Define the particle masses that we are interested in.

In[49]:=
```
toMeV = UnitConvert[#, "Megaelectronvolts" / "SpeedOfLight"^2] &;
masses =
 {m_n -> Entity["Particle", "Neutron"]["Mass"],
  m_p -> Entity["Particle", "Proton"]["Mass"],
  m_d → toMeV[Entity["Particle", "Deuteron"]["Mass"]],
  m_h → toMeV[Entity["Particle", "Helion"]["Mass"]]
 }
```

Out[50]= $\{m_n \to 939.5654133\,\text{MeV}/c^2,\ m_p \to 938.2720813\,\text{MeV}/c^2,\ m_d \to 1875.612928\,\text{MeV}/c^2,\ m_h \to 2808.391586\,\text{MeV}/c^2\}$

Compute the kinetic energies of outgoing particles in $d + d \to {}^3\text{He} + n$
Assume a head-on collision of the deuterons in the lab frame, each with kinetic energy $\mathbb{E}$. The ^{3}He nucleus is also called a helion, and we refer to it sometimes as h.

We will begin with a non-relativistic treatment, which is not completely correct, but should be approximately correct.

1) Conserve energy. Note that particle masses are part of the energy balance.

In[5]:= $\text{energy1} = 2\, m_d\, c^2 + 2\,\mathbb{E} == (m_h + m_n)\, c^2 + \frac{1}{2} m_h\, v_h{}^2 + \frac{1}{2} m_n\, v_n{}^2$

Out[5]= $2\,\mathbb{E} + 2\, c^2\, m_d == c^2\,(m_h + m_n) + \frac{1}{2} m_h\, v_h^2 + \frac{1}{2} m_n\, v_n^2$

2) Conserve momentum.

In[6]:= $\text{momentum1} = 0 == m_h\, v_h + m_n\, v_n$

Out[6]= $0 == m_h\, v_h + m_n\, v_n$

In[7]:= `solution1 = Solve[{energy1, momentum1}, {v_n, v_h}]`

Out[7]= $$\left\{\left\{v_n \to -\frac{i\sqrt{2}\sqrt{m_h}\sqrt{2\,\mathbb{E} + 2\,c^2\,m_d - c^2\,m_h - c^2\,m_n}}{\sqrt{-m_h\,m_n - m_n^2}},\ v_h \to \frac{i\sqrt{2}\,m_n\sqrt{2\,\mathbb{E} + 2\,c^2\,m_d - c^2\,m_h - c^2\,m_n}}{\sqrt{m_h}\sqrt{-m_n\,(m_h + m_n)}}\right\},\right.$$
$$\left.\left\{v_n \to \frac{i\sqrt{2}\sqrt{m_h}\sqrt{2\,\mathbb{E} + 2\,c^2\,m_d - c^2\,m_h - c^2\,m_n}}{\sqrt{-m_h\,m_n - m_n^2}},\ v_h \to -\frac{i\sqrt{2}\,m_n\sqrt{2\,\mathbb{E} + 2\,c^2\,m_d - c^2\,m_h - c^2\,m_n}}{\sqrt{m_h}\sqrt{-m_n\,(m_h + m_n)}}\right\}\right\}$$

In[8]:= $\mathbb{E}_n = \text{Simplify}[1/2\, m_n\, v_n{}^2\ /.\ \text{solution1}[[1]]]$

Out[8]= $$-\frac{m_h\left(-2\,\mathbb{E} - 2\,c^2\,m_d + c^2\,m_h + c^2\,m_n\right)}{m_h + m_n}$$

In[9]:=
```
Eh = Simplify[1/2 mh vh^2 /. solution1[[1]]]
```

Out[9]= $-\frac{m_n \left(-2\,\mathbb{E} - 2\,c^2\,m_d + c^2\,m_h + c^2\,m_n\right)}{m_h + m_n}$

These solutions are not difficult to obtain yourself. The math trick is to change the momentum conservation equation into $m_h^2\, v_h^2 = m_n^2\, v_n^2$ and then solve for v_h^2 and v_n^2.

In[10]:=
```
Simplify[En /. masses /. c → Quantity["SpeedOfLight"]]
```

Out[10]= $1.498625297\,\mathbb{E} + 2.449396$ MeV

In[11]:=
```
Simplify[Eh /. masses /. c → Quantity["SpeedOfLight"]]
```

Out[11]= $0.501374703\,\mathbb{E} + 0.819461$ MeV

Suppose that our fusor is running at 30 KeV. Substitute this value for $\mathbb{E}$.

In[12]:=
```
En /. masses /. c → Quantity["SpeedOfLight"] /. E → Quantity[30, "keV"]
```

Out[12]= 2494.354 keV

Now solve the problem in the fully relativistic way. The differences are:

1) The total energy of a massive particle is γmc^2, where $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$. (That symbol is a gamma.)

The kinetic energy part of this is $(\gamma - 1)\, mc^2$, so the total energy of the incoming deuteron is still $E + m_d\, c^2$.

2) The momentum of a massive particle is γmv. This has two variables, γ and v, that determine velocity, so we will use a convenient identity: $\gamma v = c(\gamma^2 - 1)^{\frac{1}{2}}$.

In[13]:= `energy2 = 2 md c^2 + 2 E == (γh mh + γn mn) c^2`

Out[13]= $2\,E + 2\,c^2\,m_d == c^2\,(m_h\,\gamma_h + m_n\,\gamma_n)$

In[14]:= `momentum2 = mh (γh^2 - 1)^(1/2) c == mn (γn^2 - 1)^(1/2) c`

Out[14]= $c\,m_h\,\sqrt{-1 + \gamma_h^2} == c\,m_n\,\sqrt{-1 + \gamma_n^2}$

In[15]:= `solution2 = Solve[{energy2, momentum2}, {γn, γh}]`

Out[15]= $\left\{\left\{\gamma_n \to \frac{4\,E^2 + 8\,c^2\,E\,m_d + 4\,c^4\,m_d^2 - c^4\,m_h^2 + c^4\,m_n^2}{4\,c^2\,(E + c^2\,m_d)\,m_n},\ \gamma_h \to \frac{4\,E^2 + 8\,c^2\,E\,m_d + 4\,c^4\,m_d^2 + c^4\,m_h^2 - c^4\,m_n^2}{4\,c^2\,(E + c^2\,m_d)\,m_h}\right\}\right\}$

In[16]:= `Enrel = Simplify[mn (γn - 1) c^2 /. solution2[[1]]]`

Out[16]= $\frac{4\,c^4\,m_d^2 - c^4\,m_h^2 + (-2\,E + c^2\,m_n)^2 + m_d\,(8\,c^2\,E - 4\,c^4\,m_n)}{4\,(E + c^2\,m_d)}$

In[17]:= `Ehrel = Simplify[mh (γh - 1) c^2 /. solution2[[1]]]`

Out[17]= $-c^2\,m_h + \frac{4\,E^2 + 8\,c^2\,E\,m_d + 4\,c^4\,m_d^2 + c^4\,m_h^2 - c^4\,m_n^2}{4\,E + 4\,c^2\,m_d}$

In[18]:= `Enrel /. masses /. c → Quantity["SpeedOfLight"] /. E → Quantity[0, "keV"]`

Out[18]= 2448.686 keV

In[48]:=
```
Framed[
 Row[{"En=", Enrel /. masses /. c → Quantity["SpeedOfLight"] /. E → Quantity[30, "keV"],
   ", for a grid voltage of 30 KeV"}]]
```

Out[48]= $\boxed{E_n = 2493.618\ \text{keV},\ \text{for a grid voltage of 30 KeV}}$

Now look at p+n -> d+γ. This is what happens when the neutron is finally slow enough to be captured by a proton. An energetic gamma is emitted.
A captured neutron is usually “thermalized”, meaning it has a kinetic energy less than 0.1 eV. This is so small that the calculation assumes both the neutron and proton are at rest.

In[30]:= **energy3 = $E_\gamma + \gamma_d m_d c^2 == m_p c^2 + m_n c^2$**

Out[30]= $c^2 m_d \gamma_d + E_\gamma == c^2 m_n + c^2 m_p$

In[31]:= **momentum3 = $E_\gamma == \sqrt{\gamma_d{}^2 - 1}\ m_d c^2$**

Out[31]= $E_\gamma == c^2 m_d \sqrt{-1 + \gamma_d^2}$

In[33]:= **sol3 = Solve[{energy3, momentum3}, {E_γ, γ_d}]**

Out[33]= $\left\{\left\{E_\gamma \to \frac{c^2\left(-m_d^2 + m_n^2 + 2\, m_n m_p + m_p^2\right)}{2\,(m_n + m_p)},\ \gamma_d \to \frac{m_d^2 + m_n^2 + 2\, m_n m_p + m_p^2}{2\, m_d\,(m_n + m_p)}\right\}\right\}$

In[47]:= **Framed[Row[{"E_γ=", E_γ /. sol3[[1]] /. masses /. c → Quantity["SpeedOfLight"]}]]**

Out[47]= $\boxed{E_\gamma = 2.223249\ \text{MeV}}$