7. By Mathematical induction.

For $n=1$, $2^1 = 2^2 - 2$, which is true as both side are equal to 2.

Assume the identity holds for n, i.e. $2+2^2+2^3+\cdots+2^n = 2^{n+1}-2$. (*)

Then add the $n+1$ term, i.e. 2^{n+1}, to both sides of (*).

$$2+2^2+2^3+\cdots+2^n+2^{n+1} = (2^{n+1}-2)+2^{n+1} \quad \text{(by ~~mathematical~~ induction hypothesis)}$$

$$= 2^{n+1}+2^{n+1}-2 \quad \text{(rearranging the equation)}$$

$$= 2 \cdot 2^{n+1}-2$$

$$= 2^{n+2}-2$$

Hence, by the ~~principal~~ principle of mathematical induction, the identity holds for all natural number n.