EE4C10 Analog Circuit Design Fundamentals

Homework Assignment II

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Contents

Problem 1

(a) Overdrive voltage, V_{gt}, for:

1. M1

$$I_{D1} = \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_1 (V_{GS_1} - V_{TH_1})^2 (1 + \lambda_1 V_{DS_1})$$
$$I_{D1} \approx \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_1 (V_{gt_1})^2$$
$$V_{gt_1} \approx \sqrt{\frac{2 I_{D_1}}{\mu_n C_{OX}}(\frac{L}{W})_1}$$

$$V_{gt_1} \approx 109.11 mV$$

2. M2

$$V_{gt_2} \approx \sqrt{\frac{2 I_{D_2}}{\mu_p C_{OX}}(\frac{L}{W})_2}$$

$$V_{gt_2} \approx 377.96 mV$$

(b) Small-signal gain

$$
\begin{aligned}
g_{m1}V_{in} &= \frac{-V_{out}}{r_{o1}//r_{o2}} \\
\frac{V_{out}}{V_{in}} &= -g_{m1}(r_{o1}//r_{o2})
\end{aligned}
$$

$$
\begin{aligned}
g_{m1} &= \mu_n C_{OX}(\frac{W}{L})_1 V_{gt_1} \\
&= 4.582mS
\end{aligned}
$$

$$
\begin{aligned}
r_{o1} &= \frac{1}{I_{D1}\lambda_n} \\
&= 20k\Omega
\end{aligned}
$$

$$
\begin{aligned}
r_{o2} &= \frac{1}{I_{D2}\lambda_p} \\
&= 40k\Omega
\end{aligned}
$$

$$
\frac{V_{out}}{V_{in}} \approx -61.09
$$

(c) V_{out} output swing

For M_1 to be in saturation,

$$
\begin{aligned}
V_{DS1} &\geq V_{gt1} \\
V_{out} &\geq 0.109V
\end{aligned}
$$

For M_2 to be in saturation,

$$
\begin{aligned}
V_{DS2} &\geq V_{gt2} \\
V_{DD} - V_{out} &\geq 0.377V \\
V_{out} &\leq 3.3V - 0.377V \\
V_{out} &\leq 2.923V
\end{aligned}
$$

Swing of V_{out},

$$
0.109V < V_{out} < 2.923V
$$

$$
\begin{aligned}
V_{out,pp} &= 2.923V - 0.109V \\
&= 2.814V
\end{aligned}
$$

(d)

Problem 2

(a) For M1 to be 100mV from triode,

$$
\begin{aligned}
V_{DS1} &= V_{GS1} - V_{TH,N} + 100mV \\
X &= V_{in} - V_{TH,N} + 100mV
\end{aligned}
$$

V_{in} for M1 to be in saturation with I_{D1} of 0.35 mA,

$$
\begin{aligned}
I_{D1} &= \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_1(V_{GS1}-V_{TH,N})^2 \\
I_{D1} &= \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_1(V_{in}-V_{TH,N})^2 \\
V_{in} &= \sqrt{\frac{2I_{D1}}{\mu_n C_{OX}}(\frac{L}{W})_1} + V_{TH,N} \\
&= 0.653V
\end{aligned}
$$

$$
\begin{aligned}
X &= \sqrt{\frac{2I_{D1}}{\mu_n C_{OX}}(\frac{L}{W})_1} + 100mV \\
&\approx 0.253V
\end{aligned}
$$

V_{b} for M2 to be in saturation with I_{D2} of 0.35 mA,

$$
\begin{aligned}
I_{D2} &= \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_2(V_{GS2}-V_{TH,N})^2 \\
I_{D2} &= \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_2(V_b - X - V_{TH,N})^2 \\
V_b &= \sqrt{\frac{2I_{D2}}{\mu_n C_{OX}}(\frac{L}{W})_2} + X + V_{TH,N} \\
&\approx 0.906V
\end{aligned}
$$

(b) Small-signal gain

$$
\begin{aligned}
G_m &= \frac{g_{m1}(g_{m2}r_{o1}r_{o2}+r_{o1})}{g_{m2}r_{o1}r_{o2}+r_{o1}+r_{o2}} \\
&\approx g_{m1} \\
R_{out} &= (g_{m2}r_{o1}r_{o2}+r_{o1}+r_{o2})//R_d
\end{aligned}
$$

Small-signal gain,

$$\begin{aligned}\frac{V_{out}}{V_{in}} &= -G_m R_{out} \\ &= -g_{m1}[(g_{m2}r_{o1}r_{o2} + r_{o1} + r_{o2})//R_d]\end{aligned}$$

$$\begin{aligned}g_{m1} &= \mu_n C_{OX}(\frac{W}{L})_1(V_{GS1} - V_{TH,N}) \\ &= \mu_n C_{OX}(\frac{W}{L})_1(V_{in} - V_{TH,N}) \\ &= 4.583mS\end{aligned}$$

$$\begin{aligned}g_{m2} &= \mu_n C_{OX}(\frac{W}{L})_2(V_{GS2} - V_{TH,N}) \\ &\approx \mu_n C_{OX}(\frac{W}{L})_2(V_b - X - V_{TH,N}) \\ &= 4.583mS\end{aligned}$$

$$\begin{aligned}r_{o1} &= \frac{1}{I_{D1}\lambda_n} \\ &= 28.571k\Omega\end{aligned}$$

$$\begin{aligned}r_{o2} &= \frac{1}{I_{D2}\lambda_p} \\ &= 28.571k\Omega\end{aligned}$$

$$\frac{V_{out}}{V_{in}} \approx -22.88$$

(c) Assume V_b to be 1.65V, For M2 to be in saturation,

$$\begin{aligned}V_{out} - X &\geq V_b - X - V_{TH,N} \\ V_{out} &\geq 1.15V\end{aligned}$$

When $I_D \geq 0$,

$$\begin{aligned}V_{out} &\leq V_{DD} \\ 1.15V \leq V_{out} &\leq 3.3V\end{aligned}$$

$$V_{out,pp} = 2.15V$$

(d) X_{pp}

Gain of X,

$$\begin{aligned}\frac{X}{V_{in}} &= \frac{-g_{m1}}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}} \\ &\approx \frac{-g_{m1}}{g_{m2}} \\ &\approx -1\end{aligned}$$

X_{pp}

$$\begin{aligned}\frac{X}{V_{out}} &= \frac{X}{V_{in}}\frac{V_{in}}{V_{out}} \\ &= \frac{1}{22.88}\end{aligned}$$

$$X_{pp} = 54.63mV$$

Problem 3

(a) Sketch of:

1. Output voltage

img/q3/output-voltage-sketch.pdf

Figure 1: Output voltage sketch

2. Small-signal gain

img/q3/small-signal-sketch.pdf

Figure 2: Small-signal gain

(b) Small-signal model

img/q3/small-signal-model.pdf

Figure 3: Small-signal model of folded-cascode stage

$$R_{out} = g_{m2}r_{o1}r_{o2} + r_{o1} + r_{o2}$$
$$\approx g_{m2}r_{o1}r_{o2}$$
$$G_m = \frac{-g_{m1}(g_{m2} + \frac{1}{r_{o1}})}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$
$$\approx -g_{m1}$$

$$\frac{V_{out}}{V_{in}} = g_{m1}g_{m2}r_{o1}r_{o2}$$

Problem 4

(a) V_{out} - V_{in} characteristics

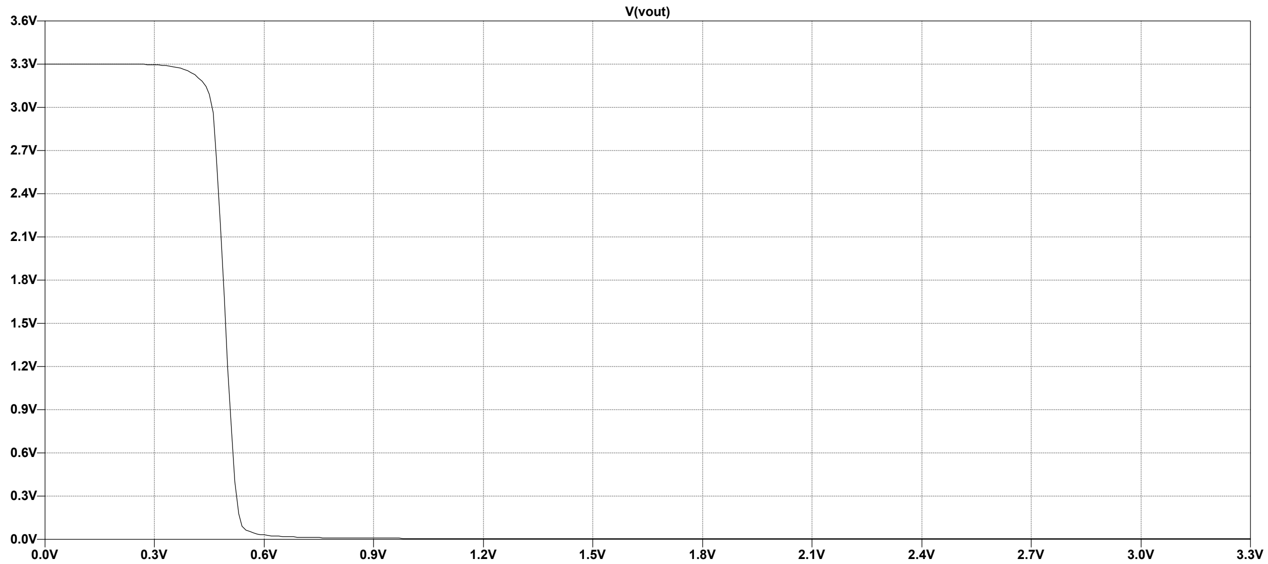

Figure 4: V_{out} - V_{in} characteristics

(b) Small-signal gain, $\frac{dV_{out}}{dV_{in}}$

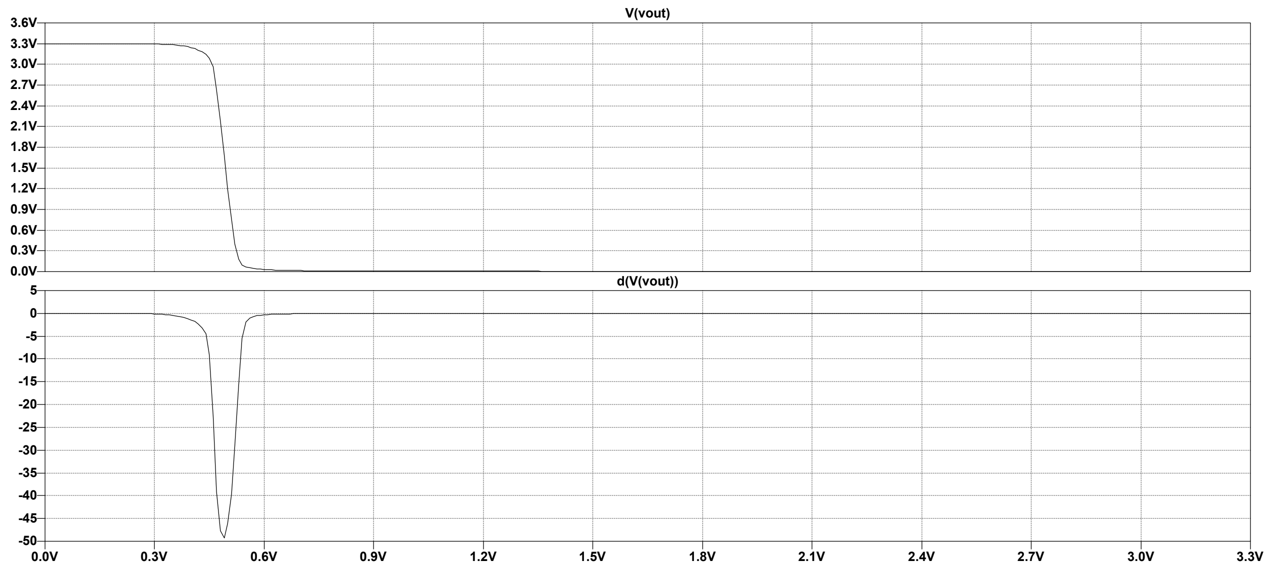

Figure 5: Small-signal gain, $\frac{dV_{out}}{dV_{in}}$

From figure 5 the gain when:

1. $V_{out} = 0.6$ V
   $\frac{dV_{out}}{dV_{in}} = -35.59$
2. $V_{out} = 2.8$ V
   $\frac{dV_{out}}{dV_{in}} = -32.77$

(c) From figure 5, the input voltage, V_{in}, for maximum gain, $max(|\frac{dV_{out}}{dV_{in}}|)$ is given to be:

$$max(|\frac{dV_{out}}{dV_{in}}|) = 50.07$$
$$V_{in} = 489mV$$

(d) Output voltage swing for gain of 1,

$$V_{out,max} = 3.24V$$
$$V_{out,min} = 56mV$$
$$V_{out,pp} = 3.184V$$

Output peak to peak voltage

$$V_{out,pp} = 3.184V$$

(e) Small-signal voltage gain when:

1. $V_{out} = 0.6$ V, $V_{in} = 0.514$ V

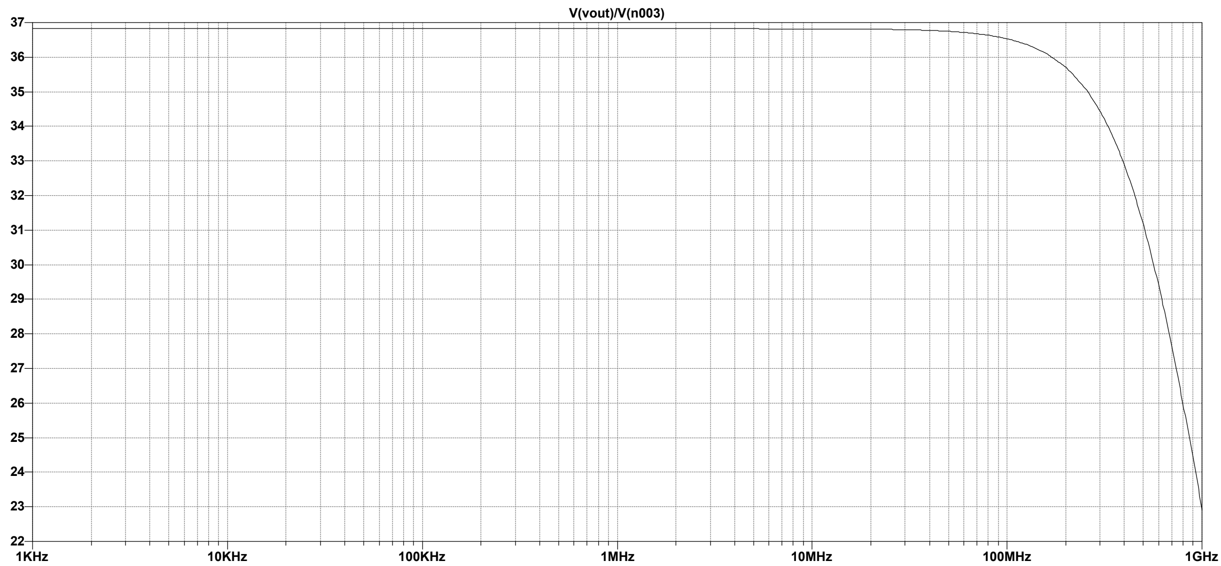

Figure 6: Small-signal gain, $|\frac{V_{out}}{V_{in}}|$, $V_{out} = 0.6$ V, $V_{in} = 0.514$ V

   Gain = 36.82
2. $V_{out} = 2.8$ V, $V_{in} = 0.464$ V

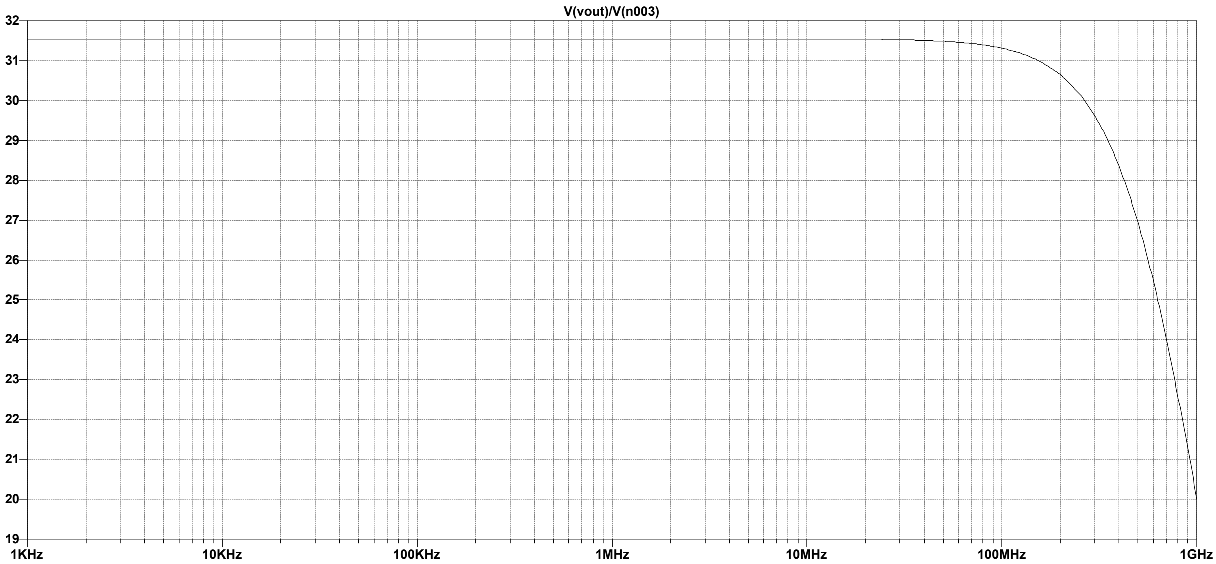

Figure 7: Small-signal gain, $|\frac{V_{out}}{V_{in}}|$, $\mathrm{V_{out}} = 2.8$ V, $\mathrm{V_{in}} = 0.464$ V

Gain = 31.54

Problem 5

(a) Procedure for designing $\mathrm{V_b}$ for M1 to be 100mV away from triode,

(a) Select an initial value $\mathrm{V_b}$ for M1 and M2 to be in saturation when $\mathrm{V_{in}} > \mathrm{V_{TH1}}$. In figure 8, $\mathrm{V_b} = 1.5$ V. From the error logfile 9, $\mathrm{V_{DS1}} > \mathrm{V_{GS1}} - \mathrm{V_{TH1}}$ and $\mathrm{V_{DS2}} > \mathrm{V_{GS2}} - \mathrm{V_{TH2}}$.

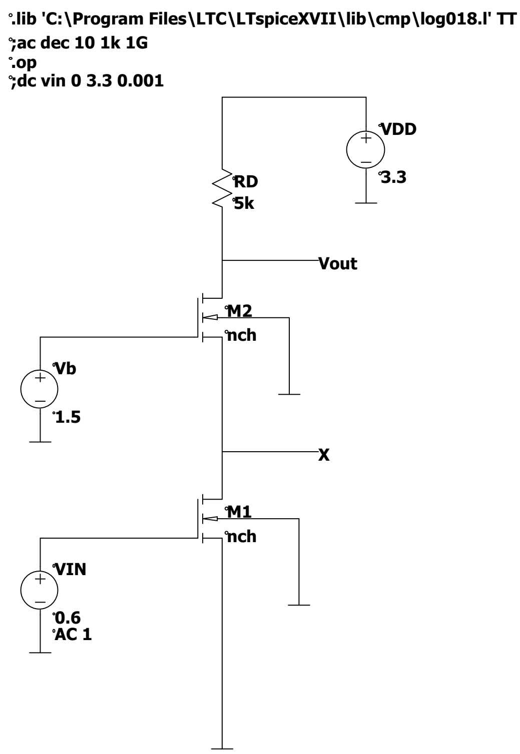

Figure 8: Testbench for Q5

```
Semiconductor Device Operating Points:
                         --- BSIM3 MOSFETS ---
Name:          m2          m1
Model:       nch.3       nch.3
Id:        3.74e-04    3.74e-04
Vgs:       7.73e-01    6.00e-01
Vds:       7.05e-01    7.27e-01
Vbs:      -7.27e-01    0.00e+00
Vth:       6.78e-01    4.97e-01
Vdsat:     1.18e-01    1.35e-01
Gm:        4.21e-03    4.42e-03
Gds:       1.29e-04    4.49e-05
Gmb        7.98e-04    1.27e-03
Cbd:       0.00e+00    0.00e+00
Cbs:       0.00e+00    0.00e+00
Cgsov:     6.59e-15    1.32e-14
Cgdov:     6.59e-15    1.32e-14
Cgbov:     3.18e-17    6.47e-17
dQgdVgb:   3.21e-14    1.00e-13
dQgdVdb:  -6.47e-15   -1.32e-14
dQgdVsb:  -2.27e-14   -7.78e-14
dQddVgb:  -6.61e-15   -1.33e-14
dQddVdb:   6.60e-15    1.32e-14
dQddVsb:   2.09e-17    9.01e-17
dQbdVgb:  -3.63e-15   -1.24e-14
dQbdVdb:   1.50e-17    2.88e-18
dQbdVsb:  -2.22e-15   -1.44e-14
```

Figure 9: Semiconductor Device Operating Points

(b) Doing a DC-sweep for I_{D1}-V_{in}, $V_{in} \approx 0.6V$ when $I_{D1} = 0.35mA$.

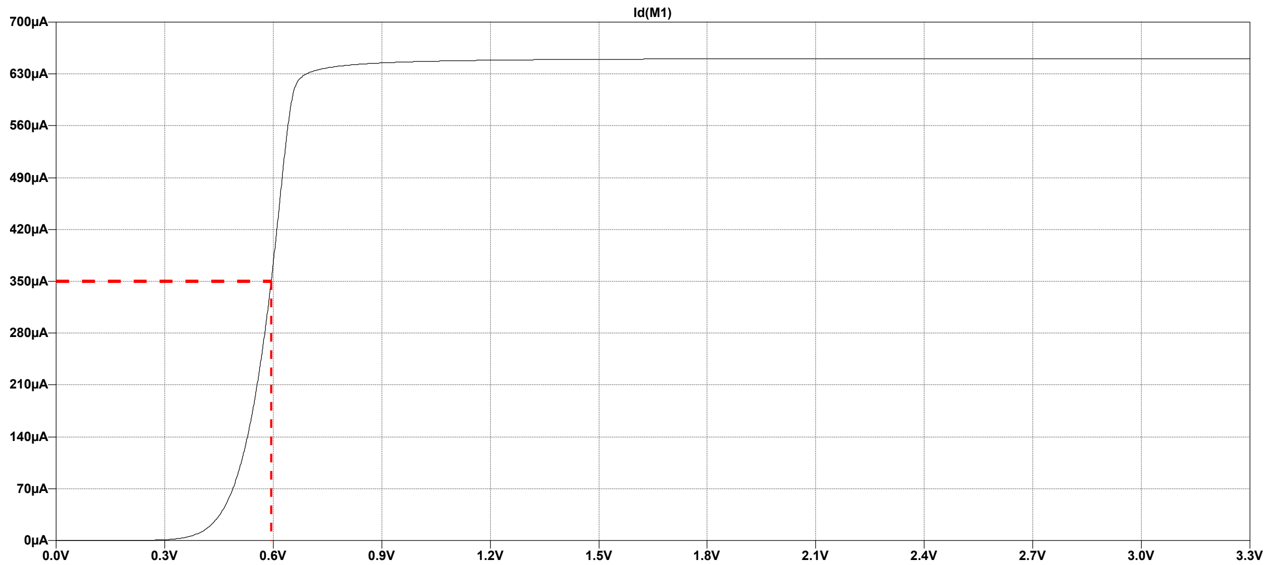

Figure 10: I_{D1}-V_{in}

(c) From the operation point simulation int figure 9, V_{TH1} is determined to be 0.497V. For M1 to be 100mV from triode region,

$$\begin{aligned} V_x &= V_{in} - V_{TH1} + 100mV \\ &\approx 0.2V \end{aligned}$$

(d) DC-sweeping V_x-V_b and determining V_b for $V_x = 0.2V$. From the figure 11 , $V_b = 0.825V$.

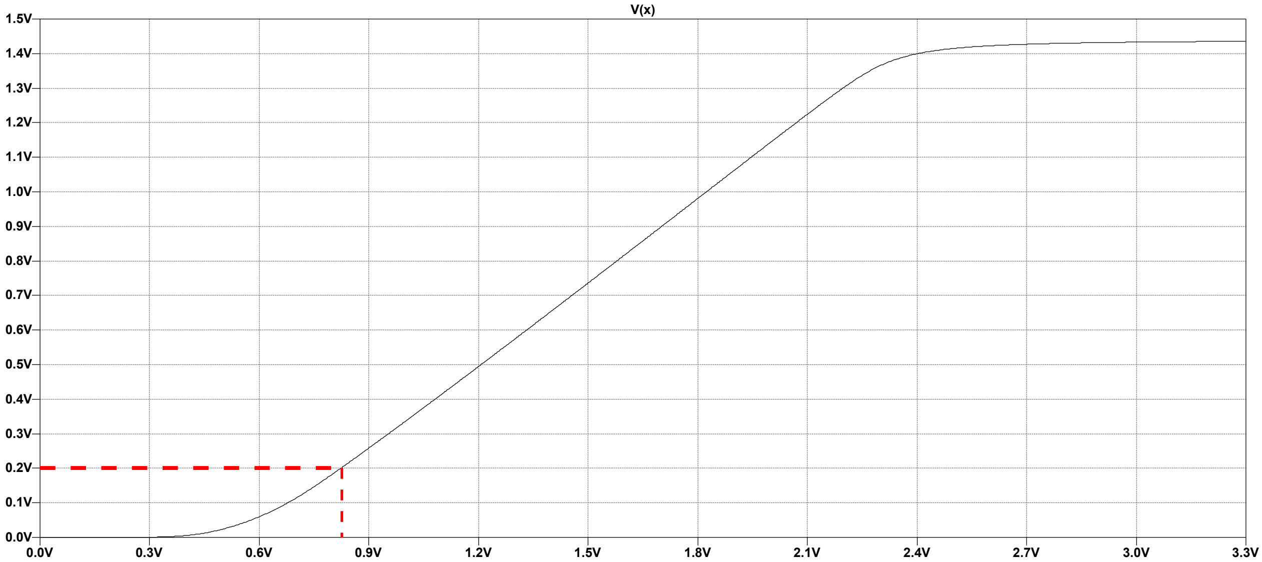

Figure 11: V_x-V_b