Representation of Wells in Numerical Reservoir Simulation

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Summary

In reservoir simulation, linear approximations generally are used for well modeling. However, these types of approximations can be inaccurate for fluid-flow calculation in the vicinity of wells, leading to incorrect well-performance predictions. To overcome such problems, a new well representation[1] has been proposed that uses a "logarithmic" type of approximation for vertical wells. In this paper, we show how the new well model can be implemented easily in existing simulators through the conventional productivity index (PI). We discuss the relationship between wellbore pressure, wellblock pressure, and flow rate in more detail, especially for the definition of wellblock pressure. We present an extension of the new approach to off-center wells and to flexible grids. Through this extension, the equivalence of various gridding techniques for the well model is emphasized. The key element is the accurate calculation of flow components in the vicinity of wells.

Introduction

The well model plays an important role in reservoir simulation because the precision of calculation in well-production rate or bottomhole pressure is directly related to this well model. The main difficulty of well modeling is the problem of singularity because of the difference in scale between the small wellbore diameter (less than 0.3 m) and the large wellblock grid dimensions used in the simulation (from tens to hundreds of meters), and to the radial nature of the flow around the well (i.e., nonlinear but logarithmic variation of the pressure away from the well). Thus, the wellblock pressure calculated by standard finite-difference methods is not the wellbore pressure. Peaceman[2,3] first demonstrated that wellblock pressure calculated by finite difference in a uniform grid corresponds to the pressure at an equivalent wellblock radius, r_0, related to gridblock dimensions. Assuming a radial flow around the well, he demonstrated that this radius could be used to relate the wellblock pressure to the wellbore pressure. However, there are problems with this approach in many practical reservoir simulation studies:

1. For routinely used nonuniform Cartesian grids,[4] there is no easy means to determine an r_0 value.
2. In three-dimensional (3D) cases with non-fully-penetrating wells, the basic radial flow assumption does not apply,[5] whereas vertical flow effects must be included.[6]
3. Off-center wells are not correctly treated.[7,8]
4. Treatment of the well model is much more complicated with non Cartesian or flexible grids.[9-11]

The aim of this paper is to show that the new well representation[1] proposed in a previous paper can handle these problems accurately.

Wellblock Pressure Calculation

A previous paper[1] presented a new approach particularly well-suited to nonuniform grids for the modeling of vertical wells in numerical simulation. The principle of this new approach, which is based on a finite-volume method, is to calculate new interblock distances that improve the modeling of flow in the vicinity of wells. Because the new approach was originally presented for two-dimensional (2D)-XY problems, it was shown that for such problems the wellbore pressure could be calculated without both the intermediate computation of the wellblock pressure and introduction of an equivalent wellblock radius. However, for at least two reasons, it is convenient to keep this standard method commonly used in numerical models, which consists of relating the wellbore pressure and wellblock pressure through the use of a numerical PI and equivalent wellblock radius. One reason is practical. To implement the new approach more easily into standard numerical models, it is better to keep their internal structure unchanged. The other reason is dictated by the necessity of having a wellblock pressure in particular 3D simulation studies. When a well partially penetrates the reservoir or when there is communication between different layers, there is a vertical flow component in the vicinity of the well that necessitates that the wellblock pressure be calculated.

How should the new approach be implemented in standard reservoir simulators? In these simulators, a numerical PI is used in the well model to relate the wellbore pressure, p_w, to the wellblock pressure, p_0. Usually, this PI is written as

$$\mathrm{PI} = \frac{q}{p_0 - p_w} = \frac{2\pi kh}{\mu}\frac{1}{\ln(r_0/r_w)}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where r_0 is the equivalent wellblock radius at which the pressure is equal to p_0.

Within the new well representation,[1] to obtain a pressure p_0 corresponding to a radius r_0, it is sufficient to use equivalent wellblock transmissibilities relating p_0 to the pressures of adjacent blocks through equivalent interblock distances, $L_{eq,i}$ (**Fig. 1**):

$$\left.\begin{aligned} T_{eq,i} &= kh\frac{\Delta y_0}{L_{eq,i}}\,(i = 1,3) \\ \text{and}\quad T_{eq,i} &= kh\frac{\Delta x_0}{L_{eq,i}}\,(i = 2,4) \end{aligned}\right\}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where Δx_0, Δy_0 are the wellblock dimensions.

For instance, in the x+ direction, $L_{eq,1}$ is written

$$L_{eq,1} = \Delta y_0\frac{\ln\left(\dfrac{\Delta x_{1/2}}{r_0}\right)}{\theta_1}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where $\theta_1 = 2\ \mathrm{arctg}\ (\Delta y_0/\Delta x_0)$ is the angle formed by the right wellblock interface seen from the well.

Because wellblock transmissibilities in standard models are conventionally expressed by

$$\left.\begin{aligned} T_i &= kh\frac{\Delta y_0}{\Delta x_{\pm 1/2}}\,(i = 1,3) \\ \text{and}\quad T_i &= kh\frac{\Delta x_0}{\Delta y_{\pm 1/2}}\,(i = 2,4) \end{aligned}\right\}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

the new approach can be implemented easily in standard models multiplying the conventional wellblock transmissibilities by constant factors. For instance, in the x+ direction, this factor is

$$\alpha_1 = \frac{T_{eq,1}}{T_1} = \frac{\Delta x_{1/2}}{\Delta y_0}\frac{\theta_1}{\ln(\Delta x_{1/2}/r_0)}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

By use of equivalent transmissibilities, the calculated wellblock pressure, p_0, should correspond to the equivalent wellblock radius, r_0, which is involved in transmissibility calculations (Eq. 3). Then, the wellblock pressure can be related to the wellbore pressure with the conventional PI (Eq. 1).

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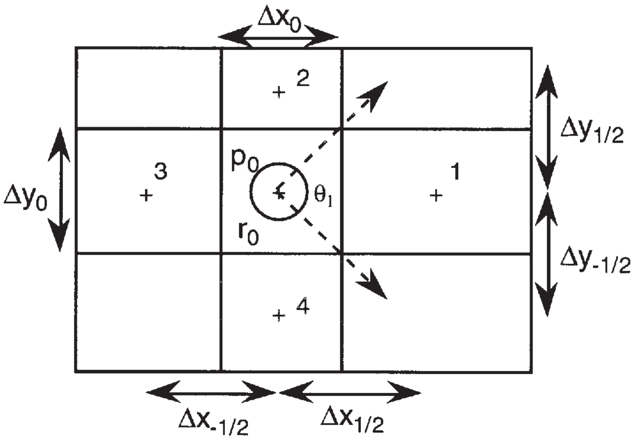

Fig. 1—Wellblock pressure calculation.

Within the new approach, which value for r_0 or wellblock pressure must be considered? In 2D-XY cases, as indicated in the next example, this can be any value.

Example 1. An isolated well ($r_w = 0.07$ m) is located at the center of a square grid (11×11 blocks, $\Delta x = 100$ m). Steady-state flow is considered and pressures at the boundary nodes are set equal to values that satisfy radial flow from the well:

$$p(r) = p(r_w) + \frac{q\mu}{2\pi kh} \ln \frac{r}{r_w} . \quad \text{(6)}$$

The wellbore pressure is specified; the analytical-derived flow rate is 100 m^3/d. Only wellblock transmissibilities are modified with the new method (it could be possible to extend it to a wider area, i.e., $2\Delta x$ for instance). Flow rates are calculated with various r_0 values from 0.5 to 50 m. The results show that the flow rates are always 99.477 m^3/day independent on r_0, the relative error on flow rate being equal to 0.5%.

This example merely illustrates that within the new well representation, in 2D-XY problems, the well results are not dependent on r_0. Though any value could be considered for r_0, it is better to choose a physical one that corresponds to the physical problem. With this point of view, a reasonable definition is to consider the wellblock pressure to be a volumetric average pressure:

$$p_0 = \frac{1}{V} \int_{\Omega} p d\Omega , \quad \text{(7)}$$

where V is the wellblock volume and pressure, p, is assumed to satisfy steady-state flow. Other authors[12,13] already have suggested using this kind of definition. However, they used the standard finite-difference method to calculate fluid-flow components through wellblock interfaces. Therefore, as Peaceman[2] clearly showed later, the wellblock pressure they obtained was not the volumetric average pressure in the wellblock.

TABLE 1—GRID DISCRETIZATIONS FOR EXAMPLE 2 (CONING PROBLEM)

Grid	x,y (10x10) or r,θ (17x1)	Vertical discretization: coarse (7 layers)	Vertical discretization: refined (20 layers)
Cartesian*	8(50.); 850.; 4150.		
Radial	0.15; 0.3; 0.6; 1.; 1.4; 2.; 3.; 5.; 10.; 15.; 30.; 60.; 120.; 200.; 375.; 1225.; 5375.	6(5.);15.	5(2.5); 5(1.25); 4(0.625); 3(1.25); 2(2.5); 15.

*The well is located in the corner block ($i = 1$, $j = 1$). Symmetry is taken into account in the simulation.

Within Cartesian grids with square blocks, assuming radial flow (Eq. 6), the average pressure in wellblocks corresponds to the pressure at an equivalent r_0 equal to $\frac{1}{\sqrt{2}} e^{-3/2 + \pi/4} \Delta x \cong 0.346\ \Delta x$. (It is better to consider an average pressure in surrounding blocks for the calculations of transmissibilities in so far as r_0 is greater than the size of the wellblock in x or y, for instance, in rectangular grids).

In 3D problems, a vertical component of flow usually has to be considered. This component is particularly important as far as partially penetrating wells are concerned. However, its evaluation will strongly depend on the wellblock pressure definition as we will show in the next example. As 3D effects occur at the tail end of a well, radial flow no longer takes place and flow calculations would have to be evaluated with 3D solutions. Therefore, previous values of equivalent wellblock radius would not be correct because they were calculated assuming pure radial flow. And so, a more sophisticated calculation of the average wellblock pressure and of the vertical transmissibility is required. The next example shows how, in 3D, calculations are dependent on r_0 as opposed to 2D cases because vertical components of flow are now calculated from the wellblock pressure that is directly related to an r_0 value.

Example 2. Here we consider a vertical well non fully penetrating an oil zone (25 m thick) underlain by an aquifer. The well is open for the first 20 m. A total well flow rate is imposed in the simulation. In a first set of simulations, non vertically refined grids, both Cartesian ($10 \times 10 \times 7$) and radial ($17 \times 1 \times 7$), are considered (**Table 1**). The new well representation, with various r_0 values, is compared to the conventional approach. Results plotted in **Fig. 2** clearly show that the value of the equivalent radius r_0 greatly influences the well performance. At the same time, they indicate that the new method with $r_0 = 0.2\ \Delta x$ is equivalent to Peaceman's approach, and with $r_0 = 0.28\ \Delta x$ is equivalent to radial grid approach. Therefore, Cartesian grids with Peaceman's well model, or radial grids, can be considered special cases of the new well representation. As the vertical

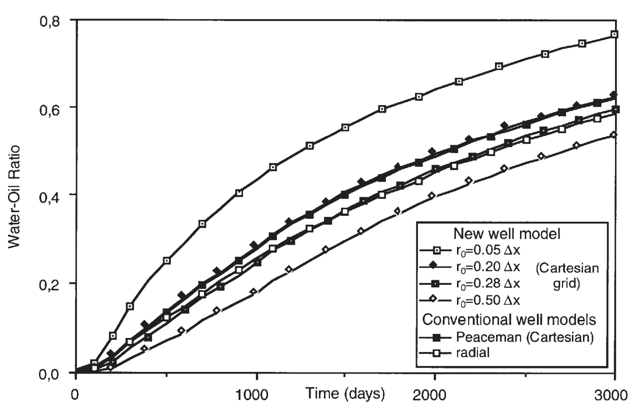

Fig. 2—Coning problem (Example 2)—Influence of the equivalent wellblock radius. Non vertically refined grids.

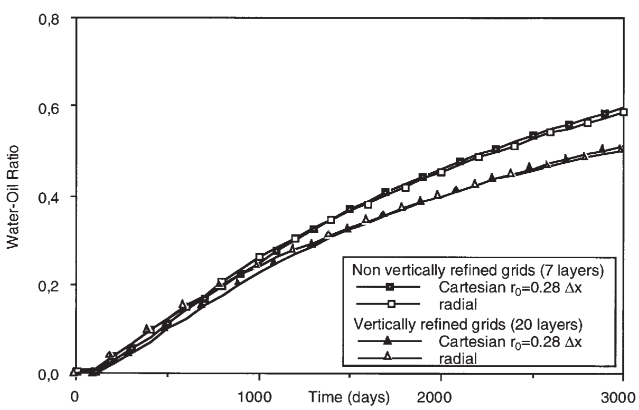

Fig. 3—Coning problem (Example 2)—Influence of vertical grid refinement.

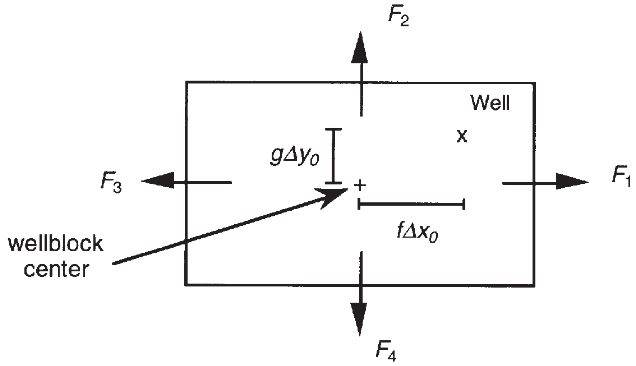

Fig. 4—Off-center well representation.

flow is not accurately approximated, it can be thought that results are not correct because modeling of coning requires a fine vertical discretization. We performed a second set of simulations with Cartesian and radial grids that were both vertically refined. **Fig. 3** plots results compared to those from coarse grids used in the first set of simulations. The trend between the two sets is clear. Refining Cartesian or radial grids in the vertical direction modifies the results. However, for a given type of discretization, vertically refined or not, the results of Cartesian (with $r_0 = 0.28\ \Delta x$) and radial grids are very similar even if the radial grids used in this example are much finer than those of Cartesian grids in the *x-y* plane, around the well. As coning is essentially a vertical phenomenon, grid refinement in the *x-y* plane is not helpful.

This example shows that neither Cartesian nor radial non vertically refined grids can be used to restore the coning phenomenon accurately. It could be assumed that considering very refined Cartesian or radial grids, both vertically and horizontally, could solve the problem, even keeping the assumption of radial flow in the vicinity of wells. However, for practical purposes (i.e., limitation of computer resources), it is not possible to do so and is necessary to consider coarser grids. The problem with coarser grids is that 3D problems usually involve a vertical component of flow that is not accurately taken into account when the assumption of radial flow is used. This point, already outlined by Muskat,[5] has been investigated recently by Sharpe *et al*.[6] These authors proposed to use an empirical formula initially published by Kozeny[14] to take the vertical-flow component into account. However, it is difficult to ascertain if this formula can be used, whatever the problem is. For this reason, more work is needed to find an accurate way to incorporate vertical flow component in the calculations when 3D effects are important. First results obtained with the integral representation[15] makes us confident in this approach to accurately model non fully penetrating vertical, deviated, or horizontal wells. Near well scaling-up technique can also be used to handle these problems.[16] The use of hybrid grids will not give a definite answer to 3D problems if the vertical discretization is not fine enough. They are much more adapted to the discretization around wells in 2D but cannot solve the problem of vertical-flow calculation.

Off-Center Wells

With standard finite differences, it is not possible to simulate off-center wells accurately because a well is always considered as being at the center of a block even if its true location is elsewhere in the block. Improvement is required to simulate the real physical problem correctly.

The treatment of off-center wells has been considered by several authors. Peaceman[17] tested the single-phase flow problem and indicated that the numerical PI should not be greatly affected by the location of the well within its grid cell. Nolen[7] found that the equivalent radius r_0 did vary for noncentered wells. In fact, the real problem for an off-center well is not the numerical PI but the components of flow in the vicinity of the well that are not approximated correctly. Recently, Su[8] studied the influence of flow pattern and proposed a formula to improve the representation of off-center wells, but this formula is complicated and its implementation is difficult. It is shown here how the new well representation[1] can provide a good, simple approximation for the fluid flow in the vicinity of an off-center well.

Consider an arbitrary location for a well within its wellblock, as **Fig. 4** depicts. The well position can be defined by $(f\Delta x_0, g\Delta y_0)$ relative to the center of the block. Physically, the fluid-flow components F_1, F_2, F_3, and F_4 will depend on the well location, but this is not taken into account by the conventional finite-difference method. Applying the new well representation, it is possible to improve the fluid-flow calculations with the following equivalent transmissibilities for the wellblock:

$$T_{eq,i} = kh\frac{\theta_i}{\ln(r_i/r_0)}(i = 1, \ldots, 4), \qquad (8)$$

where θ_i is the angle formed by the i-th wellblock interface viewed from the well, and r_i is the distance of the well to its i-th neighboring block center.

To facilitate its implementation to standard simulators, this formula can be written as

$$T_{eq,i} = \alpha_i T_i (i = 1, \ldots, 4). \qquad (9)$$

T_i is the conventional transmissibility and α_i a correction factor that depends only on the grid geometry. For instance, α_1 is defined by

$$\alpha_1 = \frac{\Delta x_{1/2}}{\Delta y_0}\frac{\theta_1}{\ln(r_1/r_0)}. \qquad (10)$$

Of course, the calculation of equivalent transmissibilities can be extended to a larger area in the vicinity of wellblocks.[1] The next example illustrates the improvement of flow calculations by use of these equivalent transmissibilities.

Example 3. We consider the five well problem given in **Fig. 5.** The initial grid is 11×11 square blocks. Four production wells are

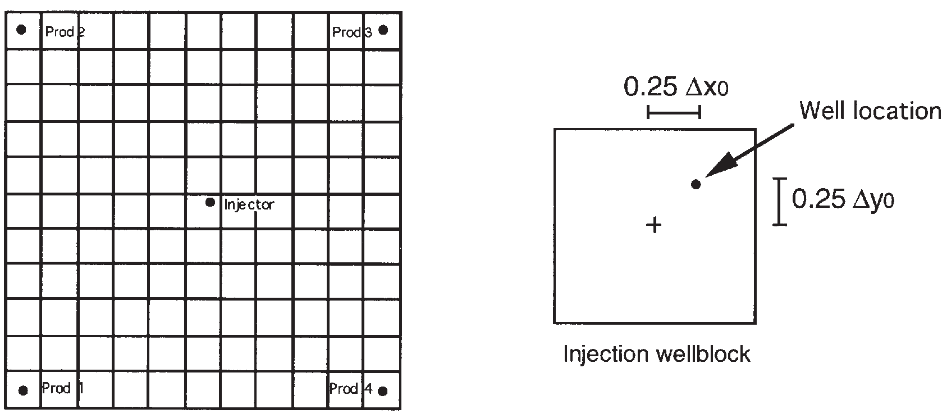

Fig. 5—First grid used for Example 3 (off-center well).

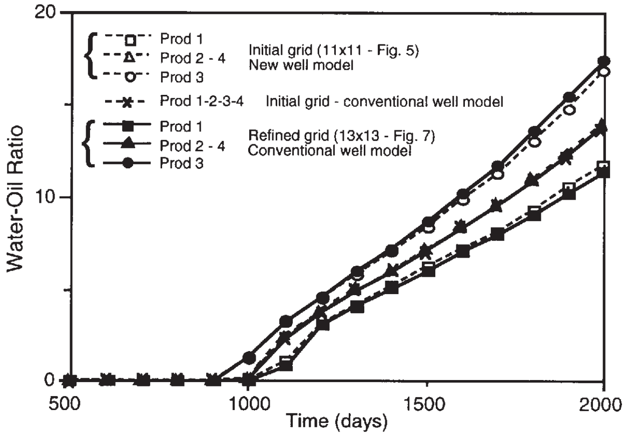

Fig. 6—Results of off-center well simulations.

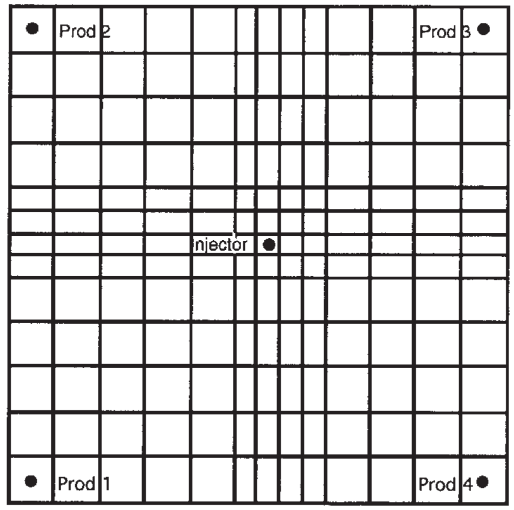

Fig. 7—Second grid used for Example 3.

placed at the center of a block in each corner and an injection well is located in the central block. The real production forecast will depend on the position of the injection well within its block. If the well is located at the center of the block, the responses for the four producers will be identical, owing to the symmetry of the pattern. On the contrary, if the injection well is not exactly at the center of the block, different responses will occur at the production wells.

Let us suppose, for instance, that the injection well is located at the point defined by $f=0.25$ and $g=0.25$. **Fig. 6** presents the evolution of the water/oil ratio (WOR) for a two-phase flow problem (unit mobility ratio). The first set of curves on Fig. 6 presents well productions calculated with the new method. At a first glance, results are coherent with the physical problem. Breakthrough time occurs first for the producer that is closer to the injector (Prod 3), and the WOR increases more rapidly than in the other wells (the contrary is observed in the producer Prod 1, which is further from the injector). However, when the simulation is performed with the conventional well model, identical results are obtained for the four production wells, which are similar to those of Prod 2 or Prod 4 obtained by the new well model (Fig. 6).

To validate the results quantitatively, we used a refined 13×13 grid (**Fig. 7**). In this grid, all the wells are at the center of their gridblock. Simulating the same problem with the conventional PI within this grid gives the second set of curves in Fig. 6. It is clear that off-centering a well has an influence on the fluid-flow calculations in the vicinity of this well and that the modifications of transmissibilities with the new approach restores this influence quite well.

Well Representation in Flexible Grids

Flexible grids, such as triangular grids, Voronoi grids, or Cartesian locally refined grids, are used more and more in reservoir simulation. One of the purposes of using flexible grids is to improve well modeling, especially for multiphase fluid flow. However, as the grid size decreases rapidly in the near well region, linear approximations are not suitable for numerical discretization. Some improvement can be obtained by use of hybrid grids,[9,18] but these grids require special well discretization and transmissibility calculations in the transition zones between the different grids (e.g., cylindrical and Cartesian).

In this section, we present a general formula for fluid-flow approximation near a well. This formula is well suited to all types of grids. In general, cylindrical grids are used in hybrid grids to improve well modeling successfully because this modeling is based on the properties of radial flow that satisfies the logarithmic law (Eq. 6). These properties can be handled easily in transmissibility calculations for cylindrical grids, but their handling has not been extended to other kinds of grids. The new well representation explains the main principle of applying these properties to every type of grid. On this basis, equivalence of well models for different grids can be considered. Moreover, when using the new model, there is no problem of handling transition zones.

Using the example of triangular grids (the same considerations would be valid for other types of grids), the transmissibilities in the near well region are calculated according to the well position and the nature of radial flow. As shown by Forsyth[19] and Fung *et al.*,[20] the principle of numerical methods for triangular grids is to approximate the fluid-flow components across a control-volume boundary. This control-volume is usually constructed from the barycentre of each triangle and the midpoints on its sides (**Fig. 8**). In a triangular element (**Fig. 9**), the flow between the Nodes A and B is calculated as

$$F_{AB} = -\int_{e_c} \frac{kh}{\mu} \vec{\nabla} P \vec{n}\, ds = -\frac{T_{AB}}{\mu}(p_A - p_B), \quad \ldots\ldots\ldots \quad (11)$$

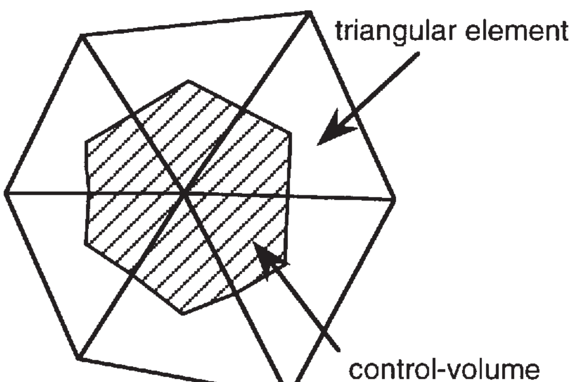

Fig. 8—A control volume in a triangular finite-element mesh.

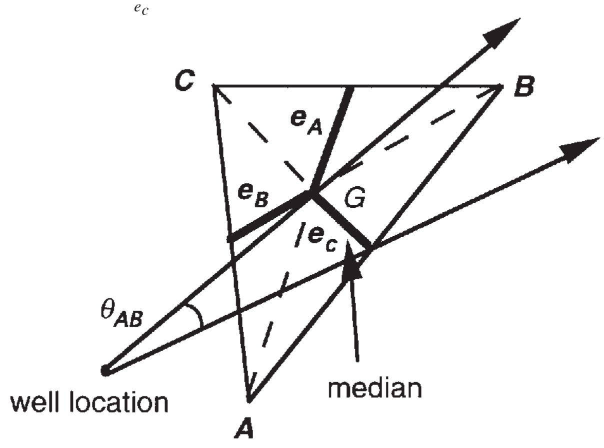

Fig. 9—Transmissibility calculation for triangular grids in the vicinity of wells.

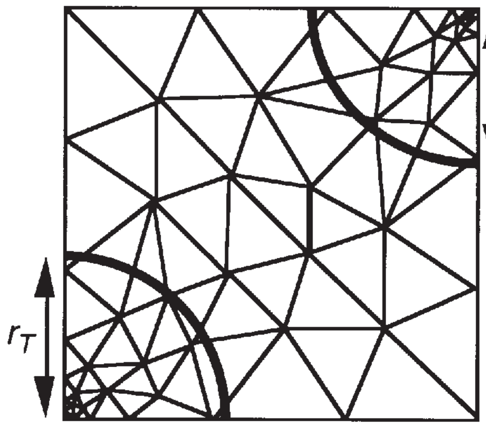

Fig. 10—Triangular grid used in Example 4 (repeated five-spot).

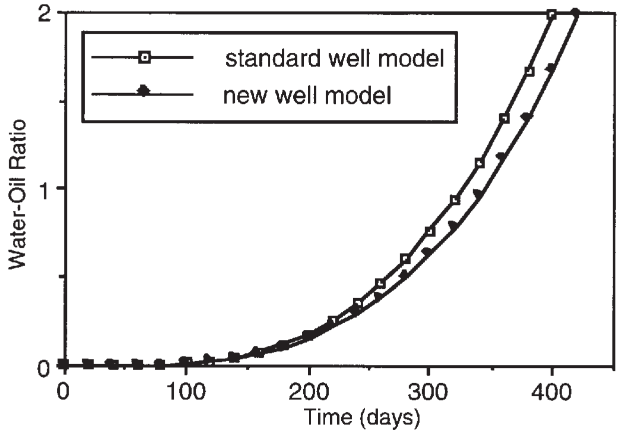

Fig. 11—Two-phase flow simulation (Example 4).

where e_c is the segment that joins the barycentre G to the midpoint of side $\overline{AB}$.

In the near well region, an accurate formula should take into account the nature of radial flow; i.e., through the segment e_c, the flow F_{AB} is equal to $(\theta_{AB}/2\pi)q$ (where θ_{AB} is the angle of the segment e_c open to the well). Therefore, a more accurate transmissibility formula should be written as (Appendix A)

$$T_{eq,AB} = kh\frac{\theta_{AB}}{\ln\frac{r_B}{r_A}}, \qquad (12)$$

where r_A and r_B are the distances between Nodes A and B and the well center. If the well coincides with one of the nodes, the transmissibility is calculated as

$$T_{eq,AB} = kh\frac{\theta_{AB}}{\ln\frac{r_{AB}}{r_0}}, \qquad (13)$$

where r_0 is an equivalent wellblock radius and r_{AB} is the distance between Nodes A and B.

To apply the new well model, a transmissibility modification area around the well should be defined. This area is usually a circle with radius r_T centered at the well location. Inside this circle, all the transmissibilities will be modified and calculated with the new formula (Eq. 12 or Eq. 13).

Example 4. A triangular grid is used in a repeated five-spot pattern (**Fig. 10**). Well models are validated by steady-state flow simulation where the analytical solution is known[5]:

$$\Delta p = \frac{q\mu}{\pi kh}\left[\ln\left(\frac{d_{well}}{r_w}\right) - 0.6190\right]. \qquad (14)$$

Pressure conditions are set at wellbores to evaluate well flow rates, and relative errors are evaluated as $(1-q_{num}/q_{ana})$. As the grid used in the vicinity of the wells is not "uniform," the use of standard triangular grid well models[20,21] produces an error of 3%, while using the new well model with a transmissibility modification area of radius $r_T = 0.2\sqrt{2}\, d_{well}$ reduces the error to 0.26%, ten times smaller. The improvement by the new well model is significant. A two-phase flow simulation also shows the difference between standard well models and the new well model (**Fig. 11**).

Finally, we will show an example of a Cartesian locally refined grid. This kind of grid has been greatly used in reservoir simulation even if it has been shown as not always suited for well modeling.[18] Here again, the use of the new well representation can improve the accuracy of the computation.

Example 5. Data and grid are the same as in Example 1, but the wellblock is locally refined, as **Fig. 12** shows. The flow rate calculated by the classical linear approximation[18] associated with Peaceman's

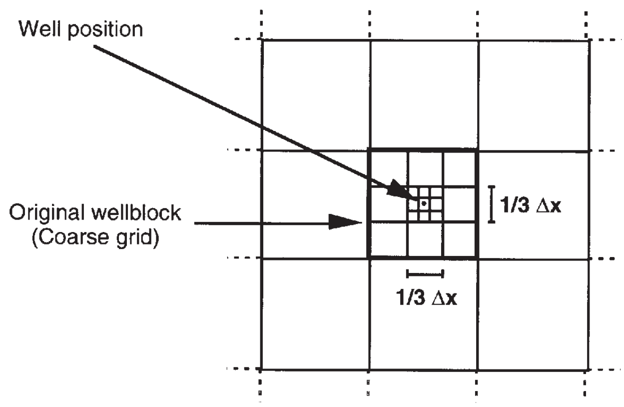

Fig. 12—Local grid refinement for wellblock (Example 5).

well model is 94.480 m^3/d, which corresponds to a relative error of 5.5%. Using the new well model with a circular transmissibility modification area of radius $r_T = \Delta x$, the flow rate is 99.477 m^3/d, corresponding to an error of only 0.5%.

Conclusions

It has been shown how a new well representation[1] can be easily implemented in standard reservoir simulators with the conventional PI. It is emphasized that in 2D-XY problems, the well model is not dependent on r_0. Therefore, the equivalent wellblock radius could be chosen so that the wellblock pressure represents the effective average volumetric pressure.

In 3D, as opposed to 2D, the well model is dependent on the r_0 value because the vertical component of flow is calculated from the wellblock pressure that is directly related to r_0. It has been shown that cylindrical grids (thus hybrid grids) are not the answer to solve coning problems if the vertical discretization is not fine enough. In 3D, more work is required to approximate vertical fluid-flow calculations accurately.

It has been clearly illustrated that off centering a well in its block has an influence on the fluid-flow components in the vicinity of the well. These components can be approximated correctly with the new well model that provides a good modeling of off-center wells.

A general formula for near well fluid-flow approximations has been proposed for flexible grids, such as triangular grids, Voronoi grids, or Cartesian locally refined grids. On the basis of radial flow, (i.e., logarithmic law), these grids can be considered equivalent with this general formula.

Nomenclature

d_{well} = distance between wells, L
$\Delta x_0, \Delta y_0$ = wellblock dimensions, L

e = control-volume boundary
F = fluid flow, L^3/t
f,g = off center well coordinates, $[-0.5, 0.5]$
h = block thickness, L
k = permeability, L^2
L_{eq} = equivalent interblock distance, L
p = pressure, m/Lt^2
p_0 = wellblock pressure, m/Lt^2
p_w = wellbore pressure, m/Lt^2
PI = productivity index, L^4t/m
q = well flow rate, L^3/t
q_{ana} = analytical well flow rate, L^3/t
q_{num} = numerical well flow rate, L^3/t
r = distance, L
r_0 = equivalent wellblock radius, L
r_T = transmissibility modification radius, L
r_w = wellbore radius, L
T = conventional transmissibility
T_{eq} = equivalent transmissibility
V = wellblock volume, L^3
α = transmissibility correction factor
θ = angle viewed from well location
μ = fluid viscosity, m/Lt

Subscripts

A,B,C = triangle nodes
i,j = block indices

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Appendix A

The fluid flow F_{AB} (Fig. 9) between the two Nodes A and B of the triangle ABC is calculated as follows:

$$F_{AB} = \frac{T_{eq,AB}}{\mu}(p_B - p_A), \qquad \text{(A-1)}$$

where $T_{eq,AB}$ is the equivalent transmissibility between these nodes. In a homogeneous media, the fluid flow F_{AB} through the control-volume boundary e_c should be equal to $(\theta_{AB}/2\pi)q$, with θ_{AB} the angle formed by the interface e_c viewed from the well. Assuming pressures p_A and p_B satisfy the logarithmic variation of pressure (i.e., Eq. 6), we obtain

$$\frac{\theta_{AB}}{2\pi}q = T_{eq,AB}\frac{q}{2\pi kh}\ln\frac{r_B}{r_A} \qquad \text{(A-2)}$$

or

$$T_{eq,AB} = kh\frac{\theta_{AB}}{\ln\frac{r_B}{r_A}}, \qquad \text{(A-3)}$$

where r_A and r_B are the distances of the well to Nodes A and B. Therefore, we obtain Eq. 12.

SI Metric Conversion Factors

bbl/D × 1.589 873	E − 01	= m^3/d
ft × 3.048*	E − 01	= m

*Conversion factor is exact.

SPEREE

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