Exercise 2

April 11, 2021

1. Consider set G of 3×3 matrices defined as follows:

   $$G = \{ \begin{bmatrix} 1 & x & z \\ 0 & 1 & y \\ 0 & 0 & 1 \end{bmatrix} \in R^{3 \times 3} | x, y, z \in R\}$$

   We define $\cdot$ as the standard matrix multiplication.
   Is (G,$\cdot$) a group? If yes, is it Abelian? Justify your answer

2. Which of the following sets are subspaces of R^3:

   (a) $A = \{(\lambda, \lambda + \mu^3, \lambda - \mu^3) | \lambda, \mu \in R\}$

   (b) $B = \{(\lambda^2, -\lambda^2, 0) | \lambda \in R)\}$

   (c) $C = \{(\xi_1, \xi_2, \xi_3) \in R^3 | \xi_2 \in Z\}$

3. Consider two subspaces of R^4:

   $$\text{U\_1} = Span[\begin{bmatrix} 1 \\ 1 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}], U_2 = \text{Span}[\begin{bmatrix} -1 \\ -2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 6 \\ -2 \\ -1 \end{bmatrix}]$$

   Determine a basis of $U_1 \cap U_2$

4. Are the following sets of vectors linearly independent?

(a) $x_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, $x_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

5. Write $y = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}$ as linear combination of

   $x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $x_3 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

6. Proof: rank(A) = rank(A^T)