Modeling Off-Center Wells in Reservoir Simulation

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Summary

In conventional reservoir simulation, wells are assumed to be at cell centers. Thus, the fluid flow pattern around an off-center well is represented incorrectly. This is especially true for large fieldwide models, which are gridded by very coarse blocks resulting in numerous off-center wells.This paper describes a new, rigorous method to model single or multiple off-center wells in a gridblock. Fluids can be drawn directly from the four closest neighboring cells into an off-center producer. Well location is honored exactly. With the new method, the calculated wellblock pressure can be interpreted at cell center rather than away from cell center as in the conventional approach. The new method is formulated fully implicitly, and although it adds more nonzero elements to the Jacobian matrix, its dimension remains unchanged. Therefore, implementation of this method is not difficult and the increase in computing costs is minimal. This work has been validated by fine-grid simulation results. Several single- and multiple-phase test cases are presented. We show that the new method can improve the accuracy of numerical results for large fieldwide models.

Introduction

Refs. 1 through 3 give excellent reviews of well representation in reservoir simulation. In fieldwide simulation models, gridblocks are usually very coarse and most wells are not located at gridblock centers. Unfortunately, in conventional simulation formulation, wells are assumed to be at cell centers; instead of flowing toward the off-center wells as it should, fluids flow toward the cell centers in the simulation calculation.

Williamson and Chappelear[1] realized the problem and presented a local analytical solution for the well source term. On the basis of off-center well location, geometric weighting factors can be derived to quantify the contribution from neighboring cells to the source term. The well rate can be calculated according to the well pressure, the neighboring cell pressures, and the weighting factors. However, they suggest that once the rate is determined, the simulator still assumes that fluids are produced only from the cell containing the well. Thus, the location of the source within the wellblock has no influence on the finite-difference-scheme results. In other words, they honored the exact off-center well location when calculating the well rate and did not take the exact well location into account when solving the finite-difference equations. The cell pressure is still calculated under the same old assumption that the wellbore fluids are produced from the cell center. Therefore, the fluid flow pattern around an off-center well is incorrectly represented. They did not present any numerical test results in their work.

Peaceman[3] derived a correct formula for evaluating the equivalent wellblock radius for a gridblock containing an off-center well. He demonstrated that a well-pressure value, very close to the analytical solution, can be obtained with the formula in a single-phase test case.

Abou-Kassem and Aziz[4] derived a general analytical expression for the equivalent wellblock radius applicable to off-center wells, and their solution matched well with the numerical results presented by Kuniansky and Hillestad.[5]

Because the conventional well equation was still used in the finite-difference scheme from Refs. 3 and 5, the flow pattern was not represented correctly as explained in the second paragraph. We show later that the cell-pressure values are not calculated correctly with the conventional well equation in a single-phase test case, although a very good well-pressure solution can be obtained.

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Siu and Nghiem[6] proposed a well equation and validated their work by comparing well-pressure values with an analytical solution for a single-phase, 2D test case. They did not present the match for cell pressures. The geometric weighting factor used in their well equation is a function of the distance between the well and the neighboring cells and the angle span for each neighboring cell.

An alternative way to model off-center wells is to use the local-grid-refinement method. However, for a truly large field with thousands of wells, the CPU cost under the current hardware technology is too high to use the local-grid-refinement method. Even in the future when the local-grid approach is affordable, the coarse-grid approach can still be useful for carrying out quick sensitivity studies after calibration of the coarse-grid model against the local grid model.

Theory

Peaceman[2] presented the following conventional well equation that is widely accepted in the petroleum industry.

$$q = (2\pi kh/\mu)\left[(p_o - p_w)/\ln(r_{eq,o}/r_w)\right], \quad \ldots\ldots\ldots\ldots (1)$$

where $r_{eq,o}$ is the equivalent wellblock radius at which the steady-state flowing pressure is equal to the numerically calculated pressure, p_o, of the block containing the well.

In this work, we propose the following well equation for multiple-phase 2D models:

$$q = 2\pi \sum_{i=1}^{4} \left\{ F_i \left(k_r kh/\mu B\right)_i \left[(p_i - p_w)/\ln(r_i/r_w)\right] \right\}, \quad \ldots\ldots\ldots (2)$$

where q = well-production rate for a phase and subscript i is the index for the four neighboring cells in **Fig. 1.** The weighting factor, F_i, quantifies the influence from the neighboring cells to the well production (Fig. 1) and is calculated by

$$F_1 = (1 - x_D)(1 - y_D),$$

$$F_2 = (1 - x_D)y_D,$$

$$F_3 = x_D y_D,$$

and $$F_4 = x_D(1 - y_D). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Eq. 2 is similar in form to those of Williamson and Chappelear[1] and Siu and Nghiem,[6] but the weighting factors and the selection of the neighboring cells are different (**Fig. 2** shows Williamson and Chappelear's neighboring-cell-selection scheme). In addition, the new well equation is included in the finite-difference equations of reservoir cells, and produced-fluid properties are evaluated at the upstream cells. Fluids are drawn directly from all the specified neighboring cells. We believe that this is the only way to represent a correct flow pattern around an off-center well. To draw the fluids only from the wellblock implies that the fluids move toward the cell center.

In the conventional formula, only the wellblock has the production term. Using the new well equation in the following governing equations as the source term will give more reservoir cells having the partial well-production term.

$$\nabla \cdot \frac{kk_r}{\mu B} \nabla(p - \rho gh) - F_i \left(\frac{k_r kh}{\mu B}\right)_i \frac{(p_i - p_w)}{\ln(r_i/r_w)} = \frac{\partial}{\partial t}\left(\frac{\phi S}{B}\right). \quad \ldots\ldots\ldots\ldots\ldots (4)$$

If the fixed rate option is specified for a well, then the following implicit well-constraint equation is used to ensure that the well produces exactly the specified rate at the end of the timestep.

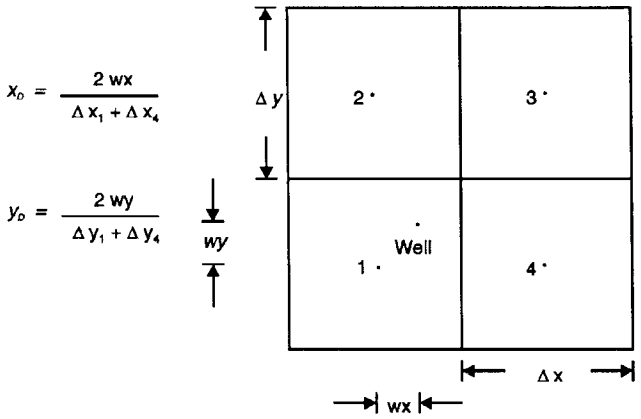

Fig. 1—Off-center well and neighboring cells.

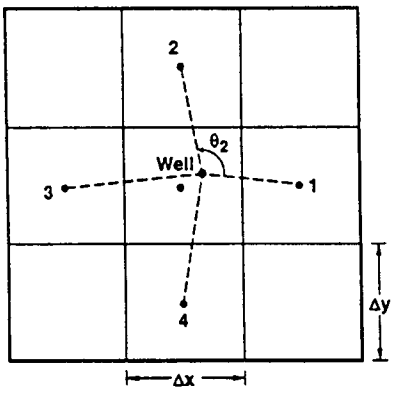

Fig. 2—Neighboring cell selection (from Ref. 1).

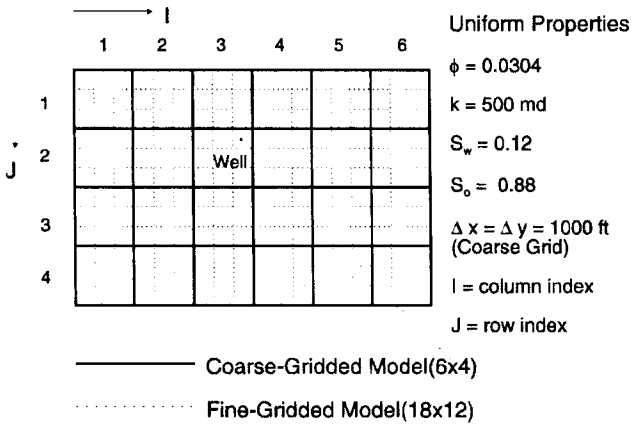

Fig. 3—Base-case description; oil/gas, two-phase, 2D, single-well primary depletion.

$$q^{l+1} = \sum_{i=1}^{4}\left(q_i^l + \frac{\partial q_i^l}{\partial p_i}\delta p_i + \frac{\partial q_i^l}{\partial S_{oi}}\delta S_{oi} + \frac{\partial q_i^l}{\partial S_{wi}}\delta S_{wi} + \frac{\partial q_i^l}{\partial p_w}\delta p_w\right), \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where superscripts $l+1$ and l = current and previous Newton-Raphson iterations, respectively, and δ = change of a variable value between iterations. One implicit well-constraint equation still exists for each well, but it contains more variables than the conventional formula. If the constant-wellbore-pressure option is specified, then the constraint equation becomes

$$\delta p_w = 0. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

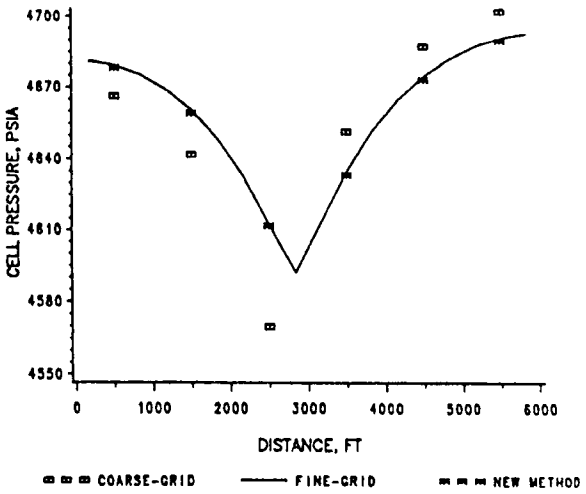

Fig. 4—Pressure comparison for base case (t=0.02 year).

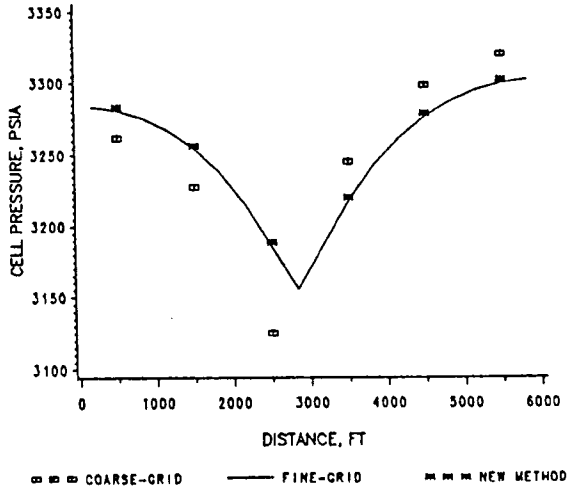

Fig. 5—Pressure comparison for base case. (t=0.8 year).

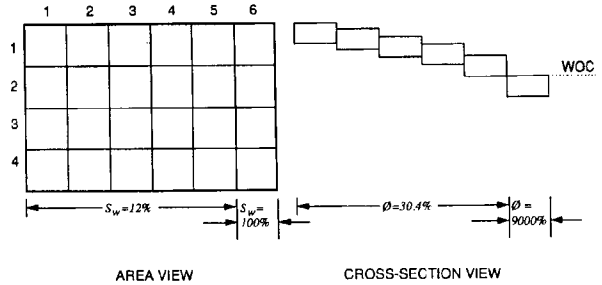

Fig. 6—Second test case description; three-phase, 2D, edgewater encroachment.

Because the total number of the finite-difference equations for cells and wells remains unchanged and no new primary variables occur, implementation of the above scheme into an existing fully implicit reservoir simulator is not difficult. Although we add more nonzero elements to the Jacobian matrix, its dimension is the same. Tests show that the CPU time increase resulting from implementation of the new scheme is minimal (< 5% of the total CPU time).

The new method can be applied to the case of multiple wells in a gridblock without any modification in the well equation. One reservoir cell can have several partial well-production terms in its finite-difference equation, one for each neighboring well.

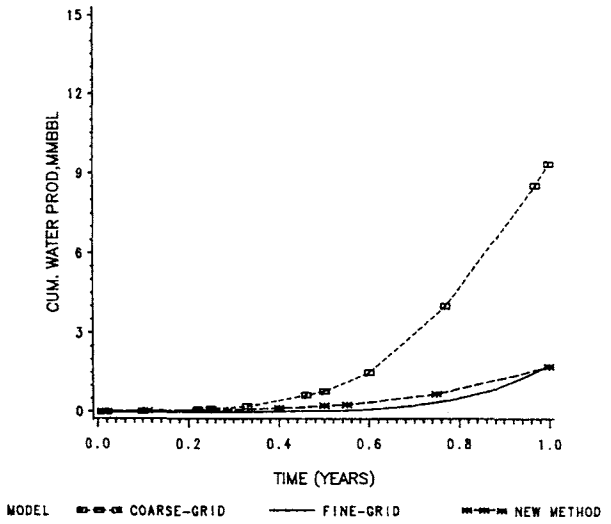

Fig. 7—Water production comparison for Well 1, edgewater-encroachment test case.

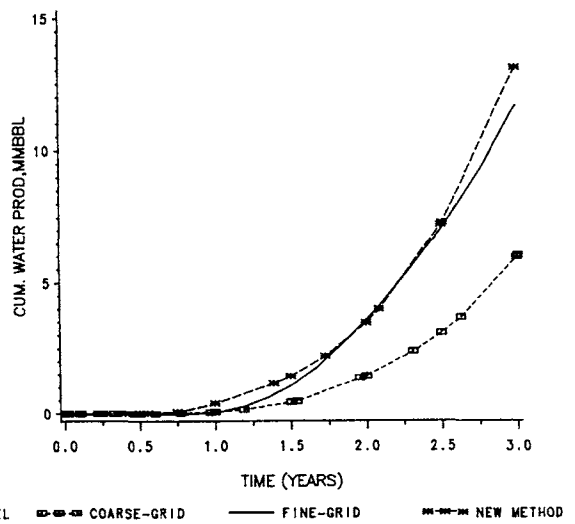

Fig. 8—Water production comparison for Well 2, edgewater-encroachment test case.

Eq. 2, the new well equation, can be readily extended for 3D models. For example, if a well has open perforations in three layers, the only modification required is to change the neighboring cell index from "1 to 4" to "1 to 12" in the summation notation. Although the new scheme is presented for producers in a black-oil-type simulator, it can be applied to injectors or a compositional formulation in a same manner.

Results

Fig. 3 shows the base case. The solid and dashed lines represent the coarse- and the fine-grid systems, respectively. The PVT properties and relative permeabilities of the oil and gas phases are taken from Case 2 of the first SPE comparative study.[7] The initial water saturation is at the irreducible water saturation. Only one producer is located at an off-center position; therefore, it is a two-phase (oil and gas) primary-depletion model. The 3-in.-radius well produces at a constant oil rate of 2,000 STB/D. Initial reservoir pressure is 4,793.5 psia, and initial bubblepoint pressure is 4,014.7 psi; therefore, the reservoir is initially undersaturated.

Two simulation runs of the coarse-grid system were made with the new and conventional approaches, respectively. In the conventional approach, Peaceman's[3] formula was used to obtain $r_{eq,o}$. In fact, a different $r_{eq,o}$ value will not change the cell-pressure solution of the finite-difference method; it affects only the wellbore-pressure value. A simulation run was made for the fine-grid system, in which the well is located at cell center.

Fig. 4 shows the calculated cell-pressure profile from the three simulation runs at 0.02 years along the center line of the second row ($J=2$) of the coarse-grid system. At 0.02 years, the reservoir is still undersaturated and can be considered a single-phase system. Fig. 4 shows that using the new well equation with the coarse-grid model, we can generate a pressure solution almost identical to the fine-grid simulation results. Note that the calculated wellblock pressure is exactly referenced at the gridblock center with the new method, while the calculated wellblock pressure is at a distance, $r_{eq,o}$, from the block center in the conventional approach. The coarse-grid pressure results of the conventional approach do not match the fine-grid pressure solution because the conventional well equation can not take the exact well location into account. At the end of the base case run (0.8 years), $\approx 10\%$ free gas saturation is present in the reservoir. **Fig. 5** shows the pressure match.

The calculated wellbore pressures from the three runs are almost identical. This is understandable because a change in well location should have negligible effect on the wellbore pressure if the reservoir has homogeneous properties and produces at a constant rate. The poor cell-pressure prediction from the conventional approach is not caused by the fact that the well is too close to the reservoir boundary. In fact, we have increased the coarse-grid system area by nine times and the off-center well is at least four cells away from the reservoir boundary. Similar poor results are obtained.

Fig. 6 shows the model configuration for the second test case, which is a dipping reservoir with a strong aquifer support at the right boundary. A large porosity value was assigned to the aquifer to represent a near-constant-pressure boundary condition. Gross and net thicknesses are 300 and 100 ft, respectively. The cell-top depths in the x direction are 8,000, 8,100, 8,250, 8,450, 8,700, and 9,000 ft. The pressure is 4,800 psi at the reference depth of 8,400 ft. Both wells produce at a fixed oil rate of 10,000 STB/D. The well closer to the aquifer is shut in after 1 year of production owing to high water cut. Water viscosity is 0.31 cp, and a typical water-wet relative permeability curve is used. Other fluid properties are the same as in the base case.

Figs. 7 and 8 show individual well cumulative water production. Assume that the fine-grid-model results are very close to the true solution. Compared with the fine-grid-model results, the new method predicts an earlier water breakthrough time for both wells owing to its ability to draw water directly from the neighboring cell of higher water saturation. Overall, the new method can predict a water-production history reasonably well, while the conventional approach either over- or underestimates water production by a wide margin. All the coarse-grid models suffer the truncation-error problem as a result of large grid size. If the pseudo-relative-permeability curves are used with the new method, prediction accuracy should improve further. In particular, the problem of predicting too-early breakthrough times, as mentioned above, can be alleviated significantly.

These results with the new method were obtained by use of the upstream k_r value. Using the upstream value is not exactly correct because the well should not produce any water before the water actually reaches the well. Therefore, the upstream k_r value of a phase is set to zero if the wellblock does not have a mobile saturation value for that phase. We have tried other schemes to evaluate k_r, such as bilinear interpolation among neighboring cells, harmonic averaging between upstream and downstream cells, and exponential curve fitting at the cell boundary. They all gave poor results.

The third case investigated whether the new method can model multiple wells in a gridblock accurately. Peaceman[3] described an exact pressure solution derived by Muskat for a multiple-well, single-phase, steady-state reservoir system:

$$p(x,y) = (\mu/2\pi kh)\sum_k (q_k \ln L_{xyk}), \quad \ldots\ldots\ldots\ldots\ldots (7)$$

where $p(x,y)$ = pressure at (x,y) and L_{xyk} = distance between Point (x,y) and the kth well location, and

$$p_w = (\mu/2\pi kh)\left[q_m \ln r + \sum_{k\neq m}(q_k \ln L_{km})\right], \quad \ldots\ldots\ldots (8)$$

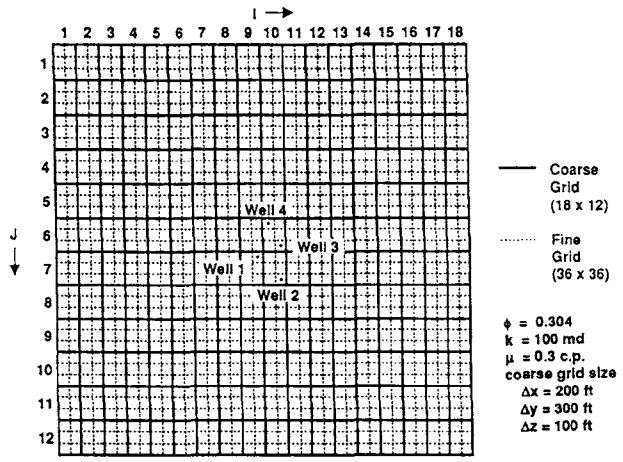

Fig. 9—Grid system for third test case.

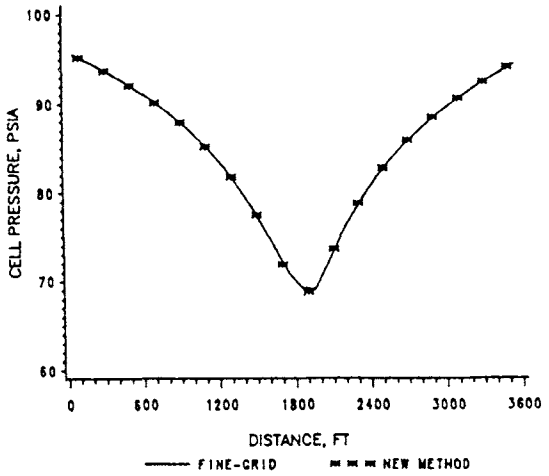

Fig. 10—Pressure comparison for multiple-well case.

where p_w = wellbore flowing pressure for the mth well and L_{km} = distance between the kth and mth wells.

We developed a small computer program based on Eq. 7 to calculate the pressure values along the boundary of the grid system shown in **Fig. 9.** The boundary cells have a very large porosity so that we can mimic a constant-pressure boundary condition. The production rates for Wells 1 through 4 are 1,000, 1,000, 800, and 200 STB/D, respectively. Fig. 9 shows the other properties. The calculated wellbore pressure with the new method, the analytical solution of Eq. 8, and the wellbore pressure calculated from the fine-grid model are in excellent agreement. **Fig. 10** shows the cell-pressure results along the sixth row ($J=6$) of the coarse-grid model.

Discussion

To test the generality of the new method, the off-center well in Test Case 1 is relocated to a position near the edge of the boundary (**Fig. 11**). **Fig. 12** shows the cell-pressure comparison for this new well location. Although intuitively the new well equation is proposed with the superposition concept of adding several radial flows together, the good match here suggests that the new method may work for nonradial flow.

To test the new method on a heterogeneous system, we reduced the permeability from 500 to 100 md for cells from Columns 4 through 6 ($I=4$ to 6) in the base case. The harmonic-averaging technique is used to obtain the average permeability value between the off-center well and the neighboring cell centers. All other conditions in this example are the same as in the base case. **Fig. 13** shows the cell-pressure results.

To test the new method for a 3D system, we added two layers to the base case. The off-center well is completed through all three layers. The match between the new method and the fine-grid model in the 3D test case is as good as the one in the 2D base case.

F_i is the most important parameter in the new well equation. Williamson and Chappelear[1] derived an analytical method to evaluate this geometric weighting factor. First, they proposed a truncated Fourier series containing only four terms to approximate the pressure solution around a well.

$$p - p_w = a_1 \ln(r/r_w) + r(b_1 \sin\theta + c_1 \cos\theta) + r^2 c_2 2\theta, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

where the origin is at the well center, the angle θ is measured from the line joining the well center to Node 1 (Fig. 2), and r is the distance from the well center. Then, the following matrix is inverted to obtain the weighting factor, g_{ij}:

$$\begin{bmatrix} \ln(r_1/r_w) & 0 & r_1 & r_1^2 \\ \ln(r_2/r_w) & r_2\sin\theta_2 & r_2\cos\theta_2 & r_2^2\cos 2\theta_2 \\ \ln(r_3/r_w) & r_3\sin\theta_3 & r_3\cos\theta_3 & r_3^2\cos 2\theta_3 \\ \ln(r_4/r_w) & r_4\sin\theta_4 & r_4\cos\theta_4 & r_4^2\cos 2\theta_4 \end{bmatrix}.$$

In this work, g_{i1} is related to F_i by

$$g_{i1} = F_i/\ln(r_i/r_w). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Applying the Williamson and Chappelear approach to the neighboring-cell configuration in the base case (Fig. 1), we can obtain the F_i of the four neighboring cells as 0.4285, 0.2273, 0.1170, and 0.2273, which are very close to the values calculated from Eq. 3 (0.4449, 0.2221, 0.1109, and 0.2221).

The test cases described have shown that selection of the neighboring cells in Fig.1 is very appropriate for wells located far away from the wellblock center. It is a more-logical and natural choice than the selection shown in Fig. 2. For example, if a well is located at the corner of four square cells, Eq. 3 will give $F_i=0.25$ to all the cells. If all the cell properties are the same, then well production will be divided equally among the four neighboring cells. The Williamson and Chappelear approach fails to give a meaningful g_{i1} solution for that well location if the neighboring-cell selection in Fig. 2 is used.

However, for off-center wells closer to the wellblock center, the neighboring cell selection in Fig. 2 becomes a better choice and the Williamson and Chappelear approach should be used to calculate F_i. In fact, the new equation will blow up for a well located exactly at cell center if the neighboring-cell selection in Fig. 1 is used.

Future research is needed to determine a switch criterion between the two selection choices. Tentatively, we suggest use of the selection in Fig. 1 if the off-center well is located at least $r_e=0.35$ away from the block center.

If a cell contains off-center wells in addition to wells exactly at the cell center, we must use the new equation for all the wells. The conventional well equation and the new well equation are not consistent with each other in interpreting the location of the wellblock pres-

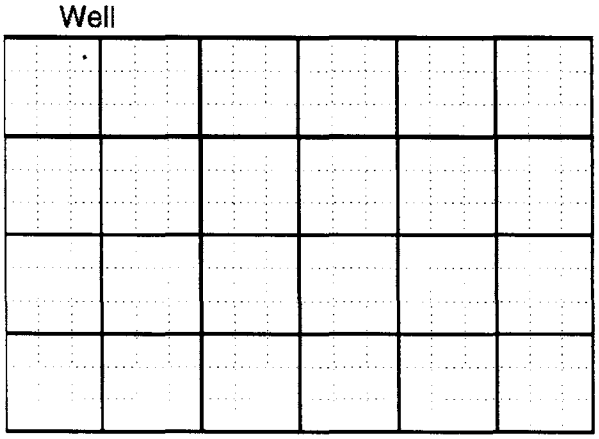

Fig. 11—Relocation of off-center well to edge cell.

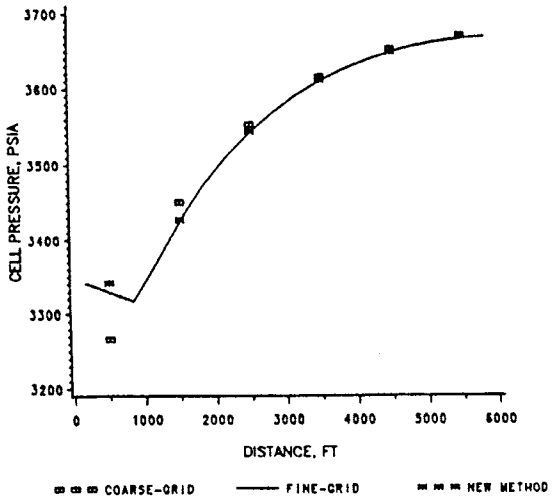

Fig. 12—Pressure comparison for test case of well in edge cell (t=0.5 year).

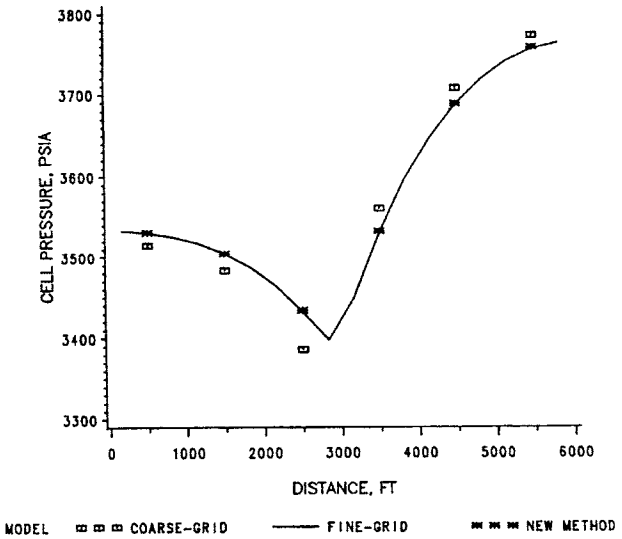

Fig. 13—Pressure comparison for varying-permeability test case.

sure. The conventional well equation may be used for center-wells located at least several cells away from any off-center well.

Conclusions

1. A new method for modeling one or more off-center wells in a cell has been proposed and has been validated by comparing it with an exact solution and results of several fine-grid-model tests.
2. The new method can be implemented easily into existing reservoir simulators.
3. The new method can improve the accuracy of coarse-grid-model simulation, as demonstrated in the edgewater-encroachment test case.
4. For off-center wells, the wellblock pressure calculated with the new method can be interpreted at the cell center.

Nomenclature

a,b,c = constants
B = FVF
F_i = geometric weighting factor for Cell i
g_{i1} = other weighting factor for Cell i
h = reservoir thickness, L
k = permeability, L^2
k_r = relative permeability, L^2
L_{km} = distance between kth and mth wells, L
L_{xyk} = distance between Point(x,y) and kth well, L
q = well rate, L^3/t, STB/D
p_i = neighboring cell pressure, m/Lt^2, psi
p_o = wellblock pressure, m/Lt^2, psi
p_w = wellbore pressure, m/Lt^2, psi
r = distance from a well center, L
r_e = wellblock drainage radius, L
$r_{eq,o}$ = equivalent wellblock radius, L
r_w = wellbore radius, L
S = saturation
t = time, t
x_D = dimensionless well location in x direction
y_D = dimensionless well location in y direction
δ = variable value changes between iterations
θ = angle measured from well center to neighboring cell center
μ = viscosity, m/Lt, cp
ρ = density, m/L^3
ϕ = porosity

Subscripts

j = matrix columns
o = oil
w = water
1,2,3,4 = neighboring cell index

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SI Metric Conversion Factors

bbl × 1.589 873	E−01 = m^3
cp × 1.0*	E−03 = Pa·s
in. × 2.54*	E+00 = cm
ft × 3.048*	E−01 = m
md × 9.869 233	E−04 = μm^2
psi × 6.894 757	E+00 = kPa

*Conversion factor is exact.

SPERE

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