Nomenclature

Variables

$\mathbf{F}$ force vector

$\mathbf{x}$ vector

$\mathbf{x}$ position vector

Subscripts

B body frame

D drag

R rotor

1 Equations of Motion

1.1 Reference

Gravity force in inertial frame

$$\mathbf{g} = -g\mathbf{k}$$

Rotor force in body frame

$$\mathbf{F}_{R,B} = k\left(\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4\right)\mathbf{k}$$

Rotor force in inertial frame

$$\mathbf{F}_R = \mathbf{R}(\mathbf{q})\mathbf{F}_{R,B}(\gamma)$$

Drag force in body frame

$$\mathbf{F}_{D,B}(\dot{\mathbf{x}}_B)$$

$$\dot{\mathbf{x}}_B = \mathbf{R}^{-1}\dot{\mathbf{x}}$$

Drag force in inertial frame

$$\mathbf{F}_D = \mathbf{R}(\mathbf{q})\mathbf{F}_{D,B}(\dot{\mathbf{x}}_B)$$

1.2 Translation

$$m\ddot{\mathbf{x}} = m\mathbf{g} + \mathbf{F}_R(\mathbf{q},\gamma) + \mathbf{F}_D(\dot{\mathbf{x}},\mathbf{q})$$

$$\ddot{\mathbf{x}} = \mathbf{g} + \frac{1}{m}\mathbf{F}_R(\mathbf{q},\gamma) + \frac{1}{m}\mathbf{F}_D(\dot{\mathbf{x}},\mathbf{q})$$

1.3 Rotation

$$I\dot{\omega} + \omega \times (I\omega) = \tau$$

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

$$\tau_B = \begin{bmatrix} Lk\left(\gamma_1 - \gamma_3\right) \\ Lk\left(\gamma_2 - \gamma_4\right) \\ b\left(\gamma_1 - \gamma_2 + \gamma_3 - \gamma_4\right) \end{bmatrix}$$

2 Control

2.1 Attitude

Tracking error

$$\mathbf{e}_1 = \mathbf{q}_{ref} - \mathbf{q}$$

$$\dot{\mathbf{e}}_1 = \dot{\mathbf{q}}_{ref} - \dot{\mathbf{q}}$$

$$\dot{\mathbf{e}}_1 = \dot{\mathbf{q}}_{ref} - \frac{1}{2}\mathbf{E}\boldsymbol{\omega}$$

The $\boldsymbol{\omega}$ is not our control input and has its own dynamic. So we set fot it a desired behavior and consider it as our virtual control

$$\boldsymbol{\omega}_{ref} = c_1\mathbf{e}_1 + \dot{\mathbf{q}}_{ref} + \lambda_1\chi_1$$

$$\mathbf{e_2} = \boldsymbol{\omega}_{ref} - \boldsymbol{\omega}$$

2.2 Position

$$\mathbf{e} = \mathbf{x}_{ref} - \mathbf{x}$$

$$\mathbf{F} = m\mathbf{a} = m\left(\mathbf{K}_p\mathbf{e} + \mathbf{K}_d\dot{\mathbf{e}}\right)$$