

# Exercise 7

January 15, 2021

1. Compute the derivative  $f'(x)$  for:

$$f(x) = \log(x^4) \sin(x^3)$$

2. Compute the derivative  $f'(x)$  of logistic sigmoid:

$$f(x) = \frac{1}{1 + \exp(-x)}$$

3. Compute the derivative  $f'(x)$  of function:

$$f(x) = \exp\left(-\frac{1}{2\sigma^2}(x - \varphi)^2\right)$$

Where  $\sigma$  and  $\varphi \in \mathbb{R}$  are constant

4. Compute the Taylor polynomial  $T_n, n = 0, \dots, 5$  of  $f(x) = \sin(x) + \cos(x)$  at  $x_0 = 0$
5. Compute the derivatives  $df/dx$  of the following functions. Provide the dimensions of every single partial derivative. Describe your steps in detail.

- (a) Use the chain rule. Provide the dimensions of every single partial derivative.

$$f(z) = \exp\left(-\frac{1}{2z}\right)$$

$$z = g(y) = y^T \tilde{S}^{-1} y$$

$$y = h(x) = x - \mu$$

- (b)  $f(x) = \text{tr}(xx^T + \sigma^2 y), x \in \mathbb{R}^D$  Here  $\text{tr}(A)$  is the trace of  $A$ , i.e., the sum of the diagonal elements  $A_{ii}$ .

Hint: Explicitly write out the outer product

- (c) Use the chain rule. Provide the dimensions of every single partial derivative. You do not need to compute the product of the partial derivatives explicitly.

$$f = \tanh(z) \in \mathbb{R}^M$$

$$z = Ax + b, x \in \mathbb{R}^N, A \in \mathbb{R}^{M \times N}, b \in \mathbb{R}^M$$

Here,  $\tanh$  is applied to every component of  $z$ .