A Well Model for Finite Element Reservoir Simulators.

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Abstract

Relatively little information has appeared in the literature about representing wells in numerical reservoir simulators. Some literatures discuss well models to be used in the Finite Difference reservoir simulator. And no literature so far has appeared on well models to be used in the Finite Element reservoir simulator.

In this report, we present a new well model to be used in the Finite Element reservoir simulator. This well model can take care of several different well constraints which is a powerful character of this well model.

Numerical experiments show that when this well model is used, pressure distribution, Darcy velocity distribution as well as mass balance can be calculated with sufficient accuracy.

1. Introduction

Recent development of high speed computers enables us to model a reservoir more complexed way and now we are dealing with reservoir models which can be used as a practical tool of reservoir engineers as discussed by Coats[1].

Relatively little infortmation has appeared in the literature about representing wells in numerical reservoir simulators. Some recent discussions are contained in Peaceman[2], and in Williamson and Chappelear[3].

Aziz and Settari[4] discuss reservoir simulation in general including treatment of wells in reservoir simulators. However, above literatures all discuss well models to be used in the finite difference method(FDM), and they are not readily applicable to the finite element method(FEM). Major drawbacks of using FDM are that, it has some restrictions in discritizing reservoirs and inflexibility in assigning boundary conditions. FEM seems to have some advantages over FDM in this respect.

Relatively small number of attempts have been made on the use of FEM in reservoir simulation. And no literature so far has appeared on "well model" suitable to FEM reservoir simulation.

In this report, we present "a well model" suitable to FEM reservoir simulator which can deal with different well constraints.

In section 2, we derive "the pressure equation" in FEM form. In section 3, we present "well model" to be used in FEM simulators. Following section shows some numerical experiments to clear the validity of the method.

2. Basic equations

2-a. Derivation of basic equations

In our reservoir simulation model, the reservoir is devided into two domains, D_r and D_w as shown in Fig.1. D_w is the domain in which wells are included. In D_r, following basic equations can be applied to analyze the steady flow of a single phase, incompressible fluid flow in porous medium.

Continuity equation;

$$\nabla \cdot U = 0 . \tag{1}$$

Momentum (Darcy) equation;

$$\rho U = -\frac{k}{\nu} \nabla P . \tag{2}$$

Let,

$$h = \frac{P}{\rho g} , \quad \sigma_p = \frac{k}{\nu} g . \tag{3}$$

Then, by substituting Eq(3) into Eq(2), we obtain;

$$U = -\sigma_p \nabla h . \tag{4}$$

Next, by substituting Eq(4) into Eq(1), we obtain "the pressure equation",

$$\nabla \cdot (\sigma_p \nabla h) = 0 . \tag{5}$$

To solve Eq(5), two boundary conditions are required. The first boundary condition must be given at the outer edge of D_r, and the second boundary condition must be given at the inner boundary of D_r. The boundary condition at the outer edge of D_r may be given as either

Dirichlet BC (h is given), or as Neumann BC (dh/dn is given). Boundary condition at the inner edge of D_r is given by "well model" which will be discussed later.

There are various numerical method to solve Eq(5). Finite Difference Method (FDM) has been used widely by petroleum enginners, and relatively small number of studies have been tried on the use of Finite Element Method (FEM). FEM has some advantages over FDM. For example, it is easier to divide domain into mesh or blocks, it has flexibility in applying BC and in giving reservoir property distribution. Thus we try FEM in our analysis.

2-b. Descritization of the basic equation

We apply Galerkin Method on Eq(5).

By multiplying an arbitrary function, $\bar{h}(\mathbf{x})$, to Eq(5), then integrating the product over the domain, Ω , and by applying Gauss' Theorem, we find;

$$\int_{\Omega} \sigma_p \nabla h \cdot \nabla \bar{h} \, d\Omega = \int_{\Gamma_u} U_n \bar{h} \, d\Gamma , \qquad (6)$$

where Γ_u is the boundary where velocity vector U_n normal to Γ_u is given. Note U_n is positive for inflow.

Now introducing shape function $N(\mathbf{x})$, we rewrite $\bar{h}$ and h as follows;

$$\bar{h} = \sum_{a=1}^{n} N_a \bar{h}^a , \quad h = \sum_{b=1}^{n} N_b h^b , \qquad (7)$$

where $\bar{h}^a$ is an arbitrary constant except when Dirichlet BC is given, and h^b is the value of h at node b.

Now by substituting Eq(7) into Eq(6), we find;

$$\sum_{b=1}^{n}\left[\int_{\Omega}\sigma_{p}\nabla N_{a}\cdot\nabla N_{b}\,d\Omega\right]h^{b}=\int_{\Gamma_{u}}U_{n}N_{a}\,d\Gamma. \quad (8)$$

Eq(8) is the descritized form of "the pressure equation" to be solved by FEM.

3. Well Model

3-a. Treatment of a well as a source term

As the radius of a well is very small compared with the entire reservoir, it is very difficult and inefficient, even if FEM is used to descritize the reservoir domain so small that we can represent the actual radius of wells to obtain steep pressure gradient near the well accurately. In order to avoid such difficulties, an analytical solution may be used to obtain pressure distribution near the well, then connect the analytical solution to the FEM solution surrounding the well.

To do this, following assumptions are needed.

Assumption 1; The analytical solution of pressure distribution is sufficiently accurate near the well, but gives large error at far away point from the well. On the other hand, the FEM solution is sufficiently accurate in the entire reservoir domain except near the well. And there exist a region where both solutions are sufficiently accurate, thus these two solutions can be connected in this region.

Assumption 2; When the well is treated as a source term, FEM solution is sufficiently accurate except near the well.

Above assumptions will be verified by numerical experiments later.

To treat the well as a source term, Γu in Eq(6) is devided into two portions, Γv and Γwi as shown in Fig.1. Then we find;

$$\int_{\Gamma_u} U_n N_a d\Gamma = \sum_{i=1}^{nw} \oint_{\Gamma_{wi}} U_n N_a d\Gamma + \oint_{\Gamma_v} U_n N_a d\Gamma, \quad (9)$$

where, Γwi is the path around the well, Γv is the path which defines the outer reservoir boundary, and nw is the number of wells.

For simplicity, we assume that wells are located on the FEM node points. Then Eq(9) may be rewritten to obtain;

$$\int_{\Gamma_u} U_n N_a d\Gamma = \begin{cases} q_{wi} & \text{node } a \text{ is on } wi, \\ \int_{\Gamma_v} U_n N_a d\Gamma & \text{node } a \text{ is on } \Gamma_v, \end{cases} \quad (10)$$

where qwi is the flow rate of fluid injected into the reservoir through well wi per unit thickness. Note that qwi is positive for injection and negative for production.

3-b. The first order approximation

In order to explain how the analytical solution may be used in FEM well model, we use an analytical solution of the following form.

$$h_w - h_e = \frac{q_w}{2\pi \sigma_p} \ln\left(\frac{r_e}{r_w}\right), \quad (11)$$

where r_w and h_w are the radius of the well and the pressure head of the well respectively. And r_e and he are the distance between the well and the point e in the reservoir and the pressure head at point e as shown in Fig.2.

There are three cases we need to consider, they are;

Case 1. Find r_w for given h_w and q_w.

Case 2. Find h_w for given q_w and r_w.

Case 3. Find q_w for given h_w and r_w.

For Case 1 and Case 2, problems are easily solved by substituting FEM solution, he , into Eq(11). Therefore, these cases can be treated separately from FEM scheme. On the other hand, for Case 3, some technique is required as q_w, which is needed in FEM schme, is unknown.

When Eq(11) is solved for q_w, we find;

$$q_w = \left\{ \frac{1}{2\pi\sigma_p} \ln\left(\frac{r_e}{r_w}\right) \right\}^{-1} (h_w - h_e) . \qquad (12)$$

Next, the pressure equation Eq(8) can be written as follows;

$$\sum_{b=1}^{n} A_{ab} \cdot h_b = f_b , \quad a = 1, 2, \cdots, n , \qquad (13)$$

when the well is at node a , above equation becomes;

$$\sum_{b=1}^{n} A_{ab} \cdot h_b = q_w . \qquad (14)$$

By substituting Eq(12) into Eq(14) and eliminating q_w, we obtain;

$$A_{a1} h_1 + \cdots + \left\{ A_{ae} - \left(\frac{1}{2\pi\sigma_p} \ln \frac{r_e}{r_w} \right)^{-1} \right\} h_e$$
$$+ \cdots + A_{an} h_n = \left(\frac{1}{2\pi\sigma_p} \ln \frac{r_e}{r_w} \right)^{-1} h_w . \qquad (15)$$

The solution of above equation, he , is then substituted back into Eq(12) to obtain q_w.

3-c. Higher order approximation

To obtain more accurate solution, we use the Fourier series solution of the following form;

$$h_w - h_e = \frac{g_w}{2\pi\sigma_p} \ln\frac{r_e}{r_w} - \sum_{k=1}^{3}\left\{b_k \cdot \cos(k\cdot\theta_e) + C_k \cdot \sin(k\theta_e)\right\}\cdot r_e^k - \left\{b_4 \cos(4\theta_e)\right\}\cdot r_e^4 \quad (16)$$

When eight nodes are used as shown in Fig.3,

$$e = 1,2,\ldots,8 .$$

The solving procedure is the same as discussed in the previous section. But we need to solve matrix equation in this case as shown below.

For Case 1, we need to solve;

$$\begin{bmatrix} \frac{1}{2\pi\sigma_p} g_w & r_1\cos\theta_1 & \cdots & r_1^4\cos 4\theta_1 \\ \vdots & \vdots & & \vdots \\ \frac{1}{2\pi\sigma_p} g_w & r_8\cos\theta_8 & \cdots & r_8^4\cos 4\theta_8 \end{bmatrix} \begin{bmatrix} \ln r_w \\ b_1 \\ c_1 \\ \vdots \\ c_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_8 \end{bmatrix}, \quad (17)$$

where

$$d_e = h_e - h_w + \frac{1}{2\pi\sigma_p} \ln r_e .$$

For Case 2, we need to solve;

$$\begin{bmatrix} 1 & r_1\cos\theta_1 & \cdots & r_1^4\cos 4\theta_1 \\ 1 & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ 1 & r_8\cos\theta_8 & & r_8^4\cos 4\theta_8 \end{bmatrix} \begin{bmatrix} h_w \\ b_1 \\ c_1 \\ \vdots \\ c_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_8 \end{bmatrix}, \quad (18)$$

where

$$d_e = h_e + \frac{q_w}{2\pi\sigma_p} \ln\left(\frac{r_e}{r_w}\right).$$

For Case 3, we need to solve;

$$\begin{bmatrix} \frac{1}{2\pi\sigma_p}\ln\frac{r_w}{r_1} & ---- & r_1^4\cos4\theta_1 \\ \vdots & & \vdots \\ \frac{1}{2\pi\sigma_p}\ln\frac{r_w}{r_8} & ---- & r_8^4\cos4\theta_8 \end{bmatrix} \begin{bmatrix} q_w \\ b_1 \\ c_1 \\ \vdots \\ c_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_8 \end{bmatrix}, \quad (19)$$

where

$$d_e = h_e - h_w.$$

Multiplying an inverse matrix W^{-1} to Eq(19) and solving for q_w, we obtain;

$$q_w = \sum_{e=1}^{8} (W^{-1})_{1e} \cdot h_e - h_w \sum_{e=1}^{8} (W^{-1})_{1e}, \quad (20)$$

where

$$W = \begin{bmatrix} \frac{1}{2\pi\sigma_p}\ln\frac{r_w}{r_1} & ---- & r_1^4\cos4\theta_1 \\ \vdots & & \vdots \\ \frac{1}{2\pi\sigma_p}\ln\frac{r_w}{r_8} & ---- & r_8^4\cos4\theta_8 \end{bmatrix}. \quad (21)$$

By substituting Eq(20) into Eq(14), we can eliminate q_w. Thus we can obtain he. Then substituting he into Eq(20), we can obtain q_w.

Before applying well model to FEM, we need to consider following points.

1. The FEM solution gives large error on well node. So the FEM solution on well node does not have significant meaning. Thus pressure head on well node must be calculated by well model.

2. In this method, a well is treated as a source term. Even in Case 1 and Case 3, hw is not given as Dirichlet BC. Therefore, it is necessary to give Dirichlet BC on any one of the nodes (except on well node) to obtain solution.

4. Numerical experiment

In order to check the validity of the assumptions stated in section 3, we solved two problems. The first problem was solved to check the accuracy of the pressure equation treating a well as a source term without well model. The second problem was solved to check the accuracy of the entire model including well model.

Problem 1: Calculation of pressure equation.

For an infinite two dimensional reservoir with one well, the analytical pressure solution is given as follows;

$$h - h_o = \frac{q_w}{2\pi \sigma_p} \ln \frac{r_o}{r} , \qquad (22)$$

where, h_o is the pressure head at r_o . Since this is a radial flow problem, only a section of the entire reservoir was cut out and descritized as shown in Fig.4. Following values are assingned for calculation.

$$h = 0 \text{ m} \qquad \text{at} \quad r = 1000 \text{ m} ,$$

$$q_w = 1 \text{ m}^3 \text{/hr/m} \qquad \text{at} \quad r = 0 \text{ m} ,$$

σ_p = 0.03528 m^3/hr/m .

The analytical solution and the FEM solution is shown in Fig.5. Except the first node (well node, not shown in Fig.5) the FEM solution gives sufficiently accurate values. Thus, the validity of treating a well as a source term was proven.

Problem 2: Analysis of the pressure and velocity vector field of a hypothetical reservoir by using well model.

A hypothetical two dimensional reservoir is shown in Fig.6. There are three wells, W1 and W3 are the production wells and W2 is the injection well. In addition to permeability(k) distributions shown in Fig.6, k=1000 mD is assigned around each well. The thickness of the reservoir is 100 m, and the reservoir outer boundary is set to be no flow boundary except the lower right portion of the reservoir where inflow velocity vector is assigned. This hypothetical reservoir is descritized to FEM mesh. Number of nodes was 1681 and number of elements was 3200.

Calculated pressure field and Darcy velocity distribution is shown in Fig.7-a,b. Calculated velocity distribution shows the significant effect of the permeability distribution. Calculated volume balance using well model is also shown in Table 1, where cases 1,2 and 3 are calculated separately. All the results shown in Table 1 including volume balance of fluid agreed quite well within desired error. Thus, the validity of well model was proven.

It should be noted that in Problem 2, any mixed combinations of cases stated in 3-b could be applied on each well, such as find q_w for W1 and find r_w , h_w for W2 and W3 respectively. This is a very powerful property of our well model.

5. Conclusions and final remarks

1. A new well model has been developed and tried on the FEM reservoir simulation. Numerical experiments show that when wells are treated as source terms, pressure distribution in the reservoir, Darcy velocity distribution as well as mass balance can be calculated with sufficient accuracy.

2. "Well model" developed here is suitable for inclusion in a FEM reservoir simulator.

Although our analysis was somewhat simple, we hope this work will stimulate others to publish their work in this very interesting field.

Nomenclature

Aab = Coefficient matrix defined in Eq(13)
b,c = Coefficients
D = Reservoir domain
fb = Right hand side vector defined in Eq(13)
g = Gravity acceleration
h = Pressure head
$\bar{h}$ = Arbitrary constant
k = Permeability
N = Shape function
n = Number of nodes
P = Pressure
q = Volume flow rate per unit thickness
r = Radius
U = Velocity vector

Greek letters

Γ = Boundary
Ω = Domain
ν = Kinematic viscosity of fluid
ρ = Density of fluid
θ = Angle
σ_p = Pressure diffusion coefficient
∇ = Gradient operator

Subscript

r = Reservoir
w = Well or inner boundary of Dr
u = Boundary where Neumann BC is given
v = Outer boundary of Dr

References

1. Coats, K.H.:"Reservoir Simulation: State of the Art," J. Pet. Tech. (Aug. 1982) 1633-1642.

2. Peaceman, D.W.:"Interpretation of Well-Block Pressure in Numerical Reservoir Simulation," Soc. Pet. Eng. J. (June 1978) 183-194.

3. Williamson, A.S. and Chappelear, J.E.:"Representing Wells in Numerical Reservoir Simulation: Part 1 -- Theory, and Part 2 -- Implementation," Soc. Pet. Eng. J. (June 1981) 323-344.

4. Aziz, K. and Settari, A.: Petroleum Reservoir Simulation, Applied Science Publishers Ltd, London (1979).

Figure captions

Fig. 1 Notation of reservoir domain and boundary.

Fig. 2 Notation of r and h.

Fig. 3 Node points to be used in Eq.(16).

Fig. 4 FEM mesh and boundary conditions for Problem 1.

Fig. 5 Comparison of the FEM solution with the analytical solution.

Fig. 6 Parameters of hypothetical reservoir for Problem 2.

Fig. 7-a Pressure (——) and Darcy velocity vector (→) solution for Problem 2. Portion of this figure is expanded and shown in Fig. 7-b.

Fig. 7-b Pressure (——) and Darcy velocity vector (→) solution for Problem 2. Expanded view.

Table 1 Calculated results using "well model."

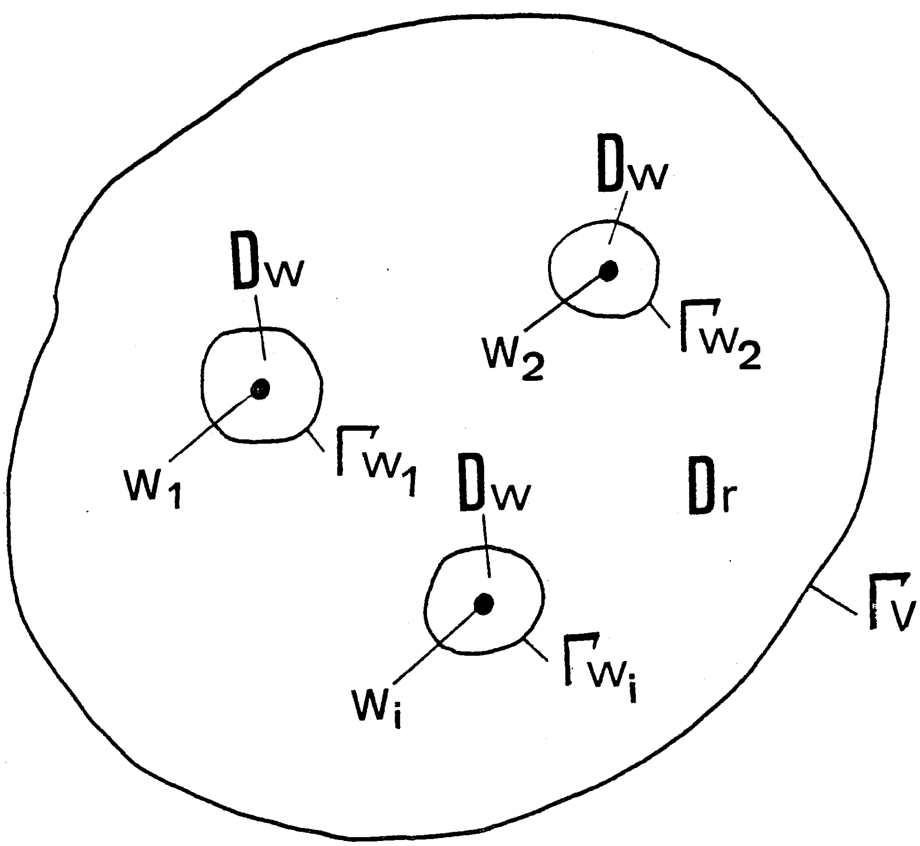

Fig. 1 Notation of reservoir domain and boundary

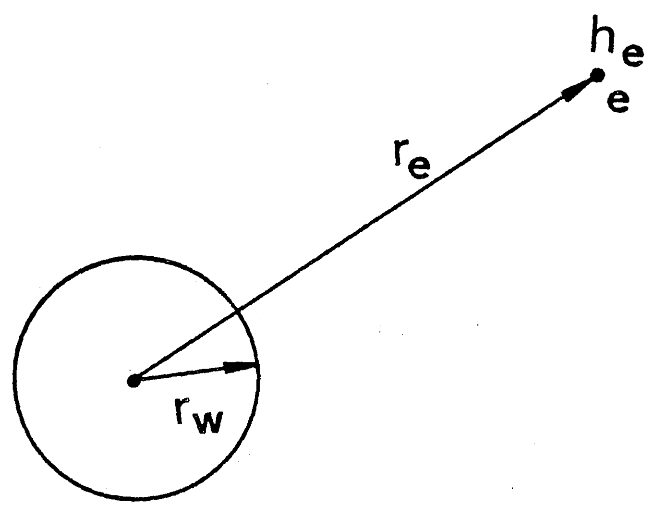

Fig. 2   Notation of r and h

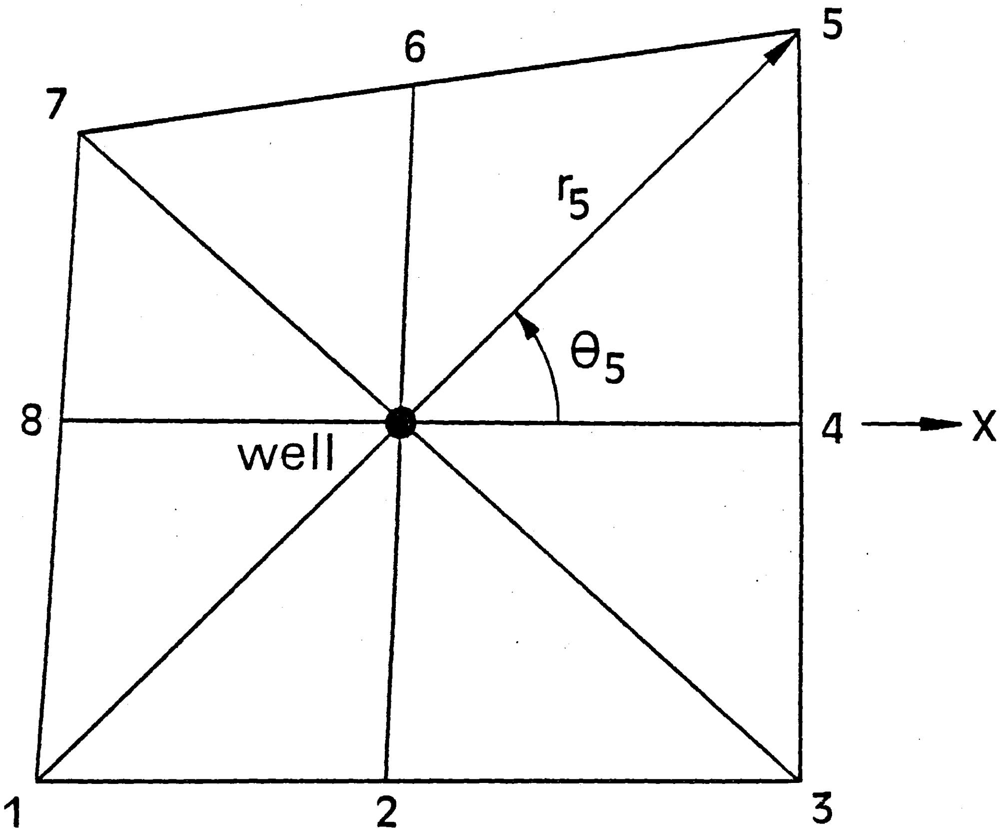

Fig. 3 Node points to be used in Eq.(16)

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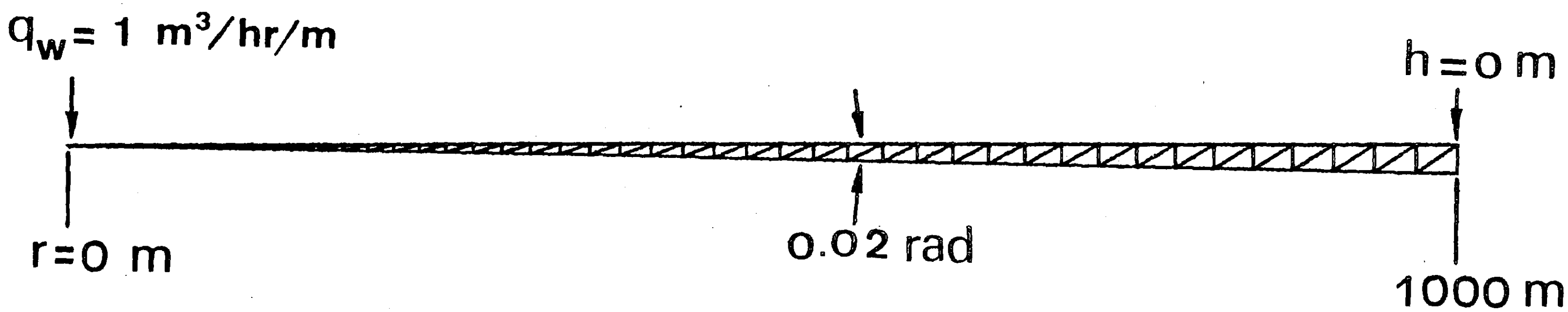

Fig. 4 FEM mesh and boundary conditions for Problem 1.

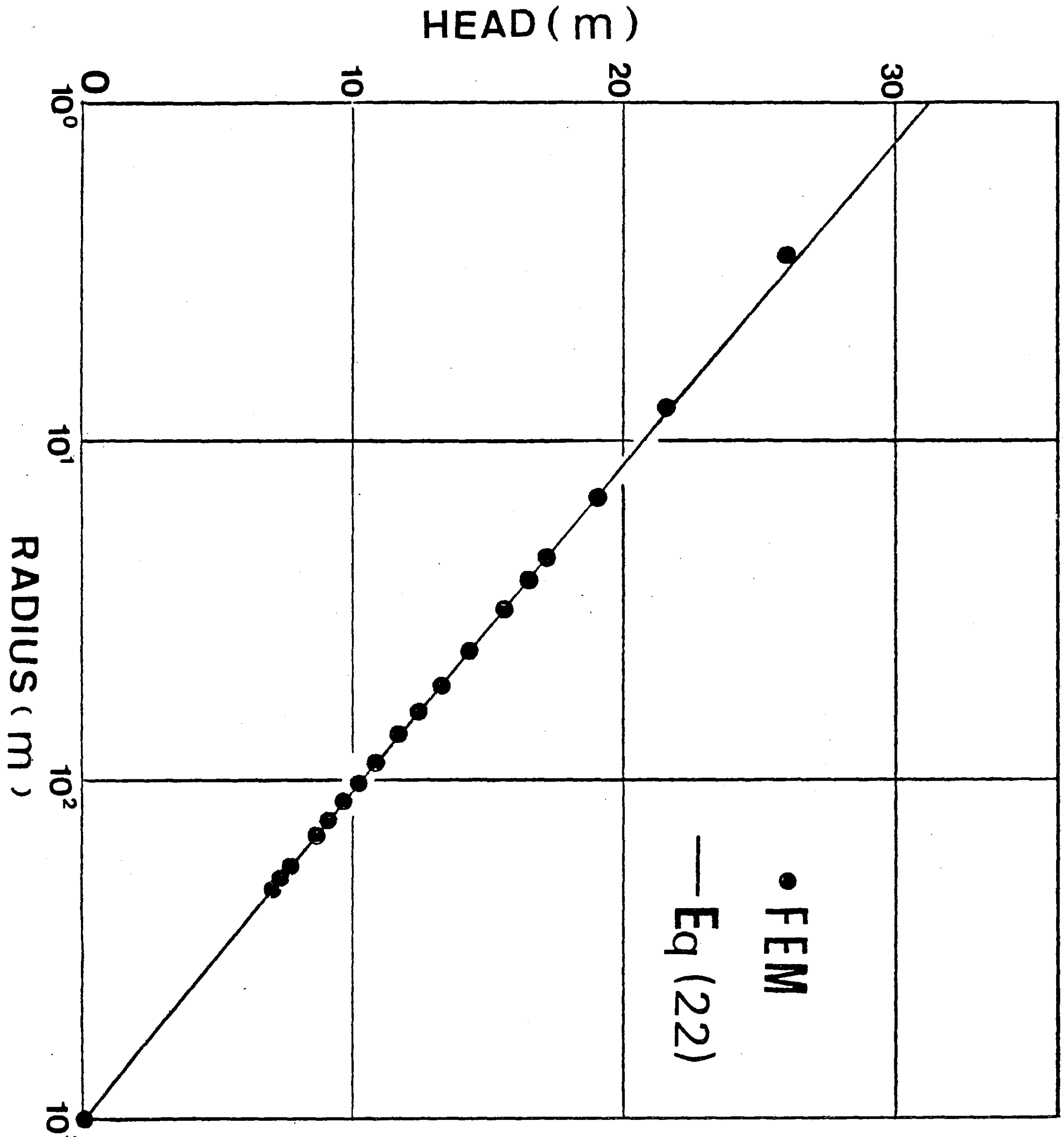

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Fig. 5 Comparison of the FEM solution with the analytical solution.

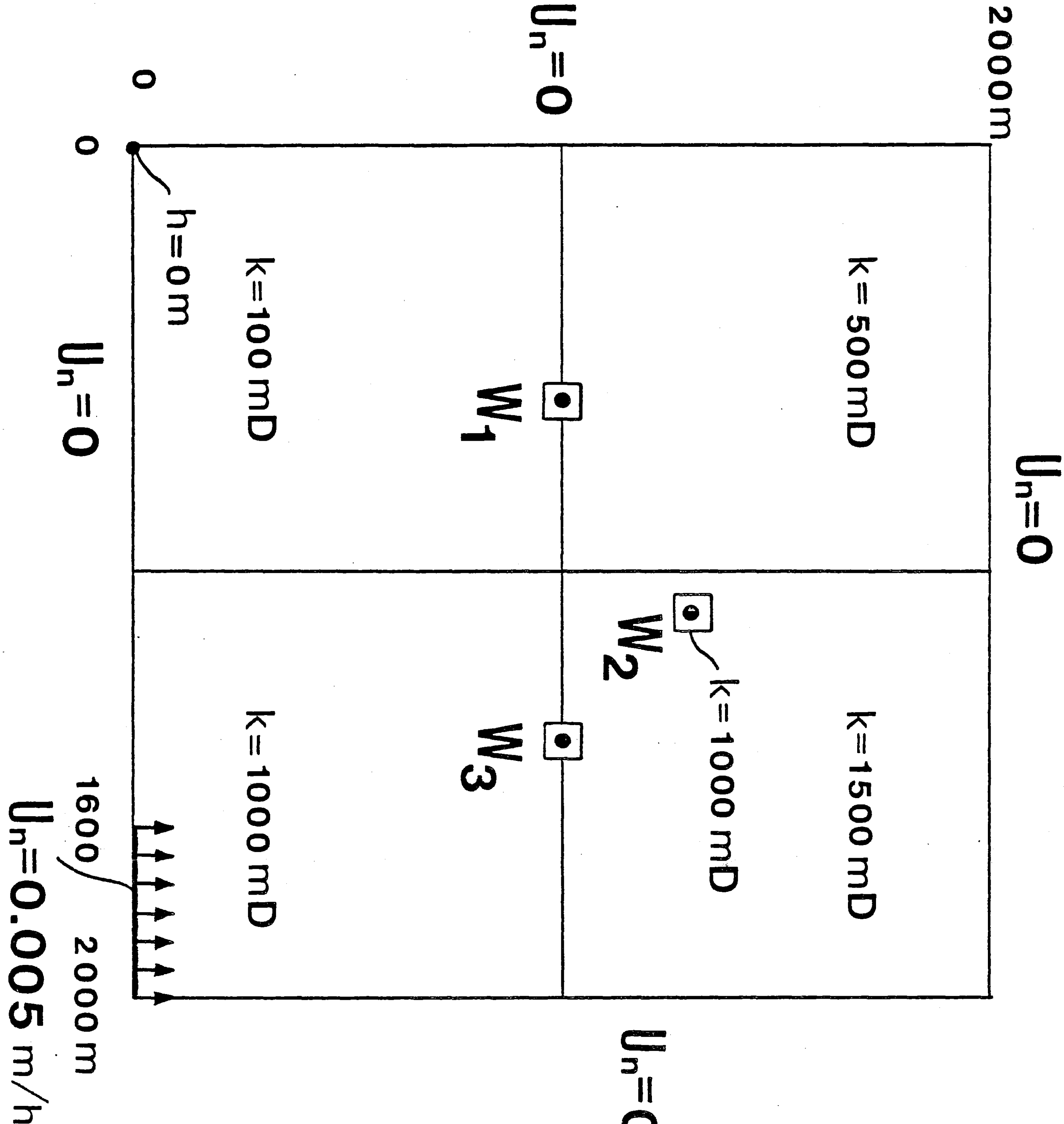

Fig. 6 Parameters of hypothetical reservoir for Problem 2.

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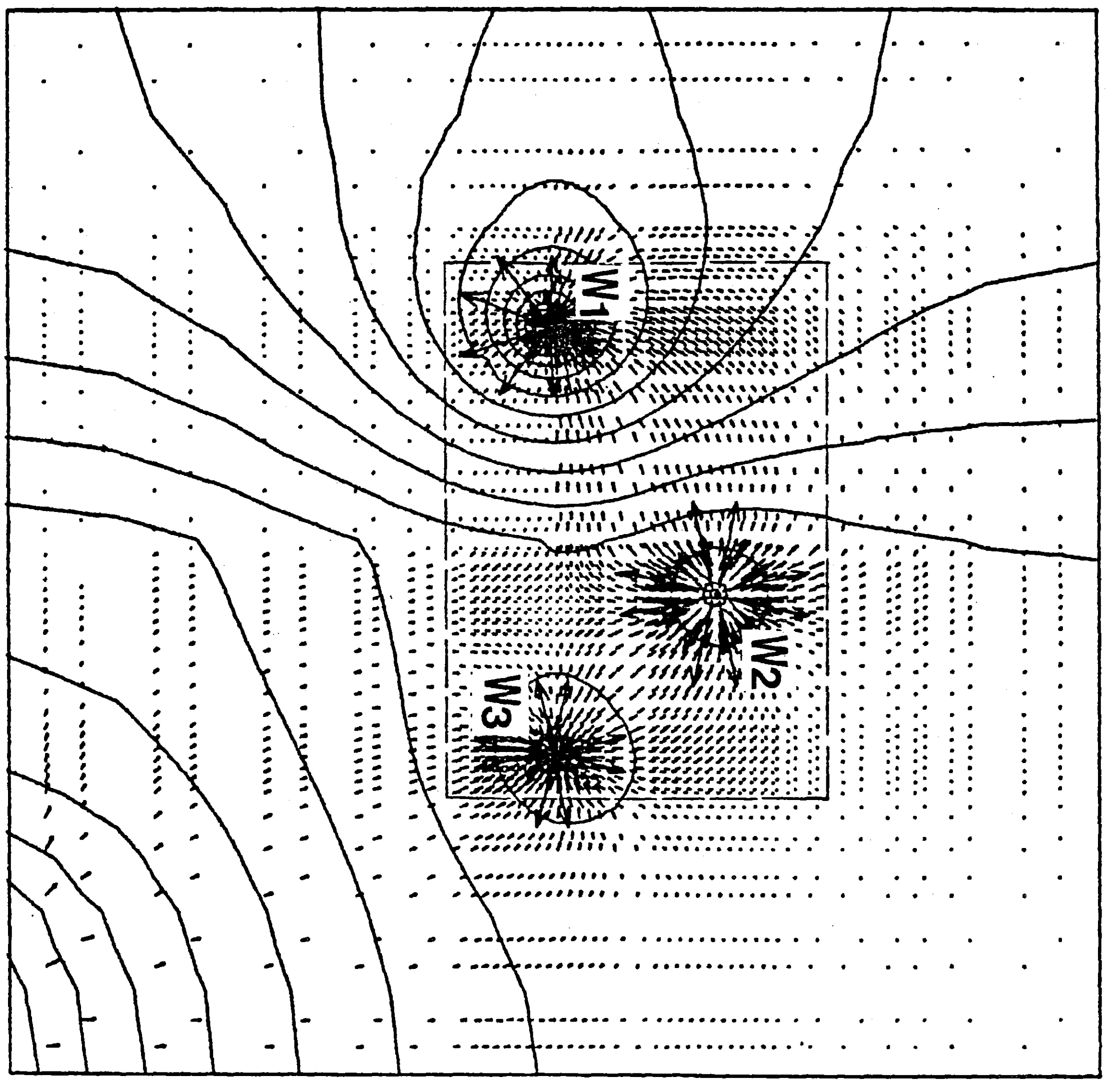

Fig. 7-a Pressure (—) and Darcy velocity vector (→) solution for Problem 2. Portion of this figure is expanded and shown in Fig.

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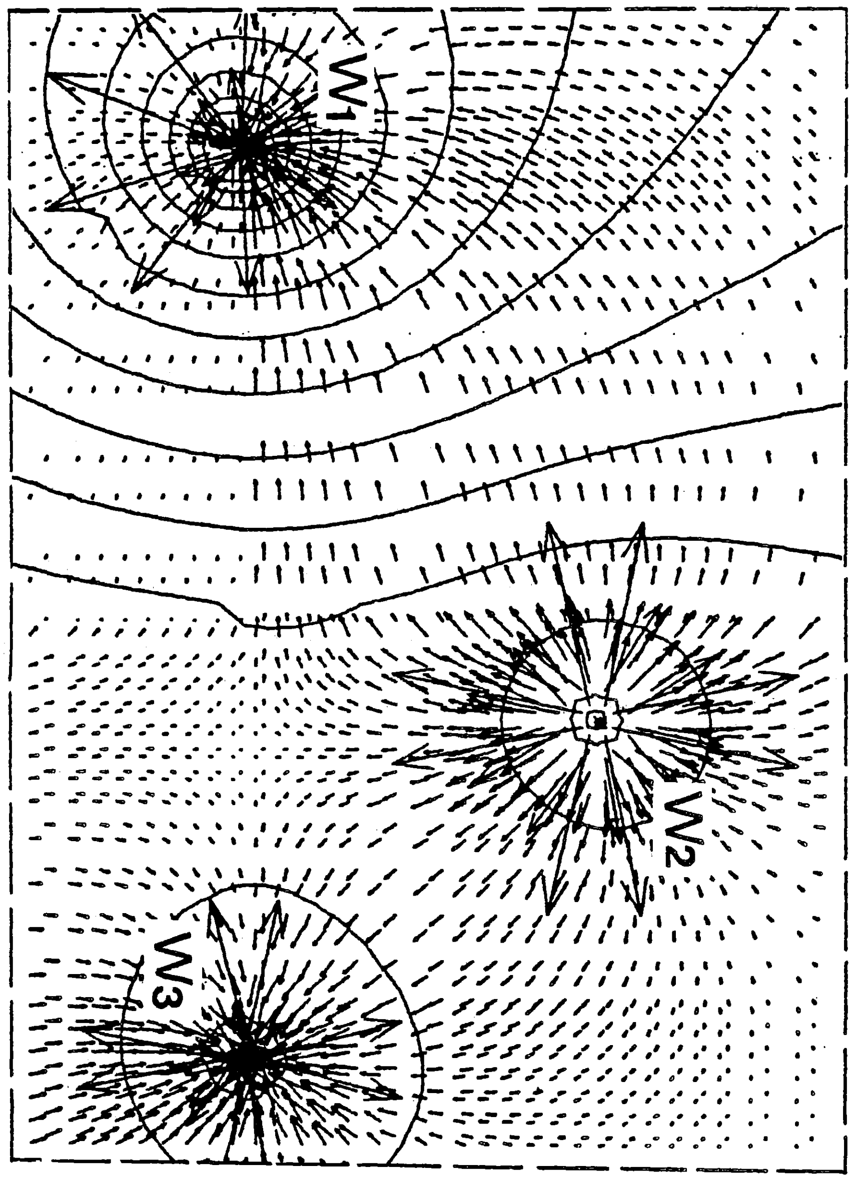

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Fig. 7-b Pressure (——) and Darcy velocity vector (→) solution for Problem 2. Expanded view.

Case No.	Well No.	Given Values		Calculated Values
		h(m)	$q_w(m^3/h)$	$r_w(m)$
Case 1	W1	-150.0	-212.0	0.10058
	W2	100.0	239.7	0.09952
	W3	-50.0	-227.7	0.10023
		$r_w(m)$	$q_w(m^3/h)$	h(m)
Case 2	W1	0.1	-212.0	-150.06
	W2	0.1	239.7	99.95
	W3	0.1	-227.7	-50.02
		h(m)	r(m)	$q_w(m^3/h)$
Case 3	W1	-150.0	0.1	-212.0
	W2	100.0	0.1	239.7
	W3	-50.0	0.1	-227.7

Inflow from reservoir boundary(q_b);

$q_b = 1 \times t \times U_n = 200.0\ m^3/h.$

Volume valance;

$\sum q_w + q_b = 0.0\ m^3/h.$

Table 1 Calculated results using "well model."

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