Modeling Vertical and Horizontal Wells With Voronoi Grid

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Summary. This paper discusses the treatment of wells in a flexible Voronoi grid. A new model problem is proposed to evaluate exact well indices for multiwell configurations and homogeneous reservoirs. A simplified model also is proposed and discussed. New exact and simplified well models for heterogeneous reservoirs are presented.

Introduction

An exact well index can be derived by comparing the solution of the differential equation (obtained analytically or numerically) for a given problem with the numerical solution of the difference equation (exact well model). Peaceman[1] first published this approach, and well models based on it are defined here as Peaceman-type models. In this sense, well models described by Kuniansky and Hillestad,[2] Abou-Kassem and Aziz,[3] and Babu *et al.*[4] also are considered to be Peaceman-type models because, although these authors used different reference solutions, the well models were based on Peaceman's[1] concept.

In this paper, we propose new exact Peaceman-type well models for Voronoi grid and multiwell configurations in homogeneous and heterogeneous reservoirs and propose and discuss a simplified model for this grid. The exact well model can be used easily to investigate the effect of different well configurations (location and rates) over the value of the well index. Because the well index is assumed to be constant during the simulation process, a good grid geometry should result in a constant (within a tolerance) value for each well, regardless of the well configuration. Therefore, this procedure is an additional tool that can be used to select the appropriate grid size for the problem of interest.

Current Well Models

Although van Poolen *et al.*[5]-type models were extensively used in the past, Peaceman[1] showed that this approach is incorrect. For this reason, only Peaceman-type models are considered here.

The relationship between wellblock pressure and bottomhole flowing pressure (BHFP) is a function of fluid rates, rock and fluid properties, and grid geometry:

$$q_P=\left[\frac{\theta kh}{\ln(r_{eq}/r_w)}\right]\frac{k_{rP}}{\mu_P B_P}(p_{P,o}-p_{P,w}) \quad \ldots\ldots (1)$$

and $I_w=\theta kh/\ln(r_{eq}/r_w)$, $\quad\ldots\ldots$ (2)

where I_w=well index, θ=angle open to flow, and r_{eq}=equivalent wellblock radius. Because only single-phase cases are discussed, the subscript P (phase index) will be dropped from all remaining equations. Peaceman[1,6] proposed the following simplified model based on the comparison between numerical and analytical solutions for the total pressure drop in a homogeneous, isotropic, repeated five-spot pattern under steady-state, single-phase flow conditions.

$$r_{eq}=0.140365\sqrt{\Delta x^2+\Delta y^2} \quad \ldots\ldots (3)$$

The conditions that must be satisfied to apply this model safely may be as important as the model itself and are discussed by Peaceman.[7,8]

In his first paper, Peaceman[1] showed that the block pressures increase logarithmically with the radial distance measured from the gridpoint to the well when a regular Cartesian grid with square blocks ($\Delta x=\Delta y$) is used to discretize an isolated one-quarter of a five-spot pattern. The extension of this concept led to the development of Kuniansky and Hillestad's[2] and Abou-Kassem and Aziz's[3] analytical well model. This model does not produce good results for wellblocks with large grid-aspect ratios ($R_y=\Delta y/\Delta x$) and may not produce good results for wells close to reservoir boundaries.[7] However, it does yield good results for isolated wells in blocks with small R_y. This model's main advantage is that it can be applied to any kind of grid geometry and discretization scheme.

Kuniansky and Hillestad[2] and Peaceman[7] proposed exact well models for multiwell configurations. They used different model problems for a reservoir with constant pressure at the external boundaries. In principle, these models can be used to derive well indices for grids of any geometry. However, conventional simulators assume closed boundaries, and the exact representation of constant pressure boundaries in these codes may be very tedious, although it is possible.

Babu *et al.*[4] proposed a well model based on the analytical solution presented by Babu and Odeh[9] for a single well producing at constant and uniform flux from a closed, box-shaped (3D) drainage volume under pseudo-steady-state conditions. The solution is valid for a well that partially penetrates the reservoir length. On the basis of a series of numerical experiments, they also presented a simplified model for regular Cartesian grids. Although their analytical solution is valid for 3D configurations, the actual well model was restricted to 2D (areal or cross-sectional) cases. Further research on fully 3D models is needed to investigate such factors as the effect of boundary conditions at the well (uniform flux, uniform pressure, or mixed boundary conditions) and partial penetration.

All the authors mentioned above have focused their attention on the treatment of wells in a Cartesian grid, which is a special case of the Voronoi grid[10,11,12] discussed in this paper (**Fig. 1**). For this reason, currently used well models were used as a foundation to develop the well models presented here for the Voronoi grid and heterogeneous reservoirs.

Exact Well Model for Homogeneous Reservoirs

By definition, the use of an exact well index in numerical simulation yields the same well pressure, p_w, as that calculated analytically for a given model problem. Because this approach was used first by Peaceman,[1] this condition characterizes the well model as Peaceman-type. A model problem consists of defining the reservoir geometry and its external and internal (well) boundary conditions. The linear, single-phase flow equation then is solved in the proposed domain to obtain the analytical solution for pressure at the well of interest, p_w. The same problem is solved numerically to compute the pressure of the block containing the well, p_o. The equivalent radius, r_{eq}, is evaluated by arranging Eq. 1 as

$$\ln(r_{eq}/r_w)=(\theta kh/qB\mu)(p_o-p_w). \quad \ldots\ldots (4)$$

This approach is also applicable to Voronoi grids because there is no restriction on grid geometry. The results presented in the literature show that different model problems produce similar well indices provided there are "enough" gridblocks between wells (or images).

While the best model problem is the one that is closest to the actual field configuration, its choice should be based on ease of use, flexibility to represent flow geometries and boundary effects, and capability of conventional simulators to model the same problem.

The model problem used in this paper, which consists of a group of wells producing (or injecting) at constant rates in a rectangular

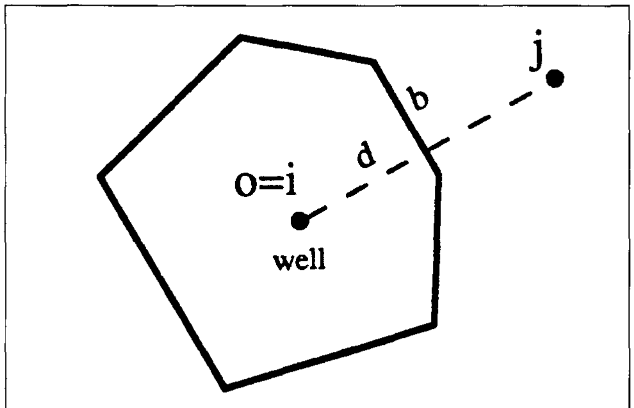

Fig. 1—Voronoi block and its connections.

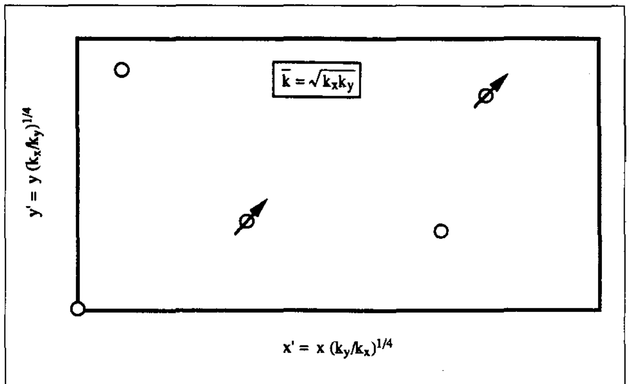

Fig. 2—Model problem, well locations in a rectangular reservoir with closed boundaries.

reservoir with closed boundaries (**Fig. 2**), takes such factors into consideration. The wells are assumed to penetrate the reservoir fully. The solution for the linear problem (single-phase, constant compressibility) at any location and time is given by superposition of the line-source solution (see Appendix A of Ref. 13). The flow equation for anisotropic (but homogeneous) reservoirs can be solved by the appropriate transformation of coordinates.[6]

In the proposed model problem, flow reaches pseudo-steady-state conditions (or steady-state conditions if the sum of flow rates is zero) at long times; therefore, the model problems proposed by Peaceman[1] and Babu *et al.*[4] are special cases of it. Actually, the model problem proposed by Babu *et al.* allows the modeling of a single well that partially penetrates the reservoir (3D problem), which cannot be considered by the model proposed in this paper. However, their model does not allow the analysis of multiwell configurations, which may significantly affect the results.

Unlike constant pressure boundaries,[2,7] closed boundaries are a standard feature of conventional simulators; therefore, the proposed model problem can be simulated easily with conventional codes. Because the analytical solution for pressure can be obtained at any position and time, numerical results can be compared with analytical results at the well under pseudo-steady-state or steady-state conditions and also at any location and time (Example 1 below).

Once the model problem is defined, the procedure to determine r_{eq} is based on the following mathematical formulation (to keep notation simple, the time dependence of pressure and r_{eq} is not explicitly expressed). Initially, Eq. 4 is arranged as

$$\ln\frac{r_{eq}}{r_w}=\frac{\theta kh}{q^*B\mu}(p_o-p_w)=\frac{\theta kh}{q^*B\mu}(\bar{p}-p_w)-\frac{\theta kh}{q^*B\mu}(\bar{p}-p_o), \quad\text{.......... (5)}$$

where the asterisk denotes the well of interest and $\bar{p}$=average reservoir pressure.

The analytical solution for the BHFP is given by (see Appendix A of Ref. 13)

$$\frac{\theta kh}{q^*B\mu}(\bar{p}-p_w)=\frac{\theta}{2\pi}\xi^w_{\text{anal}}, \quad\text{.......... (6)}$$

$$\text{with } \xi^w_{\text{anal}}=\sum_m \frac{q_m}{q^*}p_{Dm}-2\pi\frac{q_t}{q^*}t_{DA}, \quad\text{.......... (7)}$$

where q_m=rate of the mth well; p_{Dm}=dimensionless pressure at the location of the well of interest caused by the mth well and all of its images; q_t=net production rate from the reservoir, which can be positive or negative; $t_{DA}=kt/\phi\mu c_t A$=dimensionless time; and A=reservoir area.

Similarly, a numerical factor, ξ^o_{num}, is defined such that

$$\frac{\theta kh}{q^*B\mu}(\bar{p}-p_o)=\frac{\theta}{2\pi}\xi^o_{\text{num}}, \quad\text{.......... (8)}$$

$$\text{where } \xi^o_{\text{num}}=\frac{2\pi kh}{q^*B\mu}(p_{\text{ini}}-p_o)-2\pi\frac{q_t}{q^*}t_{DA}. \quad\text{.......... (9)}$$

Substituting Eqs. 6 and 8 into Eq. 5 gives

$$\ln(r_{eq}/r_w)=(\theta/2\pi)(\xi^w_{\text{anal}}-\xi^o_{\text{num}}) \quad\text{.......... (10)}$$

$$\text{and } I_w=\frac{\theta kh}{\ln(r_{eq}/r_w)}=\frac{2\pi kh}{(\xi^w_{\text{anal}}-\xi^o_{\text{num}})}. \quad\text{.......... (11)}$$

In summary, determination of the exact well index for a given flow configuration consists of three steps.

1. Determine ξ^w_{anal} (Eq. 7) at any time during pseudo-steady-state or steady-state flow conditions. A simple computer code to evaluate its value using superposition of the line-source solution (Ramey *et al.*[14] and Earlougher[15]) is outlined in Appendix A of Ref. 13. Its evaluation requires very little computer time.
2. Simulate the same linear problem numerically and evaluate ξ^o_{num} (Eq. 8 or 9) at any time during pseudo-steady-state or steady-state flow conditions.
3. Evaluate r_{eq} (Eq. 10) and I_w (Eq. 11).

The exact value of I_w actually can be obtained at any time; however, long-time reservoir predictions should be based on stabilized pseudo-steady-state or steady-state flow conditions.

Note that p_w can be obtained from Eq. 6 and p_o can be obtained numerically. These parameters can be substituted directly into Eq. 4 to obtain r_{eq}. The reason to compute ξ^w_{anal} and ξ^o_{num} is that they remain constant during pseudo-steady-state or steady-state flow conditions; therefore, evaluating them at two different times ensures that flow has stabilized.

Definition of Numerical Skin. By definition, the use of the exact I_w gives a numerical BHFP, $p_{w,\text{num}}$, equal to the analytical BHFP, $p_{w,\text{anal}}$, for a given flow configuration. However, the use of the same value of I_w in a different flow configuration (position and rate of neighboring wells) may result in a $p_{w,\text{num}}$ different from $p_{w,\text{anal}}$ for the new problem. The difference between these two values can be interpreted as being caused by a numerical skin factor, s_{num}, defined in this work as

$$\begin{aligned} s_{\text{num}}&=(\theta kh/q^*B\mu)(p_{w,\text{anal}}-p_{w,\text{num}})\\ &=\frac{\theta kh}{q^*B\mu}(p_o-p_{w,\text{num}})-\frac{\theta kh}{q^*B\mu}(p_o-p_{w,\text{anal}})\\ &=\ln(r_{eq,\text{used}}/r_w)-\ln(r_{eq,\text{anal}}/r_w)\\ &=\ln(r_{eq,\text{used}}/r_{eq,\text{anal}}), \quad\text{.......... (12)}\end{aligned}$$

where $r_{eq,\text{used}}=r_{eq}$ value actually used during the simulation process and $r_{eq,\text{anal}}=r_{eq}$ value for the configuration under study (which is not necessarily the one used to derive $r_{eq,\text{used}}$).

The concept of numerical skin is important in interpreting errors caused by well models (simplified or exact) in a more appropriate form for field applications (see Example B.2 in Appendix B of Ref.

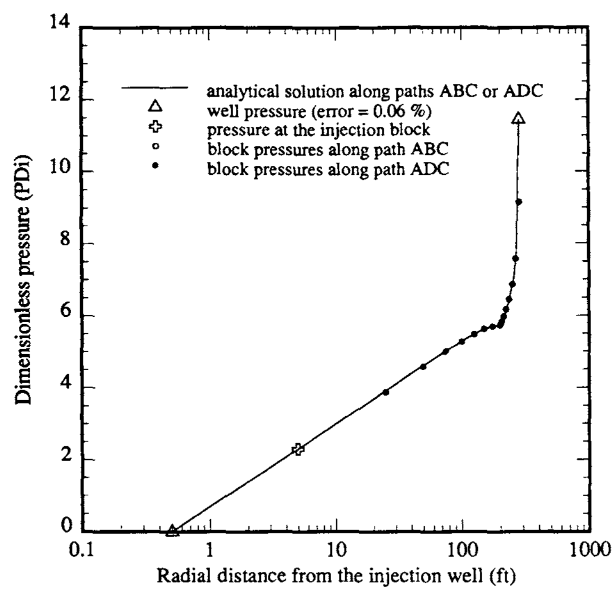

Fig. 3—Pressure values computed along no-flow boundaries of a five-spot pattern with a 9 × 9 grid, Example 1.

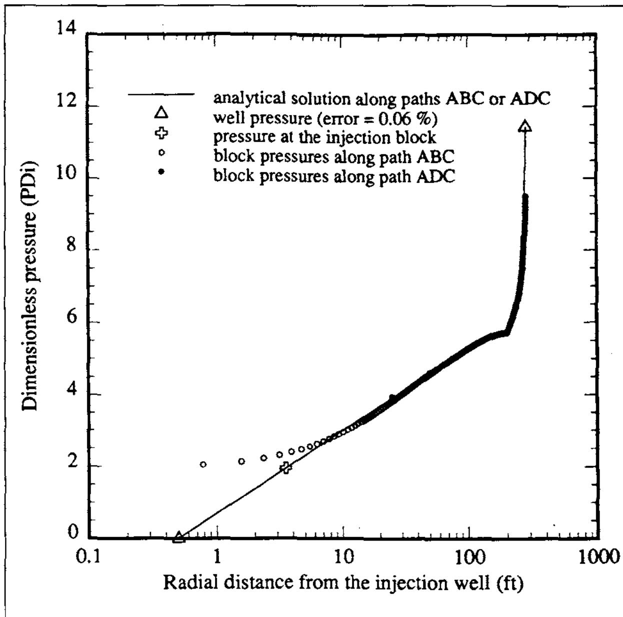

Fig. 4—Pressure values computed along the no-flow boundaries of a five-spot pattern with a 257 × 9 grid, Example 1.

13). For example, a 50% error in r_{eq} corresponds to $s_{num}=0.4$. A reservoir engineer probably can interpret the physical implications of s_{num} more easily than the r_{eq} error.

Example 1. Peaceman's[1] paper on well models analyzed well pressure and also demonstrated that the pressure at the gridblocks around the wells follows the radial solution very closely when square gridblocks are used. Except for Peaceman's paper, works presented in the petroleum literature based their conclusions only on analysis of pressure at wells.

Our example compares numerically calculated and analytical dimensionless pressure along some representative paths in a repeated five-spot pattern under steady-state flow conditions. The corners of one-quarter of the five-spot pattern are labeled as A (bottom left), B (bottom right), C (top right), and D (top left). The injector is in Corner A, and the producer is in Corner C. Segments AB and CD represent the grid x direction, while segments BC and DA represent the grid y direction. The diagonal distance between the injection and the production wells is $200\sqrt{2}$ ft, and $r_w=0.5$ ft. Flow is incompressible, with constant pressure specified at the injector and constant rate specified at the producer. The dimensionless pressure based on the injection pressure is defined as

$$p_{Di}(x,y)=(2\pi kh/qB\mu)[p_{inj}-p(x,y)]. \quad (13)$$

Pressure values along the no-flow boundaries, Paths ABC and ADC, were obtained numerically with Cartesian grids with aspect ratios, $R_y=\Delta y/\Delta x$, of 1 and 32, respectively. Because of the symmetry of the injection pattern, the actual pressures along Paths ABC and ADC are identical.

The number of gridpoints in the y direction (Segment AD) was fixed to 9 points ($\Delta y=200/8=25$ ft), and the number of gridpoints in the x direction (Segment AB) was 9 and 257 for $R_y=1$ and 32, respectively. For both grid geometries, I_w was evaluated with Peaceman's[1,6] model (Eq. 3).

Fig. 3 shows the analytical and numerical solutions for pressure at each gridpoint along Paths ABC and ADC for the symmetric grid with $R_y=1$. Pressure is plotted as a function of the distance from the gridpoint to the center of the injection well (radial distance from the injection well). Well pressures (triangles in Fig. 3) and pressure at the injection block (at Distance r_{eq}) also are plotted. The total error in Fig. 3 is given as a percentage of the total pressure drop between wells:

$$E=\frac{|\Delta p_{anal}-\Delta p_{num}|100\%}{\Delta p_{anal}}. \quad (14)$$

For small R_y (Fig. 3), the numerical solution agrees well with the analytical solution along both paths. However, for larger R_y (**Fig. 4**), the numerical results clearly differ from the analytical results at gridpoints close to the well along Path ABC (along the smaller gridblock dimension). On the basis of results obtained with intermediate values of R_y (not presented here), we concluded that the discrepancy increases as the grid is refined. However, Peaceman's[1] model always provides the required "resistance," so that the total error is almost zero for all situations, which, in his words, can be considered "truly startling."

Note that the discrepancy between the analytical and numerical solution at gridpoints adjacent to the wellblock is not caused by I_w because I_w does not affect the difference between pressures at adjacent gridblocks. Instead, the discrepancy is the result of using the assumption of linear flow to derive geometric factors for grid connections in Cartesian systems even in areas of predominantly radial flow, such as those around wells.

This example shows that numerical results obtained with large R_y do not agree well with analytical solutions at gridpoints close to the wellblock.

Simplified Well Model for Homogeneous Reservoirs

Exact I_w can be evaluated by comparing analytical and numerical results; however, this may require significant effort. For this reason, simplified models must allow quick determination of r_{eq}. The main advantage of a simplified model is that r_{eq} can be computed directly by the simulator with no human time required.

The most frequently used simplified model is Peaceman's[6] (Eq. 3), which is valid for regular Cartesian grids. The Voronoi grid has a more complex geometry; therefore, in general, the original Peaceman model cannot be used for this grid. In contrast, the so-called analytical well model can be applied to grids of any geometry, including the Voronoi grid. For this reason, we chose it as the simplified model. This model is also a Peaceman-type model because he first proposed it. The model was extended by Kuniansky and Hillestad[2] and generalized by Abou-Kassem and Aziz.[3]

The formulation described in Eqs. 15 through 17 is based on the work of Abu-Kassem and Aziz.[3] The main assumption of this model is that the pressure at gridblocks adjacent to the wellblock can be evaluated by the radial-flow equation under steady-state conditions. For a single well in a Voronoi block (Fig. 1), this assumption implies that

$$p_j-p_w=[qB\mu\ \ln(L_{ij}/r_w)]/\theta kh, \quad (15)$$

where p_j =pressure of Gridblock j adjacent to Wellblock i (Fig. 1), L_{ij} =distance between the gridpoints, and θ=angle open to flow. Because the block pressures are insensitive to the well position inside the gridblock, in this paper, flow is assumed to be radial around the gridpoint and not around the well location.

The discretized form of the single-phase, incompressible flow equation for Wellblock i is[10,11]

$$q=\sum_j \frac{T_{ij}}{B\mu}(p_j-p_o)=\sum_j \frac{T_{ij}}{B\mu}(p_j-p_w)-\sum_j \frac{T_{ij}}{B\mu}(p_o-p_w), \quad \text{(16)}$$

where T_{ij} =geometric factor for Connection ij. Combining Eqs. 16, 15, and 1 and solving for r_{eq} give

$$\ln r_{eq}=\left(\sum_j T_{ij}\ln L_{ij}-\theta kh\right)\Big/\sum_j T_{ij}. \quad \text{(17)}$$

This simplified well model can be implemented easily in the simulator code and forms the basis for the development of the well model for heterogeneous reservoirs described in the next section.

Fung *et al.*[16] proposed an expression similar to Eq. 17 for Voronoi grids and control-volume finite-element (CVFE) formulations (which is actually identical to the control-volume finite difference in this case). However, they presented no numerical results and stated no model limitations.

Eq. 17 is valid for an isolated well (i.e., a well that is far from reservoir boundaries or other wells). It also can be used for a well at a reservoir boundary when point-distributed formulation[17] is used to discretize the flow equations, as in a well in a repeated pattern. However, Peaceman[7] showed that this simplified model does not yield good results for wellblocks with large R_y (>2).

Some results obtained with this model were presented previously.[11] Additional numerical experiments[12] demonstrated that this model yields very good results under the restrictions described previously.

Palagi and Aziz[18] proposed a more general simplified model that is valid even for flexible grids with a large aspect ratio ($R_y=L_{max}/L_{min}>2$, where L is shown in Fig. 1). The model uses only the connection between the wellblock and its farthest neighbor:

$$r_{eq}=L_{j*}\exp(-\theta_{j*}L_{j*}/b_{j*}), \quad \text{(18)}$$

where $j*$=farthest wellblock neighbor.

Exact Well Model for Heterogeneous Reservoirs

Although well models for idealized homogeneous reservoirs have been discussed extensively in the petroleum literature, they have not been extended to heterogeneous reservoirs. In this paper, the use of Voronoi grid, coupled with the gridding technique described by Palagi and Aziz,[11] is proposed to determine exact I_w for wells in heterogeneous reservoirs (variable permeability, thickness, depth, and porosity). The model problem is similar to the one used for homogeneous cases (Fig. 2), except that the reference solution is obtained from a fine-grid simulation because, in general, no analytical solution exists for heterogeneous reservoirs. We propose the following procedure.

1. Arrange the well equation as

$$I_w=1/(\xi^w_{\text{fine}}-\xi^o_{\text{coarse}}), \quad \text{(19)}$$

where $\xi^w_{\text{fine}}=(\bar{p}-p_w)/q^*\mu B$ (20)

and $\xi^o_{\text{coarse}}=(\bar{p}-p_o)/q^*\mu B$. (21)

2. Perform a fine-grid simulation to determine the "exact" (reference) value of ξ^w_{fine} ("exact" is between quotation marks because even the fine-grid solution has some small errors). The fine grid should have very small symmetrically distributed gridblocks around the wells of interest, so that the analytical well model (Eq. 17), which is also simplified, can be used with confidence in the fine grid. We propose that "cylindrical" modules[11] be used around the wells of interest to generate accurate reference solutions.

3. Simulate the same configuration with the coarse grid of interest and obtain the parameter ξ^o_{coarse}.

4. Compute I_w with Eq. 19. Note that I_w, not r_{eq}, is obtained because the permeability/thickness product, kh, may actually vary inside the coarse wellblock.

Because this process is based on material-balance considerations, the reservoir volume calculated by the coarse grid should be equal to the one calculated by the fine grid.

Simplified Well Model for Heterogeneous Reservoirs

The simplified well model presented here is based on the assumption of steady-state radial flow in two concentric regions around a well with different properties. Although variable thickness also can be considered, the proposed model focuses on variable permeability only. Two values of average permeabilities are used: $\bar{k}_{wo}$= average permeability between the well and the wellblock (i.e., between r_w and r_{eq}) and $\bar{k}_{\text{conn}}$=average permeability between the wellblock and its neighbors (i.e., between r_{eq} and L_{ij}). Therefore,

$$I_w=\theta\bar{k}_{wo}h/\ln(r_{eq}/r_w), \quad \text{(22)}$$

where $\bar{k}_{wo}$ is evaluated at the well location in this work.

The assumption of radial flow between r_{eq} and L_{ij} yields

$$p_j-p_o=[qB\mu\ln(L_{ij}/r_{eq})]/\theta\bar{k}_{\text{conn}}h. \quad \text{(23)}$$

Substituting Eq. 23 into Eq. 16 and solving for r_{eq} give

$$\ln r_{eq}=\left(\sum_j T_{ij}\ln L_{ij}-\theta\bar{k}_{\text{conn}}h\right)\Big/\sum_j T_{ij}, \quad \text{(24)}$$

where $\theta\bar{k}_{\text{conn}}=\Sigma_j\theta_j\bar{k}_j$, θ_j =angle of Connection ij, $\bar{k}_j$ =harmonic average of the permeability values evaluated at points logarithmically distributed between r^*_{eq} and Gridpoint j (**Fig. 5**), and r^*_{eq}= estimated value of r_{eq} used only to determine the position of the points where permeability values are evaluated at each connection. Ref. 12 shows that, for regular polygons and homogeneous reservoirs, r^*_{eq} varies between 0.135 and $0.208L_{ij}$. Because it does not need to be exact, it is set to $r^*_{eq}=0.2L_{ij}$ (for each connection) in this work (Fig. 5). Note that $\bar{k}_{\text{conn}}$ and consequently r_{eq} (Eq. 24) are not functions of well location inside the wellblock, whereas $\bar{k}_{wo}$ and consequently I_w (Eq. 22) are.

Example 2. The example hypothetical heterogeneous reservoir is based on a modification of the permeability values of the Berea data set described by Giordano *et al.*[19] Journel[20] provides some statistics of the data set as well as a grey-scale representation. The data set consists of 1,600 (40×40) permeability value measurements in a 2×2-ft slab of Berea sandstone. To increase the data variability and to investigate a field-scale situation, we introduced the following modifications: the permeability values were modified ($k_{\text{new}}=k^3_{\text{old}}/1000$) and the distribution of points was scaled to represent a 200×200-ft geometry. **Fig. 6** shows a grey-scale representation of the modified values, and **Table 1** summarizes the other parameters used.

The purpose of this example is to evaluate "exact" and simplified I_w for wells at the corners ($\theta=\pi/2$) of this hypothetical reservoir. Initially, the fine grid depicted in Fig. 6 was constructed so that most gridblocks were homogeneous. Also, a fine "cylindrical" module with 8 gridpoints in the angular direction and 10 gridpoints in the radial direction (8×10) was used around the corner of interest. The wellblock and its neighbors are homogeneous; therefore, Eq. 17 can be used to compute r_{eq} in the fine grid. For each well location, a single-well (with constant rate) configuration was used to determine the "exact" ξ^w_{fine} value under pseudo-steady-state flow conditions. Fig. 6 shows the fine grid (40×40 plus 8×10) used to obtain the "exact" solution for Well A (bottom left).

The "exact" I_w can be computed for any kind of coarse grid. In this case, we selected a 10×11 hexagonal grid. The "exact" and simplified well models were used to evaluate I_w for wells at the corners of the reservoir geometry (Wells A, B, C, and D). All results were computed for the same time (6 days).

The most important parameter under consideration is the error in the difference, $\bar{p}-p_w$, caused by the use of the simplified well

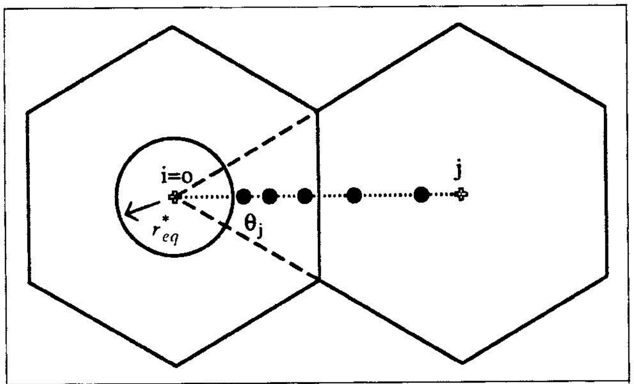

Fig. 5—Evaluation of average permeability at Connection *ij*.

model. In this case, error is 3.5%, 1.3%, 3.5%, and 1.7% for Wells A, B, C, and D, respectively.

Figs. 7 and 8 show the block and well pressures as functions of radial distance from Well A. The "exact" (reference) values obtained with the fine grid also are shown in both figures. Fig. 7 shows the well pressure computed with the coarse grid with the "exact" well model, and Fig. 8 shows the one computed with the same grid with the simplified well model. Note that the only difference between these two figures is the well pressure obtained with the coarse grid because the block pressures are insensitive to I_w value.

By definition, the "exact" I_w is such that the well pressure computed by the coarse grid is equal to the one computed by the fine grid. The "exact" r_{eq} was obtained graphically by matching fine- and coarse-grid results. Because r_{eq} is an arbitrary parameter in this case, the wellblock pressure can be "slid" in the horizontal direction in Fig. 7 until agreement is reached. Each position of r_{eq} represents a different $\bar{k}$ value around the well, which is a function of the slope of the dotted line; however, I_w remains the same. Note that r_{eq} does not need to be determined because the actual parameter of interest is I_w, which is determined directly with Eq. 19.

Discussion

The simplified well model proposed in this paper for flexible grids is based on the assumption of radial flow around the gridpoint. Because boundary effects are not considered, this simplified model should *not* be used for wells that are close to reservoir boundaries or for several wells completed in blocks close to each other. For those situations, the exact well model described in this paper should be used.

Simplified well models are valid only under the assumptions used for their derivation. Actually, the rule is the violation of some of the assumptions and the exception is the validity of all of them for field cases. Although we do not give simple solutions for all situations in this paper, we suggest the following guidelines.

1. The key to an accurate I_w is robustness; i.e., I_w should be independent (within a tolerance) of flow configuration. If this condition is not satisfied, the only probable solution is grid refinement. I_w should not be interpreted as a magic number that can produce the correct pressure even for coarse grids.

2. A useful simplified well model should be implementable in a simulation code and should require very little input from the user (only r_w and s).

3. If doubts exist about the validity of the available simplified well model, use of an exact well model like the one described that can represent a variety of well configurations probably would be better. Use of an exact well model appears to be more accurate and simpler for wells close to reservoir boundaries or to other wells than Peaceman's[7] correction factors. The exact well model may

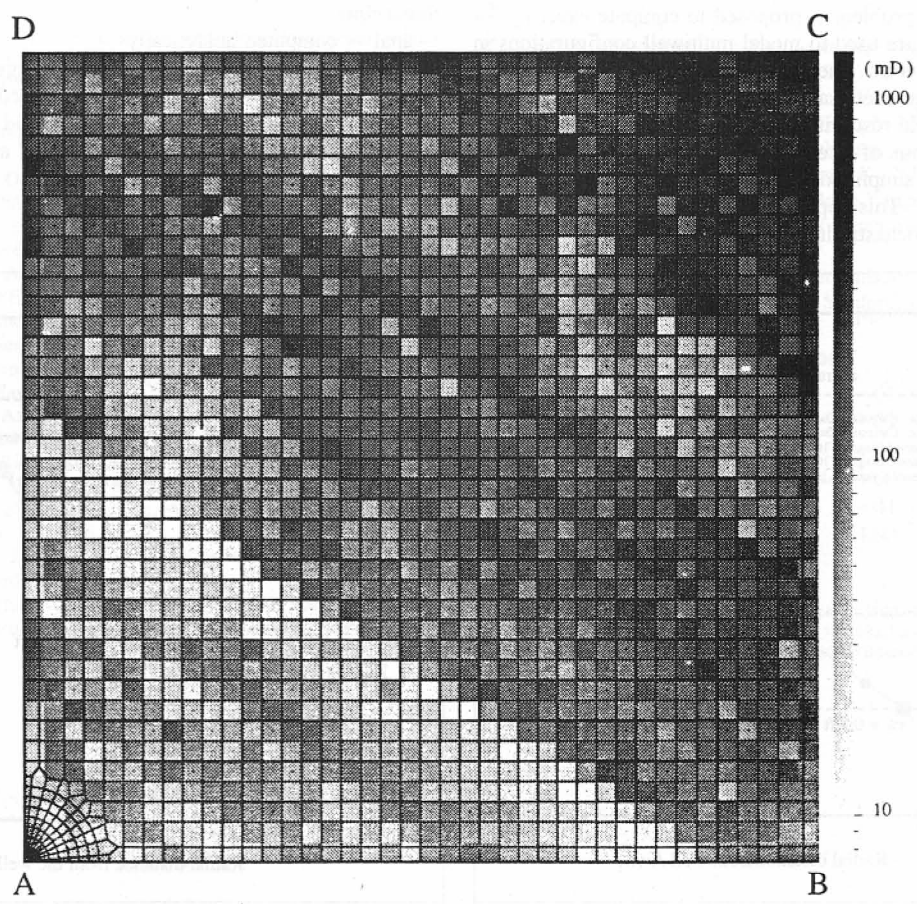

Fig. 6—Fine hybrid-Cartesian grid and modified values of the Berea data set, Example 2.

TABLE 1—ROCK AND FLUID PROPERTIES FOR EXAMPLE 2

q_w, B/D	1
p_{ini}, psi	1,000.6
r_w, ft	0.5
ϕ^o	0.25
h, ft	1
c_r, psi^{-1}	4×10^{-6}
c_w, psi^{-1}	0
B_w, bbl/STB	1
μ_w, cp	1
S_w	1

require additional computer time (usually seconds), but it enhances the solution accuracy. Of course, any model problem with a known analytical solution can be used as a reference solution. For example, Babu *et al.*[4] proposed a very useful model problem for a single well that partially penetrates the reservoir.

In heterogeneous reservoirs, $\bar{p}$ and p_w can be obtained directly from field measurements. Provided the remaining variables (e.g., fluid saturations and relative permeabilities) can be properly simulated, exact I_w can be obtained by requiring the numerical results to match the field observations. If suitable data are available, this is the most accurate way to determine I_w because it reflects the actual reservoir conditions. However, the robustness of this parameter can be determined only by comparing the results produced by the grid used to solve the field problem with accurate solutions obtained either numerically or analytically for different flow configurations.

Conclusions

1. Cartesian grids with large R_y do not model pressure behavior properly in regions close to wells; therefore, large R_y should be avoided.
2. A new model problem is proposed to compute exact I_w for Voronoi grids that are used to model multiwell configurations in homogeneous reservoirs. The model can be used to investigate I_w sensitivity to grid geometry and well locations and rates. A good grid geometry should result in robust (approximately constant) I_w for the configurations of interest.
3. New exact and simplified well models for heterogeneous reservoirs are presented. This approach can be used, for example, to simulate flow in stochastic images of permeability.

Nomenclature

A = area of rectangular reservoir, L^2, ft
b = width of connection between gridblocks, L, ft
B = phase FVF, dimensionless, bbl/STB
c = compressibility, Lt^2/m, psi^{-1}
E = total pressure drop error
h = thickness, L, ft
I_w = well index
k = permeability, L^2, md
k_r = relative permeability
L = distance between gridpoints, L, ft
p = pressure, m/Lt^2, psi
p_D = dimensionless pressure
Δp = pressure difference between injector and producer, m/Lt^2, psi
q = well rate, L^3/t, B/D
r_{eq} = equivalent wellblock radius, L, ft
r_w = well radius, L, ft
R_y = grid-aspect ratio ($\Delta y/\Delta x$ or $\Delta x/\Delta y$)
s = skin effect
S = phase saturation, dimensionless
t_{DA} = dimensionless time based on reservoir area
T = transmissibility coefficient, dimensionless
x = distance in x direction, L, ft
x' = equivalent distance in x direction, L, ft
$\Delta x, \Delta y$ = Cartesian system gridblock dimensions, L, ft
y = distance in y direction, L, ft
y' = equivalent distance in y direction, L, ft
θ = angle open to flow around well, degrees
μ = viscosity, m/Lt, cp
ξ = dimensionless pressure
ϕ = porosity, dimensionless

Subscripts

anal = computed analytically
coarse = computed with relatively coarse grid
conn = average over all wellbock connections
fine = computed with relatively fine grid
ij = connection between Gridblocks i and j
ini = initial conditions
inj = injection well
max = maximum parameter value

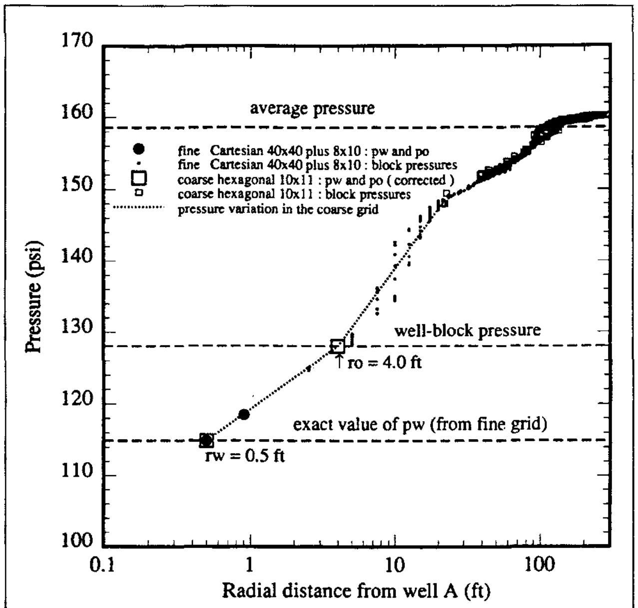

Fig. 7—Pressure values in a heterogeneous reservoir computed with the exact well model, Example 2.

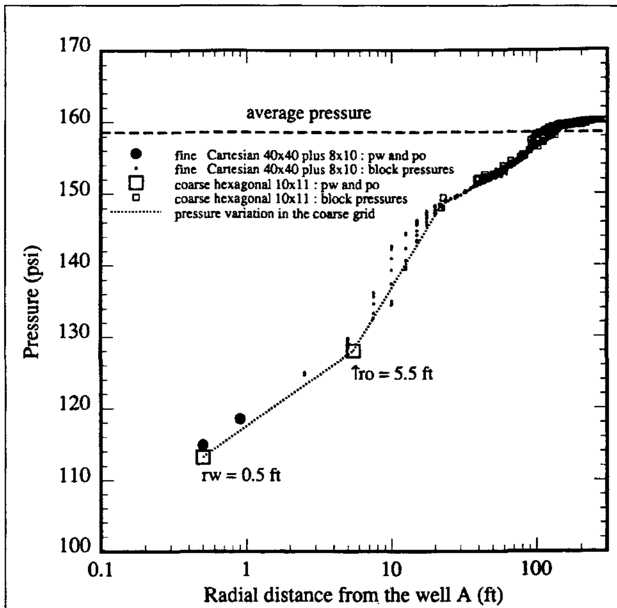

Fig. 8—Pressure values in a heterogeneous reservoir computed with the simplified well model, Example 2.

min = minimum parameter value
o = wellbock
P = phase
r = rock
t = total
used = actually used during simulation
w = bottomhole conditions at the well
W = water

Superscripts

o = wellblock
w = well
$^{-}$ = average
$*$ = well of interest

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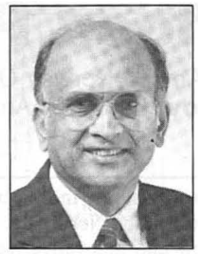

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SI Metric Conversion Factors

bbl	×	1.589 873	E−01	=	m^3
cp	×	1.0*	E−03	=	Pa·s
ft	×	3.048*	E−01	=	m
in.	×	2.54*	E+00	=	cm
md	×	9.869 233	E−04	=	μm^2
psi	×	6.894 757	E+00	=	kPa

*Conversion factor is exact.

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