

# Relative permeabilities in reservoir simulation

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## Abstract

In this note we describe the standard models of three-phase relative permeabilities in reservoir simulation. Emphasis is given to the treatment of hysteresis, which is essential in water-alternating-gas applications.

## 1 Introduction

The classical macroscopic description of multiphase flow in porous media relies on a straightforward multiphase extension of Darcy's equation [Muskat, 1949]. Underlying this extension is the concept of *relative permeability*. Relative permeabilities account for the reduction in the flow of each phase due to the mutual interaction of the different flowing phases. Relative permeabilities to water, oil and gas are arguably the most important fluid flow descriptors in reservoir engineering, yet considerably uncertain.

Applicability of the traditional formulation hinges on the validity of Darcy's equation and its extension to multiphase flow, and on the use of a proper model for the relative permeabilities. Darcy's law is an empirical relation that replaces —at the macroscopic level— the equations of momentum conservation. For single-phase flow, Darcy's law is written as:

$$\mathbf{q} = -\frac{\mathbf{k}}{\mu}(\nabla p - \rho \mathbf{g}), \quad (1)$$

where  $\mathbf{q}$  is the volumetric fluid flux,  $\mathbf{k}$  is the absolute permeability tensor of the medium,  $\mu$  is the dynamic viscosity of the fluid,  $p$  is the fluid pressure,  $\rho$  is the fluid density, and  $\mathbf{g}$  is the gravitational acceleration. Equation (1) is a *postulate*, supported for *single-phase flow* by experimental evidence and by volume averaging as a first-order approximation [Hassanizadeh, 1986].

A major assumption was introduced by Muskat [1949, Chap. 7], who extended Darcy’s equation to model multiphase flow. The constitutive relation for the volumetric fluid flux  $\mathbf{q}_\alpha$  of a given phase  $\alpha$  (water, oil, or gas) takes the form:

$$\mathbf{q}_\alpha = -\frac{\mathbf{k}k_{r\alpha}}{\mu_\alpha}(\nabla p_\alpha - \rho_\alpha \mathbf{g}), \quad \alpha = w, o, g, \quad (2)$$

where  $k_{r\alpha}$  is the relative permeability to the  $\alpha$ -phase. The multiphase extension of Darcy’s equation given by Equation (2) may be described as a *quasi-linear* relation, because the fluid flux depends linearly on the “driving force” —which includes viscous, capillary, and gravity forces— and all the nonlinearity is agglutinated in the relative permeabilities. Although it does not have a rigorous foundation, Darcy-like formulations are almost universally used in hydrogeology and petroleum engineering, because alternative approaches have not yet been reduced to a tractable form.

Traditionally, relative permeabilities have been taken as functions of fluid saturations alone. However, theoretical and experimental studies have shown that relative permeabilities cannot be understood as fixed functions of saturations, and that they depend on the wettability properties of the fluids, the fluid viscosities, the displacement process —drainage or imbibition— and the displacement history that determined the pore-scale configuration of the fluids.

## 2 Three-phase relative permeability models

Due to the strong dependence of the relative permeabilities on the saturation history and the displacement process, the only reliable way to estimate them is through laboratory experiments. These experiments should reproduce the initial reservoir conditions and the sequence of saturation changes expected in the reservoir. However, laboratory measurement of three-phase relative permeabilities is prohibitively difficult, expensive, and time-consuming.

Therefore, three-phase relative permeabilities are often estimated from two-phase data. Independently of how this estimation is carried out, it is

essential that the two-phase curves reproduce a similar saturation history to that of the three-phase relative permeabilities to be estimated. Consider, for example, the saturation path in Figure 1. This path corresponds to the history of saturations at a particular location of a black-oil reservoir, after production by internal gas drive, followed by a waterflood. In this case, the appropriate experiments for the estimation of three-phase relative permeabilities are the following (see Figure 1):

1. A water-oil flow experiment, with increasing water saturation.
2. A gas-oil flow experiment at connate water saturation, with increasing gas saturation.

In both cases, the core should be brought to initial reservoir conditions first, by injecting oil into a water-filled core.

The usual assumption to estimate three-phase relative permeabilities from two-phase data is that water and gas relative permeabilities depend on their respective saturations only, while the oil relative permeability depends on both saturations:

$$k_{rw} = k_{rw}(S_w), \quad (3)$$

$$k_{rg} = k_{rg}(S_g), \quad (4)$$

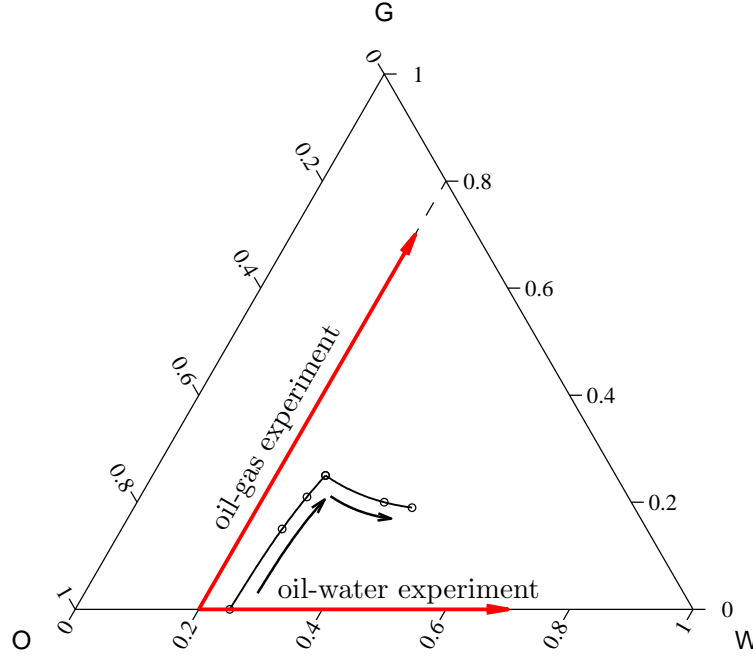
$$k_{ro} = k_{ro}(S_w, S_g). \quad (5)$$

These dependencies are based on the assumption of strict wettability: water is assumed to be the most wetting phase, and gas the least wetting phase. We shall make use of this assumption in the remainder of this section. Therefore, the water relative permeability is usually obtained from an oil-water flow experiment, and the gas relative permeability from an oil-gas flow experiment at connate water saturation (see Figure 2).

The most common methods for interpolating the oil relative permeability are due to Stone [1970; 1973]. Another classic method is the saturation-weighted interpolation originally proposed by Baker [1988]. These methods are discussed below.

## 2.1 Stone I model

This model was originally proposed by Stone [1970]. Here we describe the normalized version of Aziz and Settari [1979] implemented in most commer-



**Figure 1.** Schematic of the saturation history in a black-oil reservoir after solution gas drive followed by a waterflood. The red arrows indicate saturation changes in laboratory experiments conducted to measure relative permeabilities.

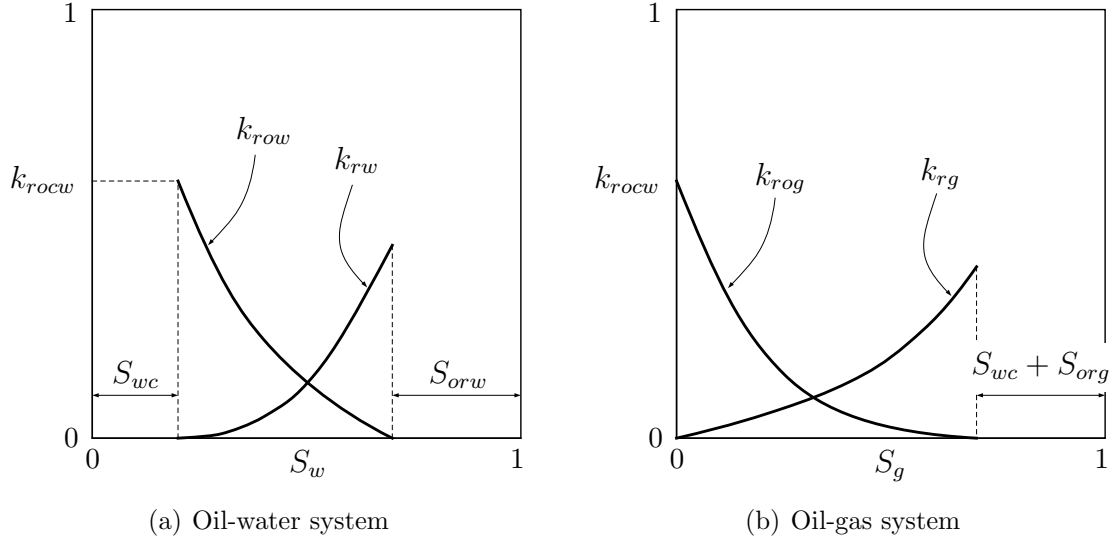
cial reservoir simulators. Let us define the normalized saturations:

$$S_w^* = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{om}}, \quad S_w > S_{wc}, \quad (6)$$

$$S_g^* = \frac{S_g}{1 - S_{wc} - S_{om}}, \quad (7)$$

$$S_o^* = \frac{S_o - S_{om}}{1 - S_{wc} - S_{om}}, \quad S_o > S_{om}, \quad (8)$$

where  $S_{wc}$  is the connate (irreducible) water saturation, and  $S_{om}$  is the minimum (residual) oil saturation. In this case, the critical gas saturation is assumed to be zero. One should note that the residual oil saturation is *not* a constant. It will take a different value depending on the amount of gas present (see Figure 3). Therefore, the residual oil saturation is in fact a (possibly nonlinear) function of gas saturation. Different models for  $S_{om}$  have



**Figure 2.** Two-phase relative permeabilities for an oil-water system (left) and an oil-gas system at connate water saturation (right).

been studied by Fayers and Matthews [1984]. They conclude that a linear interpolation between the two extreme values ( $S_{orw}$  and  $S_{org}$ ) is sufficient in most cases:

$$S_{om} = \alpha S_{orw} + (1 - \alpha) S_{org}, \quad \text{with } \alpha = 1 - \frac{S_g}{1 - S_{wc} - S_{org}}. \quad (9)$$

The relative permeability to oil is obtained from the following expression:

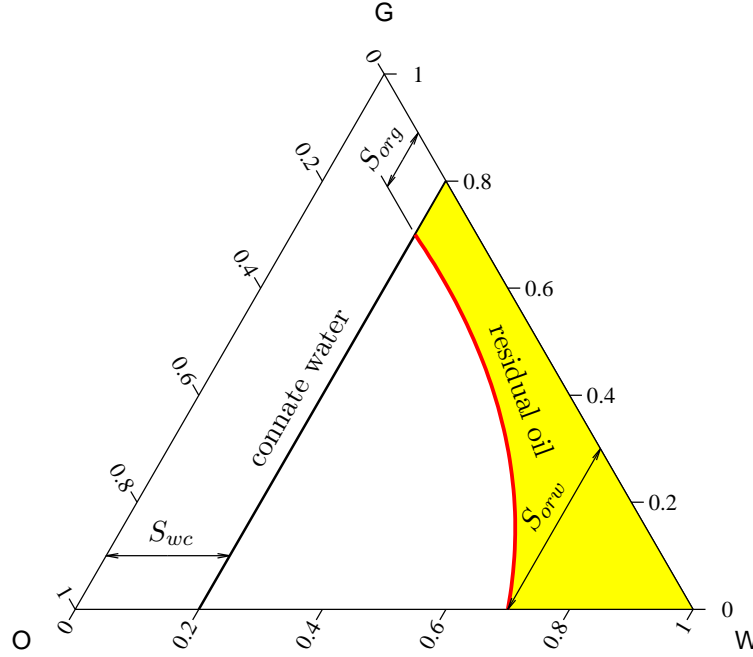
$$k_{ro}(S_w, S_g) = k_{rocw} S_o^* \beta_w \beta_g, \quad (10)$$

where  $k_{rocw}$  is the oil relative permeability at connate water and critical gas saturations, and

$$\beta_w = \frac{k_{row}(S_w)/k_{rocw}}{1 - S_w^*}, \quad (11)$$

$$\beta_g = \frac{k_{rog}(S_g)/k_{rocw}}{1 - S_g^*}. \quad (12)$$

It is important to note that  $k_{row}$  is evaluated at  $S_o = 1 - S_w$ , and  $k_{rog}$  is evaluated at  $S_o = 1 - S_g - S_{wc}$ , that is, at *different* oil saturations than the three-phase situation being predicted.



**Figure 3.** Typical behavior of the residual oil saturation. The value of  $S_{om}$  is not constant.

## 2.2 Stone II model

This interpolation method for the oil relative permeability was introduced by Stone [1973]. A separate interpolation of the residual oil saturation  $S_{om}$  is not required, and the model actually predicts this curve. In normalized form, the model is written as [Aziz and Settari, 1979]:

$$k_{ro}(S_w, S_g) = k_{rocw} \left[ \left( \frac{k_{row}}{k_{rocw}} + k_{rw} \right) \left( \frac{k_{rog}}{k_{rocw}} + k_{rg} \right) - (k_{rw} + k_{rg}) \right]. \quad (13)$$

When Equation (13) predicts  $k_{ro} < 0$ , the oil relative permeability is set to zero. The two-phase relative permeabilities are evaluated at the same saturation values as in the Stone I model.

## 2.3 Saturation-weighted models

A saturation-weighted interpolation of two-phase relative permeabilities was first proposed by Baker [1988]. It is used as the default model in the com-

mercial simulator Eclipse. In this model, the oil relative permeability is computed as:

$$k_{ro}(S_w, S_g) = \frac{S_g \bar{k}_{rog} + (S_w - S_{wc}) \bar{k}_{row}}{S_g + (S_w - S_{wc})}. \quad (14)$$

In Equation (14), the two-phase relative permeabilities are evaluated at the *same* oil saturation of the three-phase scenario, that is:

$$\bar{k}_{row} = k_{row}(1 - S_o) = k_{row}(S_w + S_g), \quad (15)$$

$$\bar{k}_{rog} = k_{rog}(1 - S_{wc} - S_o) = k_{rog}(S_g + S_w - S_{wc}). \quad (16)$$

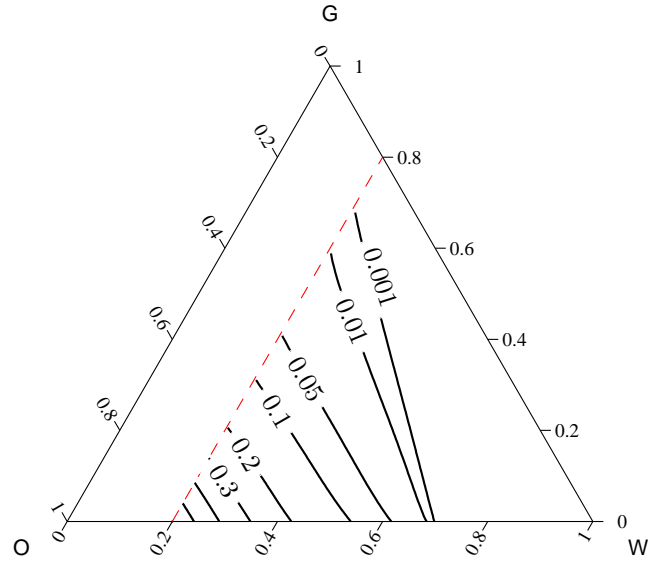
## 2.4 Comparison of the different models

We present here a comparison of the three models described above for the prediction of the three-phase relative permeabilities, given the two-phase data of Figure 2. The oil isoperms obtained with the Stone I (with linear  $S_{om}$  model), Stone II, and saturation-weighted methods are plotted in Figure 4. It is noticeable that, while all three methods give essentially the same results for high oil saturations, the predictions at low oil saturations vary wildly. For example, the oil isoperms at low oil saturations are linear in the Stone I model (due to the linear  $S_{om}$  model employed), concave for the Stone II model, and convex for the saturation-weighted interpolation. This disagreement in the predictions of relative permeabilities is typical of the behavior encountered in practice.

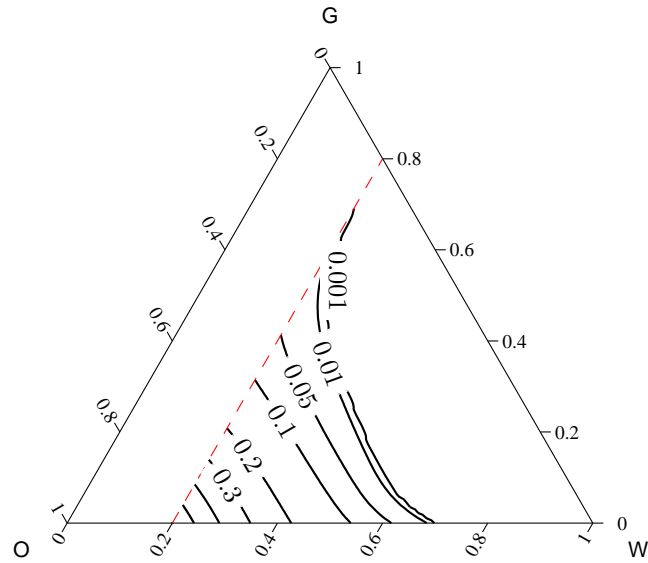
## 3 Hysteretic effects

Hysteresis refers to irreversibility, or path dependence. In multiphase flow, it manifests itself through the dependence of the relative permeabilities and capillary pressures on the saturation path. From the point of view of pore-scale processes, hysteresis has at least two sources:

1. *Contact angle hysteresis.* A graphical representation of the contact angle  $\theta$  in a water-oil system is sketched in Figure 5. Most porous rocks exhibit contact angle hysteresis (see Figure 6). The *advancing* contact angle  $\theta_a$  (of wetting phase displacing a nonwetting phase — imbibition) is larger, sometimes much larger, than the *receding* contact angle  $\theta_r$  (of wetting phase retreating by nonwetting phase invasion — drainage).



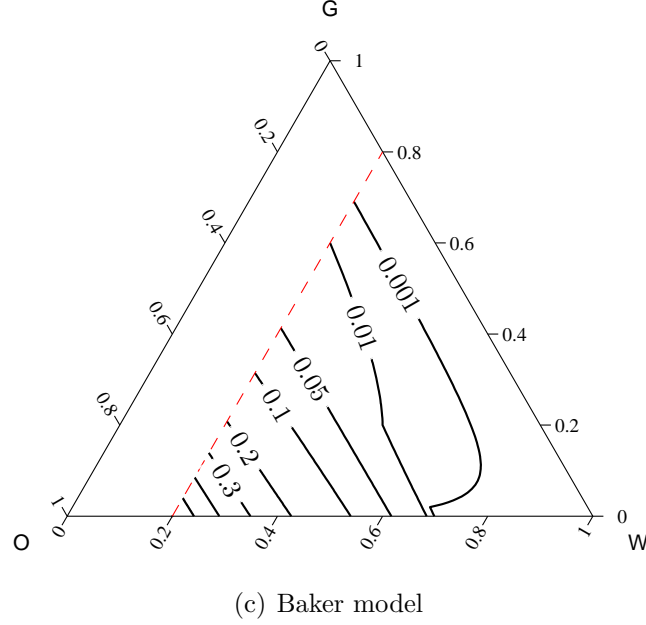
(a) Stone I model



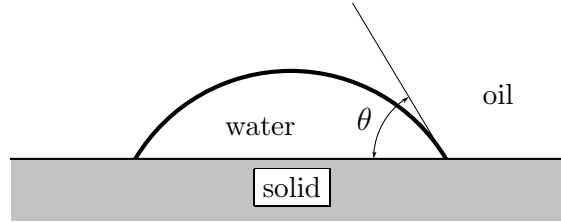
(b) Stone II model

**Figure 4.** Three-phase oil isoperms predicted by the different models, when applied to the two-phase data of Figure 2.



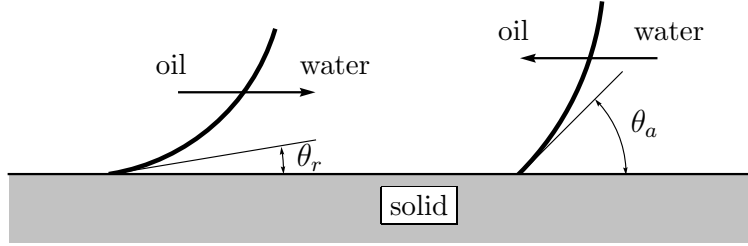


**Figure 4.** Three-phase oil isoperms predicted by the different models, when applied to the two-phase data of Figure 2 (continued).



**Figure 5.** Contact angle in a water-oil system.

2. *Trapping of the nonwetting-phase.* During an imbibition process (increasing saturation of the most wetting phase), a fraction of the nonwetting phase gets disconnected in the form of blobs and ganglia. This fraction is referred to as *trapped* nonwetting phase saturation, because it is effectively immobile.



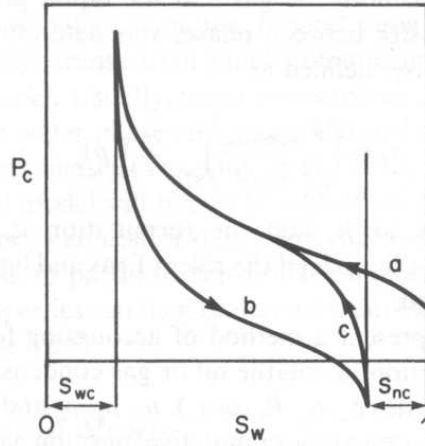
**Figure 6.** Contact angle hysteresis. Receding (left) and advancing (right) contact angles.

Modeling of hysteresis is particularly important in situations involving flow reversals, such as water-alternating-gas processes. In this section we describe the standard treatment of hysteresis in reservoir simulators. It closely follows the presentation in Aziz and Settari [1979].

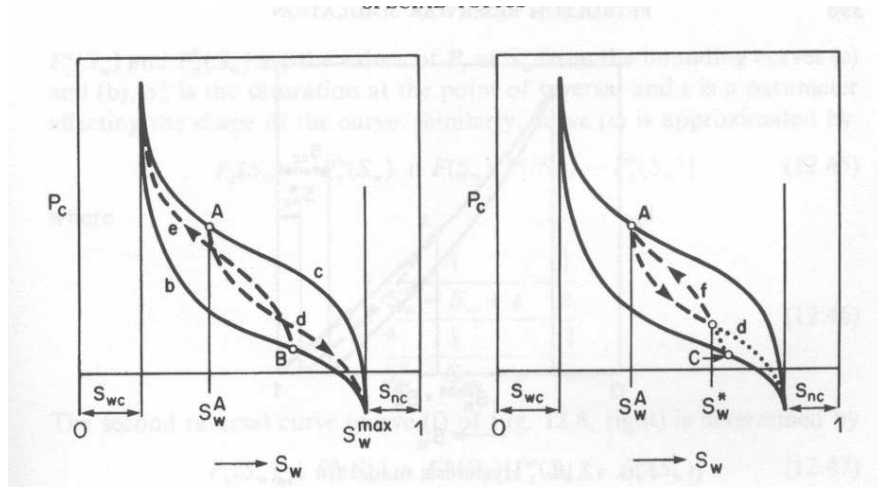
### 3.1 Capillary pressure hysteresis

It has long been recognized that capillary pressure curves exhibit a strong dependence on the direction of saturation changes. A typical situation for a water-oil system is sketched in Figure 7. Three *bounding* curves need to be defined: (a) primary drainage, (b) pendular imbibition, and (c) secondary drainage. These three curves are obtained when the displacements in either direction are taken to their ultimate saturations: irreducible water saturation  $S_{wc}$  in drainage, and residual oil saturation  $S_{or}$  in imbibition. The imbibition capillary pressure curve is *always* below the drainage curve. Note that in a forced imbibition displacement, it is possible to reach negative values of the capillary pressure.

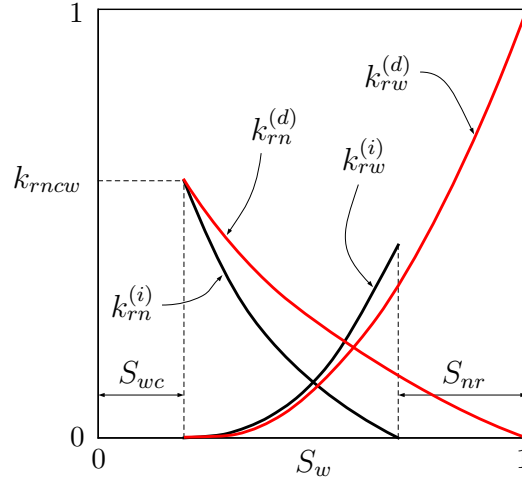
Intermediate (*scanning*) curves can then be defined if the flow reversal takes place at an intermediate saturation. The standard treatment of scanning capillary pressure curves is due to Killough [1976]. It is assumed that, if the flow reversal is completed, the scanning curve will define a closed loop (Figure 8).



**Figure 7.** Capillary pressure hysteresis: bounding curves (from Aziz and Settari [1979]).



**Figure 8.** Capillary pressure hysteresis: scanning curves (from Aziz and Settari [1979]).



**Figure 9.** Two-phase relative permeability hysteresis: bounding curves corresponding to primary drainage and pendular imbibition.

### 3.2 Two-phase relative permeability hysteresis

It is by now well established that relative permeability curves are also subject to hysteresis. In Figure 9 we show typical bounding curves (primary drainage and pendular imbibition) for a two-phase system. It is noticeable that the wetting-phase relative permeabilities exhibit small hysteretic effects. The nonwetting-phase imbibition relative permeabilities, on the other hand, display a considerable reduction due to trapping by the wetting phase.

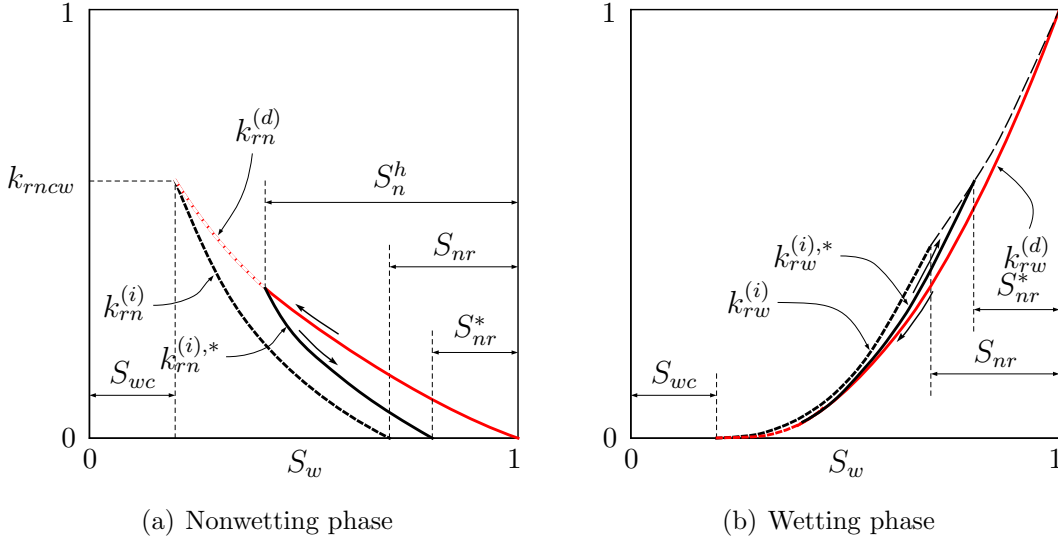
The bounding curves are used to define scanning curves, much in the same way as for capillary pressure curves. The most important quantity is the trapped nonwetting-phase saturation  $S_{nr}^*$  after a flow reversal (see Figure 10(a)). All current models of relative permeability hysteresis are based on the trapping model proposed by Land [1968]:

$$S_{nr}^* = \frac{S_n^h}{1 + CS_n^h}, \quad (17)$$

where  $S_n^h$  is the nonwetting-phase saturation at the reversal, and  $C$  is the Land parameter:

$$C = \frac{1}{S_{nr}} - \frac{1}{1 - S_{wc}}. \quad (18)$$

Different functional forms for the imbibition relative permeability curve  $k_{rn}^{(i),*}$



**Figure 10.** Two-phase relative permeability hysteresis: scanning curves.

after reversal have been proposed [Carlson, 1981; Killough, 1976; Land, 1968].

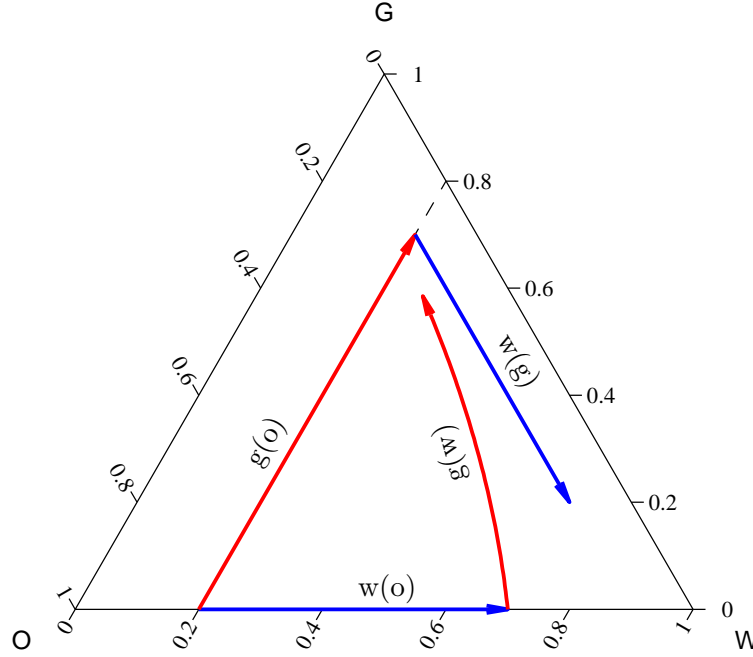
Hysteresis in the wetting-phase relative permeability can be treated similarly. However, the dependence on the trapped nonwetting-phase saturation is usually much smaller (see Figure 10(b)).

### 3.3 Three-phase relative permeability hysteresis

Three-phase flow descriptors are obviously subject to hysteresis as well. In fact, hysteretic effects in three-phase flow systems are often stronger than in two-phase flow. This issue becomes particularly relevant for the simulation of water-alternating-gas (WAG) processes.

Typically, hysteresis in the oil relative permeability is incorporated by using history-dependent two-phase data in the three-phase relative permeability models described in Section 2. However, this effect alone is insufficient to reproduce flow behavior in WAG systems. The reason is twofold:

1. Imbibition water relative permeabilities *after a gas flood* are significantly lower.
2. Drainage gas relative permeabilities *after a waterflood* are also significantly lower.



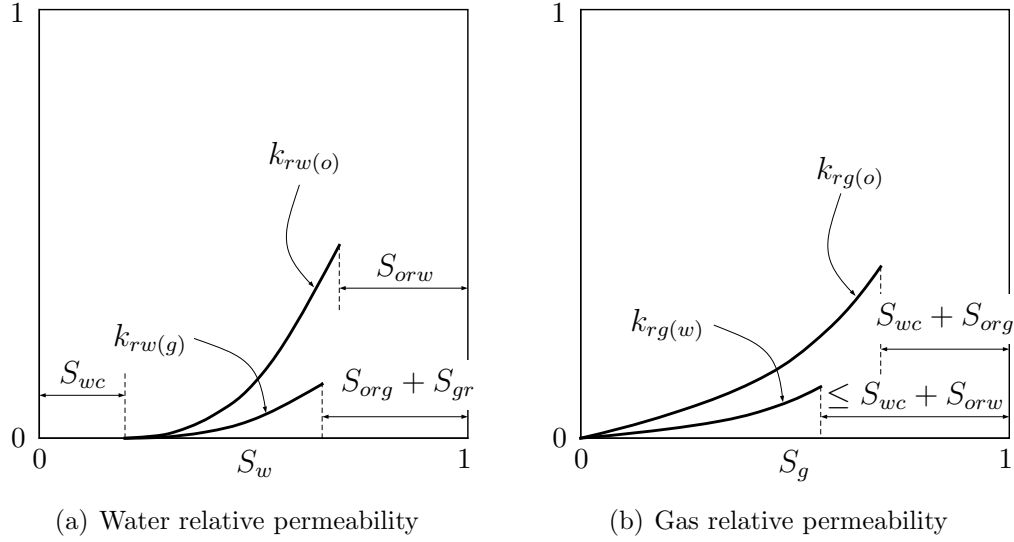
**Figure 11.** Saturation path of “two-phase” relative permeability experiments for WAG modeling. Paths  $w(o)$  and  $w(g)$  correspond to increasing water saturation, and paths  $g(o)$  and  $g(w)$  to increasing gas saturation.

From these observations, it is clear that the two-phase relative permeability data traditionally employed (Figure 1) are insufficient. For the simulation of WAG, it is imperative that relative permeabilities are measured along two additional saturation paths, corresponding to each of the processes described above (Figure 11).

In Figure 12 we show water and gas relative permeabilities typical of water-wet media for each of these saturation paths. According to the previous observations:

1. The  $k_{rw(g)}$  curve is always below the  $k_{rw(o)}$  curve.
2. The  $k_{rg(w)}$  curve is always below the  $k_{rg(o)}$  curve.

These curves can (and should) be used to estimate the relative permeabilities to water and gas in a three-phase flow situation using, for example, a



**Figure 12.** Three-phase relative permeabilities to water (left) and gas (right). Each curve corresponds to one of the paths shown in Figure 11.

saturation-weighted interpolation like the one proposed by Baker [1988] for the oil relative permeability. Hysteresis can be incorporated using the scheme described in Section 3.2 for the wetting and nonwetting phase.

## 4 Notes

We give in this section an account of existing relative permeability models, which we organize in the following categories:

1. *Capillary models.* The first three-phase relative permeability model should be attributed to Rose [1949]. He used an analogy of the Kozeny-Carman equation [Carman, 1937; Kozeny, 1927] for multiphase flow, and the capillarity theory of Leverett [1941], to obtain wetting-phase relative permeabilities. Extension to the nonwetting and intermediate-wetting phases was done using heuristic arguments. Similar expressions were obtained by Purcell [1949] using the analogy of a bundle of capillary tubes, and Rapoport and Leas [1951] using thermodynamic arguments. Later modifications of the Rose-Purcell model to

account for tortuosity of the porous medium include those of Fatt and Dykstra [1951] and Burdine [1953]. Corey, Rathjens, Henderson and Wyllie [1956] and Brooks and Corey [1966] used the model of Burdine [1953] with particularly simple capillary pressure curves.<sup>1</sup> Models of non-wetting and intermediate-wetting phase trapping were proposed by Naar and Henderson [1961], Naar and Wygal [1961] and Land [1968] for imbibition relative permeabilities. A capillary model of relative permeabilities similar to that of Rose-Purcell was derived by Mualem [1976] for two phase flow, and later used by Lenhard and Parker [Lenhard and Oostrom, 1998; Lenhard and Parker, 1987; Lenhard et al., 1989; Parker and Lenhard, 1987; Parker et al., 1987] in a sophisticated three-phase hysteretic model of multiphase flow constitutive relations.

2. *Probability models.* These models estimate three-phase oil relative permeability from two-phase data, relying on the assumption that “each fluid establishes its own tortuous paths, which form very stable channels” [Bear, 1972, p. 457]. Probably, the most popular three-phase relative permeability models in petroleum engineering are the Stone I and Stone II models [Stone, 1970, 1973], normalized by Aziz and Settari [1979, pp. 33–37]. Different interpolation schemes for the residual oil saturation, which is a free parameter in the Stone I model, are given in Fayers and Matthews [1984] and Fayers [1987]. Alemán and Slattery [1988], and Robinson and Slattery [1994] have proposed a model similar to Stone I.
3. *Power-law models.* These purely empirical models have been used both in two-phase [Irmay, 1954] and three-phase flow [Delshad and Pope, 1989].
4. *Saturation-weighted models.* The first saturation-weighted interpolation of two-phase relative permeabilities was proposed by Baker [1988]. Improved saturation-weighted models, which account for trapping, are presented in Jerauld [1997] and Blunt [2000], among others.

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<sup>1</sup>Contrary to the wide-spread terminology of “Corey-type models” as models in which the relative permeability of each phase depends solely on the saturation of that phase, Corey et al. [1956] proposed a model of oil relative permeability which depends on *all* fluid saturations, so that the oil isoperms are *not* straight lines (see Equation (1) and Figure 4 of Corey et al. [1956]).



5. *Pore-network and process-based models.* The quasi-static pore-network models of multiphase flow allow one to predict relative permeabilities in two-phase [Al-Futaisi and Patzek, 2003b; Blunt, 2001; Blunt and King, 1991; Øren et al., 1998; Patzek, 2001], and three-phase flow [Al-Futaisi and Patzek, 2003a; Fenwick and Blunt, 1998a,b; Lerdahl and Øren, 2000; Piri and Blunt, 2002; van Dijke et al., 2002], if topology and geometry of the pore space are known [Øren and Bakke, 2002]. Recently, van Dijke, McDougall and Sorbie [2001] and van Dijke, Sorbie and McDougall [2001] proposed three-phase relative permeability models with “correct physics” limited to a bundle of capillaries.

A number of papers provide a comparison of different models, and discuss their success (or lack of success) at replicating experimental data [Baker, 1988; Delshad and Pope, 1989; Hicks Jr. and Grader, 1996; Oak, 1990].

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