CS182 Assignment 5 Charles Liu cliu02@g.harvard.edu November 13, 2015

1: Assuming a uniform prior distribution, calculate the condition probability table (CPT) of Pr(X=x|E=e) for all x and e

x	e	P(E = e X = x)P(X = x)	P(X = x E = e)
A	A	.125	.5
В	A	.0625	.25
\mathbf{C}	A	.0625	.25
D	A	0	0
A	В	.0625	.25
В	В	.125	.5
\mathbf{C}	В	0	0
D	В	.0625	.25
A	\mathbf{C}	.0625	.25
В	\mathbf{C}	0	0
\mathbf{C}	\mathbf{C}	.125	.5
D	\mathbf{C}	.0625	.25
A	D	0	0
В	D	.0625	.25
\mathbf{C}	D	.0625	.25
D	D	.125	.5

2: Now let the prior distribution be:

$$\begin{array}{ccc} {\bf x} & P(X=x) \\ {\bf A} & {\bf 0.4} \\ {\bf B} & {\bf 0.2} \\ {\bf C} & {\bf 0.1} \\ {\bf D} & {\bf 0.3} \end{array}$$

Calculate the CPT P(X = x | E = e) for all x and e

X	e	P(E = e X = x)P(X = x)	P(X = x E = e)
A	A	.2	$.ar{72}$
В	A	.05	$.1\overline{8}$
\mathbf{C}	A	.025	$.\bar{9}$
D	A	0	0
A	В	.1	$.ar{36}$
В	В	.1	$.ar{36}$
\mathbf{C}	В	0	0
D	В	.075	$.2\overline{7}$
A	\mathbf{C}	.1	$.ar{4}$
В	\mathbf{C}	0	0
\mathbf{C}	\mathbf{C}	.05	$egin{array}{c} .ar{2} \ .ar{3} \end{array}$
D	\mathbf{C}	.075	$.ar{3}$
A	D	0	0
В	D	.05	$0 \over .ar{2}$
\mathbf{C}	D	.025	$.ar{1}$
D	D	.15	$.ar{6}$

3: We next consider a time series of observations. Our transition model will be (rows X_{i-1}):

$P(X_i X_{i-1})$						
	\mathbf{A}	\mathbf{B}	\mathbf{C}	\mathbf{D}		
Begin	1	0	0	0		
${f A}$	0.5	0.5	0	0		
\mathbf{B}	0.0	0.5	0.5	0		
${f C}$	0.5	0	0	0.5		
\mathbf{D}	0.25	0.25	0.25	0.25		

For this problem we are concerned with true (hidden) sequences, as opposed to observations. What is the probability under this model of the sequence of letters "A B B C D"? How about "A A B A"? What is $P(X_3 = x | X_1 = A, X_2 = B)$ for all x?

$$ABBCD = P(X_1 = A)P(X_2 = B|X_1 = A)P(X_3 = B|X_2 = B)P(X_4 = C|X_3 = B)P(X_5 = D|X_4 = C)$$

$$= 1*0.5*0.5*0.5*0.5$$

$$= .0625$$

$$AABA = P(X_1 = A)P(X_2 = A|X_1 = A)P(X_3 = B|X_2 = A)P(X_4 = A|X_3 = B)$$

$$= 1*0.5*0.5*0.0$$

$$= 0$$

$$P(X_3 = x|X_1 = A, X_2 = B) = P(X_3 = x|X_2 = B) \text{ which corresponds to row } B \text{ of the table:}$$

$$A B C D$$

$$B 0.0 0.5 0.5 0.5 0$$

4: Finally we consider the full filtering problem $P(X_n|E_1,...,E_n)$. Let "A B B C D" be the sequence of observed key strokes. What is the current belief state of the model? That is compute $P(X_n = x|E_1 = A, E_2 = B, E_3 = B, E_4 = C, E_5 = D)$ for all x.

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We build a beliefs table below (row i corresponds to belief b_i):
 i
            Α
 1
           1.0
                             0.0
                                               0.0
                                                           0.0
 2
                                                           0.0
     0.3333333333333
                       0.666666666667
                                               0.0
 3
     0.142857142857
                       0.857142857143
                                                           0.0
                                               0.0
 4
    0.0769230769231
                                         0.923076923077
                                                           0.0
                             0.0
 5
                             0.04
           0.0
                                               0.0
                                                          0.96
Table produced by following Python code:
#Print latex table row
def printBelief(i, b):
     print "{} _&_{} _&_{} _&_{} _. \\\". format(i, b["A"], b["B"], b["C"], b["D"])
\#Initial distribution
def initProb(x):
     if x = "A":
          return 1.0
     return 0.0
\#Pr(E_i = e \mid X_i = x)
\mathbf{def} \ \mathbf{E}_{-}\mathbf{cond}_{-}\mathbf{X}(\mathbf{e}, \mathbf{x}):
     if e == x:
          return 0.5
     if (e = "A" \text{ and } x = "D") \text{ or } (e = "D" \text{ and } x = "A") \text{ or } \setminus
         (e = "B" \text{ and } x = "C") \text{ or } (e = "C" \text{ and } x = "B"):
              return 0.0
     return 0.25
\#Pr(X_{-}i = x \mid X_{-}i-1 = x_{-}1)
\operatorname{def} X_{-\operatorname{cond}_{-}}X_{-1}(x, x_{-1}):
     if x_1 = "A":
          if x = A:
                return 0.5
          if x = B:
                return 0.5
          if x = "C":
                return 0.0
          if x = "D":
                return 0.0
     if x_1 = B:
          if x = "A":
                return 0.0
          if x = "B":
                return 0.5
          if x = "C":
                return 0.5
          if x = "D":
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return 0.0
    if x_1 = "C":
        if x = "A":
             return 0.5
        if x = "B":
             return 0.0
        if x = "C":
             return 0.0
        if x = "D":
             return 0.5
    if x_1 = "D":
        \mathbf{i}\,\mathbf{f}\ x == "A":
             return 0.25
        if x = "B":
             return 0.25
        if x = "C":
             return 0.25
        if x = "D":
             return 0.25
#Normalize beliefs
def normalize(b):
    total = sum(b.values())
    for k in b:
        b[k] /= total
if __name__ = '__main__':
    states = ["A", "B", "C", "D"]
    E = ["A", "B", "B", "C", "D"]
    b = \{\}
    for x in states:
        b[x] = initProb(x)*E_cond_X(E[0], x)
    normalize(b)
    printBelief(1, b)
    for i in range (1, len(E)):
        temp = \{\}
        for x in states:
             temp[x] = 0.0
             for x_1 in states:
                 temp[x] += E_{cond}X(E[i], x)*X_{cond}X_{1}(x, x_{1})*b[x_{1}]
        normalize (temp)
        b = temp
        printBelief(i+1, b)
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