such that, as $T \rightarrow \infty, [< u, y >_T \ / < u, u >_T] \rightarrow \delta(u) > 0$. For example, for the scalar case, if $u(t) = sin(\omega_0 t)$, then for large t, $y(t) = |H(j\omega_0)| sin(\omega_0 t + \phi)$ where $\phi = arg[H(j\omega_0)]$, and as $T \rightarrow \infty, [< u, y >_T \ / < u, u >_T] \rightarrow Re[H(j\omega_0)] > 0$. The remaining arguments in the proof given[1] are valid in the LTI case. An alternate Lyapunov proof of Lemma 1 for the LTI case can also be found in [2].

In conclusion, [1] infers that Lemma 1 is incorrect because it does not prove that the feedback interconnection of a general passive plant and a weak SPR system is stable. This indeed remains an important open problem. If Lemma 1 is to be used to solve this problem, one would first have to prove that the input of the weak SPR controller (satisfying (6a)) is regular so that (6b) is satisfied. Although we have not been able to prove this stability result, it is our conjecture that it would be valid for large classes of nonlinear systems, as well as linear time-varying systems. We present it as an interesting challenge to the controls community to prove or disprove this conjecture.

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Robust Attitude Stabilization of Spacecraft Using Nonlinear Quaternion Feedback

S. M. Joshi, A. G. Kelkar, and J. T.-Y. Wen

Abstract— **This paper considers the problem of three-axis attitude stabilization of a rigid spacecraft. A nonlinear control law which uses the feedback of the unit quaternion and the measured angular velocities is proposed and is shown to provide global asymptotic stability. The control law does not require the knowledge of the system parameters and is, therefore, robust to modeling errors. The significance of the control law is that it can be used for large-angle maneuvers with guaranteed stability.**

I. Introduction

Attitude control of a free-flying spacecraft has long been known as an important problem and has been the subject of many papers since the 1950's and 1960's [1]–[3]. It is also a unique problem in dynamics because of the fact that the finite rotation of a rigid body does not obey the laws of vector addition (in particular, commutativity) and, as a result, the angular velocity of the body cannot be integrated to give the attitude of the body. The most widely used method of defining the rotation of a body between two different orientations is an Euler angle description. A 3×3 direction cosine matrix (of Euler rotations) is used to describe the orientation of the body (achieved by three successive rotations) with respect to some fixed frame of reference. There is, however, an inherent geometric singularity in the Euler representation. This problem can be avoided by using a four-parameter description of the orientation [2]–[4], known as "quaternions," which can be used to describe all possible orientations. The quaternion approach uses Euler's theorem which states that any rotation of a rigid body can be described by a single rotation about a fixed axis. The advantage of using quaternions is that successive rotations result in successive multiplications of 4×4 quaternion matrices which are commutative [2]. Some early results on the use of quaternion feedback for attitude error representation and automatic control of the attitude can be found in [1]. Quaternions were used for simulation of the rotational motion of rigid bodies as early as the 1950's [5]. The use of quaternion feedback for controlling robotic manipulators can be found in [6] and [7] and for spacecraft control can be found in [8]–[12].

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Various linear and nonlinear quaternion-based control laws have been proposed [10]–[12] for the attitude control of a single-body rigid spacecraft. A linear decoupled model-independent control law was considered in [10]. The control laws proposed in [11] require the knowledge of the system's moments of inertia and also constrain the choice of the gain matrices. In [12], both model-dependent and model-independent control laws were presented; however, most of the control laws used scalar gains. Recently, in [9], a globally stabilizing nonlinear control law using quaternion feedback was given for a class of multibody flexible space structures. This control law also used a scalar feedback gain for the quaternion vector.

In this note, a model-independent, nonlinear control law is presented which uses quaternion feedback and symmetric and positive definite gain matrices. Global asymptotic stability of the proposed control law is shown by using the Lyapunov method. The Lyapunov function used here does not need a cross term similar to the one used in [12] and leads to a simpler stability proof.

II. Quaternion Feedback Control

The rotational equations of motion of a rigid spacecraft are given by

$$J\dot{\omega} + \omega \times (J\omega) = u \tag{1}$$

where J is the 3×3 inertia matrix, ω is the 3×1 angular velocity vector, and u is the 3×1 vector of actuator torques. The objective of the control system is to bring the spacecraft to the desired attitude (orientation) starting from any initial condition.

For the reasons stated earlier, a globally nonsingular quaternion representation will be used in this paper to describe the attitude of a rigid spacecraft. For the sake of completeness and for later use, the necessary quaternion equations [9] are given next.

The unit quaternion α is defined as follows

$$\alpha = \{\bar{\alpha}^T, \alpha_4\}^T, \quad \bar{\alpha} = \begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \\ \hat{\alpha}_3 \end{bmatrix} \sin\left(\frac{\phi}{2}\right), \quad \alpha_4 = \cos\left(\frac{\phi}{2}\right). \tag{2}$$

$\hat{\alpha} = (\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3)^T$ is the unit vector along the eigen-axis of rotation and ϕ is the magnitude of rotation. The quaternion is also subjected to the norm constraint

$$\bar{\alpha}^T\bar{\alpha} + \alpha_4^2 = 1. \tag{3}$$

The quaternion obeys the following kinematic differential equations [13]

$$\dot{\bar{\alpha}} = \frac{1}{2}(\omega \times \bar{\alpha} + \alpha_4\omega) \tag{4}$$

$$\dot{\alpha}_4 = -\frac{1}{2}\omega^T\bar{\alpha}. \tag{5}$$

The equilibrium solutions of the open-loop system, given by (1), (4), and (5), can be obtained by setting all the derivatives to zero. That is

$$\omega \times (J\omega) = 0 \tag{6}$$
$$\omega \times \bar{\alpha} + \alpha_4\omega = 0 \tag{7}$$
$$\omega^T\bar{\alpha} = 0. \tag{8}$$

Taking the dot product of ω with both sides of (7)

$$\alpha_4(\omega \cdot \omega) = 0.$$

That is, either $\alpha_4 = 0$, or $\omega = 0$, or both are zero. If $\alpha_4 \neq 0$, then $\omega = 0$. If $\alpha_4 = 0$, then from (7) and (8), $\omega \times \bar{\alpha} = 0$, and $\omega \cdot \bar{\alpha} = 0$, i.e., $\omega = 0$, or $\bar{\alpha} = 0$, or both are zero. From (3), however, $\alpha_4 = 0 \Rightarrow \bar{\alpha} \neq 0$; therefore, $\omega = 0$ when the system is in equilibrium. The system has multiple equilibrium solutions: $(\bar{\alpha}_{ss}^T, \alpha_{4ss})$, where, the subscript "ss" denotes the constant steady-state value.

Consider the control law u, given by

$$u = -\frac{1}{2}[(\tilde{\bar{\alpha}} + \alpha_4 I)G_p + \gamma(1-\alpha_4)I]\bar{\alpha} - G_r\omega \tag{9}$$

where G_p and G_r are symmetric positive definite (3×3) matrices, γ is a positive scalar, and $\tilde{\bar{\alpha}}$ represents the 3×3 cross product matrix of the vector $\bar{\alpha}$. Equation (9) represents a nonlinear control law. The control law given by (9) was stated in [12]; however, conditions for the existence of the closed-loop equilibrium solutions were not investigated, and explicit stability proof was not given.

The following result gives the closed-loop equilibrium solutions.

Lemma 1: Suppose G_p is symmetric and positive definite and $0 < \lambda_M(G_p) \leq 2\gamma$, where $\lambda_M(\cdot)$ denotes the largest eigenvalue. Then the closed-loop system given by (1), (4), (5) and (9) has exactly two equlibrium solutions: $[\bar{\alpha} = \omega = 0, \alpha_4 = 1]$ and $[\bar{\alpha} = \omega = 0, \alpha_4 = -1]$.

Proof: The closed-loop system is in equilibrium when the derivatives in (1), (4), and (5) are zero. Proceeding as in the open-loop case, the closed-loop equilibrium solution is given by: $\omega = 0, \bar{\alpha} = \bar{\alpha_{ss}}, \alpha_4 = \alpha_{4ss}$. From (1), $\omega = 0 \Rightarrow u = 0$. From (9), we have

$$[(\tilde{\bar{\alpha}} + \alpha_4 I)G_p + \gamma(1-\alpha_4)I]\bar{\alpha} = 0. \tag{10}$$

Premultiplying the above equation by $\bar{\alpha}^T$ and noting that the first term vanishes, we have the following

$$u = 0 \Rightarrow \bar{\alpha}^T M \bar{\alpha} = 0$$

where

$$M = \alpha_4 G_p + \gamma(1-\alpha_4)I. \tag{11}$$

The eigenvalues of M are given by: $\lambda_i(M) = \alpha_4\lambda_i(G_p) + \gamma(1 - \alpha_4) = \alpha_4(\lambda_i(G_p) - \gamma) + \gamma$. M is singular when $\lambda_i(M) = 0$, i.e., when $\alpha_4 = \frac{-\gamma}{\lambda_i(G_p)-\gamma}$. There are four different subcases that need to be examined:

a) $0 < \lambda_i(G_p) < \gamma$,
b) $\lambda_i(G_p) = \gamma$,
c) $\gamma < \lambda_i(G_p) < 2\gamma$, and
d) $\lambda_i(G_p) = 2\gamma$.

In subcases a) and c), $\lambda_i(M) = 0$ only if $|\alpha_4| > 1$, which is not feasible, since $-1 \leq \alpha_4 \leq 1$. That means, for subcases a) and c), $\lambda_i(M) \neq 0$ for any feasible values of α_4. In subcase b), i.e., when $\lambda_i(G_p) = \gamma$, $\lambda_i(M) = \gamma > 0$. Therefore, M is nonsingular for subcases a), b), c), and $\bar{\alpha} = 0$. For subcase d), $\lambda_i(M) = 0 \Rightarrow (\alpha_4 + 1)\gamma = 0$, i.e., $\alpha_4 = -1$, and $\bar{\alpha} = 0$. Therefore, we have: $\bar{\alpha} = 0$ and $\alpha_4 = 1$ or -1. □

Note that if $\lambda_i(G_p) > 2\gamma$, then there are feasible nonzero values of α_4 for which $\lambda_i(M) = 0$. These are not the desired equilibrium points, however, since they correspond to the states of rest (i.e., zero velocities) in any arbitrary orientation.

From Lemma 1, there appear to be two closed-loop equilibrium points corresponding to $\alpha_4 = 1$ and $\alpha_4 = -1$ (all other state variables being zero). From (2), however, $\alpha_4 = 1 \Rightarrow \phi = 0$ and $\alpha_4 = -1 \Rightarrow \phi = 2\pi$ (or more generally $2n\pi$), i.e., there is only one equilibrium point in the physical space. We shall define the desired equilibrium state as: $\bar{\alpha} = \omega = 0, \alpha_4 = 1$. To make the origin of the state space the desired state, define $\beta = (\alpha_4 - 1)$. Equations (4) and (5) can then be rewritten as

$$\dot{\bar{\alpha}} = \frac{1}{2}(\omega \times \bar{\alpha} + (\beta + 1)\omega) \tag{12}$$
$$\dot{\beta} = -\frac{1}{2}\omega^T\bar{\alpha}. \tag{13}$$

The system represented by (1), (12), and (13) can be expressed in the state-space form as

$$\dot{x} = f(x, u) \tag{14}$$

where $x = (\bar{\alpha}^T, \beta, \omega^T)^T$. Note that the dimension of x is seven, which is one more than the dimension of the system. One constraint (3), however, is now present. It can be easily verified from (4) and (5) that the constraint (3) is satisfied for all $t > 0$ if it is satisfied at $t = 0$.

If the objective of the control law is to transfer the state of the system from one orientation (equilibrium) position to another orientation (i.e., a rest-to-rest maneuver), then without loss of generality, the target orientation can be defined to be zero. The initial orientation, given by $(\bar{\alpha}(0), \beta(0))$, can always be defined in such a way that $-1 \leq \beta \leq 0$ (i.e., $0 \leq \alpha_4(0) \leq 1$), corresponding to $|\phi| \leq \pi$.

The control law given by equation can be rewritten as

$$u = -\frac{1}{2}[(\tilde{\bar{\alpha}} + (\beta+1)I)G_p - \gamma\beta I]\alpha - G_r\omega. \tag{15}$$

The following theorem establishes the global asymptotic stability of the physical equilibrium state (the origin of the state space) of the system.

Theorem 1: Suppose G_p and G_r are symmetric and positive definite, and $0 < \lambda_M(G_p) \leq 2\gamma$. Then, the closed-loop system given by (1), (12), (13), and (15) is globally asymptotically stable (g.a.s.).

Proof: Consider the candidate Lyapunov function

$$V = \omega^T J\omega + \bar{\alpha}^T G_p\bar{\alpha} + \gamma\beta^2. \tag{16}$$

V is clearly positive definite and radially unbounded with respect to the state vector $x = \{\bar{\alpha}^T, \beta, \omega^T\}^T$. Taking the time derivative of V, we have

$$\dot{V} = 2\omega^T[-\omega \times (J\omega) + u] + \bar{\alpha}^T G_p(\omega \times \alpha + (\beta+1)\omega) - \gamma\beta\omega^T\bar{\alpha}. \tag{17}$$

Noting that $\omega^T[\omega \times (J\omega)] = 0$ and substituting for u from (15), after simplification, we get: $\dot{V} = -2\omega^T G_r\omega$, i.e., $\dot{V}$ is negative semidefinite. $\dot{V} = 0$ only when $\omega = 0$. Following the same procedure as Lemma 1, it can be shown that $\dot{V} = 0$ only at the two equilibrium points, $\bar{\alpha} = \omega = 0$, $\beta = 0$ (corresponding to $\alpha_4 = 1$) and $\bar{\alpha} = \omega = 0$, $\beta = -2$ (corresponding to $\alpha_4 = -1$).

Consistent with the previous discussion, these values correspond to two equilibrium points representing the same physical equilibrium state. It can be easily verified, from (16), that any small perturbation ϵ in β from the equilibrium point corresponding to $\beta = -2$ will cause a decrease in the value of V (ϵ has to be > 0 because $-2 \leq \beta \leq 0$). Thus, in the mathematical sense, $\beta = -2$ corresponds to an isolated equilibrium point such that $\dot{V} = 0$ at that point, and $\dot{V} < 0$ in a neighborhood of that point (i.e., $\beta = -2$ is a "repeller" and not an "attractor"). It has been already shown that $\dot{V}$ is negative everywhere in the feasible state space except at the two equilibrium points. That is, if the system's initial condition lies anywhere in the state space except at the equilibrium point corresponding to $\beta = -2$, then the system will asymptotically approach the origin ($x = 0$); if the system is at the equilibrium point corresponding to $\beta = -2$ at $t = 0$, then it will stay there for all $t > 0$. This is the same equilibrium point in the physical space, however; hence, it can be concluded by LaSalle's theorem that the system is globally asymptotically stable. □

Theorem 1 gives a condition on G_p and γ which assures the global asymptotic stability of the desired equilibrium state. It can also be shown that, for any choice of $G_p > 0$ and $\gamma > 0$, the desired equilibrium is always locally asymptotically stable. To see this, first note that the equilibrium value of α_4 cannot be zero; if $\alpha_4 = 0$ then $\lambda_i(M) = \gamma > 0$, which implies that the equilibrium value of $\bar{\alpha} = 0$. This is not possible in view of (3). Therefore, when the system is in equilibrium we have (see the proof of Lemma 1)

$$\bar{\alpha}^T G_p \bar{\alpha} = -\frac{\gamma(1-\alpha_4)}{\alpha_4}\bar{\alpha}^T\bar{\alpha}. \tag{18}$$

From the Lyapunov analysis in the proof of Theorem 1 above, V at the equilibrium state is always bounded from above by the initial V, defined by $V_0 = \omega^T(0)J\omega(0) + \bar{\alpha}^T(0)G_p\bar{\alpha}(0) + \gamma\beta^2(0)$. Therefore, for all $t \geq 0$

$$\bar{\alpha}^T G_p \bar{\alpha} + \gamma(\alpha_4 - 1)^2 \leq V_0. \tag{19}$$

When the system is in equilibrium, the left-hand side of (19) can be simplified as follows by using (18)

$$\bar{\alpha}^T G_p \bar{\alpha} + \gamma(\alpha_4 - 1)^2 = \gamma\left(2 - \alpha_4 - \frac{1}{\alpha_4}\right). \tag{20}$$

Substituting this expression in (19) and rearranging terms, we obtain

$$\frac{V_0}{2\gamma} \geq 1 - \frac{1+\alpha_4^2}{2\alpha_4}. \tag{21}$$

Hence, if

$$V_0 \leq 2\gamma \tag{22}$$

then $\alpha_4 > 0$, which implies that the only solution of (18) is $\bar{\alpha} = 0$ (since G_p is positive definite); hence $\alpha_4 = 1$. Therefore, the region given by

$$\{(\bar{\alpha}, \beta, \omega) : \omega^T J\omega + \bar{\alpha}^T G_p \bar{\alpha} + \gamma\beta^2 \leq 2\gamma\} \tag{23}$$

contains only one equilibrium point, which is the desired equilibrium point. Since $V > 0$ and $\dot{V} < 0$ along all trajectories in this region, (23) represents the domain of attraction. In the case where the initial velocity is zero (i.e., $\omega(0) = 0$), the condition on the initial attitude to guarantee asymptotic stability becomes

$$\bar{\alpha}^T(0)G_p\bar{\alpha}(0) + \gamma\beta^2(0) \leq 2\gamma. \tag{24}$$

Note that $\beta(0)$ is always nonpositive by definition. The left-hand side can be bounded as

$$\begin{aligned} &\bar{\alpha}^T(0)G_p\bar{\alpha}(0) + \gamma\beta^2(0) \\ &\quad\leq \lambda_M(G_p)\|\bar{\alpha}(0)\|^2 + \gamma\beta^2(0) \\ &\quad\leq \lambda_M(G_p)(1-\alpha_4^2(0)) + \gamma\beta^2(0) \\ &\quad\leq \lambda_M(G_p)(1-(\beta(0)+1)^2) + \gamma\beta^2(0) \\ &\quad\leq -(\lambda_M(G_p)-\gamma)\beta^2(0) - 2\beta(0)\lambda_M(G_p). \end{aligned} \tag{25}$$

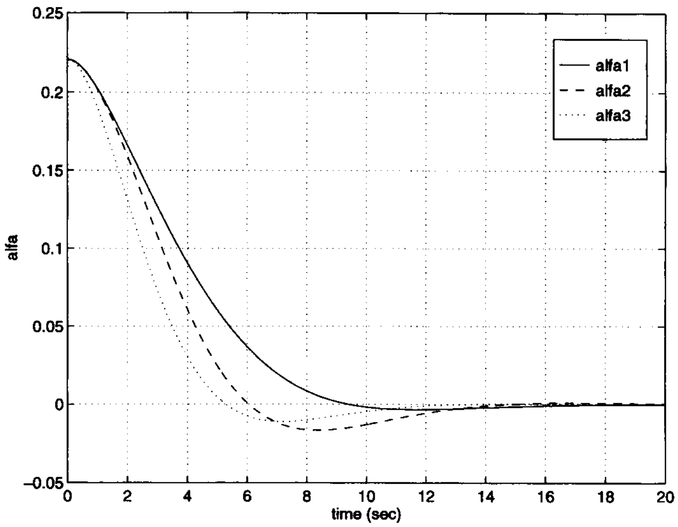

Fig. 1. Time histories of quaternions.

Hence, a sufficient condition for convergence to the desired equilibrium (in the case that $\omega(0) = 0$) is

$$-(\lambda_M(G_p) - \gamma)\beta^2(0) - 2\beta(0)\lambda_M(G_p) \leq 2\gamma. \tag{26}$$

We need only to consider the case that $\lambda_M(G_p) > 2\gamma$ as Theorem 1 has already shown that $\lambda_M(G_p) \leq 2\gamma$ guarantees global asymptotic stability. Under this inequality, a sufficient condition for asymptotic stability is

$$|\beta(0)| \leq \frac{\gamma}{\lambda_M(G_p)}. \tag{27}$$

This bound is always ≤ 0.5 and becomes smaller as the ratio between $\lambda_M(G_p)$ and γ increases. Since $\beta(0)$ is always nonpositive the permissible range for $\beta(0)$ is: $-0.5 \leq \beta(0) \leq 0$.

A numerical example is given next to demonstrate the result of Theorem 1.

III. Numerical Example

The spacecraft considered for a numerical simulation is a representative of a realistic spacecraft in terms of its mass and inertia properties. The mass of the spacecraft was 2042.11 kg, and its principal inertias about x-, y-, and z-axis were 800.027 kg/m^2, 839.93 kg/m^2, and 289.93 kg/m^2, respectively. The spacecraft also had significant cross-products of inertia due to asymmetric configuration. A rest-to-rest multi-axis maneuver was considered, where the initial orientation of the spacecraft corresponds to the Euler parameters, $\bar{\alpha} = (0.221, 0.221, 0.221)^T$ and $\alpha_4 = 0.924$. The objective of the control law was to bring the spacecraft to its zero state, i.e., $\bar{\alpha} = 0$. This was accomplished by using the nonlinear control law given by (9). The feedback gains were chosen as: $G_p = \mathrm{diag}[750, 800, 400]$, $G_r = \mathrm{diag}[600, 550, 250]$, and $\gamma = 700$. The choice of gains was made by trial and error since no systematic synthesis procedure is known to date. These values of gains were found to give satisfactory response as shown in Fig. 1. In general, increasing the gains resulted in faster responses. The control torque time histories corresponding to the chosen gains are shown in Fig. 2. It should be noted that the responses given in Fig. 1 by no means represent the best possible design. Systematic synthesis techniques for such controllers remains a topic of future research.

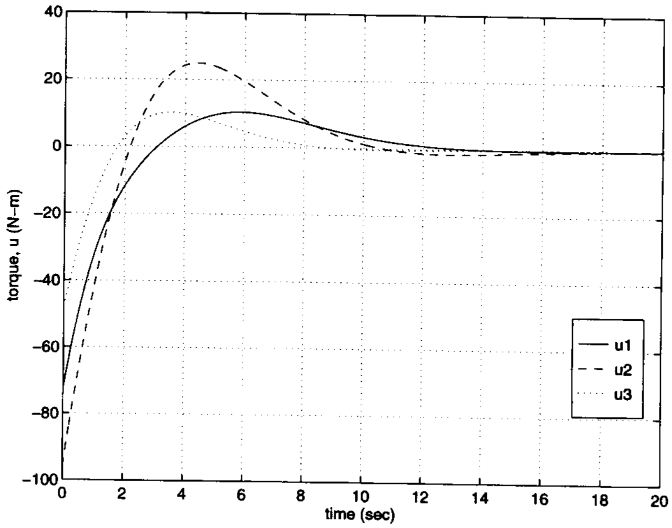

Fig. 2. Time histories of control inputs.

IV. Concluding Remarks

The problem of three-axis attitude stabilization of a spacecraft was considered. A nonlinear quaternion-based feedback control law was given and was shown to provide global asymptotic stability. A numerical example was also given to demonstrate the control law. The control law does not depend on the knowledge of the system parameters (i.e., moments of inertia) and is therefore robust to modeling errors and parametric uncertainties.

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D-Stability of Continuous Time-Delay Systems Subjected to a Class of Highly Structured Perturbations

Chien-Hua Lee

***Abstract*—The D-stability testing problem for continuous time-delay systems subjected to a class of highly structured parametric perturbations is addressed in this note. By means of the Lyapunov stability theorem, Razumikhin-type theorem, Gersgorin theorem, concept of spectral radius, and norm and matrix measure techniques, we have developed several new sufficient conditions for guaranteeing that all characteristic roots of the above systems are located inside a specified disk $D(\alpha, r)$ with center at $\alpha + j0$ and radius r in the left-half complex plane. For the present results, it is not necessary to solve any Lyapunov equation which may be unsolvable though the Lyapunov stability theorem is utilized.**

Nomenclature

$\mathbf{R}$	Real number field.
$\mathbf{R}^+$	Real positive number field.
$\mathbb{C}$	Complex number field.
$\|\|x\|\|$	Vector norm of $x \in \mathbf{R}^n$.
$\lambda(A)$	Eigenvalue of square matrix A.
$\underline{\lambda}(A)$	The minimal eigenvalue of square matrix A.
$\overline{\lambda}(P)$	The maximal eigenvalue of square matrix A.
$\mathrm{Re}\,(s)$	Real part of complex number s.
$\rho(A)$	Spectral radius of matrix $A \in \mathbb{C}^{n \times n}$.
$\|a\|$	Absolute value of complex number a.
$\|\|A\|\|$	Induced norm of matrix A.
$\mu(A)$	Matrix measure of matrix A.
$\|A\|$	$\{\|a_{ij}\|\}$ for matrix $A = \{a_{ij}\}$.
$A \geq B$	$a_{ij} \geq b_{ij}$ for all i, j where matrices $A = \{a_{ij}\}$ and $B = \{b_{ij}\}$.
I	Identity matrix with appropriate dimensions.

I. Introduction

It is well known that the location of all characteristic roots is an important indicator for the system dynamic performance of linear controlled systems. In practice, all characteristic roots cannot be assigned in fixed locations but can be only located inside some restricted regions due to the unavoidable parametric perturbations which occur as a result of aging, changes in environmental conditions, data errors, etc. Much has been written about the robust pole-assignment or root-clustering problem [7], [8], [10]–[13], [19], [21], [22] for linear systems with parametric perturbation. Recently, the D-stability problem which guarantees all characteristic roots of controlled systems to be located inside a specified disk in the complex plane has also become an attractive area of research for the mentioned systems [3], [4], [6], [15], [18]. Time delay which occurs as a result of data computation, information transmission, etc., however, may also be integrated into a system model in addition to parametric perturbations. The time delay will change the system characteristic equation (there will be $e^{-s\tau}$ and z^{-i} term, respectively,

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