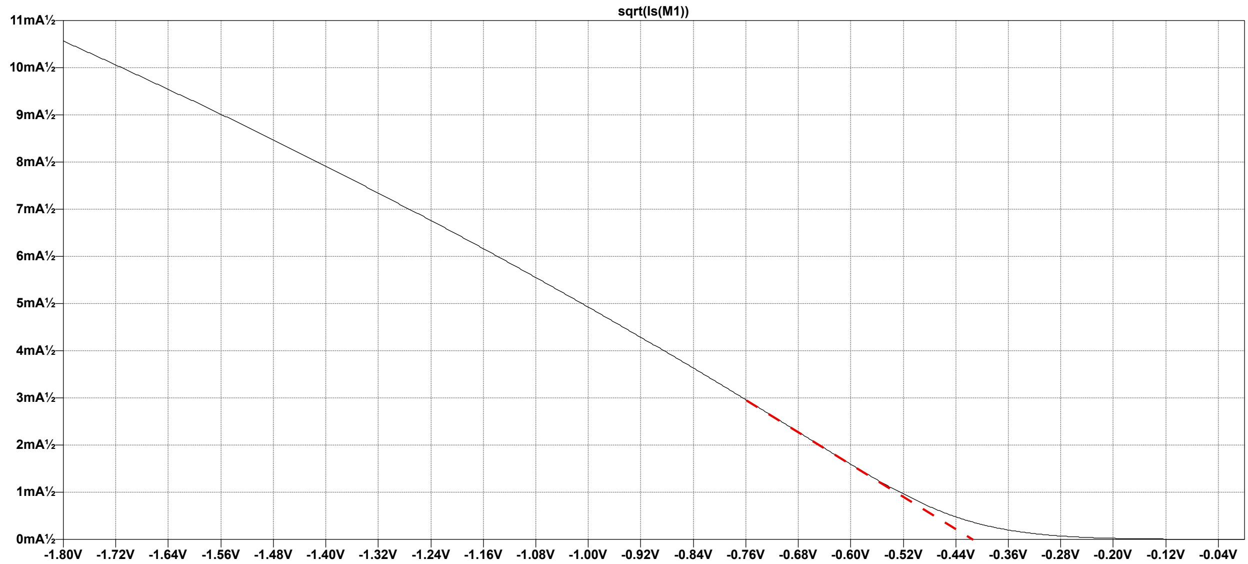

Figure 8: PMOS $\sqrt{I_S} - V_{GS}$

Problem 4

1. Testbench and I_D-V_{DS} characteristics of NMOS and PMOS
   (a) NMOS
      i. Testbench

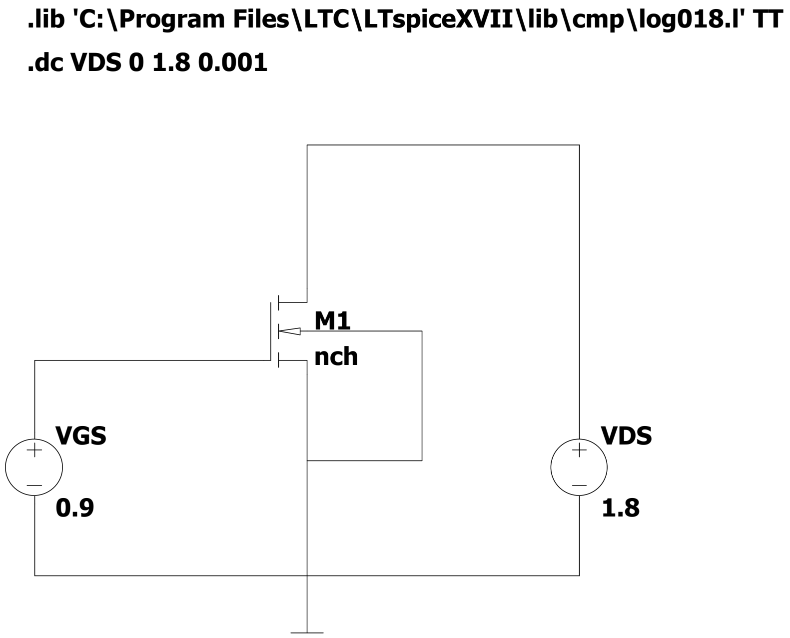

Figure 9: NMOS Testbench

      ii. I_D-V_{DS} characteristics

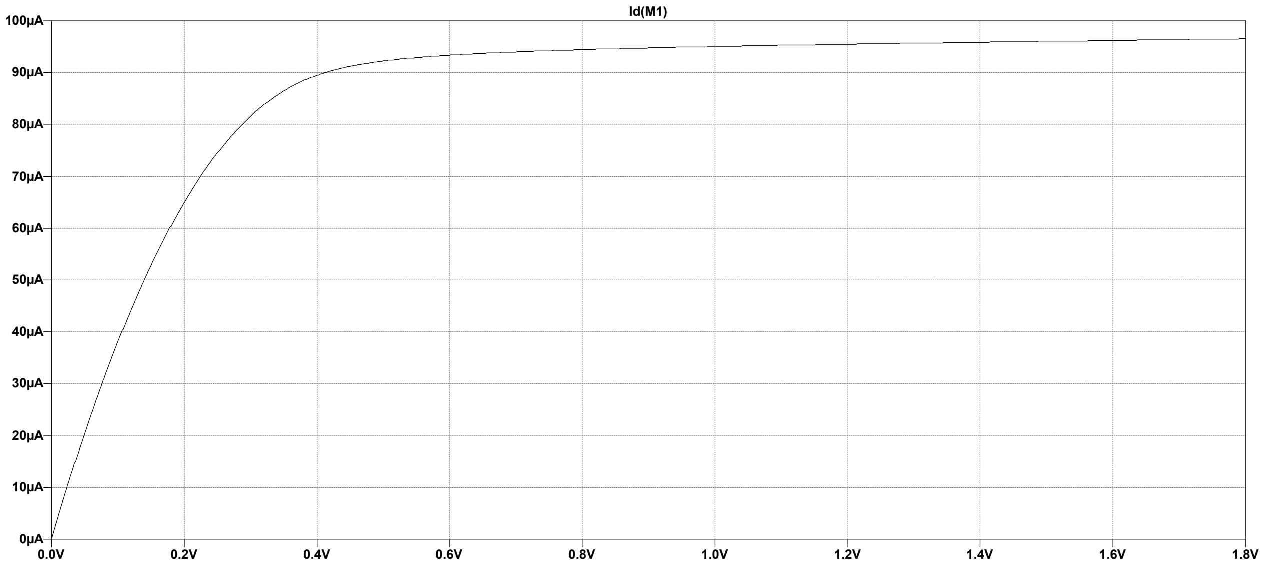

Figure 10: NMOS I_D-V_{DS}

(b) PMOS

i. Testbench

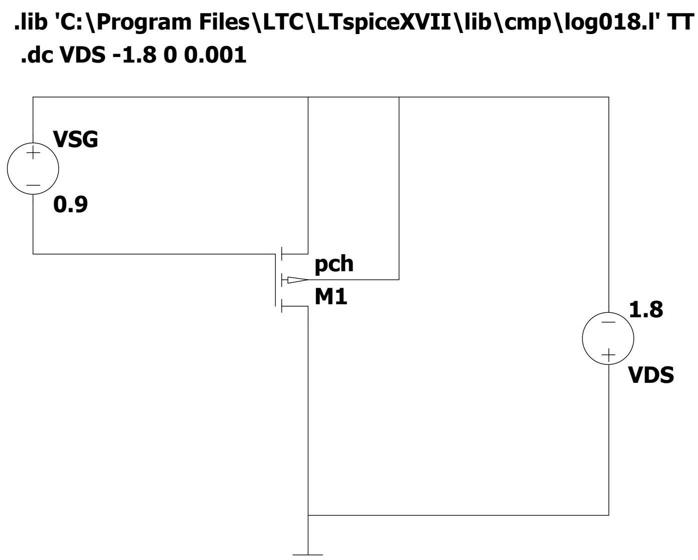

Figure 11: PMOS Testbench

ii. I_S-V_{DS} characteristics

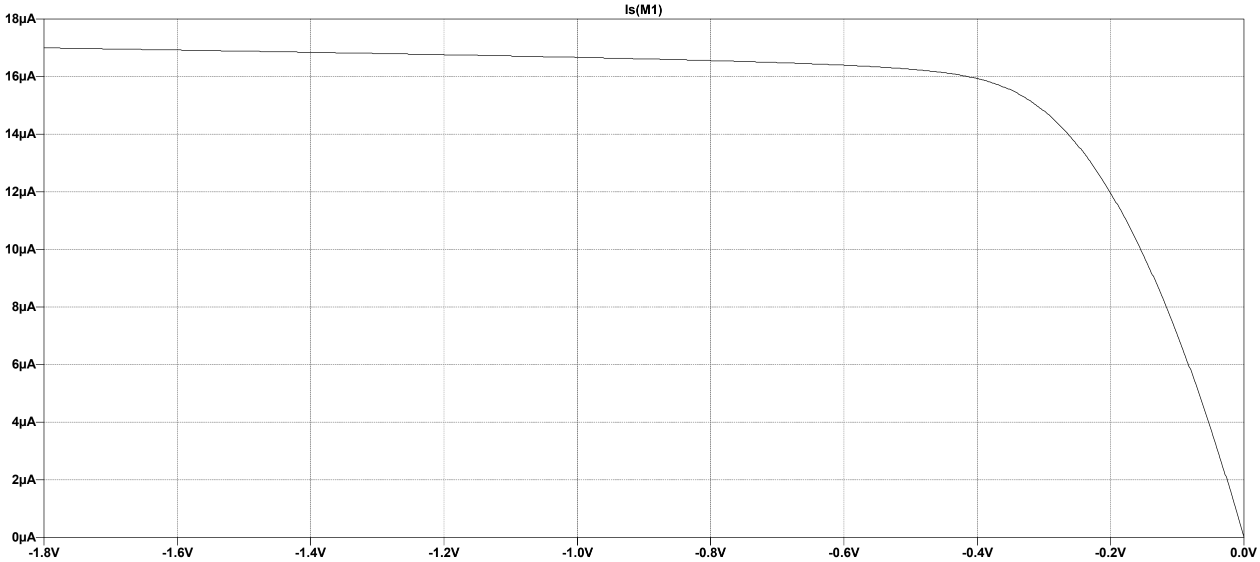

Figure 12: PMOS I_S-V_{DS}

2. $\lambda_{n(p)}$

   Drain current characteristics for NMOS under saturation conditions:

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda_n V_{DS})$$

   Differentiating both side with respect to V_{DS}.

$$\begin{aligned}\frac{dI_D}{dV_{DS}} &= \frac{d}{dV_{DS}}(\frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda_n V_{DS})) \\ \frac{dI_D}{dV_{DS}} &= \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_{TH})^2\lambda_n\end{aligned}$$

   Assuming that the body-effect is small:

$$I_D \approx \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_{TH})^2$$

$$\begin{aligned}\frac{dI_D}{dV_{DS}} &\approx I_D\lambda_n \\ \lambda_n &\approx \frac{1}{I_D}\frac{dI_D}{dV_{DS}}\end{aligned}$$

   In the case of PMOS:

$$\lambda_p \approx \frac{1}{I_S}\frac{dI_S}{dV_{SD}}$$

   (a) $\lambda_n = 0.18V^{-1}$

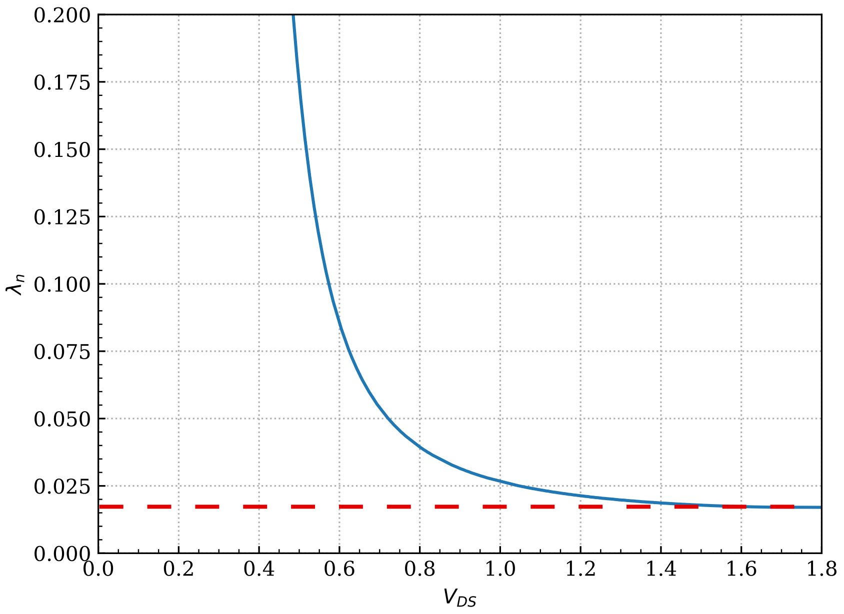

Figure 13: NMOS $\lambda_n - V_{DS}$

(b) $\lambda_p = -0.022V^{-1}$

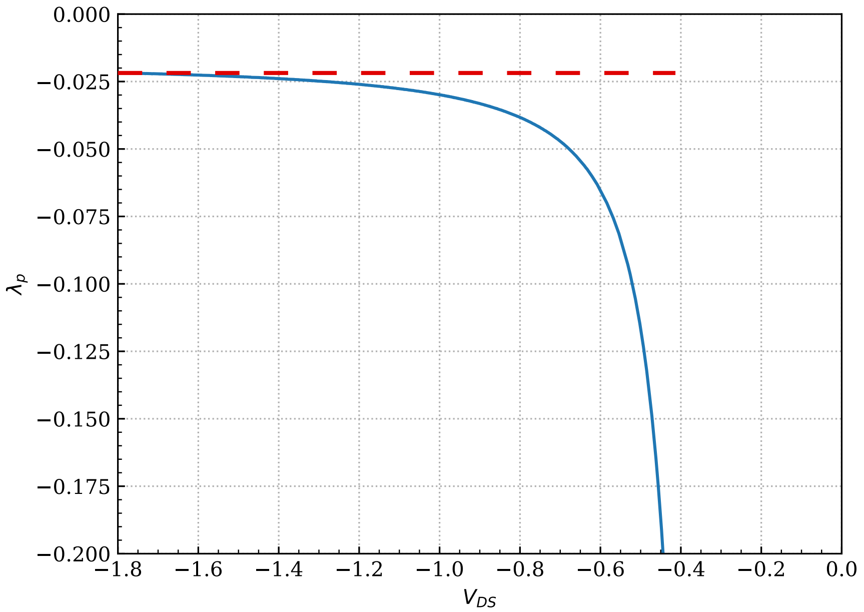

Figure 14: PMOS $\lambda_p - V_{DS}$

Problem 5

g_m for NMOS is approximately:

$$g_m \approx \frac{\partial I_D}{\partial V_{GS}}$$

For PMOS:

$$g_m \approx \frac{\partial I_S}{\partial V_{SG}}$$

1. $\frac{g_m}{I_D} - V_{GS}$

   (a) NMOS

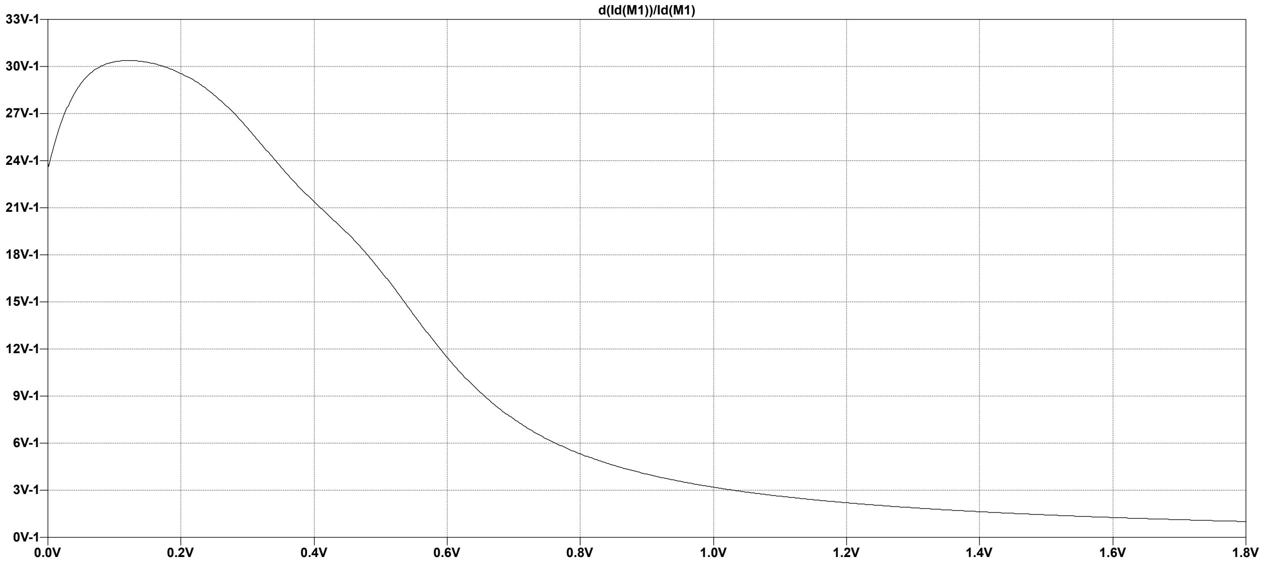

Figure 15: NMOS $\frac{g_m}{I_D} - V_{GS}$

   (b) PMOS

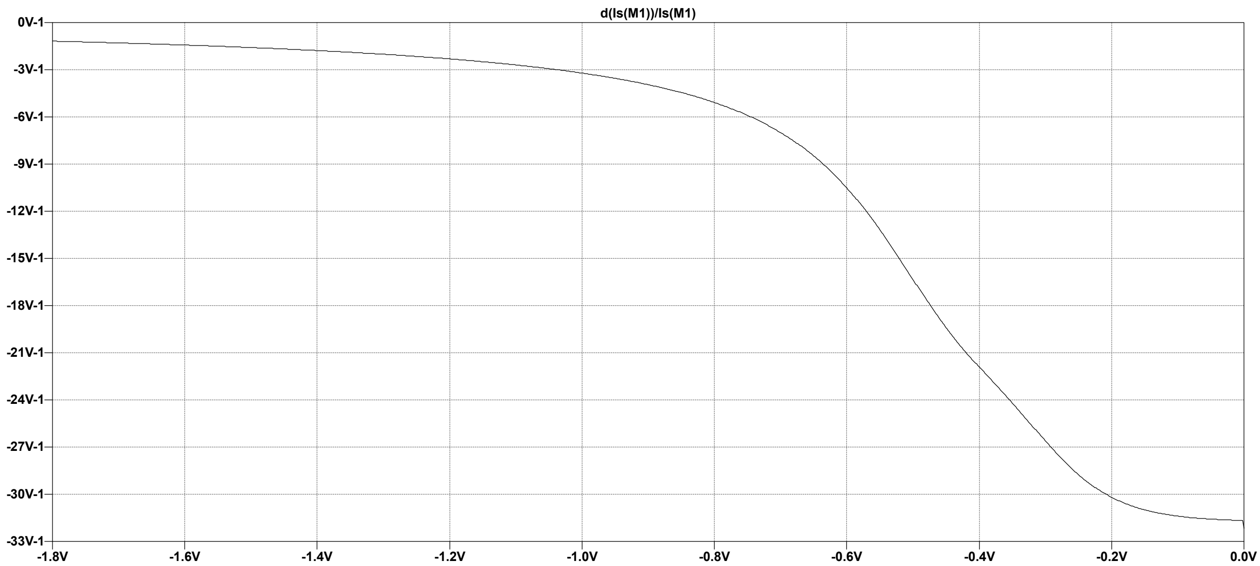

Figure 16: PMOS $\frac{g_m}{I_S} - V_{GS}$

2. $max(|\frac{g_m}{I_{D(S)}}|)$

   (a) NMOS
   $max(|\frac{g_m}{I_D}|) = 30.4V^{-1}$

   (b) PMOS
   $max(|\frac{g_m}{I_S}|) = 31.7V^{-1}$

3. Slope factor, n

(a) NMOS

$$max(|\frac{g_m}{I_D}|) = 30.4V^{-1}$$

$$\frac{1}{nV_t} = 30.4V^{-1}$$
$$n = \frac{1}{0.026 \times 30.4}$$
$$n = 1.27$$

(b) PMOS

$$max(|\frac{g_m}{I_S}|) = 31.7V^{-1}$$

$$\frac{1}{nV_t} = 31.7V^{-1}$$
$$n = \frac{1}{0.026 \times 31.7}$$
$$n = 1.21$$

Problem 6

1. Small-signal Model

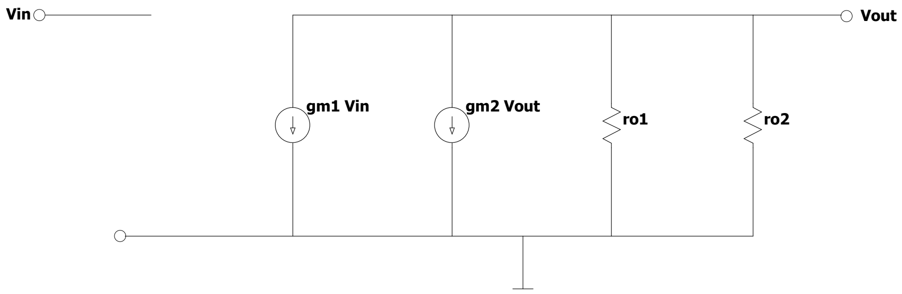

Figure 17: Small signal model

2. $\lambda = 0V^{-1}$

(a) $A_V = \frac{v_{out}}{v_{in}}$

$$(g_{m1}v_{in} + g_{m2}v_{out}) = 0$$
$$A_V = \frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{m2}}$$

(b) R_{out}

$$R_{out} = \frac{1}{g_{m2}}$$

3. $\lambda \neq 0V^{-1}$

(a) $A_V = \frac{v_{out}}{v_{in}}$

$$-v_{out} = (g_{m1}v_{in} + g_{m2}v_{out})(r_{o1}//r_{o2})$$
$$-v_{in}g_{m1}(r_{o1}//r_{o2}) = (1 + g_{m2}(r_{o1}//r_{o2}))v_{out}$$
$$A_V = \frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$

(b) R_{out}

$$R_{out} = \frac{1}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$

Problem 7

1. V_{out}-V_{in} relation when:

   (a) M_1 and M_2 under subthreshold conditions

$$\begin{aligned} V_{TH_n} &= 0.44V \\ V_{TH_p} &= -0.42V \end{aligned}$$

$$\begin{aligned} \mu_n C_{OX_n} &= 306\mu AV^{-2} \\ \mu_p C_{OX_p} &= 49\mu AV^{-2} \end{aligned}$$

$$\begin{aligned} n_n &= 1.27 \\ n_p &= 1.21 \end{aligned}$$

$$\begin{aligned} I_{D_1} &= I_{D_2} \\ (\mu_n C_{OX_n}(n-1)\frac{W_n}{L_n}V_T^2)e^{\frac{V_{in}-V_{TH_n}}{n_n V_T}} &= (\mu_p C_{OX_p}(n-1)\frac{W_p}{L_p}V_T^2)e^{\frac{V_{DD}-V_{out}+V_{TH_p}}{n_p V_T}} \\ 247e^{\frac{V_{in}-0.44}{0.033}} &= 154e^{\frac{1.8-V_{out}-0.42}{0.031}} \\ ln(1.6) + \frac{V_{in}-0.44}{0.033} &= \frac{1.38-V_{out}}{0.031} \\ 0.015 + V_{in} - 0.44 &\approx 1.38 - V_{out} \\ V_{out} &\approx 1.8V - V_{in} \end{aligned}$$

$$V_{in} < 0.44V$$

   (b) M_1 and M_2 at saturation

$$\begin{aligned} I_{D_1} &= I_{D_2} \\ \frac{\mu_n C_{OX_n}}{2}\frac{W_n}{L_n}(V_{GS_1} - V_{TH_n})^2 &= \frac{\mu_p C_{OX_p}}{2}\frac{W_p}{L_p}(V_{SG_2} + V_{TH_p})^2 \\ 918(V_{in} - 0.44)^2 &= 735(1.8 - V_{out} - 0.42)^2 \\ 1.12(V_{in} - 0.44) &= 1.38 - V_{out} \\ V_{out} &= 1.87V - 1.12V_{in} \end{aligned}$$

   Saturation conditions for M_1:

$$\begin{aligned} V_{GS_1} - V_{TH_1} &< V_{DS} \\ V_{in} - 0.44V &< V_{out} \\ V_{in} - 0.44V &< 1.87V - 1.12V_{in} \\ 2.12V_{in} &< 2.31V \\ V_{in} &< 1.09V \end{aligned}$$

   Condition for M_1 and M_2 at saturation:

$$0.44V < V_{in} < 1.09V$$

   (c) M_1 at triode and M_2 at saturation

$$\begin{aligned} I_{D_1} &= I_{D_2} \\ \mu_n C_{OX_n}\frac{W_n}{L_n}[(V_{GS_1} - V_{TH_n})V_{DS} - \frac{V_{DS}^2}{2}] &= \frac{\mu_p C_{OX_p}}{2}\frac{W_p}{L_p}(V_{SG_2} + V_{TH_p})^2 \\ 2.50[(V_{in} - 0.44V)V_{out} - \frac{V_{out}^2}{2}] &= (V_{DD} - V_{out} - 0.42V)^2 \end{aligned}$$

$$V_{out} = \frac{(2.5V_{in} + 1.66) - \sqrt{(2.5V_{in} + 1.66)^2 - 17.1}}{4.5}$$

Condition:

$$V_{in} > 1.09V$$

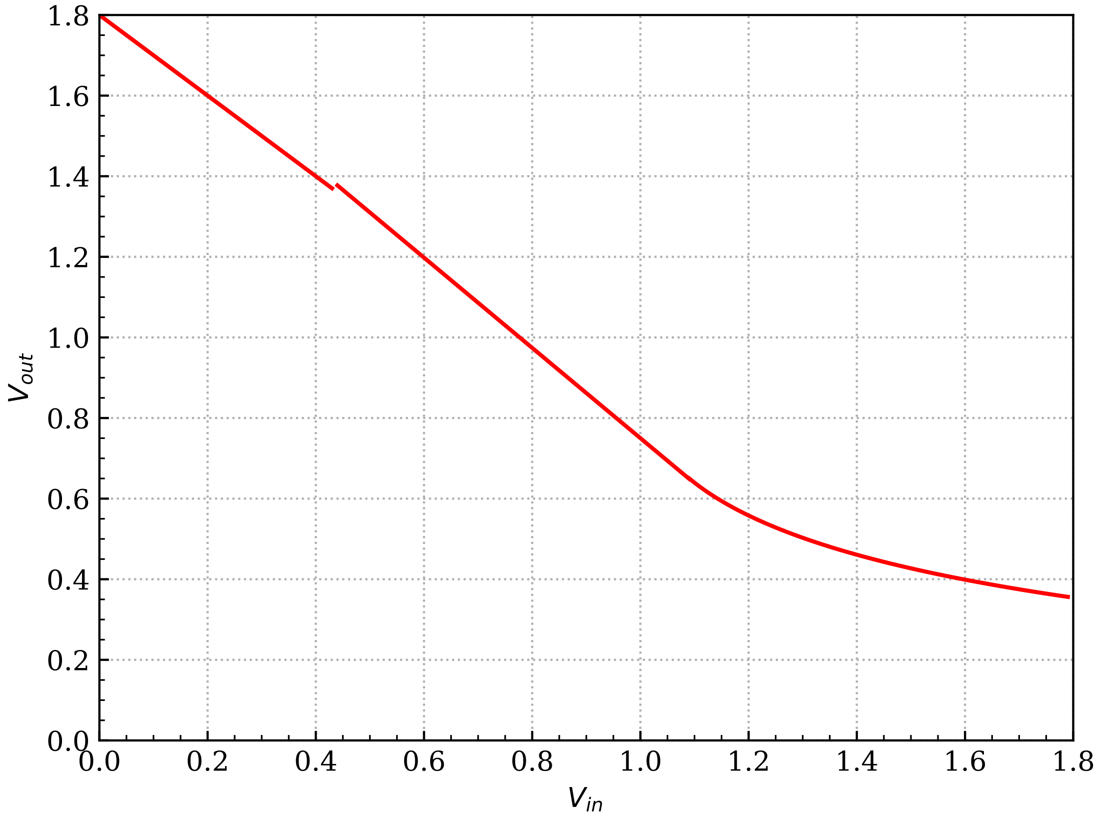

Figure 18: Calculated $V_{out} - V_{in}$

2. Simulated V_{out}-V_{in} relation using LTSpice

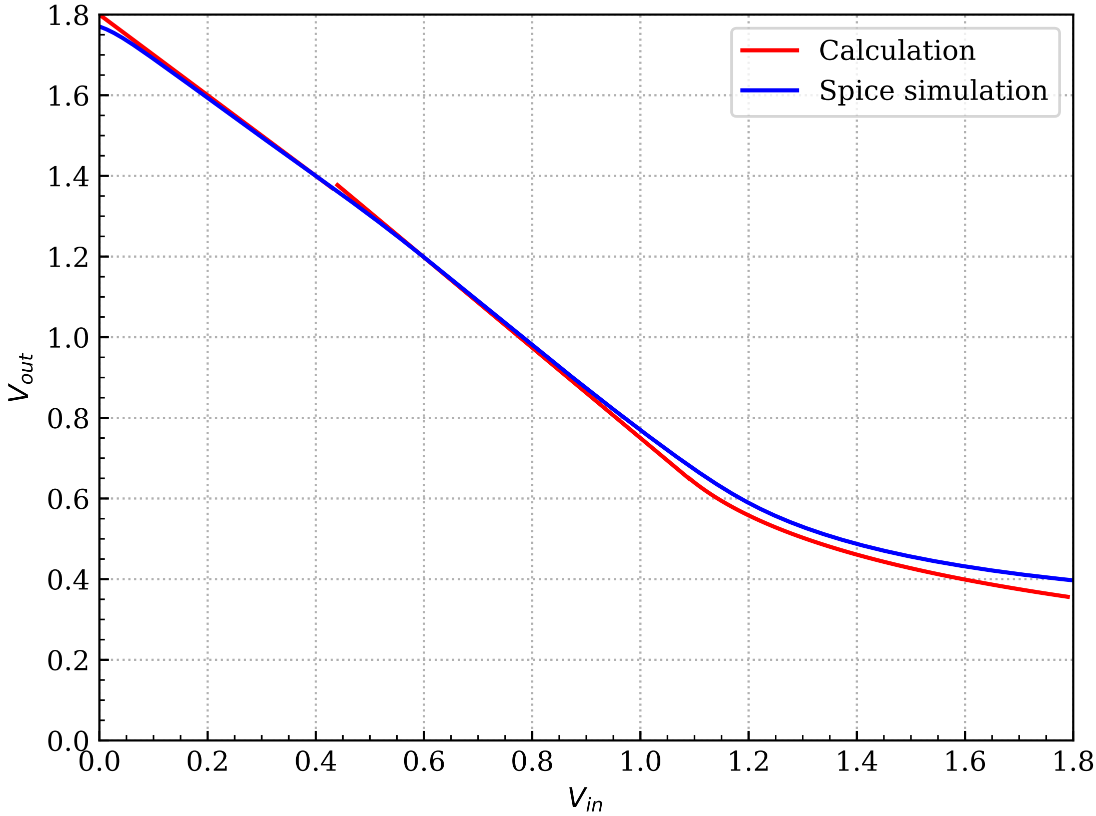

Figure 19: Simulated and calculated $V_{out} - V_{in}$

3. Maximum small-signal gain For maximum small-signal gain:

$$max(|A_V|) = max(|\frac{\partial V_{out}}{\partial V_{in}}|)$$

$$V_{in} \approx 0.69V$$

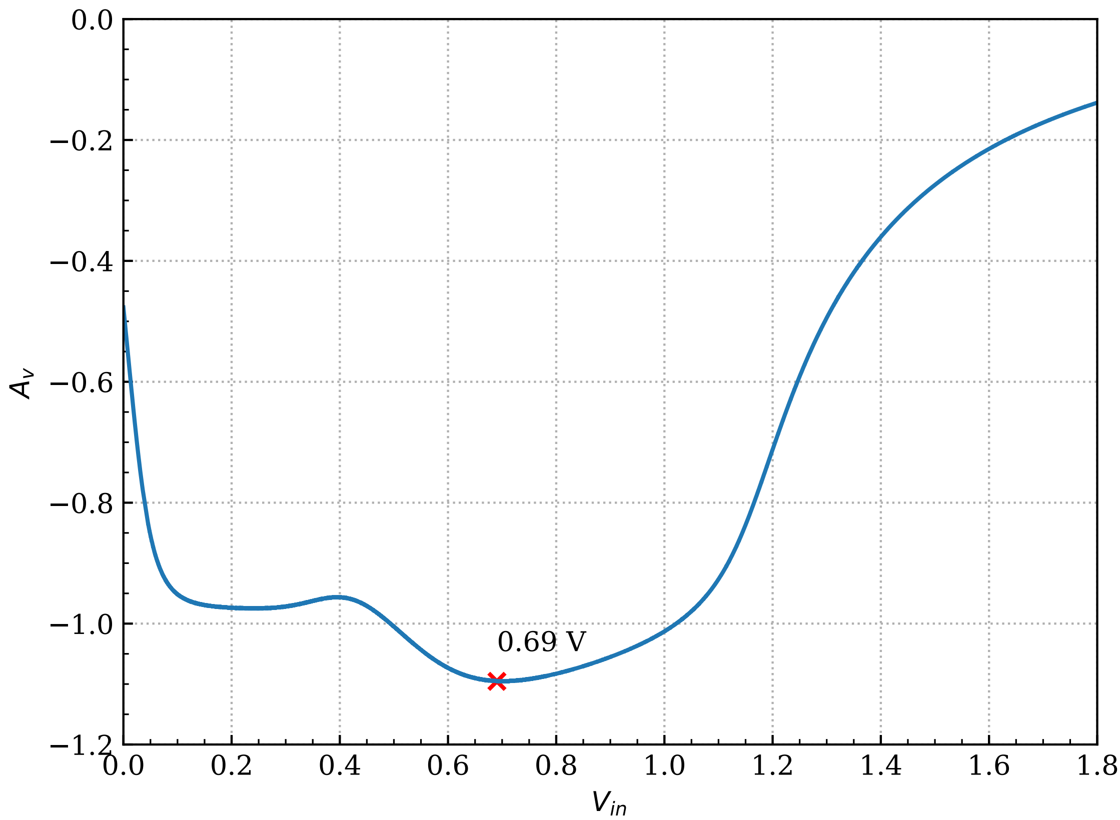

Figure 20: $A_v - V_{in}$

4. Small signal parameters:

   (a) $|A_v| = 787.83mdB = 1.09$

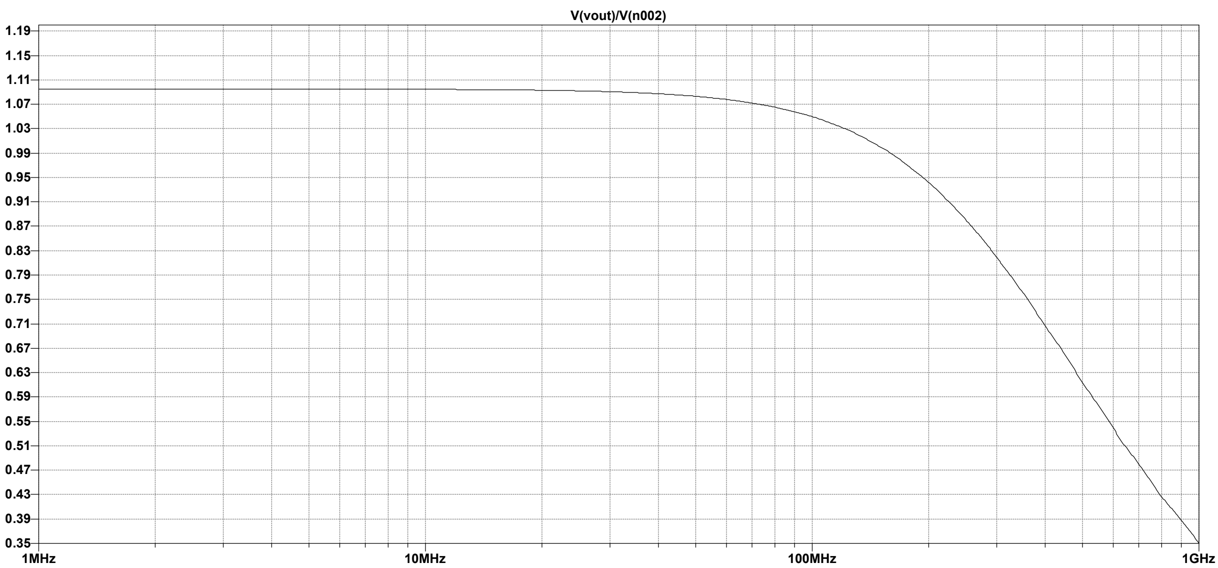

Figure 21: A_v at $V_{in} = 0.69V$

   (b) $R_{out} = 4.73k\Omega$

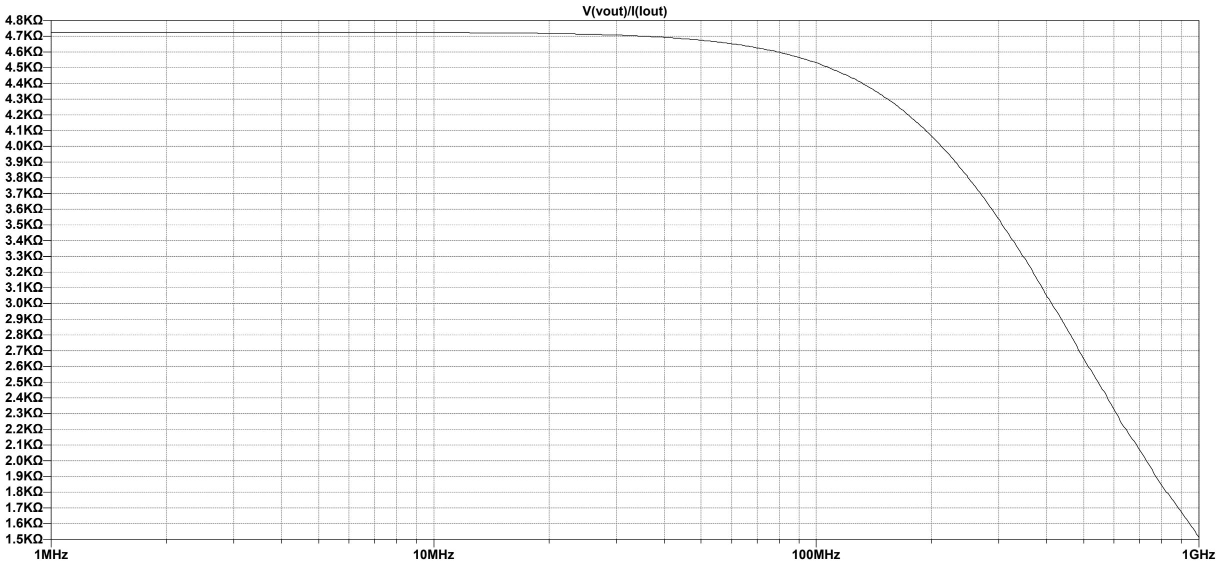

Figure 22: R_{out} at $V_{in} = 0.69V$

5. $\mathrm{g_m}$ and $\mathrm{g_{ds}}$ From 7(d),

$$A_V = 1.09$$
$$R_{out} = 4.73k\Omega$$

From 6(c),

$$g_{m_1} = 2.32e-04\Omega^{-1}$$
$$g_{m_2} = 2.09e-04\Omega^{-1}$$
$$g_{DS_1} = \frac{1}{r_{o_1}} = 8.66e-07\Omega^{-1}$$
$$g_{DS_2} = \frac{1}{r_{o_2}} = 1.28e-06\Omega^{-1}$$

$$\begin{aligned} A_V = \frac{v_{out}}{v_{in}} &= -\frac{g_{m1}}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}} \\ &\approx 1.10 \\ R_{out} &= \frac{1}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}} \\ R_{out} &= 4.74k\Omega \end{aligned}$$