Solution 1

December 24, 2020

1. (a) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

   (b) 14

   (c) can not be multiplied

   (d) $\begin{bmatrix} 5 & 7 & 9 \\ 11 & 13 & 15 \\ 8 & 10 & 12 \end{bmatrix}$

   (e) $\begin{bmatrix} 14 & 6 \\ -21 & 2 \end{bmatrix}$

2. Using Gaussian elimination:

   (a) $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 1 \\ 1 & 1 & -4 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

   We start with: $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 2 & 1 & 1 \\ 1 & 1 & -4 & 0 \end{array}\right]$

   $\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & 1/2 & 1/2 \\ 0 & 0 & 1 & 1/3 \end{array}\right] R_2 \times 1/2, (-R_3 + R_1) \times 1/3$

   $\Rightarrow \begin{cases} x_1 + x_2 + x_3 & = 1 \\ x_2 + 1/2x_3 & = 1/2 \\ x_3 & = 1/3 \end{cases}$

$\Rightarrow (x_1, x_2, x_3) = (1, 1/3, 1/3)$
$\Rightarrow X = \begin{bmatrix} 1 & 1/3 & 1/3 \end{bmatrix}^T$

(b) We start with: $\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 1 \\ 2 & 5 & -7 & 5 & -2 \\ 2 & -1 & 1 & 3 & 4 \\ 5 & 2 & -4 & 2 & 6 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 1 \\ 0 & 3 & -5 & -3 & -4 \\ 0 & -3 & 3 & 5 & 2 \\ 0 & -3 & 1 & 7 & 1 \end{array}\right] R_2 - 2R_1, R_3 - 2R_1, R_4 - 5R_1$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 1 \\ 0 & 3 & -5 & -3 & -4 \\ 0 & 0 & -2 & 2 & -2 \\ 0 & 0 & -4 & 4 & -3 \end{array}\right] R_3 + R_2, R_4 + R_2$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 1 \\ 0 & 3 & -5 & -3 & -4 \\ 0 & 0 & -2 & 2 & -2 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right] R_4 - 2R_3$

No solution

(c) We start with: $\left[\begin{array}{ccc|c} 2 & 1 & 1 & 3 \\ -1 & 2 & 1 & 6 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 2 & 9 \\ -1 & 2 & 1 & 6 \end{array}\right] R_1 + R_2$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 2 & 9 \\ 0 & 1 & 3/5 & 3 \end{array}\right] (R_2 + R_1) \times 1/5$

We see: $X = \begin{bmatrix} 0 & 3 & 0 \end{bmatrix}^T$ is a solution
Again: $c_3 = 1/5c_1 + 3/5c_2 \Rightarrow \alpha(3c_1 + 9c_2 - 15c_3) = 0$
$\Rightarrow X = \begin{bmatrix} 0 & 3 & 0 \end{bmatrix}^T + \alpha \begin{bmatrix} 3 & 9 & -15 \end{bmatrix}^T \alpha \in R$ is general solution

3. Determine the inverse matrix:

(a) $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

We perform Gaussian elimination on $\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 6 & 0 & 0 & 1 \end{array}\right]$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/6 \end{array}\right] R_1 \times 1/2, R_2 \times -1, R_3 \times 1/6$$

The matrix to the right of the vertical line is the inverse of A.

(b) $A = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$

We perform Gaussian elimination on $\left[\begin{array}{cc|cc} 4 & 1 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right]$

$$\Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 3 & 1 & 0 & 1 \end{array}\right] R_1 - R_2$$

$$\Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & -3 & 4 \end{array}\right] R_2 - 3R_1$$

The matrix to the right of the vertical line is the inverse of A.