A New Representation of Wells in Numerical Reservoir Simulation

Yu Ding, SPE, and **Gérard Renard,** SPE, Inst. Français du Pétrole

Summary. Numerical PI's are used to relate wellblock and wellbore pressures and the flow rate of a well in reservoir simulations by finite differences. This approach is based on an "equivalent wellblock radius," $r_{eq,o}$. When nonuniform grids are used, $r_{eq,o}$ may create an error in wellbore pressure or oil rate. This paper presents a new well representation. The analytical solution for near-well pressure is included by modifying the transmissibilities between gridblocks so that flow around a well is described fully. The new method is applicable to nonuniform grids and nonisolated wells.

Introduction

Because finite-difference methods are used in reservoir simulation, well models are used to relate wellbore to wellblock pressure.[1–9] These models assume a radial flow around the well so that a logarithmic relation is satisfied between the wellblock pressure, p_o, and the wellbore pressure, p_w, by

$$p_w - p_o = \frac{q\mu}{2\pi kh}\ln\frac{r_w}{r_{eq,o}}, \quad (1)$$

where $r_{eq,o}$ = equivalent wellblock radius.

Peaceman[6] first demonstrated numerically that the $r_{eq,o}$ should be equal to $0.2\Delta x$ for square gridblocks and derived an analytical expression for $r_{eq,o}$. In a later study,[2] he showed that this analytical expression could not be used in cases of grids with nonsquare blocks. However, he provided other formulas to calculate $r_{eq,o}$ in uniform grids in isotropic and anisotropic systems. Babu *et al.*[3] gave a general analytical expression for $r_{eq,o}$ in uniform grids when the drainage area of the well is a rectangle. As pointed out by both Peaceman[1,2] and Babu *et al.*,[3] the formulas they provided are irrelevant to simulations on nonuniform grids.

Wellblock pressure calculated by finite-difference methods depends mainly on grid configuration[6,9] and numerical scheme.[7] Therefore, in nonuniform grids, a general expression cannot be found for $r_{eq,o}$ that relates p_o to p_w with certainty.

In this paper, we propose a new approach based on the finite-volume method to compute p_w. It does not involve the $r_{eq,o}$ concept but does require that the near-well pressure distribution be known. The new approach implies a modification in the calculation of transmissibilities between the wellblock and its neighboring blocks. The approach is general. It can be applied to nonuniform grids and can be viewed as a generalization of Peaceman's[1] analytical method for $r_{eq,o}$.

Following Peaceman's approach, the new well representation is shown for a 2D single-phase flow problem for which the pressure distribution is known. Expressions of the new block transmissibilities are derived for isolated and nonisolated wells in nonuniform grids. Both isotropic and anisotropic cases are treated. The numerical results are more accurate with the new approach than with the numerical PI method, especially in cases of nonuniform grids and nonisolated wells.

New Isolated Well Representation

To illustrate the new approach, we consider the steady-state flow of a single-phase fluid toward an isolated well in a homogeneous porous media. The flow is governed by

$$\frac{kh}{\mu}\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right) = u. \quad (2)$$

A standard five-point finite-difference scheme is generally used for its discretization:

$$\frac{kh}{\mu}\left(\Delta y_j\frac{p_{i+1,j}-p_{i,j}}{\Delta x_{i+\frac{1}{2}}} + \Delta y_j\frac{p_{i-1,j}-p_{i,j}}{\Delta x_{i-\frac{1}{2}}}\right.$$

$$\left. + \Delta x_i\frac{p_{i,j+1}-p_{i,j}}{\Delta y_{j+\frac{1}{2}}} + \Delta x_i\frac{p_{i,j-1}-p_{i,j}}{\Delta y_{j-\frac{1}{2}}}\right) = q. \quad (3)$$

In particular, for the wellblock **(Fig. 1)**, we have

$$\frac{kh}{\mu}\left(\Delta y_o\frac{p_1-p_o}{\Delta x_{\frac{1}{2}}} + \Delta y_o\frac{p_3-p_o}{\Delta x_{-\frac{1}{2}}}\right.$$

$$\left. + \Delta x_o\frac{p_2-p_o}{\Delta y_{\frac{1}{2}}} + \Delta x_o\frac{p_4-p_o}{\Delta y_{-\frac{1}{2}}}\right) = q, \quad (4)$$

where, p_o = wellblock pressure.

Assuming the isolated well is at (x_o,y_o), the pressure at a distance r from the wellbore is given by

$$p(x,y) = p_w + \frac{q\mu}{2\pi kh}\ln\frac{r}{r_w}, \quad (5)$$

where $r = \sqrt{(x-x_o)^2 + (y-y_o)^2}$.

As pressure varies rapidly (logarithmically) near the well, its gradient cannot be approximated by linear relation. Because of the singularity created by the well, the calculated p_o is very different from p_w. To improve pressure-gradient calculations, a finite-volume method[10] is considered.

Formulation With Finite-Volume Method

The finite-volume method applied to the flow equation for the wellblock yields

$$\frac{kh}{\mu}\left(\int_{\Gamma_1}\frac{\partial p}{\partial x}\mathrm{d}y + \int_{\Gamma_2}\frac{\partial p}{\partial y}\mathrm{d}x + \int_{\Gamma_3}\frac{\partial p}{\partial x}\mathrm{d}y + \int_{\Gamma_4}\frac{\partial p}{\partial y}\mathrm{d}x\right) = q. \quad (6)$$

We must find a good approximation for flow terms

$$q_{\Gamma i} = \frac{kh}{\mu}\int_{\Gamma i}\frac{\partial p}{\partial x}\mathrm{d}y \ (i = 1,3) \text{ and } q_{\Gamma i} = \frac{kh}{\mu}\int_{\Gamma i}\frac{\partial p}{\partial y}\mathrm{d}x \ (i = 2,4).$$

When the near-well pressure distribution is known, as in an isolated well case, good approximations can be obtained.

Flow-Term Approximations

This section shows the derivation of $q_{\Gamma 1}$. Extension to the three other directions can be obtained easily.

There are many ways to approximate near-well flow terms; a finite-difference method implies that $q_{\Gamma 1}$ is calculated by

$$\int_{\Gamma 1}\frac{\partial p}{\partial x}\mathrm{d}y = \Delta y_o\frac{p_1-p_w}{\Delta x_{\frac{1}{2}}}. \quad (7)$$

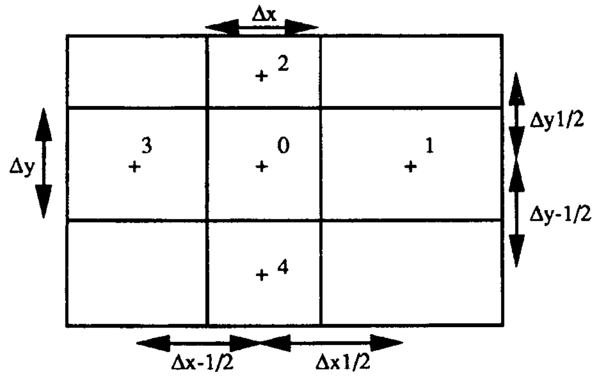


Fig. 1—Wellblock.

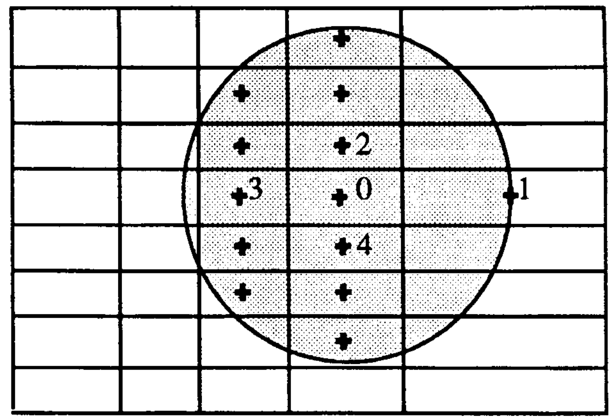


Fig. 2—Extension of L_{eq} around the well.

As a result of the singularity created by the well, this approximation is usually poor but can be improved by use of equivalent wellblock radius, $r_{eq,o}$, or equivalent interblock distance, L_{eq}.

Equivalent Wellblock Radius. The first flow-term approximation involves use of a wellblock pressure, p_o, instead of p_w:

$$\int_{\Gamma_1} \frac{\partial p}{\partial x} dy = \Delta y_o \frac{p_1 - p_o}{\Delta x_{1/2}}. \qquad (8)$$

p_o corresponds to a pressure at $r_{eq,o}$. To get a good approximation for Eq. 8, $r_{eq,o}$ should be given by

$$r_{eq,o1} = \Delta x_{1/2} \exp\left(-2 \frac{\Delta x_{1/2}}{\Delta y_o} \operatorname{arctg} \frac{\Delta y_o}{\Delta x_o} \right). \qquad (9)$$

Appendix A gives the derivation of Eq. 9. The subscript 1 added to $r_{eq,o}$ indicates that this radius depends on the calculated flow term. In the three other directions, radii will be indicated by $r_{eq,o2}$, $r_{eq,o3}$ and $r_{eq,o4}$, respectively. In the special case of square blocks, these four values are equal to $r_{eq,o} = e^{(-\pi/2)} \Delta x \approx 0.208\, \Delta x$. This radius is identical to Peaceman's[1] analytical equivalent wellblock radius. In nonuniform grids, these four radii are usually different.

Equivalent Interblock Distance. The second flow-term approximation method is a substitution of an equivalent interblock distance L_{eq1} to $\Delta x_{1/2}$. Writing the flow-term balance through Γ_1 leads to

$$\int_{\Gamma_1} \frac{\partial p}{\partial x} dy = \Delta y_o \frac{p_1 - p_w}{L_{eq1}}. \qquad (10)$$

As Appendix B shows, L_{eq1} must be equal to

$$L_{eq1} = \frac{\Delta y_o}{2} \frac{\ln \dfrac{\Delta x_{1/2}}{r_w}}{\operatorname{arctg} \dfrac{\Delta y_o}{\Delta x_o}}. \qquad (11)$$

This flow-term approximation allows direct computation of the wellbore pressure. To implement this approximation in a numerical model, $\Delta x_{i+1/2}$, $\Delta x_{i-1/2}$, $\Delta y_{j+1/2}$, and $\Delta y_{j-1/2}$ in Eq. 4 must be replaced by their corresponding equivalent interblock distances L_{eq1} through L_{eq4}, respectively.

In uniform grids with square blocks, the equivalent interblock distance becomes $L_{eq} = (2/\pi) \Delta x \ln(\Delta x / r_w)$.

Comparison of Approximations. These two flow-term approximations give identical results in square-block grids. However, in nonuniform grids, the first approximation is not practical and cannot be used directly. In contrast, the second approach can be applied directly to nonuniform grids and can be generalized as shown in the next section. Therefore, the proposed new representation of wells is based on the second approach.

Extension of L_{eq}

Calculation of L_{eq1} detailed in Appendix B is based on the assumption that the flow is radial only near the well. In fact, depending on grid discretization, the singularity created by a well can affect the pressure-gradient approximation not only for the wellblock itself, as assumed first, but also for several blocks around the wellblock. **Fig. 2** shows that the new approach can be generalized by calculating new transmissibilities between blocks in an area close to the wellblock because of features of the finite-volume method.

Consequently, distances $\Delta x_{i+1/2}$, $\Delta x_{i-1/2}$, $\Delta y_{j+1/2}$, $\Delta y_{j-1/2}$ in Eq. 3 must be replaced by $L_{eq,i+1/2,j}$, $L_{eq,i-1/2,j}$, $L_{eq,i,j+1/2}$, $L_{eq,i,j-1/2}$, respectively. Expressions of these quantities can be easily derived. For instance, the equivalent distance between Blocks (i,j) and $(i+1,j)$ is given by

$$L_{eq,i+1/2,j} = \Delta y_j \frac{\ln\left(r_{i+1,j}/r_{i,j}\right)}{\operatorname{arctg}\left(\dfrac{y_j + 1/2\Delta y_j - y_o}{x_i + 1/2\Delta x_i - x_o}\right) - \operatorname{arctg}\left(\dfrac{y_j - 1/2\Delta y_j - y_o}{x_i + 1/2\Delta x_i - x_o}\right)}, \qquad (12)$$

where $r_{i,j} = \sqrt{(x_i - x_o)^2 + (y_j - y_o)^2}$.

Using new interblock distances is equivalent to replacing linear pressure-gradient approximations by better radial approximations. Note that farther away from the well the ratio of new to substitute distance rapidly approaches one in principal flow directions. This indicates that linear approximation of radial flow is adequate and can be used as a substitution criterion.

In practice, if ε_o is a small positive quantity, we can use new distance L_{eq} around a well instead of the usual interblock distance L_{xy} if $|(L_{eq}/L_{xy}) - 1| \geqq \varepsilon_o$ is verified.

Numerical Results

The new approach is compared with the conventional Peaceman[2] method. **Fig. 3** shows grids used to simulate the radial flow to an isolated well. The well is at or near the center of the grid. The pressures at the boundary nodes are set equal to values given by Eq. 5.

In the following examples, the well flow rate, q, is specified; therefore, calculated wellbore pressures, $p_{w,\text{num}}$, are used to compute the relative error in PI. This error is defined by

$$\varepsilon = \frac{J_{\text{ana}} - J_{\text{num}}}{J_{\text{ana}}}, \qquad (13)$$

where J_{ana} = analytically derived PI [$J_{\text{ana}} = q/(p_z - p_w)$, where p_z = pressure set at the lower-right corner gridblock (Fig. 3)] and J_{num} = numerically derived PI [$J_{\text{num}} = q/(p_z - p_{w,\text{num}})$].

Example 1. ***Uniform Grids With Square Blocks.*** **Fig. 4** shows results of relative PI error for a 41×41-mesh grid. The extended approach was applied successively in circular areas around the well. The radius of the area was varied from Δx to $5\Delta x$. As discussed previously, the new approach is equivalent to Peaceman's[1] analytical method for a radius equal to Δx. For this radius, however, the error observed in Fig. 4 is greater than that obtained with the conventional

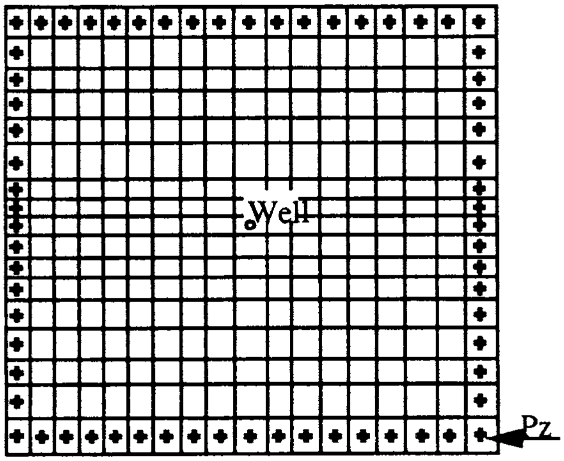


Fig. 3—Finite-difference grid with fixed outer boundary (radial flow for an isolated well).

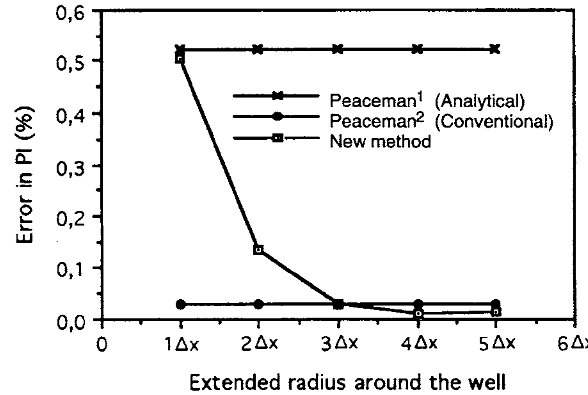


Fig. 4—PI error for square grid.

Peaceman method because the new method assumes radial flow only near the well and the Peaceman method assumes radial flow in the whole domain. If we consider a larger scale, the error decreases. For square blocks, a linear approximation of pressure gradients is adequate if the distance to the well is $>3\Delta x$.

Example 2. ***Nonuniform Grids.*** PI errors were also computed with 13×13-block nonuniform grids **(Fig. 5).** In this figure, most of the blocks are square, with only the few blocks around the well having nonuniform grid spacing. The conventional method gives an error of up to 10.3%, while the new method gives errors of $\approx 2.25\%$ and 1.14% with extended radii r_{ext1} and r_{ext2}, respectively.

Nonisolated Well

In this section, we discuss the new well representation for particular configurations (e.g., n-well systems or corner well). In these cases, flow is not radial; therefore, p_o and p_w need special treatment because they do not satisfy the logarithmic relation (Eq. 5).

***n*-Well Systems.** If $(x_k,y_k)(k=1\ \dots\ n)$ = position of n wells, $q_k\ (k=1\ \dots\ n)$ = well flow rates, and $p_{w,k}\ (k=1\ \dots\ n)$ = wellbore pressures, the pressure in the domain can be written as[11]

$$p(x,y) = c + \frac{\mu}{2\pi kh}\sum_{k=1}^{n} q_k \ln r_k\,, \quad \dots\dots\dots\dots\dots\dots (14)$$

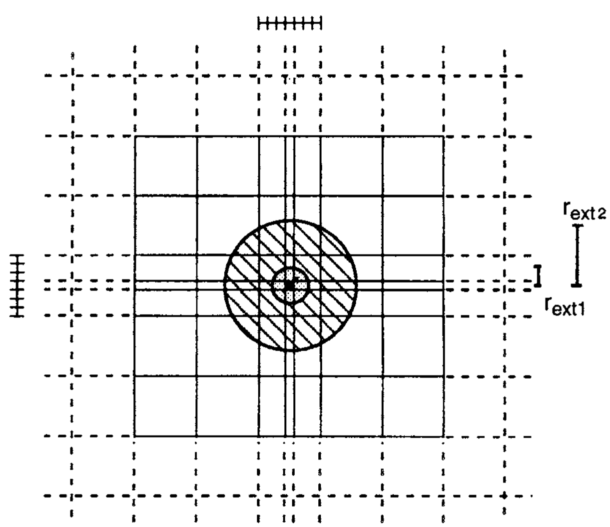


Fig. 5—Nonuniform grid.

where c = a constant and $r_k = \sqrt{(x-x_k)^2 + (y-y_k)^2}$. In particular, p_w at Well m satisfies

$$p_{w,m} = c + \frac{\mu}{2\pi kh}\left(q_m \ln r_{w,m} + \sum_{k\neq m} q_k \ln L_{k,m}\right), \quad \dots\dots (15)$$

where $L_{k,m} = \sqrt{(x_k-x_m)^2 + (y_k-y_m)^2}$.

Using these relations, we can find a good flow-term approximation by changing transmissibilities.

Two-Well Systems. Fig. 6 shows two production wells positioned at the center of two arbitrary blocks, Blocks A and B. Assuming that the two wells have the same rate, we can find that L_{eq} between adjacent Blocks $m1$ and $m2$ should be[12]

$$L_{eq,\,m1,m2} = \frac{\Delta y_{m1}\left(\ln\dfrac{L_{m2,A}}{L_{m1,A}} + \ln\dfrac{L_{m2,B}}{L_{m1,B}}\right)}{d_A + d_B}, \quad \dots\dots\dots\dots\dots (16)$$

where

$$d_k = \operatorname{arctg}\left(\frac{y_{m1} + \tfrac{1}{2}\Delta y_{m1} - y_k}{x_{m1} + \tfrac{1}{2}\Delta x_{m1} - x_k}\right)$$

$$-\operatorname{arctg}\left(\frac{y_{m1} - \tfrac{1}{2}\Delta y_{m1} - y_k}{x_{m1} + \tfrac{1}{2}\Delta x_{m1} - x_k}\right) \qquad (k = A, B),$$

and

$$L^2_{mi,j} = (x_{mi}-x_k)^2 + (y_{mi}-y_k)^2 \qquad (i = 1,2; k = A, B).$$

Example 3. To illustrate application of the new approach to nonisolated wells, we present the PI error for a two-production-well system in uniform grid with rectangular blocks (aspect ratio $\alpha = \Delta x/\Delta y$).

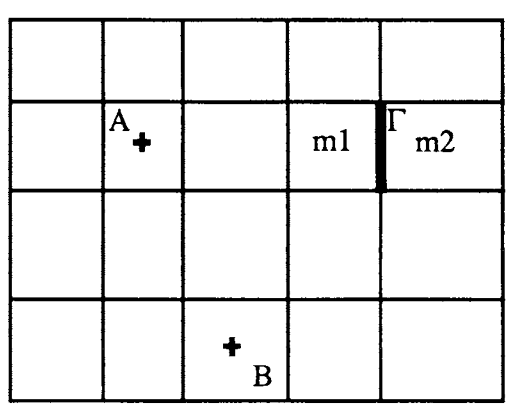
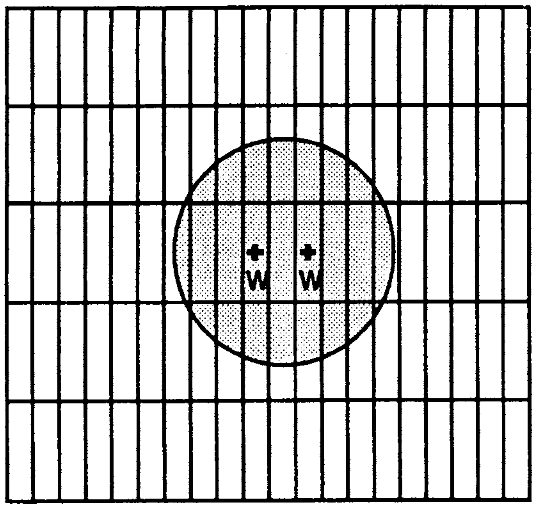


Fig. 6—Two-well system.



(S = 2)

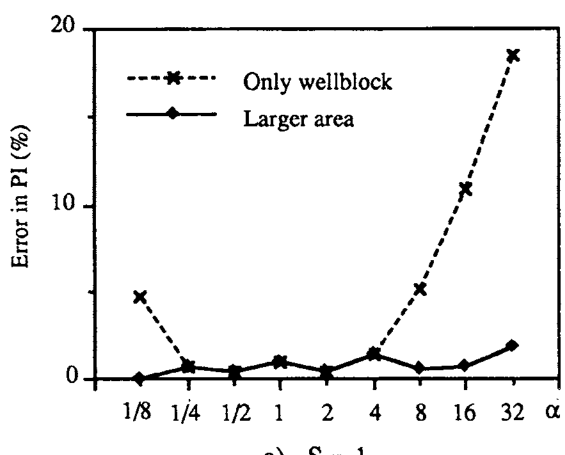
Fig. 7—Two wells in uniform grid.

Wells A and B are assumed to be in the same horizontal row so that $j_A = j_B$ (**Fig. 7**). Let S be an integer that describes the distance between the blocks (i.e., $S = |i_B - i_A|$). **Fig. 8** shows the relative PI error with respect to α for $S = 1$ and 2. The dashed lines show the errors calculated when only L_{eq} at the wellblock is considered, and the solid lines present those calculated with a larger-scale equivalent interblock distance (circle in Fig. 7). Results indicate that for a large aspect ratio, the larger-scale approach must be considered to avoid important errors in calculation of wellbore pressure or flow rate.

Boundary and Corner Wellblocks. If a wellblock is at the boundary or corner of the reservoir, it can be treated by use of the method of images.[11] A well in a boundary block is equivalent to two adjacent wells in the whole domain, while a well in a corner block is equivalent to four adjacent wells in the whole domain.

Anisotropic Systems. The new representation of wells can also be applied to anisotropic systems. The expression of the L_{eq1} and $L_{eq,i+½,j}$ for an isolated well in such systems is[12]

$$L_{eq1} = \frac{\Delta y_o}{2}\sqrt{k_y/k_x}\frac{\ln\dfrac{\Delta x_{½}}{r_w\left(1+\sqrt{k_x/k_y}\right)/2}}{\operatorname{arctg}\dfrac{k_x^{½}\Delta y_o}{k_y^{½}\Delta x_o}} \quad \text{(17)}$$



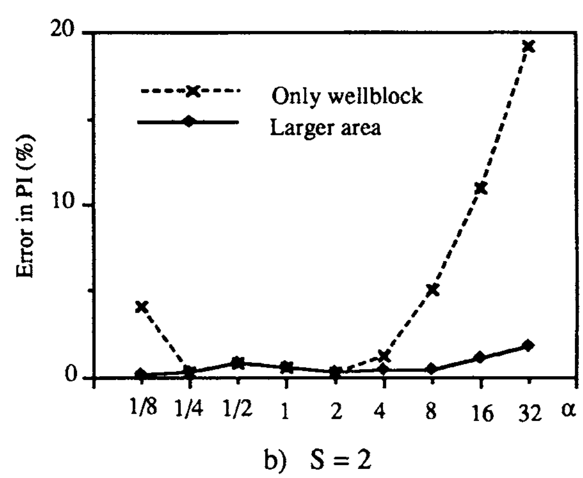


Fig. 8—PI error for a two-well system.

and $$L_{eq,i+1/2,j} = \Delta y_j\sqrt{k_y/k_x}\frac{\ln\left(\bar{r}_{i+1,j}/\bar{r}_{i,j}\right)}{\operatorname{arctg}(\theta^+)-\operatorname{arctg}(\theta^-)}, \quad \text{(18)}$$

where

$$\bar{r}_{i,j}^2 = \sqrt{k_y/k_x}\,(x_i-x_o)^2 + \sqrt{k_x/k_y}\,(y_i-y_o)^2$$

and $$\theta^{\pm} = \sqrt{k_x/k_y}\frac{y_j \pm ½\Delta y_j - y_o}{x_i + ½\Delta x_i - x_o}.$$

Conclusions

1. A new method is proposed to represent wells in numerical simulations that is easy to implement in any standard reservoir simulator.
2. The new method is more accurate and more general than a well model with numerical PI. It does not require use of the $r_{eq,o}$ concept.
3. The method is particularly well-suited for nonuniform grids and can be applied to nonisolated wells and anisotropic systems.

Nomenclature

d_A, d_B = defined by Eq. 16
h = reservoir thickness, L
J_{ana} = analytical PI defined by Eq. 13, L^4t/m
J_{num} = numerical PI defined by Eq. 13, L^4t/m
k = permeability, L^2
k_x, k_y = permeability in x and y directions, L^2
L_{eq} = equivalent interblock distance, L
$L_{eq,i+½,j}$ = equivalent interblock distance between Blocks (i,j) and $(i+1,j)$, L
$L_{eq,m1,m2}$ = equivalent interblock distance between Blocks $m1$ and $m2$, L
$L_{k,m}$ = distance between Gridblocks k and m defined by Eq. 15, L
L_{xy} = interblock distance, L
n = number of wells
p = pressure, m/Lt^2
p_o = wellblock pressure, m/Lt^2
p_w = wellbore pressure, m/Lt^2
$p_{w,k}$ = wellbore pressure in Well k, m/Lt^2
$p_{w,num}$ = numerical wellbore pressure, m/Lt^2
p_z = pressure at grid corner, m/Lt^2
q = well flow rate, L^3/t
q_Γ = flux through Interface Γ, L^3/t
r = distance from well center defined by Eq. 5, L
$r_{eq,o}$ = equivalent wellblock radius, L
$r_{eq,o1}, r_{eq,o2}, r_{eq,o3}, r_{eq,o4}$ = equivalent wellblock radii (Eq. 9), L
r_{ext1}, r_{ext2} = extended radii defined in Fig. 5, L

r_k = distance from the kth well defined by Eq. 14, L
r_w = well radius, L
$r_{w,m}$ = mth well radius, L
S = separation between two wellblocks = $|i_B - i_A|$
u = well flow rate per unit volume, L/t
x,y = coordinates, L
$\Delta x, \Delta y$ = grid spacing in x and y directions, L
$\Delta x_{i+1/2}$ = $\frac{1}{2}(\Delta x_i + \Delta x_{i+1})$, L
x_o, y_o = well coordinates, L
$\Delta x_o, \Delta y_o$ = wellblock dimensions, L
$\Delta y_{j+1/2}$ = $\frac{1}{2}(\Delta y_j + \Delta y_{j+1})$
α = $\Delta x / \Delta y$ grid aspect ratio
Γ = interface between two blocks, L
$\Gamma_1, \Gamma_2,$
Γ_3, Γ_4 = wellblock boundaries, L
ε = relative PI error
μ = fluid viscosity, m/Lt
$\theta^{\pm}$ = defined by Eq. 18

Subscripts

A,B = well indices
i,j = block indices in x and y directions
k,m = well indices
$m1, m2$ = grid indices
1,2,3,4 = adjacent blocks of wellblock

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Appendix A—Determination of $r_{eq,o}$

Let us assume that the well is centered at the origin of coordinates. From Eq. 5, we have

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial x}\left(p_w + \frac{q\mu}{2\pi kh}\ln\frac{\sqrt{x^2+y^2}}{r_w}\right) = \frac{q\mu}{2\pi kh}\frac{x}{x^2+y^2}; \quad \text{(A-1)}$$

therefore,

$$\int_{\Gamma_1}\frac{\partial p}{\partial x}dy = \frac{q\mu}{\pi kh}\operatorname{arctg}\frac{\Delta y_o}{\Delta x_o}. \quad \text{(A-2)}$$

If we assume that the pressure p_1 in adjacent Block 1 of the wellblock (Fig. 1) satisfies radial flow, we can write

$$p_1 = p_o + \frac{q\mu}{2\pi kh}\ln\frac{\Delta x_{1/2}}{r_{eq,o1}}. \quad \text{(A-3)}$$

Combining Eqs. A-2, A-3, and Eq. 8, we obtain

$$\operatorname{arctg}\frac{\Delta y_o}{\Delta x_o} = \Delta y_o \frac{\ln\frac{\Delta x_{1/2}}{r_{eq}}}{2\Delta x_{1/2}}, \quad \text{(A-4)}$$

or Eq. 9:

$$r_{eq,o1} = \exp\left(-2\frac{\Delta x_{1/2}}{\Delta y_o}\operatorname{arctg}\frac{\Delta y_o}{\Delta x_o}\right)\Delta x_{1/2}.$$

Appendix B—Determination of L_{eq}

Using the same assumption as in Appendix A, we can write the following relation between p_1 and p_w:

$$p_1 = p_w + \frac{q\mu}{2\pi kh}\ln\frac{\Delta x_{1/2}}{r_w}. \quad \text{(B-1)}$$

Combining Eqs. A-2, B-1, and Eq. 10, we obtain

$$\operatorname{arctg}\frac{\Delta y_o}{\Delta x_o} = \Delta y_o\frac{\ln\frac{\Delta x_{1/2}}{r_w}}{2L_{eq1}}, \quad \text{(B-2)}$$

or, Eq. 11:

$$L_{eq1} = \frac{\Delta y_o}{2}\frac{\ln\frac{\Delta x_{1/2}}{r_w}}{\operatorname{arctg}\frac{\Delta y_o}{\Delta x_o}}.$$

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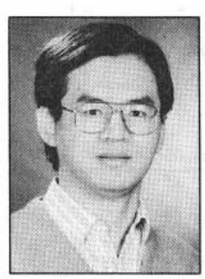
Yu Ding is a research engineer at Inst. Français du Pétrole (IFP) in Rueil Malmaison, France. His research interests include numerical modeling and reservoir simulation and characterization. Yu Ding holds a BS degree in mathematics from Peking U. and MS and PhD degrees in applied mathematics from U. de Paris (Sud). Gérard Renard is a research engineer at IFP in Rueil Malmaison with research interests in reservoir engineering aspects of horizontal wells and EOR. Renard holds a degree in fluid mechanics from Ecole Natl. Supérieure d'Electronique, d'Electrotechnique, d'Informatique, & d'Hydraulique de Toulouse and a degree in petroleum engineering from Ecole Natl. Supérieure du Pétrole & des Moteurs.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