Exercise 8

January 16, 2021

1. Consider the univariate function:

   $f(x) = x^3 + 6x^2 - 3x - 5$

   Find its stationary points and indicate whether they are maximum, minimum, or saddle points.

2. Express the following optimization problem as a standard linear program in matrix notation
   $\max_{x \in R^2, \xi \in R} p^T x + \xi$

3. Consider the linear program:

   $$\min_{x \in R^2} - \begin{bmatrix} 5 \\ 3 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ subject to } \begin{bmatrix} 2 & 2 \\ 2 & -4 \\ -2 & 1 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 33 \\ 8 \\ 5 \\ -1 \\ 8 \end{bmatrix}$$

   Derive the dual linear program using Lagrange duality

4. Consider the following convex optimization problem:

   $\min_{x \in R^D} \frac{1}{2} w^T w$ subject to $w^T w \geq 1$

   Derive the Lagrangian dual by introducing the Lagrange multiplier λ.

5. Consider the quadratic program:

$$\min_{x\in R^2} \frac{1}{2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 5 \\ 3 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{subject to } \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \end{bmatrix} \leq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Derive the dual quadratic program using Lagrange duality.