

8. Given that

$$(\forall \varepsilon > 0) (\exists n \in \mathbb{N}) (\forall m \geq n) [|a_m - L| < \varepsilon]. \quad (*)$$

We want to show that for any fixed number $M > 0$,

$$(\forall \varepsilon_1 > 0) (\exists n_1 \in \mathbb{N}) (\forall m_1 \geq n_1) [|Ma_{m_1} - ML| < \varepsilon_1].$$

To do so, ~~we~~ given $\varepsilon_1 > 0$, we need to pick an n_1 such that

$$m_1 \geq n_1 \Rightarrow |Ma_{m_1} - ML| < \varepsilon_1$$

$$\Rightarrow M |a_{m_1} - L| < \varepsilon_1 \quad (\because M > 0)$$

$$\Rightarrow |a_{m_1} - L| < \frac{\varepsilon_1}{M}$$

So, we need to pick an n_1 so large that $|a_{m_1} - L| < \frac{\varepsilon_1}{M}$.

To find this n_1 , we can set $\frac{\varepsilon_1}{M} = \varepsilon$ and work backward from the given information (*).