

UNIT I

1

APPLIED MATH AND MACHINE LEARNING BASICS

Unit Structure

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1.0 OBJECTIVES

This chapter will help to you understand the:

- Concepts of basic Mathematics useful in Machine learning and deep Learning
- Basic concepts related scalar, vectors, matrix and tensor
- Different types of matrix and its operations
- Decomposition of matrix

1.1 INTRODUCTION AND OVERVIEW OF APPLIED MATH AND MACHINE LEARNING BASICS

This section of the book tells some basic mathematical concepts which helps to understand the deep Learning. Deep learning is a subdomain of machine learning which is concerned with algorithms, mathematical functions and artificial neural network.

1.2 LINEAR ALGEBRA

Linear Algebra is one of the widely used branches of mathematics related with mathematical structures which is continuous rather than discrete mathematics. It includes operations like addition, scalar multiplication that helps to understand concepts like linear transformations, vector spaces, linear equations, matrices and determinants. A good knowledge of Linear Algebra is important to understand and working with many essential machine learning algorithms, especially algorithms related with deep learning.

1.2.1 Scalars, Vectors, Matrices and Tensors:

Let's start with some basic definitions:

Scalar	Vector	Matrix	Tensor
1	$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$	$\begin{bmatrix} [2 \ 3] & [8 \ 2] \\ [3 \ 6] & [2 \ 7] \end{bmatrix}$

(1.1)

Understanding of a scalar, a vector, a matrix, a tensor

Scalar: A scalar is a single number represent 0^{th} order tensor. Scalars are written in lowercase and italics. For Example: n . there is many different sets of numbers with interest in deep learning. The notation $x \in \mathbb{R}$ represents x is a scalar belonging to a real values numbers i.e. \mathbb{R} . \mathbb{N} states the set of positive integers (1, 2, 3, ...). \mathbb{Z} states the integers, which is combination of positive, negative and zero values. Rational numbers are representing by notation \mathbb{Q}

Python code to explains arithmetic operations on Scalars:

Code: # In-Built Scalars

```
a = 15
b = 3
print(type(a))
print(type(b))
print(a + b)
print(a - b)
print(a * b)
print(a / b)
```

Output:

```
<class 'int'>
<class 'int'>
15
```

```
12
45
5
```

```
# Python code snippet checks if the given variable is scalar or not.
import numpy as np
# Is Scalar Function
def isscalar(num):
    if isinstance(num, generic):
        return True
    else:
        return False
print(np.isscalar(2.4))
print(np.isscalar([4.1]))
print(np.isscalar(False))
Output:
True
False
True
```

Vector: A vector is an ordered array of single numbers represent 1st order tensor. Vectors should be written in lowercase, bold, and italics. For Example: \mathbf{x} .

$$\mathbf{x} = [\mathbf{x}_1 \ \mathbf{x}_2 \ \mathbf{x}_3 \ \dots \ \mathbf{x}_n] \quad \text{or} \quad \mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_n \end{bmatrix} \quad (1.2)$$

Here, first element of \mathbf{x} is \mathbf{x}_1 , the next second element is \mathbf{x}_2 and so on element gets listed. To understand the necessary element of a vector index wise, the scalar element of a vector positioned i^{th} is written as $\mathbf{x}[i]$. Suppose $S = \{1, 2, 4\}$ then $\mathbf{x}_1 = 1$, $\mathbf{x}_2 = 2$, $\mathbf{x}_4 = 4$. The $-$ sign to index is used to indicate the complement of a S , like for example \mathbf{x}_{-1} is the vector consisting of all elements of \mathbf{x} except \mathbf{x}_1 , and \mathbf{x}_{-s} is the vector consist of all elements of \mathbf{x} except \mathbf{x}_1 , \mathbf{x}_2 and \mathbf{x}_4 . Vectors are pieces of objects known as vector spaces, which can be considered as collection of all possible vectors of a particular dimension.

```
#Python code demonstrating Vectors
import numpy as np
# Declaring Vectors
x = [2, 4, 6]
y = [3, 5, 1]
print(type(x))
# Vector addition using Numpy
```

```

z = np.add(x, y)
print(z)
print(type(z))
Output:
<class 'list'>
[2, 4, 6, 3, 5, 1]
[5 9 7]
<class 'numpy.ndarray'>

```

Matrix: Matrix is a 2-D rectangular array consisting of numbers represents 2nd order tensor. Matrices should be written in uppercase, bold and italics. For Example: ***X***. If p and q are positive integers, that is $p, q \in \mathbb{N}$ then the $p \times q$ matrix contains $p \cdot q$ numbers, with p rows and q columns

$$A = \begin{bmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \dots & \mathbf{a}_{1q} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \dots & \mathbf{a}_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{a}_{p1} & \mathbf{a}_{p2} & \dots & \mathbf{a}_{pq} \end{bmatrix} \quad (1.3)$$

Full matrix component can be express as follows:

$$A = [\mathbf{a}_{ij}]_{p \times q} \quad (1.4)$$

Some of the operations of matrices are as follows:

- **Matrix Addition:**

We can do addition of Matrices to scalars, vectors and other matrices. These precise techniques are often used in machine learning and deep learning.

```
# Python code for Matrix Addition
```

```
import numpy as np
```

```
x = np.matrix([[5, 3], [2, 6]])
```

```
sum = x.sum()
```

```
print(sum)
```

Output: 16

- **Matrix-Matrix Addition:**

$$Z = X + Y$$