Effective Solvers for Reservoir Simulation

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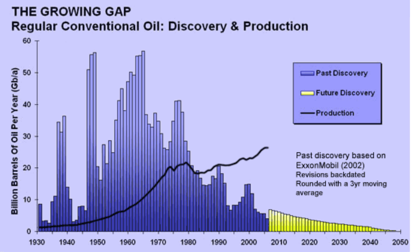
- Collaboration with:
  - ExxonMobil Upstream Research Company
  - China National Offshore Oil Cooperation
  - Monix Energy Solutions
- Main Collaborators:
  - James Brannick (PSU), Yao Chen (PSU), Chunsheng Feng (XTU), Panayot Vassilevski (LLNL), Jinchao Xu (PSU), Chensong Zhang (CAS), Shiquan Zhang (Fraunhofer ITWM), and Ludmil Zikatanov (PSU)

Outline

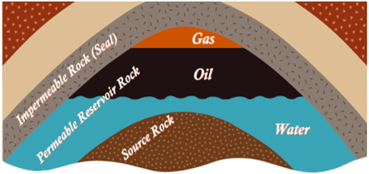
World without Oil?

List of Oil Products:
Antiseptics, Aspirin, Auto Parts, Ballpoint pens, Candles, Cosmetics, Crayons, Eye Glasses, Fishing Line, Food Packaging, Glue, Hand Lotion, Insect Repellant, Insecticides, Lip Stick, Perfume, Shampoo, Shaving Cream, Shoes, Toothpaste, Trash Bags, Vitamin Capsules, Water Pipes, $\cdots$.

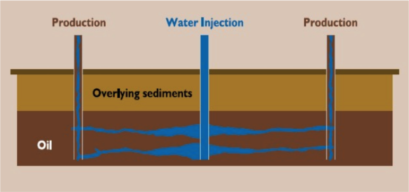


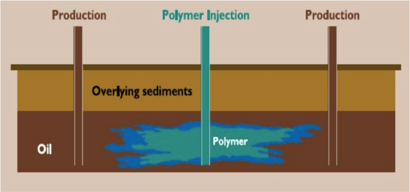
Global oil discovery peaked in the late 1960s. Now we consume much more than the amount we discover and this gap is still growing!
(http://www.aspo-ireland.org/)

Oil Recovery



- Primary Recovery
- Secondary Recovery
- Enhanced Oil Recovery

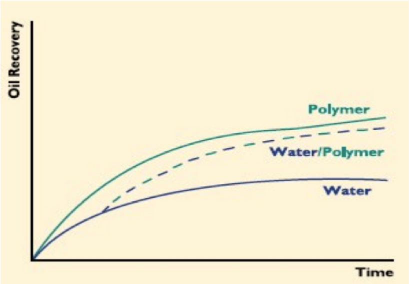




Enhanced Oil Recovery

Enhanced Oil Recovery (EOR) is a generic term for techniques for increasing the oil recovery rate from an oil field:

- Thermal Method
- Microbial Injection
- Gas Injection
- Chemical Flooding
  - Alkaline
  - Polymer
  - Surfactant
  - Gel
  - ...
- ...



Recovery Rate:

- Primary and Secondary recovery: 20-40%
- Enhanced oil recovery: 30-60%

Black Oil Model

Mass Conservation:

$$\frac{\partial}{\partial t}\left[\phi\left(\frac{S_o}{B_o}+\frac{R_v S_g}{B_g}\right)\right] = -\nabla\cdot\left(\frac{1}{B_o}u_o+\frac{R_v}{B_g}u_g\right)+\tilde{q}_O$$
$$\frac{\partial}{\partial t}\left[\phi\left(\frac{S_g}{B_g}+\frac{R_s S_o}{B_o}\right)\right] = -\nabla\cdot\left(\frac{1}{B_g}u_g+\frac{R_s}{B_o}u_o\right)+\tilde{q}_G$$
$$\frac{\partial}{\partial t}\left(\phi\frac{S_w}{B_w}\right) = -\nabla\cdot\left(\frac{1}{B_w}u_w\right)+\tilde{q}_W$$

Darcy's Law:

$$u_\alpha = -\frac{k k_{r\alpha}}{\mu_\alpha}(\nabla P_\alpha - \rho_\alpha \mathfrak{g}\nabla z), \quad \alpha = o, g, w$$

Constitutive Laws:

$$S_o + S_g + S_w = 1, \quad P_{cow} = P_o - P_w, \quad P_{cog} = P_g - P_o,$$

Modified Black Oil Model

Add polymer and sodium chloride:

$$\frac{\partial}{\partial t}\left(\frac{\phi S_w^* C_P}{B_r B_w}\right) + \frac{\partial}{\partial t}\left(\rho_r (1-\phi) C_a\right) = -\nabla\left(\frac{C_P}{B_w} u_P\right) + \tilde{q}_W C_P$$

$$\frac{\partial}{\partial t}\left(\frac{\phi S_w C_N}{B_r B_w}\right) = -\nabla\left(\frac{C_N}{B_w} u_N\right) + \tilde{q}_W C_N$$

Effects on Viscosity:

$$u_w = -\frac{k k_{rw}}{\mu_{w,eff} R_k}(\nabla P_w - \rho_w \mathfrak{g} \nabla z)$$

Add more components (brine, gel, surfactant, $\cdots$)

Numerical Method

- **Discretization in time:**
  - Fully Implicit Method (FIM)
    (Douglas, Peaceman & Rachford 1959)
  - Implicit Pressure Explicit Saturation (IMPES)
    (Sheldon, Zondek & Cardwell 1959; Stone & Garder 1961)
  - Sequential Solution Method (SSM)
    (MacDonald & Coats 1970)
  - Adaptive Implicit Method (AIM)
    (Thomas & Thurnau 1983)
- **Discretization in space:** finite difference, finite volume, finite element, etc.
- **Linearization:** Newton-Raphson Method.
- **Grid:** Structured grids are still used by the main-stream; but unstructured grids are catching up.

Fully Implicit Method

$$\frac{1}{\Delta t}\left\{\left[\phi\left(\frac{S_g}{B_g}+\frac{R_s S_o}{B_o}\right)\right]^{n+1}-\left[\phi\left(\frac{S_g}{B_g}+\frac{R_s S_o}{B_o}\right)\right]^{n}\right\}=$$
$$\nabla\cdot\left(T_g^{n+1}\nabla\Phi_\alpha^{n+1}+R_s^{n+1}T_o^{n+1}\nabla\Phi_o^{n+1}\right)$$

$$\frac{1}{\Delta t}\left[\left(\frac{\phi S_w}{B_w}\right)^{n+1}-\left(\frac{\phi S_w}{B_w}\right)^{n}\right]=\nabla\cdot\left(T_w^{n+1}\nabla\Phi_w^{n+1}\right)$$

$$\frac{1}{\Delta t}\left[\left(\frac{\phi S_o}{B_o}\right)^{n+1}-\left(\frac{\phi S_o}{B_o}\right)^{n}\right]=\nabla\cdot\left(T_o^{n+1}\nabla\Phi_o^{n+1}\right)$$

Potential:

$$\Phi_\alpha := P_\alpha - \rho_\alpha \mathfrak{g} z, \quad \alpha = w, o, g,$$

Transmissibility:

$$T_\alpha := \frac{k_{r\alpha}k}{\mu_\alpha B_\alpha}, \quad \alpha = w, o, g.$$

Newton-Raphson Method

In each Newton Iteration, we need to solve the following Jacobian system:

$$\begin{bmatrix} J_{gP} & J_{gS_w} & J_{gS_o} \\ J_{wP} & J_{wS_w} & J_{wS_o} \\ J_{oP} & J_{oS_w} & J_{oS_o} \end{bmatrix} \begin{bmatrix} \delta P \\ \delta S_w \\ \delta S_o \end{bmatrix} = \begin{bmatrix} R_g \\ R_w \\ R_o \end{bmatrix}$$

Different blocks have different properties, for example

$$J_{gP} = \frac{1}{\Delta t} c_{gP} - \nabla \cdot \mathcal{D}_{gP} \nabla - \mathcal{C}_{gP} \cdot \nabla - \mathcal{R}_{gP},$$

usually the diffusion term in J_{gP} is dominating, which makes J_{gP} like an elliptic problem. However,

$$J_{oS_o} = \frac{1}{\Delta t} c_{oS_o} - \mathcal{C}_{oS_o} \cdot \nabla - \mathcal{R}_{oS_o},$$

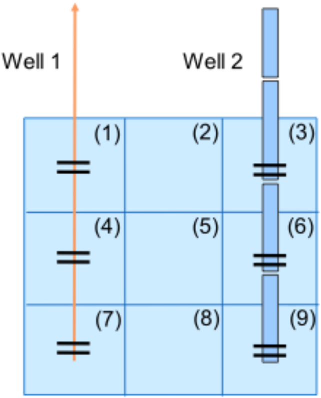
here convection term is dominating, which makes J_{oS_o} like a transport problem.

Including Wells

Peaceman Model

- One-dimentional radial flow
- Single-phase flow in the wellbore

$$q_{i,\alpha} = -\sum_{j}^{N_{perf}} \frac{2\pi h_{perf} K k_{r\alpha}}{\ln(\frac{r_e}{r_w+s})\mu_\alpha}(P_j - P_{\text{bhp}} - H_{wj})$$



Jacobian system with wells:

$$\begin{bmatrix} J_{RR} & J_{RW} \\ J_{WR} & J_{WW} \end{bmatrix} \begin{bmatrix} \delta_R \\ \delta_W \end{bmatrix} = \begin{bmatrix} R_R \\ R_W \end{bmatrix}$$

Numerical Challenges

- Strongly coupled and complicated PDE system
- High nonlinearity
- Complicated geometry: irregular domain with faults, pinch-out and erosion
- Heterogeneous porous media; large permeability jumps
- Complex wells
- Unstructured grids: corner point grid, anisotropic mesh
- Highly non-symmetric and indefinite large-scale Jacobian system
- Different properties of the physical quantities (the equation that describes the pressure is mainly elliptic, the equation that describes saturation is mainly hyperbolic)
- Commercial Simulators set a high bar: ECL2009 (Schlumberger), STARS (CMG), VIP (Halliburton), $\cdots$

Goal: develop fast solvers, deliver a solver package

Solvers

"For a reservoir simulation with a number of gridblocks of order 100,000, about 80% – 90% of the total simulation time is spent on the solution of linear systems with the Jacobian."

—- Chen, Huan, & Ma, *Computational Methods for Multiphase Flows in Porous Media*, 2005

Solvers

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- Direct solvers: (Price & Coats 1974)
- Incomplete factorization preconditioner: (Watts 1981, Behie & Vinsome 1982; Meyerink 1983; Appleyard & Cheshire 1983)
- Constrained Pressure Residual (CPR) preconditioner: (Wallis 1983; Wallis,Kendall, Little & Nolen 1985; Aksoylu & Klie 2009)
- Algebraic Multigrid (AMG) Method :
  - Used in CPR preconditioner (pressure equation): (Klie 1997; Lacroix, Vassilevski & Wheeler 2001, 2003; Scheichl, Masson & Wendebourg 2003; Cao, Tchelepi, Wallis & Yardumian 2005; Hammersley & Ponting 2008; Dubois, Mishev & Zikatanov 2009; Al-Shaalan, Klie, Dogru & Wheeler 2009; Jiang & Tchelepi 2009)
  - For whole Jacobian system: (Papadopoulos & Tchelepi 2003; Stüben 2004; Clees & Ganzer 2007)

Auxiliary Space Preconditioning

1. construct preconditioner via auxiliary (simpler) problems
2. solve the auxiliary problems by efficient solvers (such as multigrid)
3. apply preconditioned Krylov subspace methods

Examples:

- Fictitious Domain Methods (Nepomnyaschikh 1992)
- Method of Subspace Correction (Xu 1992)
- Auxiliary Space Method (Xu 1996)
- Nodal Auxiliary Space Preconditioning in $H(curl)$ and $H(div)$ spaces (Hiptmair & Xu 2007)
- ······

Auxiliary Space Method (Xu 1996)

- Abstract problem: $A : V \to V$ is SPD

$$Au = f$$

- Auxiliary spaces:

$$\bar{V} = V \times W_1 \times \cdots \times W_J$$

- Additive preconditioner:

$$B = S + \sum_{j=1}^{J} \Pi_j \bar{A}_j^{-1} \Pi_j^*$$

With $\Pi_j : W_j \mapsto V$ (transfer) and $S : V \mapsto V$ (smoother). It can be shown that under some assumptions

$$\kappa(BA) \leq C$$

Auxiliary Space Method for Reservoir Simulation

Fast Auxiliary Space Preconditioning

Auxiliary Spaces:

$$\bar{V} = V \times W_P \times W_S \times W_{well}$$

FASP Algorithm: Given u_0, $Bu_0 := u_4$ where

$$
\begin{aligned}
u_1 &= u_0 + \Pi_{well} A_{well}^{-1} \Pi_{well}^* (f - Au_0) \\
u_2 &= u_1 + \Pi_S A_S^{-1} \Pi_S^* (f - Au_1) \\
u_3 &= u_2 + \Pi_P A_P^{-1} \Pi_P^* (f - Au_2) \\
u_4 &= u_3 + S(f - Au_3)
\end{aligned}
$$

Auxiliary problems A_{well}, A_S and A_P are solved approximately.

Single Phase Flow (pressure equation)

Intergrated Reservoir Performance Prediction
ExxonMobil Upstream Research Company

Possion-like Model Problem (After linearization):

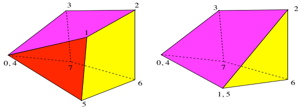
$$
\begin{aligned}
-\nabla \cdot (a \nabla p) + cp = f, &\quad \text{in } \Omega \\
p = g_D, &\quad \text{on } \partial\Omega_D, \\
(a \nabla p) \cdot \boldsymbol{n} = g_N &\quad \text{on } \partial\Omega_N,
\end{aligned}
$$

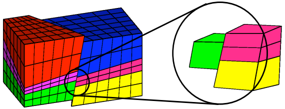
Discretization: Hybrid-Mixed FEM (Kuznetsov & Repin 2003)
Linear system: Symmetric and Positive definite

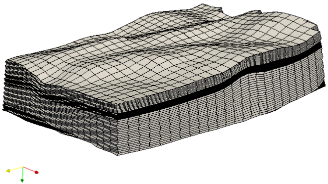
Difficulties for the solver

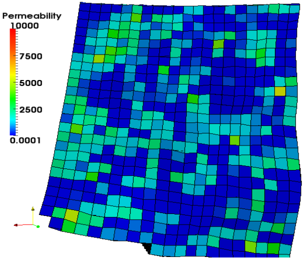
Models are from real problems, the geometry is complicated

- Mesh (hexahedral grid): Highly unstructured; Degenerated, nonmatching, and distorted gridblock.
- Permeability: Large jumps: vary form 10^{-4} to 10^{4}; Anisotropic.









AMG with ILU smoother?

Preconditioner	#iter	Setup time	Solve time
ILU(0)	3458	0.16	96.19
AMG	362	0.85	40.32
ILU(0)/1+AMG	2255	1.00	305.17
ILU(0)/2+AMG	-	1.18	-

- ILU or AMG alone does not work well
- Using ILU as smoother in AMG may not work

References about ILU smoother: Wittum 1989; Stevenson 1994

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Why AMG with ILU smoother does not work in this case?

Consider (B denotes ILU and S denotes AMG)

$$u \leftarrow u + B(f - Au), u \leftarrow u + S(f - Au), u \leftarrow u + B(f - Au).$$

This gives:

$$I - \bar{B}A = (I - BA)(I - SA)(I - BA).$$

$\bar{B}$ might not be positive definite, and cannot be applied to PCG

References about ILU smoother: Wittum 1989; Stevenson 1994

AMG + ILU!

Consider

$$u \leftarrow u + S(f - Au), u \leftarrow u + B(f - Au), u \leftarrow u + S(f - Au).$$

This gives:

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Theorem (Falgout, H., Wu, Xu, Zhang, Zhang & Zikatanov 2011)

Assume that $S : V \to V$ satisfies $\|(I - SA)x\|_A \leq \|x\|_A, \forall x \in V$ and operator $B : V \to V$ is SPD. Then the operator $\widetilde{B}$ is SPD.

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Assume that $((I - SA)v, v)_A \leq \rho(v, v)_A$, $\rho \in [0, 1)$, B is SPD and the condition number of BA is $\kappa(BA) = m_1/m_0$. If $m_1 > 1 \geq m_0 > 0$. then $\kappa(\widetilde{B}A) \leq \kappa(BA)$, Furthermore, if $\rho \geq 1 - \frac{m_0}{m_1 - 1}$, then $\kappa(\widetilde{B}A) \leq \kappa(SA)$.

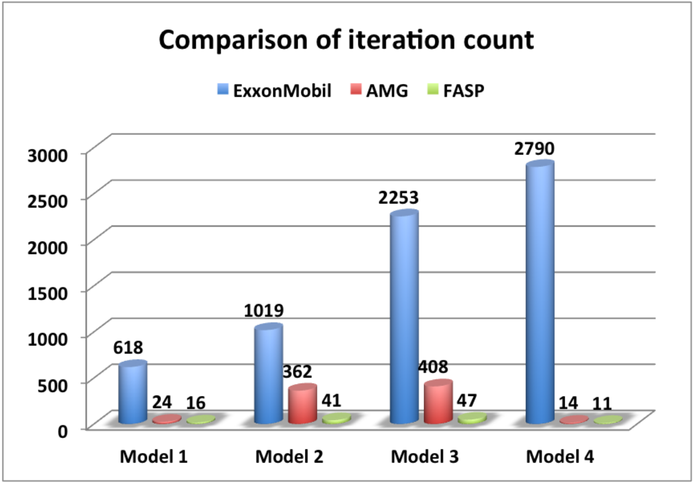
ExxonMobil Test Problems

Preconditioner	#iter	Setup time	Solve time
ILU(0)	3458	0.16	96.19
AMG	362	0.85	40.32
ILU(0)/1+AMG	2255	1.00	305.17
ILU(0)/2+AMG	-	1.18	-
FASP (AMG+ILU)	41	0.99	5.83

	Model 1	Model 2	Model 3	Model 4
size	388,675	156,036	287,553	312,851
nnz	4,312,175	1,620,356	3,055,173	3,461,087

	#iter	Setup time (s)	Solve time (s)	Total time (s)
FASP(AMG+ILU) + CG				
Model 1	16	2.12	4.63	6.75
Model 2	41	0.99	5.83	6.82
Model 3	47	1.97	12.81	14.78
Model 4	11	1.75	2.68	4.43

ExxonMobil Test Problems



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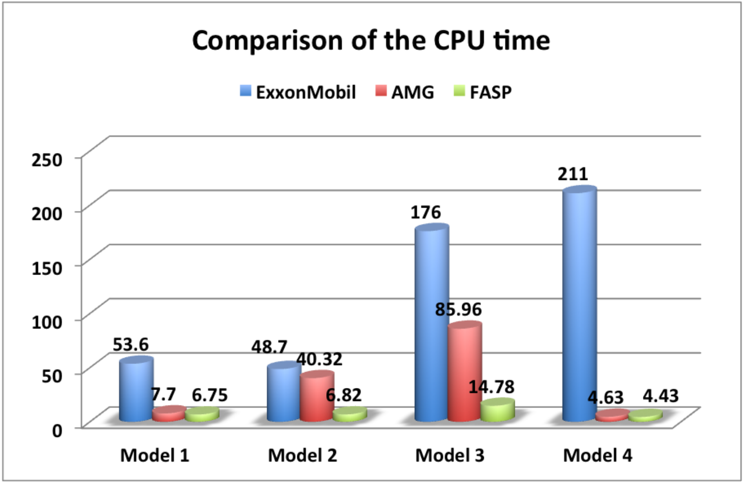
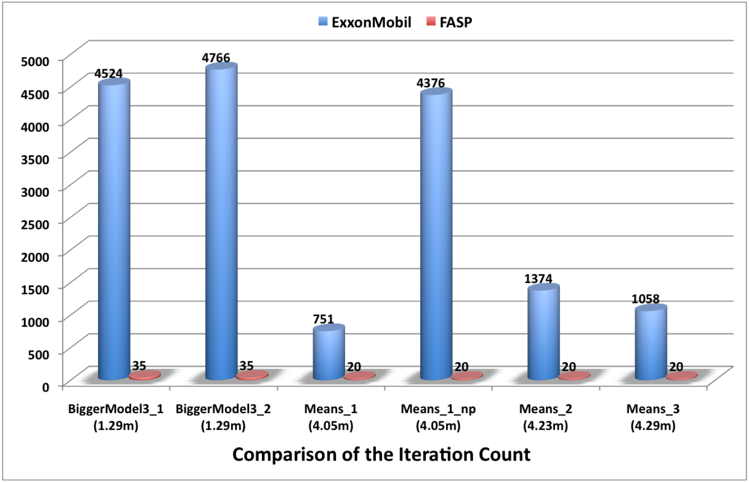


ExxonMobil Large Test Problems



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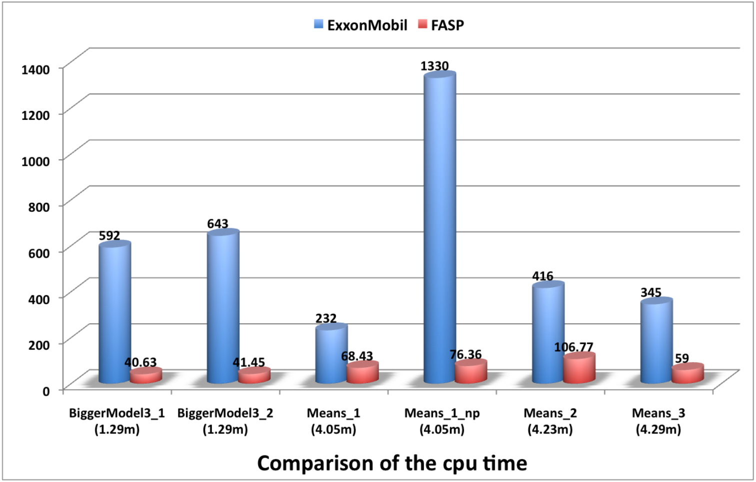


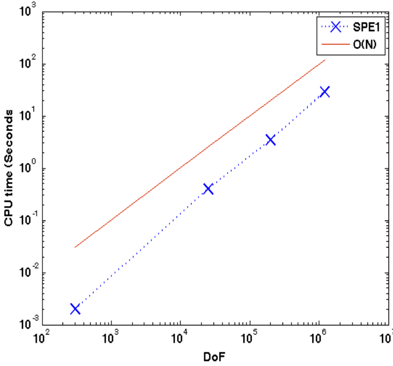
(Modified) Black Oil Model

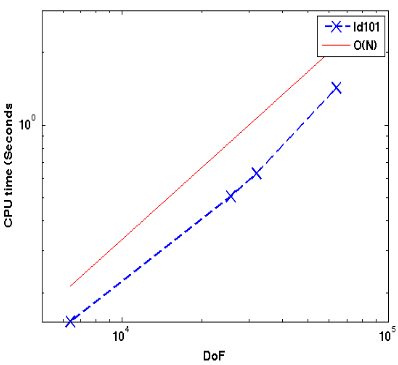
CNOOC and Monix: SOCF Simulator

- Decoupling: Alternative Block Factorization (Bank, Chan, Coughran & Smith 1989)
- Auxiliary problems:
  - W_P: pressure (+ bottom hole pressure)
  - W_S: saturation (+ concentrations)
  - W_{well}: wells (+ perforations)
- Solvers for each auxiliary problems:
  - A_P: AMG (+ILU)
  - A_S: Block Gauss-Seidel with downwind ordering and crosswind blocks (Bey & Wittum 1996; Hackbush & Probst 1997; Wang & Xu 1999; Kwok 2007)
  - A_{well}: ILU/Direct solver
- Smoother S: Block Gauss-Seidel with downwind ordering and crosswind blocks.
- Krylov iterative method: GMRes.

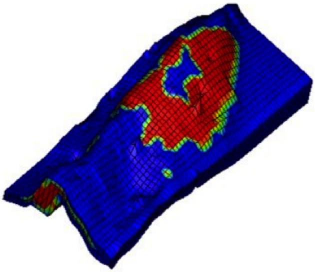
Ref: H., Liu, Qin, Xu, Yan and Zhang 2011, SPE 148388

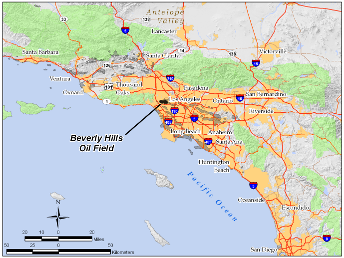
Optimality





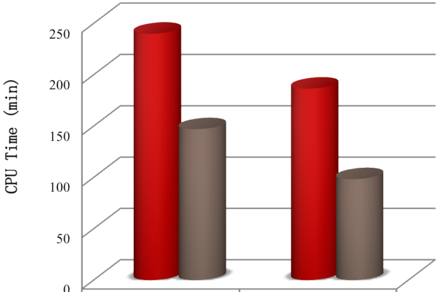
East Beverly Oil Field Test





- Black Oil Model
- Unstructured grid
- 83,592 grid blocks
- 169 wells
- Faults: 994 non-local connections
- 40 years of water flooding

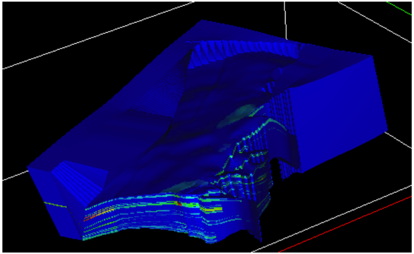
East Beverly Oil Field Test



	1-Core 2.4GHz	4-Core 2.8GHz
SOCF+ILU	240	186.5
SOCF+FASP	147	98.3

We use sequential version of FASP in this table. OpenMP version of FASP costs 46 min (8-core, 2.8GHz).

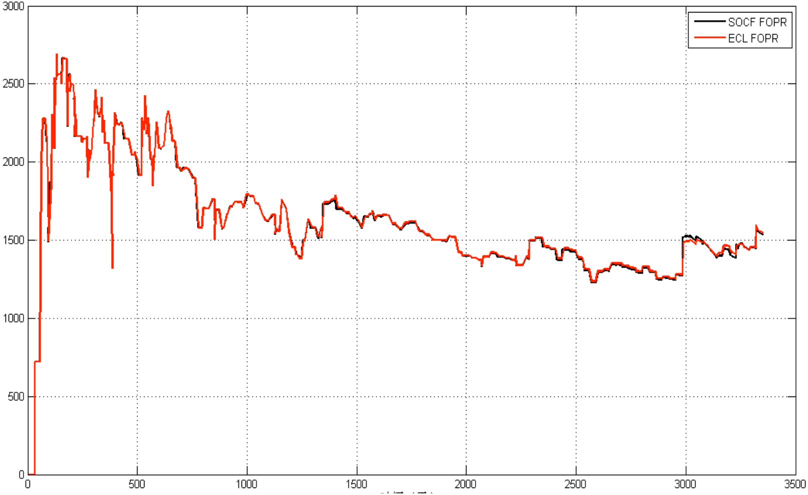
JZ9-3 Oil Field Test



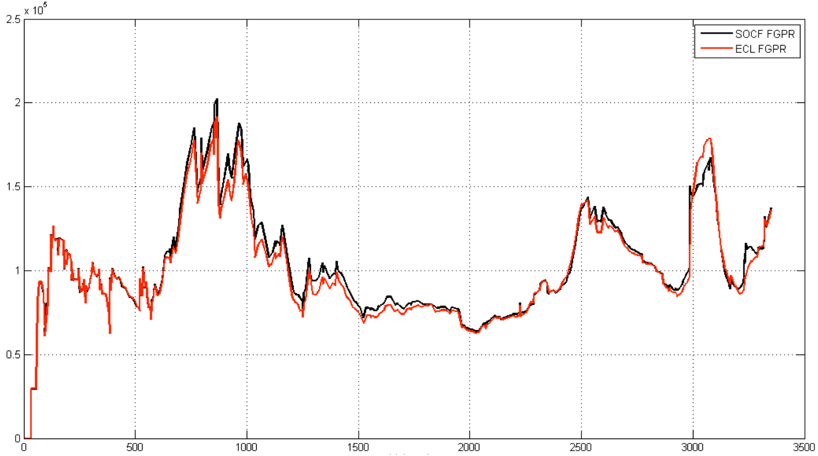
- Polymer flooding (5-compoment)
- Number of grids: 157x53x57
- Lots of inactive grids
- Several faults
- 37 wells
- 10 years simulation in total
- 3 years of polymer flooding

Simulator	# Newton Iter	Total CPU Time	CPU Time/Newton
ECL2009	1562	81.0 (min)	3.11 (sec)
SOCF(FASP)	1653	40.0 (min)	1.45 (sec)

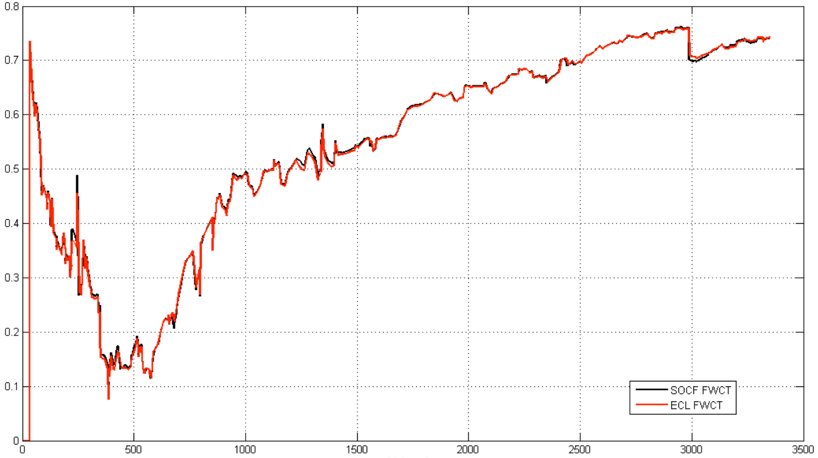
JZ9-3 Results Comparison: Oil Production Rate



JZ9-3 Results Comparison: Gas Production Rate



JZ9-3 Results Comparison: Water Cut



ld101-1 7-component Test

PENNSTATE

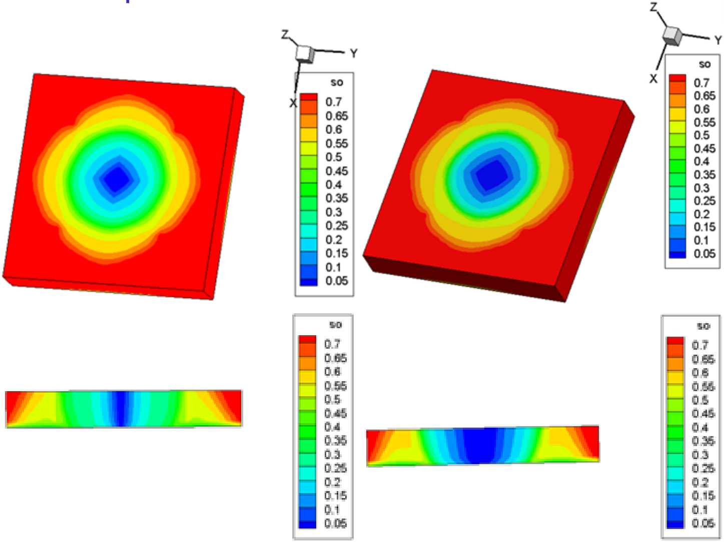
- Polymer-Gel flooding
- 5 wells: 1 inject well, 4 product wells
- 32 years simulation in total

No well				
	ILU		FASP	
Size	# Iter	Time (s)	# Iter	Time (s)
40x40x4	37	2.20	12	0.47
40x40x20	153	19.59	15	2.86
40x40x40	296	56.05	13	4.11

5 wells				
	ILU		FASP	
	# Iter	Time (s)	# Iter	Time (s)
40x40x4	55	2.70	13	0.50
40x40x20	236	28.21	15	2.90
40x40x40	319	65.04	14	5.78

PENNSTATE

Id101-1 7-component Test



Oil Saturation (Left: Polymer; Right: Polymer/Gel)

Move to GPU

NVIDIA

Parallel AMG algorithm suits GPU:

- Regular sparsity pattern
- Low complexity
- Low setup cost
- Fine-grain parallelism
- Take advantage of hierarchical memory structure

Our choice: Unsmoothed Aggregation AMG (UA-AMG)

- Controllable sparsity pattern
- Low complexity (less fill-in)
- Low setup cost (no triple matrices multiplication)
- Large potential of parallelism

UA-AMG with Nonlinear AMLI-cycle

Problem: UA-AMG with V-cycle usually dose not converge uniformly.

UA-AMG with Nonlinear AMLI-cycle

Problem: UA-AMG with V-cycle usually dose not converge uniformly.

Solution: UA-AMG with Nonlinear AMLI-cycle (Variable AMLI-cycle / K-cycle).

Nonlinear AMLI-cycle: $\hat{B}_k[\cdot] : V_k \longrightarrow V_k$.

Assume $\hat{B}_1[f] = A_1^{-1} f$, and $\hat{B}_{k-1}[\cdot]$ has been defined, then for $f \in V_k$

Pre-smoothing $u_1 = R_k f$;

Coarse grid correction $u_2 = u_1 + \tilde{B}_{k-1}[Q_{k-1}(f - A_k u_1)]$

Post-smoothing $\hat{B}_k[f] := u_2 + R_k^t (f - A_k u_2)$.

Nonlinear coarse grid correction: $\tilde{B}_{k-1}[\cdot] : V_{k-1} \longrightarrow V_{k-1}$

Defined by n iterations of a Krylov subspace iterative methods using $\hat{B}_{k-1}[\cdot]$ as the preconditioner.

Ref: Axelsson & Vassilevski 1994; Kraus 2002; Kraus & Margenov 2005; Notay & Vassilevski 2008.

Convergence Analysis of Nonlinear AMLI-cycle (SPD)

Theorem (H., Vassilevski & Xu 2011)

Assume the convergence factor of V-cycle MG with fixed level difference k_0 is bounded by $\delta_{k_0} \in [0,1)$, if n is chosen such that $(1-\delta^n)\delta_{k_0} + \delta^n \leq \delta$ has a solution $\delta \in [0,1)$, which is independent of k, then we have

$$\|v - \hat{B}_k[A_k v]\|_{A_k}^2 \leq \delta \|v\|_{A_k}^2 \text{ and } \|v - \tilde{B}_k[A_k v]\|_{A_k}^2 \leq \delta^n \|v\|_{A_k}^2,$$

A sufficient condition of n: $n > \frac{1}{1-\delta_{k_0}} > 1$.

This is a generalization of the result in Notay & Vassilevski 2008.

Primary Results

	1024×1024			2048×2048		
	Setup	Solve	Total	Setup	Solve	Total
CPU	0.80	7/0.69	1.49	3.28	7/2.75	6.03
GPU (CUSP)	0.63	36/0.35	0.98	2.38	41/1.60	3.98
GPU (FASP)	0.13	19/0.47	0.60	0.62	19/2.01	2.63

- Processor:
  - CPU: Intel i7-2600 3.4 GHz
  - GPU: Tesla C2070
- Algorithm:
  - CPU: Classical AMG + V-cycle;
  - GPU(CUSP): Smoothed Aggregation AMG + V-cycle;
  - GPU(FASP): UA-AMG + Nonlinear AMLI-cycle

Ref: Brannick, Chen, H., Xu, Zikatanov, 2011

Other ongoing work

- Compositional Model (Equation of State)
- Complex wells
- Improve robustness of FASP solvers
- Improve robustness of nonlinear iteration
- Heterogeneous parallel computing

Conclusions

- We developed effective solvers for reservoir simulation, which works well for reservoirs from real world.
- We developed a solver package (FASP). Part of the solver package has be used for the chemical flooding simulator (SOCF) by Monix Energy Solutions and CNOOC.
- We are working on parallelization of FASP package, including OpenMP, MPI and CUDA.
- We are working on the robustness of our package.

Thank you!