# Interpretation of Well-Block Pressures in Numerical Reservoir Simulation

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## ABSTRACT

*Examination of grid pressures obtained in the numerical simulation of single-phase flow into a single well shows that the well-block pressure is essentially equal to the actual flowing pressure at a radius of 0.2 $\Delta x$. Using the equation for steady-state .radial flow then allows calculation of the flowing bottom-hole pressure.*

*The relation between pressures measured in a buildup test and the simulator well-block pressure is derived. In particular, the buildup pressure and the well-block pressure are shown equal at a shut-in time of 67.5 $\phi\mu c_t \Delta x^2/k$. This is about one-third the shut-in time stated by previous authors, who derived their results from an erroneous assumption concerning the significance of the well-block pressure.*

*When only a single buildup pressure is observed at a different shut-in time, an adjustment to the observed pressure can be made for matching with the simulator well-block pressure.*

## INTRODUCTION

When modeling reservoir behavior by numerical methods, inevitably the horizontal dimensions of any grid block containing a well are much larger than the wellbore radius of that well. It long has been recognized that the pressure calculated for a well block will be greatly different from the flowing bottom-hole pressure of the modeled well, but the literature contains few specific guides as to how to make the correction.

In this study, we confine our attention to single-phase flow in two dimensions. Consider the five blocks abstracted from a regular grid system (Fig. 1) with the center block containing a well producing at rate $q$. Schwabe and Brand[1] proposed the relationship

$$q = \frac{2\pi kh}{\mu} \; \frac{p_e - p_{wf}}{\ln(r_e/r_w) + s}, \quad \ldots \ldots (1)$$

where $r_e$ is taken equal to $\Delta x$, and $p_e$ is an effective pressure at the "drainage radius," $r_e$, obtained from

$$p_e = p_o + F_i \sum_{i=1}^{4} (p_i - p_o).$$

Schwabe and Brand did not define $F_i$, but seemed to imply that it be taken as zero. Thus, in the absence of a skin effect, Eq. 1 reduces to

$$q = \frac{2\pi kh}{\mu} \; \frac{p_o - p_{wf}}{\ln(\Delta x/r_w)} \quad \ldots \ldots \ldots (2)$$

The most significant treatment of this subject until now was that of van Poollen *et al.*[2] They stated that the calculated pressure for a well block should be the *areal average* pressure in the portion of the reservoir represented by the block. Assuming steady state, the pressure distribution

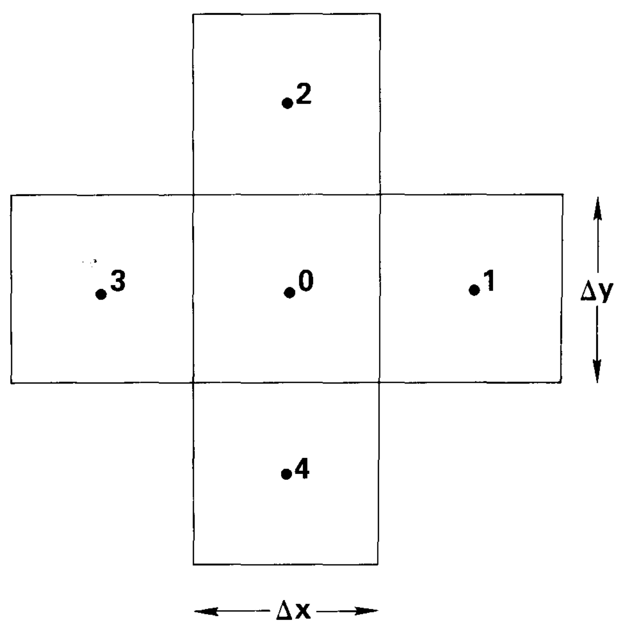


FIG. 1 — BLOCK 0 CONTAINING A WELL AND ITS FOUR NEIGHBORING BLOCKS.

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about a well is given by

$$p = p_{wf} + \frac{q\mu}{2\pi kh} \ln \frac{r}{r_w} . \quad \ldots\ldots\ldots (3)$$

Integration over a circle with an area equal to that of the block (assuming negligible $r_w$) gives, for the average pressure,

$$\bar{p} = p_{wf} + \frac{q\mu}{2\pi kh}\left[\ln \frac{r_b}{r_w} - \frac{1}{2}\right], \quad \ldots\ldots (4)$$

where

$$\pi r_b^2 = \Delta x \cdot \Delta y .$$

Let $\Delta x = \Delta y$. Then,

$$r_b = \Delta x/\sqrt{\pi} \quad \ldots\ldots\ldots\ldots\ldots (5)$$

Assuming $\bar{p}$ equal to $p_o$, in accordance with the basic assumption of van Poollen *et al.* that the well-block pressure is the same as the areal average pressure, then

$$q = \frac{2\pi kh}{\mu} \frac{p_o - p_{wf}}{\ln \frac{\Delta x}{r_w\sqrt{\pi}} - \frac{1}{2}} \quad \ldots\ldots (6)$$

Coats *et al.*[3] used a productivity index to relate the block pressure to wellbore flowing pressure in their steam simulator. For single-phase flow, an equivalent relation would be

$$q = PI\ (p_o - p_{wf})/\mu . \quad \ldots\ldots\ldots (7)$$

Their specification for PI (neglecting the dimensional constants that they used) was

$$PI = \frac{2\pi kh}{\ln \frac{\sqrt{\Delta x \Delta y/\pi}}{r_w} - \frac{1}{2}} . \quad \ldots\ldots\ldots (8)$$

Since Eqs. 7 and 8 combine to give Eq. 6, we see that the Coats *et al.* approach also is equivalent to assuming that the block pressure equals the areal average pressure.

When deriving the finite-difference equations for transient flow, which come from material balances written for each block, it is natural to regard the accumulation term, $\Delta p_{ij}/\Delta t$, as the change in the average pressure for the block $(i, j)$. This leads to the association of $p_{ij}$ with the average pressure of the block. And, indeed, for the majority of blocks, those which contain no wells, such an association is appropriate. For a block containing a well, however, we show that the block pressure is not an average pressure. This study presents the proper interpretation of the well-block pressure, and shows how it relates to the flowing bottom-hole pressure.

Finally, we show how the buildup pressures measured in a shut-in test may be related to the simulator well-block pressure. While the buildup analysis is similar to that of van Poollen *et al.*,[2] the result is significantly different because of the new interpretation of well-block pressure.

## EQUIVALENT RADIUS OF A WELL BLOCK

It is convenient to associate an equivalent radius, $r_o$, with the well block. This is the radius at which the steady-state flowing pressure for the actual well is equal to the numerically calculated pressure for the well block. This definition for $r_o$ gives

$$p = p_o + \frac{q\mu}{2\pi kh} \ln \frac{r}{r_o} , \quad \ldots\ldots\ldots (9)$$

and

$$q = \frac{2\pi kh}{\mu} \frac{p_o - p_{wf}}{\ln(r_o/r_w)} . \quad \ldots\ldots\ldots (10)$$

## EVALUATION OF EQUIVALENT RADIUS OF WELL BLOCK

By examining the numerical solution for Laplace's equation near a single well, we can get the correct value for $r_o$. This is done by solving for the steady-state pressure distribution in a repeated five-spot pattern using the $10 \times 10$ square grid shown in Fig. 2. Details of the calculation are given in Appendix A. The numerical solution for the various blocks is plotted as a function of radius in Fig. 3. On this semilog plot, a straight line with slope $\frac{1}{2}\pi$ fits very well through all the points up to a radius of 6 $\Delta x$. Similar runs were made for larger grids ($20 \times 20$ and $30 \times 30$) and the results are practically identical.

It is probably not a new observation that the numerical solution near a single well behaves so much like the solution to the radial flow equation. What appears to have escaped notice is the

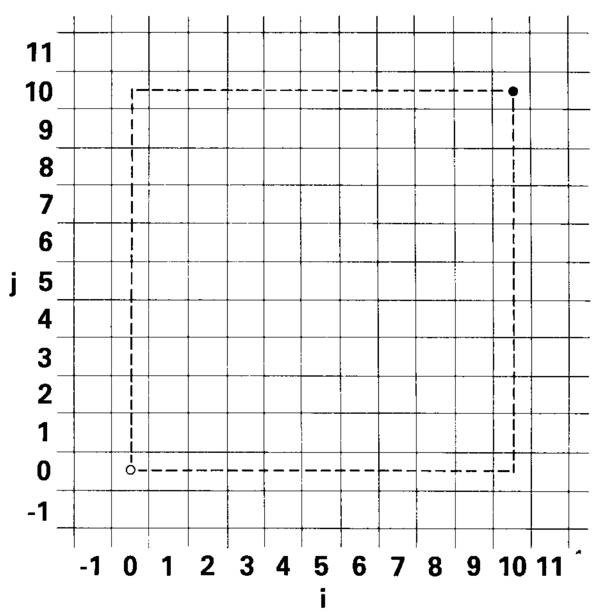


FIG. 2 — 10 × 10 COMPUTING GRID FOR REPEATED FIVE-SPOT PATTERN.

significance of extrapolating the straight line of Fig. 3 to the horizontal line $p - p_o = 0$. From Eq. 9, it follows that this intersection occurs where $r = r_o$, so we have the important result from Fig. 3 that

$$r_o = 0.2\ \Delta x \cdot \qquad (11)$$

This, then, is the new interpretation for well-block pressure; it equals the steady-state flowing pressure at a radius of 0.2 $\Delta x$.

This result contrasts with that obtained by assuming that the well-block pressure equals the areal average pressure. Comparison of Eq. 6 with Eq. 10 gives

$$\ln \frac{\Delta x}{r_w \sqrt{\pi}} - \frac{1}{2} = \ln \frac{r_o^A}{r_w},$$

or

$$r_o^A = \frac{\Delta x}{\sqrt{\pi}} \exp(-1/2) = 0.342\ \Delta x \cdot \cdot \qquad (12)$$

Thus, that assumption leads to the result that the well-block pressure is equal to the flowing pressure at a radius of 0.342 $\Delta x$. This ratio of $r_o^A/\Delta x$ also is shown on the $p - p_o = 0$ line of Fig. 3. The deviation from the true intercept is quite apparent.

## APPROXIMATE CALCULATION OF EQUIVALENT RADIUS

Another observation whose significance appears to have escaped notice is that the pressure calculated for the block adjacent to the well block (that is, $p_{1,o}$ in Fig. 2, or $p_1$ in Fig. 1) is practically on the straight line. Thus, we can substitute $r = \Delta x$ into Eq. 9 to obtain

$$p_1 = p_o + \frac{q\mu}{2\pi kh} \ln \frac{\Delta x}{r_o} . \qquad (13)$$

In addition, we have available the difference equation for the well block (see Appendix A) of

$$(kh/\mu)(p_1 + p_2 + p_3 + p_4 - 4p_o) = q \cdot \qquad (14)$$

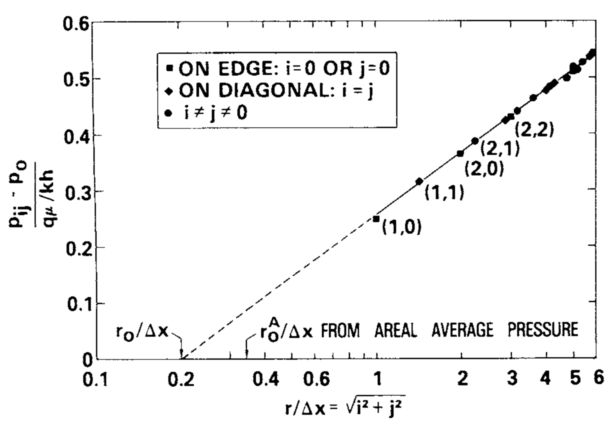


FIG. 3 — NUMERICAL SOLUTIONS FOR PRESSURE PLOTTED VS RADIUS.

By symmetry,

$$p_1 = p_2 = p_3 = p_4 . \qquad (15)$$

Combination of Eqs. 13 through 15 gives

$$\ln(\Delta x/r_o) = \pi/2 ,$$

or

$$r_o = \Delta x \exp(-\pi/2) = 0.208\ \Delta x . \qquad (16)$$

This result depends only on the assumption that the pressure of the adjacent block lies *exactly* on the straight line. Fig. 3 indicates that this assumption is not exactly correct; hence, Eq. 16 is only an approximation.

## EXACT CALCULATION OF EQUIVALENT RADIUS

For any square grid of arbitrary size, we can, in fact, calculate $r_o$ exactly without using the graphical construction of Fig. 3. This can be done by solving numerically for the steady-state pressure distribution (as outlined in Appendix A) and then using the equation for the pressure drop between injection and producing wells in a repeated five-spot pattern given by Muskat[4] of

$$\Delta p = \frac{q\mu}{\pi kh} \left[ \ln(d/r_w) - 0.6190 \right], \qquad (17)$$

where $d$ is the "diagonal" distance between wells. If we take $\Delta p$ to be the difference in pressure between the injection and production well blocks, then the $r_w$ of Eq. 17 should be replaced by $r_o$. Further, we have

$$d = \sqrt{2}\ M\ \Delta x \cdot$$

Then, Eq. 17 may be rewritten as

$$\frac{\pi kh}{q\mu} (p_{M,M} - p_{o,o}) = \ln(\sqrt{2}\ M\ \Delta x/r_o) - 0.6190$$

or

$$(r_o/\Delta x) = \sqrt{2}\ M \exp\left[ - 0.6190 - \frac{\pi kh}{q\mu} (p_{M,M} - p_{o,o}) \right]. \qquad (18)$$

Calculations for grids ranging in size from 1 × 1 to 32 × 32 are shown in Table 1. The second column lists the dimensionless pressure drop, $(kh/q\mu)(p_{M,M} - p_{o,o})$, obtained in the numerical calculation for each grid; the third column lists the values of $r_o/\Delta x$ calculated using Eq. 18. As expected, the 1 × 1 and 2 × 2 grids are anomalous. The 1 × 1 grid has no diagonal point between the wells, while the 2 × 2 grid has only one interior diagonal point. But for $M \geq 3$, we see that the value of $r_o/\Delta x$ is between 0.194 and 0.198, with 0.1982 being the apparent limit as $M \to \infty$. The rule of thumb that $r_o = 0.2\ \Delta x$ is, therefore, well substantiated by these numerical experiments.

## EXTENSION TO UNSTEADY STATE

The validity of the rule of thumb that $r_o = 0.2\ \Delta x$ so far has been demonstrated for steady-state conditions. However, a numerical calculation shows that this rule also is true for unsteady state.

Appendix B outlines the numerical solution for the unsteady-state problem of production of a compressible fluid from a single well in an infinite region. Results of the calculations are summarized in Table 2. The numerically calculated solution for the well-block pressure drawdown is listed in the second column of Table 2 for various values of the dimensionless time, $t_D = kt/(\phi\mu c_t\ \Delta x^2)$. The third column gives the "radius of investigation," defined by

$$r_{inv} = \left[4kt/(\phi\mu c_t)\right]^{\frac{1}{2}} = \Delta x(4t_D)^{\frac{1}{2}} \quad . \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (19)$$

The last column gives the equivalent radius of the well block obtained by comparison with the analytical solution to this unsteady-state problem (also given in Appendix B). For $t_D$ greater than 1.0, $r_o = 0.2\ \Delta x$ is indeed a satisfactory rule of thumb.

(Although Table 2 shows $r_o$ to be considerably greater than $0.2\ \Delta x$ for $t_D$ less than 1.0, this does not detract significantly from the utility of this rule. For this short period of time after a rate change, the radius of investigation is less than 2 $\Delta x$, and well-block pressure cannot be expected to relate to bottom-hole pressure in a simple way. Where a numerical calculation must reflect realistically the pressure changes occurring so soon after a rate change, a smaller value of $\Delta x$ must be used.)

The equation, $r_o = 0.208\ \Delta x$, previously derived for steady state by the approximate theoretical argument leading to Eq. 16, also can be derived for pseudosteady state, which is done in Appendix C. Because most transient situations can be described well by pseudosteady-state behavior near a well, this derivation provides additional support for the conclusion that $r_o = 0.2\ \Delta x$ also is true for transient conditions.

## EXTENSION TO GAS FLOW

While the presented derivations and calculations for the equivalent radius of a well block are for incompressible or slightly compressible fluids, identical results would be obtained for highly compressible liquids and gases. Further, situations where the Darcy permeability is a function of pressure (as in the case of the Klinkenberg effect) also can be included. It suffices merely to define a quantity, $m(p)$, by the integral

$$m(p) = \int_{p_{ref}}^{p} \frac{\rho(p)\ k(p)}{\mu(p)}\, dp$$

analogous to the real gas potential defined by Al-Hussainy and Ramey.[5]

If one uses $m(p)$ for the dependent variable, then the differential and difference equations for both steady state and pseudosteady state have exactly the same form as the differential and difference equations for incompressible or slightly compressible fluids using $p$ as the dependent variable, although with different constants. Thus, the numerical calculations for steady state, the derivation leading to Eq. 16, and Appendix C go through in exactly the same way, leading to the conclusion that $r_o = 0.2\ \Delta x$ also is true for highly compressible liquids and gases.

## COMPARING PRESSURE BUILDUP DATA WITH SIMULATOR RESULTS

In theory it would be sufficient, for history matching, to relate the simulator well-block pressure to the flowing well pressure by the equation

TABLE 1 — NUMERICAL CALCULATION OF PRESSURE DROP FOR REPEATED FIVE-SPOT PATTERN AND OF THE WELL-BLOCK EQUIVALENT RADIUS

| $M$ | $\frac{kh}{q\mu}(p_{M,M} - p_{o,o})$ | $r_o/\Delta x$ |
|---|---|---|
| 1 | 0.50000 | 0.1583 |
| 2 | 0.66667 | 0.1876 |
| 3 | 0.78571 | 0.1936 |
| 4 | 0.87395 | 0.1956 |
| 5 | 0.94346 | 0.1965 |
| 6 | 1.00067 | 0.1970 |
| 7 | 1.04925 | 0.1973 |
| 8 | 1.09143 | 0.1975 |
| 9 | 1.12870 | 0.1977 |
| 10 | 1.16208 | 0.1978 |
| 12 | 1.21991 | 0.1979 |
| 14 | 1.26885 | 0.1980 |
| 16 | 1.31128 | 0.1980 |
| 18 | 1.34871 | 0.1981 |
| 20 | 1.38220 | 0.1981 |
| 22 | 1.41251 | 0.1981 |
| 24 | 1.44018 | 0.1981 |
| 26 | 1.46564 | 0.1981 |
| 28 | 1.48921 | 0.1982 |
| 30 | 1.51115 | 0.1982 |
| 32 | 1.53168 | 0.1982 |

TABLE 2 — NUMERICAL CALCULATION OF PRESSURE DRAWDOWN FOR AN ISOLATED WELL IN AN INFINITE REGION AND THE WELL-BLOCK EQUIVALENT RADIUS

| $\frac{kt}{\phi\mu c_t \Delta x^2}$ | $\frac{kh}{q\mu}(p_i - p_{o,o})$ | $r_{inv}/\Delta x$ | $r_o/\Delta x$ |
|---|---|---|---|
| 0.1 | 0.0829 | 0.63 | 0.281 |
| 0.2 | 0.1407 | 0.89 | 0.277 |
| 0.5 | 0.2383 | 1.41 | 0.237 |
| 1.0 | 0.3092 | 2.00 | 0.215 |
| 2.0 | 0.3714 | 2.83 | 0.205 |
| 5.0 | 0.4477 | 4.47 | 0.201 |
| 10.0 | 0.5040 | 6.32 | 0.200 |
| 20.0 | 0.5596 | 8.94 | 0.199 |

$$p_{wf} = p_o + \frac{q\mu}{2\pi kh} \ln \frac{r_w}{0.2\Delta x}, \quad . . . . (20)$$

which was obtained by combining Eqs. 10 and 11. However, the flowing well pressure is difficult to measure, and Eq. 20 requires exact knowledge of the effective wellbore radius, $r_w$, which also can be difficult to obtain. Normally, the well is shut in and pressure at the bottom of the well is recorded, either at a single point in time or as a function of time. Previous analyses of buildup curves (e.g., Miller *et al.*,[6] Dietz,[7] van Poollen *et al.*,[2] and Kazemi[8]) focused on the average pressure in some bounded region, or at least required the specification of some drainage radius or external radius. Here, we attempt a more straightforward approach that avoids specification of $r_d$ or $r_e$.

Fig. 4 shows a typical plot of pressure vs the logarithm of $r$ — first at shut-in time, $t_s$ (the straight line), and then at two subsequent times after shut-in. After shut-in, the slope of the pressure curve is zero at the wellbore radius, $r_w$, which explains why buildup pressure is insensitive to $r_w$.

If a horizontal line is projected from the shut-in pressure, $p_{ws}$, to the straight line, it intersects at some radius, $r_m$ (which we call the "probe radius"). This probe radius is the radius at which the steady-state flowing pressure (before shut-in) *was* equal to the current well pressure after shut-in.

Note that the probe radius, $r_m$, is a function of shut-in time. By two independent derivations, Appendix D shows that the probe radius and the shut-in time are related by the equation

$$\frac{k\,\Delta t_s}{\phi\mu c_t\, r_m^2} = 0.445 \cdot \quad . . . . . . . . . . . . (21)$$

Now, the shut-in pressure of the actual well will be equal to the simulator well-block pressure at a shut-in time, $\Delta t_s^o$, when the probe radius is equal to the equivalent radius of the well block. Thus, we have

$$\frac{k\,\Delta t_s^o}{\phi\mu c_t\, r_o^2} = 0.445, \quad . . . . . . . . . (22)$$

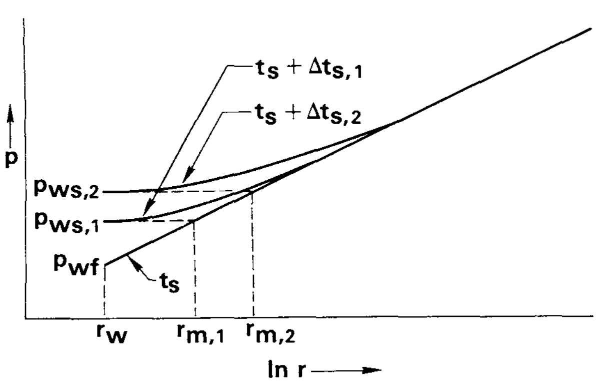


FIG. 4 — PRESSURE DISTRIBUTION BEFORE AND AFTER SHUT-IN.

and

$$r_m = r_o = 0.2\,\Delta x \quad . . . . . . . . . (23)$$

Substitution of Eq. 23 into Eq. 22 for this shut-in time gives

$$\Delta t_s^o = 0.445\,\phi\mu c_t (0.2\Delta x)^2/k,$$

or

$$\Delta t_s^o = 0.0178\,\phi\mu c_t \Delta x^2/k \quad . . . . . (24)$$

In practical units, Eq. 24 becomes

$$(\Delta t_s^o)' = 67.5\,\phi\mu c_t' (\Delta x')^2/k' \quad . . . (25)$$

This may be compared with the corresponding equation reported by Earlougher[9] (which he obtained directly from the work of van Poollen *et al.*[2]),

$$(\Delta t_s^o)_A' = 200\,\phi\mu c_t' (\Delta x')^2/k' \quad . . . (26)$$

Comparison with Eq. 25 shows that this time is too large by a factor of three. As this is the time at which the observed buildup pressure is expected to agree with the simulator well-block pressure, using this overly large time will result in an error in the history-matching process because there will be an error in the matching pressure equal to

$$p_o^A - p_o = [q\mu/(2\pi kh)]\ln(r_o^A/r_o).$$

But, from Eq. 22,

$$\ln(r_o^A/r_o) = \frac{1}{2}\ln[(\Delta t_s^o)_A/(\Delta t_s^o)]$$

$$= \frac{1}{2}\ln(200/67.5) = 0.543 \cdot$$

Then, in practical units,

$$(p_o')^A - p_o' = 0.543\,(141)(q'\mu/k'h')$$

$$= 76.6\,q'\mu/k'h'.$$

An example calculation given in van Poollen *et al.*[10] shows how large this error can be. In that example, $(k/\mu) = 235$ md/cp, $h' = 12$ ft, $q' = 1{,}350$ B/D. The error in the pressure to be matched would be 36.7 psi.

## USE OF SINGLE-POINT BUILDUP PRESSURE

Previously, we assumed that the buildup curve was available so that a shut-in pressure equal to the simulator well-block pressure could be measured at the shut-in time given by Eq. 25. However, as Earlougher[9] pointed out, if only one pressure is observed, Eq. 25 cannot be used, and an adjustment must be made. If the observed pressure, $p_{ws}^{obs}$, is measured at shut-in time, $\Delta t_s^{obs}$, then the corresponding probe radius is given by Eq. 21 as

$$(r_m^{obs})^2 = k \, \Delta t_s^{obs}/(0.445 \, \phi\mu c_t). \quad \text{. . .(27)}$$

By Eq. 9:

$$p_{ws}^{obs} = p_o + \frac{q\mu}{2\pi kh} \ln (r_m^{obs}/r_o) \quad \text{. . . . . . . . . . . . . . . . . . . . . (28)}$$

Substitution of Eqs. 22 and 27 into Eq. 28 then gives

$$p_o = p_{ws}^{obs} + \frac{q\mu}{4\pi kh} [\ln \Delta t_s^o - \ln \Delta t_s^{obs}]. \quad \text{. . . . . . . . . . . . . . . . . . . . . . (29)}$$

In practical units, Eq. 29 becomes

$$p_o' = (p_{ws}')^{obs} + \frac{162.6 \, q'\mu}{k'h'} \left[\log_{10}(\Delta t_s^o)' - \log_{10}(\Delta t_s^{obs})'\right]. \quad \text{. . . . . . . . (30)}$$

which is consistent with the standard well-test equation that relates well pressure with the logarithm of time. Substitution of Eq. 25 into Eq. 30 gives the more convenient form

$$p_o' = (p_{ws}')^{obs} + \frac{162.6 \, q'\mu}{k'h'} \log_{10}\left[\frac{67.5 \, \phi\mu c_t'(\Delta x')^2}{k' \, (\Delta t_s^{obs})'}\right] \quad \text{. . . . . . . . . . . . . . . . . . . . . . (31)}$$

This is the equation reported by Earlougher, except that his constant, 200, in the argument of the log, is replaced by the correct value of 67.5. As Earlougher pointed out, the chief requirement for using Eq. 31 is that the observed pressure be on the straight-line portion of the semilog buildup plot.

Further qualifications ought to be made. If there is only one measurement, at time $t_s + \Delta t_s^{obs}$, then $r_m^{obs}$ obtained from Eq. 27 should not be too small or too large. Not being too small means that $r_m^{obs}$ should be considerably larger than $r_w$, so that the effect of near-well inhomogeneities will not be important.

The more likely situation is that $r_m^{obs}$ may be too large, relative to either $\Delta x$ or to some neighboring well or fault that may cause interference. Unfortunately, it is frequently the practice to report only the 72-hour buildup pressure with the mistaken notion that the object of the buildup test is to measure a stabilized pressure corresponding to an "average" pressure near the well. In a high-permeability reservoir containing slightly compressible fluid, such a long shut-in time can be excessive, resulting in a probe radius larger than $\Delta x$ and possibly larger than the distance to the nearest well or fault, or the distance to the reservoir boundary.

## DISCUSSION

While the derivations in this study are based on single-phase flow, the concept of an equivalent radius of a well block equal to 0.2 $\Delta x$ has been useful in more complicated reservoir simulations involving multiphase flow. Where the well block is not square, the empirical relation $r_o = 0.2 \sqrt{\Delta x \cdot \Delta y}$ has been used. Further theoretical and numerical study has been conducted for $\Delta x$ not equal to $\Delta y$, as well as for the case of anisotropic permeability. This research will be reported later.

## CONCLUSIONS

1. In numerical reservoir simulation, the pressure calculated for a well block is the same as the flowing pressure at an equivalent radius, $r_o$. For a square grid, $r_o = 0.2 \Delta x$. This equation is valid for both steady state and transient conditions.
2. The conventional equation for steady-state radial flow then can be used to relate the well-block pressure (at radius $r_o$) to the flowing bottom-hole pressure (at radius $r_w$).
3. Buildup pressures should be measured at a shut-in time equal to $67.5 \, \phi\mu c_t' (\Delta x')^2/k'$ to obtain a pressure for matching with simulator well-block pressure.
4. The previously reported value for this shut-in time is too large by a factor of three and use of the corresponding buildup pressure can result in a significant error in history matching of pressures.
5. If one buildup pressure is measured at a time different from that given in Conclusion 3, an adjustment can be made to obtain the value to be matched with simulator well-block pressure.

## NOMENCLATURE

- $c_t$ = total compressibility of rock and fluid, atm$^{-1}$
- $c_t'$ = compressibility in field units, psi$^{-1}$
- $C$ = constant of integration
- $d$ = diagonal distance between injection and production wells in five-spot pattern, cm
- $Ei$ = exponential integral function
- $h$ = reservoir thickness, cm
- $h'$ = reservoir thickness in field units, ft
- $J$ = Bessel function
- $k$ = permeability, darcy
- $k'$ = permeability in field units, md
- $M$ = number of blocks on each side of computing grid
- $p$ = pressure, atm
- $p'$ = pressure in field units, psi
- $\bar{p}$ = average pressure, atm

$\Delta p$ = pressure drop between injection and production wells in five-spot pattern, atm
$p_D$ = dimensionless pressure = $(kh/q\mu)p$
$p_e$ = pressure at external boundary, atm
$p_i$ = initial pressure, atm
$p_o$ = simulator well-block pressure, atm
$p_{wf}$ = flowing bottom-hole pressure, atm
$p_{ws}$ = bottom-hole pressure after shut-in, atm
PI = productivity index
$q$ = production rate, cc/sec
$q'$ = production rate in field units, B/D
$r$ = radius, cm
$r'$ = radius in field units, ft
$r_b$ = radius of circle whose area = $\Delta x \cdot \Delta y$, cm
$r_e$ = radius of external boundary, cm
$r_{inv}$ = radius of investigation, defined by Eq. 19
$r_m$ = "probe" radius from buildup test, cm
$r_o$ = equivalent radius of well block, cm
$r_w$ = wellbore radius, cm
$t$ = time, seconds
$t'$ = time in field units, hours
$t_D$ = dimensionless time = $kt/(\phi\mu c_t \Delta x^2)$ in Appendix B
$t_D$ = dimensionless shut-in time = $k\Delta t_s/(\phi\mu c_t r_e^2)$ in Appendix E
$t_s$ = time at shut-in, seconds
$\Delta t_s$ = time since well was shut in, seconds
$\Delta t_s^o$ = shut-in time when $r_m = r_o$, seconds
$x_n$ = $n$th zero of Bessel function, $J_o$
$\Delta x$ = grid spacing in $x$ direction, cm
$\Delta x'$ = grid spacing in field units, ft
$\Delta y$ = grid spacing in $y$ direction, cm
$\alpha$ = constant of integration, defined by Eq. D-17
$\gamma$ = Euler's constant = 0.5772157 . . . .
$\mu$ = viscosity, cp
$\sigma$ = summation (function of $t_D$) defined by Eq. D-14
$\rho$ = density, gm/cc
$\phi$ = porosity, fraction

SUBSCRIPTS AND SUPERSCRIPTS

$A$ = based on the assumption that well-block pressure equals *areal average* pressure
$i$ = grid index in $x$ direction
$j$ = grid index in $y$ direction
$obs$ = observed

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## APPENDIX A

### NUMERICAL SOLUTION FOR REPEATED FIVE-SPOT PATTERN

Fig. 2 shows a portion of the repeated five-spot pattern that stretches to infinity in all directions. Because of symmetry, we need calculate only the quarter five-spot that is enclosed within the dashed lines. We divide this area within the dashed lines into $M \times M$ blocks, using half-blocks on the boundaries. For all blocks, $0 \leq i \leq M$, $0 \leq j \leq M$, the difference equation for the steady-state pressure distribution is

$$\frac{kh\Delta y}{\mu\Delta x}(p_{i+1,j} - 2p_{ij} + p_{i-1,j}) + \frac{kh\Delta x}{\mu\Delta y}(p_{i,j+1} - 2p_{ij} + p_{i,j-1}) = q_{ij} . \quad \text{(A-1)}$$

We assume production at rate $q$ at the lower left corner, and injection at rate $q$ at the upper right corner. Thus,

$$q_{o,o} = q$$

$$q_{M,M} = -q$$

$$q_{ij} = 0 \quad \text{for } i,j \neq 0,0 \text{ or } M,M$$

If we take $\Delta x = \Delta y$, and define

$$p_D = (kh/q\mu)p,$$

then Eq. A-1 simplifies to

$$(p_D)_{i-1,j} + (p_D)_{i+1,j} + (p_D)_{i,j-1} + (p_D)_{i,j+1} - 4(p_D)_{ij} = \delta_{ij} \quad \text{(A-2)}$$

where

$$\delta_{ij} = 0 \quad \text{for } i,j \neq 0,0 \text{ or } M,M,$$

$$\delta_{o,o} = 1,$$

$$\delta_{M,M} = -1.$$

The following reflection conditions are used:

$$\left. \begin{array}{l} p_{-1,j} = p_{1,j} \\ p_{M+1,j} = p_{M-1,j} \end{array} \right\} \text{for } 0 \leq j \leq M \quad \text{(A-3)}$$

and

$$\left. \begin{array}{l} p_{i,-1} = p_{i,1} \\ p_{i,M+1} = p_{i,M-1} \end{array} \right\} \text{for } 0 \leq i \leq M \quad \text{(A-4)}$$

Substitution of these reflection conditions yields a system of $(M+1) \times (M+1)$ equations. These equations were solved using direct solution.

## APPENDIX B

### NUMERICAL CALCULATION OF EQUIVALENT RADIUS FOR UNSTEADY STATE

The unsteady-state extension of Eq. A-1 is

$$\frac{kh\Delta y}{\mu\Delta x}(p_{i+1,j} - 2p_{ij} + p_{i-1,j}) + \frac{kh\Delta x}{\mu\Delta y}(p_{i,j+1} - 2p_{ij} + p_{i,j-1}) = q_{ij} + \phi c_t h\Delta x\Delta y \frac{\partial p_{ij}}{\partial t} \quad \text{(B-1)}$$

If we take $\Delta x = \Delta y$, and define dimensionless time and pressure drawdown by

$$t_D = kt/(\phi\mu c_t \Delta x^2), \quad \text{(B-2)}$$

$$p_D = (kh/q\mu)(p_i - p), \quad \text{(B-3)}$$

then Eq. B-1 simplifies to

$$(p_D)_{i-1,j} + (p_D)_{i+1,j} + (p_D)_{i,j-1} + (p_D)_{i,j+1} - 4(p_D)_{ij} = \delta_{ij} + \frac{\partial(p_D)_{ij}}{\partial t_D} \quad \text{(B-4)}$$

where

$$\delta_{ij} = -1 \quad \text{for } i,j = 0,0,$$

$$\delta_{ij} = 0 \quad \text{for } i,j \neq 0,0.$$

The initial condition is

$$(p_D)_{i,j} = 0 \quad \text{for all } i,j,$$

while the boundary conditions are given by the reflection conditions of Eqs. A-3 and A-4.

The resulting system of difference-differential equations was solved with a sufficiently large number of grid points ($M = 20$) that, for $t_D \leq 20$, $(p_D)_{M,i}$ is less than 0.001. Thus, the solution obtained near the well is the same as that which would be obtained for an infinite region. Further, a small enough time step was used that the time truncation error was negligible. (Both direct solution and alternating direction were used to solve the equations at each time step, with essentially identical results.) The numerical solution for the dimensionless pressure drawdown of the well block is given in Col. 2 of Table 2 for selected values of dimensionless time.

To obtain an expression for the equivalent radius, we need to compare the numerical solution for the well-block pressure with the exact solution for the radial transient problem. For an infinite reservoir initially at pressure $p_i$, producing at a constant rate, $q$, from a line sink at $r = 0$ from time $t = 0$ to time $t$, the solution satisfies (Muskat[4] and Matthews and Russel[11]) the equation

$$p(r,t) = p_i - \frac{q\mu}{4\pi kh}\left[-\,\mathrm{Ei}\left(-\frac{\phi\mu c_t r^2}{4kt}\right)\right]. \quad \text{(B-5)}$$

For sufficiently small $r$, the $Ei$ function can be approximated by

$$-\,\mathrm{Ei}(-x) = -\gamma - \ln x \quad \text{(B-6)}$$

Thus, the wellbore pressure is approximated well by

$$p_{wf}(t) = p_i - \frac{q\mu}{4\pi kh}\left[-\gamma - \ln\left(\frac{\phi\mu c_t r_w^2}{4kt}\right)\right] \quad \text{(B-7)}$$

From the definition of the equivalent radius of a well block (Eq. 9), we have

$$p_{wf}(t) = p_{o,o} + \frac{q\mu}{2\pi kh} \ln \frac{r_w}{r_o} \quad \ldots (B\text{-}8)$$

Substitution of Eq. B-8 into Eq. B-7 and rearranging gives

$$p_i - p_{o,o} = \frac{q\mu}{4\pi kh}\left[-\gamma - \ln\left(\frac{\phi\mu c_t r_o^2}{4kt}\right)\right].$$

Using Eqs. B-2 and B-3 simplifies this to

$$(p_D)_{o,o} = \frac{1}{4\pi}\left[-\gamma - \ln\left(\frac{r_o^2}{4t_D \Delta x^2}\right)\right],$$

and this may be solved for the equivalent radius as

$$r_o = \Delta x \left[4t_D \exp\left\{-\gamma - 4\pi(p_D)_{o,o}\right\}\right]^{1/2} \quad \ldots (B\text{-}9)$$

using $(p_D)_{o,o}$ listed in Col. 2 of Table 2. The ratio of $r_o$ to $\Delta x$ is listed in the last column of Table 2.

## APPENDIX C

## APPROXIMATE CALCULATION OF EQUIVALENT RADIUS FOR PSEUDOSTEADY STATE

Under transient conditions, the following differential equation describes the flow of a slightly compressible fluid in a radial system:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = \frac{\phi\mu c_t}{k}\frac{\partial p}{\partial t}. \quad \ldots (C\text{-}1)$$

The usual definition of pseudosteady state is that the rate of pressure decline throughout a closed region is a linear function of time.[11] A less restrictive definition can be used here, namely that $\partial p/\partial t$ simply be independent of $r$. Eq. C-1 then can be integrated to

$$r\frac{\partial p}{\partial r} = \frac{\phi\mu c_t r^2}{2k}\frac{\partial p}{\partial t} + C_1. \quad \ldots (C\text{-}2)$$

Use of the boundary condition,

$$q = \frac{2\pi khr_w}{\mu}\left(\frac{\partial p}{\partial r}\right)_w,$$

gives

$$C_1 = \frac{q\mu}{2\pi kh} - \frac{\phi\mu c_t r_w^2}{2k}\frac{\partial p}{\partial t}. \quad \ldots (C\text{-}3)$$

Dividing Eq. C-2 through by $r$ and integrating once more gives

$$p = \frac{\phi\mu c_t r^2}{4k}\frac{\partial p}{\partial t} + C_1 \ln r + C_2$$

$C_2$ can be eliminated using the boundary condition $p = p_{wf}$ at $r = r_w$. Then,

$$p = p_{wf} + \frac{\phi\mu c_t (r^2 - r_w^2)}{4k}\frac{\partial p}{\partial t} + C_1 \ln\frac{r}{r_w} \quad \ldots (C\text{-}4)$$

Substituting Eq. C-3 into Eq. C-4 and noting that the contribution of $r_w$ is negligible (except in the log term) gives

$$p = p_{wf} + \frac{\phi\mu c_t r^2}{4k}\frac{\partial p}{\partial t} + \frac{q\mu}{2\pi kh}\ln\frac{r}{r_w}. \quad \ldots (C\text{-}5)$$

In Appendix B, the numerical solution for unsteady-state conditions was outlined. Take $\Delta x = \Delta y$ and use the notation of Fig. 1. Then, Eq. B-1 written for the well block, and the symmetry conditions of Eq. 15 lead to

$$4(p_1 - p_o) = \frac{q\mu}{kh} + \frac{\phi\mu c_t \Delta x^2}{k}\frac{\partial p_o}{\partial t}. \quad \ldots (C\text{-}6)$$

To derive the equivalent radius of the well block, we now make two assumptions. First, $p_1$ is assumed to be the same as the radial solution given by Eq. C-5 evaluated at $r = \Delta x$. (This corresponds to the assumption that was used in the derivation of Eq. 16.) Thus,

$$p_1 = p_{wf} + \frac{\phi\mu c_t \Delta x^2}{4k}\frac{\partial p}{\partial t} + \frac{q\mu}{2\pi kh}\ln\frac{\Delta x}{r_w} \quad \ldots (C\text{-}7)$$

Second, we assume that the rate of decline of the well-block pressure, $\partial p_o/\partial t$, is the same as the uniform rate of decline, $\partial p/\partial t$, in the pseudosteady-state radial solution. Combining Eqs. B-8, C-6, and C-7 then yields Eq. 16.

Thus, Eq. 16 can be derived not only for the steady-state conditions used in the text, but also for pseudosteady state, which adequately describes most transient situations near a well.

## APPENDIX D

## DERIVATION OF PROBE RADIUS AS A FUNCTION OF SHUT-IN TIME

We seek the solution to the following differential equation for flow of a slightly compressible fluid in a radial system:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi\mu c_t}{k}\frac{\partial p}{\partial t} \quad \text{(D-1)}$$

Two approaches using different initial and boundary conditions are presented.

USING *Ei* FUNCTION

Assume an infinite reservoir initially at pressure $p_i$ producing at constant rate $q$ from a line sink at $r = 0$ from time $t = 0$ to shut-in time $t_s$. Then, the solution at $t_s$ satisfies (Muskat[4] and Matthews and Russell[11]) the equation

$$p(r,t_s) = p_i - \frac{q\mu}{4\pi kh}\left[-\mathrm{Ei}\left(-\frac{\phi\mu c_t r^2}{4kt_s}\right)\right] \quad \text{(D-2)}$$

After shut-in, by superposition, the bottom-hole pressure is

$$p_{ws}(t_s + \Delta t_s) = p_i - \frac{q\mu}{4\pi kh}\left[-\mathrm{Ei}\left(-\frac{\phi\mu c_t r_w^2}{4k(t_s+\Delta t_s)}\right) + \mathrm{Ei}\left(-\frac{\phi\mu c_t r_w^2}{4k\,\Delta t_s}\right)\right] \quad \text{(D-3)}$$

By combining the definition of $r_m$,

$$p(r_m,t_s) = p_{ws}(t_s + \Delta t_s) \quad \text{(D-4)}$$

with Eqs. D-2 and D-3, we obtain

$$-\mathrm{Ei}\left(-\frac{\phi\mu c_t r_m^2}{4kt_s}\right) = -\mathrm{Ei}\left(-\frac{\phi\mu c_t r_w^2}{4k(t_s+\Delta t_s)}\right) + \mathrm{Ei}\left(-\frac{\phi\mu c_t r_w^2}{4k\,\Delta t_s}\right) \quad \text{(D-5)}$$

For sufficiently large $t_s$ and $\Delta t_s$, the *Ei* function can be approximated by

$$-\mathrm{Ei}(-x) = -\gamma - \ln x,$$

where $\gamma = 0.5772157$ is Euler's constant. Then Eq. D-5 becomes

$$-\gamma - \ln\left(\frac{\phi\mu c_t r_m^2}{4kt_s}\right) = -\gamma - \ln\left(\frac{\phi\mu c_t r_w^2}{4k(t_s+\Delta t_s)}\right) + \gamma + \ln\left(\frac{\phi\mu c_t r_w^2}{4k\,\Delta t_s}\right)$$

or

$$\ln\left(\frac{4k\,\Delta t_s}{\phi\mu c_t r_m^2}\;\frac{t_s}{t_s+\Delta t_s}\right) = \gamma$$

Then,

$$\frac{k\,\Delta t_s}{\phi\mu c_t r_m^2} = \frac{t_s+\Delta t_s}{t_s}\;\frac{\exp(\gamma)}{4} = 0.44527(t_s+\Delta t_s)/t_s \quad \text{(D-6)}$$

For sufficiently large $t_s$ (greater than 2,000 $\Delta t_s$), Eq. D-6 is the same as Eq. 21.

USING BESSEL SERIES

Here, assume a bounded reservoir with internal radius $r_w$ and external radius $r_e$. The pressure at the external boundary is maintained constant at $p_e$. (The nature of the external boundary condition is not important, as van Poollen *et al.*[2] found when they compared solutions obtained by assuming constant pressure at $r_e$ with solutions obtained by assuming no flow across $r_e$. Further, we show that the solution is, in fact, insensitive to the choice of $r_e$.) Suppose that steady-state radial flow exists just before shut-in. Then by Eq. 3, we have

$$p_e = p_{wf} + [q\mu/(2\pi kh)]\ln(r_e)/r_w), \quad \text{(D-7)}$$

while the initial condition can be written as

$$p(r,t_s) = p_e + [q\mu/(2\pi kh)]\ln(r/r_e). \quad \text{(D-8)}$$

The solution to Eq. D-1 subject to this initial condition and to the boundary conditions

$$p = p_e \quad \text{at } r = r_e, \quad \text{(D-9)}$$

and

$$\left(\frac{\partial p}{\partial r}\right)_{r_w} = 0 \quad \text{as } r_w \to 0, \quad \text{(D-10)}$$

is given by Muskat[4] as

$$\frac{p_e - p(r,t_s+\Delta t_s)}{p_e - p_{wf}} = \frac{2}{\ln(r_e/r_w)}$$

$$\cdot \sum_{n=1}^{\infty} \frac{J_o(x_n r/r_e)\exp(-x_n^2 t_D)}{x_n^2 J_1^2(x_n)},$$

. . . . . . . . . . . . . . . . . . . . . (D-11)

where $J_o$ and $J_1$ are Bessel functions of the first kind, $x_n$'s are the zeros of $J_o$; that is,

$$J_o(x_n) = 0 \qquad n=1,2,\ldots$$

and $t_D$ is a dimensionless shut-in time defined by

$$t_D = \frac{k\,\Delta t_s}{\phi\mu c_t r_e^2}. \qquad \text{(D-12)}$$

The use of the limiting condition $r_w \to 0$ at the interior boundary (Eq. D-10) is justified by the observation that $p_w(t_s + \Delta t_s)$ is insensitive to the value of $r_w$. To avoid the apparent dependence of the solution on $r_w$ and $p_{wf}$, we substitute Eq. D-7 into Eq. D-11 to obtain

$$p_e - p(r,t_s+\Delta t_s) =$$

$$\frac{q\mu}{\pi kh} \sum_{n=1}^{\infty} \frac{J_o(x_n r/r_e)\exp(-x_n^2 t_D)}{x_n^2 J_1^2(x_n)}.$$

In particular, by letting $r \to r_w \to 0$, and noting that $J_o(0) = 1$, we get

$$p_e - p_{ws}(t_s+\Delta t_s) = [q\mu/(\pi kh)]\,\sigma(t_D),$$

. . . . . . . . . . . . . . . . . . . . . (D-13)

where

$$\sigma(t_D) = \sum_{n=1}^{\infty} \frac{\exp(-x_n^2 t_D)}{x_n^2 J_1^2(x_n)} \qquad \text{(D-14)}$$

By substituting $r_m$ into Eq. D-8 and combining with the definition of $r_m$, Eq. D-4, we get

$$p_{ws}(t_s+\Delta t_s) = p_e$$

$$+ [q\mu/(2\pi kh)]\ln(r_m/r_e) \qquad \text{(D-15)}$$

Substitution of Eq. D-15 into Eq. D-13 then gives the simple relation

$$\tfrac{1}{2}\ln(r_e/r_m) = \sigma(t_D). \qquad \text{(D-16)}$$

It seems reasonable to assume tentatively that $r_m$ is independent of $r_e$, at least for some range of $t_D$. It follows that

$$\frac{d}{dr_e}(\sigma - \tfrac{1}{2}\ln r_e) = 0,$$

$$(d\sigma/dt_D)(dt_D/dr_e) = 1/(2r_e).$$

But,

$$dt_D/dr_e = -2k\Delta t_s/(\phi\mu c_t r_e^3) = -2t_D/r_e.$$

Thus,

$$(d\sigma/dt_D) = -1/(4t_D),$$

$$\sigma = \tfrac{1}{4}\ln(\alpha/t_D), \qquad \text{(D-17)}$$

where $\alpha$ is an undetermined constant. The assumption that $r_m$ is independent of $r_e$ is validated if $\alpha$ is indeed found constant.

Substitution of Eq. D-17 into Eq. D-16 yields

$$r_m^2/r_e^2 = t_D/\alpha = k\,\Delta t_s/(\alpha\phi\mu c_t r_e^2),$$

or

$$\frac{k\,\Delta t_s}{\phi\mu c_t r_m^2} = \alpha. \qquad \text{(D-18)}$$

The constant, $\alpha$, can be evaluated precisely from the series of Eq. D-14 by rewriting Eq. D-17 as

$$\alpha = t_D\exp(4\sigma).$$

Table 3 summarizes the calculations. For each value of $t_D$, the series was terminated when the last term was less than $10^{-8}$.

Clearly, for $t_D < 0.1$, $\alpha$ may be taken constant at 0.445. Eq. D-18 is, therefore, the same as Eq. 21.

## RANGE OF VALIDITY

We have established that Eq. 21 is valid only if $t_D < 0.1$. Combining Eqs. 21 and D-12 gives

TABLE 3 — EVALUATION OF THE CONSTANT, $\alpha$, FROM BESSEL SERIES

| $t_D$ | Number of Terms | $\sigma(t_D)$ | $\alpha$ |
|---|---|---|---|
| 1.0 | 2 | 0.00198 | 1.0079 |
| 0.7 | 2 | 0.01120 | 0.7321 |
| 0.5 | 3 | 0.03561 | 0.5765 |
| 0.3 | 3 | 0.11324 | 0.4719 |
| 0.2 | 4 | 0.20249 | 0.44956 |
| 0.1 | 5 | 0.37346 | 0.44542 |
| 0.03 | 8 | 0.67445 | 0.44541 |
| 0.01 | 13 | 0.94907 | 0.44535 |
| 0.003 | 23 | 1.2500 | 0.44529 |
| 0.001 | 39 | 1.5247 | 0.44525 |
| 0.0003 | 68 | 1.8256 | 0.44520 |
| 0.0001 | 115 | 2.1003 | 0.44517 |
| 0.00003 | 205 | 2.4012 | 0.44509 |
| 0.00001 | 348 | 2.6758 | 0.44498 |

$$t_D = 0.445\ (r_m/r_e)^2 < 0.1\ ,$$

or

$$\frac{r_m}{r_e} < 0.47\ .$$

Thus we can use the buildup curve to "probe" to a radius up to one-half that of an external boundary.

## APPENDIX E

### FURTHER COMPARISON WITH van POOLLEN *ET AL.*

van Poollen *et al.*[2] obtained the following equation for the shut-in time at which buildup pressure should equal the simulator well-block pressure:

$$(\Delta t_s^o)_A = 0.167\ \phi\mu c_t r_b^2/k\ .$$

Substitution of Eq. 5 gives

$$(\Delta t_s^o)_A = 0.0532\ \phi\mu c_t \Delta x^2/k \quad \text{. . . . . . . . . . . . . . . . . . . . . (E-1)}$$

We can derive this result directly by substituting Eq. 12,

$$r_m = r_o^A = 0.342\ \Delta x\ ,$$

into Eq. 22 to get

$$(\Delta t_s^o)_A = 0.0520\ \phi\mu c_t \Delta x^2/k\ \text{. . . (E-2)}$$

[The difference between Eqs. E-1 and E-2 results from van Poollen *et al.* using graphical methods based on the graphs of Miller *et al.*[6]) to obtain Eq. E-1, while the more exact methods of Appendix D were used to obtain Eq. 21 and, from this, Eq. E-2.]

In practical units, Eq. E-1 is essentially the same as Eq. 26, which gives a time too large by a factor of three. Similarly, Eq. E-2 is too large by a factor of three, compared with our Eq. 24.

Another error that can occur from using Ref. 2 is when calculating the difference between the well-block pressure and the flowing well pressure. The relative error in this difference is (by Eqs. 10 through 12),

$$\frac{(p_o - p_{wf}^A) - (p_o - p_{wf})}{(p_o - p_{wf})}$$

$$= \frac{\ln(r_o^A/r_w) - \ln(r_o/r_w)}{\ln(r_o/r_w)}$$

$$= \frac{\ln(0.342/0.2)}{\ln(0.2\ \Delta x/r_w)}$$

Assume $\Delta x/r_w$ ranges from 200 to 3,000. Then, this relative error will range from 15 to 8 percent.

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Discussion of
(SPE 6893)

Interpretation of Well-Block Pressures in Numerical Reservoir Simulation

by Donald W. Peaceman

Discussion

by
I. Mrosovsky

A very interesting treatment of well block pressures has been presented by Peaceman in the above paper. In particular, he demonstrates that in applying the usual formula

$$p_o = p_{wf} + \frac{q\mu}{2\pi kh} \ln \frac{r_o}{r_w}$$

to well productivity, $r_o$ should be set equal to $0.2\Delta x$, where $\Delta x$ is the length of the side of a block. (The same symbols as used by Peaceman will be employed here as far as possible.) Another approach to this problem indicates that this rule is not universally true, not even for single phase flow in square blocks.

For semi-steady state compressible flow, the average pressure $\bar{p}$ in a closed reservoir, of area A and producing through a single well, is given by the equation[1]

$$\bar{p} = p_{wf} + \frac{q\mu}{4\pi kh} \left[ \ln \frac{A}{C_A r_w^2} + .809 \right]$$

where $C_A$ is a shape factor. For a circle $C_A$ is 31.6 and the equation reduces to the familiar form

$$\bar{p} = p_{wf} + \frac{q\mu}{2\pi kh}\left[\ln\frac{r_b}{r_w} - \frac{3}{4}\right]$$

For a square $C_A = 30.9$ and the term in square brackets becomes $[\ln(r_b/r_w) - .7385]$ where $r_b = \Delta x/\sqrt{\pi}$. If, nevertheless, well productivity is to be described by the formula

$$\bar{p} = p_{wf} + \frac{q\mu}{2\pi kh}\ln\frac{r_o}{r_w}$$

then, in order to obtain correct answers, one must put

$$\ln\frac{r_o}{r_w} = \ln\frac{\Delta x/\sqrt{\pi}}{r_w} - .7385$$

This readily yields $r_o = 0.27\Delta x$, a result midway between that of Peaceman and van Poollen[2]. $\bar{p}$ is here the average block pressure in the sense that it is equal to the fully built-up static pressure that would be obtained if the well were closed in. $\bar{p}$ is also the pressure of the block that would be obtained in numerical reservoir simulation.

The case just discussed is not only a special one, but a trivial one as far as simulation is concerned. Nevertheless, it is closely related to cases of practical importance. In modeling a field with a large number of wells, it is frequently necessary to have wells in all of many adjacent blocks (figure 1a). For the purpose of well productivity, each block may be treated as a sealed cell. Of course, the block boundaries are not sealed, but in general, it is unknown in which directions the flows across these boundaries will occur, or in what quantities. It seems therefore reasonable to take the middle condition in which flow is in neither one direction nor the other, and apply as an average rule of thumb $r_o = 0.27\Delta x$ to all blocks. A block pressure is the average pressure of a block in the sense that it is the static pressure that would be obtained if the well were closed in and the block were simultaneously sealed at its boundaries.

Suppose, however, that some more detail can be afforded in the simulator, as in figure 1b. The boundaries of the small blocks in which the wells are situated, cannot even approximately be considered as no flow boundaries. If the well block is small enough, the pressure distribution over it can be described by

$$p = p_{wf} + \frac{q\mu}{2\pi kh} \ln \frac{r}{r_w}$$

The average pressure is obtained by integration over the block square, but the result is close to the formula used by van Poollen:

$$\bar{p} = p_{wf} + \frac{q\mu}{2\pi kh} \left[ \ln \frac{\Delta x/\sqrt{\pi}}{r_w} - \frac{1}{2} \right]$$

However, Peaceman has shown that the block pressure is not the average pressure of a block when there is a well in it. This is disturbing since a simulator is set up to maintain material balance. If the block pressure is not the average pressure of the material in the block, then a material balance error is incurred.

Peaceman's three demonstrations that a well block pressure is not the average pressure of the block, all are based on incompressible fluid flow. No material balance error is therefore involved since the amount of fluid in a block is then independent of pressure. Some comment on this problem would be welcome.

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[1]Matthews, C.S., and Russell, D.G., 'Pressure Buildup and Flow Tests in Wells'. SPE Monograph 1967.

[2]van Poollen, H.K., Breitenbach, E.A., and Thurnau, D.H.: "Treatment of Individual Wells and Grids in Reservoir Modeling". Soc. Pet. Eng. J. December 1968.

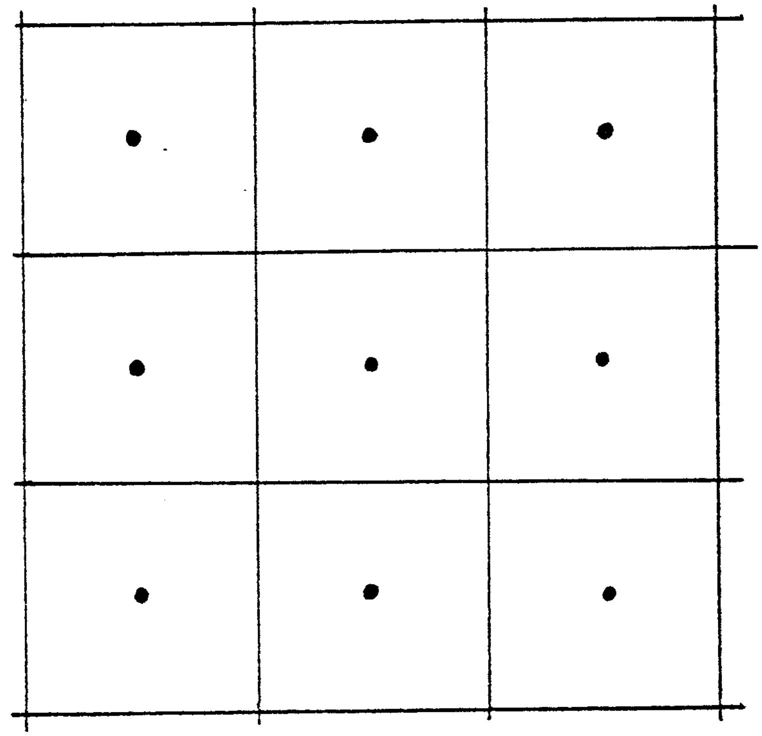

Fig. 1a. Adjacent Well-Blocks

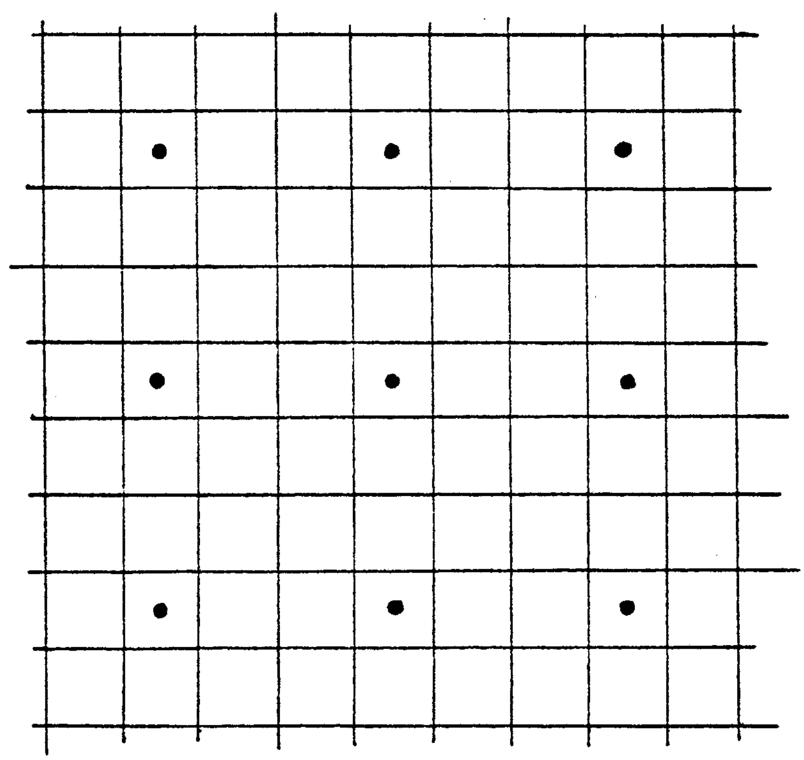

Fig 1b. More finely gridded simulator