1 Quaternions as Rotations in $\mathbb{R}^3$

1.1 Time Derivative

$$\dot{q} = \omega q$$

$$\dot{q}^* = q^* \omega^*$$

where

$$\omega = \frac{1}{2} \begin{bmatrix} 0 \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

2 Matrix Representaion

Define

$$A = \begin{bmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{bmatrix}$$

$$B = \begin{bmatrix} a & -b & -c & -d \\ b & a & -d & c \\ c & d & a & -b \\ d & -c & b & a \end{bmatrix}$$

2.1 Multiplication

$$ab = c$$

$$B(a)[b] = [c]$$