

CS182 Assignment 5

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1: Assuming a uniform prior distribution, calculate the condition probability table (CPT) of $Pr(X = x|E = e)$ for all x and e

x	e	$P(E = e X = x)P(X = x)$	$P(X = x E = e)$
A	A	.125	.5
B	A	.0625	.25
C	A	.0625	.25
D	A	0	0
A	B	.0625	.25
B	B	.125	.5
C	B	0	0
D	B	.0625	.25
A	C	.0625	.25
B	C	0	0
C	C	.125	.5
D	C	.0625	.25
A	D	0	0
B	D	.0625	.25
C	D	.0625	.25
D	D	.125	.5

2: Now let the prior distribution be:

x	$P(X = x)$
A	0.4
B	0.2
C	0.1
D	0.3

Calculate the CPT $P(X = x|E = e)$ for all x and e

x	e	$P(E = e X = x)P(X = x)$	$P(X = x E = e)$
A	A	.2	.72
B	A	.05	.18
C	A	.025	.9
D	A	0	0
A	B	.1	.36
B	B	.1	.36
C	B	0	0
D	B	.075	.27
A	C	.1	.4
B	C	0	0
C	C	.05	.2
D	C	.075	.3
A	D	0	0
B	D	.05	.2
C	D	.025	.1
D	D	.15	.6

3: We next consider a time series of observations. Our transition model will be (rows X_{i-1}):

	$P(X_i X_{i-1})$			
	A	B	C	D
Begin	1	0	0	0
A	0.5	0.5	0	0
B	0.0	0.5	0.5	0
C	0.5	0	0	0.5
D	0.25	0.25	0.25	0.25

For this problem we are concerned with true (hidden) sequences, as opposed to observations. What is the probability under this model of the sequence of letters “A B B C D”? How about “A A B A”? What is $P(X_3 = x|X_1 = A, X_2 = B)$ for all x ?

$$\begin{aligned}
 ABBCD &= P(X_1 = A)P(X_2 = B|X_1 = A)P(X_3 = B|X_2 = B)P(X_4 = C|X_3 = B)P(X_5 = D|X_4 = C) \\
 &= 1 * 0.5 * 0.5 * 0.5 * 0.5 \\
 &= .0625
 \end{aligned}$$

$$\begin{aligned}
 AABA &= P(X_1 = A)P(X_2 = A|X_1 = A)P(X_3 = B|X_2 = A)P(X_4 = A|X_3 = B) \\
 &= 1 * 0.5 * 0.5 * 0.0 \\
 &= 0
 \end{aligned}$$

$P(X_3 = x|X_1 = A, X_2 = B) = P(X_3 = x|X_2 = B)$ which corresponds to row B of the table:

	A	B	C	D
B	0.0	0.5	0.5	0


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        return 0.0
    if x_1 == "C":
        if x == "A":
            return 0.5
        if x == "B":
            return 0.0
        if x == "C":
            return 0.0
        if x == "D":
            return 0.5
    if x_1 == "D":
        if x == "A":
            return 0.25
        if x == "B":
            return 0.25
        if x == "C":
            return 0.25
        if x == "D":
            return 0.25

#Normalize beliefs
def normalize(b):
    total = sum(b.values())
    for k in b:
        b[k] /= total

if __name__ == '__main__':
    states = ["A", "B", "C", "D"]
    E = ["A", "B", "B", "C", "D"]
    b = {}
    for x in states:
        b[x] = initProb(x)*E_cond_X(E[0], x)
    normalize(b)
    printBelief(1, b)
    for i in range(1, len(E)):
        temp = {}
        for x in states:
            temp[x] = 0.0
            for x_1 in states:
                temp[x] += E_cond_X(E[i], x)*X_cond_X_1(x, x_1)*b[x_1]
        normalize(temp)
        b = temp
        printBelief(i+1, b)

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