A polynomial-time algorithm for finding degenerate ground states of gapped 1D quantum systems

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Due to the exponential increase in the number of variables required to entirely describe a system, many properties that are efficiently calculable for classical systems cease to be for their quantum analogues. This complexity poses a significant problem to classical simulation of general quantum systems.

The necessity of coherence makes low temperature quantum systems important models in quantum information theory. At low temperatures most of the dynamics of a system are captured by its ground state(s) and the properties thereof, especially in the case of gapped systems. As such the efficient calculation of these ground state(s) is an important step towards the classical simulation of these systems.

The simplest non-trivial ground state problem that can be raised is that of a 1D system. Here the problem is in P for a classical system, but the quantum case is QMA-Complete, and remains so even when structure such as translation invariance is imposed [1].

Whilst 1D systems in general do not admit an efficient ground state algorithm, in practice the Density Matrix Renormalisation Group (DMRG) has proven to be an effective and typically efficient algorithm, but is known to fail for some systems [2]. Though DMRG has known failure modes, its typical efficiency suggests that many 1D systems may possess extra structure that allows for a provably polynomial-time algorithm.

Due to their importance in low-temperature systems, the existence of a spectral gap provides a natural choice for this additional structure. For a general state, the entanglement entropy on an arbitrary region R can exhibit, at worst, an asymptotic volume law $S_R \leq \mathcal{O}(|R|)$. It is conjectured [3] however that all local gapped systems exhibit an area law $S_R \leq \mathcal{O}(|\partial R|)$, which in 1D results in constant entanglement between simply connected regions. Because of this bound, 1D ground states can be effectively described by Matrix Product States (MPS) [4].

The first proof of the 1D area law [5] came from use of monogamy of entanglement arguments in conjunction with the Lieb-Robinson bound. For a system of quidits of spectral gap ϵ the entanglement entropy is bounded by a constant $S \leq 2^{\mathcal{O}(\log d/\epsilon)}$. This approximation is efficiently represented by an MPS with polynomial bond dimension, this in turn serves as a possible witness, demonstrating that 1D unique ground state problem is in NP. Using an approximate ground state projector (AGSP) this bound was exponentially improved by Arad, Landau, Vazarani [6] and Kitaev [7] to give $S \leq \tilde{\mathcal{O}}(\log^3 d/\epsilon)$ and generalised to frustrated models. This in turn yielded the first subexponential-time algorithm.

By introducing methods of 'approximate decoupling', stochastic matrix sampling, and convex optimsation, this line of work lead to the first fully polynomial-time stochastic algorithm for unique ground states by Landau, Vazarani, and Vidick [8]. This work culminated in a deterministic polynomial-time algorithm [9] with an exponential improvement on the ϵ-dependence, running in time $n^{\mathcal{O}(1/\epsilon)}$ and giving an inverse polynomial approximation.

Here we present ongoing work on an algorithm for calculating degenerate solutions to gapped 1D systems and the logical operators of the associated quantum codes. This is enabled by the recent extension of the area law (with the same bounds as previously) to degenerate 1D systems [10].

Our main result is to show that if the entire ground space is assumed to obey an area law, then the techniques of Refs. [8, 9] can be used to *sample* from the degenerate ground space. When this sampling is sufficiently close to uniform, then all of the ground states can be found with high probability in polynomial time, provided constant degeneracy. This shows that such systems are essentially no harder to simulate classically than nondegenerate ones.

The extension of the degenerate 1D area law of Ref. [10] to the entire ground space is however an open problem. As in the nondegenerate case the presence of an area law here also implies the existence of an efficient ground state algorithm. As well as implying the existence, the shared use of AGSPs show that problems in entanglement structure and computational complexity may be linked, and that AGSPs serve as a powerful tool in both arenas. The power of various forms of AGSPs and the contexts in which they are useful provides a natural route for further research.

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