EE4C10 Analog Circuit Design Fundamentals

Homework Assignment I

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Contents

Problem 1

For $I_D = 40\mu A$:

$$
\begin{aligned}
I_D &= \frac{1.8V - V_D}{R} \\
V_D &= 1.8V - I_D R \\
\underline{V_D} &= 1.0V
\end{aligned}
$$

Saturation region:

$$
\begin{aligned}
V_{GS} = 1.0V > V_{TH} \\
V_{GS} - V_{TH} = 0.4V < V_{DS}
\end{aligned}
$$

1. $\lambda = 0V^{-1}$

$$
\begin{aligned}
I_D &= \frac{\mu_n C_{OX}}{2} \frac{W}{L} (V_{GS} - V_{TH})^2 \\
L &= \frac{\mu_n C_{OX}}{2} \frac{W}{I_D} (V_{GS} - V_{TH})^2 \\
\underline{L} &= 0.39\mu m
\end{aligned}
$$

2. $\lambda = 0.06V^{-1}$

$$
\begin{aligned}
I_D &= \frac{\mu_n C_{OX}}{2} \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS}) \\
L &= \frac{\mu_n C_{OX}}{2} \frac{W}{I_D} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS}) \\
\underline{L} &= 0.41\mu m
\end{aligned}
$$

Problem 2

1. Bulk of the transistors are connected to the source, $V_B = V_S$

$$
\begin{aligned}
V_{TH} &= V_{TH0} + \gamma(\sqrt{2\varphi_F + V_{BS}} - \sqrt{|2\varphi_F|}) \\
V_{TH} &= V_{TH0} = 0.33V
\end{aligned}
$$

(a) Transistor M_1

$$V_{SG} = 2.5V - 1.7V = 0.8V$$

$$\begin{aligned} I_D &= \frac{\mu_p C_{OX}}{2}\frac{W}{L}(V_{SG} - V_{TH})^2 \\ W &= \frac{2LI_D}{\mu_p C_{OX}}\frac{1}{(V_{SG} - V_{TH})^2} \\ W_1 &= 2.72\mu m \end{aligned}$$

(b) Transistor M_2

$$V_{SG} = 1.7V - 1V = 0.7V$$

$$\begin{aligned} W &= \frac{2LI_D}{\mu_p C_{OX}}\frac{1}{(V_{SG} - V_{TH})^2} \\ W_2 &= 4.38\mu m \end{aligned}$$

(c) Transistor M_3

$$V_{SG} = 1V$$

$$\begin{aligned} W &= \frac{2LI_D}{\mu_p C_{OX}}\frac{1}{(V_{SG} - V_{TH})^2} \\ W_3 &= 1.37\mu m \end{aligned}$$

2. Bulk terminals are attached to the V_{DD}, $V_B = V_{DD}$.

(a) Transistor M_1

$$V_{BS} = 2.5V - 2.5V = 0V$$

$$\begin{aligned} V_{TH} &= V_{TH0} + \gamma(\sqrt{2\varphi_F + V_{BS}} - \sqrt{|2\varphi_F|}) \\ V_{TH} &= V_{TH0} = 0.33V \end{aligned}$$

$$\begin{aligned} W &= \frac{2LI_D}{\mu_p C_{OX}}\frac{1}{(V_{SG} - V_{TH})^2} \\ W_1 &= 2.72\mu m \end{aligned}$$

(b) Transistor M_2

$$V_{BS} = 2.5V - 1.7V = 0.8V$$

$$\begin{aligned} V_{TH} &= V_{TH0} + \gamma(\sqrt{2\varphi_F + V_{BS}} - \sqrt{|2\varphi_F|}) \\ V_{TH} &= V_{TH0} = 0.43V \end{aligned}$$

$$\begin{aligned} W &= \frac{2LI_D}{\mu_p C_{OX}}\frac{1}{(V_{SG} - V_{TH})^2} \\ W_2 &= 8.23\mu m \end{aligned}$$

(c) Transistor M_3

$$V_{BS} = 2.5V - 1.0V = 1.5V$$

$$\begin{aligned} V_{TH} &= V_{TH0} + \gamma(\sqrt{2\varphi_F + V_{BS}} - \sqrt{|2\varphi_F|}) \\ V_{TH} &= V_{TH0} = 0.49V \end{aligned}$$

$$\begin{aligned} W &= \frac{2LI_D}{\mu_p C_{OX}}\frac{1}{(V_{SG} - V_{TH})^2} \\ W_3 &= 2.31\mu m \end{aligned}$$

Problem 3

1. Testbench and I_D-V_{GS} characteristics of NMOS and PMOS

   (a) NMOS

      i. Testbench

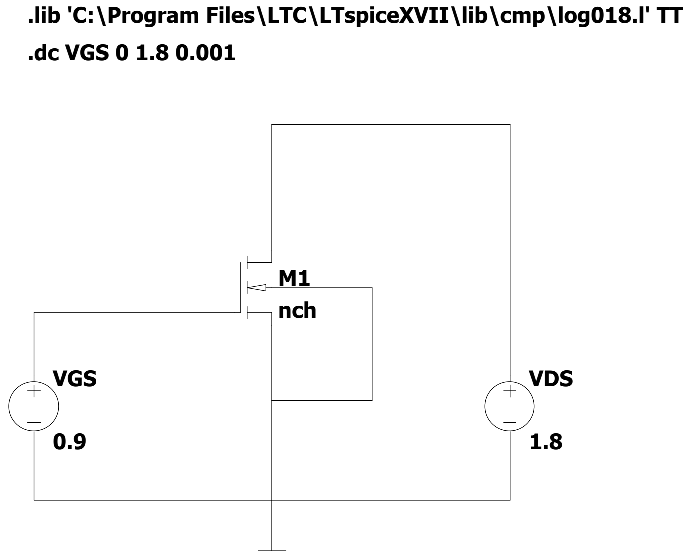

Figure 1: NMOS Testbench

      ii. I_D-V_{GS}

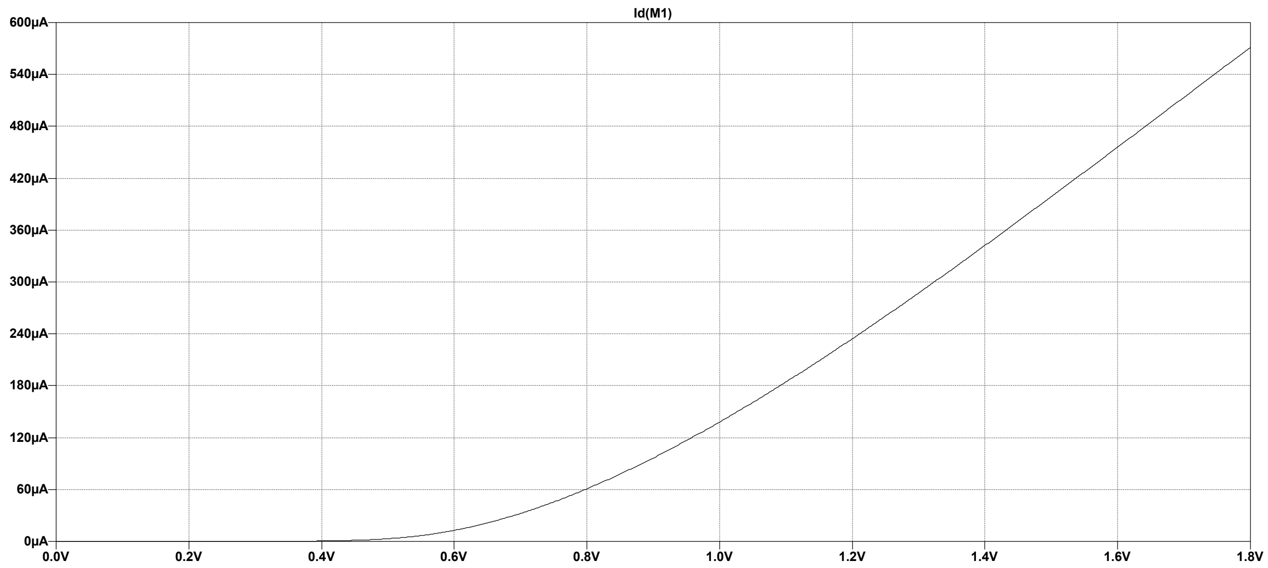

Figure 2: NMOS I_D-V_{GS}

   (b) PMOS

      i. Testbench

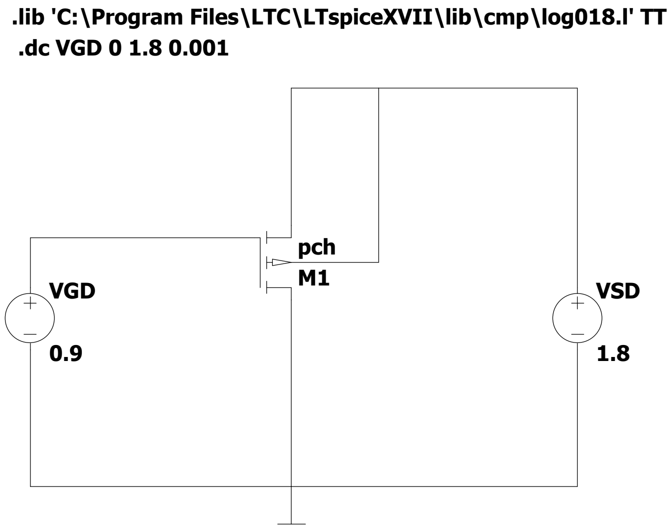

Figure 3: PMOS Testbench

ii. I_S-V_{GD}

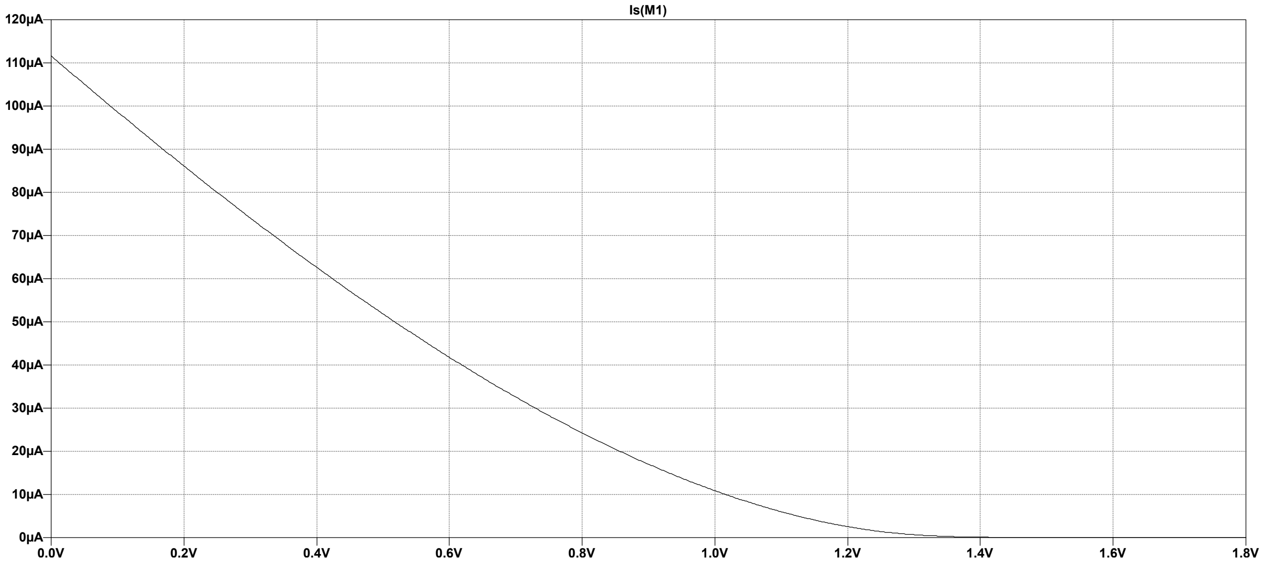

Figure 4: PMOS I_S-V_{GD}

2. $\mu_{n(p)}C_{OX}$ and $V_{THn(p)}$

Assuming that channel length modulation is negligible, V_{THn} for NMOS can be derived from the following relation:

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_{THn})^2$$
$$\frac{2I_D}{\mu_n C_{ox}}\frac{L}{W} = (V_{GS} - V_{THn})^2$$
$$\sqrt{\frac{2I_D}{\mu_n C_{ox}}\frac{L}{W}} = V_{GS} - V_{THn}$$

V_{THn} is the x-axis intercept when the saturation region is extrapolated. In the case of PMOS, the relation becomes:

$$\sqrt{\frac{2I_S}{\mu_p C_{ox}}\frac{L}{W}} = V_{SG} - V_{THp}$$

For deriving $\mu_n C_{OX}$, since $V_{THn(p)}$ is constant at specific temperatures. Differentiating both sides with respect to

$V_{GS(SG)}$ will give:

$$\frac{d}{dV_{GS}}\sqrt{\frac{2I_D}{\mu_n C_{ox}}\frac{L}{W}} = \frac{d}{dV_{GS}}(V_{GS} - V_{THn})$$

$$\frac{1}{2}\frac{dI_D}{dV_{GS}}\sqrt{\frac{2}{I_D\mu_n C_{ox}}\frac{L}{W}} = 1$$

$$\sqrt{\mu_n C_{ox}} = \frac{1}{2}\frac{dI_D}{dV_{GS}}\sqrt{\frac{2}{I_D}\frac{L}{W}}$$

$$\mu_n C_{ox} = \frac{1}{2}\frac{L}{W}\frac{1}{I_D}(\frac{dI_D}{dV_{GS}})^2$$

$$\mu_n C_{ox} = \frac{1}{6I_D}(\frac{dI_D}{dV_{GS}})^2$$

In the case for PMOS, the relation becomes:

$$\mu_p C_{ox} = \frac{1}{6I_S}(\frac{dI_S}{dV_{SG}})^2$$

(a) NMOS

i. $\mu_n C_{OX} = 306\mu AV^{-2}$

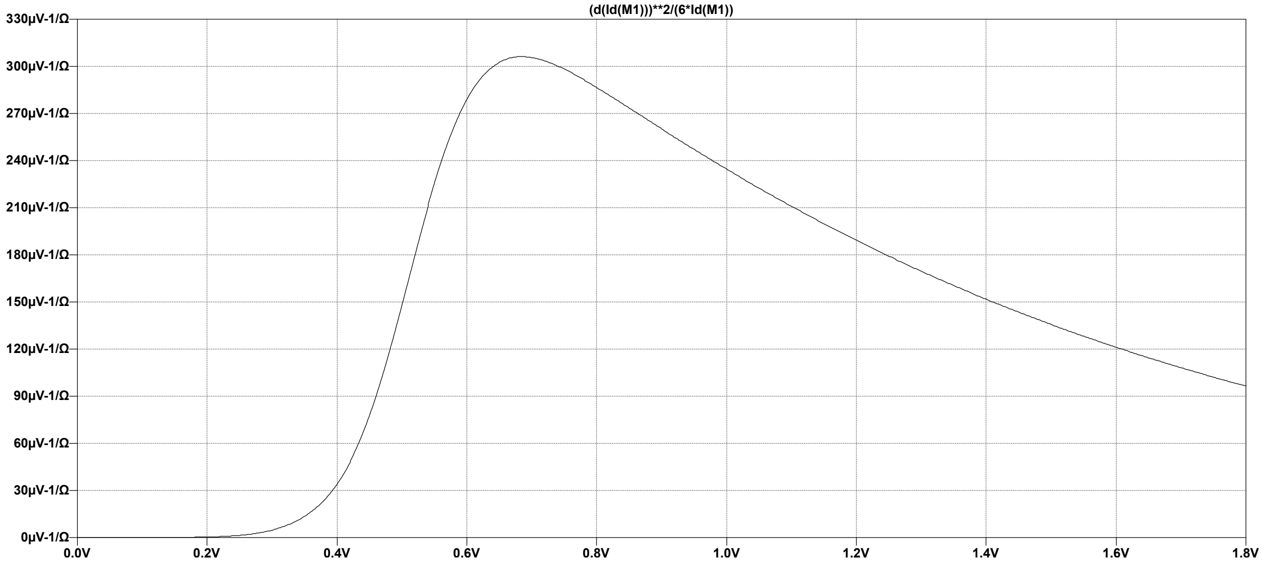

Figure 5: NMOS $\mu_n C_{OX}$-V_{GS}

ii. $V_{THn} = 0.44V$

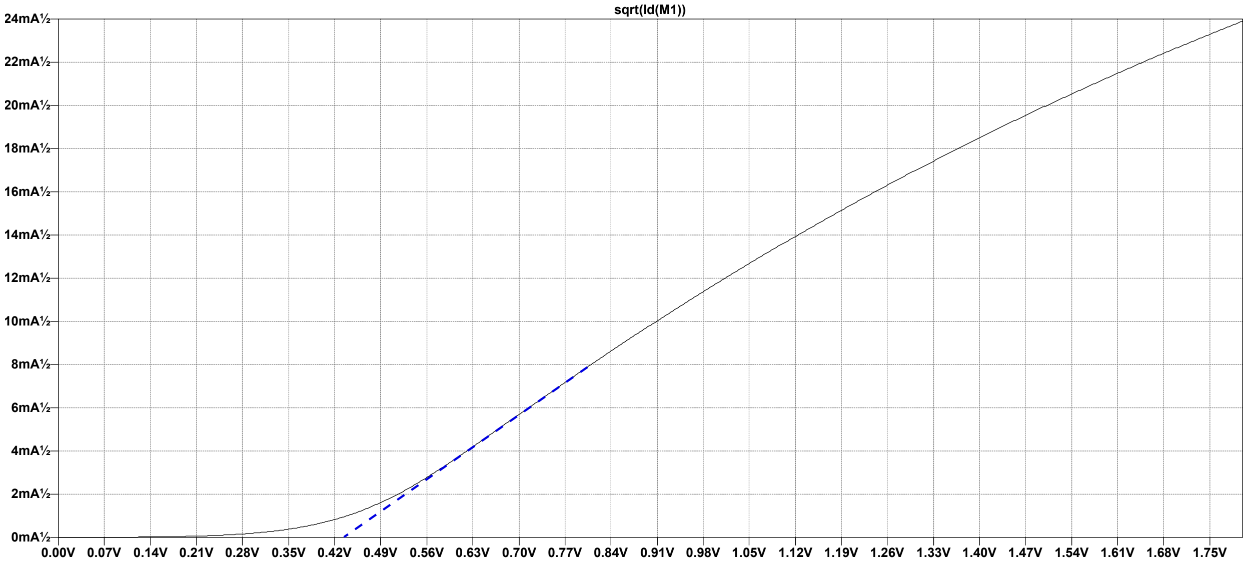

Figure 6: NMOS $\sqrt{I_D} - V_{GS}$

(b) PMOS

i. $\mu_p C_{OX} = 294\mu AV^{-2}$

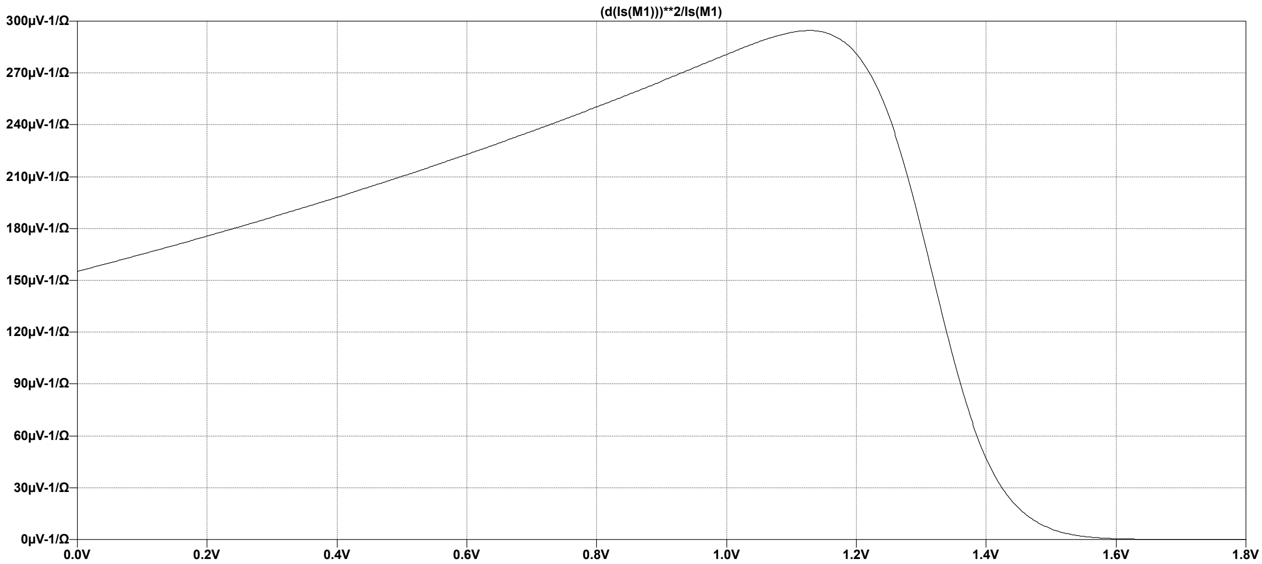

Figure 7: PMOS $\mu_p C_{OX}$-V_{GD}

ii. $V_{THp} = 1.8V - 1.38V = 0.42V$

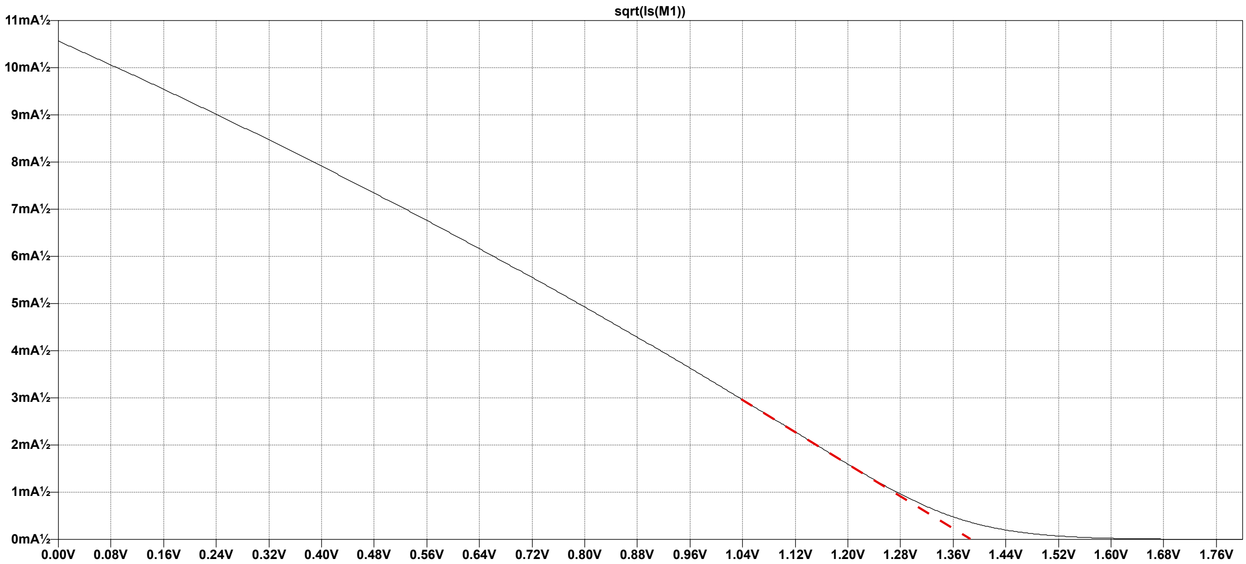

Figure 8: PMOS $\sqrt{I_S} - V_{GD}$

Problem 4

1. Testbench and I_D-V_{DS} characteristics of NMOS and PMOS
   (a) NMOS
      i. Testbench

```
.lib 'C:\Program Files\LTC\LTspiceXVII\lib\cmp\log018.l' TT
.dc VDS 0 1.8 0.001
```

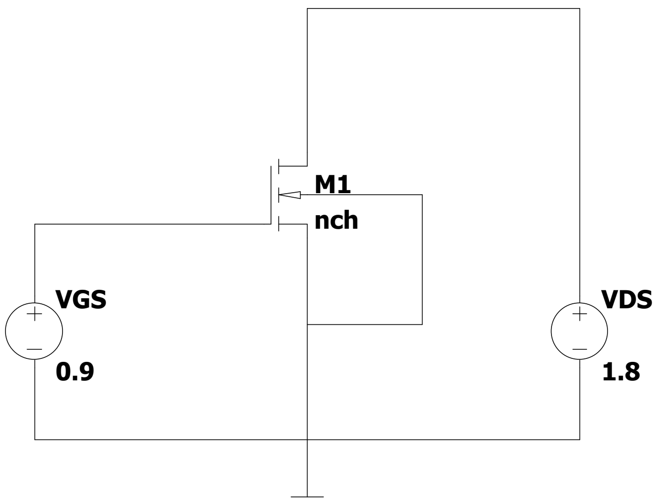

Figure 9: NMOS Testbench

      ii. I_D-V_{DS} characteristics

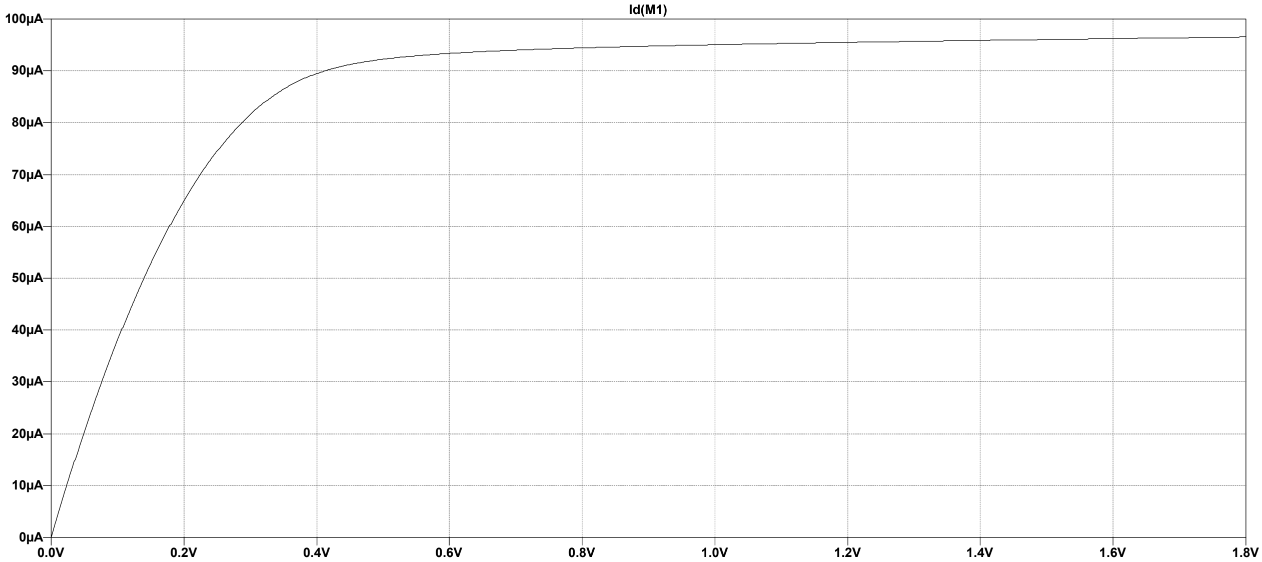

Figure 10: NMOS I_D-V_{DS}

(b) PMOS

i. Testbench

ii. I_D-V_{DS} characteristics

2. $\lambda_{n(p)}$

Drain current characteristics for NMOS under saturation conditions:

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda_n V_{DS})$$

Differentiating both side with respect to V_{DS}.

$$\frac{dI_D}{dV_{DS}} = \frac{d}{dV_{DS}}(\frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda_n V_{DS}))$$
$$\frac{dI_D}{dV_{DS}} = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_{TH})^2\lambda_n$$

Assuming that the body-effect is small:

$$I_D \approx \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_{TH})^2$$

$$\frac{dI_D}{dV_{DS}} \approx I_D\lambda_n$$
$$\lambda_n \approx \frac{1}{I_D}\frac{dI_D}{dV_{DS}}$$

In the case of PMOS:

$$\lambda_p \approx \frac{1}{I_S}\frac{dI_S}{dV_{SD}}$$

(a) $\lambda_n = 0.18V^{-1}$

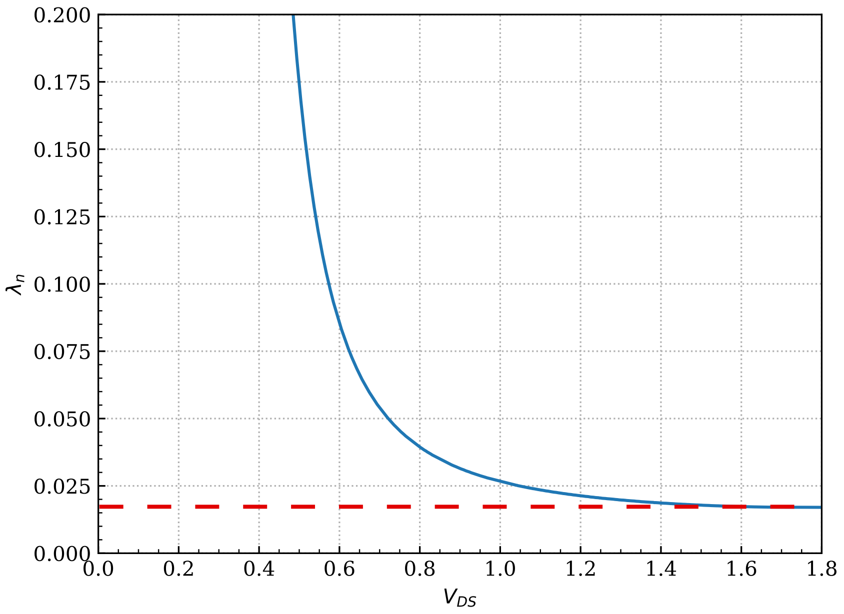

Figure 11: NMOS Testbench

(b) λ_p

Problem 5

Problem 6

1. Small-signal Model

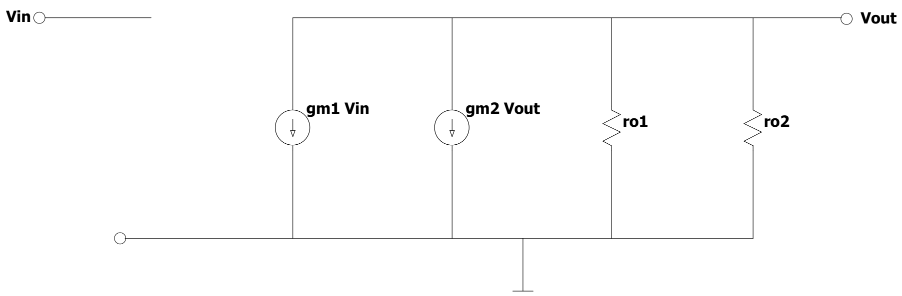

Figure 12: Small signal model

2. $\lambda = 0V^{-1}$
3. $\lambda \neq 0V^{-1}$
   (a) $A_V = \frac{v_{out}}{v_{in}}$

$$-v_{out} = (g_{m1}v_{in} + g_{m2}v_{out})(r_{o1}//r_{o2})$$
$$-v_{in}g_{m1}(r_{o1}//r_{o2}) = (1 + g_{m2}(r_{o1}//r_{o2}))v_{out}$$
$$A_V = \frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$

(b) R_{out}

$$R_{out} = \frac{1}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$

Problem 7