EE4C10 Analog Circuit Design Fundamentals

Homework Assignment II

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Contents

Problem 1

(a) Overdrive voltage, V_{gt}, for:

1. M1

$$I_{D1} = \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_1 (V_{GS_1} - V_{TH_1})^2 (1 + \lambda_1 V_{DS_1})$$
$$I_{D1} \approx \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_1 (V_{gt_1})^2$$
$$V_{gt_1} \approx \sqrt{\frac{2I_{D_1}}{\mu_n C_{OX}}(\frac{L}{W})_1}$$

$$V_{gt_1} \approx 109.11mV$$

2. M2

$$V_{gt_2} \approx \sqrt{\frac{2I_{D_2}}{\mu_p C_{OX}}(\frac{L}{W})_2}$$

$$V_{gt_2} \approx 377.96mV$$

(b) Small-signal gain

$$g_{m1}V_{in} = \frac{-V_{out}}{r_{o1}//r_{o2}}$$
$$\frac{V_{out}}{V_{in}} = -g_{m1}(r_{o1}//r_{o2})$$

$$g_{m1} = \mu_n C_{OX}(\frac{W}{L})_1 V_{gt_1}$$
$$= 4.582mS$$

$$r_{o1} = \frac{1}{I_{D1}\lambda_n}$$
$$= 20k\Omega$$

$$r_{o2} = \frac{1}{I_{D2}\lambda_p}$$
$$= 40k\Omega$$

$$\frac{V_{out}}{V_{in}} \approx -61.09$$

(c) V_{out} output swing

For M_1 to be in saturation,

$$V_{DS1} \geq V_{gt1}$$
$$V_{out} \geq 0.109V$$

For M_2 to be in saturation,

$$V_{DS2} \geq V_{gt2}$$
$$V_{DD} - V_{out} \geq 0.377V$$
$$V_{out} \leq 3.3V - 0.377V$$
$$V_{out} \leq 2.923V$$

Swing of V_{out},

$$0.109V < V_{out} < 2.923V$$

$$V_{out,pp} = 2.923V - 0.109V$$
$$= 2.814V$$

(d)

Problem 2

(a) For M1 to be 100mV from triode,

$$V_{DS1} = V_{GS1} - V_{TH,N} + 100mV$$
$$X = V_{in} - V_{TH,N} + 100mV$$

V_{in} for M1 to be in saturation with I_{D1} of 0.35 mA,

$$
\begin{aligned}
I_{D1} &= \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_1(V_{GS1} - V_{TH,N})^2 \\
I_{D1} &= \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_1(V_{in} - V_{TH,N})^2 \\
V_{in} &= \sqrt{\frac{2I_{D1}}{\mu_n C_{OX}}(\frac{L}{W})_1} + V_{TH,N} \\
&= 0.653V
\end{aligned}
$$

$$
\begin{aligned}
X &= \sqrt{\frac{2I_{D1}}{\mu_n C_{OX}}(\frac{L}{W})_1} + 100mV \\
&\approx 0.253V
\end{aligned}
$$

V_b for M2 to be in saturation with I_{D2} of 0.35 mA,

$$
\begin{aligned}
I_{D2} &= \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_2(V_{GS2} - V_{TH,N})^2 \\
I_{D2} &= \frac{\mu_n C_{OX}}{2}(\frac{W}{L})_2(V_b - X - V_{TH,N})^2 \\
V_b &= \sqrt{\frac{2I_{D2}}{\mu_n C_{OX}}(\frac{L}{W})_2} + X + V_{TH,N} \\
&\approx 0.906V
\end{aligned}
$$

(b) Small-signal gain

$$
\begin{aligned}
G_m &= \frac{g_{m1}(g_{m2}r_{o1}r_{o2} + r_{o1})}{g_{m2}r_{o1}r_{o2} + r_{o1} + r_{o2}} \\
&\approx g_{m1} \\
R_{out} &= (g_{m2}r_{o1}r_{o2} + r_{o1} + r_{o2})//R_d
\end{aligned}
$$

Small-signal gain,

$$\begin{aligned}\frac{V_{out}}{V_{in}} &= -G_m R_{out} \\ &= -g_{m1}[(g_{m2} r_{o1} r_{o2} + r_{o1} + r_{o2})//R_d]\end{aligned}$$

$$\begin{aligned}g_{m1} &= \mu_n C_{OX}(\frac{W}{L})_1 (V_{GS1} - V_{TH,N}) \\ &= \mu_n C_{OX}(\frac{W}{L})_1 (V_{in} - V_{TH,N}) \\ &= 4.583mS\end{aligned}$$

$$\begin{aligned}g_{m2} &= \mu_n C_{OX}(\frac{W}{L})_2 (V_{GS2} - V_{TH,N}) \\ &\approx \mu_n C_{OX}(\frac{W}{L})_2 (V_b - X - V_{TH,N}) \\ &= 4.583mS\end{aligned}$$

$$\begin{aligned}r_{o1} &= \frac{1}{I_{D1}\lambda_n} \\ &= 28.571k\Omega\end{aligned}$$

$$\begin{aligned}r_{o2} &= \frac{1}{I_{D2}\lambda_p} \\ &= 28.571k\Omega\end{aligned}$$

$$\frac{V_{out}}{V_{in}} \approx -22.88$$

(c) Assume V_b to be 1.65V, For M2 to be in saturation,

$$\begin{aligned}V_{out} - X &\geq V_b - X - V_{TH,N} \\ V_{out} &\geq 1.15V\end{aligned}$$

When $I_D \geq 0$,

$$\begin{aligned}V_{out} &\leq V_{DD} \\ 1.15V \leq V_{out} &\leq 3.3V\end{aligned}$$

$$V_{out,pp} = 2.15V$$

(d) X_{pp}

Gain of X,

$$\begin{aligned}\frac{X}{V_{in}} &= \frac{-g_{m1}}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}} \\ &\approx \frac{-g_{m1}}{g_{m2}} \\ &\approx -1\end{aligned}$$

X_{pp}

$$\begin{aligned}\frac{X}{V_{out}} &= \frac{X}{V_{in}}\frac{V_{in}}{V_{out}} \\ &= \frac{1}{22.88}\end{aligned}$$

$$X_{pp} = 54.63mV$$

Problem 3

(a) Sketch of:

1. Output voltage

img/q3/output-voltage-sketch.pdf

Figure 1: Output voltage sketch

2. Small-signal gain

img/q3/small-signal-sketch.pdf

Figure 2: Small-signal gain

(b) Small-signal model

img/q3/small-signal-model.pdf

Figure 3: Small-signal model of folded-cascode stage

$$
\begin{aligned}
R_{out} &= g_{m2}r_{o1}r_{o2} + r_{o1} + r_{o2} \\
&\approx g_{m2}r_{o1}r_{o2} \\
G_m &= \frac{-g_{m1}(g_{m2} + \frac{1}{r_{o1}})}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}} \\
&\approx -g_{m1}
\end{aligned}
$$

$$
\frac{V_{out}}{V_{in}} = g_{m1}g_{m2}r_{o1}r_{o2}
$$

Problem 4

(a) V_{out} - V_{in} characteristics

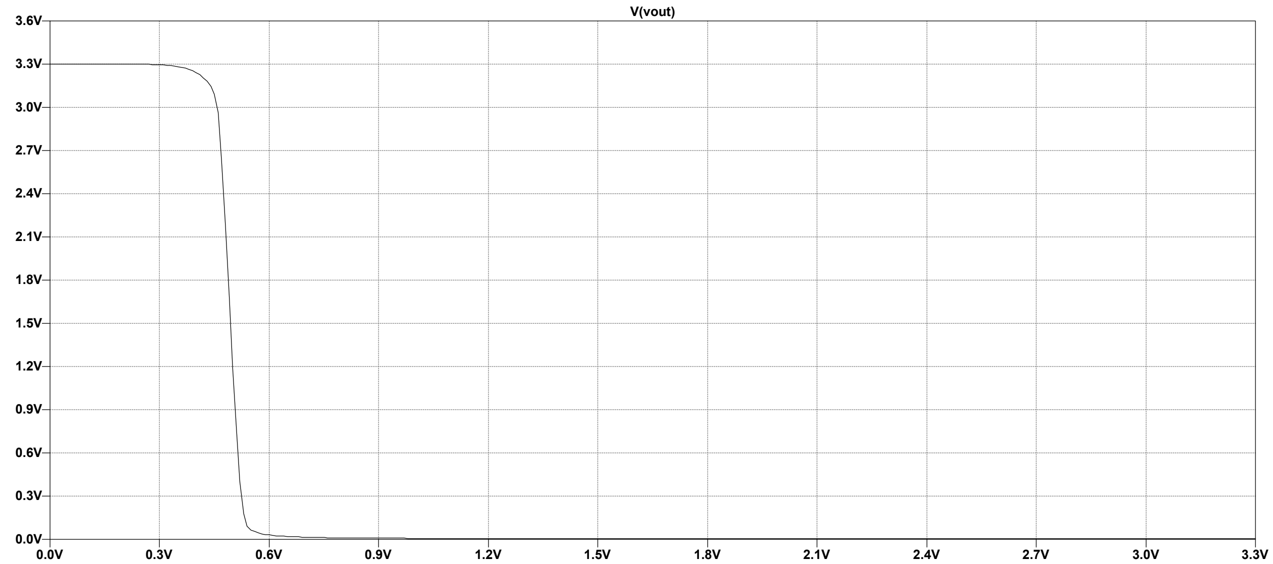

Figure 4: V_{out} - V_{in} characteristics

(b) Small-signal gain, $\frac{dV_{out}}{dV_{in}}$

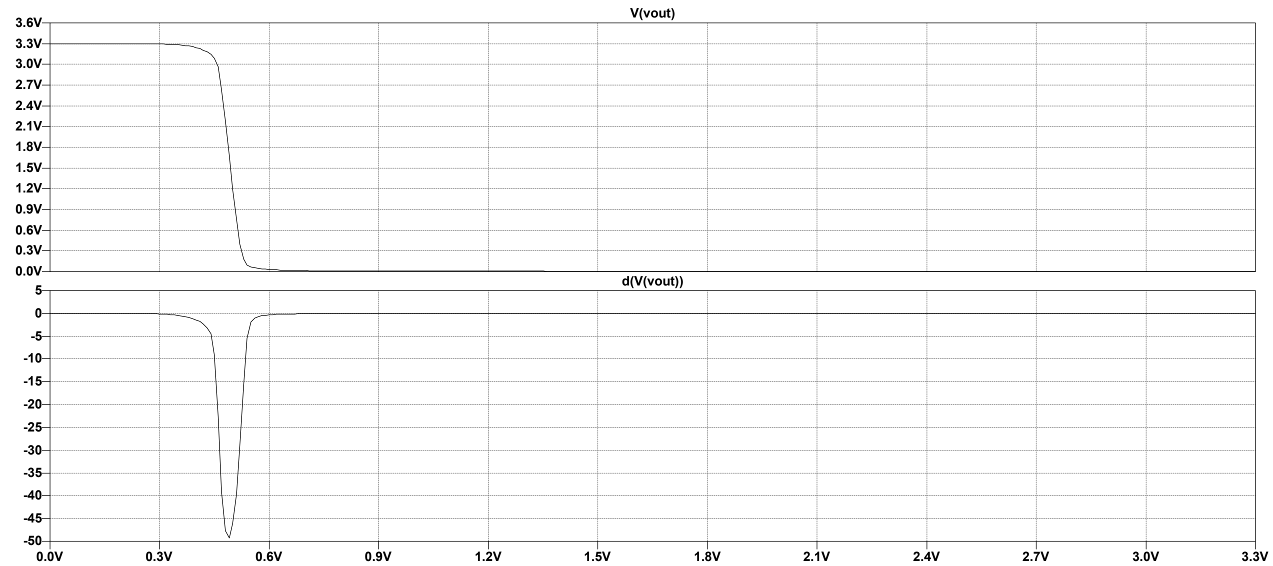

Figure 5: Small-signal gain, $\frac{dV_{out}}{dV_{in}}$

From figure 5 the gain when:

1. $V_{out} = 0.6$ V
   $\frac{dV_{out}}{dV_{in}} = -35.59$
2. $V_{out} = 2.8$ V
   $\frac{dV_{out}}{dV_{in}} = -32.77$

(c) From figure 5, the input voltage, V_{in}, for maximum gain, $max(|\frac{dV_{out}}{dV_{in}}|)$ is given to be:

$$max(|\frac{dV_{out}}{dV_{in}}|) = 50.07$$
$$V_{in} = 489mV$$

(d) Output voltage swing for gain of 1,

$$V_{out,max} = 3.24V$$
$$V_{out,min} = 56mV$$
$$V_{out,pp} = 3.184V$$

Output peak to peak voltage

$$V_{out,pp} = 3.184V$$

(e) Small-signal voltage gain when:

1. $V_{out} = 0.6$ V, $V_{in} = 0.514$ V

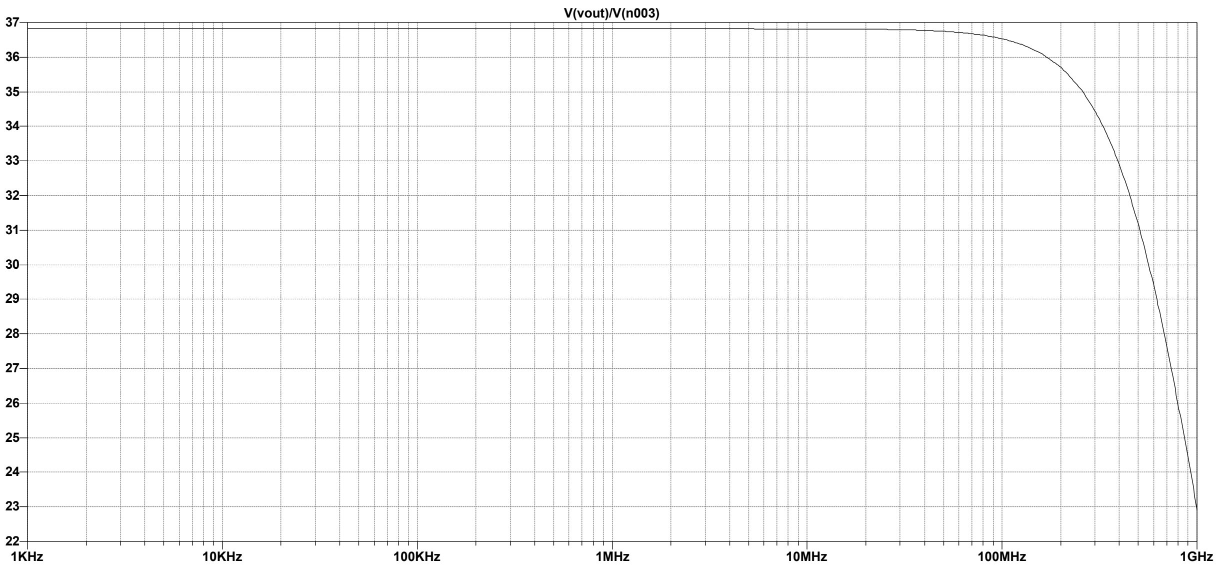

Figure 6: Small-signal gain, $|\frac{V_{out}}{V_{in}}|$, $V_{out} = 0.6$ V, $V_{in} = 0.514$ V

   Gain = 36.82

2. $V_{out} = 2.8$ V, $V_{in} = 0.464$ V

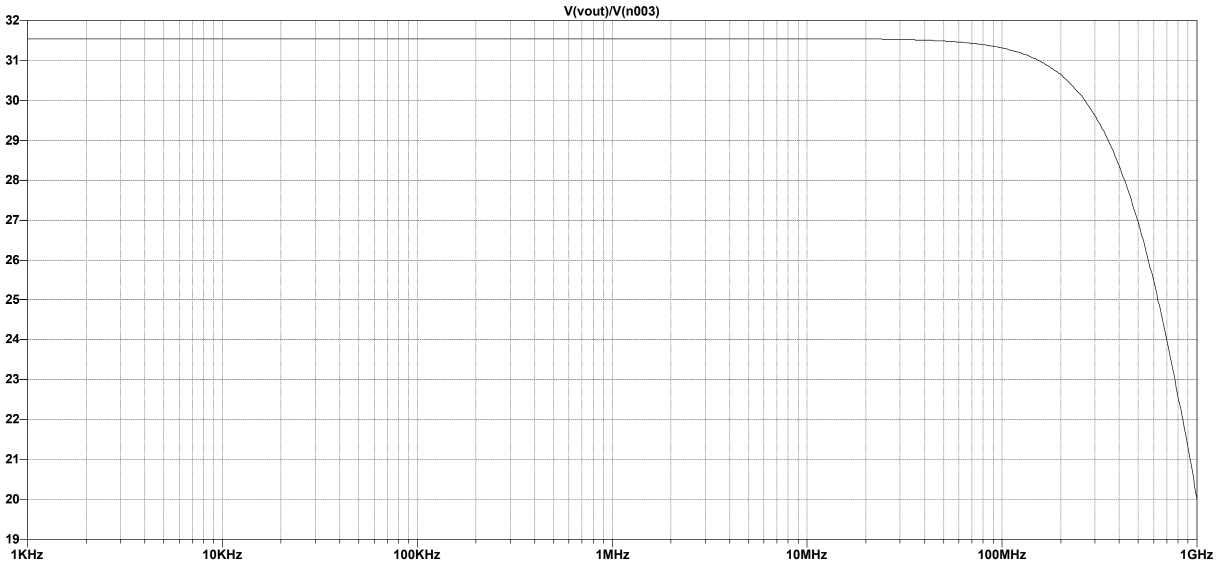

Figure 7: Small-signal gain, $|\frac{V_{out}}{V_{in}}|$, $\mathrm{V_{out}} = 2.8$ V, $\mathrm{V_{in}} = 0.464$ V

Gain $= 31.54$

Problem 5