THE UNIVERSITY OF SYDNEY

NUS National University of Singapore

UTS:QSI CENTRE FOR QUANTUM SOFTWARE AND INFORMATION

Moderate Deviation Analysis for Classical-Quantum Channels

Speaker: Hao-Chung Cheng

arXiv:1701.03114

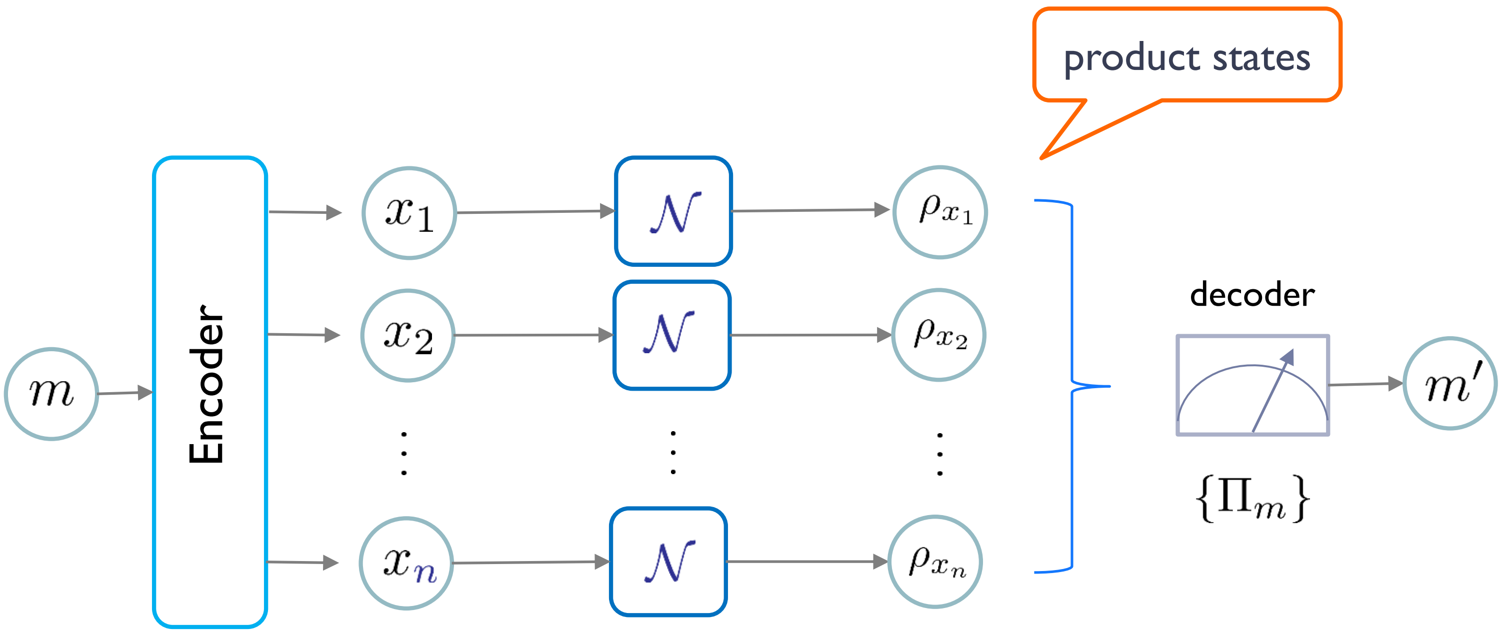
Moderate deviation analysis for classical communication over quantum channels
by Christopher Chubb (USyd), Vincent Tan (NUS), and Marco Tomamichel (UTS)

arXiv:1701.03195

Moderate deviation analysis for classical-quantum channels and quantum hypothesis testing
by Hao-Chung Cheng and Min-Hsiu Hsieh (UTS)

Motivations

- Communications over a classical-quantum channel:



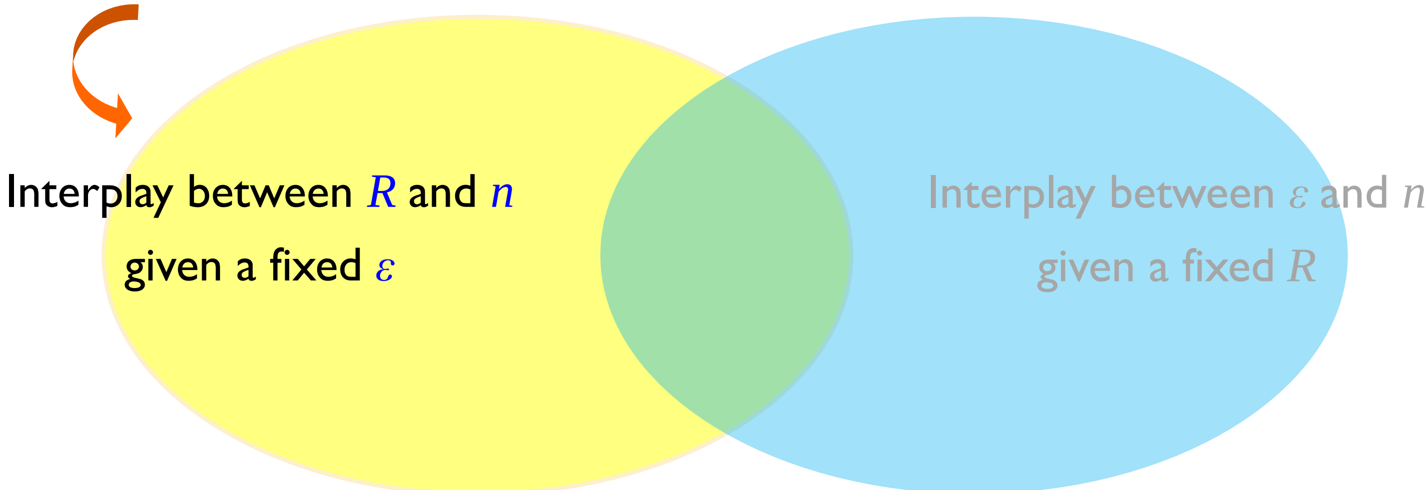
Three Parameters

trade-off

- *Blocklength* – the total number of channel uses
- *Rate* – the amount of information (bits) transmitted per channel use
- *Optimal error probability* – $\Pr\{m' \neq m\}$ of the best coding scheme

- Investigate the interplay between the optimum probability of error ε, transmission rate R, and the blocklength n

Interplay of Two Parameters (1/2)



- Small deviation regime
  - $R^*(n, \epsilon)$: maximum rate using n times channel with error below ϵ

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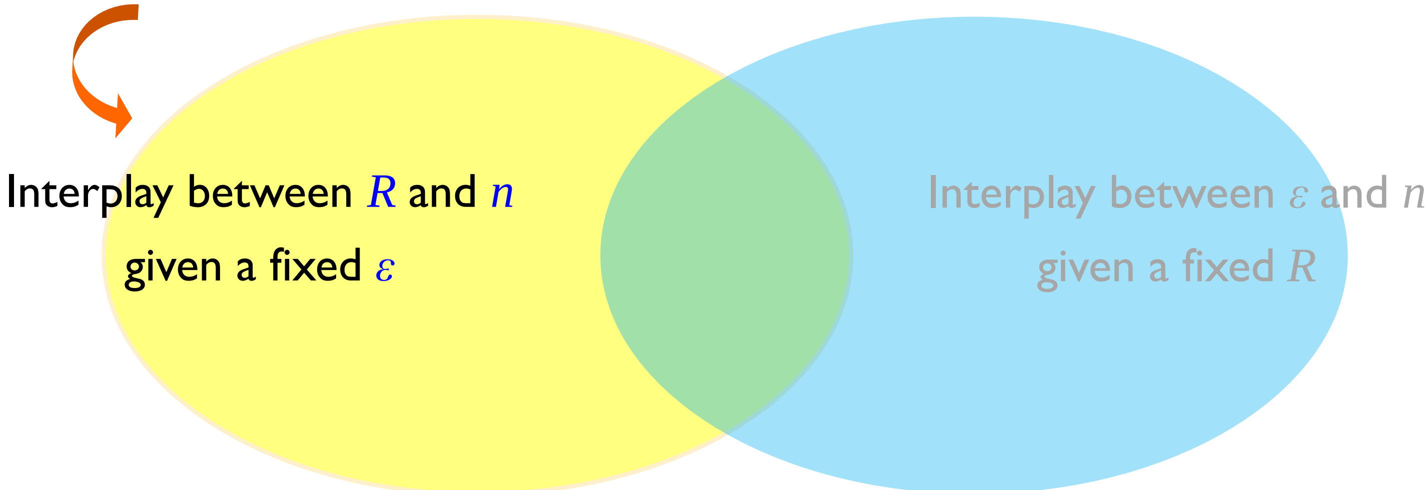

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$$\lim_{n \to \infty} R^*(n, \epsilon) = \mathsf{C}, \ \forall \epsilon \in (0, 1)$$

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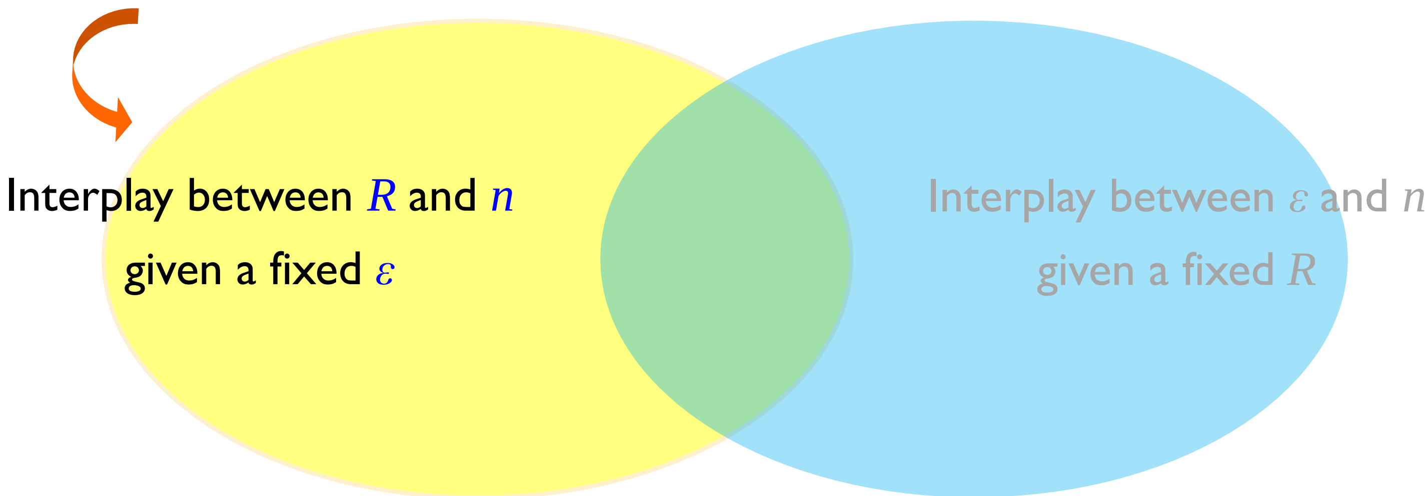

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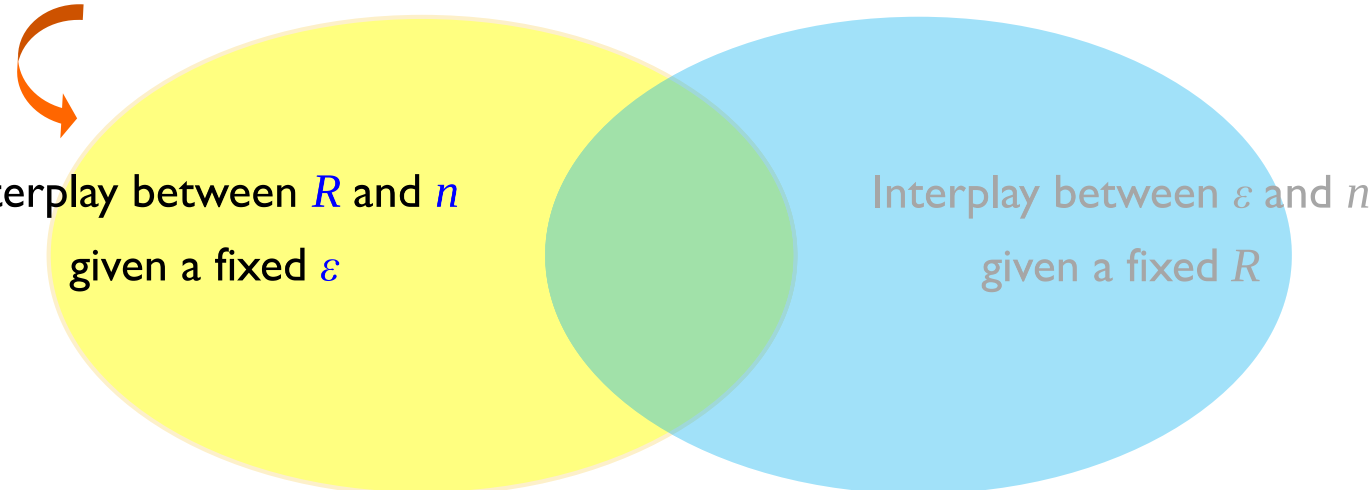

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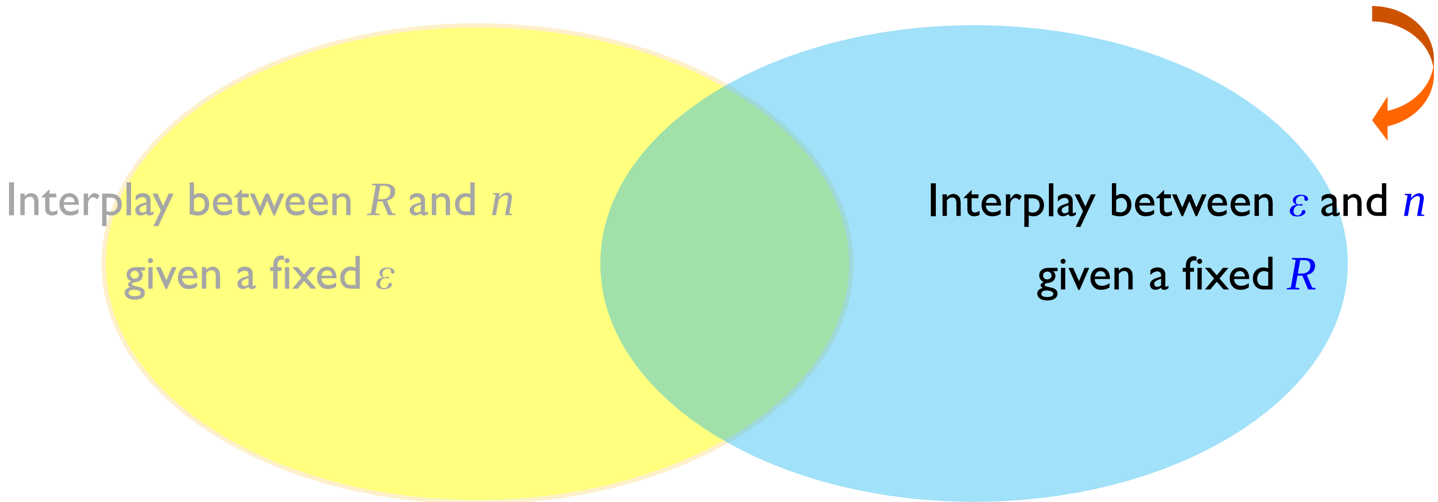
$$\Rightarrow R^*(n,\epsilon) = \mathsf{C} + \sqrt{\frac{\mathsf{V}}{n}}\Phi^{-1}(\epsilon) + O\left(\frac{\log n}{n}\right)$$

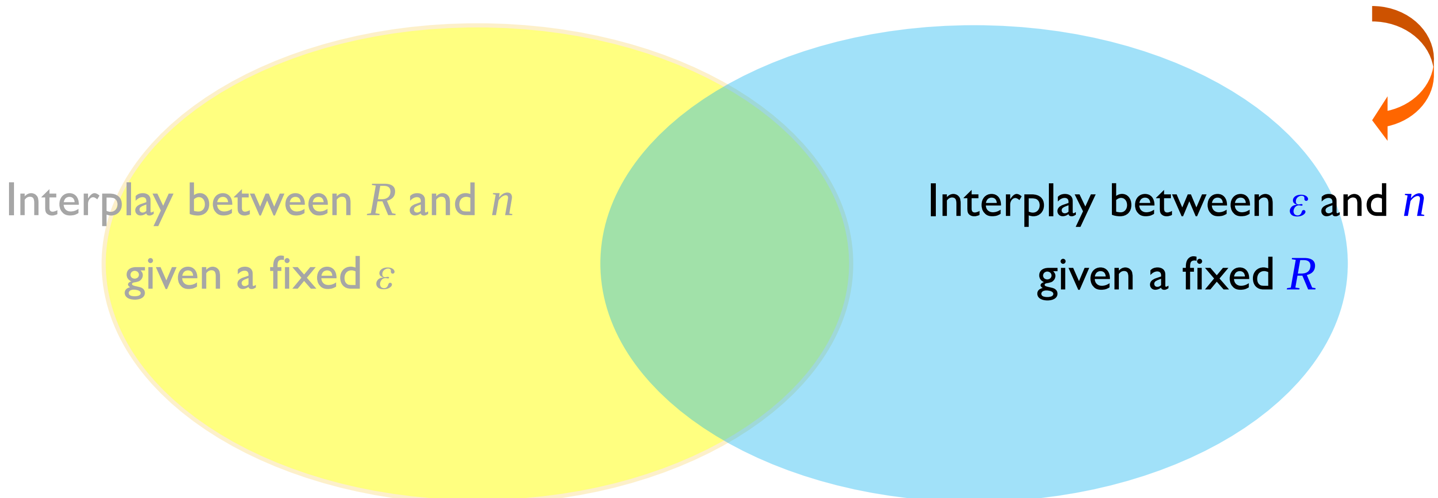
Second-order analysis

Interplay of Two Parameters (2/2)



- Large deviation regime

Interplay of Two Parameters (2/2)



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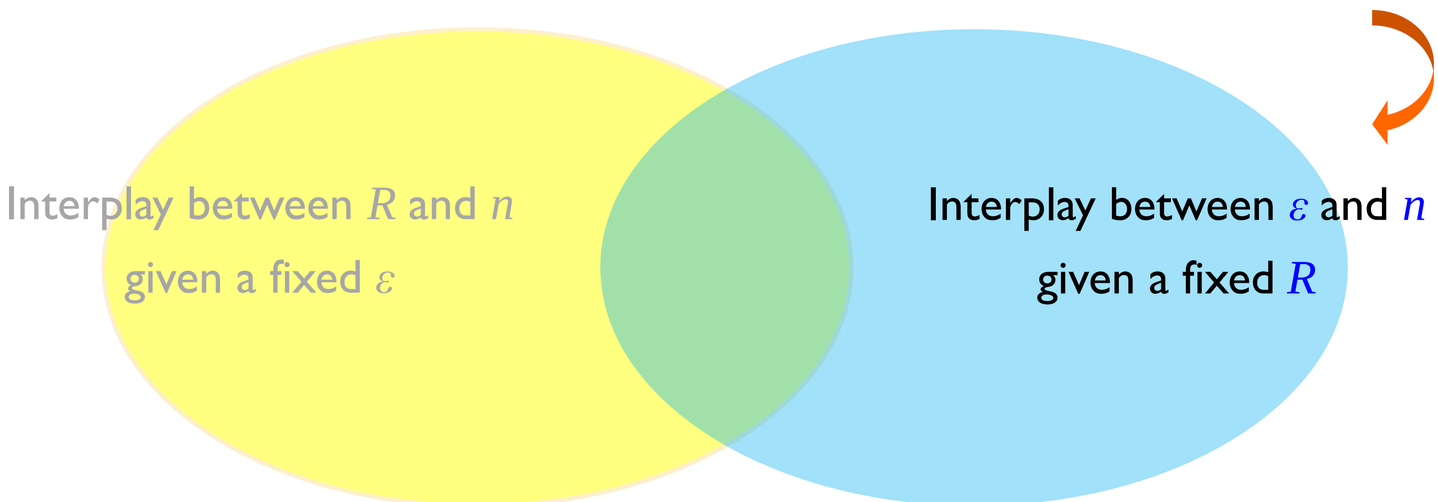

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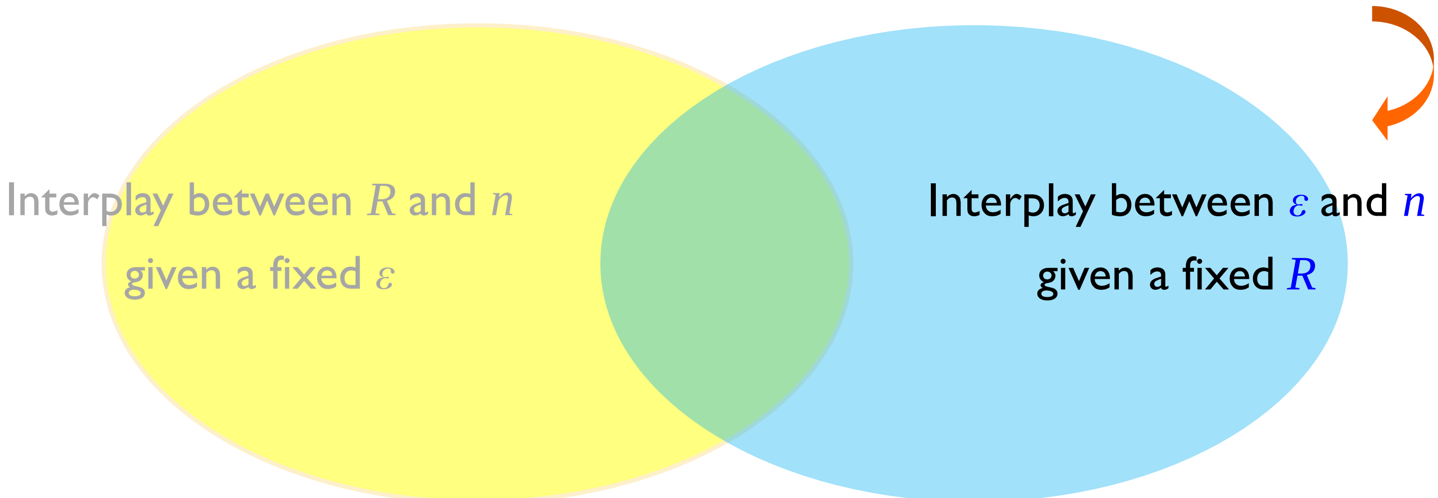

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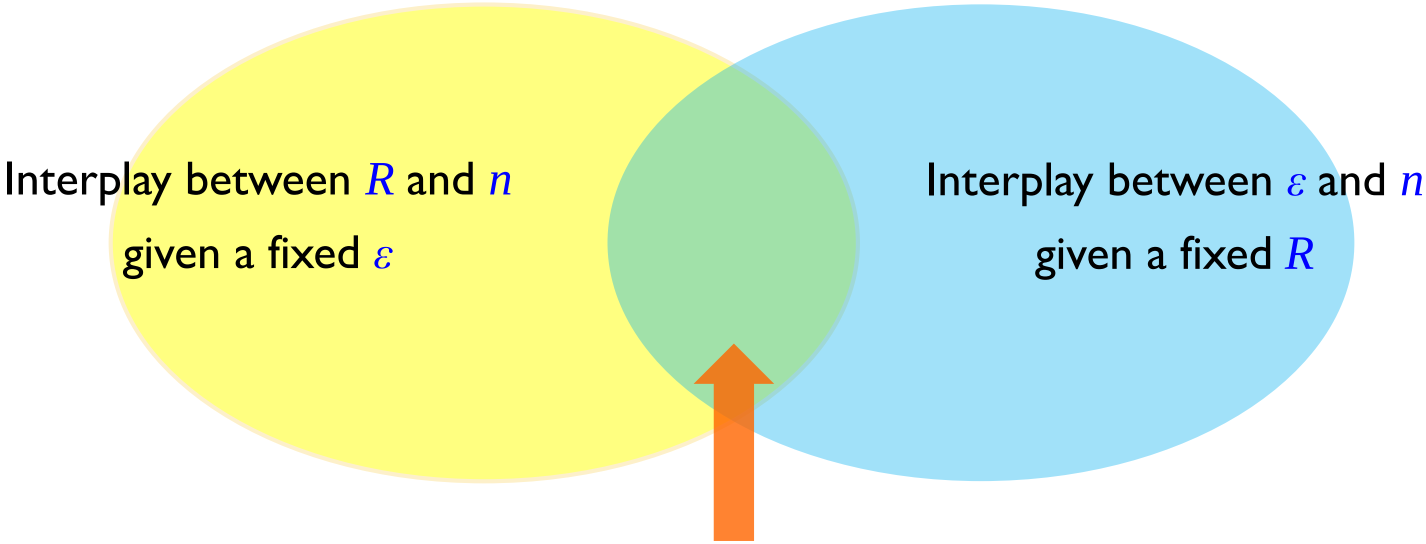
$$\Rightarrow \epsilon^*(n,R) = \exp\{-n\mathsf{E}(R) + o(n)\}$$

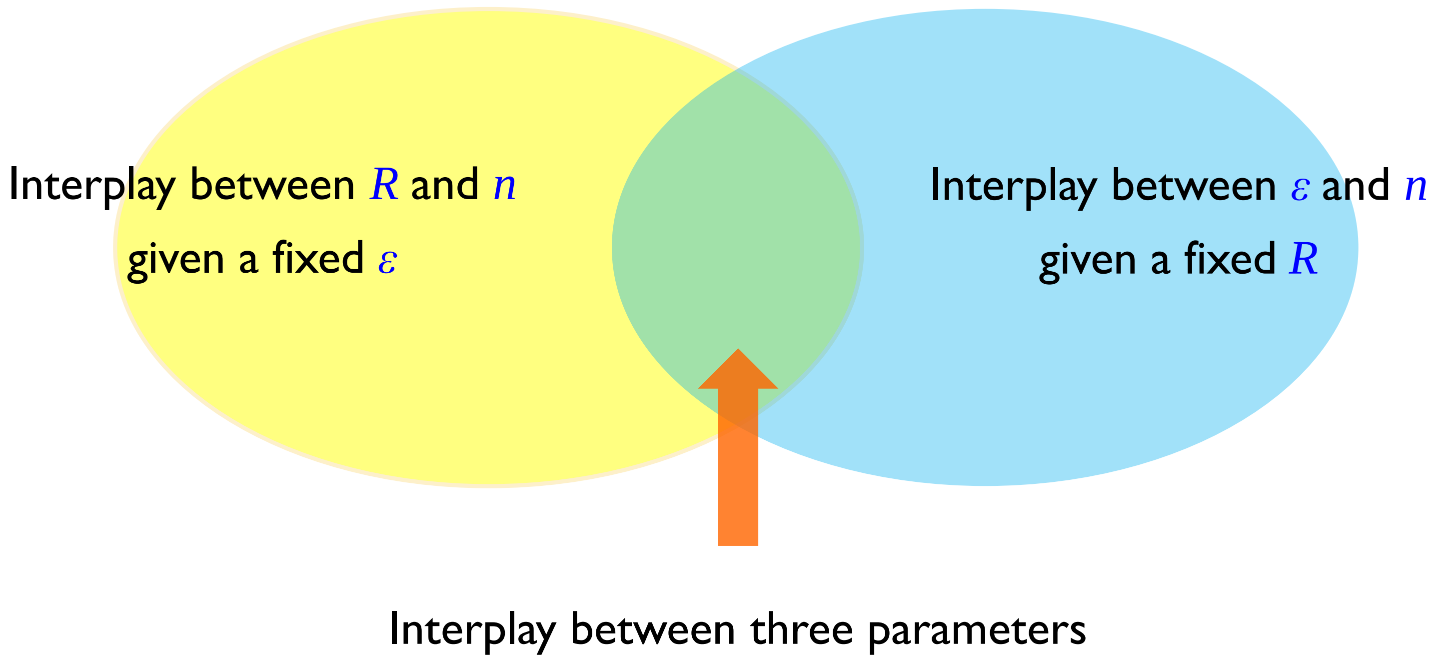
Error exponent analysis

Interplay of Three Parameters



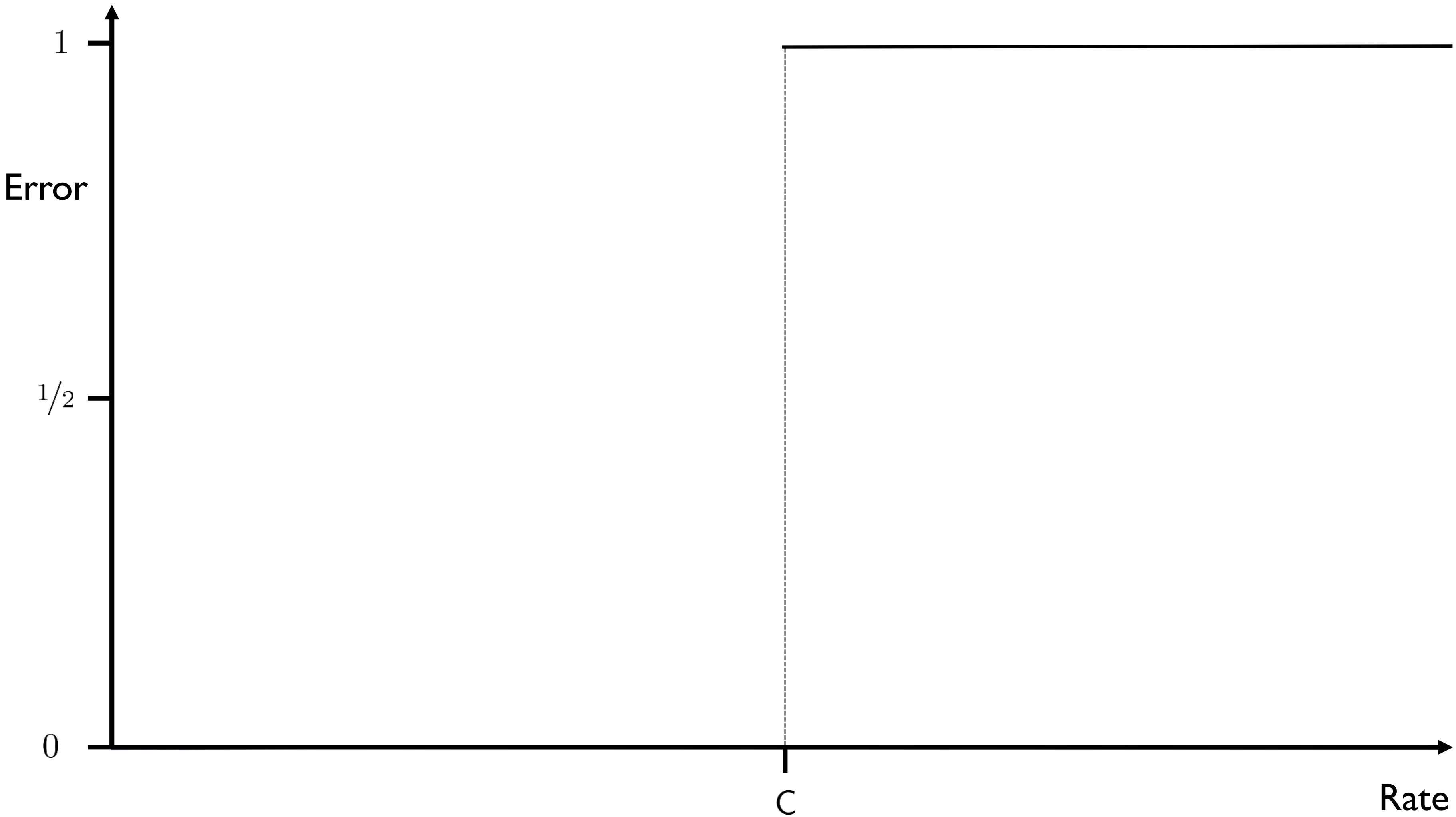
Interplay between three parameters

Interplay of Three Parameters



- Q: Is the reliable communication possible as rate approaches capacity?

Illustrations



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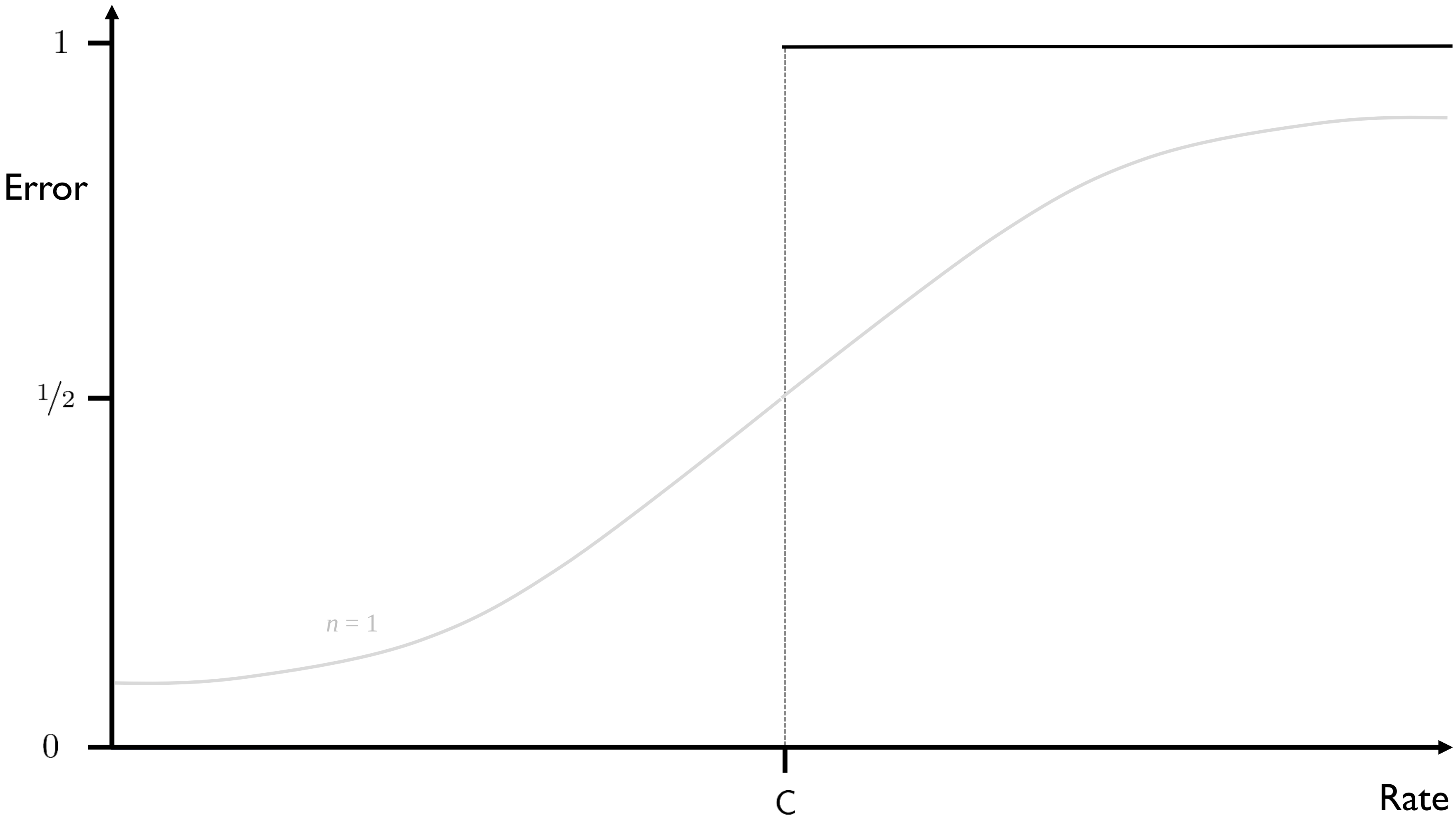

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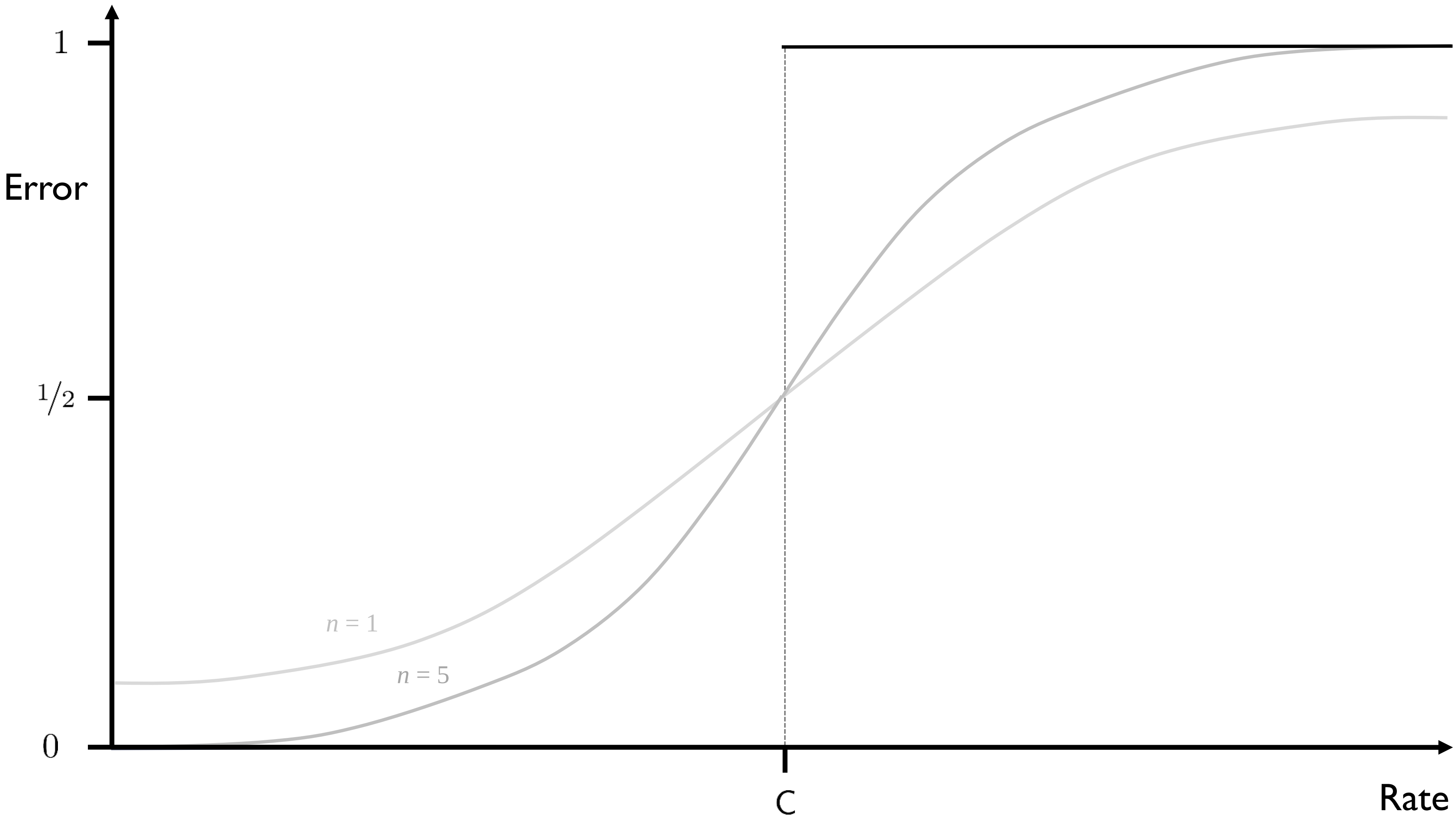

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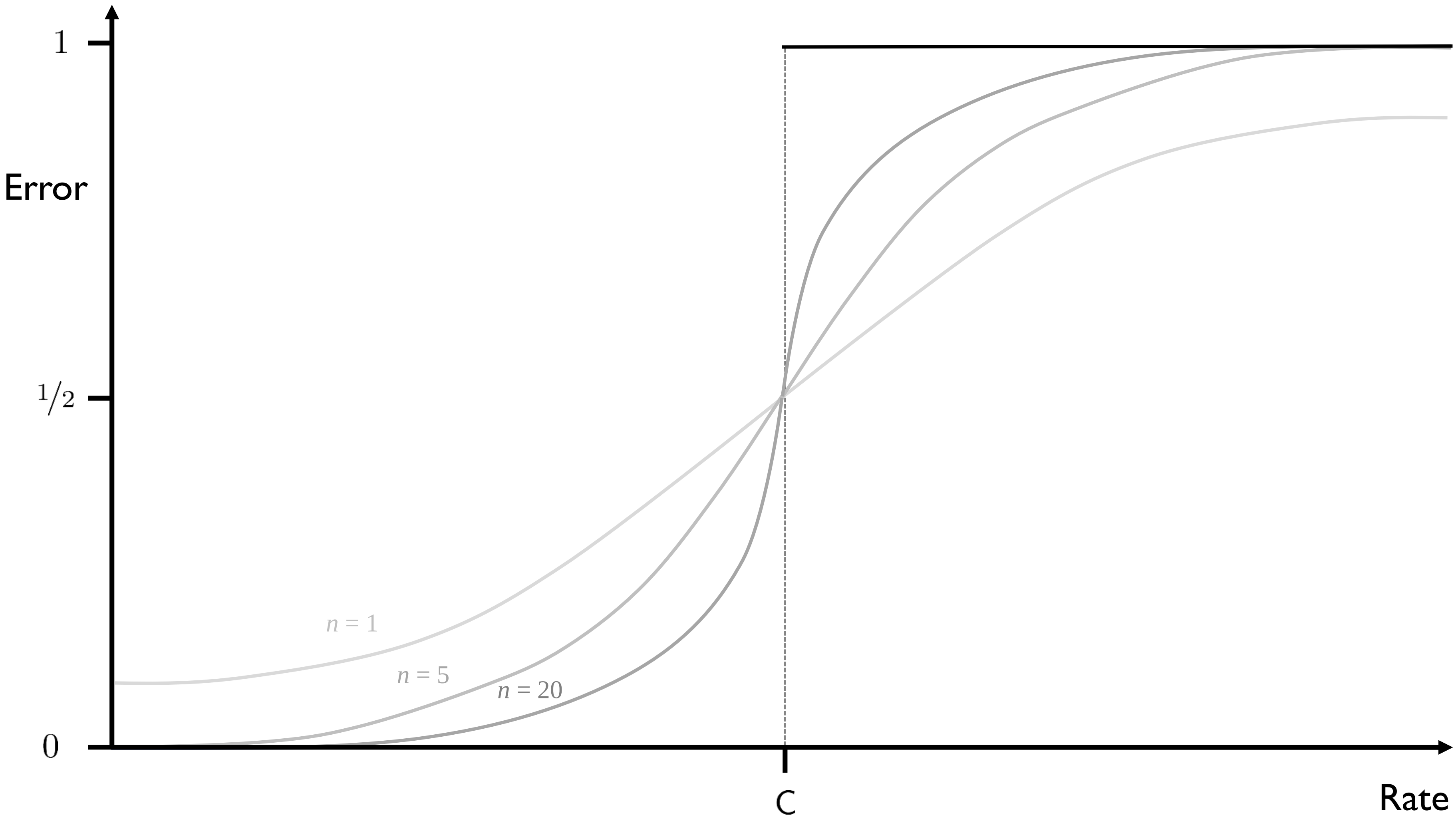

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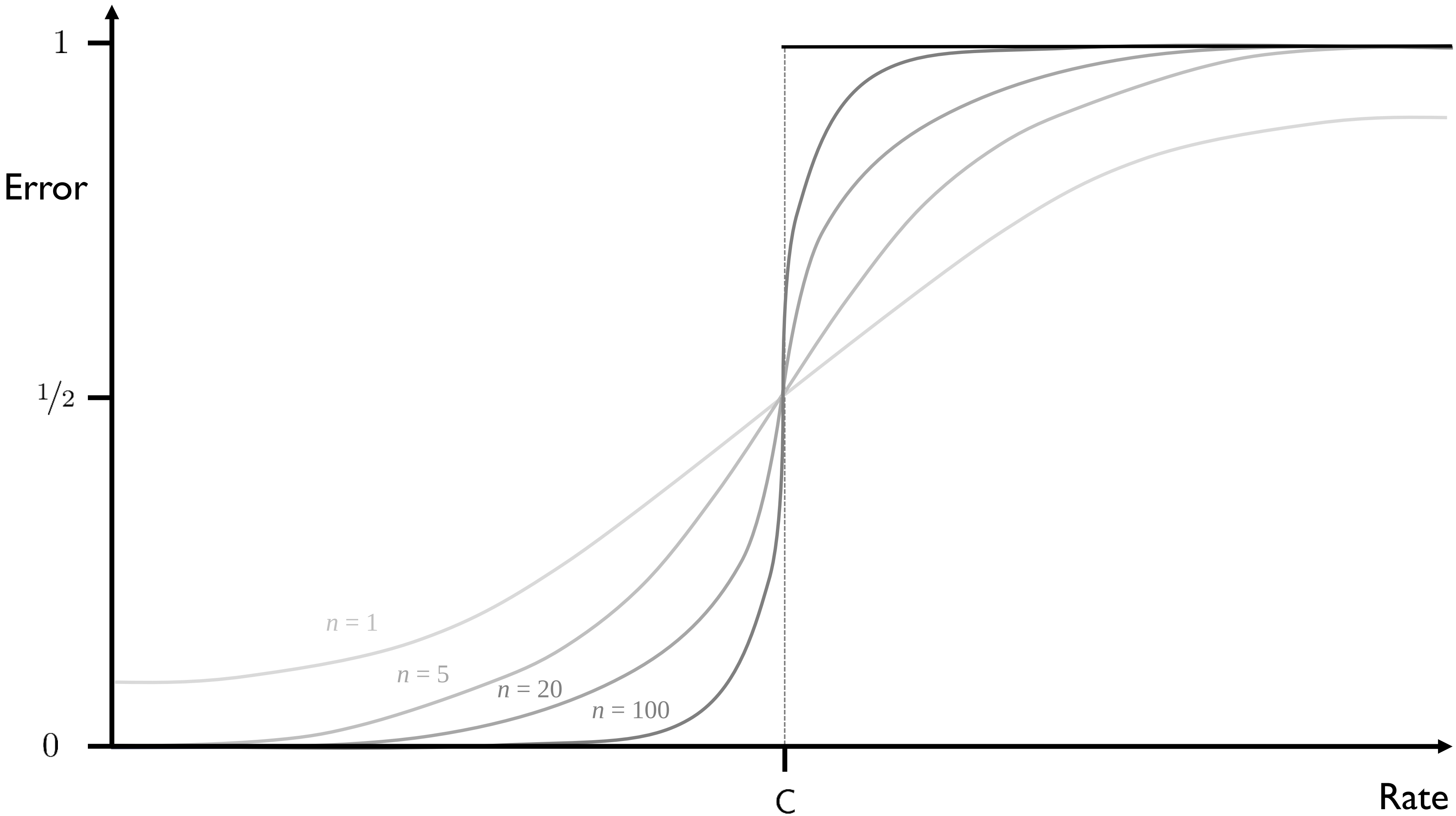

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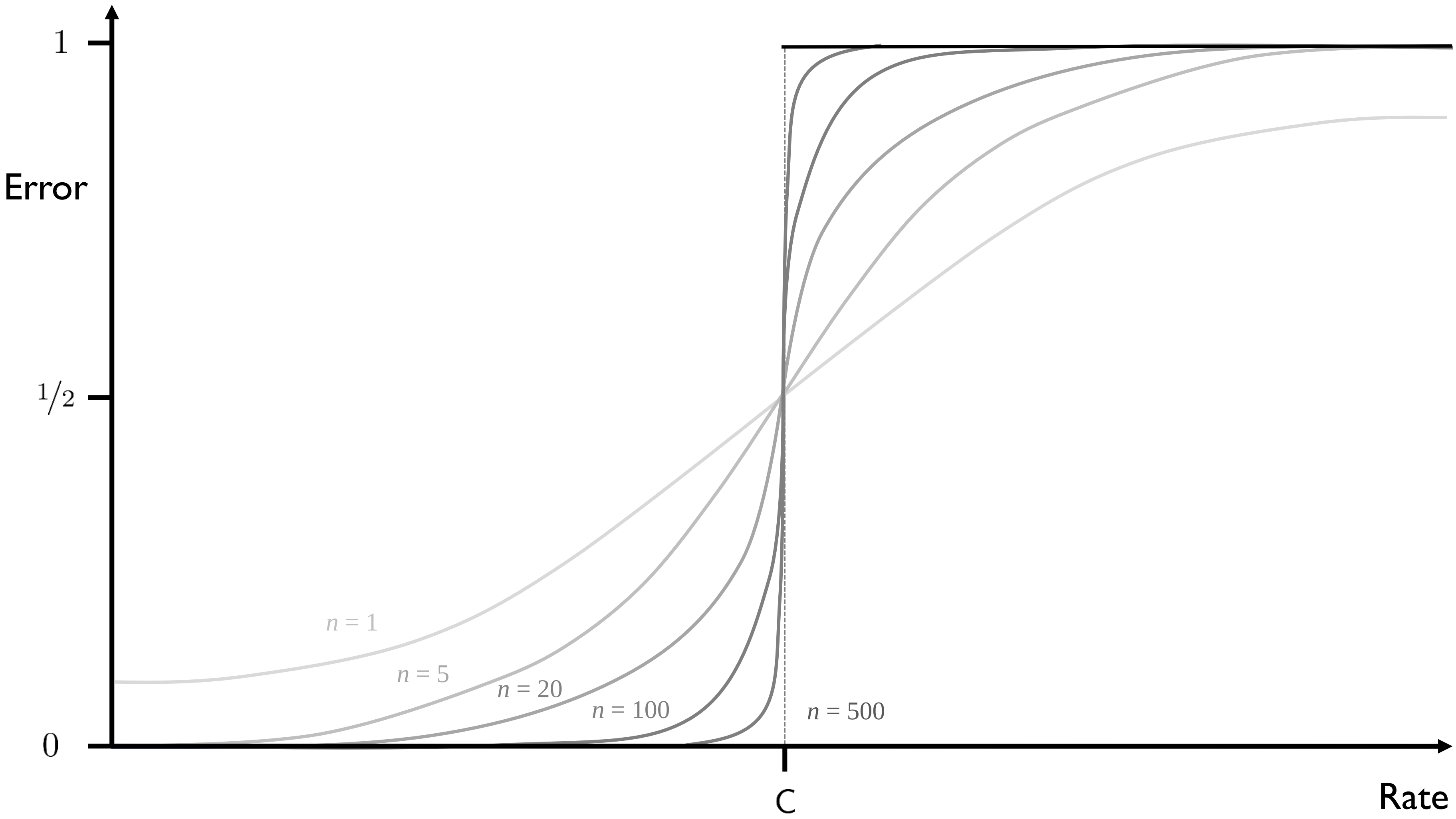

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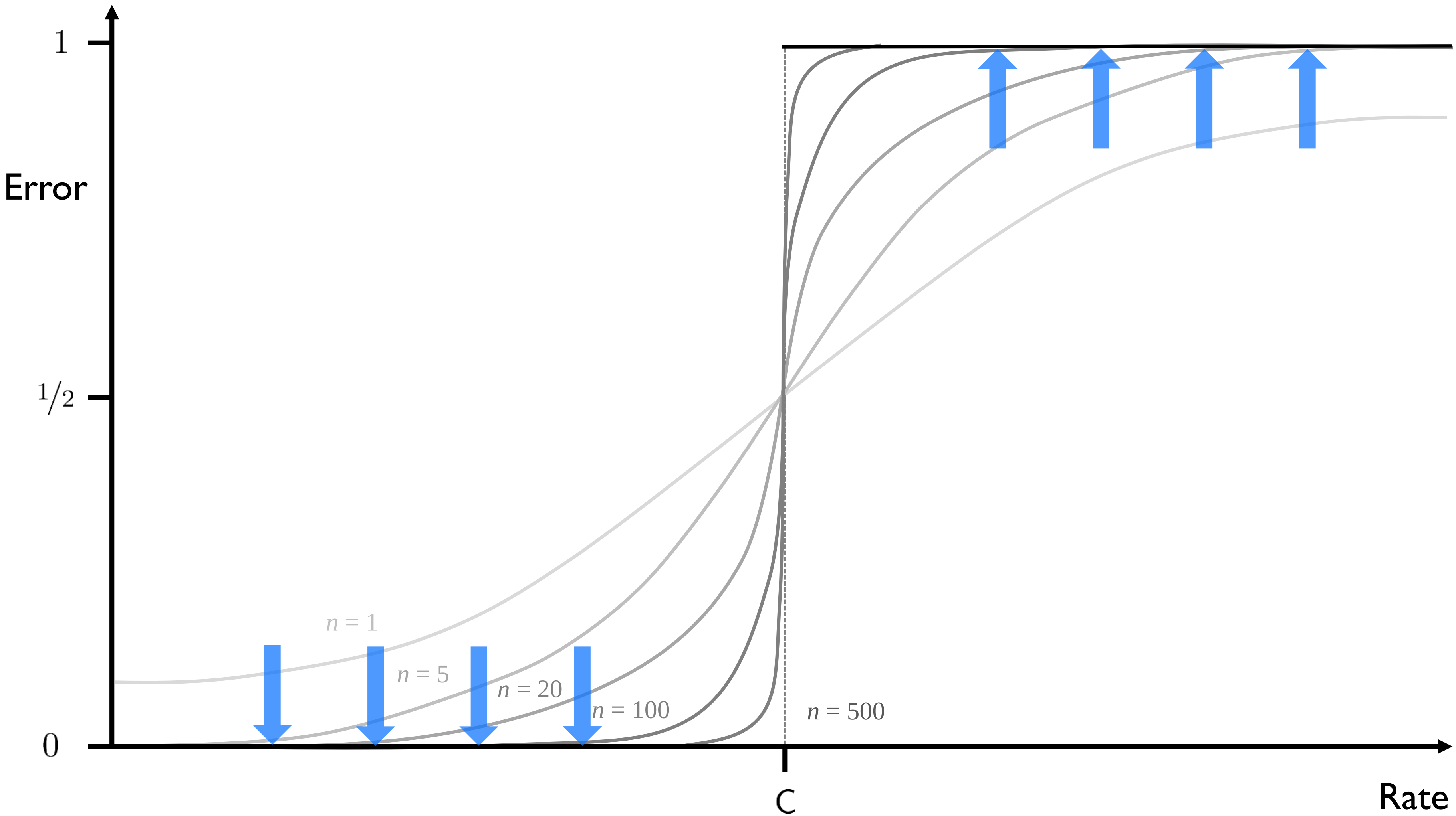

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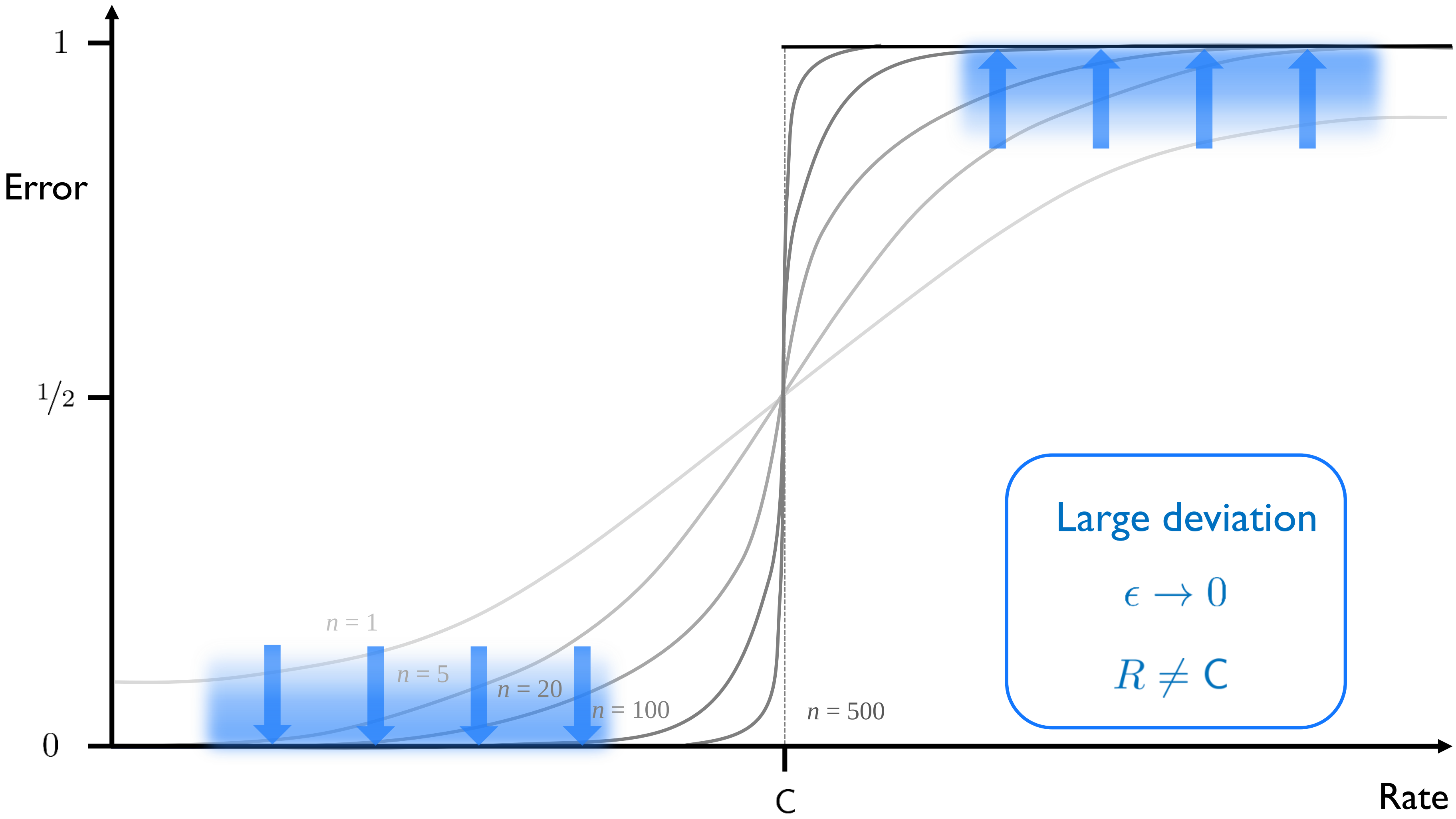

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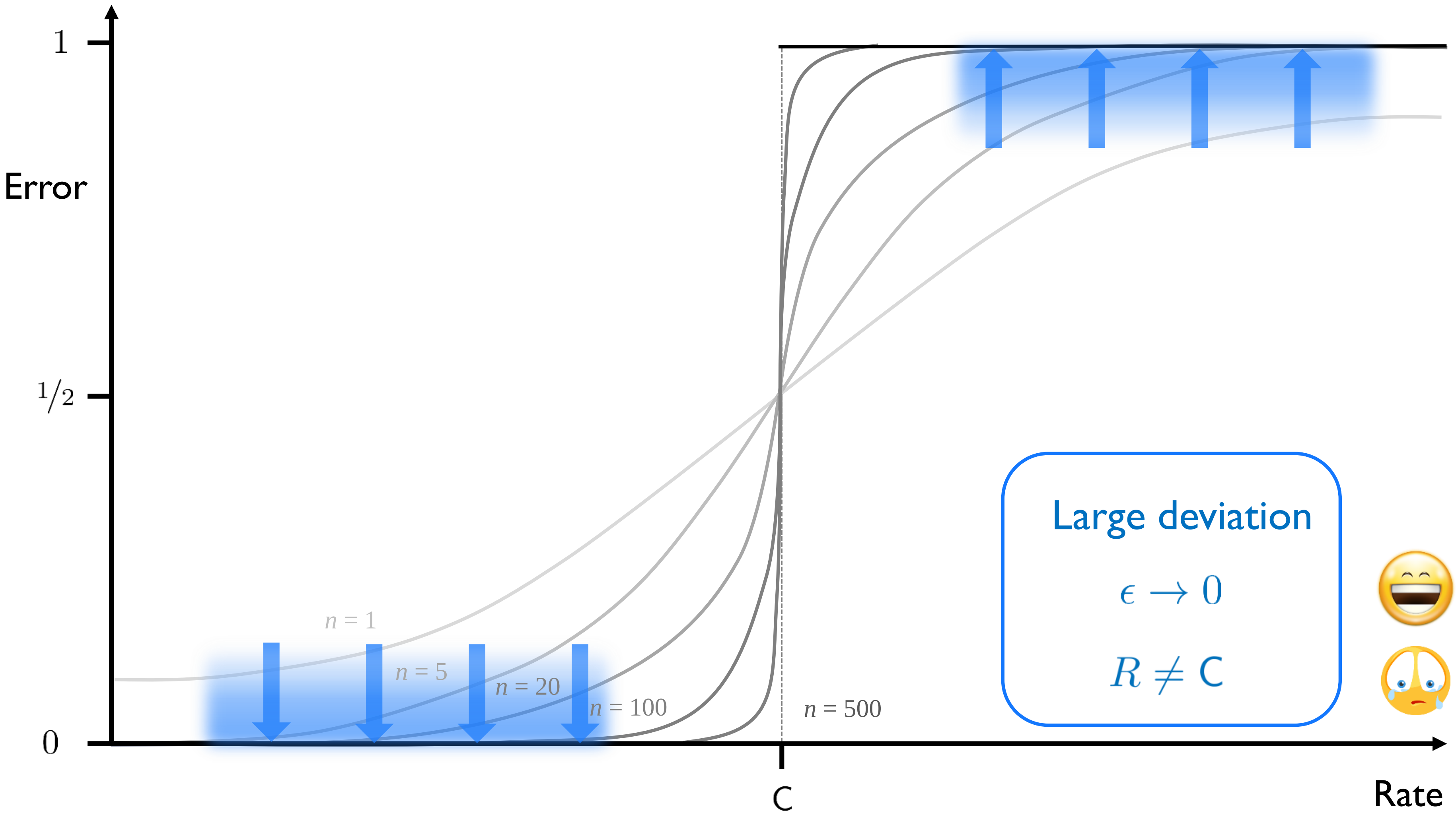

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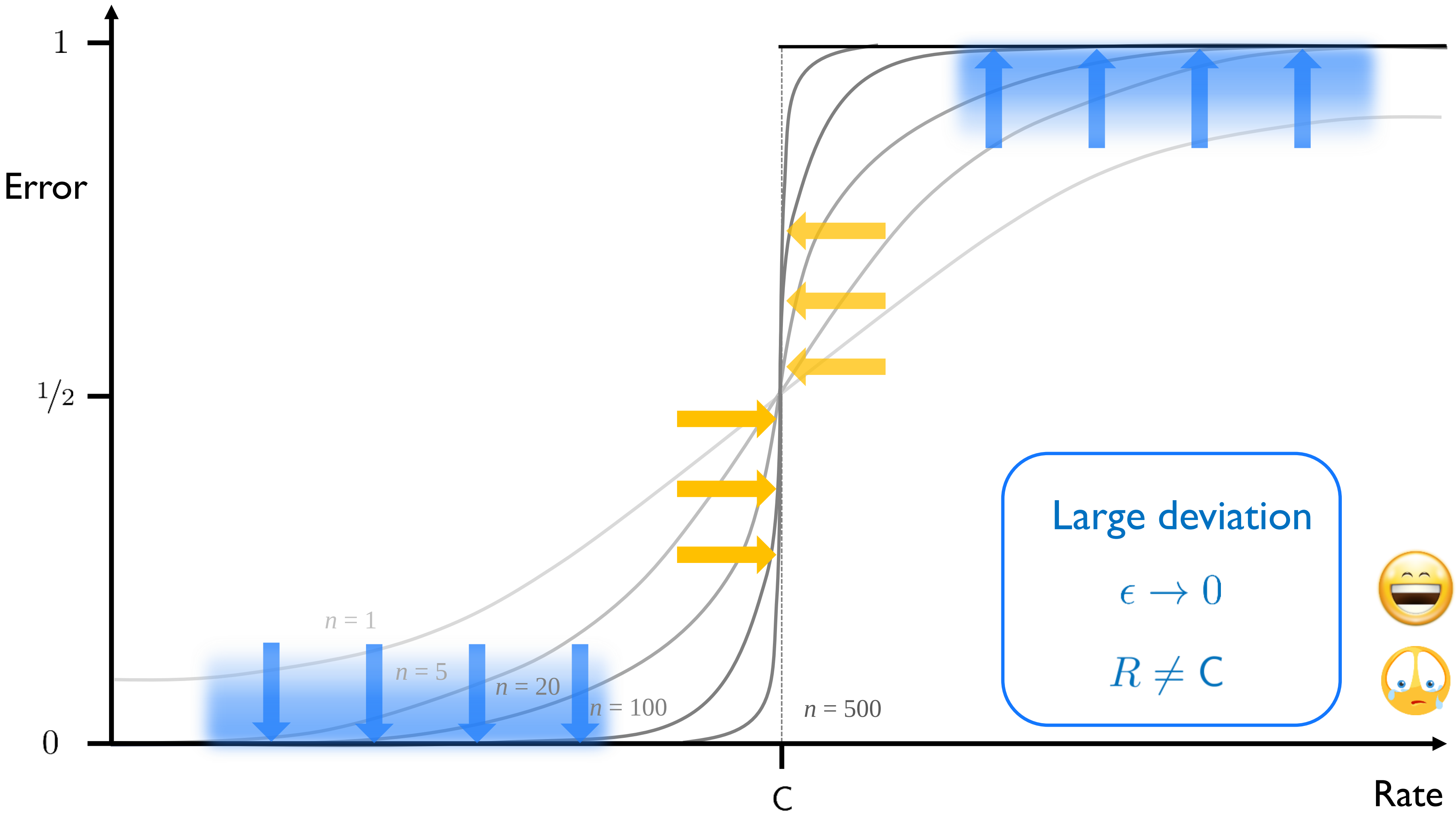

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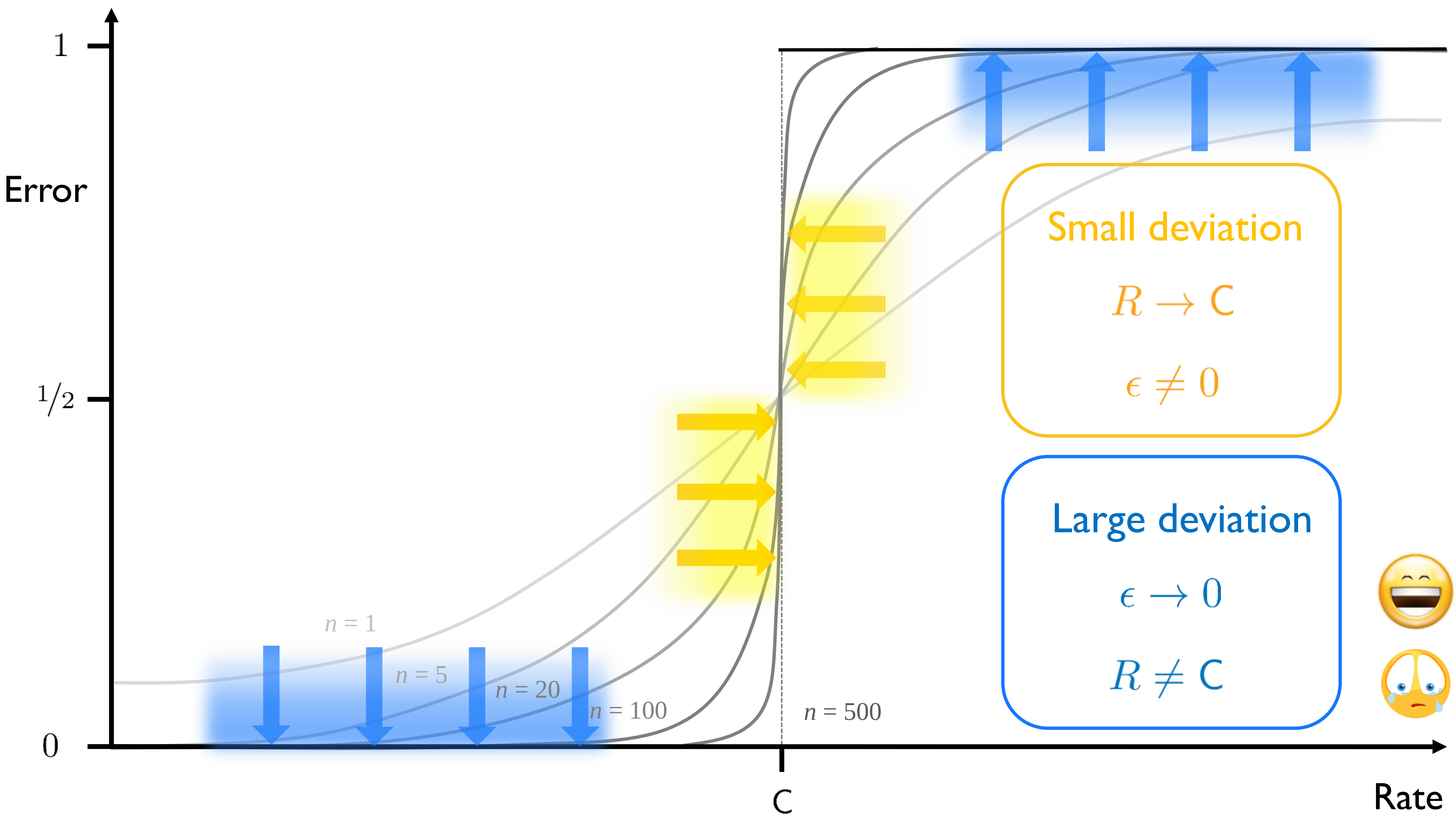

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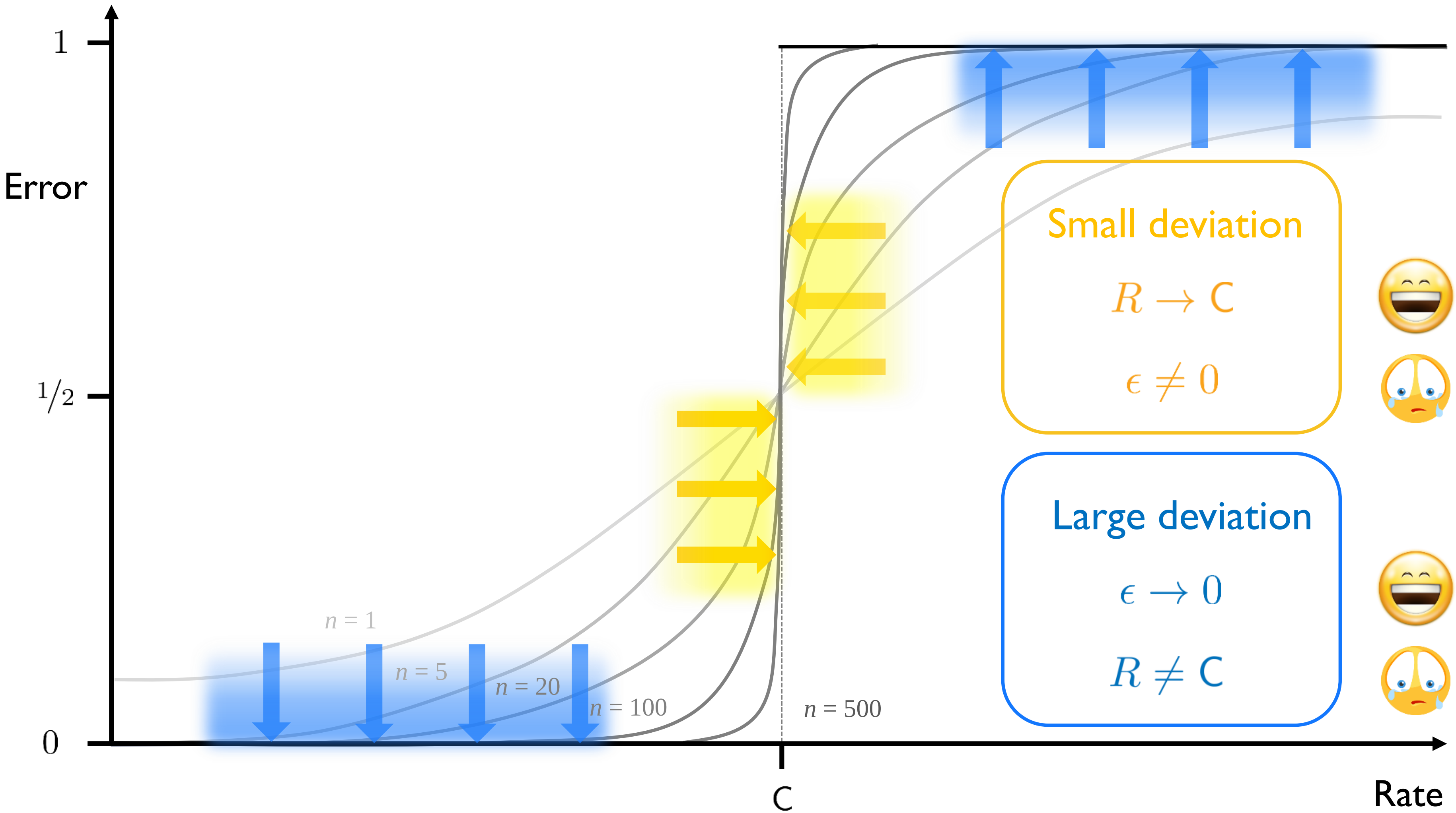

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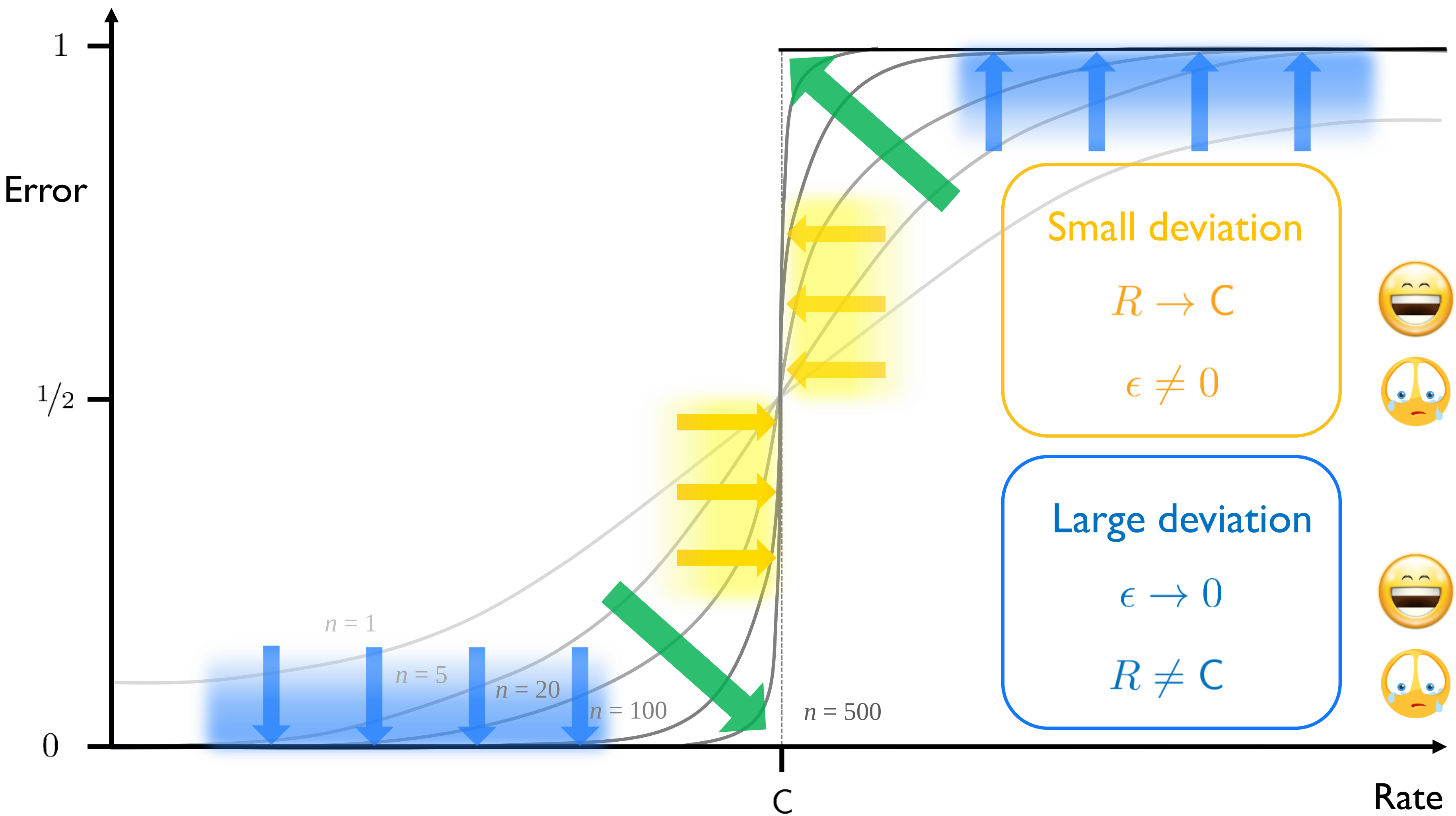

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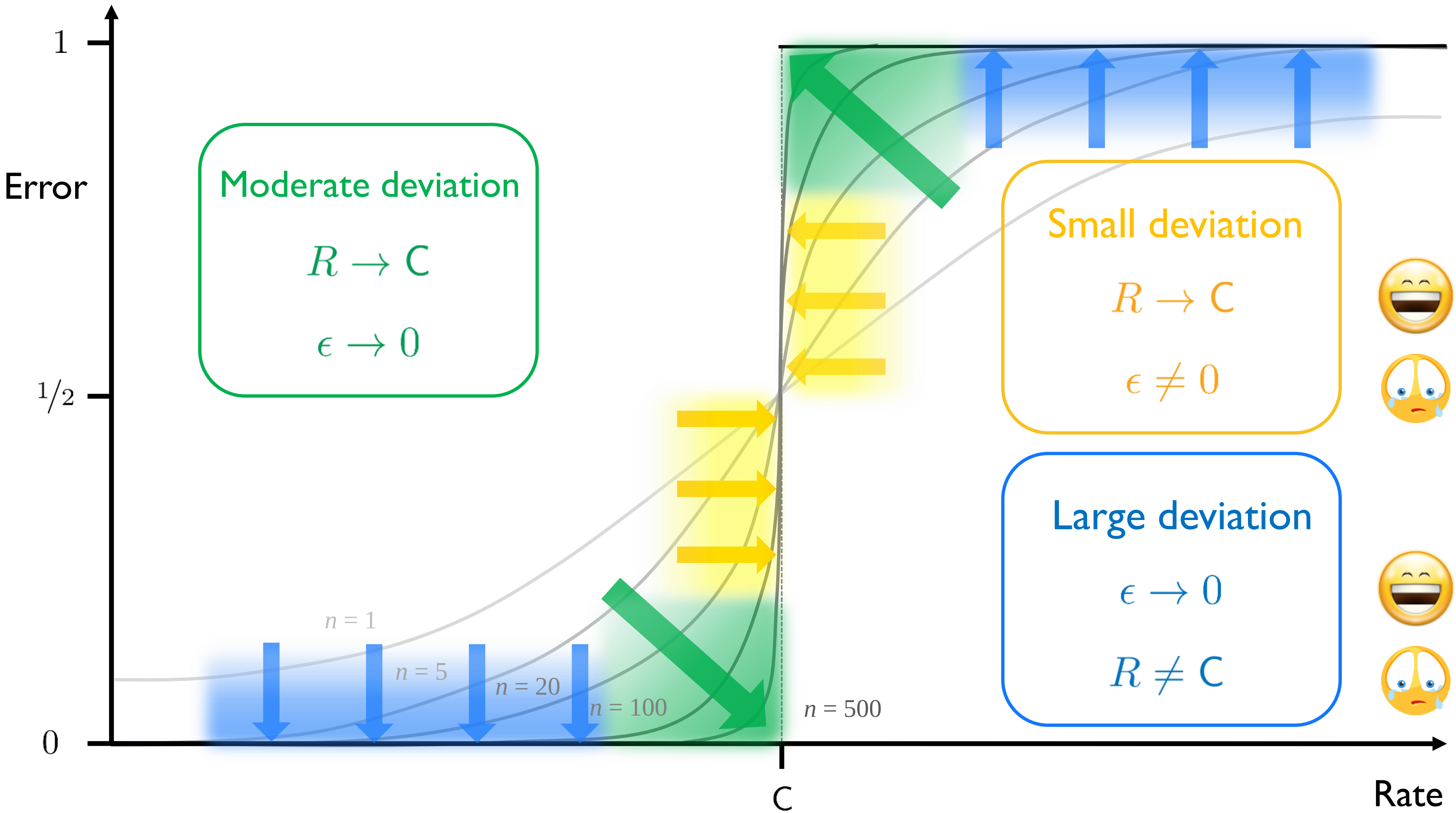

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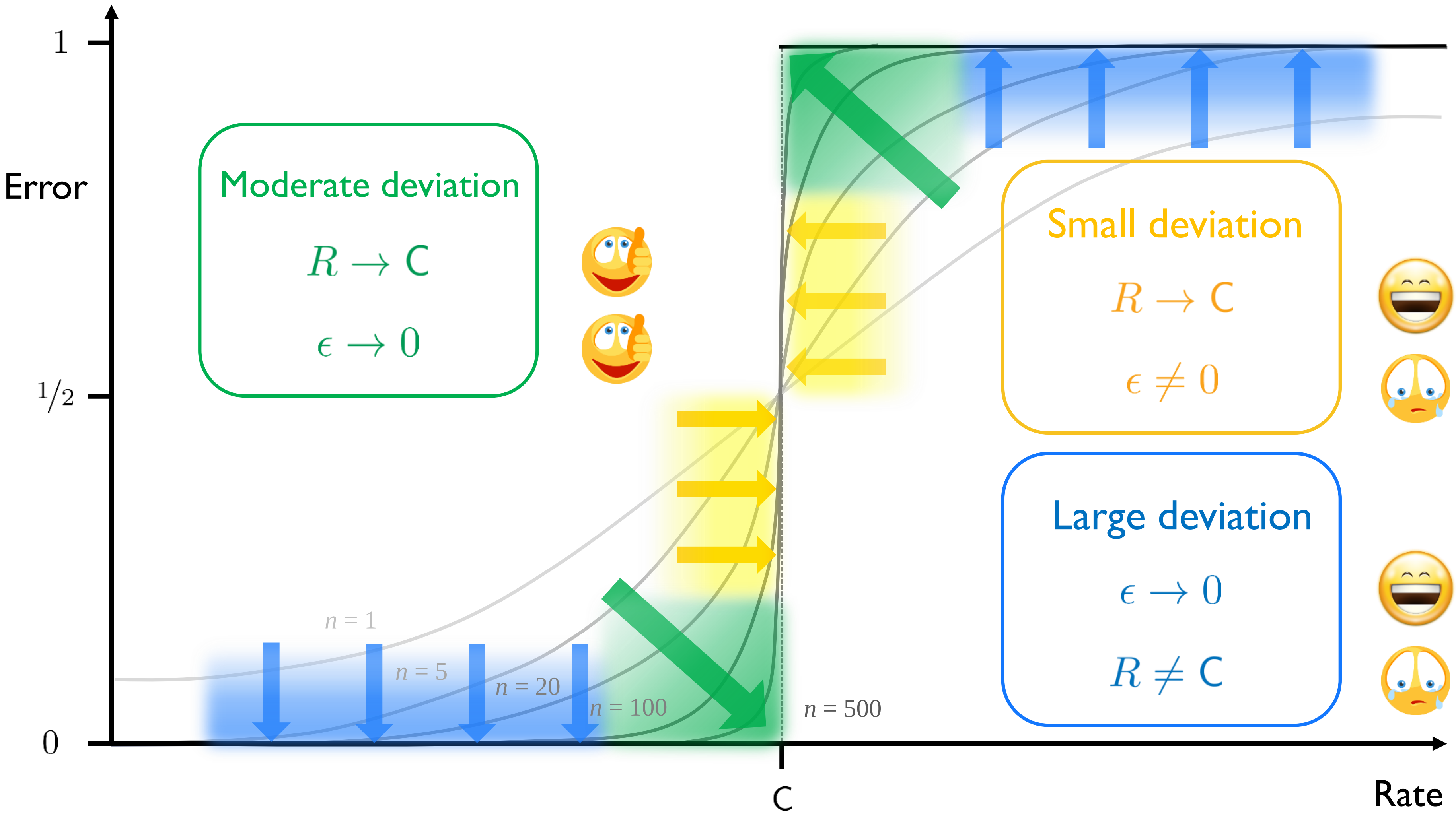

Main Results

- $$\begin{cases} R_n \to \mathsf{C} \\ \epsilon_n \to 0 \end{cases}$$

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Small deviation	Large deviation
$R^*(n,\epsilon) = \mathsf{C} + \sqrt{\frac{\mathsf{V}}{n}}\Phi^{-1}(\epsilon) + O\left(\frac{\log n}{n}\right)$	

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Let $\epsilon_n = \exp\{-n a_n^2\}$ $\Rightarrow R^*(n,\epsilon_n) = \mathsf{C} - \sqrt{2\mathsf{V}} a_n + o(a_n)$	
arXiv:1701.03114	

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Let $\epsilon_n = \exp\{-na_n^2\}$ $\Rightarrow R^*(n,\epsilon_n) = \mathsf{C} - \sqrt{2\mathsf{V}}a_n + o(a_n)$	Let $R_n = \mathsf{C} - a_n$ $\Rightarrow \epsilon^*(n,R_n) = \exp\left\{-\frac{na_n^2}{2\mathsf{V}} + o(na_n^2)\right\}$
`arXiv:1701.03114`	`arXiv:1701.03195`

Proof Ideas

Channel coding

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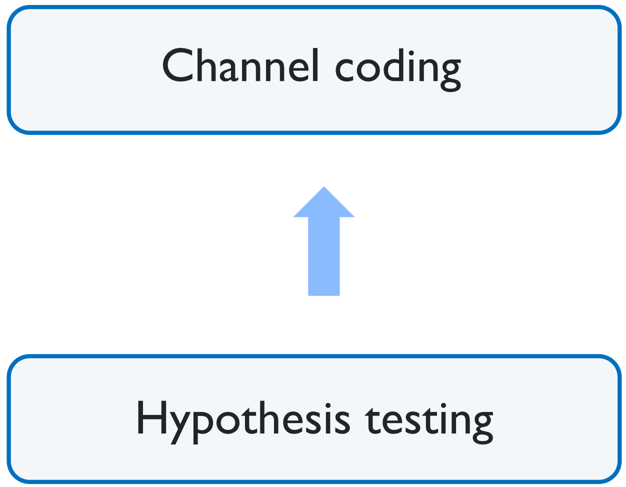

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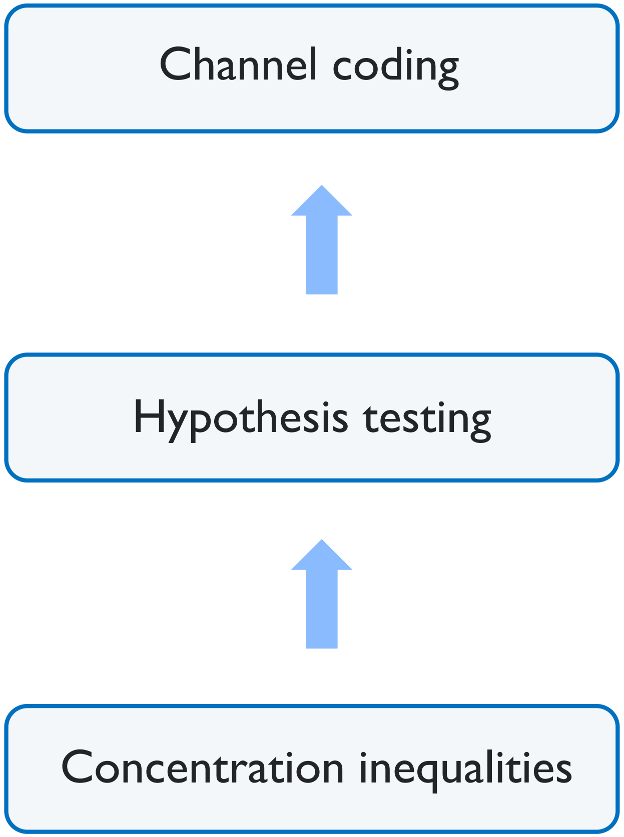
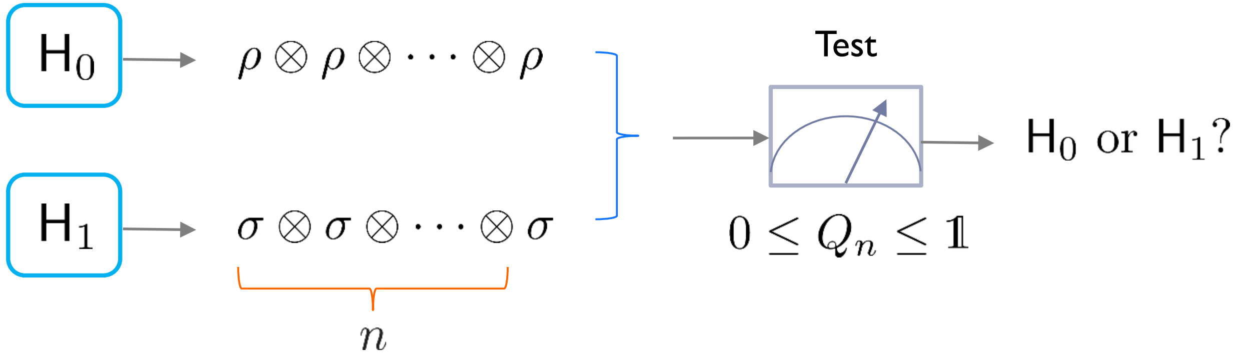

Channel coding
Hypothesis testing
Concentration inequalities

Moderate Deviations for Hypothesis Testing

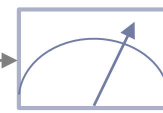


Moderate Deviations for Hypothesis Testing

$\mathsf{H}_0 \rightarrow \rho \otimes \rho \otimes \cdots \otimes \rho$

$\mathsf{H}_1 \rightarrow \underbrace{\sigma \otimes \sigma \otimes \cdots \otimes \sigma}_{n}$

Test



$0 \leq Q_n \leq \mathbb{1}$

H_0 or H_1?

trade-off

- **Type-I, -II errors:**

$$\alpha_n := \Pr(\text{reject } \mathsf{H}_0 | \mathsf{H}_0) = \mathrm{Tr}\left[(\mathbb{1} - Q_n)\rho^{\otimes n}\right]$$

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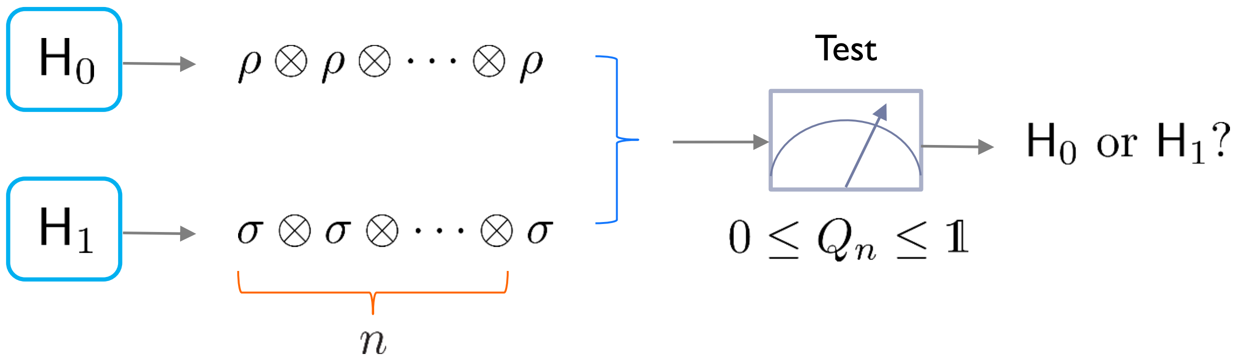
Moderate Deviations for Hypothesis Testing



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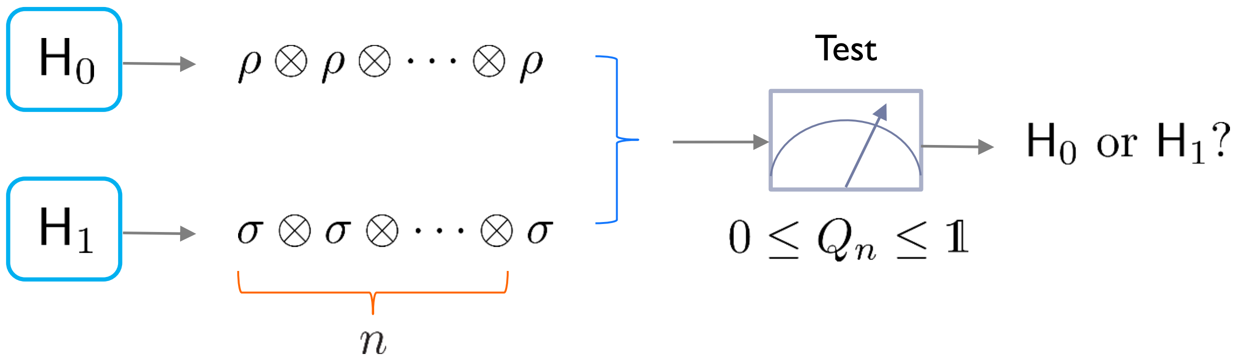

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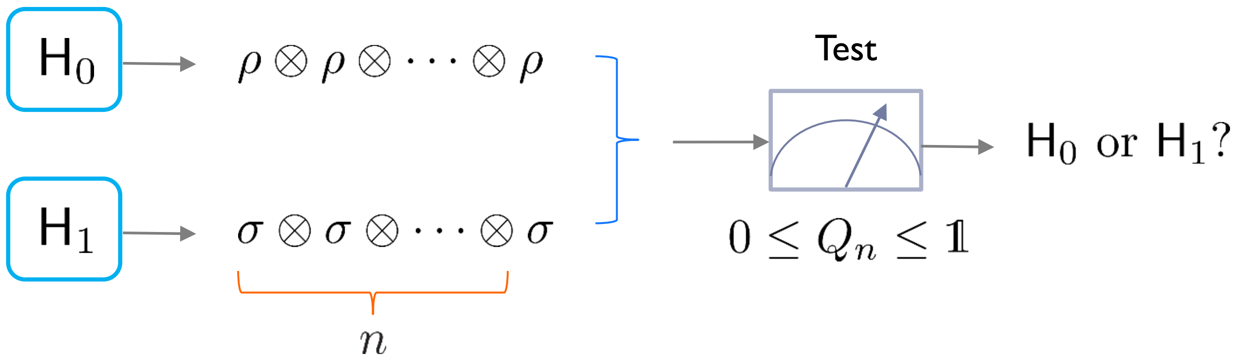

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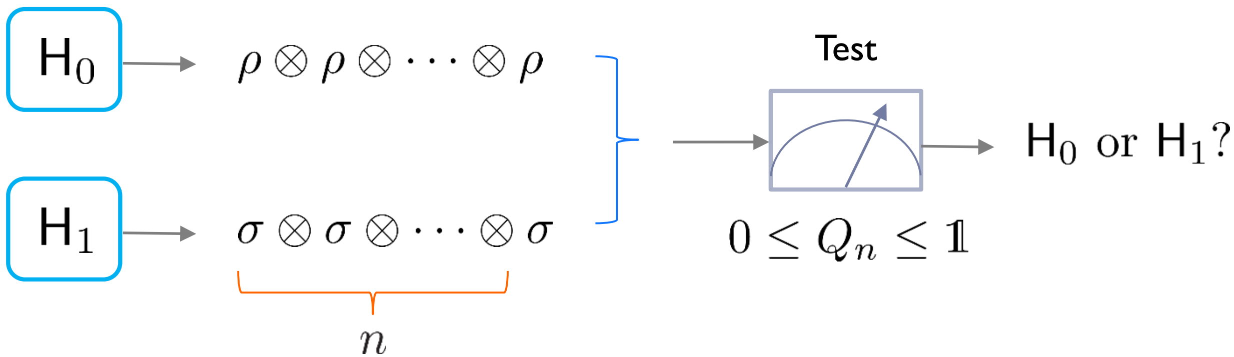

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    $\Rightarrow \alpha_n^* \to \exp\{-na_n^2/2V(\rho\|\sigma)\}$, given $\beta_n = \exp\{-n[D(\rho\|\sigma) - a_n]\}$

Proof Ideas

$$-\frac{1}{n}\log\beta_n^* \to D - \sqrt{2V}a_n,$$

$$\alpha_n = \exp\{-na_n^2\}$$

[Small deviation]

Hypothesis testing

↑

Concentration inequalities

$$\alpha_n^* \to \exp\left\{-\frac{na_n^2}{2V}\right\},$$

$$-\frac{1}{n}\log\beta_n = D - a_n$$

[Large deviation]

Proof Ideas

$$R^*(n, \epsilon_n) = \mathsf{C} - \sqrt{2\mathsf{V}}a_n + o(a_n)$$

$$\epsilon_n = \exp\{-na_n^2\}$$

Channel coding

$$\epsilon^*(n, R_n) = \exp\left\{-\frac{na_n^2}{2\mathsf{V}} + o(na_n^2)\right\}$$

$$R_n = \mathsf{C} - a_n$$

$$-\frac{1}{n}\log \beta_n^* \to D - \sqrt{2V}a_n,$$

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Hypothesis testing

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[Small deviation]

Concentration inequalities

[Large deviation]

From Large Deviation: Achievability

- Hoeffding bound: $\alpha_n \leq \exp\{-n\mathsf{E}(R)\}$, given $\beta_n \leq \exp\{-nR\}$

K. Audenaert, M. Nussbaum, A. Szkola, F. Verstraete, "Asymptotic error rates in quantum hypothesis testing," *CMP*, **279**:251-283, 2008

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Let $R_n = D(\rho\|\sigma) - a_n$

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$$\frac{\mathsf{E}(D(\rho\|\sigma) - a_n)}{na_n^2} \to \frac{1}{2V(\rho\|\sigma)}$$

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$$\frac{\mathsf{E}(D(\rho\|\sigma) - a_n)}{na_n^2} \to \frac{1}{2V(\rho\|\sigma)}$$

$$\Rightarrow \alpha_n \lesssim \exp\left\{-\frac{na_n^2}{2V(\rho\|\sigma)}\right\}$$

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From Large Deviation: Conerse

- **Hoeffding bound:** $\alpha_n \geq \exp\left\{-n\mathsf{E}(R_n) + O(\sqrt{n})\right\}$, given $\beta_n \leq \exp\left\{-nR_n\right\}$

H. Nagaoka, "The converse part of the theorem for quantum Hoeffding bound," arXiv:quant-ph/0611289
Chaganty, Sethuraman, "Strong large deviation and local limit theorems," *Ann. Prob.*, **21**:1671-1690, 1993

From Large Deviation: Conerse

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- Refined polynomial prefactor
  - A strong large deviation inequality $\Pr\left\{\frac{1}{n}\sum_{i=1}^{n} Z_i \geq z_n\right\} \geq \frac{1}{t_n^\star\sqrt{2\pi n \mathrm{Var}}} \exp\{-n\Lambda_n^*(z_n)\}$

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- Refined polynomial prefactor
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- A strong small deviation inequality

$$\log \Pr\left\{\frac{1}{n}\sum_{i=1}^{n} Z_i \geq a_n\right\} \geq \log \Phi\left(-\sqrt{\frac{n a_n^2}{\mathsf{V}}}\right) + o(n a_n^2)$$

L. Rozovsky, "Estimate from below for large-deviation probabilities of a sum of independent random variables with finite variances," *J. Math. Sci.*, **109**(6):2192-2209, 2002

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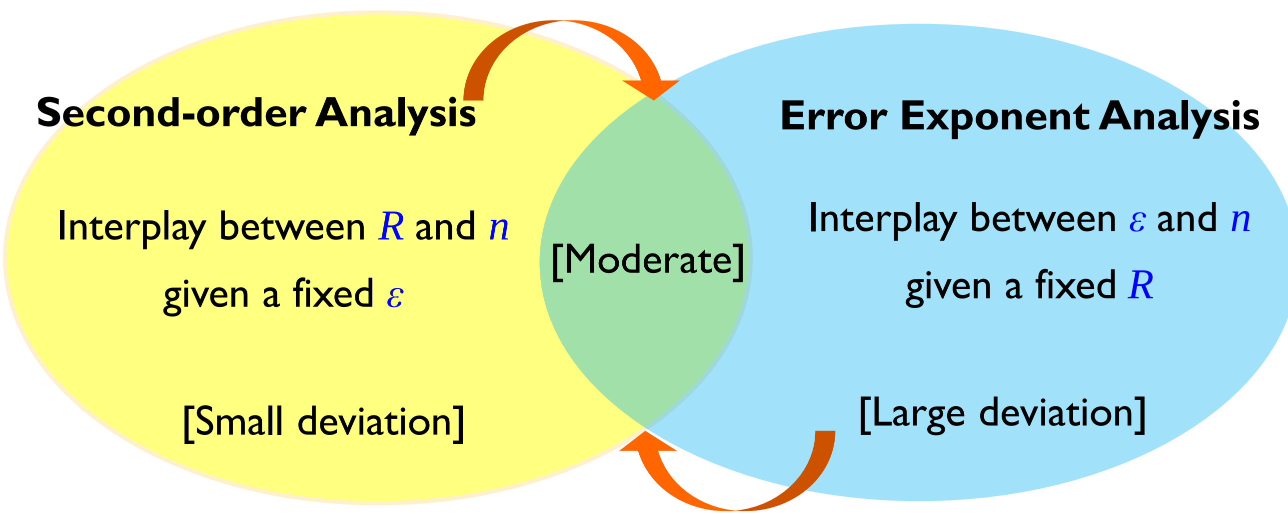
In Summary

(i) $\lim_{n\to\infty} a_n = 0$

(ii) $\lim_{n\to\infty} \sqrt{n} a_n = \infty$

Regimes	Channel Coding	Concentration
Small deviation	$\epsilon^*\left(n, \mathsf{C} - \frac{A}{\sqrt{n}}\right) \sim \Phi\left(\frac{A}{\sqrt{\mathsf{V}}}\right)$	$\Pr\left(S_n \geq \sqrt{n}x\right) \sim 1 - \Phi\left(\frac{x}{\sqrt{\mathsf{V}}}\right)$
Moderate deviation	$\epsilon^*(n, \mathsf{C} - a_n) = \mathrm{e}^{-\frac{n a_n^2}{2\mathsf{V}} + o(n a_n^2)}$	$\Pr\left(S_n \geq n a_n x\right) = \mathrm{e}^{-\frac{n a_n^2}{2\mathsf{V}} x + o(n a_n^2)}$
Large deviation	$\epsilon^*(n, R) = \mathrm{e}^{-n\mathsf{E}(R) + o(n)}$	$\Pr\left(S_n \geq nx\right) = \mathrm{e}^{-n\Lambda^*(x) + o(n)}$



Future Works

- Beyond classical-quantum channels:
  - Image-additive quantum channels

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- Entanglement-assisted classical communications over quantum channels
- Other applications of moderate deviation analysis in quantum information science


Thanks!