CS284 Problem Set 3

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1: Direct Shooting and Transcription

(a) N decision variables, $u_0, ..., u_{N-1}$

(b) $c_g(x_N), c_o(x_k) \forall k \in [1, N]$ so $N+1$ altogether, and $x(0) = x_0$ if that's considered a constraint

(c) The same as (a) and additional $x_1, ..., x_N$ so $2N$ altogether

(d) The same as (b) and N constraints of the forward Euler integration for each step, so $2N+1$ altogether

(e) Changing any u_i will directly affect x_{i+1} in the shooting method, so $\frac{\delta c_g(x_N)}{\delta u_i} = \frac{\delta c_g(x_N)}{\delta x_N} \frac{\delta x_N}{\delta x_{N-1}} ... \frac{\delta x_{i+1}}{\delta u_i} \neq 0$, so N non-zero entries

(f) Unlike in the shooting method the $x_i's$ are fixed decision variables, so only x_N is non-zero

(g) As noted in part e, any change in a decision variable will cause a change in subsequent $x_i's$ so it will be (i)

(h) Similar to (f) only the x_i variable can affect the constraint corresponding to that i so it will be (iv)

2: Direct Collocation

(a)

Given the hint of the cubic term being $(\frac{2}{h^3}(x_0 - x_1) + \frac{1}{h^2}(\dot{x}_0 + \dot{x}_1))t^3$ and $x(0) = x_0$, we get that the cubic polynomial is of the form:

$$x(t) = (\frac{2}{h^3}(x_0 - x_1) + \frac{1}{h^2}(\dot{x}_0 + \dot{x}_1))t^3 + \alpha_2 t^2 + \alpha_1 t + x_0$$

Next plugging in $x(h) = x_1$, we get:

$$\begin{aligned} \alpha_1 &= \frac{3(x_1 - x_0) - h(\dot{x}_1 + \dot{x}_0) - \alpha_2 h^2}{h} \\ x(t) &= (\frac{2}{h^3}(x_0 - x_1) + \frac{1}{h^2}(\dot{x}_0 + \dot{x}_1))t^3 + \alpha_2 t^2 + [\frac{3(x_1 - x_0) - h(\dot{x}_1 + \dot{x}_0) - \alpha_2 h^2}{h}]t + x_0 \end{aligned}$$

Next plugging in $\dot{x}(0) = \dot{x}_0$, we get:

$$\begin{aligned}
\dot{x}_0 &= \frac{3(x_1 - x_0) - h(\dot{x}_1 + \dot{x}_0) - \alpha_2 h^2}{h} \\
\alpha_2 &= \frac{3(x_1 - x_0) - 2h\dot{x}_0 - h\dot{x}_1}{h^2} \\
x(t) &= (\frac{2}{h^3}(x_0 - x_1) + \frac{1}{h^2}(\dot{x}_0 + \dot{x}_1))t^3 + (\frac{3(x_1 - x_0) - 2h\dot{x}_0 - h\dot{x}_1}{h^2})t^2 + \\
&\quad \dot{x}_0 t + x_0
\end{aligned}$$

d **(b)**

$$\begin{aligned}
x(.5h) &= \frac{x_0 - x_1}{4} + \frac{h(\dot{x}_0 + \dot{x}_1)}{8} + \frac{3(x_1 - x_0)}{4} - \frac{h\dot{x}_0}{2} - \frac{h\dot{x}_1}{4} + \frac{h\dot{x}_0}{2} + x_0 \\
&= \frac{x_1 + x_0}{2} + \frac{h(\dot{x}_0 - \dot{x}_1)}{8} \\
x'(t) &= 3(\frac{2}{h^3}(x_0 - x_1) + \frac{1}{h^2}(\dot{x}_0 + \dot{x}_1))t^2 + 2(\frac{3(x_1 - x_0) - 2h\dot{x}_0 - h\dot{x}_1}{h^2})t + \dot{x}_0 \\
x'(.5h) &= \frac{3}{2}h(x_1 - x_0) - \frac{1}{4}(\dot{x}_0 + \dot{x}_1)
\end{aligned}$$

3: Collocation Constraint Implementation

Code for g/dg are in colconstraint.m, the P3 file runs checks the approximation errors.