Let's Write a Type Checker

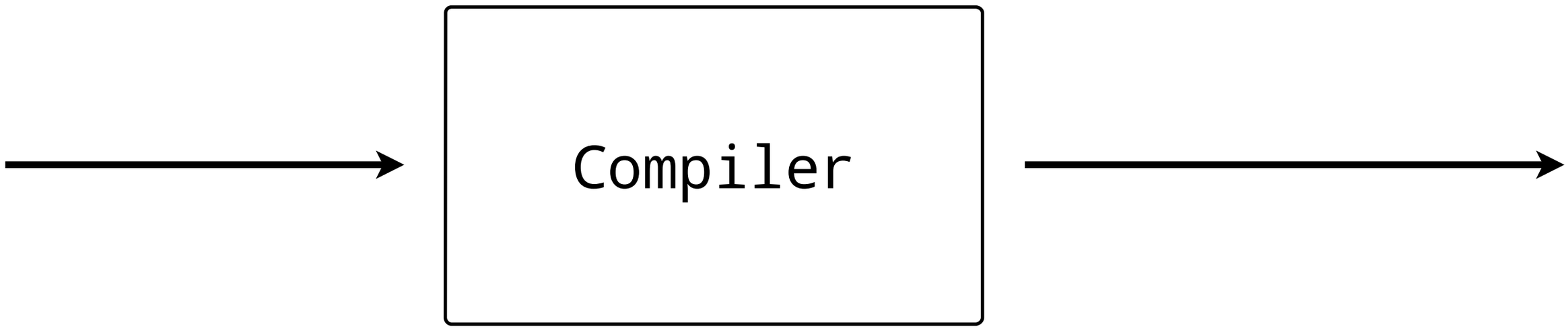
Ionuț G. Stan — I T.A.K.E. — May 2015

The Plan

- Part 1
  - Compilers Overview
  - Type Checking vs Type Inference
  - Vehicle Language
  - Wand's Type Inference Algorithm
- Part 2
  - Live Demonstration

Compilers Overview

Compilers Overview



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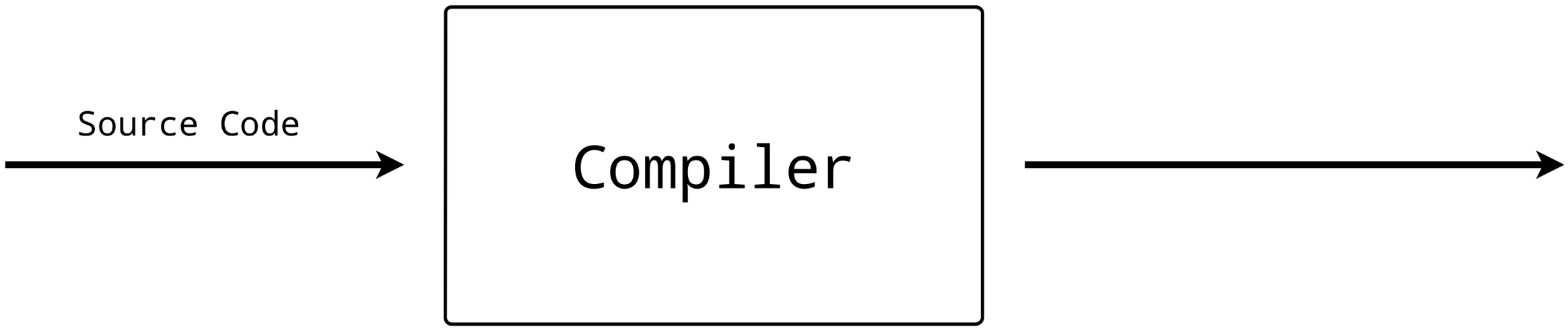

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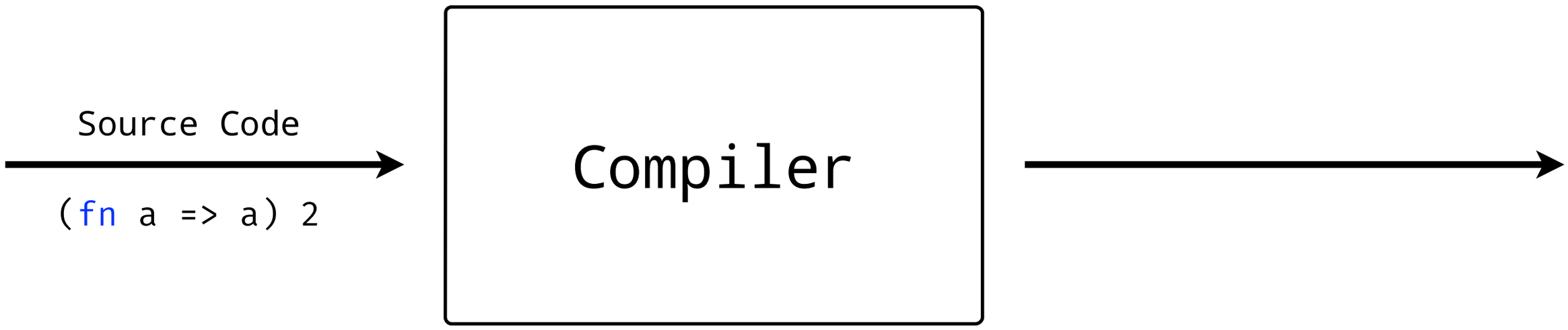

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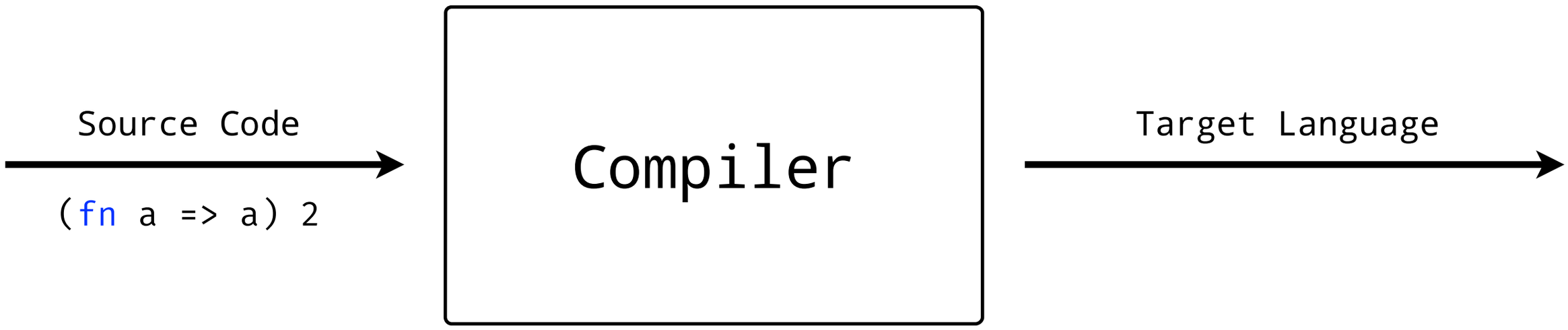

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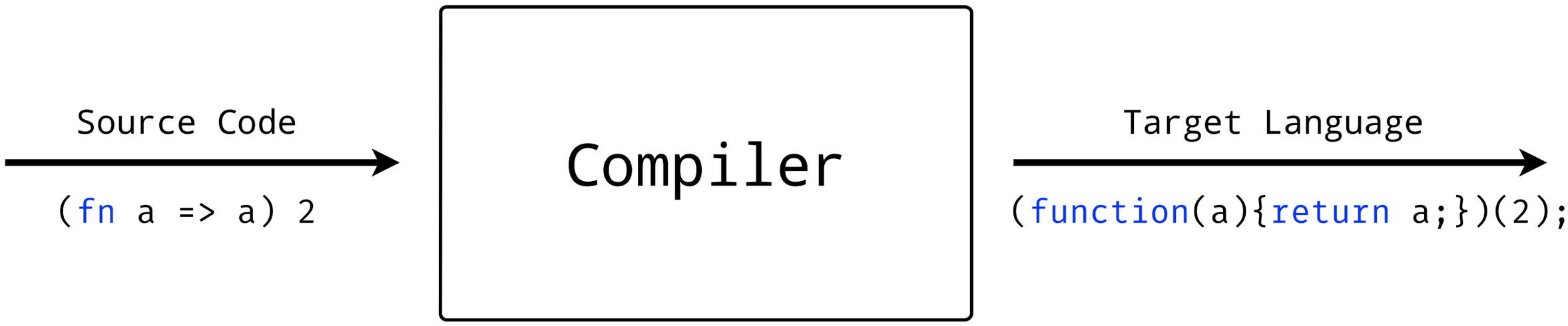

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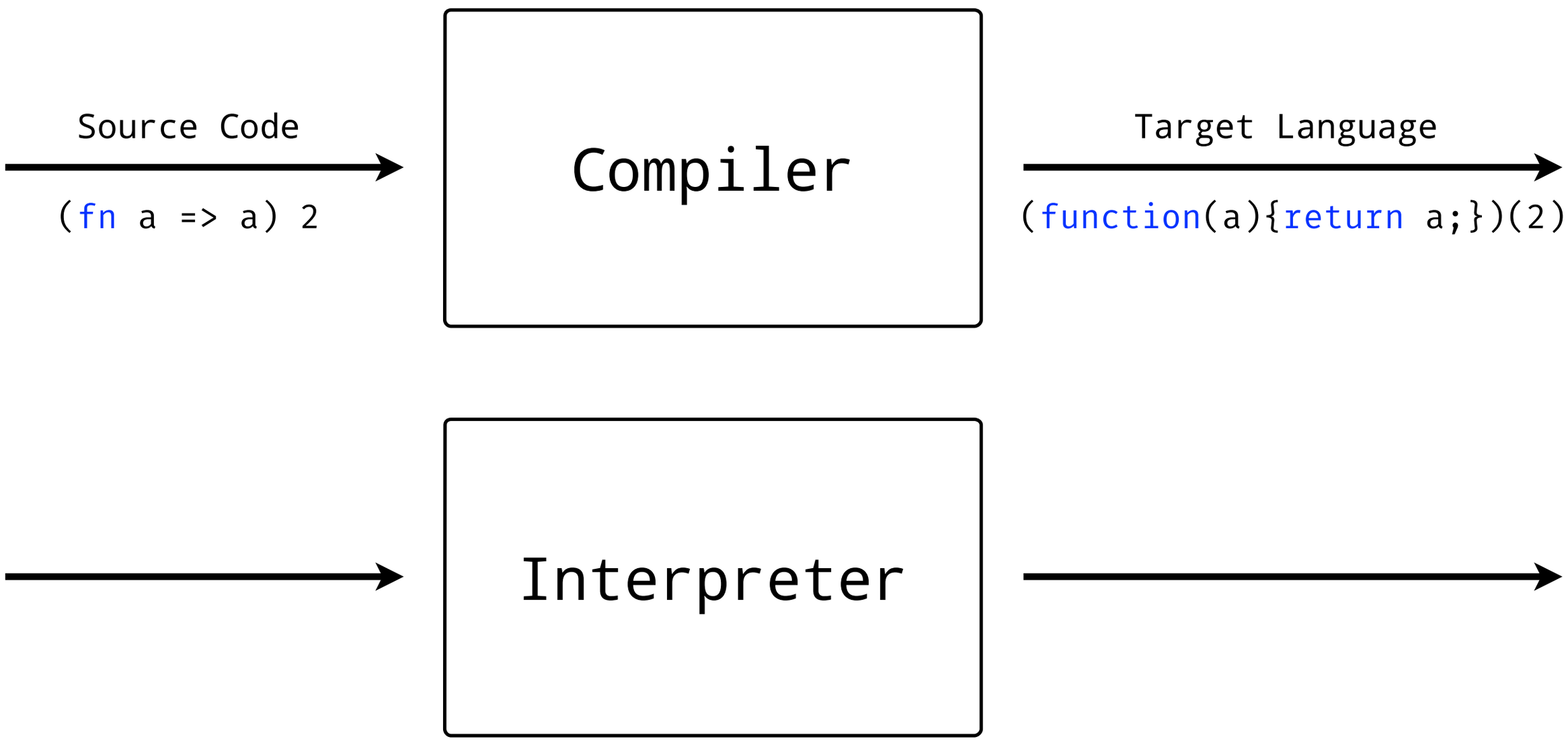

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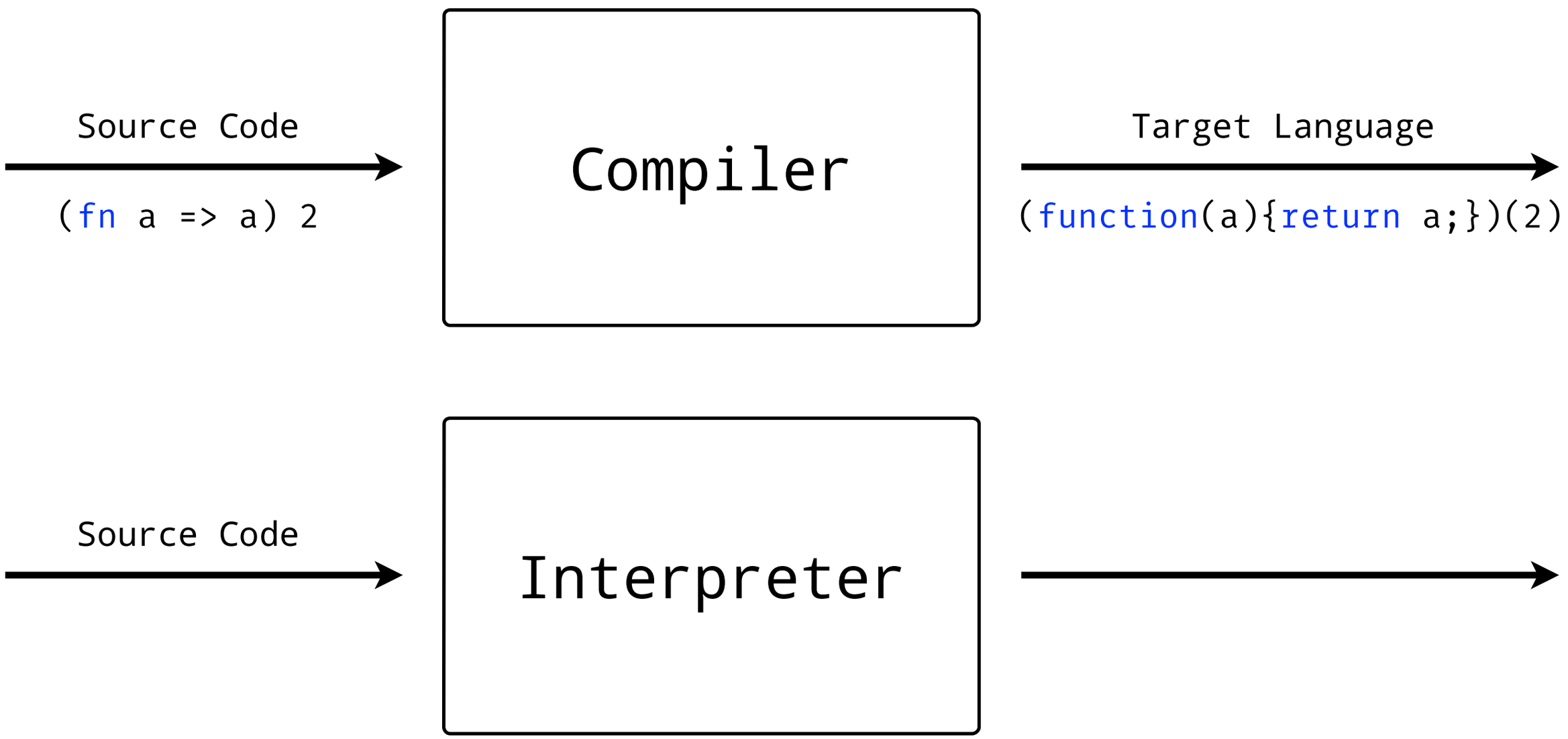

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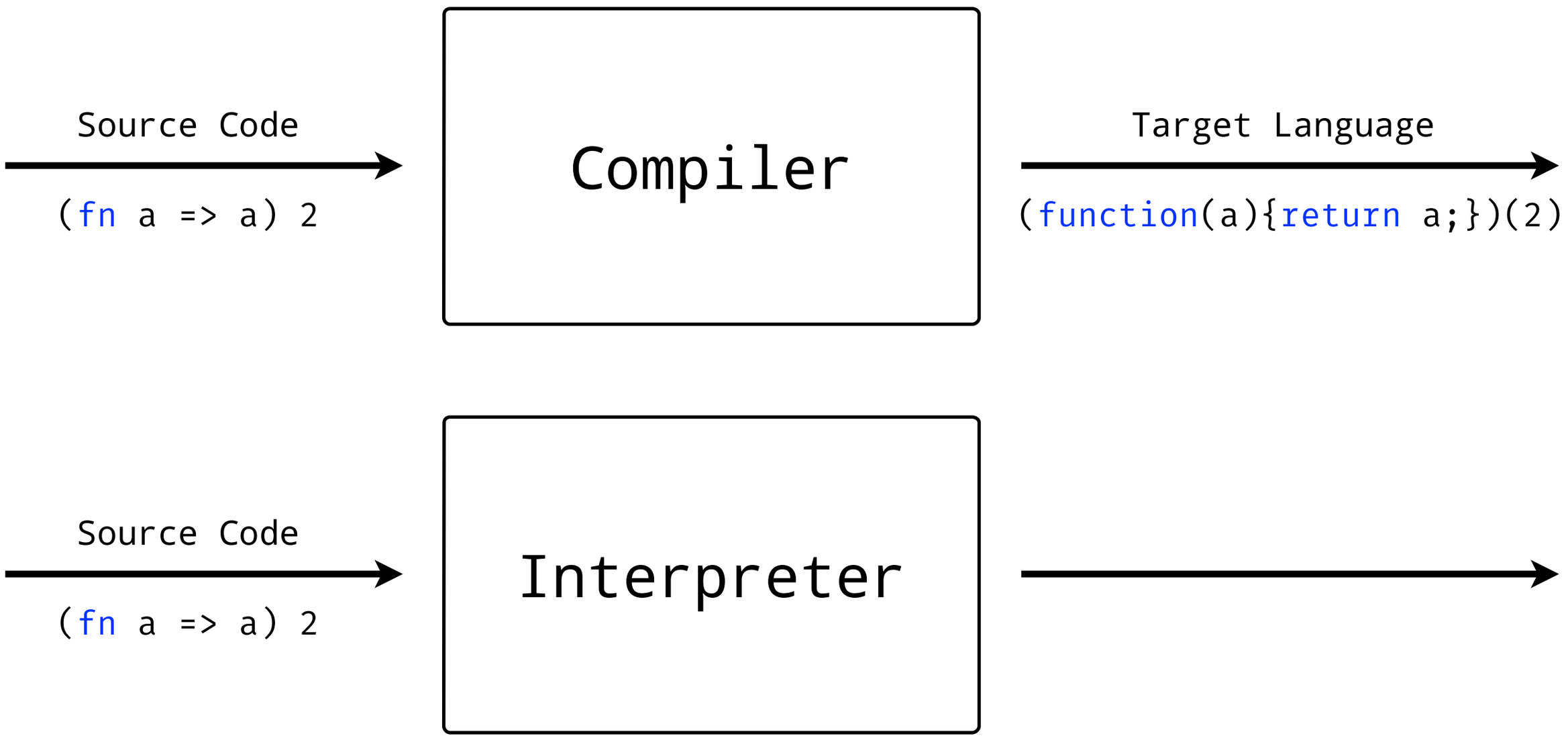

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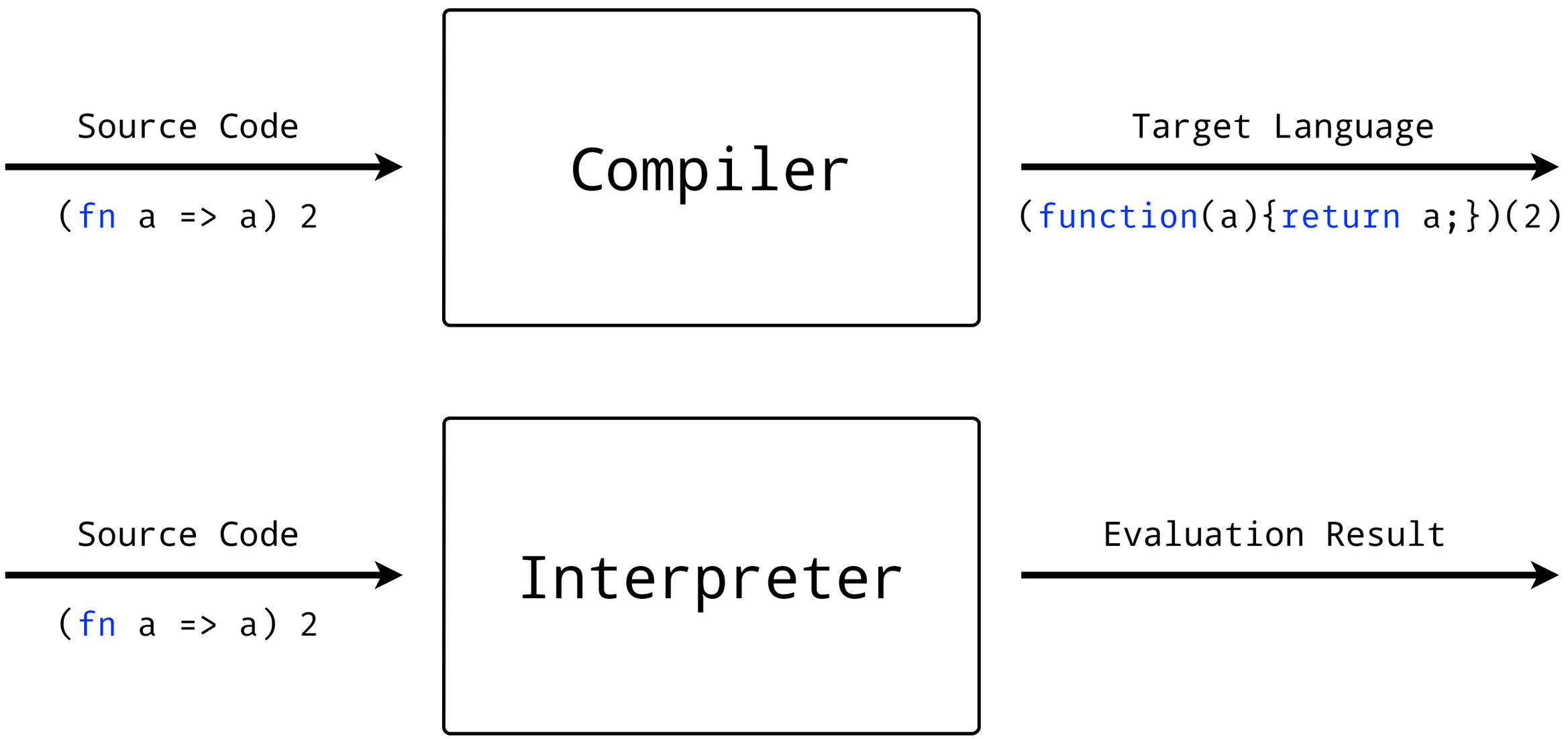

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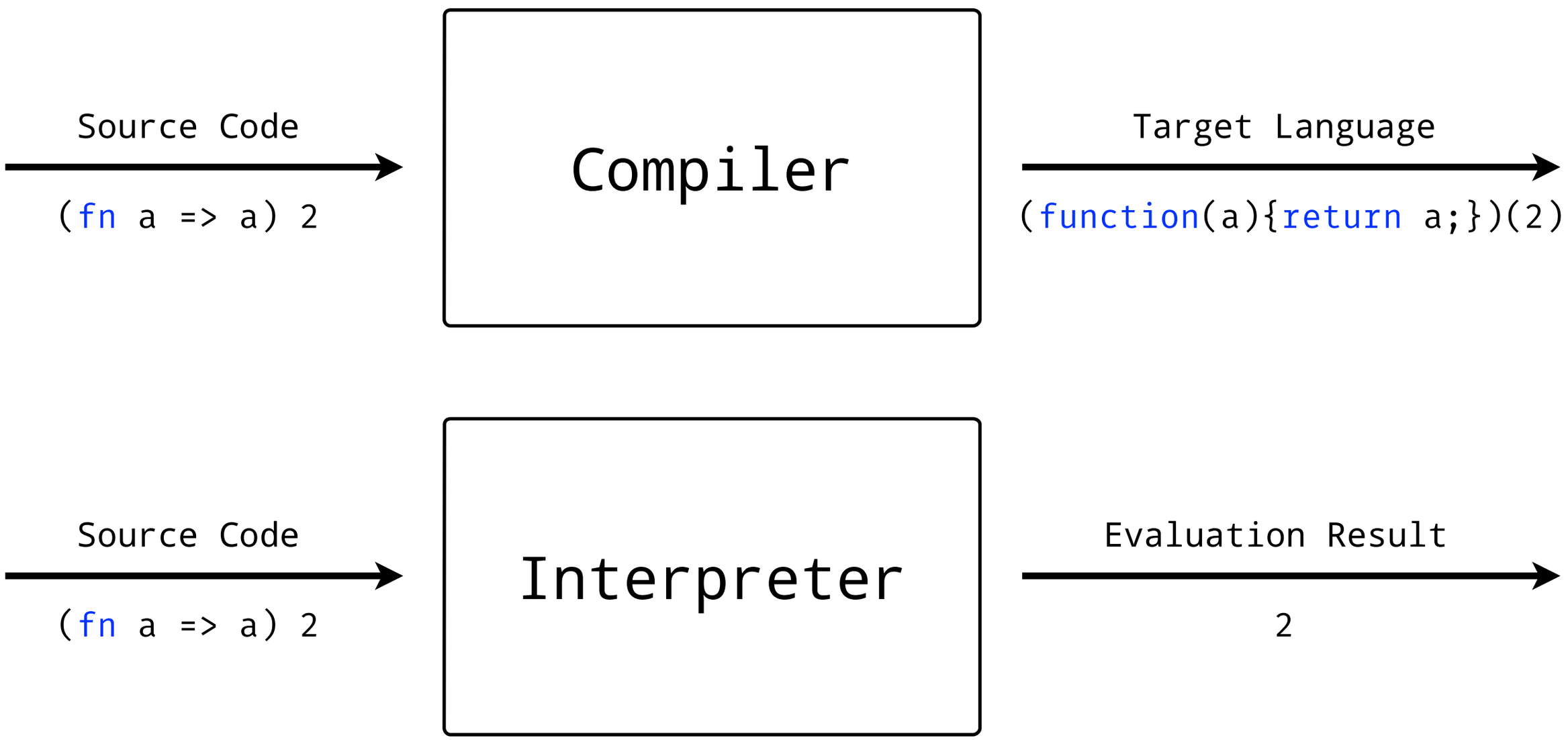

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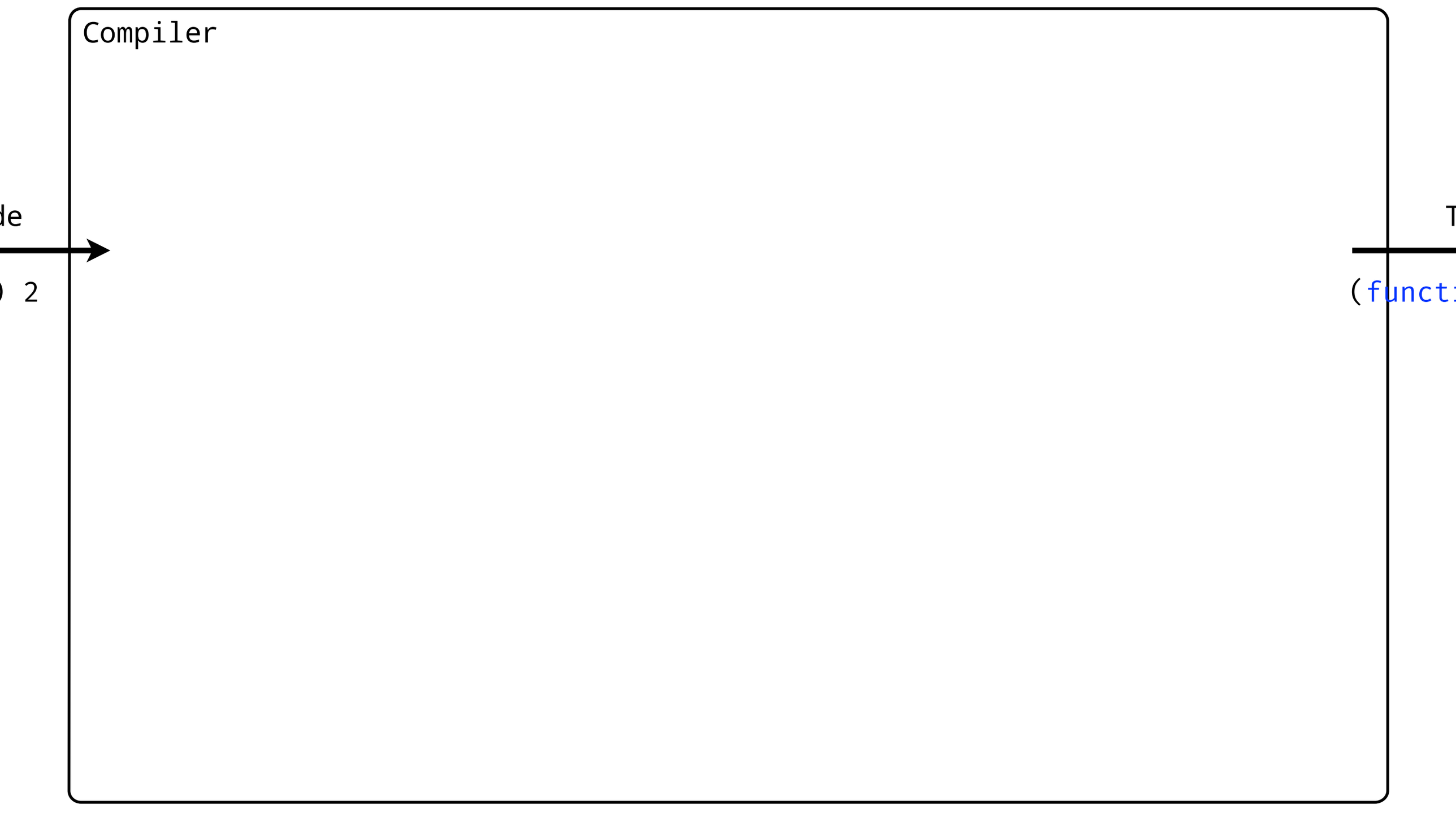
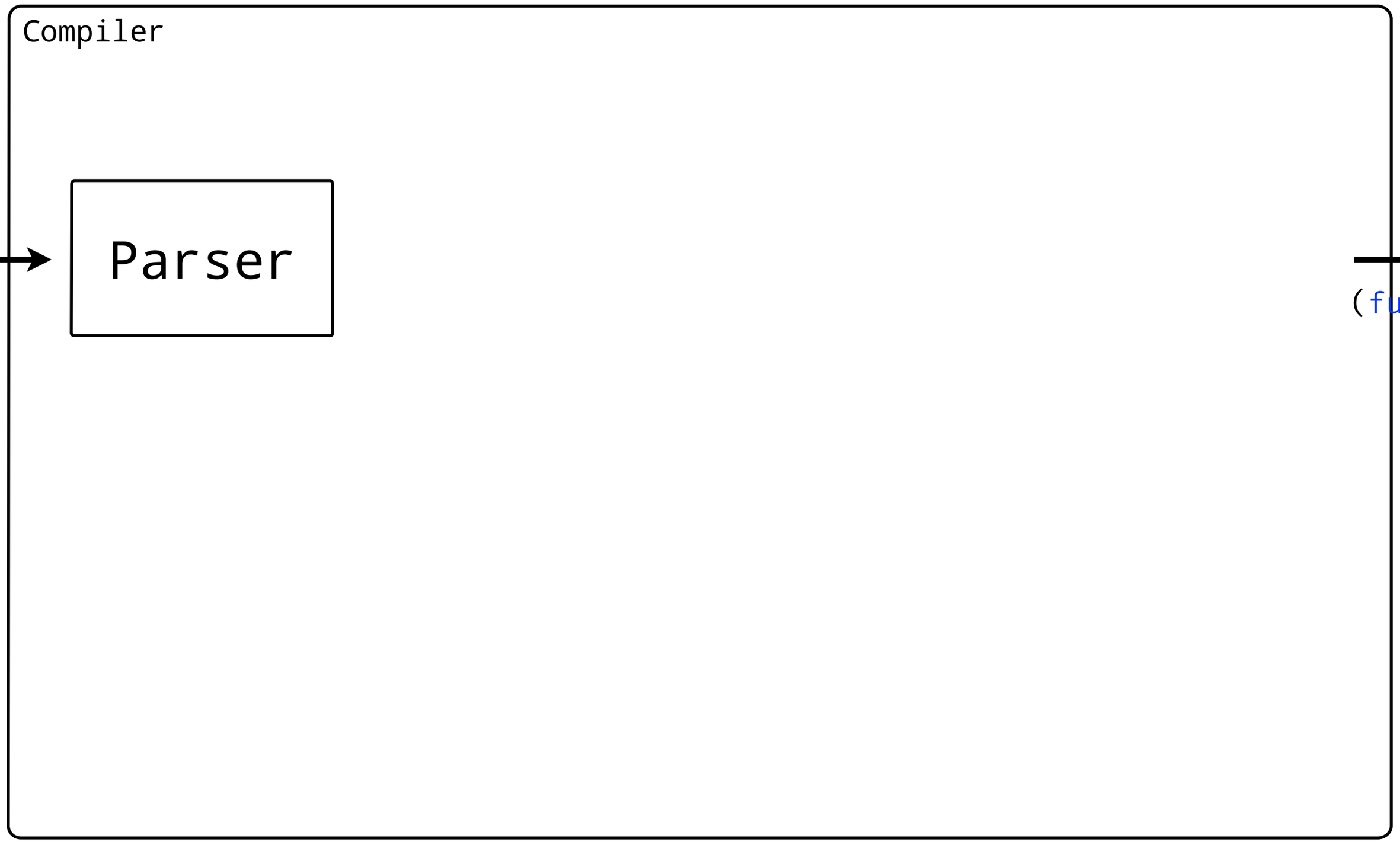
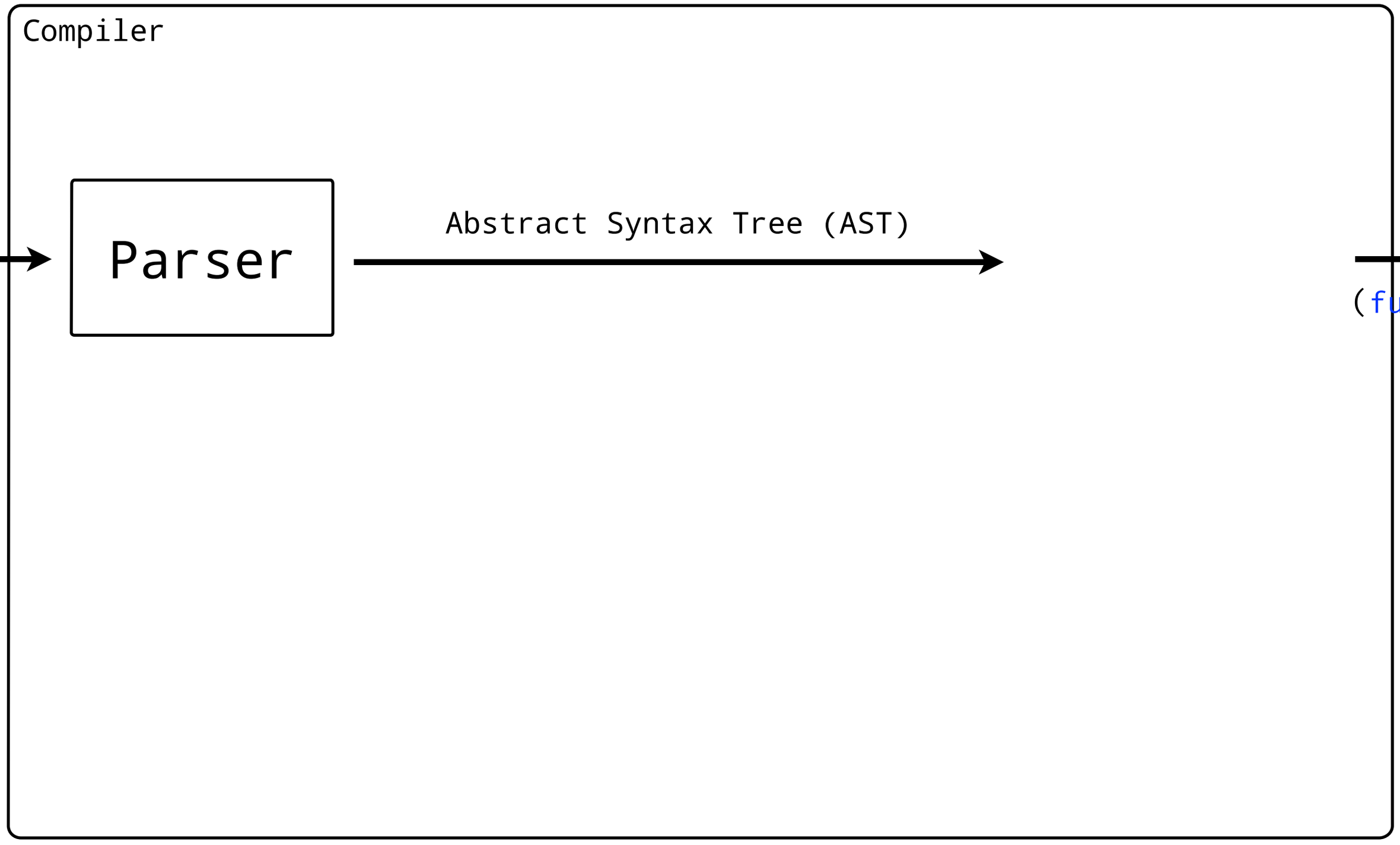


Parsing



Abstract Syntax Tree



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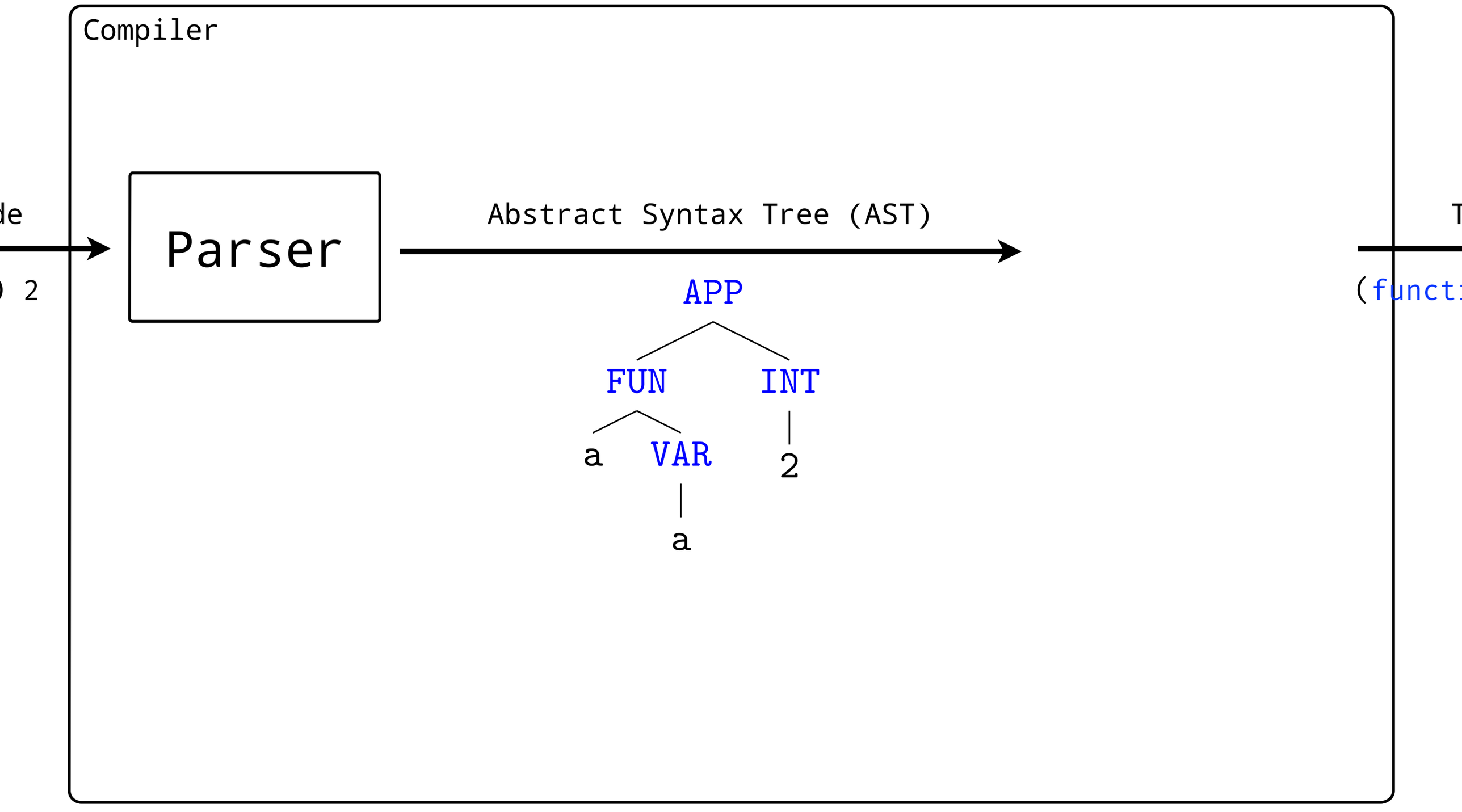
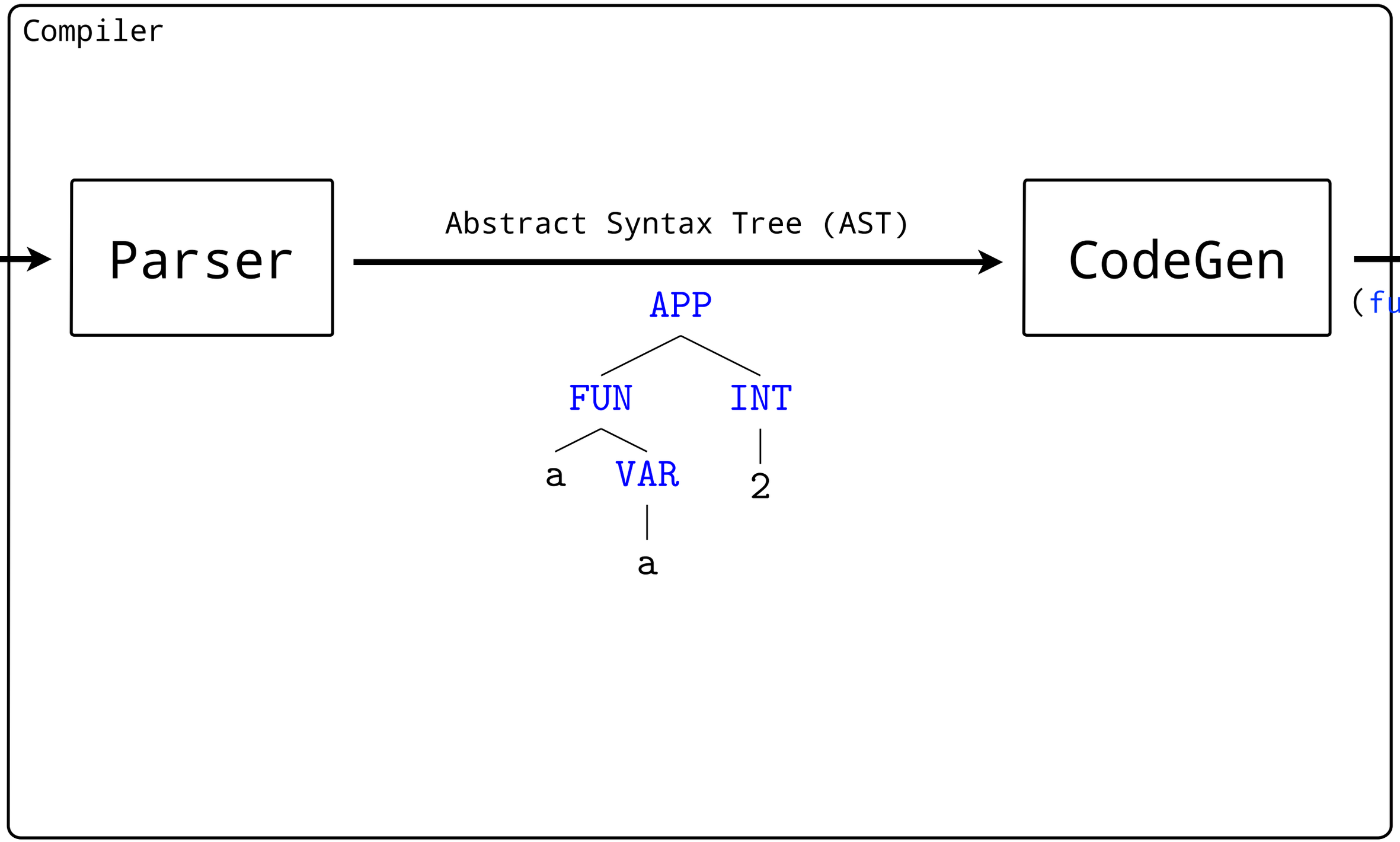
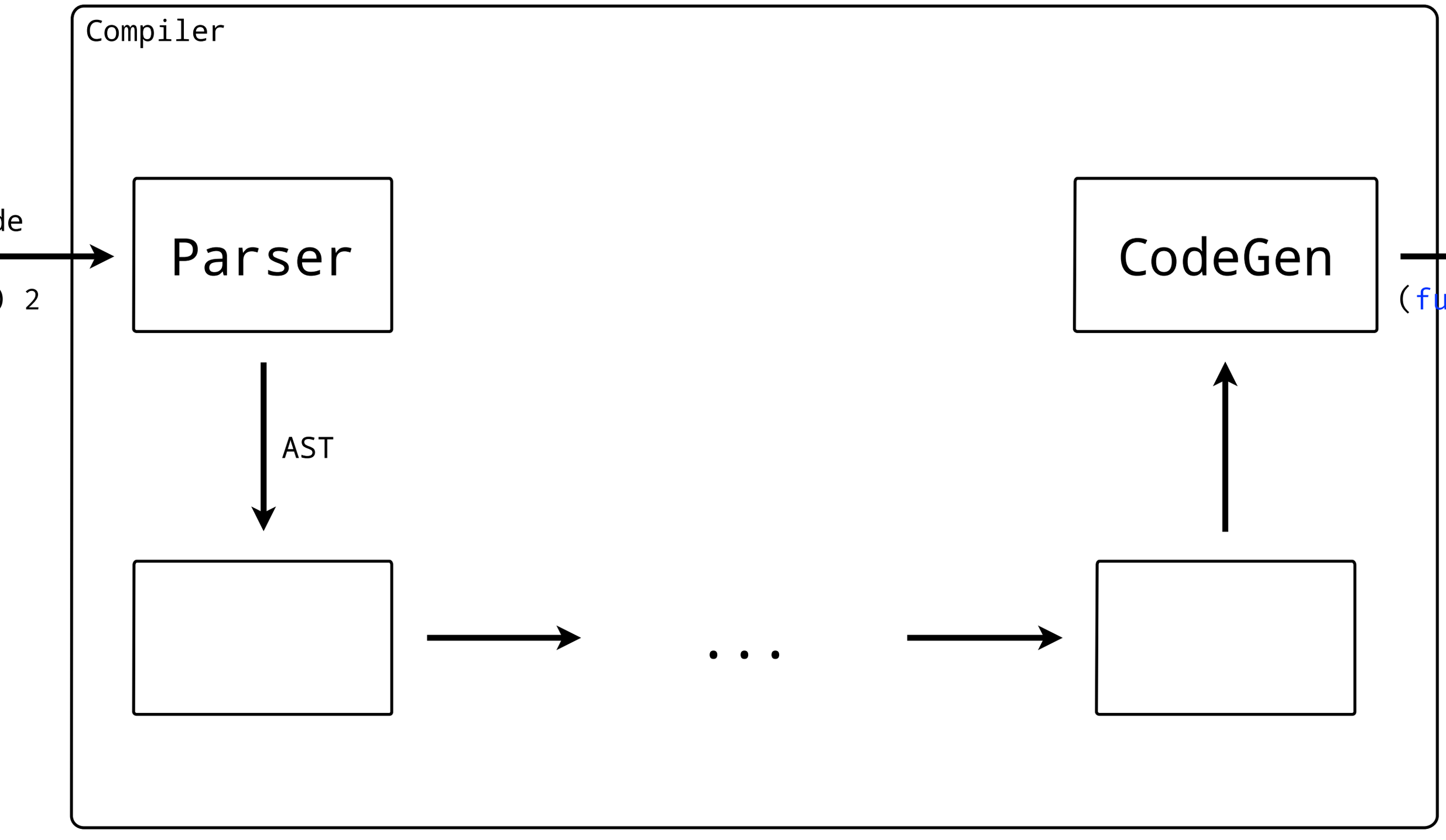


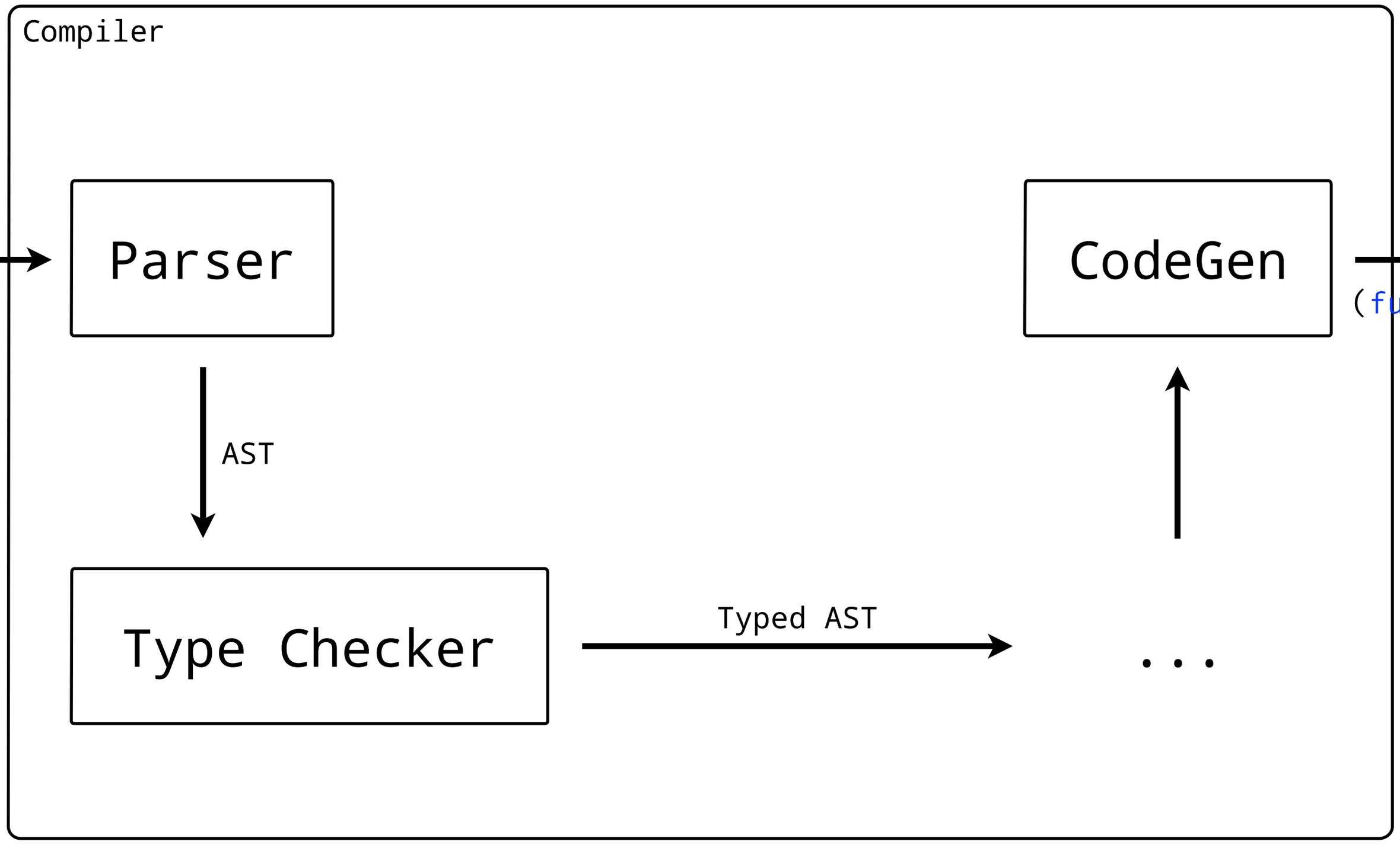
Code Generation



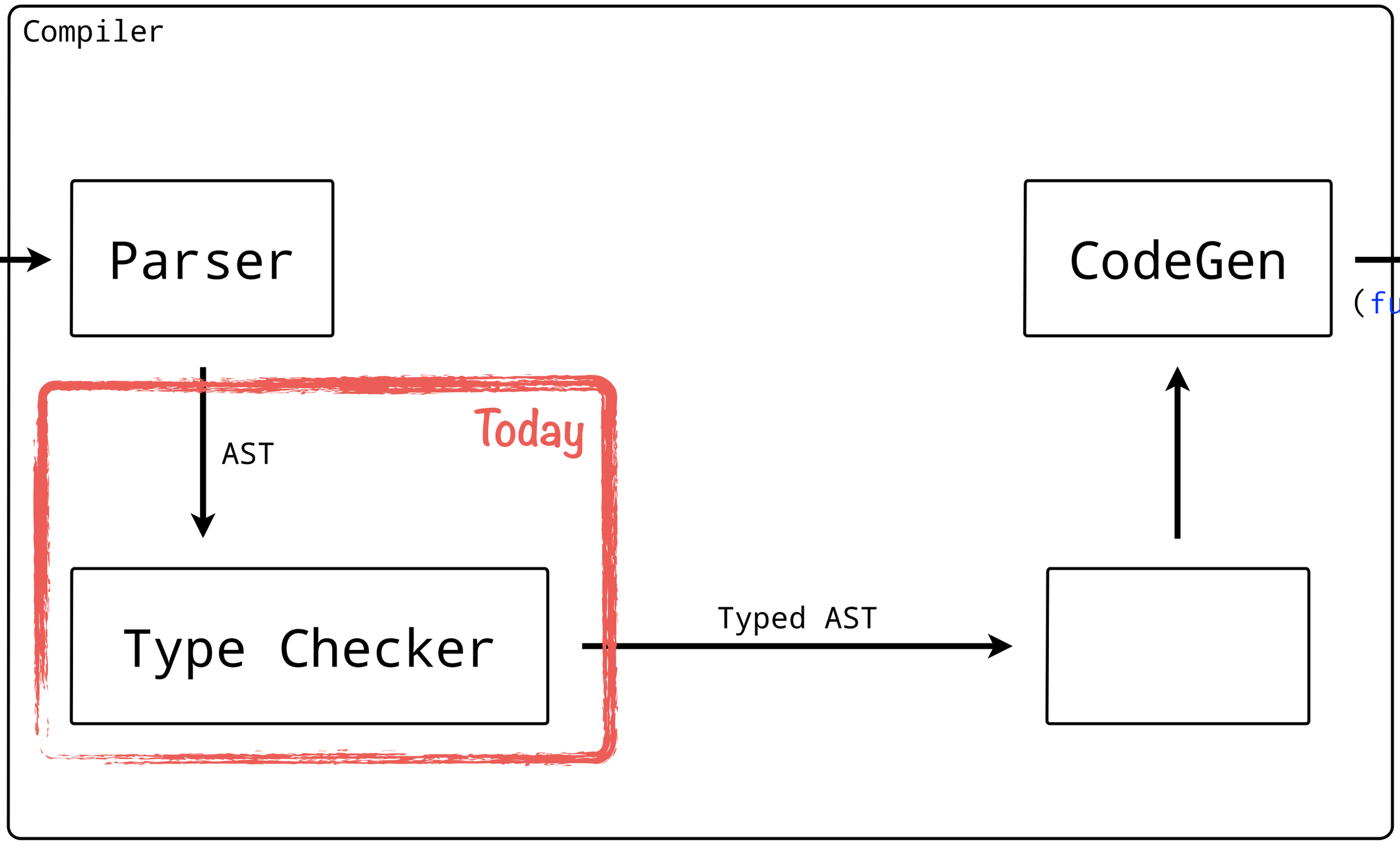
Many Intermediate Phases



Type Checking



Today's Talk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Parser
CodeGen
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AST
Today
Type Checker
Typed AST

Type Checking

vs

Type Inference

Type Checking vs Inference

- Type Checking
  - Ensures **declared** types are used consistently
  - All types must be declared
  - Traverse AST and compare def site with use site
- Type Inference
  - Ensures consistency as well
  - Types need not be declared, though; are **deduced**
  - Two main classes of algorithms
  - We'll see one instance today

Vehicle Language

Vehicle Language

- Surface Syntax
  - What's the concrete syntax of the language
- Type System
  - What types are supported by the language

Surface Syntax[1]

1. Numbers: 1, 2, 3, ...

1 — a subset of Standard ML

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2. Booleans: `true` and `false`
3. Anonymous functions (lambdas): `fn a => a`
4. Function application: `inc 42`
5. If expressions: `if cond then t else f`

1 — a subset of Standard ML

Surface Syntax[1]

6. Let blocks/expressions:

```
let
   val name = ...
in
   name
end
```

1 — a subset of Standard ML

Small Example

```
let
  val inc = fn a => a + 1
in
  inc 42
end
```

Language Types

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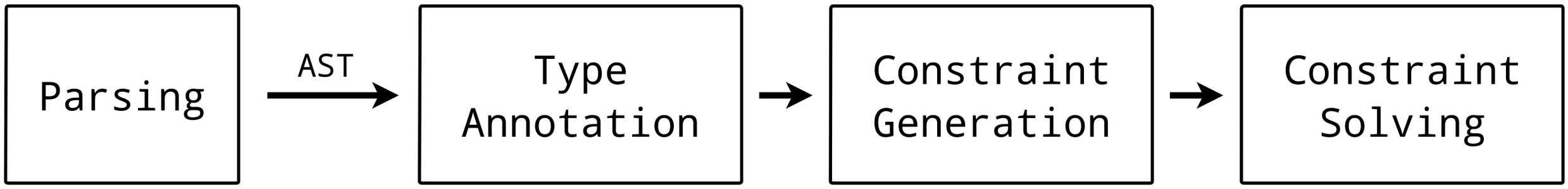
4. Generic type variables: `'a`, `'b`, `'c`, etc.

Today's Algorithm Overview

Wand's Algorithm

Overview

Wand's Algorithm Overview



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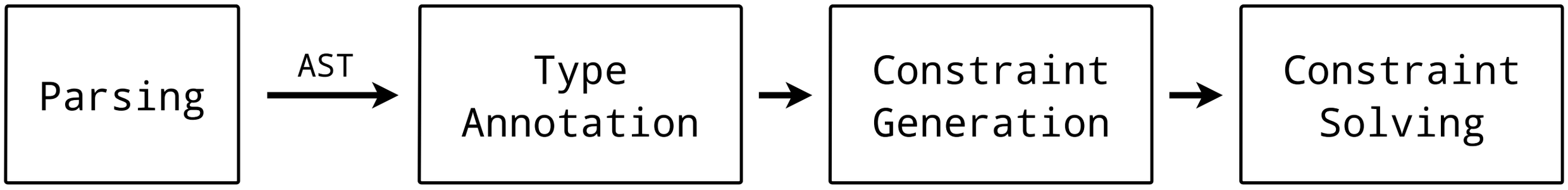


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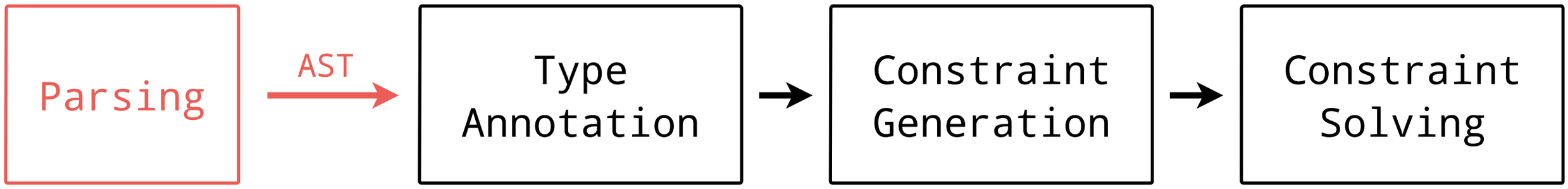


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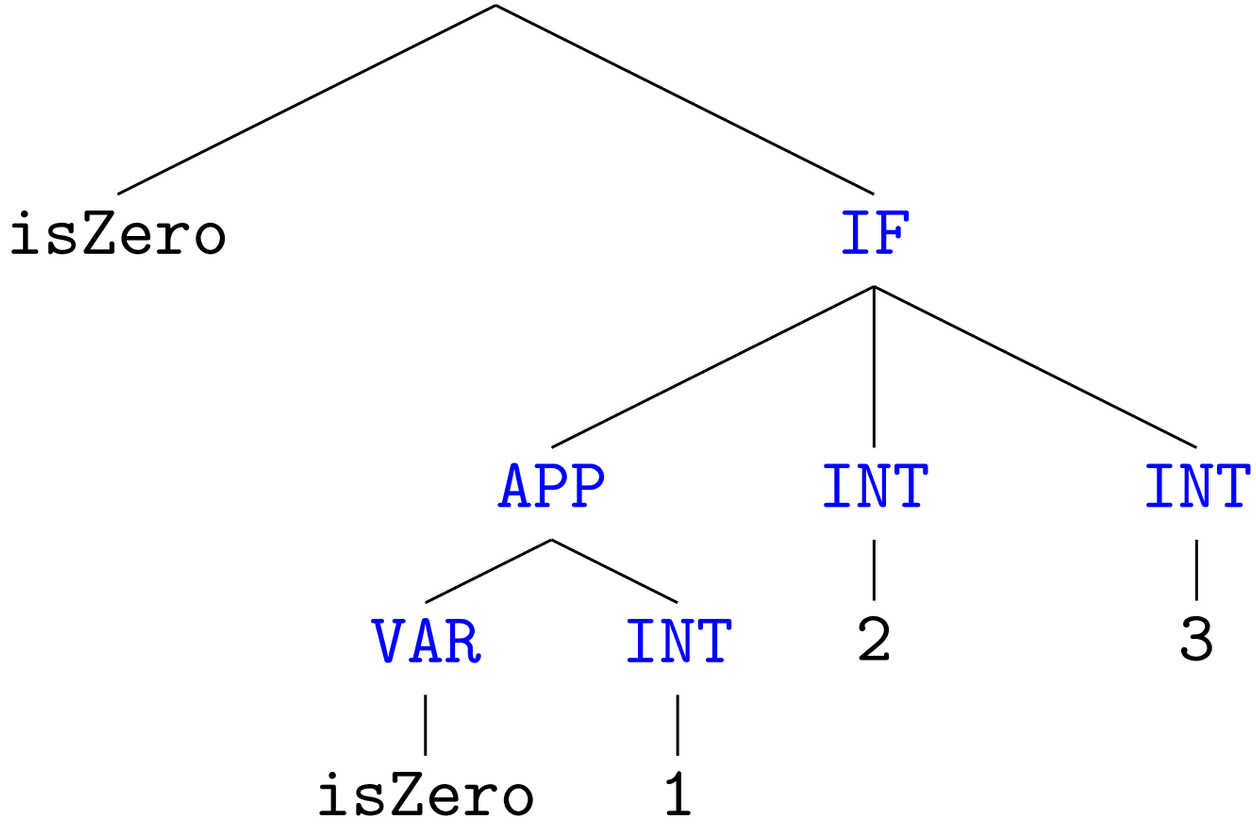

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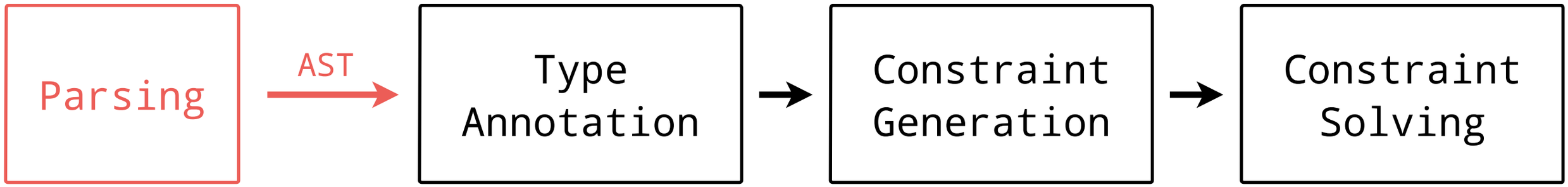


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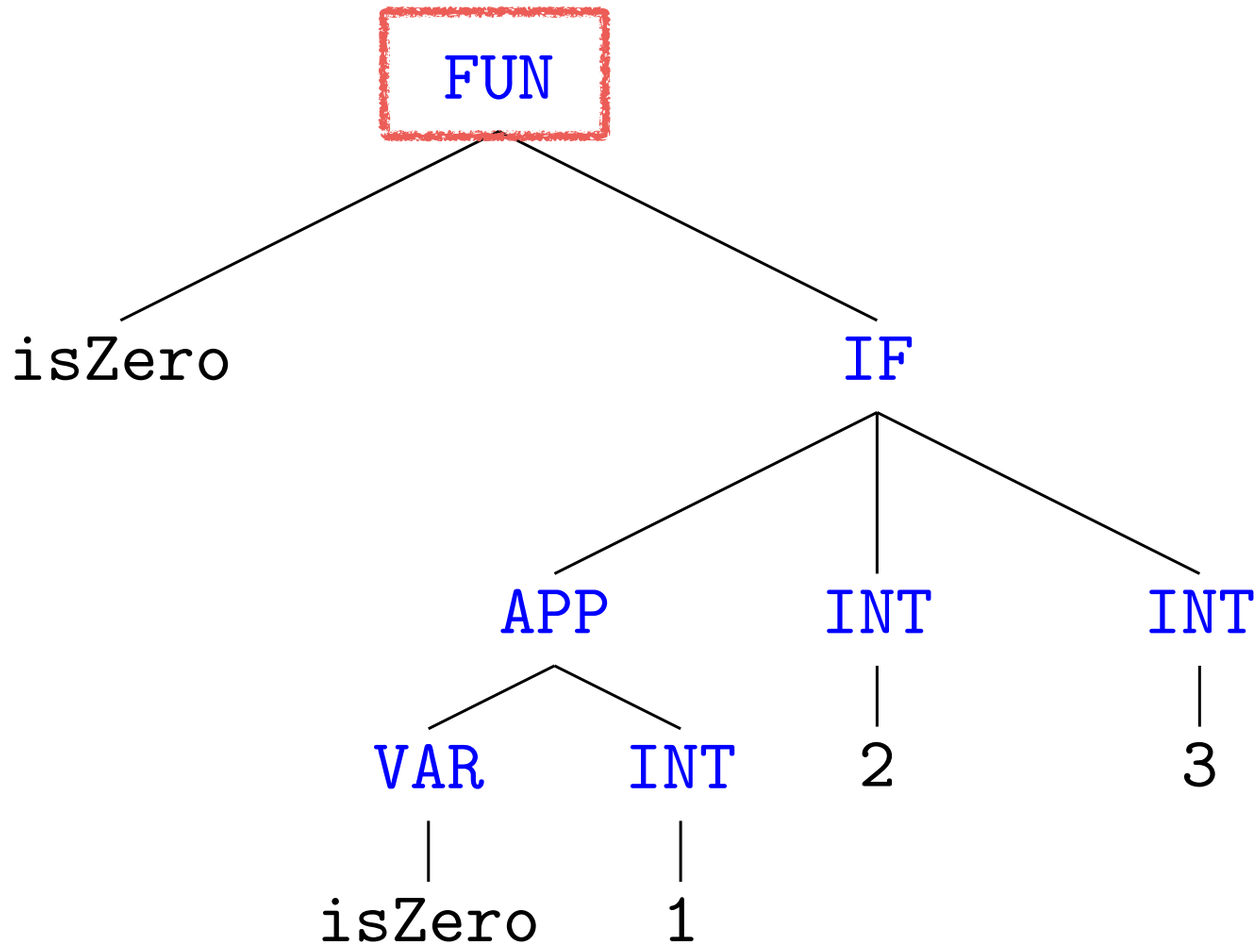

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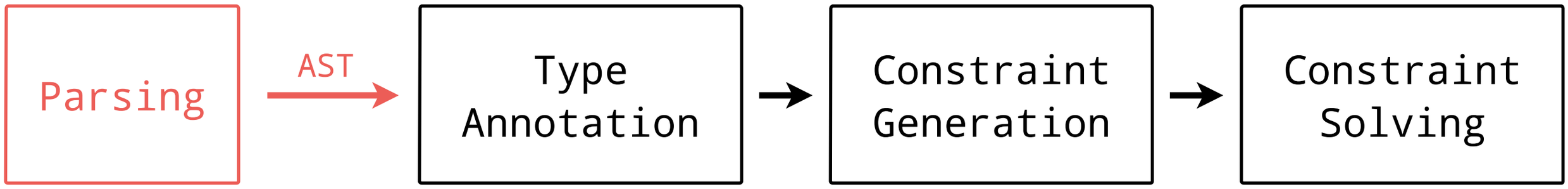


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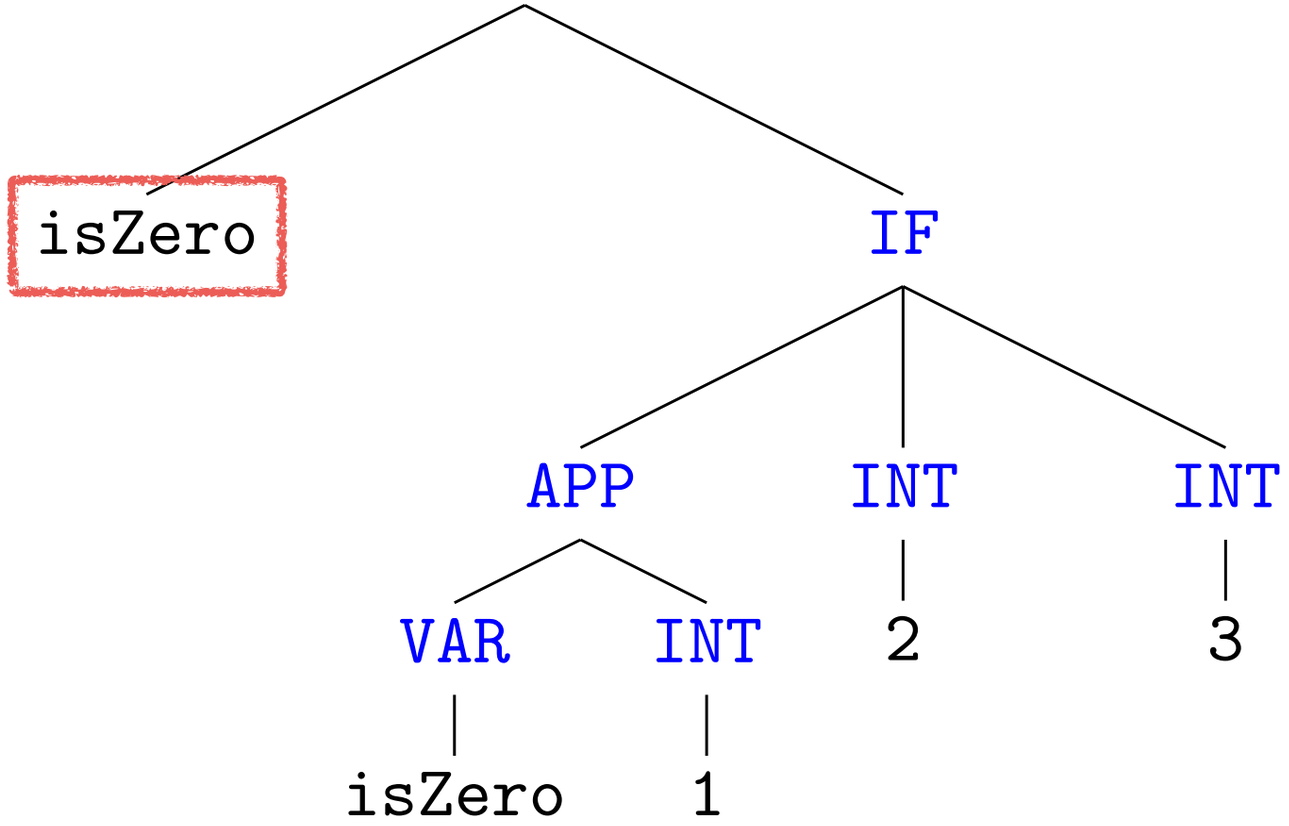

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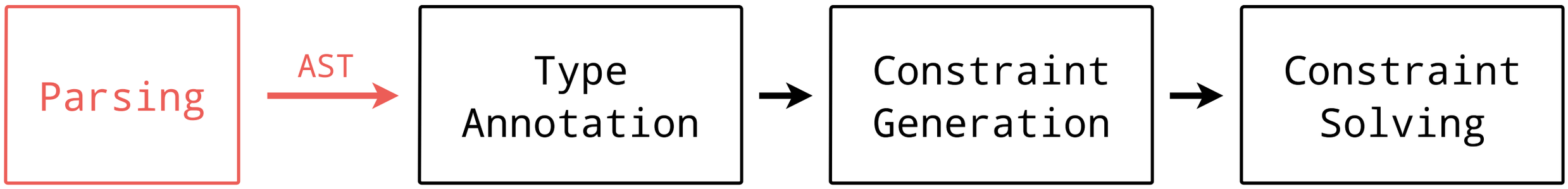


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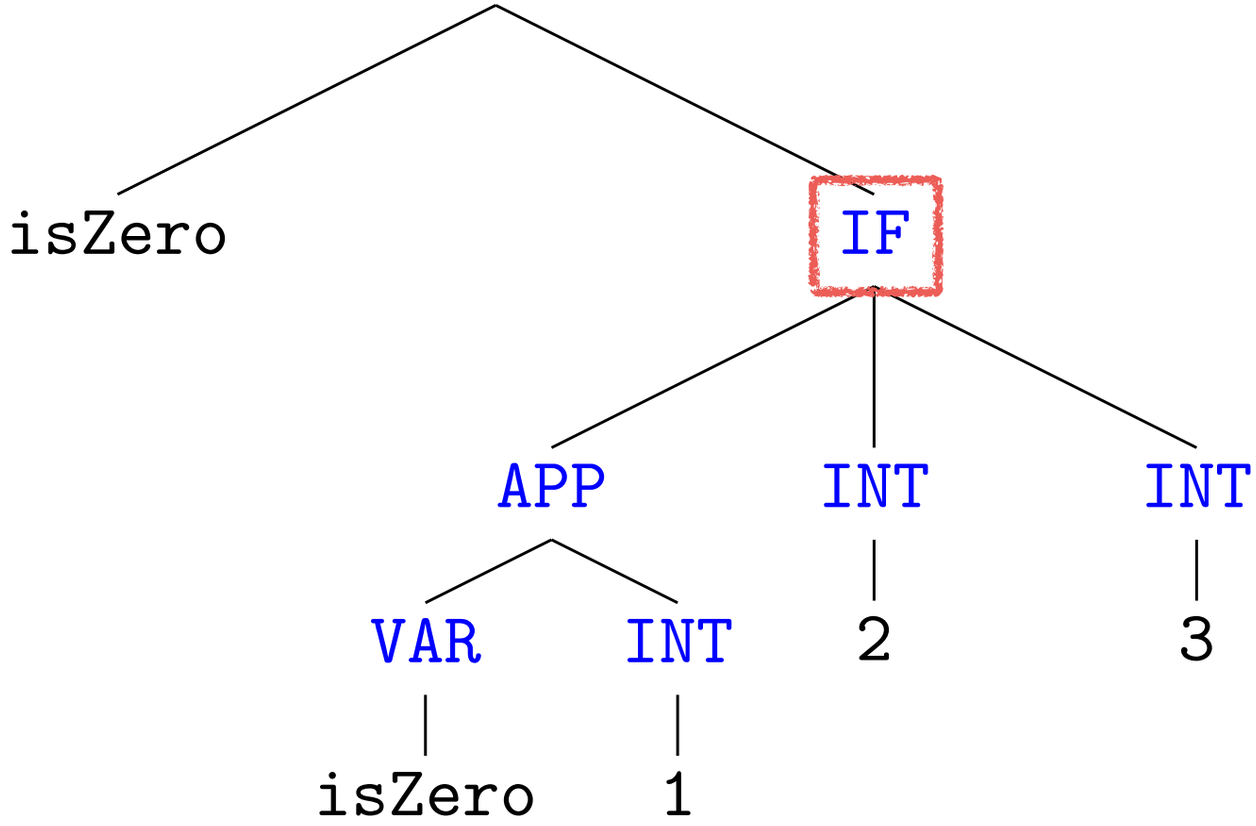

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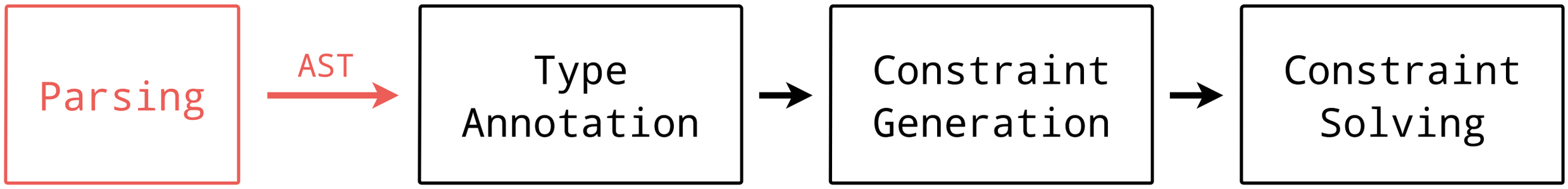


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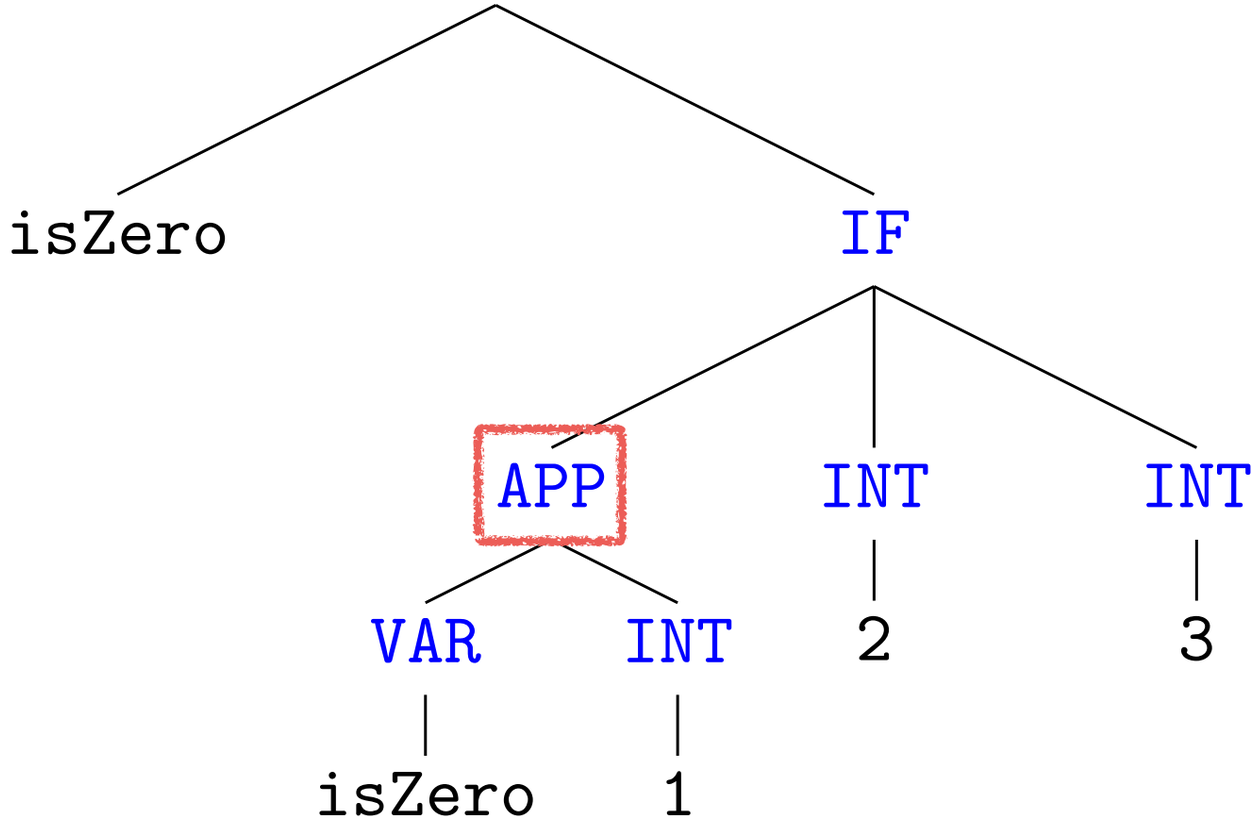

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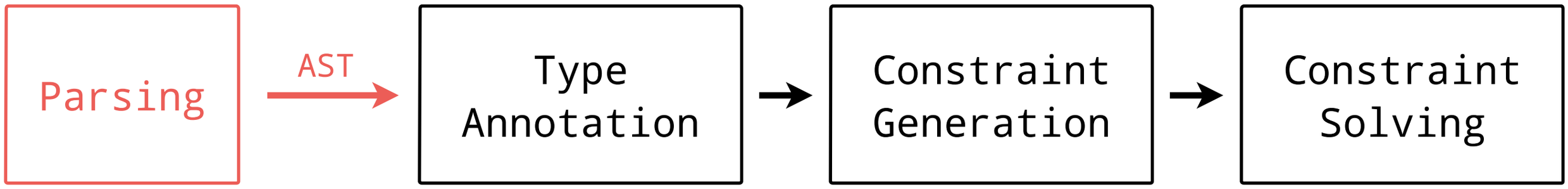


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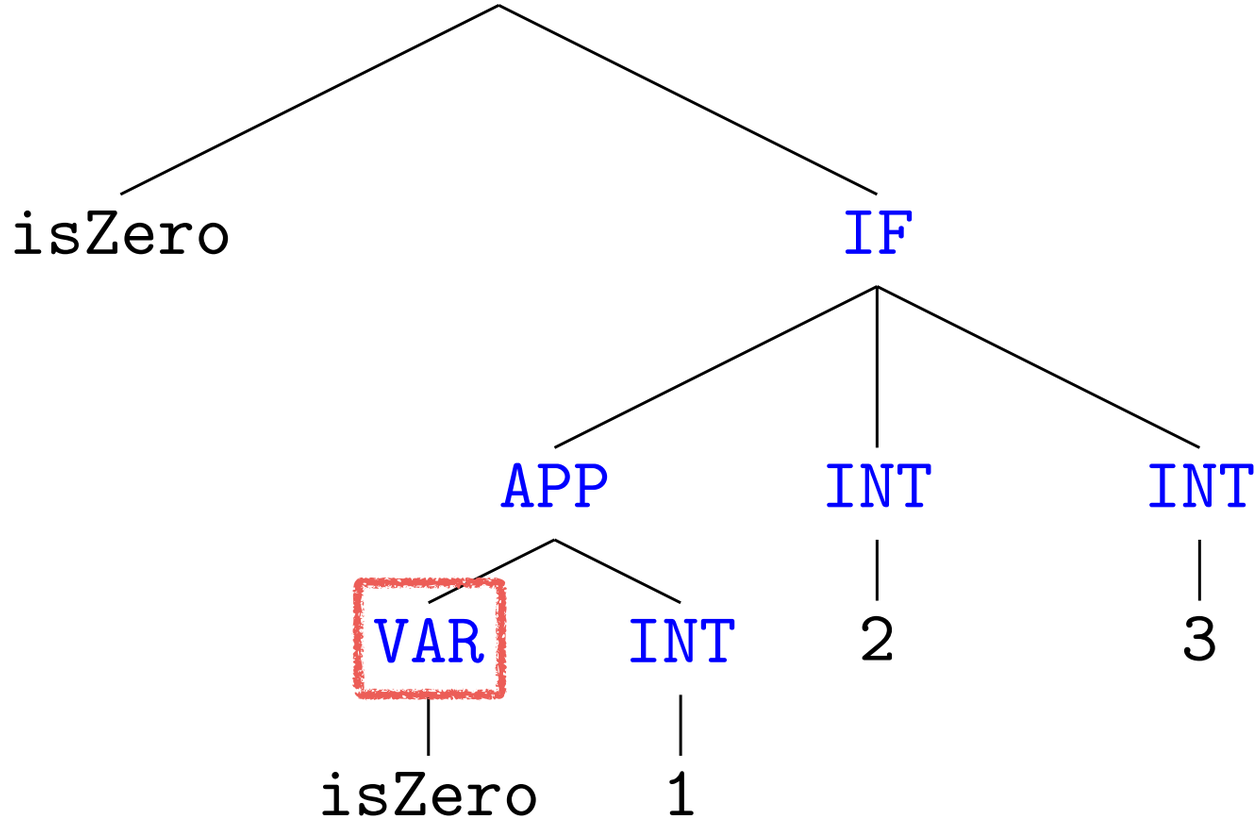

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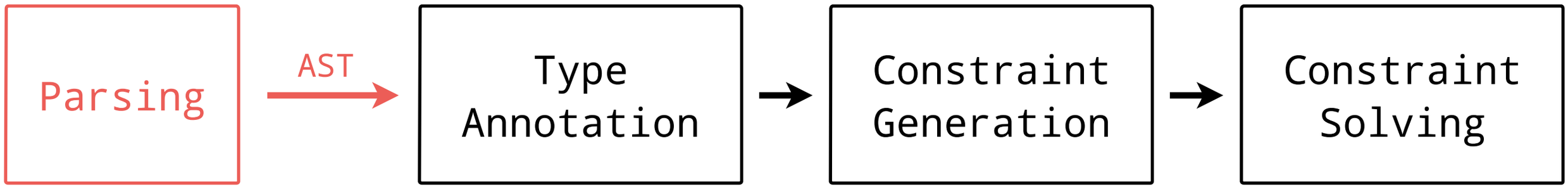


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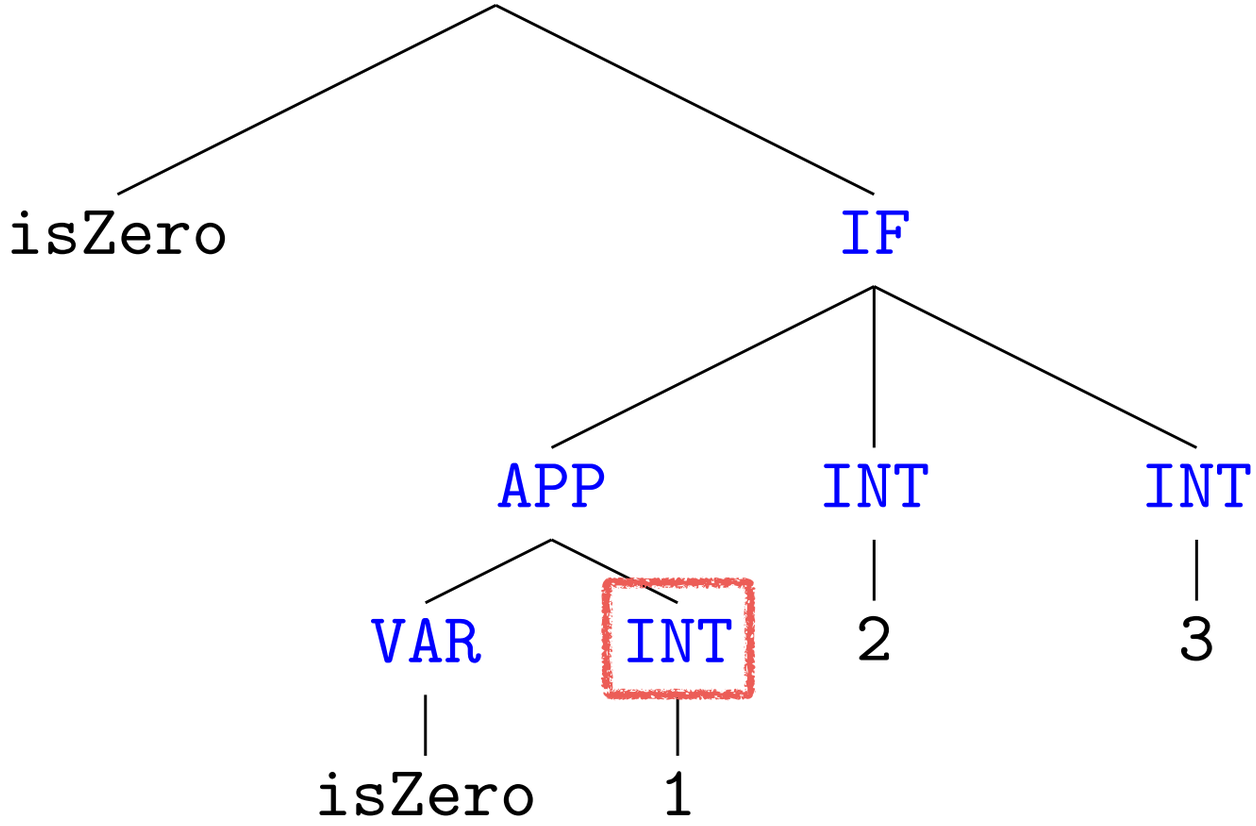

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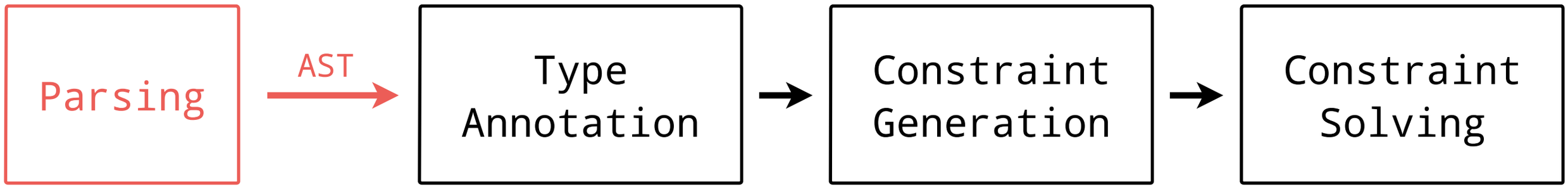


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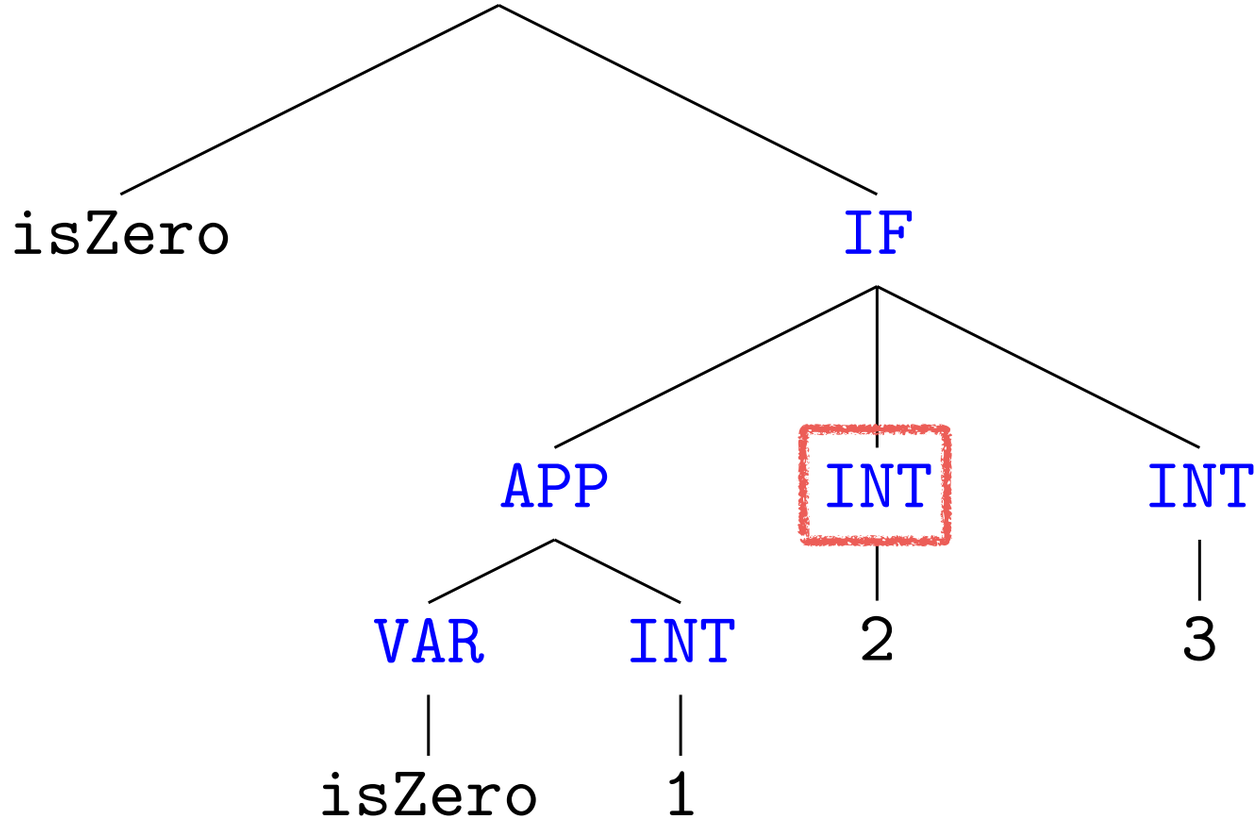

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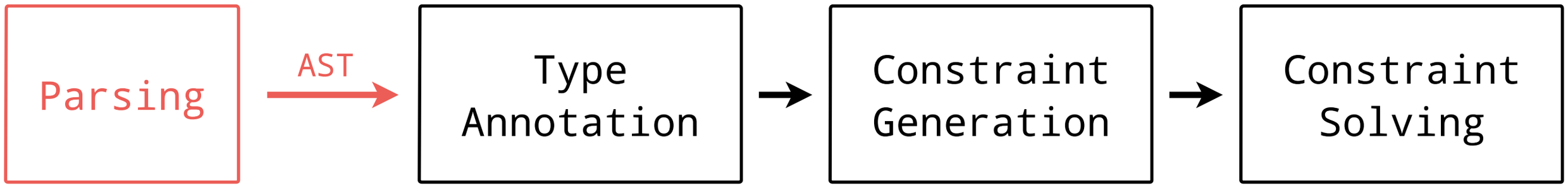


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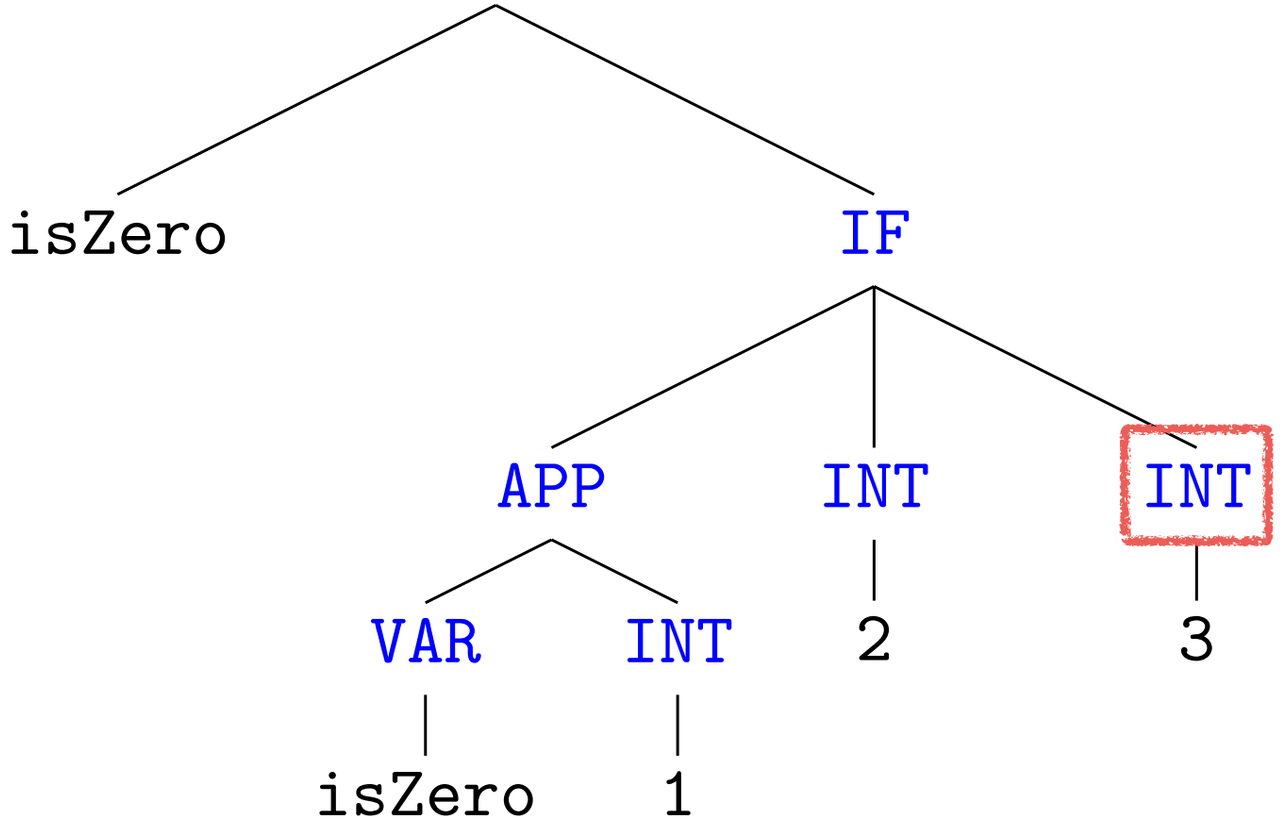

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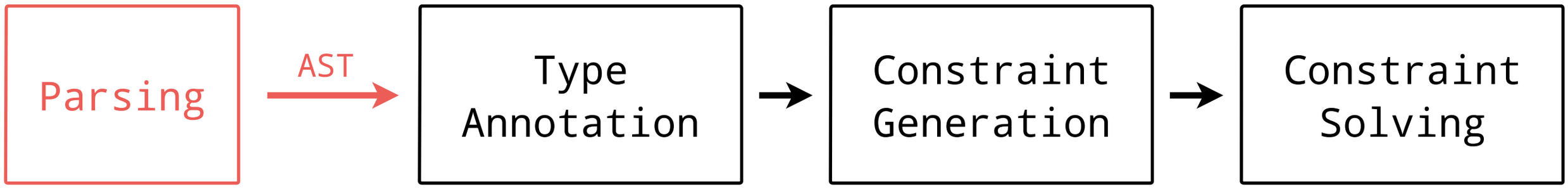


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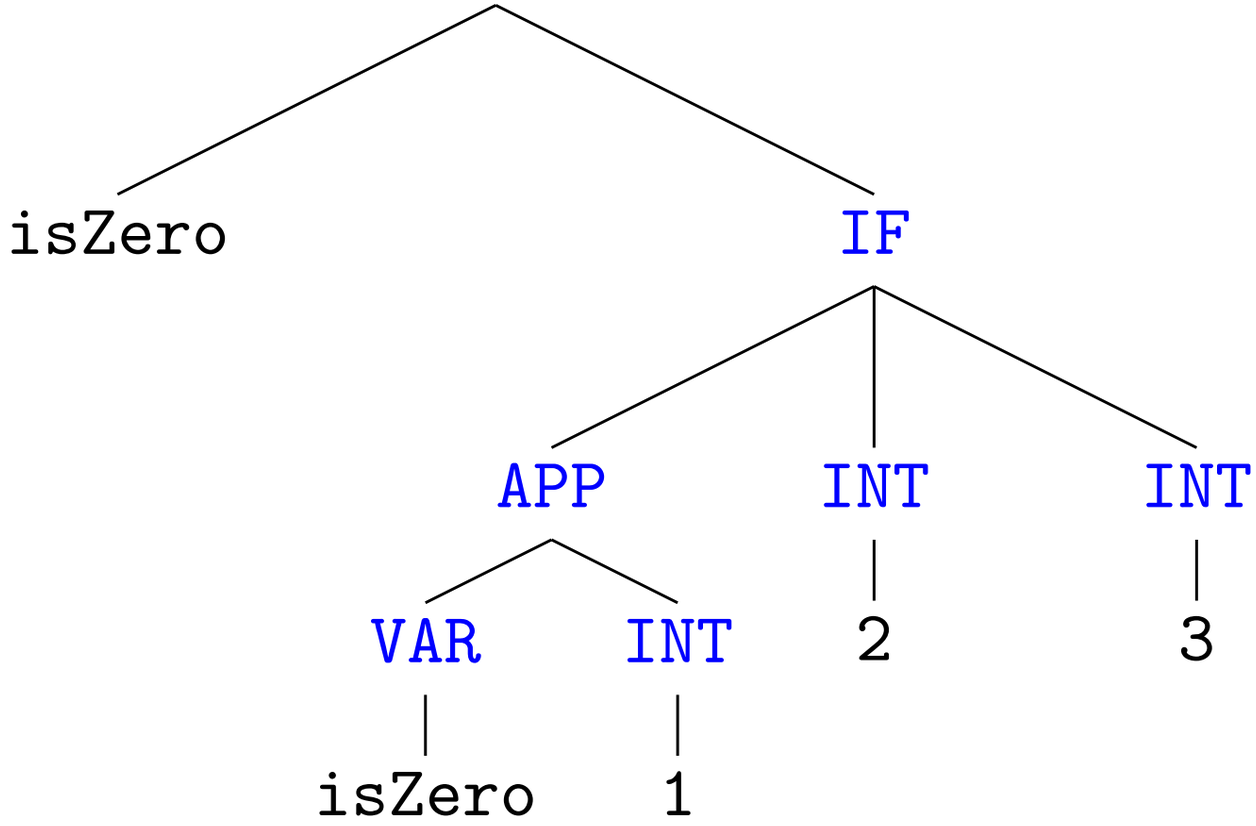

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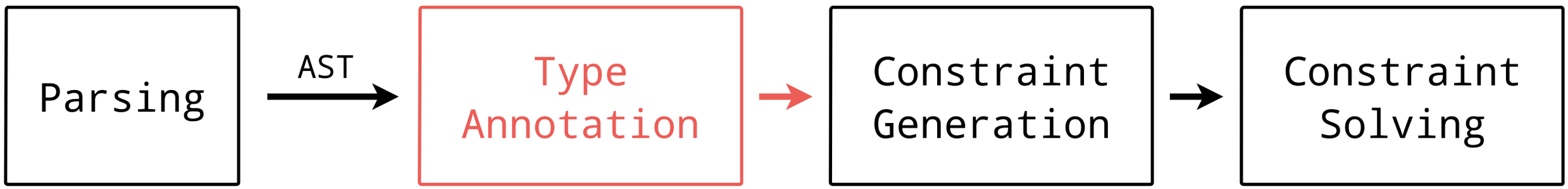

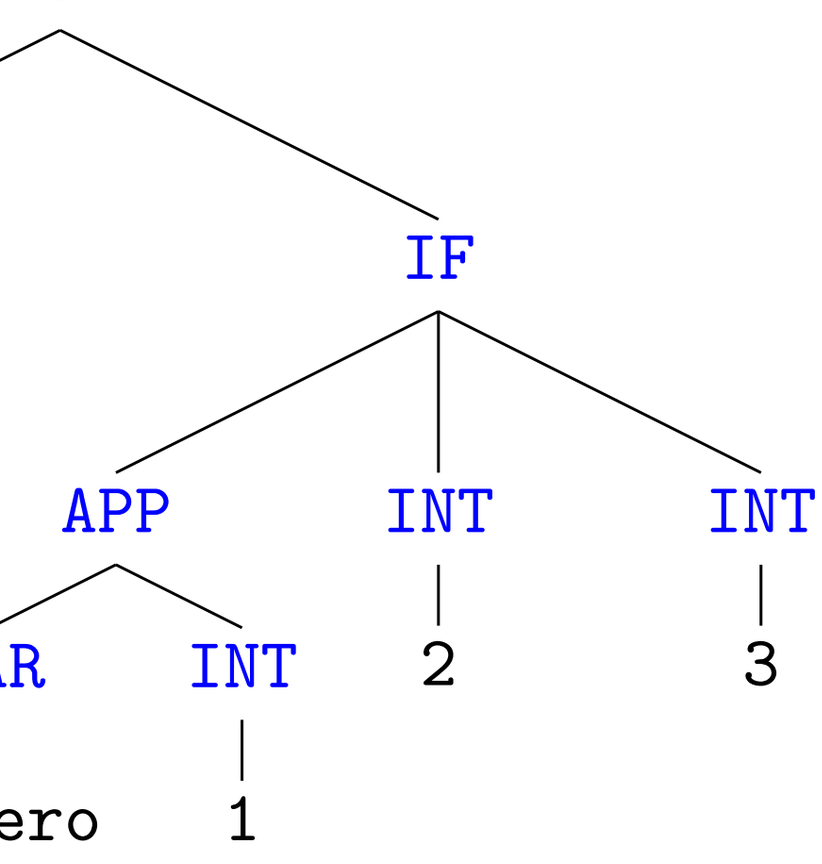

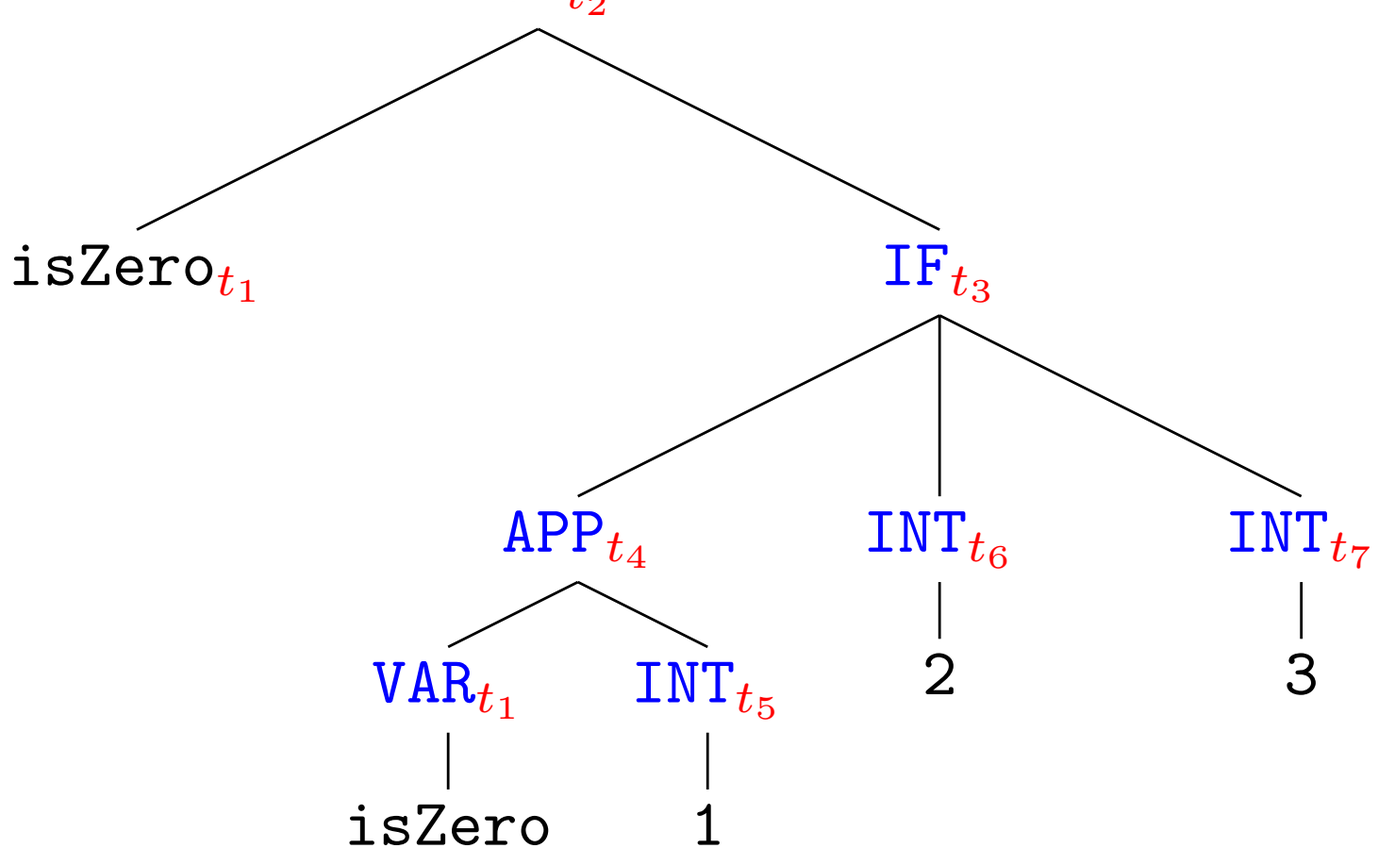

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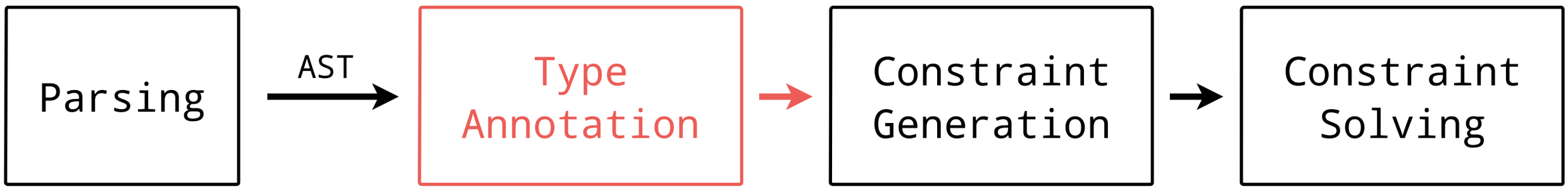

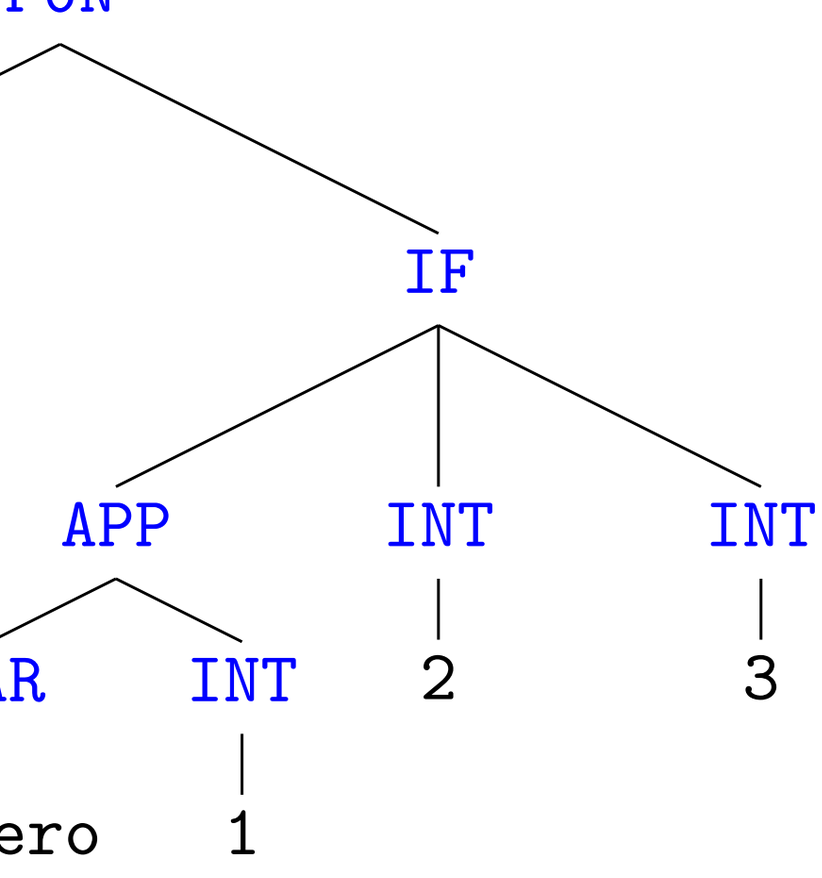

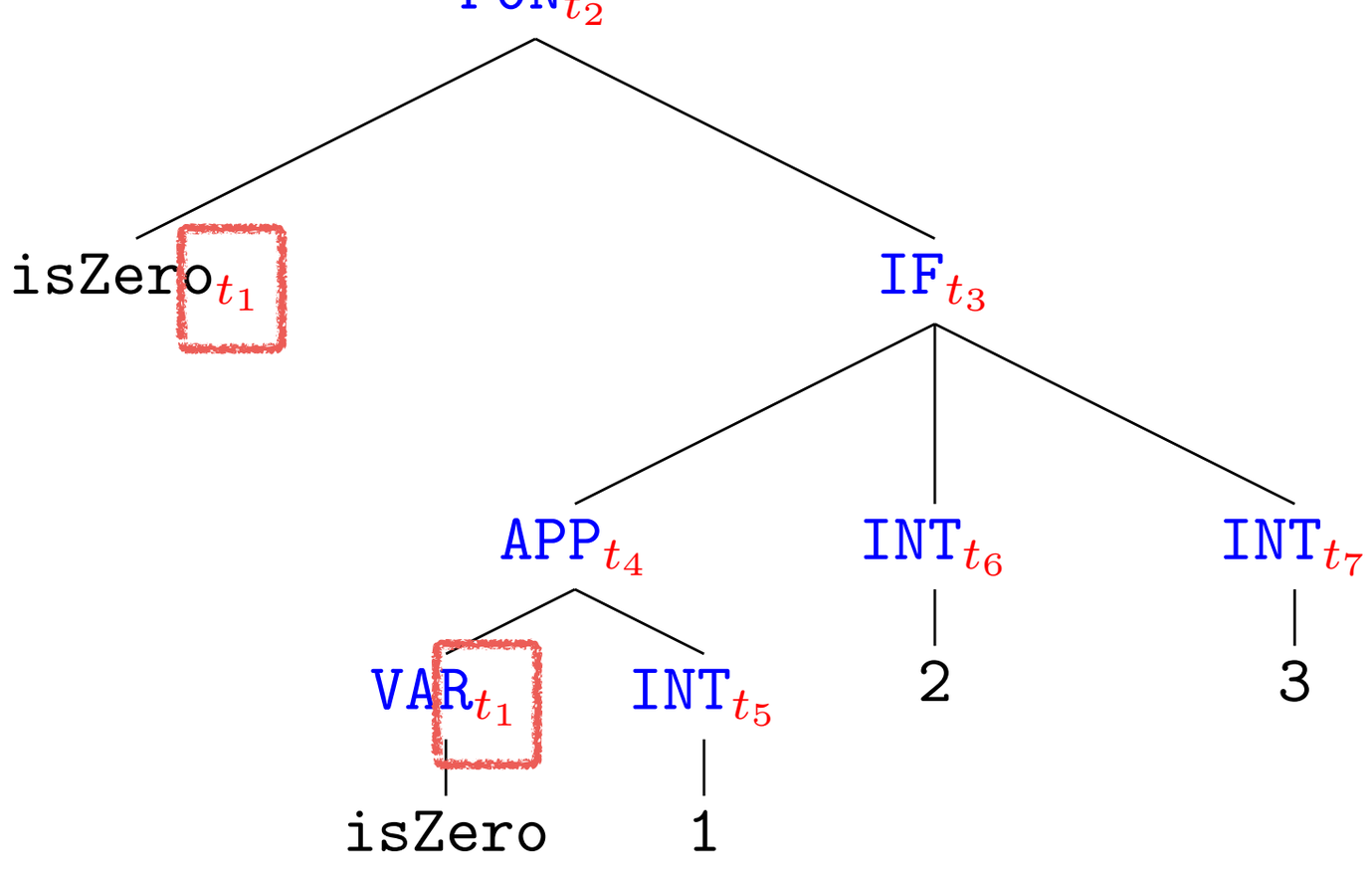

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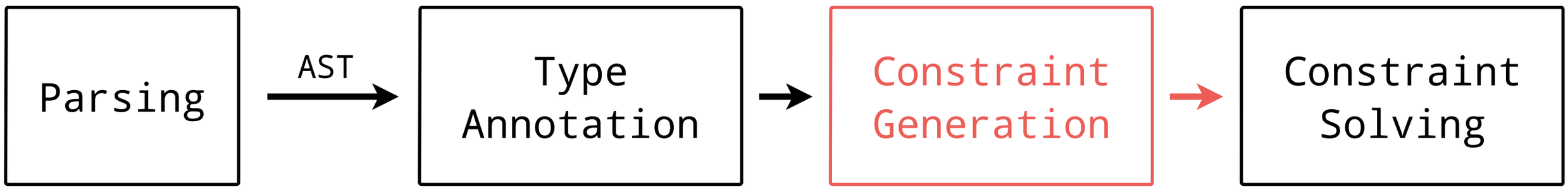
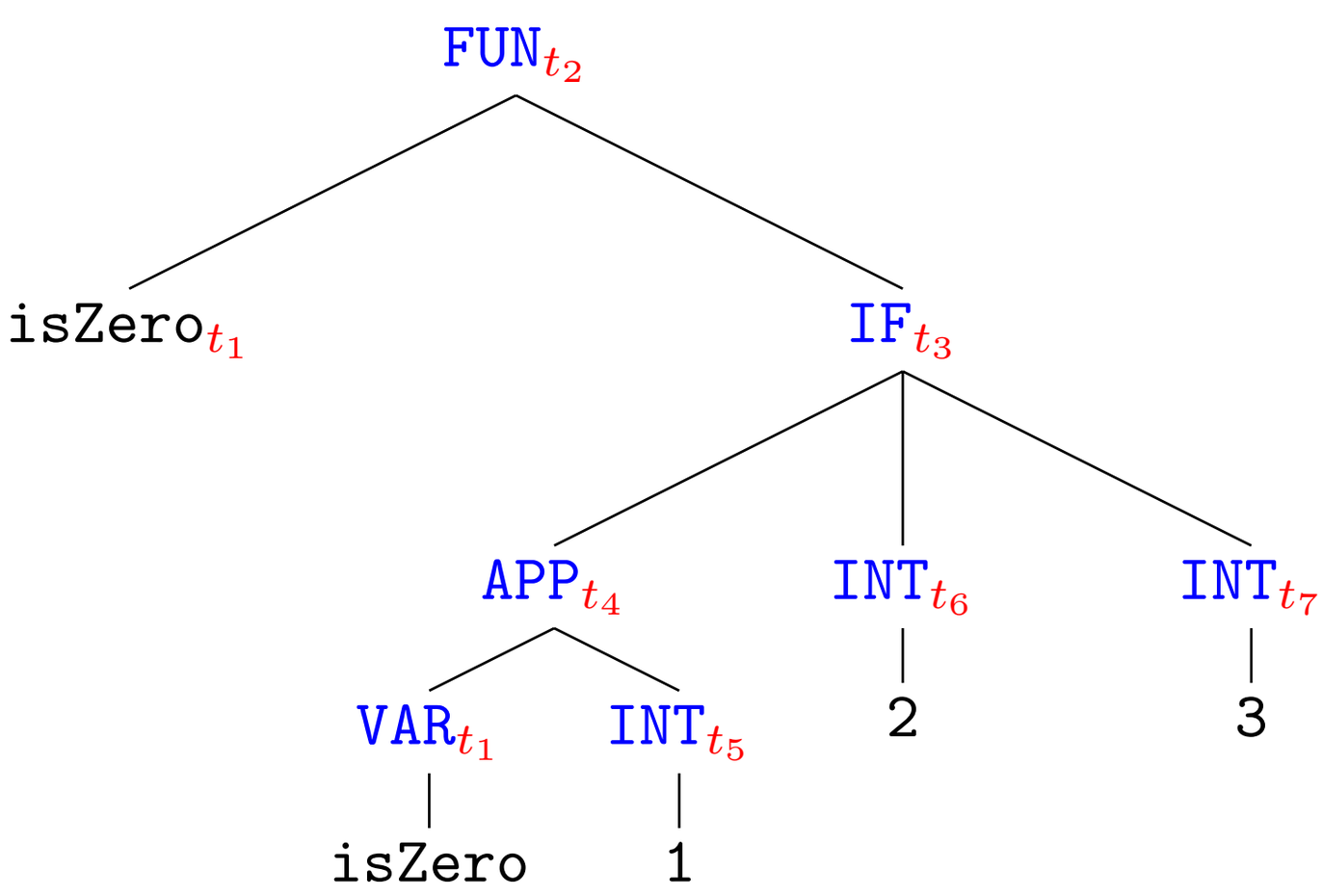
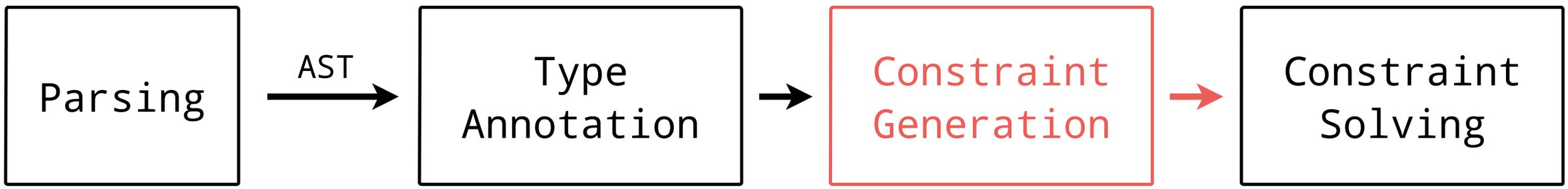
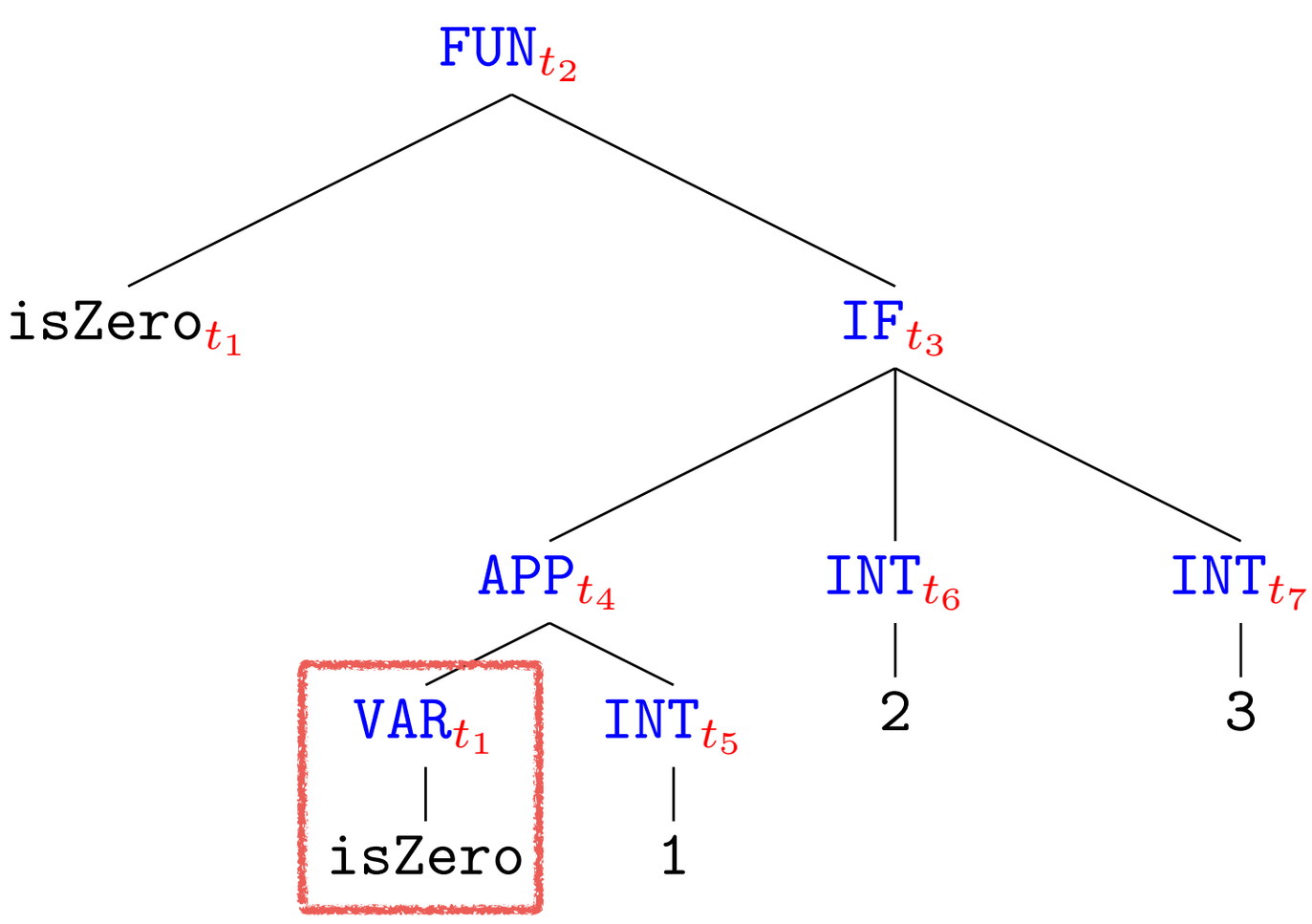




Constraint Set

Wand's Algorithm Overview





Constraint Set

```
t1 ≡ t5 → t4
```

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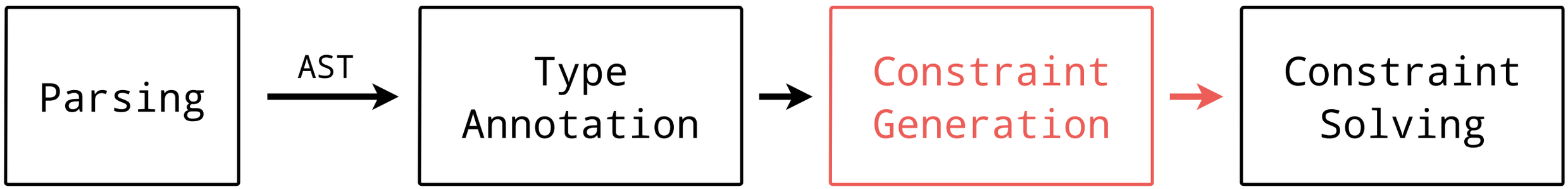
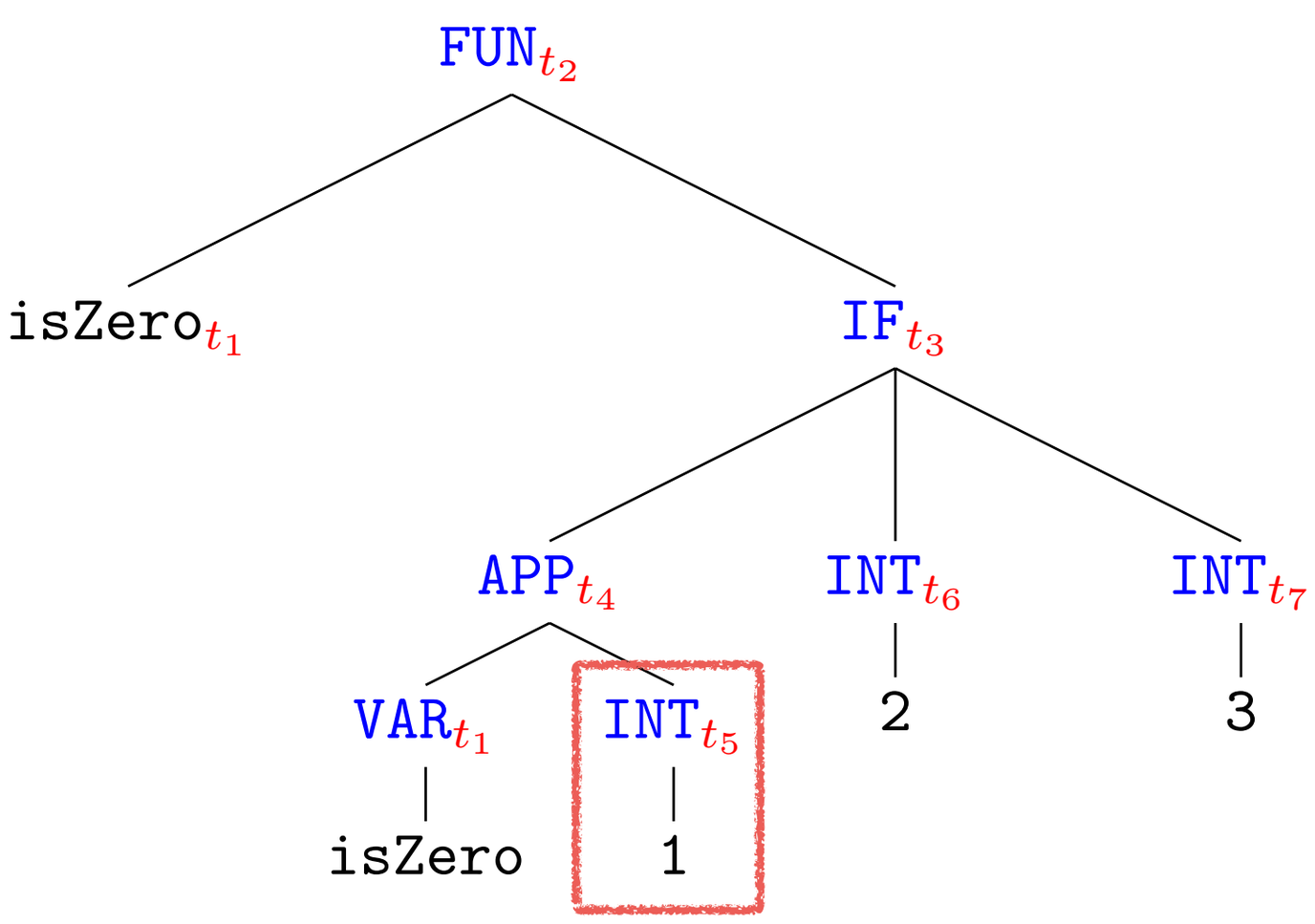
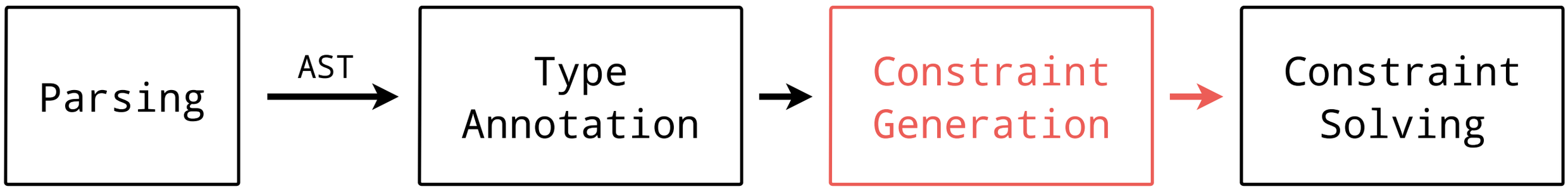


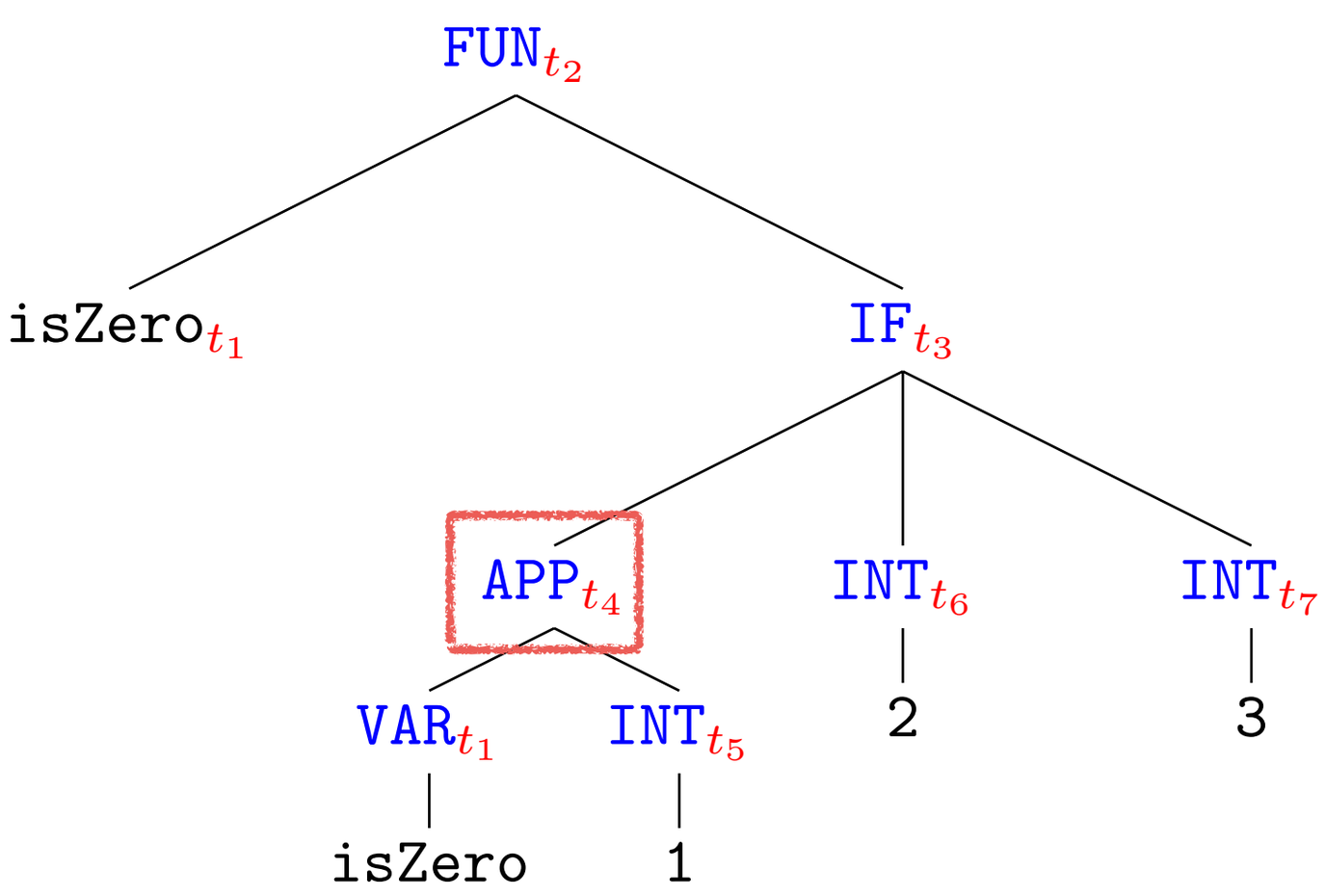


Constraint Set

```
t1 ≡ t5 → t4
t5 ≡ int
```

Wand's Algorithm Overview

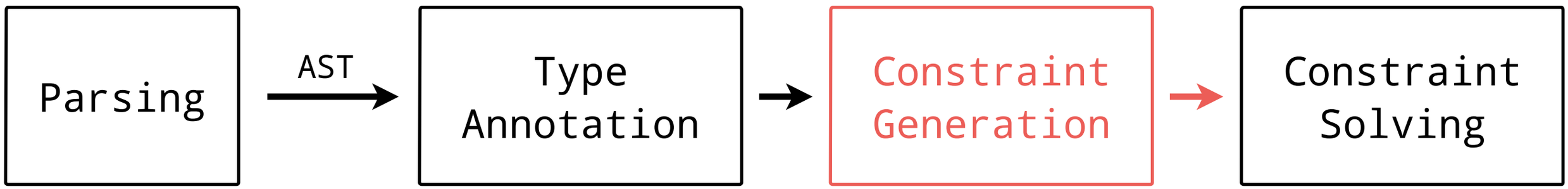


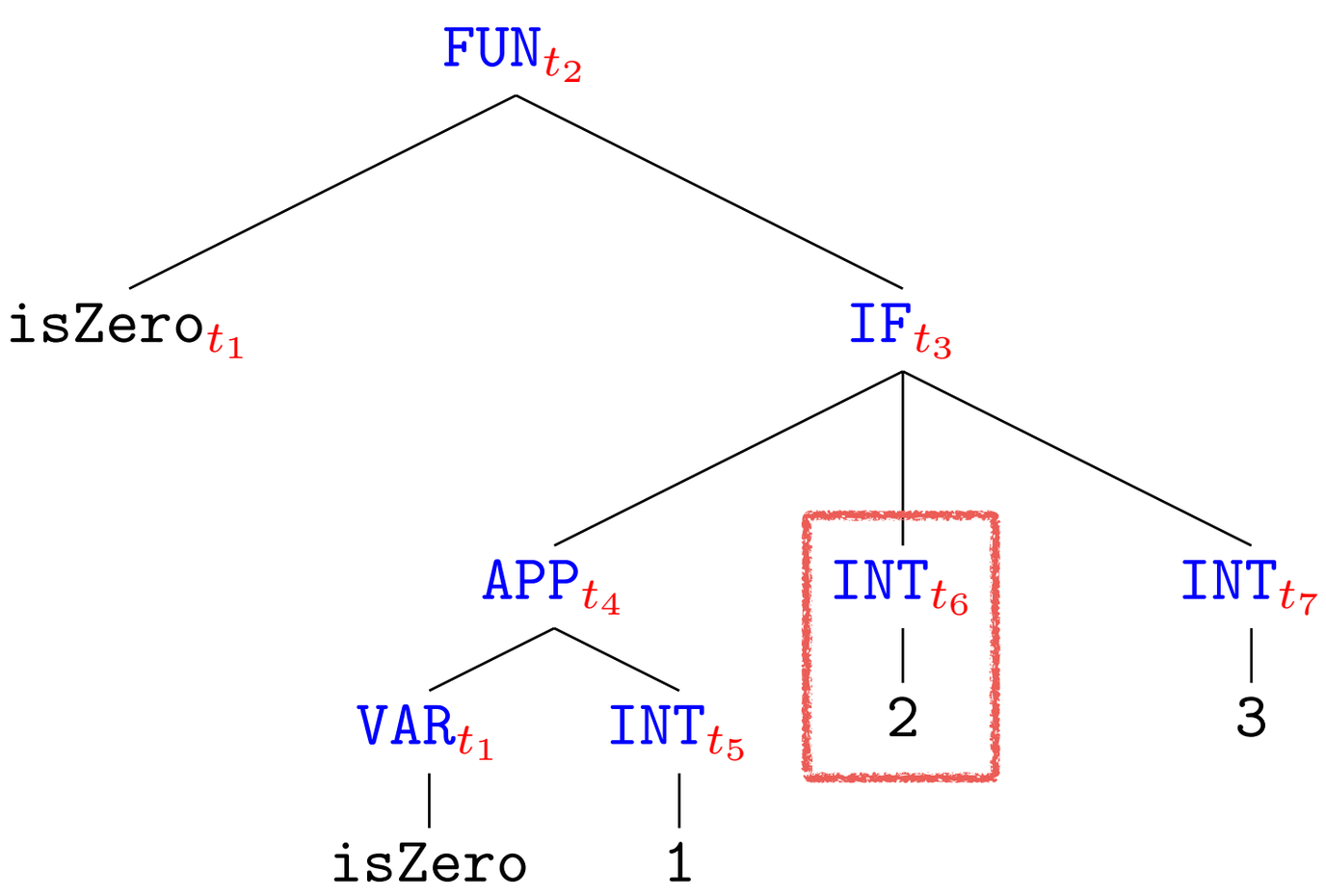


Constraint Set

```
t1 ≡ t5 → t4
t5 ≡ int
t4 ≡ bool
```

Wand's Algorithm Overview

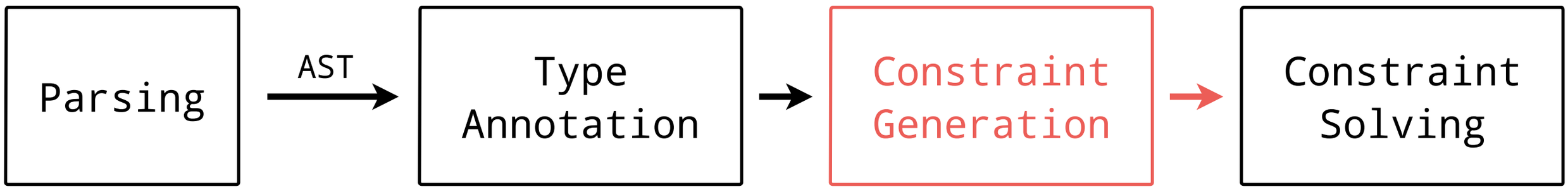


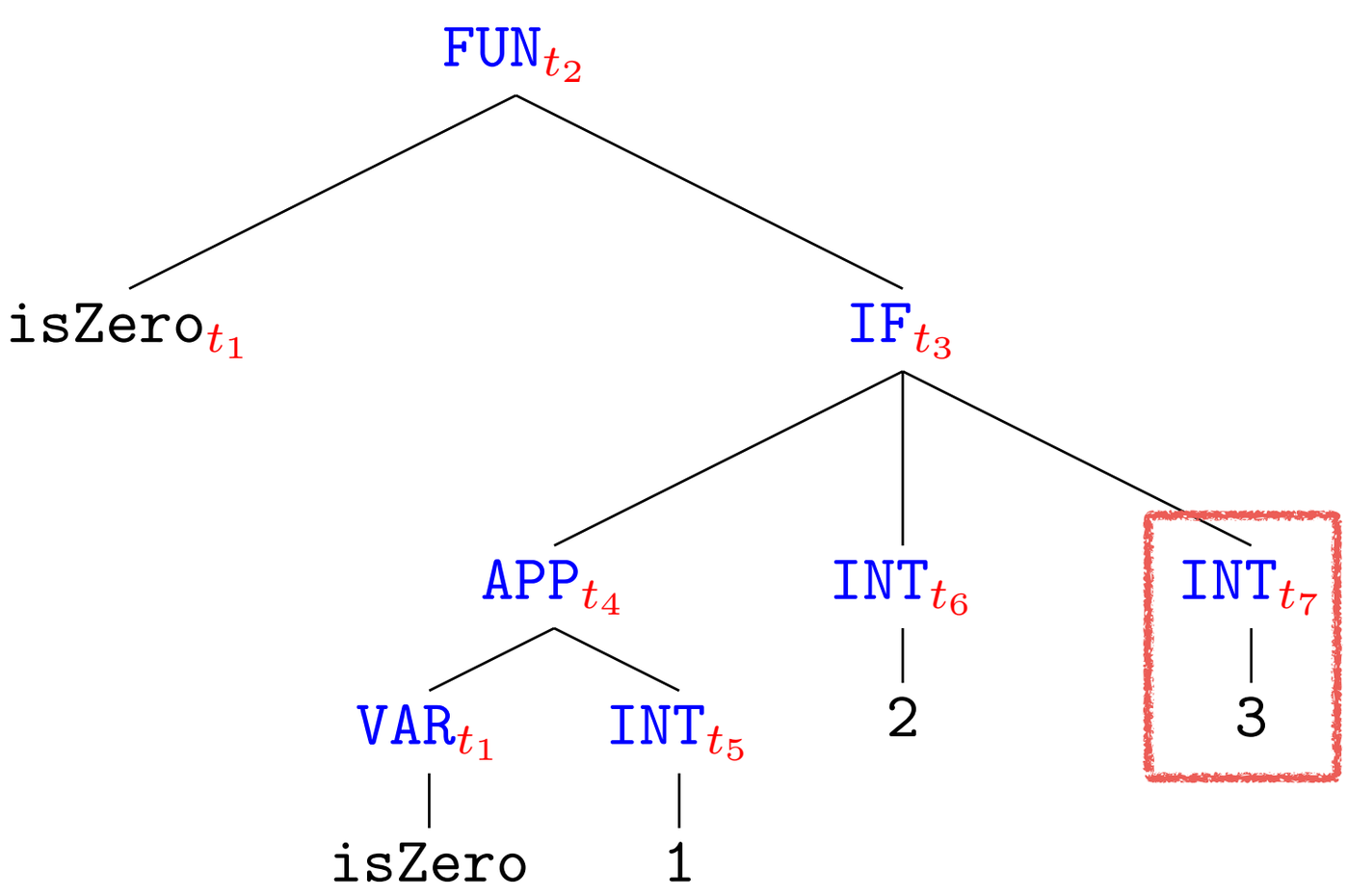


Constraint Set

t1 ≡ t5 → t4
t5 ≡ int
t4 ≡ bool
t6 ≡ int

Wand's Algorithm Overview

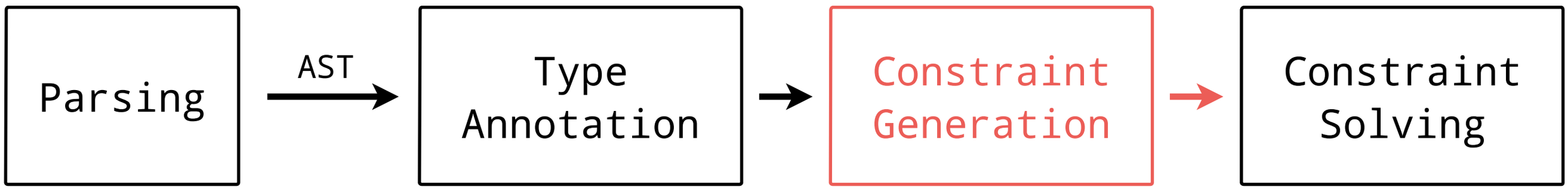


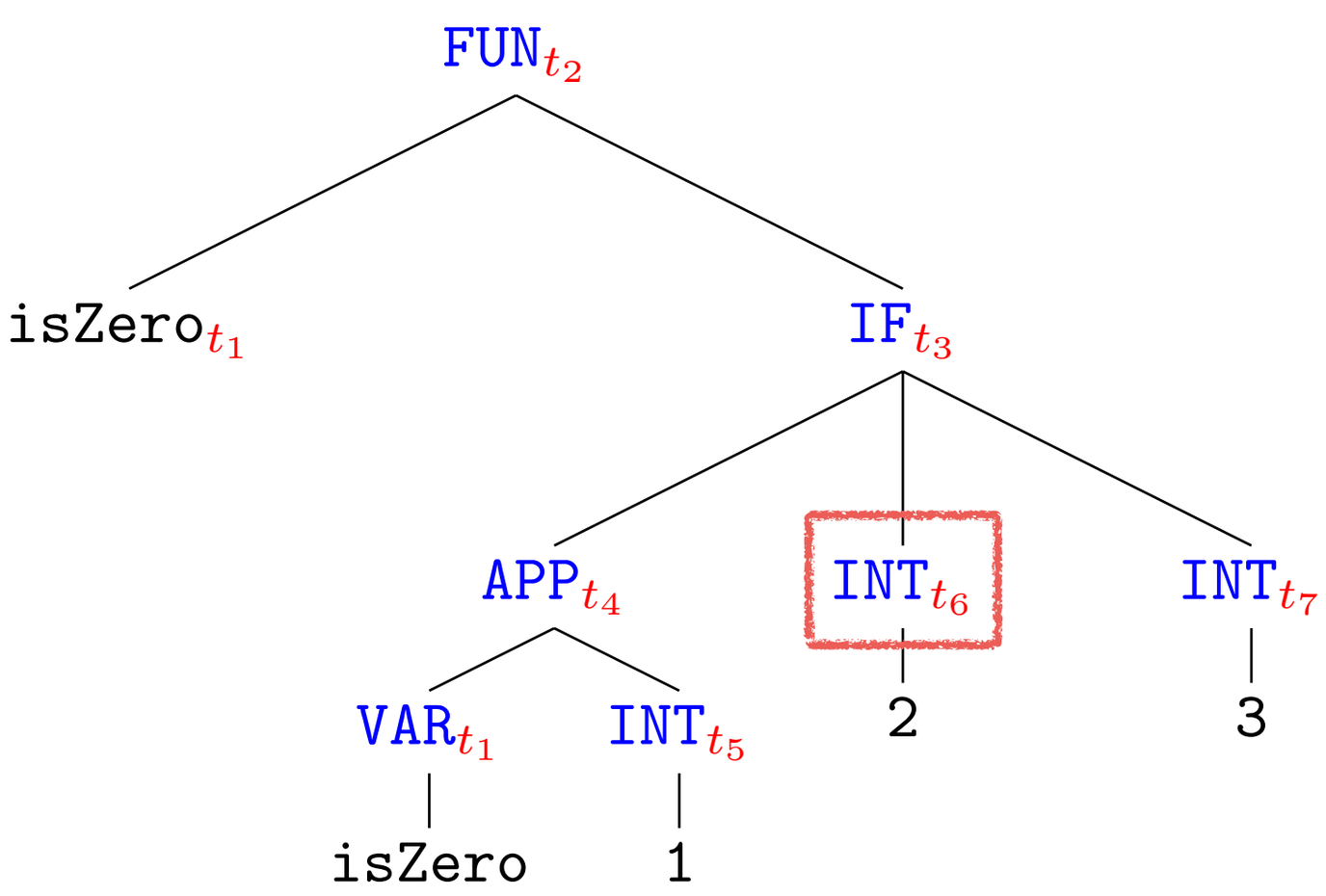


Constraint Set

```
t1 ≡ t5 → t4
t5 ≡ int
t4 ≡ bool
t6 ≡ int
t7 ≡ int
```

Wand's Algorithm Overview

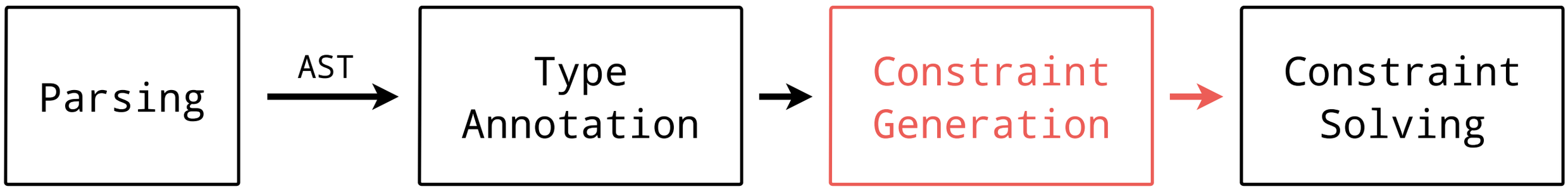


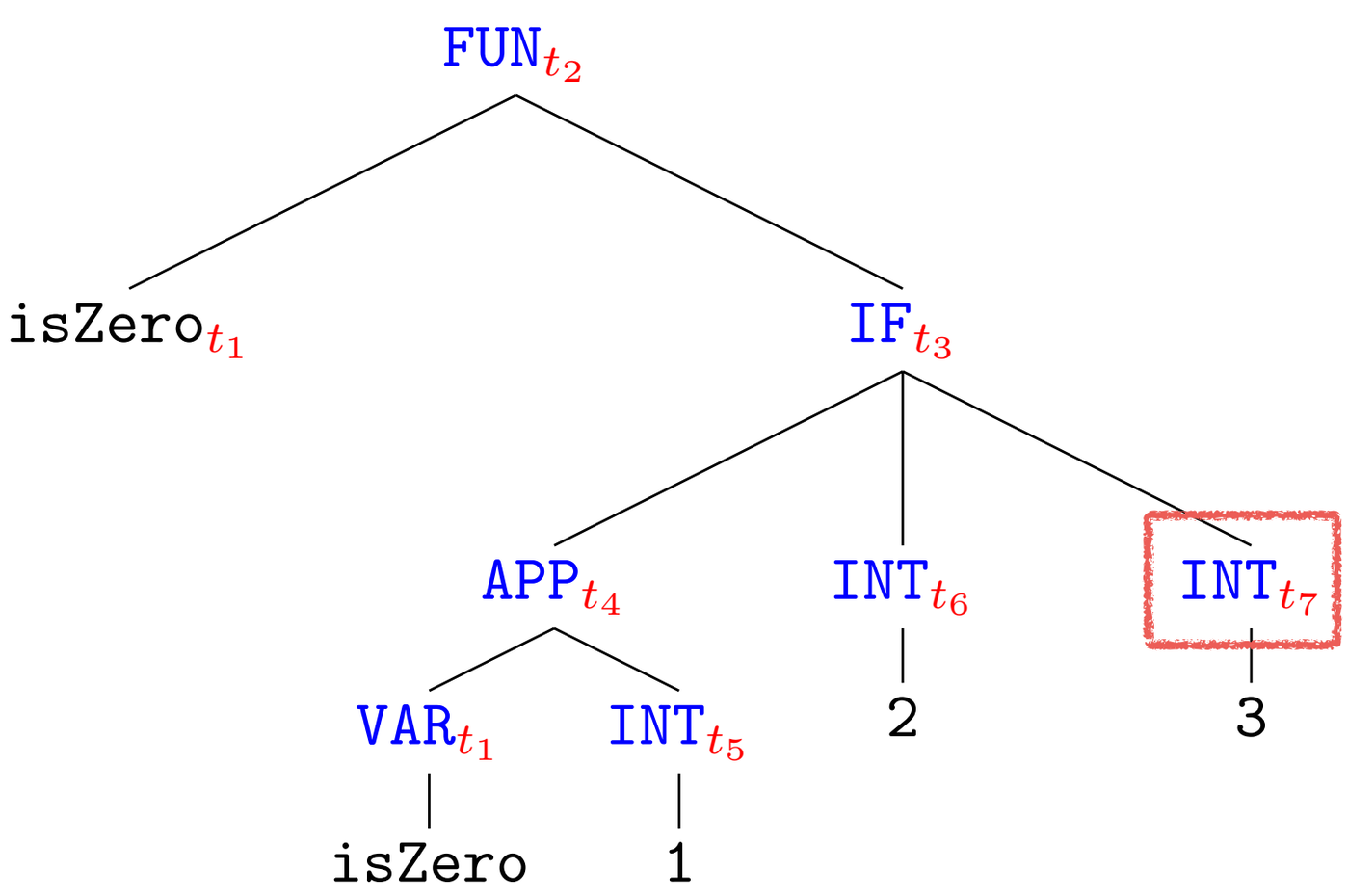


Constraint Set

```
t1 ≡ t5 → t4
t5 ≡ int
t4 ≡ bool
t6 ≡ int
t7 ≡ int
t6 ≡ t3
```

Wand's Algorithm Overview

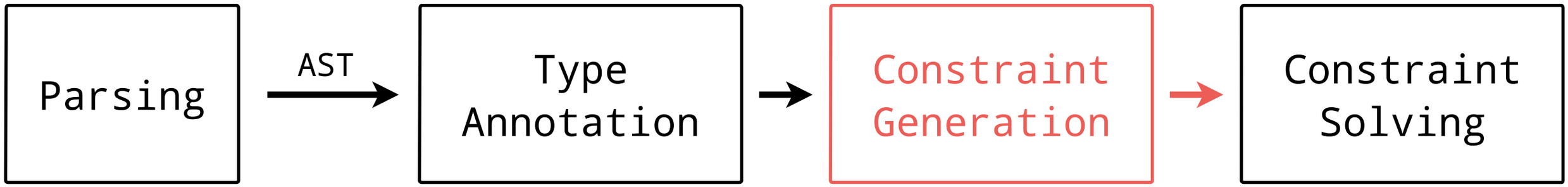


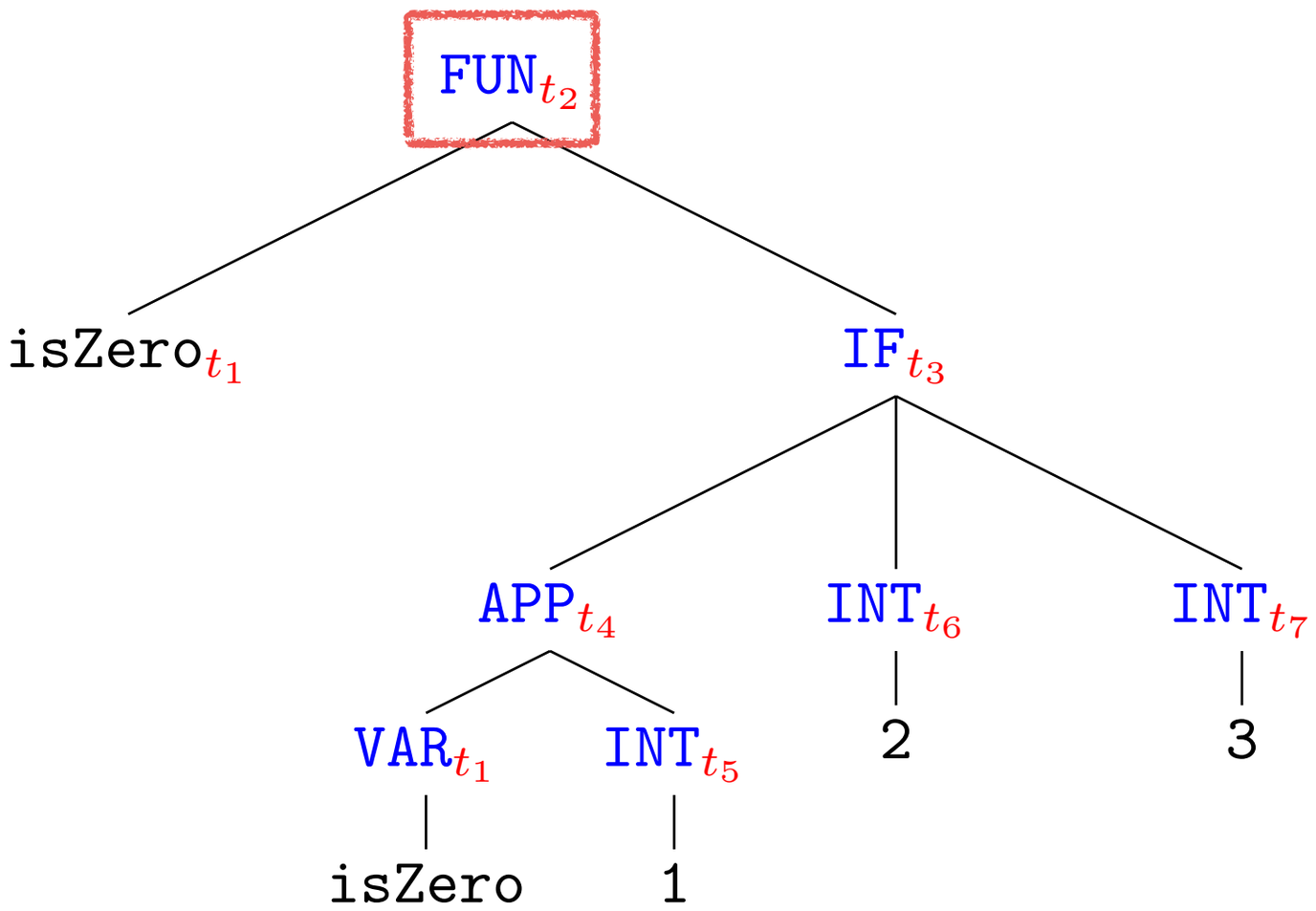


Constraint Set

```
t1 ≡ t5 → t4
t5 ≡ int
t4 ≡ bool
t6 ≡ int
t7 ≡ int
t6 ≡ t3
t7 ≡ t3
```

Wand's Algorithm Overview

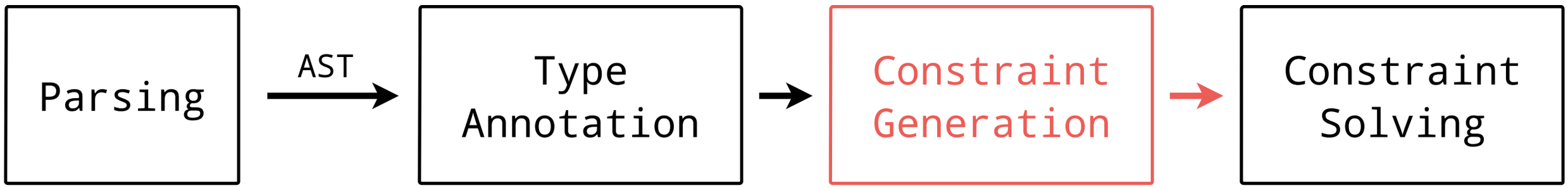


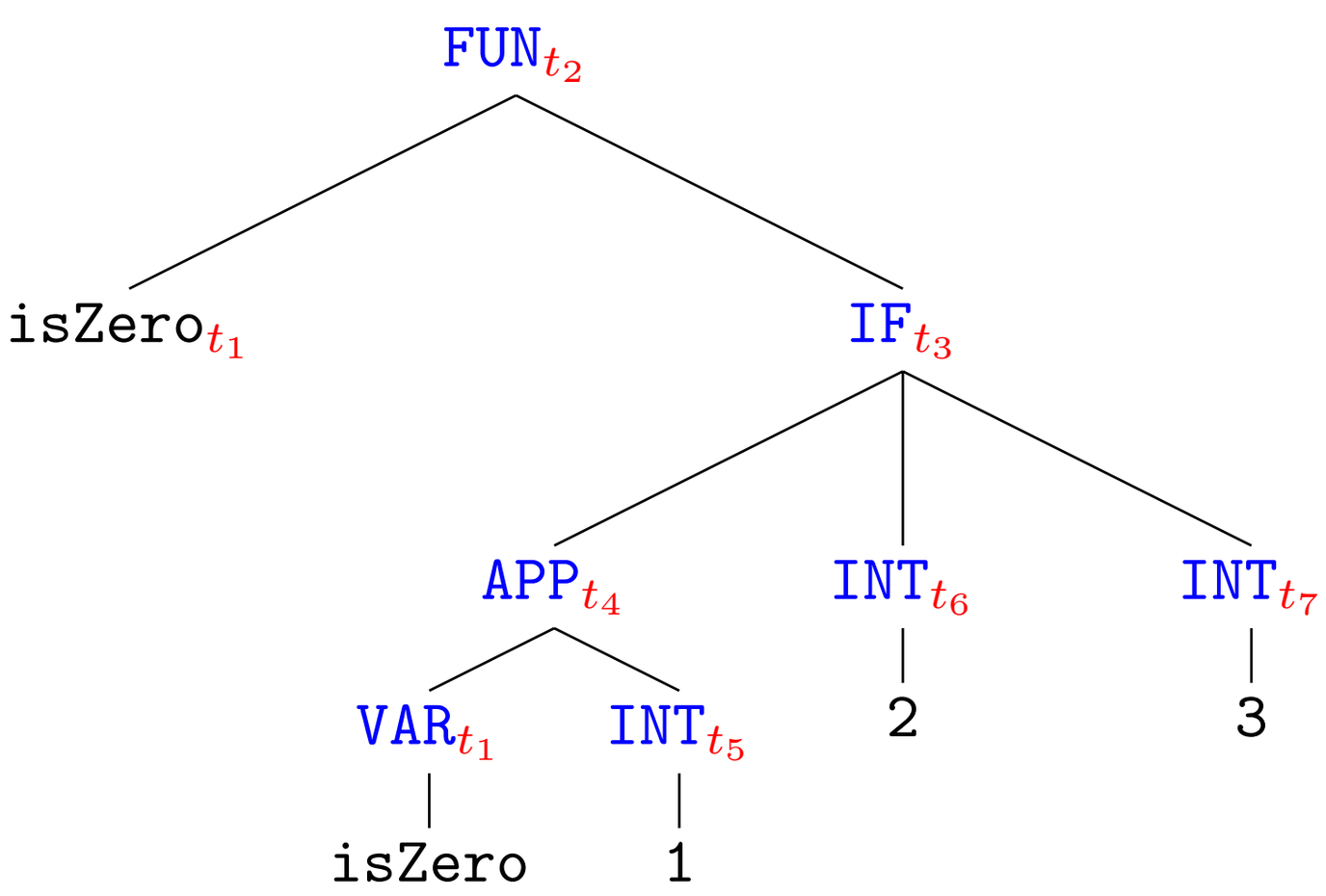


Constraint Set

```
t1 ≡ t5 → t4
t5 ≡ int
t4 ≡ bool
t6 ≡ int
t7 ≡ int
t6 ≡ t3
t7 ≡ t3
t2 ≡ t1 → t3
```

Wand's Algorithm Overview

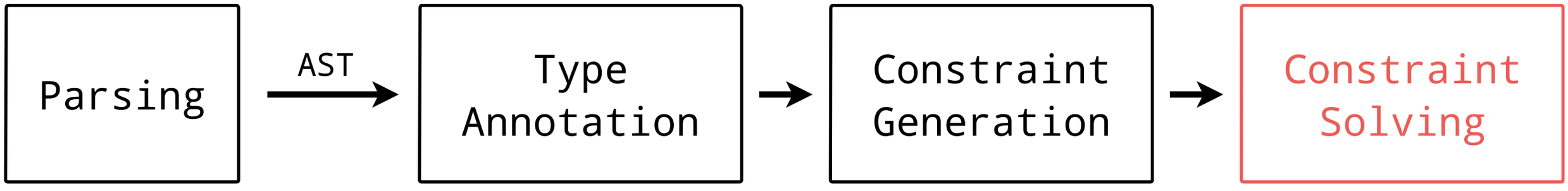




Constraint Set

```
t1 ≡ t5 → t4
t5 ≡ int
t4 ≡ bool
t6 ≡ int
t7 ≡ int
t6 ≡ t3
t7 ≡ t3
t2 ≡ t1 → t3
```

Wand's Algorithm Overview

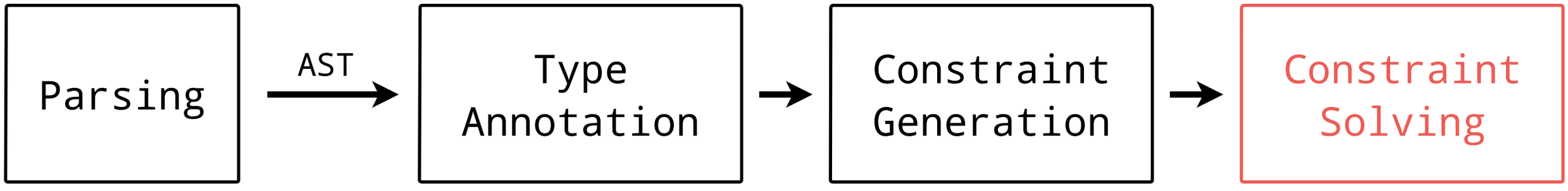


Constraint Set

```
t1 ≡ t5 → t4
t5 ≡ int
t4 ≡ bool
t6 ≡ int
t7 ≡ int
t6 ≡ t3
t7 ≡ t3
t2 ≡ t1 → t3
```

Solution Map

Wand's Algorithm Overview



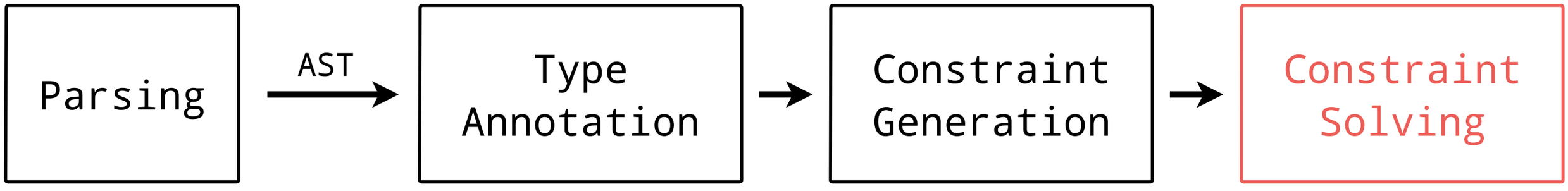
Constraint Set

```
t5 ≡ int
t4 ≡ bool
t6 ≡ int
t7 ≡ int
t6 ≡ t3
t7 ≡ t3
t2 ≡ t1 → t3
```

Solution Map

```
t1: t5 → t4
```

Wand's Algorithm Overview



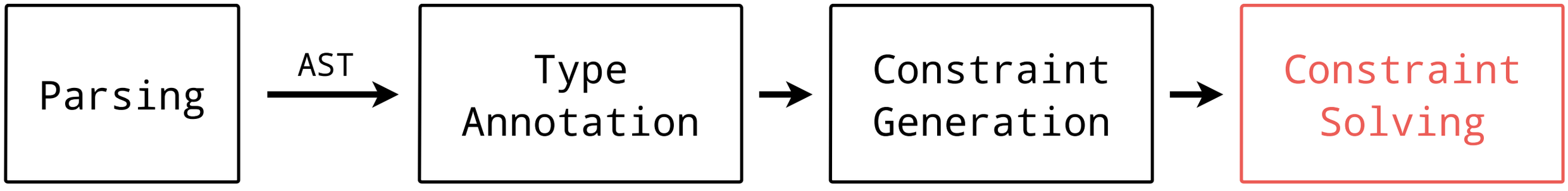
Constraint Set

```
t5 ≡ int
t4 ≡ bool
t6 ≡ int
t7 ≡ int
t6 ≡ t3
t7 ≡ t3
t2 ≡ (t5 → t4) → t3
```

Solution Map

```
t1: t5 → t4
```

Wand's Algorithm Overview



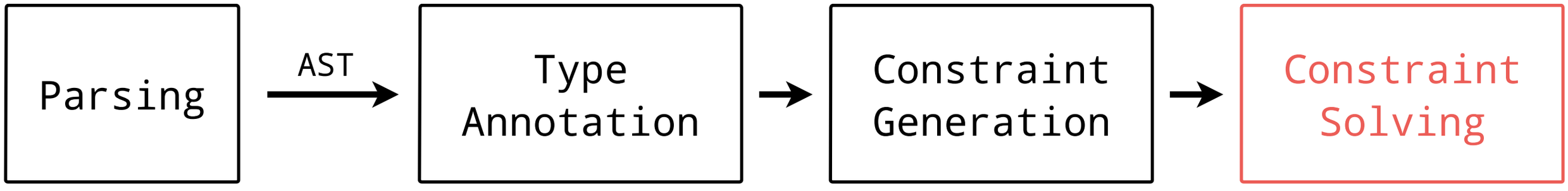
Constraint Set

```
t4 ≡ bool
t6 ≡ int
t7 ≡ int
t6 ≡ t3
t7 ≡ t3
t2 ≡ (t5 → t4) → t3
```

Solution Map

```
t1: t5 → t4
t5: int
```

Wand's Algorithm Overview



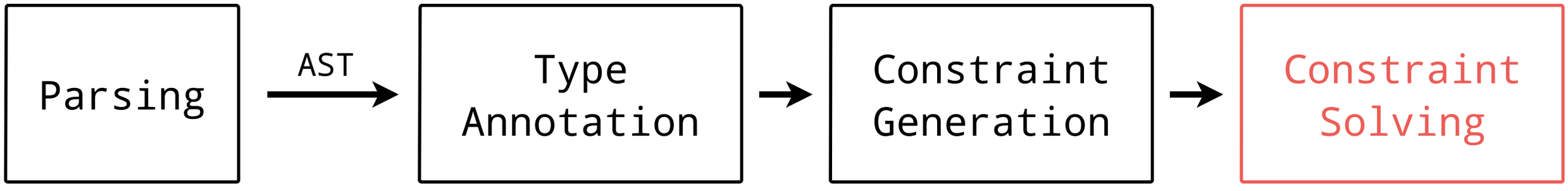
Constraint Set

```
t4 ≡ bool
t6 ≡ int
t7 ≡ int
t6 ≡ t3
t7 ≡ t3
t2 ≡ (int → t4) → t3
```

Solution Map

```
t1: int → t4
t5: int
```

Wand's Algorithm Overview



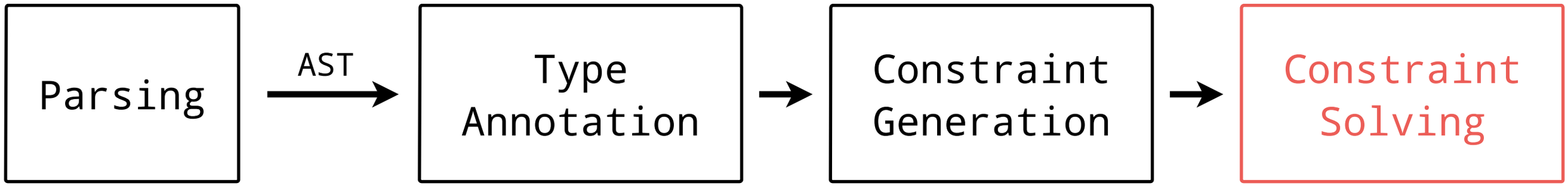
Constraint Set

t6 ≡ int
t7 ≡ int
t6 ≡ t3
t7 ≡ t3
t2 ≡ (int → t4) → t3

Solution Map

t1: int → t4
t5: int
t4: bool

Wand's Algorithm Overview



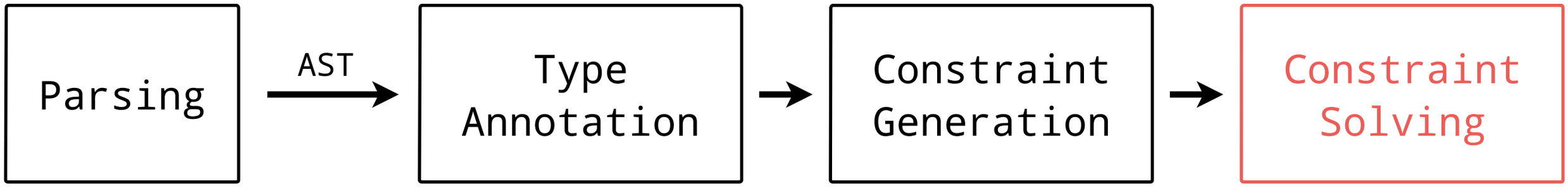
Constraint Set

```
t6 ≡ int
t7 ≡ int
t6 ≡ t3
t7 ≡ t3
t2 ≡ (int → bool) → t3
```

Solution Map

```
t1: int → bool
t5: int
t4: bool
```

Wand's Algorithm Overview



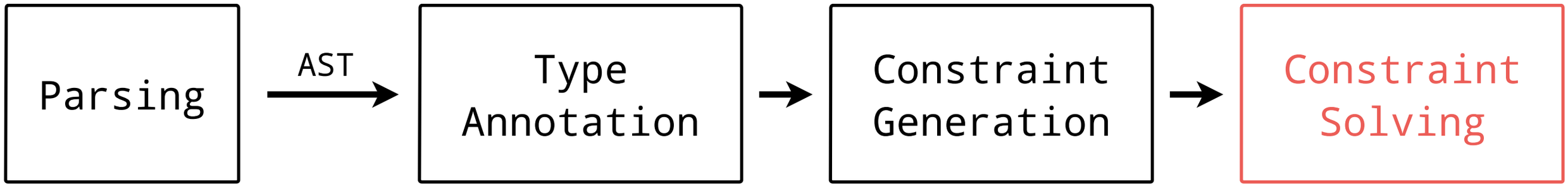
Constraint Set

t7 ≡ int
t6 ≡ t3
t7 ≡ t3
t2 ≡ (int → bool) → t3

Solution Map

t1: int → bool
t5: int
t4: bool
t6: int

Wand's Algorithm Overview



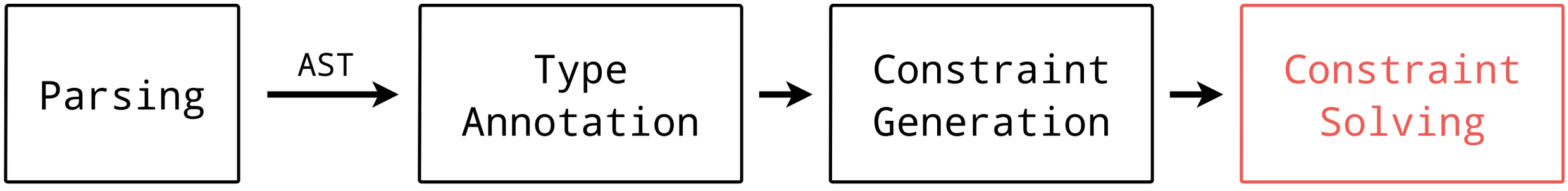
Constraint Set

```
 t7 ≡ int
int ≡ t3
 t7 ≡ t3
 t2 ≡ (int → bool) → t3
```

Solution Map

```
t1: int → bool
t5: int
t4: bool
t6: int
```

Wand's Algorithm Overview



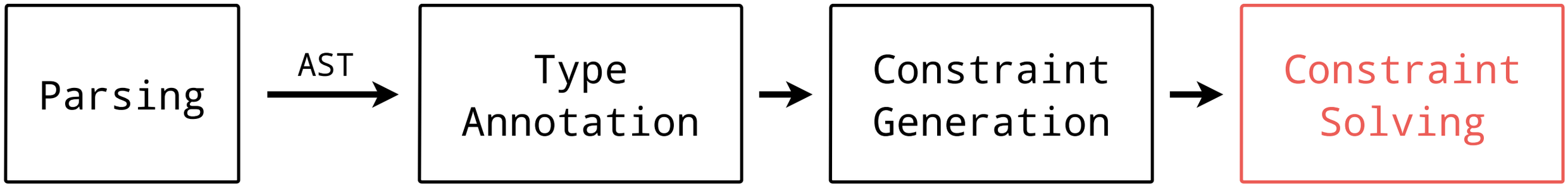
Constraint Set

```
int ≡ t3
 t7 ≡ t3
 t2 ≡ (int → bool) → t3
```

Solution Map

```
t1: int → bool
t5: int
t4: bool
t6: int
t7: int
```

Wand's Algorithm Overview



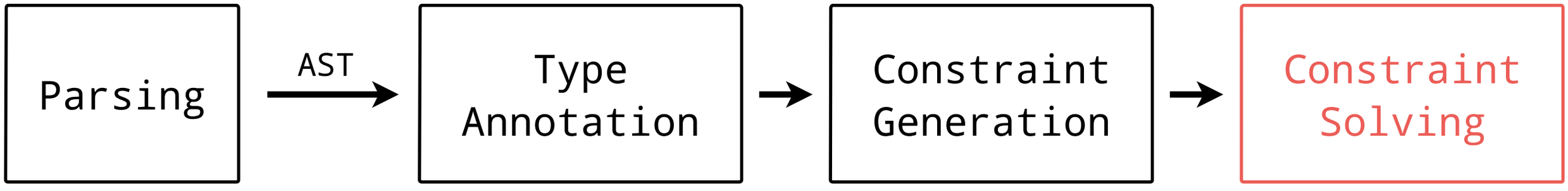
Constraint Set

```
int ≡ t3
int ≡ t3
 t2 ≡ (int → bool) → t3
```

Solution Map

```
t1: int → bool
t5: int
t4: bool
t6: int
t7: int
```

Wand's Algorithm Overview



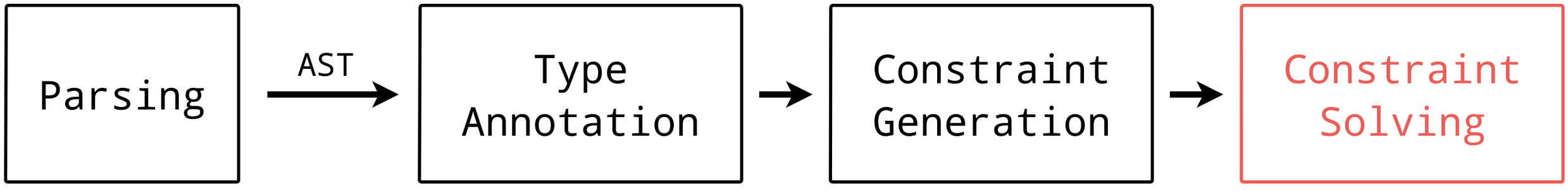
Constraint Set

```
int ≡ t3
 t2 ≡ (int → bool) → t3
```

Solution Map

```
t1: int → bool
t5: int
t4: bool
t6: int
t7: int
t3: int
```

Wand's Algorithm Overview



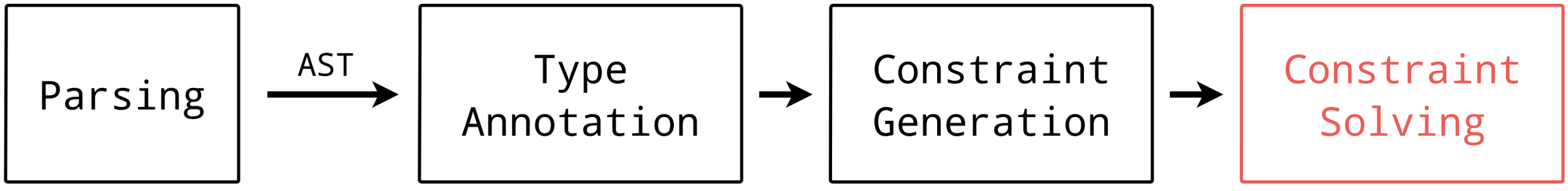
Constraint Set

```
int ≡ int
 t2 ≡ (int → bool) → int
```

Solution Map

```
t1: int → bool
t5: int
t4: bool
t6: int
t7: int
t3: int
```

Wand's Algorithm Overview



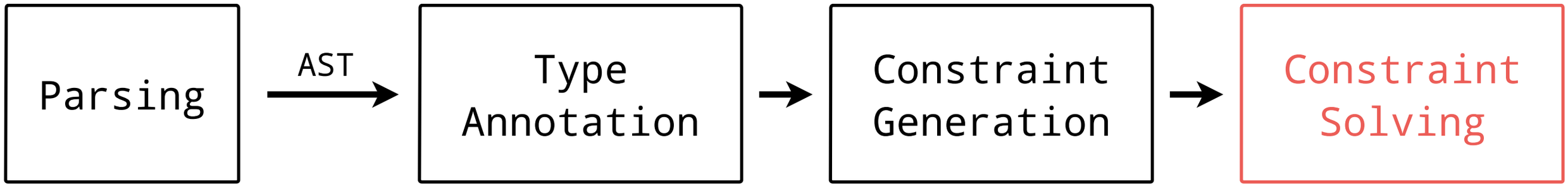
Constraint Set

```
int ≡ int
 t2 ≡ (int → bool) → int
```

Solution Map

```
t1: int → bool
t5: int
t4: bool
t6: int
t7: int
t3: int
```

Wand's Algorithm Overview



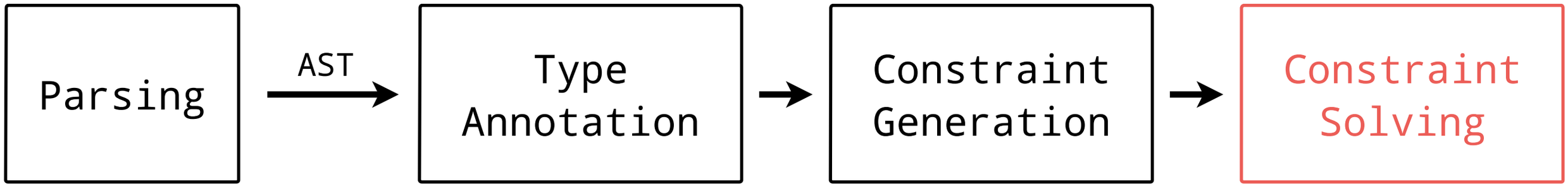
Constraint Set

t2 ≡ (int → bool) → int

Solution Map

t1: int → bool
t5: int
t4: bool
t6: int
t7: int
t3: int

Wand's Algorithm Overview

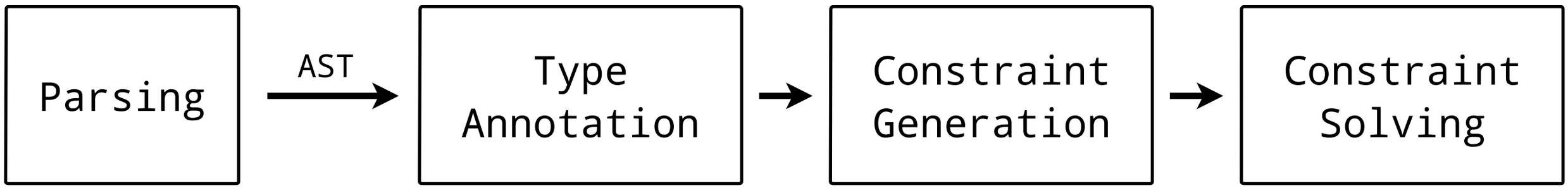


Constraint Set

Solution Map

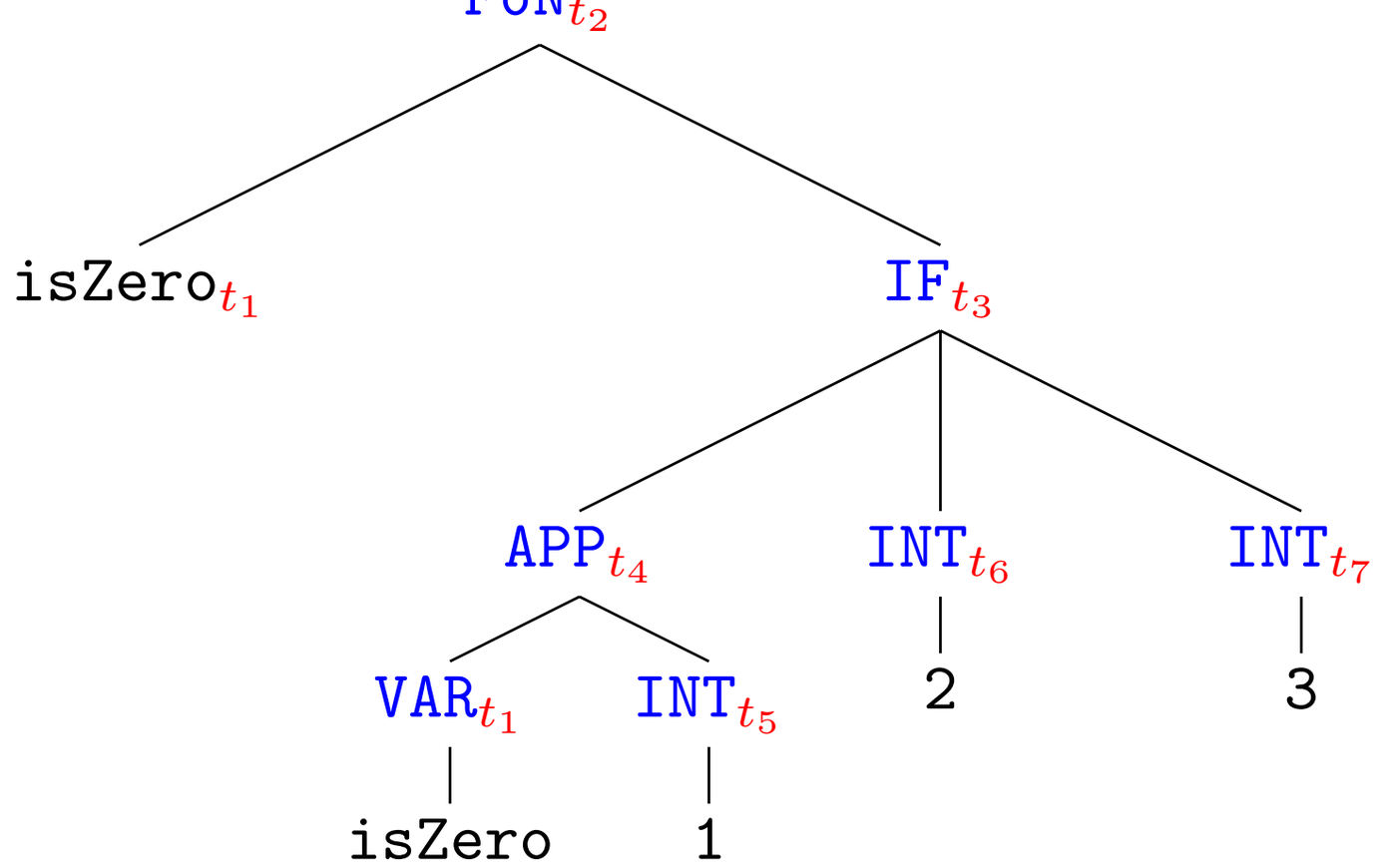
```
t1: int → bool
t5: int
t4: bool
t6: int
t7: int
t3: int
t2: (int → bool) → int
```

Wand's Algorithm Overview

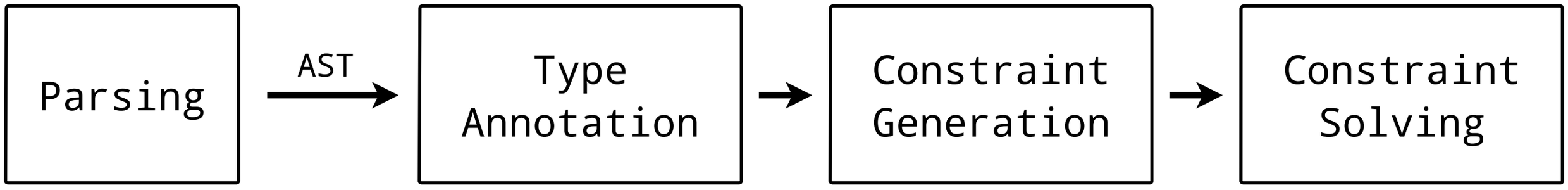


Solution Map

t1: int → bool
t5: int
t4: bool
t6: int
t7: int
t3: int
t2: (int → bool) → int



Wand's Algorithm Overview




```
fn isZero =>
  if isZero 1
  then 2
  else 3
```

➔ `(int -> bool) -> int`

Type Checking Algorithms

- As I said, there are two main classes
- Constraint-based ones. We've just seen one example — Wand's algorithm
- Substitution-based ones, where constraint generation and solving are not two separate processes, they are interleaved. Example: the classic Hindley-Milner Algorithm W.

Live Demonstration

https://github.com/igstan/itake-2015

Thank You!

Resources

- **Sunil Kothari and James L. Caldwell**. Type Reconstruction Algorithms - A Survey
- **Mitchell Wand**. A simple algorithm and proof for type inference.
- **Bastiaan Heeren, Jurriaan Hage and Doaitse Swierstra**. Generalizing Hindley-Milner Type Inference Algorithms
- **Oleg Kiselyov and Chung-chieh Shan**. Interpreting Types as Abstract Values
- **Shriram Krishnamurthi**. Programming Languages: Application and Interpretation, chapter 15
- **Shriram Krishnamurthi**. Programming Languages: Application and Interpretation, lecture 24
- **Shriram Krishnamurthi**. Programming Languages: Application and Interpretation, lecture 25
- **Bastiaan Heeren**. Top Quality Type Error Messages
- **Stephen Diehl**. Write You a Haskell, chapter 6
- **Andrew Appel**. Modern Compiler Implementation in ML, chapter 16
- **Benjamin Pierce**. Types and Programming Languages, chapter 22
- **Martin Odersky**. Scala by Example, chapter 16
- **Danny Gratzer**. https://github.com/jozefg/hm
- **Arlen Cox**. ML Type Inference and Unification
- **Radu Rugină.** CS 312, Type Inference