

Exercise 6

January 10, 2021

1. Are the following matrices diagonalizable? If yes, determine their diagonal form and a basis with respect to which the transformation matrices are diagonal. If no, give reasons why they are not diagonalizable.

(a) $A = \begin{bmatrix} 0 & 1 \\ -8 & 4 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 5 & 4 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ -1 & -1 & 3 & 0 \\ 1 & 1 & -1 & 2 \end{bmatrix}$

(d) $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$

2. Find the SVD of the matrix

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$

3. Find the singular value decomposition of $A = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$

4. Find the rank-1 approximation of $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$

5. Show that for $x \neq 0$
- $$\max_x \frac{\|Ax\|_2}{\|x\|_2} = \theta_1, \text{ where } \theta_1 \text{ is the largest singular value of } A \in R^{m \times n}$$