1 Multiplication

$$q_1 q_2 = A_1 q_2$$

$$A_1 = \begin{bmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{bmatrix}$$

2 Quaternions as Rotations in R^3

2.1 Time Derivative

$$\dot{q} = \frac{1}{2} B \omega$$

$$\omega = 2 B^T \dot{q}$$

$$B = \begin{bmatrix} -b & -c & -d \\ a & -d & c \\ d & a & -b \\ -c & b & a \end{bmatrix}$$

2.1.1 Time Derivative of Conjugate

$$\dot{q}^* = C \omega$$

$$C = \begin{bmatrix} -b & -c & -d \\ -a & d & -c \\ -d & -a & b \\ c & -b & -a \end{bmatrix}$$