Formal Methods in SE: Homework #3

Due on 3/6 at 11:59 a.m.

Professor Ciardo

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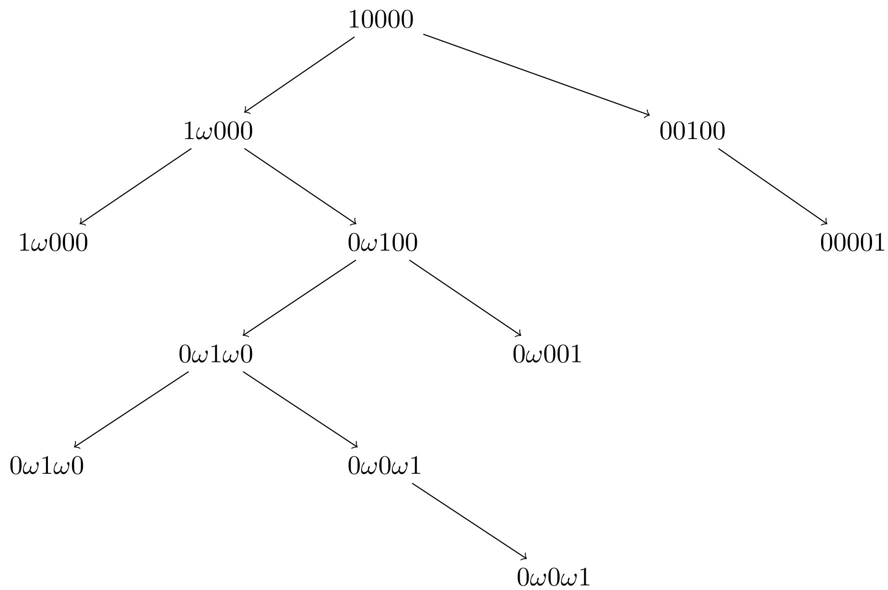
Problem 1



$\{1\omega000, 0\omega1\omega0, 0\omega0\omega1\}$

Problem 2

$([03110] \vDash \mathcal{AF}(p_4 = 0)) \Leftrightarrow \texttt{tt}$
$([03110] \vDash \mathcal{AF}(p_2 = 0 \wedge p_4 = 0)) \Leftrightarrow \texttt{ff}$

Problem 3

$\mathcal{L} = \{a^x b^y c^y | x, y \in \mathbb{N}\}$

Problem 4

Proof $\mathcal{L}$ (as defined above) is non-regular:

> Suppose $\mathcal{L}$ was a regular language. Let $\mathcal{L}' \subset \mathcal{L}$ such that $\mathcal{L}' = \{a^x b^n c^n \ |x = 0, y \in \mathbb{N}\}$. Additionally, let $\mathcal{L}'' = \{b^n c^n\}$. Clearly $\mathcal{L}' \equiv \mathcal{L}''$, therefore $\mathcal{L}'' \subset \mathcal{L}$. It is known that $\mathcal{L}''$ is a non-regular language, therefore we have a non-regular language $\mathcal{L}''$ which is subset of regular language $\mathcal{L}$. Here we have reached a contradiction as, for language $\mathcal{P}$, ($\mathcal{P} \in$ Regular Languages) $\Leftrightarrow$ ($\neg\exists\mathcal{P}' \subseteq \mathcal{P}$ such that $\mathcal{P}' \notin$ Regular Languages). Thus the assumption that $\mathcal{L}$ was a regular language was false, and $\mathcal{L}$ must be non-regular.

Proof $\mathcal{L}$ is a context-free language by defining context-free grammar $\mathcal{G}$

$\mathcal{G} = (V, T, S, P)$ such that
$V = \{X, Y, Z\}$,
$T = \{a, b, c\}$,
$S = X$,
$P = \{X \to YZ,\ Y \to Ya \,|\, a \,|\, \epsilon,\ Z \to bZc \,|\, bc \,|\, \epsilon\}$