3. We want to show that for any integer n, the number n^2+n+1 is odd.

For all integers, we can express them in the following forms:

① $n=2k$ for $k \in \mathbb{Z}$ for all even integers

② $n=2k+1$ for $k \in \mathbb{Z}$ for all odd integers.

If $n=2k$ for some $k \in \mathbb{Z}$,

$$n^2+n+1 = 4k^2+2k+1$$
$$= 2(2k^2+k)+1$$, which is an odd integer.

If $n=2k+1$ for some $k \in \mathbb{Z}$,

$$n^2+n+1 = (2k+1)^2 + (2k+1)+1$$
$$= 4k^2+4k+1 + 2(k+2)$$
$$= 2(2k^2+2k+k+2)+1$$
$$= 2(2k^2+3k+2)+1$$, which is also an odd integer.

So for any integer n, the number n^2+n+1 is odd.