9. Consider ~~$A_n = \left(\frac{n}{n+1}, 1\right)$~~.

$$A_n = \left(\frac{n}{n+1}, 1\right). \quad \text{when } n \in \mathbb{N}.$$

Then we want to prove that $A_{n+1} \subset A_n$.

$$A_{n+1} = \left(\frac{n+1}{n+2}, 1\right) \quad \text{and} \quad A_n = \left(\frac{n}{n+1}, 1\right)$$

Consider
$$\frac{\cancel{n+1}\, n}{n+1} - \frac{n+1}{n+2} = \frac{n(n+2)-(n+1)^2}{(n+1)(n+2)}$$
$$= \frac{\cancel{n^2} + \cancel{2n} - \cancel{n^2} - \cancel{2n} - 1}{(n+1)(n+2)}$$
$$= -\frac{1}{(n+1)(n+2)} < 0 \quad \text{as } n \in \mathbb{N}$$

So $\frac{n}{n+1} < \frac{n+1}{n+2} \Rightarrow A_{n+1} \subset A_n$.

~~However~~ Next, we want to show that $\frac{n}{n+1} \to 1$ as $n \to \infty$

such that $\bigcap_{n=1}^{\infty} A_n = \emptyset$ because A_n does not include 1.

We want to pick an n so large that

$$m \geq n \Rightarrow \cancel{\frac{n}{n+1}} \left|\frac{n}{n+1} - 1\right| < \varepsilon$$

$$\Rightarrow \left|\frac{n-n-1}{n+1}\right| < \varepsilon$$

$$\Rightarrow \frac{1}{n+1} < \varepsilon$$

$$\Rightarrow n > \frac{1}{\varepsilon} - 1$$

So given $\varepsilon > 0$. we pick $n > \frac{1}{\varepsilon} - 1$ such that

$$(\forall \varepsilon > 0)(\exists n \in \mathbb{N})(\forall m \geq n)\left[\left|\frac{n}{n+1} - 1\right| < \varepsilon\right]$$

Hence $\frac{n}{n+1} \to 1$ as $n \to \infty$.

This proves that $\bigcap_{n=1}^{\infty} A_n = \emptyset$ as A_n is an open interval that does not include 1.