EE4C10 Analog Circuit Design Fundamentals

Homework Assignment II

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Contents

Simulation Files

Each question with simulation files will have their respective subfolder. Except for `q6`, the other questions will have separate subfolders for each sub questions.

Running the simulation files should be able to directly plot the graphs used (configured in the *.plt file). The folders for each question are arranged as follows after extracting:

```
spice
        q4
                a
                        step3
                        step4
                b
                e
        q5
                a
                b
        q6
```

Problem 1

(a) Overdrive voltage, V_{gt}, for:

    1. M1

$$I_{D1} = \frac{\mu_n C_{OX}}{2} \left(\frac{W}{L}\right)_1 (V_{GS_1} - V_{TH_1})^2 (1 + \lambda_1 V_{DS_1})$$

$$I_{D1} \approx \frac{\mu_n C_{OX}}{2} \left(\frac{W}{L}\right)_1 (V_{gt_1})^2$$

$$V_{gt_1} \approx \sqrt{\frac{2I_{D_1}}{\mu_n C_{OX}} \left(\frac{L}{W}\right)_1}$$

$$V_{gt_1} \approx 109.11 mV$$

2. M2

$$V_{gt_2} \approx \sqrt{\frac{2I_{D_2}}{\mu_p C_{OX}}(\frac{L}{W})_2}$$

$$V_{gt_2} \approx 377.96mV$$

(b) Small-signal gain

$$g_{m1}V_{in} = \frac{-V_{out}}{r_{o1}//r_{o2}}$$
$$\frac{V_{out}}{V_{in}} = -g_{m1}(r_{o1}//r_{o2})$$

$$g_{m1} = \mu_n C_{OX}(\frac{W}{L})_1 V_{gt_1}$$
$$= 4.582mS$$

$$r_{o1} = \frac{1}{I_{D1}\lambda_n}$$
$$= 20k\Omega$$

$$r_{o2} = \frac{1}{I_{D2}\lambda_p}$$
$$= 40k\Omega$$

$$\frac{V_{out}}{V_{in}} \approx -61.09$$

(c) V_{out} output swing

For M_1 to be in saturation,

$$V_{DS1} \geq V_{gt1}$$
$$V_{out} \geq 0.109V$$

For M_2 to be in saturation,

$$V_{DS2} \geq V_{gt2}$$
$$V_{DD} - V_{out} \geq 0.377V$$
$$V_{out} \leq 3.3V - 0.377V$$
$$V_{out} \leq 2.923V$$

Swing of V_{out},

$$0.109V < V_{out} < 2.923V$$

$$V_{out,pp} = 2.923V - 0.109V$$
$$= 2.814V$$

(d) The up-swing is limited by the PMOS leaving saturation, while the down swing is limited by the NMOS leaving saturation. The up-swing can be increased by connecting the base of the PMOS to the drain to form a diode connected structure, therefore V_{SD2} will always be larger than V_{gt2} by V_{TH2}. As a result, V_{out} can be increased to V_{DD} - V_{TH2} without PMOS leaving saturation.

Problem 2

(a) For M1 to be 100mV from triode,

$$V_{DS1} = V_{GS1} - V_{TH,N} + 100mV$$
$$X = V_{in} - V_{TH,N} + 100mV$$

V_{in} for M1 to be in saturation with I_{D1} of 0.35 mA,

$$I_{D1} = \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_1(V_{GS1} - V_{TH,N})^2$$

$$I_{D1} = \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_1(V_{in} - V_{TH,N})^2$$

$$V_{in} = \sqrt{\frac{2I_{D1}}{\mu_n C_{OX}}(\frac{L}{W})_1} + V_{TH,N}$$

$$= 0.653V$$

$$X = \sqrt{\frac{2I_{D1}}{\mu_n C_{OX}}(\frac{L}{W})_1} + 100mV$$

$$\approx 0.253V$$

V_b for M2 to be in saturation with I_{D2} of 0.35 mA,

$$I_{D2} = \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_2(V_{GS2} - V_{TH,N})^2$$

$$I_{D2} = \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_2(V_b - X - V_{TH,N})^2$$

$$V_b = \sqrt{\frac{2I_{D2}}{\mu_n C_{OX}}(\frac{L}{W})_2} + X + V_{TH,N}$$

$$\approx 0.906V$$

(b) Small-signal gain

$$G_m = \frac{g_{m1}(g_{m2}r_{o1}r_{o2} + r_{o1})}{g_{m2}r_{o1}r_{o2} + r_{o1} + r_{o2}}$$

$$\approx g_{m1}$$

$$R_{out} = (g_{m2}r_{o1}r_{o2} + r_{o1} + r_{o2})//R_d$$

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Small-signal gain,

$$\frac{V_{out}}{V_{in}} = -G_m R_{out}$$
$$= -g_{m1}[(g_{m2}r_{o1}r_{o2} + r_{o1} + r_{o2})//R_d]$$

$$g_{m1} = \mu_n C_{OX}(\frac{W}{L})_1(V_{GS1} - V_{TH,N})$$
$$= \mu_n C_{OX}(\frac{W}{L})_1(V_{in} - V_{TH,N})$$
$$= 4.583mS$$

$$g_{m2} = \mu_n C_{OX}(\frac{W}{L})_2(V_{GS2} - V_{TH,N})$$
$$\approx \mu_n C_{OX}(\frac{W}{L})_2(V_b - X - V_{TH,N})$$
$$= 4.583mS$$

$$r_{o1} = \frac{1}{I_{D1}\lambda_n}$$
$$= 28.571k\Omega$$

$$r_{o2} = \frac{1}{I_{D2}\lambda_p}$$
$$= 28.571k\Omega$$

$$\frac{V_{out}}{V_{in}} \approx -22.88$$

(c) Assume V_b to be 1.65V, For M2 to be in saturation,

$$V_{out} - X \geq V_b - X - V_{TH,N}$$
$$V_{out} \geq 1.15V$$

When $I_D \geq 0$,

$$V_{out} \leq V_{DD}$$
$$1.15V \leq V_{out} \leq 3.3V$$

$$V_{out,pp} = 2.15V$$

(d) X_{pp}

Gain of X,

$$\frac{X}{V_{in}} = \frac{-g_{m1}}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$
$$\approx \frac{-g_{m1}}{g_{m2}}$$
$$\approx -1$$

X_{pp}

$$\frac{X}{V_{out}} = \frac{X}{V_{in}}\frac{V_{in}}{V_{out}}$$
$$= \frac{1}{22.88}$$

$$X_{pp} = 54.63mV$$

Problem 3

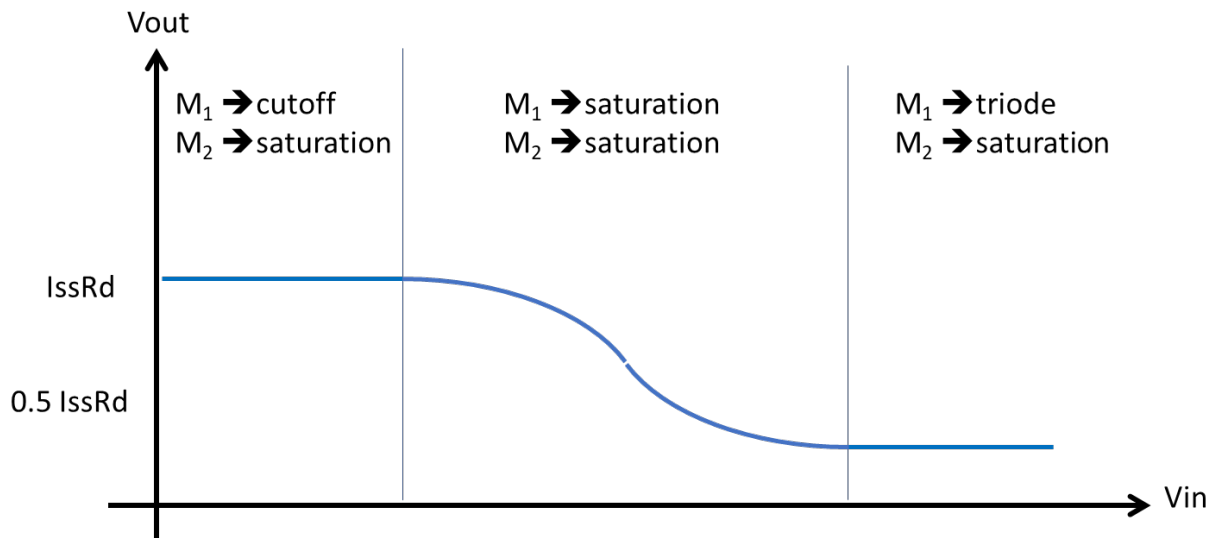(a) Sketch of:

    1. Output voltage [[

Figure 1: Output voltage sketch

    2. Small-signal gain
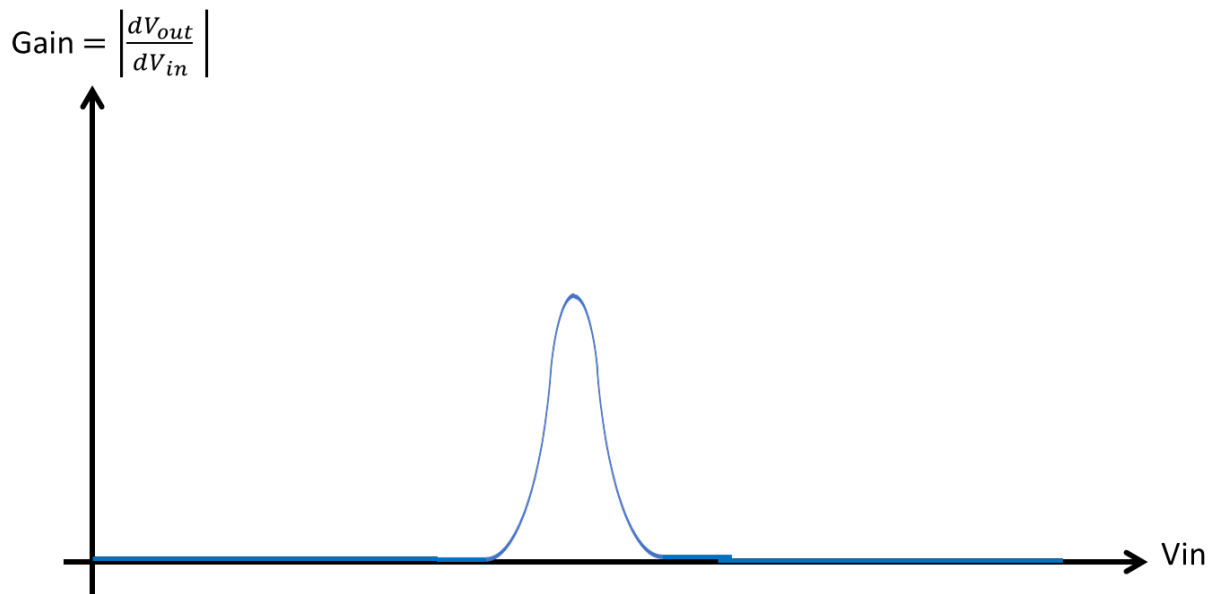

Figure 2: Small-signal gain
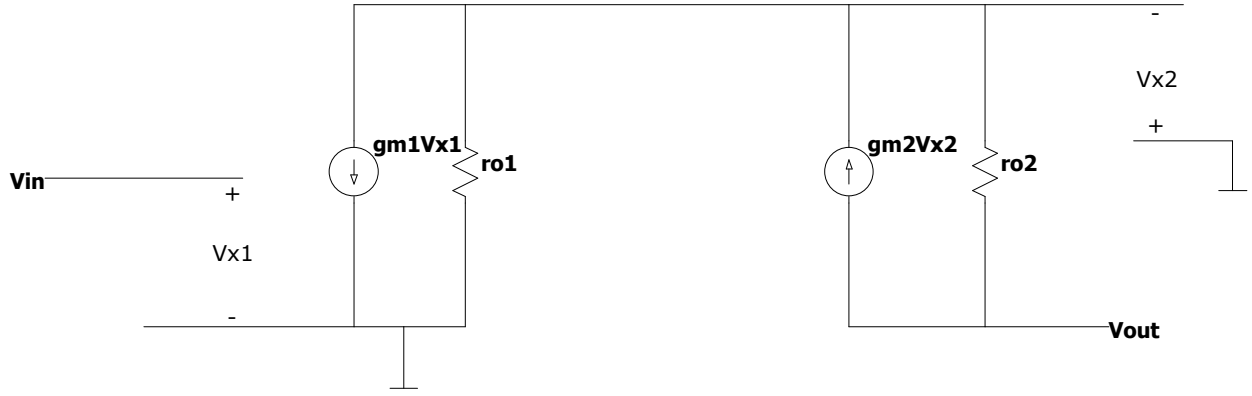
(b) Small-signal model

Figure 3: Small-signal model of folded-cascode stage

$$R_{out} = g_{m2}r_{o1}r_{o2} + r_{o1} + r_{o2}$$
$$\approx g_{m2}r_{o1}r_{o2}$$
$$G_m = \frac{-g_{m1}\left(g_{m2} + \frac{1}{r_{o1}}\right)}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$
$$\approx -g_{m1}$$

$$\frac{V_{out}}{V_{in}} = g_{m1}g_{m2}r_{o1}r_{o2}$$

Problem 4

(a) Testbench

.lib 'C:\Program Files\LTC\LTspiceXVII\lib\cmp\log018.l' TT
;ac dec 10 1Meg 1G
;op
;step param Vb 0.5 2.5 0.25
.dc VIN 0 3.3 10m

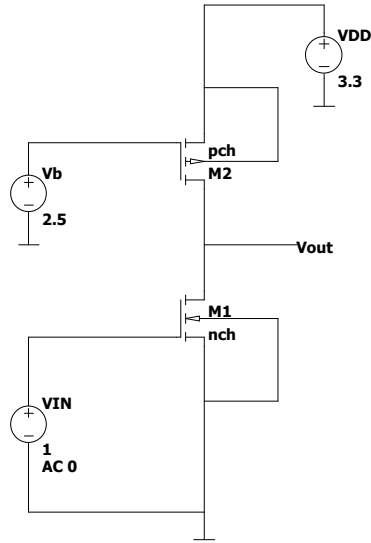Figure 4: Testbench

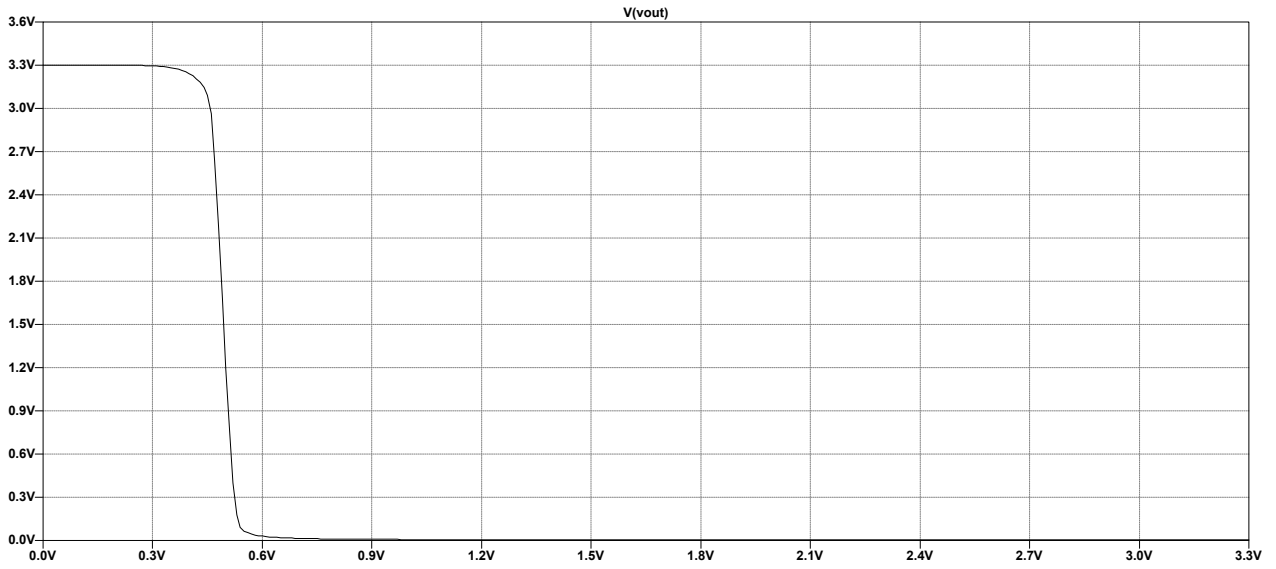## V_{out} - V_{in} characteristics

Figure 5: V_{out} - V_{in} characteristics
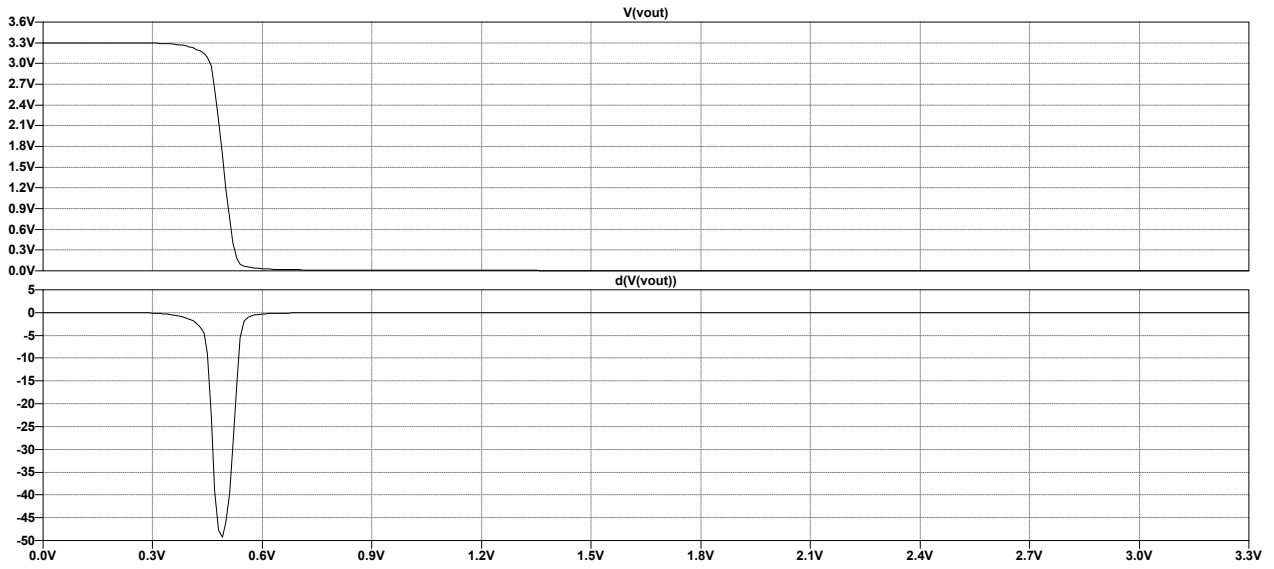
(b) Small-signal gain, $\frac{dV_{out}}{dV_{in}}$

Figure 6: Small-signal gain, $\frac{dV_{out}}{dV_{in}}$

From figure 6 the gain when:

1. $V_{out} = 0.6$ V

   $\frac{dV_{out}}{dV_{in}} = -35.59$

2. $V_{out} = 2.8$ V

   $\frac{dV_{out}}{dV_{in}} = -32.77$

(c) From figure 6, the input voltage, V_{in}, for maximum gain, $max(|\frac{dV_{out}}{dV_{in}}|)$ is given to be:

$$max(|\frac{dV_{out}}{dV_{in}}|) = 50.07$$
$$V_{in} = 489mV$$

(d) Output voltage swing for gain of 1,

$$V_{out,max} = 3.24V$$
$$V_{out,min} = 56mV$$
$$V_{out,pp} = 3.184V$$

Output peak to peak voltage

$$V_{out,pp} = 3.184V$$

(e) Small-signal voltage gain when:
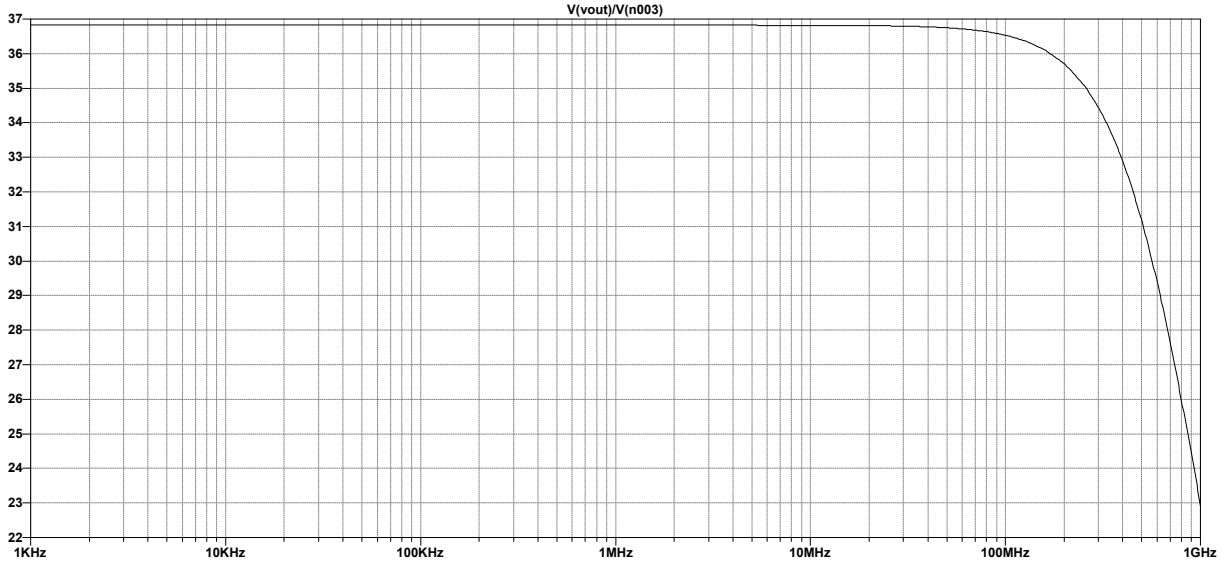
1. $V_{out} = 0.6$ V, $V_{in} = 0.514$ V

Figure 7: Small-signal gain, $|\frac{V_{out}}{V_{in}}|$, $V_{out} = 0.6$ V, $V_{in} = 0.514$ V

Gain = 36.82

2. $V_{out} = 2.8$ V, $V_{in} = 0.464$ V
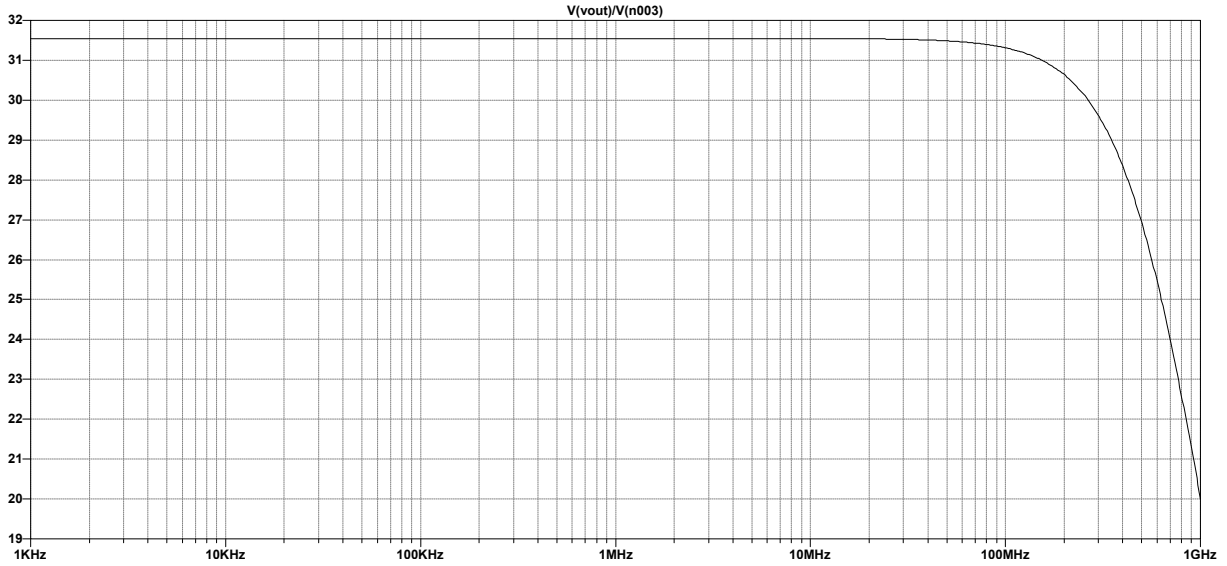
Figure 8: Small-signal gain, $|\frac{V_{out}}{V_{in}}|$, $V_{out} = 2.8$ V, $V_{in} = 0.464$ V

Gain = 31.54

Problem 5

(a) Procedure for designing V_b for M1 to be 100mV away from triode, $V_b = 0.825$V, is determined by the following:

1. Select an initial value V_b for M1 and M2 to be in saturation when $V_{in} > V_{TH1}$. In figure 9, $V_b = 1.5$ V. From the error logfile in figure 10, $V_{DS1} > V_{GS1}$ - V_{TH1} and $V_{DS2} > V_{GS2}$ - V_{TH2}.

```
*.lib 'C:\Program Files\LTC\LTspiceXVII\lib\cmp\log018.l' TT
;ac dec 10 1k 1G
*.op
;dc vin 0 3.3 0.001

                                              ┌──────────┐
                                              │       ┌──┴──┐ VDD
                                          RD  │       │  +  │
                                          5k  ≷       │  −  │ 3.3
                                              │       └──┬──┘
                                              ├─── Vout  │
                                           ┌──┤M2        │
                                     ┌──────┤  nch       │
                              ┌──┴──┐│      │  ┴         │
                         Vb   │  +  ││      │            │
                         1.5  │  −  ││      │            │
                              └──┬──┘│      ├─── X       │
                                 │   │   ┌──┤M1          │
                                 │   └───┤  nch          │
                          ┌──┴──┐│       │  ┴            │
                     VIN  │  +  ││       │               │
                     0.6  │  −  ││       │               │
                     AC 1 └──┬──┘│       │               │
                            ─┴─ ─┴─      │               │
                                        ─┴─             ─┴─
```
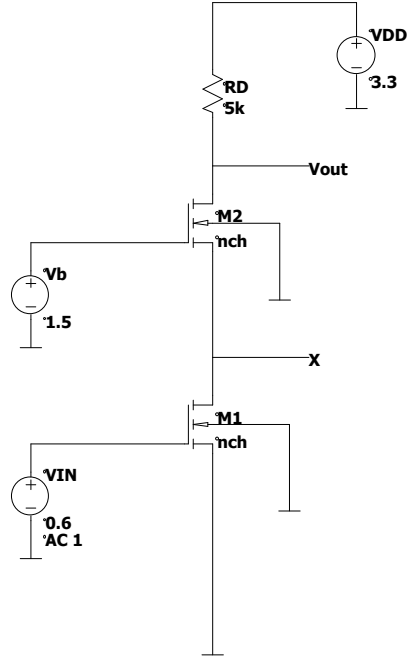
Figure 9: Testbench for Q5


```
Semiconductor Device Operating Points:
                --- BSIM3 MOSFETS ---
Name:         m2          m1
Model:        nch.3       nch.3
Id:           3.74e-04    3.74e-04
Vgs:          7.73e-01    6.00e-01
Vds:          7.05e-01    7.27e-01
Vbs:         -7.27e-01    0.00e+00
Vth:          6.78e-01    4.97e-01
Vdsat:        1.18e-01    1.35e-01
Gm:           4.21e-03    4.42e-03
Gds:          1.29e-04    4.49e-05
Gmb:          7.98e-04    1.27e-03
Cbd:          0.00e+00    0.00e+00
Cbs:          0.00e+00    0.00e+00
Cgsov:        6.59e-15    1.32e-14
Cgdov:        6.59e-15    1.32e-14
Cgbov:        3.18e-17    6.47e-17
dQgdVgb:      3.21e-14    1.00e-13
dQgdVdb:     -6.47e-15   -1.32e-14
dQgdVsb:     -2.27e-14   -7.78e-14
dQddVgb:     -6.61e-15   -1.33e-14
dQddVdb:      6.60e-15    1.32e-14
dQddVsb:      2.09e-17    9.01e-17
dQbdVgb:     -3.63e-15   -1.24e-14
dQbdVdb:      1.50e-17    2.88e-18
dQbdVsb:     -2.22e-15   -1.44e-14
```

Figure 10: Semiconductor Device Operating Points

2. Doing a DC-sweep for I_{D1}-V_{in}, $V_{in} \approx 0.6V$ when $I_{D1} = 0.35$mA.
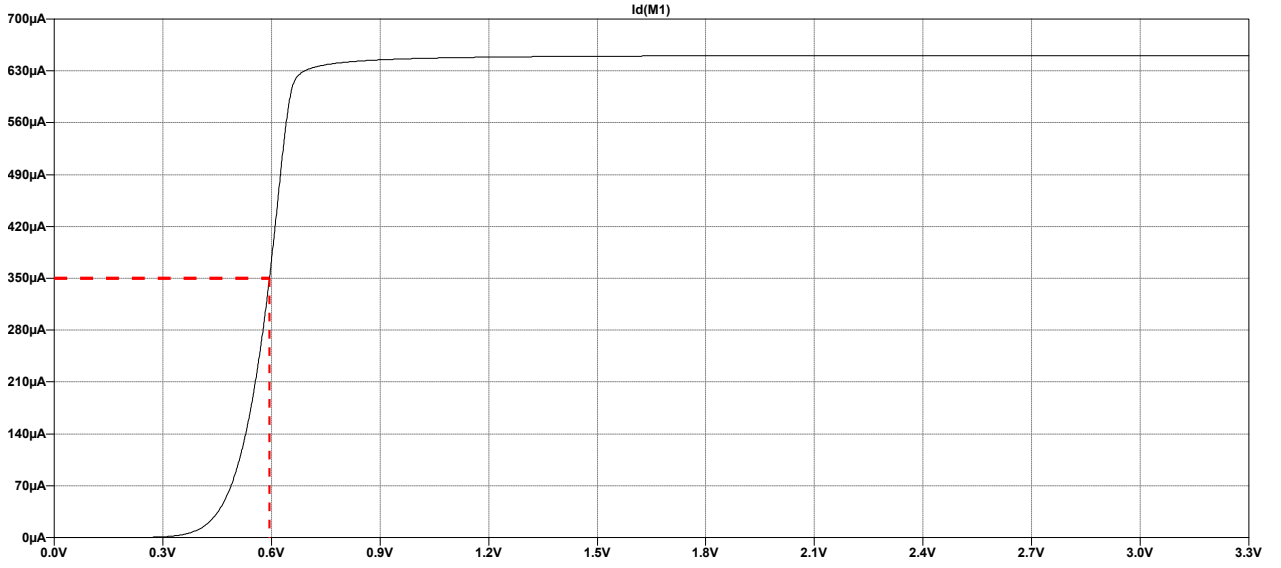
Figure 11: I_{D1}-V_{in}

3. From the operation point simulation int figure 10, V_{TH1} is determined to be 0.497V. For M1 to be 100mV from triode region,

$$V_x = V_{in} - V_{TH1} + 100mV$$
$$\approx 0.2V$$

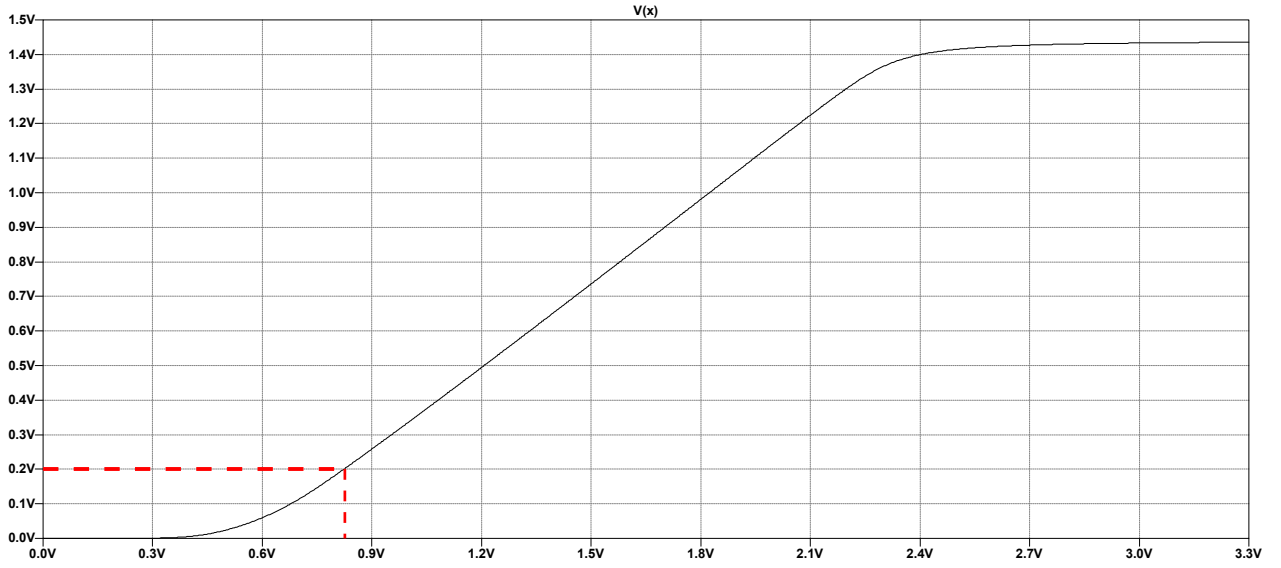4. DC-sweeping V_x-V_b and determining V_b for V_x = 0.2V. From the figure 12 , V_b = 0.825V.

Figure 12: V_x-V_b

(b) I_{OUT}-V_{OUT} when V_{IN} is fixed for I_{D1} = 0.35 mA
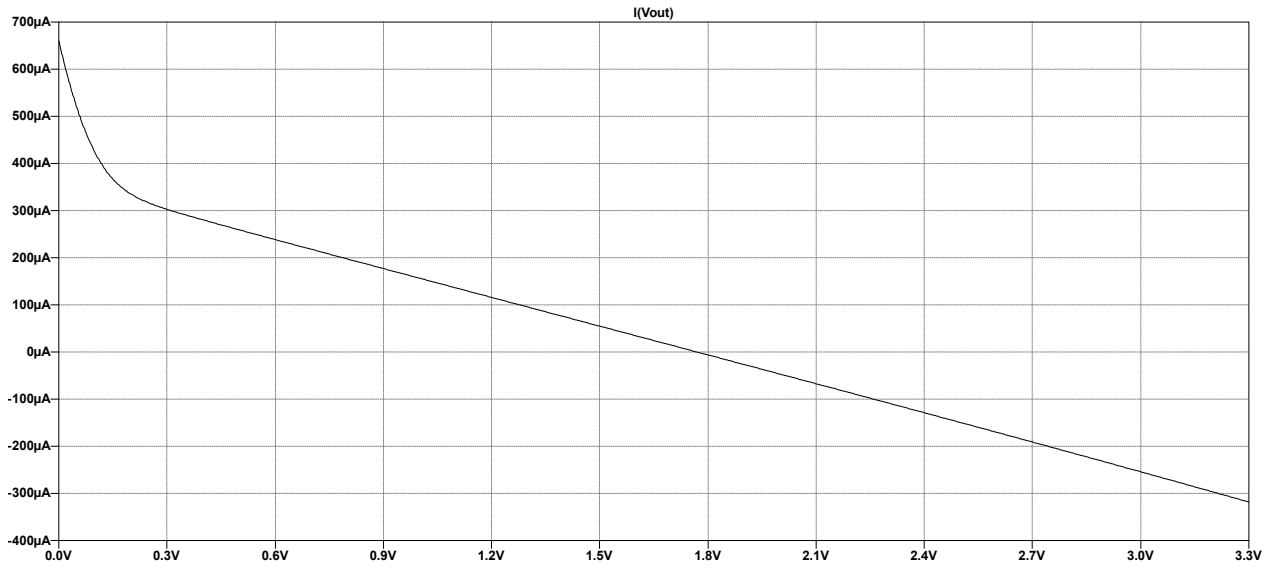
11

Figure 13: I_{OUT}-V_{OUT}

Problem 6

The output resistance, R_{out} is the reciprocal of the slope of I_{OUT}-V_{OUT},

$$R_{out} = (\frac{I_{OUT}}{V_{OUT}})^{-1}$$

The results are shown in figure 14,

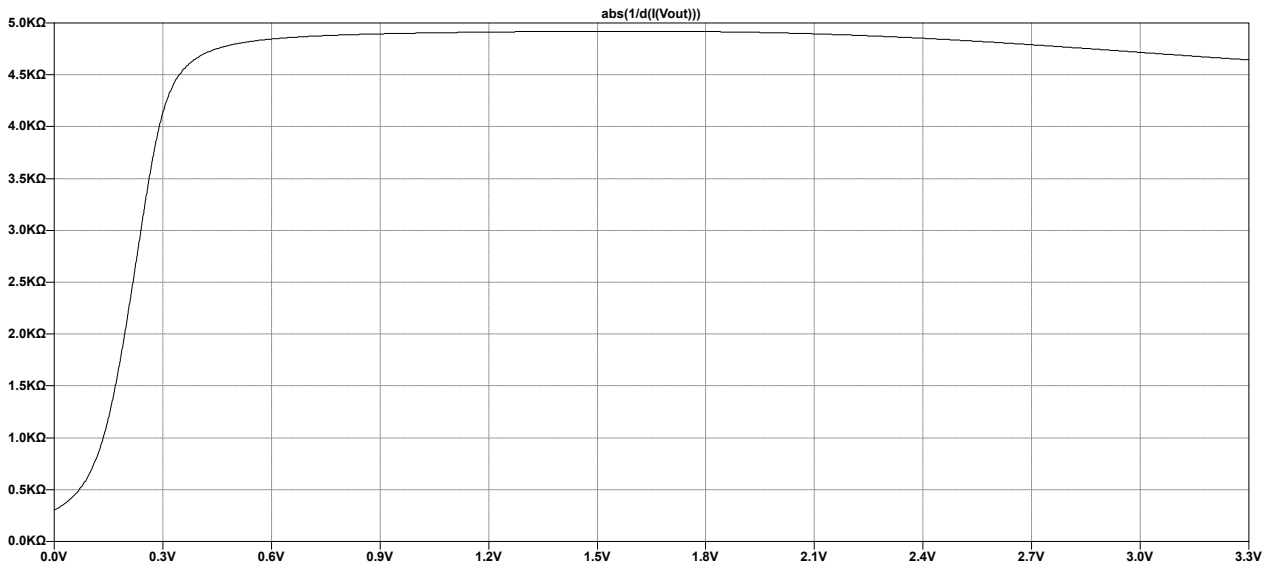Figure 14: R_{OUT}-V_{OUT}

From figure 15,
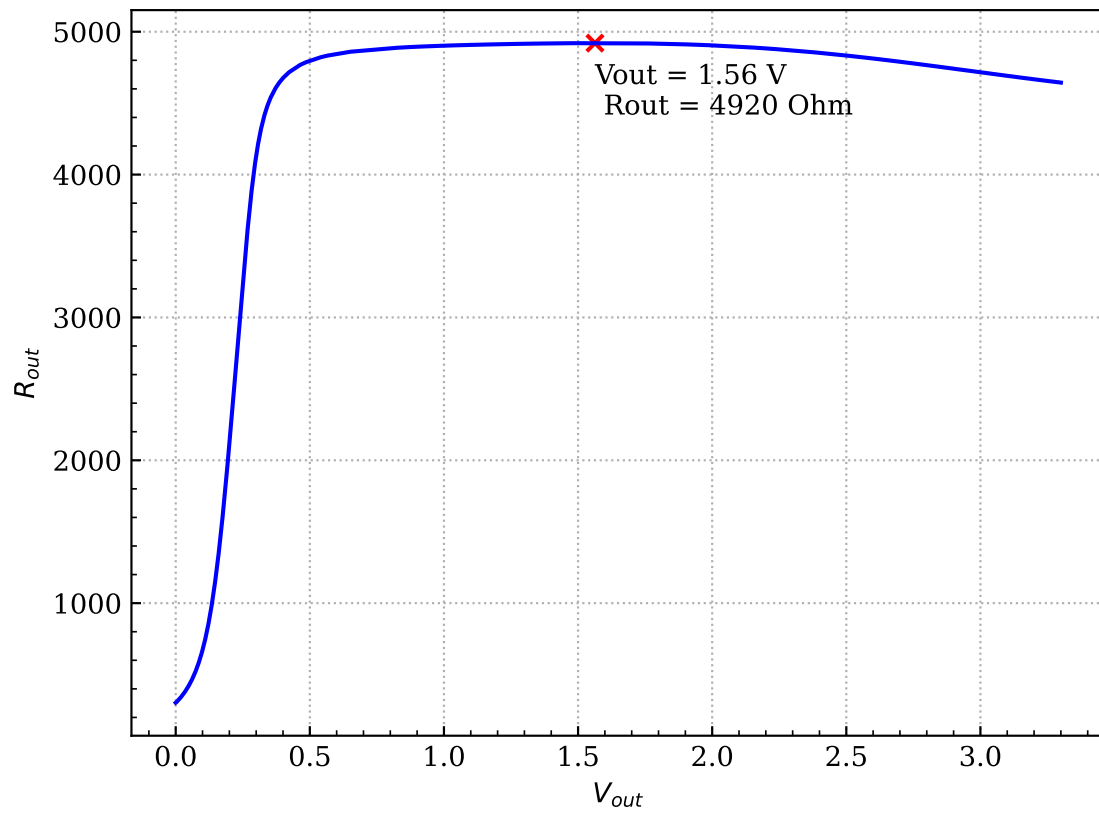max(R_{out}) = 4920 Ω
when,
V_{out} = 1.53 V

Vout = 1.56 V
Rout = 4920 Ohm

Figure 15: Maximum R$_{out}$