A survey of online learning-augmented algorithms

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Abstract

Online learning-augmented (OLA) algorithms utilize predictions from machine learning (ML) models to overcome the limitations of the worst-case analysis of online algorithms. These algorithms can achieve near-optimal performance when the predictions are accurate (consistency) while ensuring worst-case performance guarantees in the presence of high prediction errors (robustness). Numerous OLA algorithms have been developed to tackle a wide range of online problems, such as caching (Antoniadis et al., 2022) and ski rental (Anand et al., 2022c). Recognizing the accelerated development, Mitzenmacher and Vassilvitskii (2020) conducted a *problem-centric* survey of OLA algorithms, showing connections between algorithms within specific problem domains. Nevertheless, this approach fails to capture connections among algorithms across different problems. Moreover, several studies (Étienne Bamas et al., 2020; Grigorescu et al., 2022) have developed unified frameworks and demonstrated their applications across multiple online problems. Therefore, our goal is to survey OLA algorithms from a general perspective, focusing on *settings* and *algorithmic design* to provide a more comprehensive view that transcends individual problem domains.

1 Introduction

Online learning-augmented (OLA) algorithms (Kumar et al., 2024; Antoniadis et al., 2022; Anand et al., 2022c) represent the intersection of online algorithms and machine learning (ML) models. In traditional online learning problems, the input arrives sequentially, requiring the algorithm to make irrevocable decisions without knowledge about the future events. Future uncertainty requires the design of online algorithms with worst-case guarantees compared to offline optimal solutions (OPT). However, such guarantees can lead to being *overly cautious*, rendering these algorithms **impractical** in situations where worst-case events are rare or low-cost. Conversely, learning-augmented algorithms incorporate predictions from ML models about future input. While these predictions can enhance performance, they are often unreliable since there are no assumptions about their accuracy. Consequently, the primary challenge lies in leveraging accurate predictions while remaining robust against inaccurate ones. To resolve this, learning-augmented algorithms are required to satisfy two properties: *consistency* and *robustness*. The *consistency* property ensures the algorithms perform near-optimally when predictions are correct. In the case of inaccurate predictions, the *robustness* property guarantees that the performance remains competitive with traditional online algorithms. Therefore, OLA algorithms combine the advantages of learning-augmented approaches with the reliability of classical online algorithms, enabling them to perform beyond the worst-case analysis. In other words, the predictive component of learning-augmented algorithms enhances the **practicality** of online algorithms.

Due to limitations of the previous survey (Mitzenmacher and Vassilvitskii, 2020), our goal is to provide an alternative overview of OLA algorithms to better understand the current landscape of the field. The previous survey categorized OLA algorithms from a *problem-centric* perspective, including problems like ski rental, counting sketches, bloom filters, caching, and scheduling problems. However, this categorization has several limitations. First, the paper covers only a limited subset of online problems, omitting important others, such as set cover, secretary, and bipartite graph matching. Second, while Mitzenmacher and Vassilvitskii (2020) focusing on relationships among algorithms within the same problem category, they neglect to identify connections across different categories. This limitation prevents the recognition of common techniques that could apply to diverse problems. Third, new research has created generalized frameworks (Étienne Bamas et al., 2020; Grigorescu et al., 2022) or generalized the basic setting (Im et al., 2022). These methods can be used for more than one problem. This means that the problem-centric approach often unnecessarily includes the same framework in more than one problem category. Motivated by these drawbacks, we propose a new taxonomy of OLA algorithms based on the *settings* or *algorithmic design* perspective. Since our focus is on the practicality of OLA algorithms, we cover those algorithms with more *complex settings* and *generalized algorithm* paradigms. Therefore, we propose a novel way to survey OLA algorithms using the following five perspectives:

1. **Restricted predictions**: Previous OLA algorithms were designed to access predictors with an **unrestricted** number of queries. In certain applications, it may not be optimal to query predictions every round due to potentially repetitive information or the high cost associated with such queries. Therefore, the **restricted** setting aims to limit the power of advice through reducing the number of queries or constraining the size of advice.
2. **Multiple predictors**: Many OLA algorithms assume there exists a **single predictor** to query predictions. However, there are scenarios where we can easily obtain **multiple predictors** that provide advice. Therefore, the multiple-predictors setting aims to design either problem-specific or generalized OLA algorithms that can fully exploit this extra advantage.
3. **Grey-box predictors**: Previously, **black-box** assumptions about prediction models prevented the algorithms from having any knowledge about the models' construction. Due to the black-box nature of predictors, previous algorithms also cannot always trust the prediction quality to improve performance. However, in practice, it is reasonable to assume that we already know some information, such as the type of algorithm that actually trains the model. Therefore, **grey-box** predictors are used to leverage extra information about the models' structure to better design and enhance the performance of OLA algorithms.
4. **Optimal consistency-robustness trade-off**: Previous studies only characterize loosely the **non-optimal** trade-off between consistency and robustness, which leads to the sensitivity of algorithms to total prediction error. Recent studies have shown how **optimal** trade-offs can effectively address this issue.
5. **Systematic algorithmic design**: Many OLA algorithms are developed from an **ad-hoc** perspective, by extending online algorithms specialized to specific problems. These algorithms are restricted to particular problems, hence difficult to generalize to others. A more **systematic** framework, such as the primal-dual learning-augmented framework (Étienne Bamas et al., 2020), can unify these ad-hoc algorithms to solve a broader range of problems.

These five dimensions are illustrated in the figure 1. Each circle represents a distinct perspective on OLA algorithms. It is also important to note that *several perspectives can be combined to enhance the practicality of OLA algorithms*. For instance, Étienne Bamas et al. (2020) combine the last two perspectives to design a systematic primal-dual learning-augmented framework (PDLA) with optimized robustness-consistency trade-off for the ski rental problem (figure 2). The survey will be structured as follows: Section 2 will introduce key definitions used throughout this survey, while Sections 3 to 8 review papers within these five perspectives, including a smaller sub-category for each, along with suggested direcitons for future research.

Remark: In this document, we interchangeably refer to model as oracle and advice as prediction.

2 Preliminaries

In this section, we describe key definitions that will be used extensively throughout the document. Consider a given online learning problem, an algorithm ALG produces a solution $x_0, x_1, ..., x_T$ for

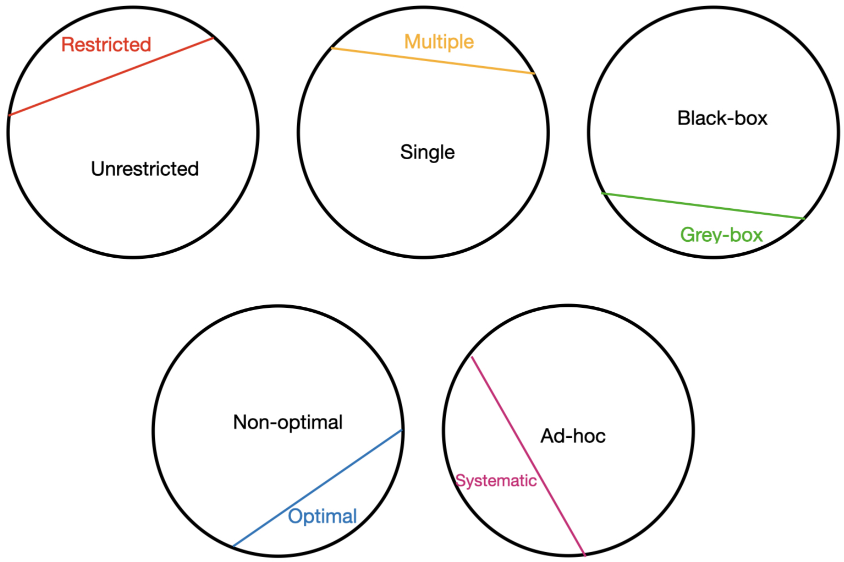

Figure 1: 5 perspectives on viewing OLA algorithms

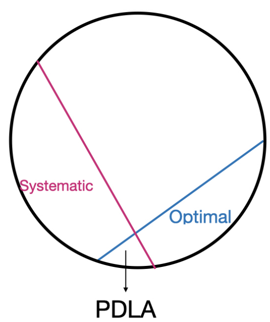

Figure 2: PDLA framework

an input instance I consisting of loss functions $l_1, ..., l_T$. Then, the total cost of the algorithm is $cost(ALG(I)) = \sum_{t=1}^{T} l_t(x_t)$. We said the algorithm is α-*competitive* if

$$cost(ALG(I)) \leq \alpha \cdot OPT(I) \quad \forall I \tag{1}$$

where $OPT(I)$ is the optimal offline solution of a given instance. Let define a function $c(\eta)$ where η is the error of the predictor. We say the algorithm is γ-*robust* if $c(\eta) \leq \gamma$ for all η, and β-*consistent* if $c(0) = \beta$. Intuitively, the *robustness* is a measure of the algorithm in the worst-case of prediction accuracy. Meanwhile, the *consistency* measures the algorithm under perfect predictions. Lastly, the *smoothness* captures the requirement that the competitive ratio smoothly interpolates between the two extremes, namely consistency and robustness. In other words, *smoothness* property prevents the algorithms from being too sensitive to the change of the predictors' error.

3 Restricted predictions

Most OLA algorithms assume unlimited access to predictors, while practical constraints, such as high prediction costs, have led to the development of frameworks that restrict predictor usage. Many studies in OLA algorithms have demonstrated efficient solutions to problems such as caching (Emek et al., 2020), matching, load balancing, and non-clairvoyant scheduling (Dinitz et al., 2022). These approaches assume **unrestricted predictions** at no cost, which can be impractical due to constrained computation resources. Moreover, not all predictions are informative and repetitive and incorrect predictions can degrade algorithms' performance. Therefore, the **restricted prediction** setting aims to design algorithms that minimize the number of prediction queries while preserving the performance guarantees. This survey primarily focuses on algorithms in this restricted prediction setting due to its practicality. There are two main strategies, including *limiting the number of prediction queries* or *constraining the size of prediction*.

3.1 Limiting the number of prediction queries

In this setting, OLA algorithms aim to maintain competitive performance while reducing the number of queries.

While most online problems contain multiple random variables for prediction, the ski rental problem only requires a single prediction. Drygala et al. (2023) propose a novel approach to optimize the usage of a single prediction. While previous algorithms query predictions immediately, the authors recommend delaying the prediction by $\sqrt{cB}$ days, where c is the prediction cost and B is the ski cost. Moreover, they propose to use statistical information, such as the mean and variance of the ski season's length, to replace the prediction query. If the prediction cost is higher than the expected value of season length and the variance is low, the prediction is redundant. Building on the previous idea, Benomar and Perchet (2023) design an algorithm that avoids explicitly representing the cost. Instead of predicting the season length as a continuous value, their approach uses binary predictions to compare it to the budget, assuming prediction accuracy improves over time. Both papers Drygala et al. (2023); Benomar and Perchet (2023) demonstrates novel algorithms in the restricted setting by proposing competitive algorithms that optimize the timing of prediction queries. Together, these studies address two important questions: (1) *What is the optimal time to query predictions?* and (2) *Can alternatives, such as statistical information, substitute for predictions?*

For problems requiring multiple predictions, recent research focuses on balancing between query predictions and performance. Drygala et al. (2023) extend their analysis to the Bahncard problem, deriving a lower bound on the necessary number of predictions to maintain algorithmic guarantees. Im et al. (2022) further advance this direction by introducing a caching algorithm employing sublinear predictions. While previous algorithms query predictions for each arrival page, this algorithm reduces the number of queries by sampling B unmarked pages uniformly without replacement for predictions. Their experimental validation on the CitiBike dataset supports these findings. Sadek and Elias (2024) extend the concept of restricted predictions to the metrical task system (MTS) problem - a generalization of the online caching problem. Moreover, the authors propose an *action prediction* setup, which contains not only the previous prediction setup (Im et al., 2022) but also cache configurations. The study also addresses the sensitivity of previous algorithms to prediction errors by establishing the relationship between prediction frequency and the smoothness property. Their results demonstrate that with $f(\log k)OPT$ predictions, algorithms can achieve 1-consistency, $O(\log k)$-robustness, and $O(f^{-1}(\eta/OPT))$-smoothness for any convex functions f with $f(0) = 0$ and $f(i) \leq 2^i - 1$. Among the functions meeting these conditions, the choice of f plays a crucial role in optimizing smoothness, affecting the sensitivity of the algorithm. They also extend these results to a *well-separated queries* setting, allowing predictions for at least every a time step. This adjustment achieves a competitive ratio of $O(a)$ times the optimal value. Empirical evaluations on CitiBike and BrightKite datasets demonstrate robust performance under noisy predictions, although extreme noise slightly degrades results. Benomar and Perchet (2024) introduce algorithms that address the challenges of restricted predictions for the non-clairvoyant scheduling problem. They introduced the Catch-up and Resume Round-Robin (CRRR) algorithm for perfect predictions, leveraging the relative order of a subset of job sizes to reduce the delays. For imperfect predictions, the Switch algorithm further enhances the performance by combining the predictions of job size with well-chosen randomized breakpoints. Finally, experimental results on synthetic benchmarks confirm the effectiveness of these algorithms. Together, these studies propose the *minimum number of predictions* as well as the *frequency of these predictions* to maintain previous performance.

There are also studies that use similar settings but look at different problems. These include online linear optimization (Bhaskara et al., 2021), clustering (Aamand et al., 2023), sorting (Bai and Coester, 2023; Benomar and Coester, 2024), and the multi-color secretary problem (Benomar et al., 2024). These algorithms generally design mechanisms to strategically delay queries while maintaining competitive performance. Notably, Drygala et al. (2023) suggest that carefully leveraging prior knowledge can sometimes replace prediction queries altogether. However, knowing the true expected value and variance is a strong assumption, which is sometimes difficult to satisfy. Therefore, an intriguing future research question is whether it is possible to maintain competitive performance with only sample statistics. For the non-clairvoyant scheduling problem, CRRR only designs for perfect action prediction setup. Also, there is a gap between the current competitive ratios and the lower bound. Therefore, addressing these issues can further enhance the performance of current algorithms.

3.2 Constraining the size of prediction

Some methods focus on restricting the amount of information per prediction query, in contrast to the above setting, which limits the prediction frequency.

Antoniadis et al. (2022) address the online caching problem by using just one bit per prediction. While previous methods (Lykouris and Vassilvitskii, 2020) rely on fully informative predictions to achieve competitive performance, this limited setting can make these algorithms susceptible to wrong predictions. Therefore, the paper introduces a novel approach using two separate loss functions. Specifically, the first quantifies the cost of wrong evictions (one cache miss), while the second captures the cost for keeping a page that should have been evicted (potentially multiple cache misses). This dual loss approach makes it easier to understand the connection between loss and competitive performance, which the algorithms use to minimize cache misses. Also, the authors adapt the algorithms to two setups: next arrival predictions and action prediction settings, and achieve competitive ratios comparable to prior methods. Similarly, Mitzenmacher (2020) extends the concept of using single-big advice to queue scheduling problems. The single-bit advice indicates whether the service time of a job is above or below a defined threshold. By modeling queues with this binary prediction, it demonstrates how even minimal, potentially inaccurate advice can significantly improve scheduling metrics like waiting and sojourn times. This approach adapts classical tools like Kleinrock's Conservation Law and mean-field analysis to derive performance equations for both single and multiple queue systems. Furthermore, it exhibits robustness across various distributions, including the heavy-tailed ones such as Weibull. To confirm theoretical results, simulations show significant performance gains even when the advice is imperfect. Lastly, Dütting et al. (2020) propose a linear programming framework for solving variations of secretary problems with small, potentially noisy advice. By identifying the structural properties, such as Non-Filtering (NF) and History-Irrelevance (HI), the framework efficiently computes optimal solutions under different variations using closed-form solutions for thresholds and asymptotic probability. In summary, these algorithms demonstrate the possibility of using minimal information to not only maintain but also improve the performance of OLA algorithms.

Overall, these studies demonstrate how online algorithms can strategically leverage limited and noisy information to maintain competitive performance. However, they also rely on specific problem structures, such as caching and queuing, which restrict their generalizability to other domains. This limitation leads to interesting research directions, such as the design of a general framework that integrates limited predictions across diverse problems. For the queue problem, the existing scheduling schemes are fundamental yet simple, so investigating other schemes would further show the practicality of the setting.

4 Multiple predictors

While OLA algorithms typically assume a **single oracle** for prediction queries, some applications can contain **multiple predictors** providing diverse advice. For instance, there are different models trained on distinct algorithms for the same task. Despite this practical relevance, most existing OLA algorithms (Emek et al., 2020; Dinitz et al., 2022) are designed under the assumption of a single predictor. Researchers have looked into the multiple predictors setting across various problem domains, including online facility location (Almanza et al., 2021), scheduling (Angelopoulos et al., 2024), portfolio optimization (Dinitz et al., 2022), and the multi-shop ski rental problem (Wang et al., 2020). Due to their practicality, this survey aims to highlight the research progress in the setting of multiple predictors. Leveraging advice from multiple predictors presents unique challenges, such as determining which advice to trust when predictors provide conflicting or even inaccurate recommendations. To deal with these issues, two main types of algorithmic approaches have come up: *problem-specific* approaches, which use the structure of specific problems, and *generalized approaches*, which offer a framework that can be used for many problems.

4.1 Problem-specific approach

Many OLA algorithms for multiple predictors build upon problem-specific methods by leveraging additional structural assumptions unique to the problem.

Gollapudi and Panigrahi (2019) critically evaluates how predictions from multiple experts can improve the performance of existing algorithms for the classical rent-or-buy problem. Their work generalizes prior single-expert methods, introducing new partitioning strategies to integrate advice from multiple experts. They demonstrate theoretically optimal bounds on consistency and robustness, achieving a golden ratio ϕ for two experts and extending this result to k-experts with generalized Fibonacci-based consistency ratios. Experimental results validate the practical effectiveness of these approaches, revealing that a small number of experts (3–5) suffices to significantly lower prediction errors and improve decision outcomes. Wang et al. (2020) studies multiple expert settings but for a generalization of the ski rental problem, called the multi-shop ski rental (MSSR) problem. Unlike the classical ski rental setting, MSSR introduces shop-specific renting and buying costs, significantly increasing decision complexity. To tackle this, they propose novel algorithms that leverage probabilistic buy timing and majority-based thresholds to manage conflicting advice from multiple predictors. Experiments on synthetic and real-world datasets demonstrate the effectiveness of these methods, with results showing new algorithms can outperform classical strategies, even when predictions are imperfect.

We also want to highlight other studies addressing a variety of problems, such as online linear optimization Bhaskara et al. (2020), online facility control (Almanza et al., 2021), matching, load balancing, and non-clairvoyant scheduling (Dinitz et al., 2022). While these methods exploit problem-specific structures or extend prior techniques to improve performance, they often lack the flexibility to generalize across different problems. The general approaches in the next section will address partially this problem.

4.2 General approach

In contrast to problem-specific methods, general approaches use optimization techniques that are independent of problem structure to tackle a broader class of problems.

Anand et al. (2022a) introduce a general framework for online covering problems in multiple machine-learned predictions settings. By incorporating box constraints, the framework is solvable for even broader problem classes. While previous papers rely on static benchmarks where the algorithms are compared against the solution suggested by a single best expert, this paper introduces the dynamic benchmark, allowing the best solution at each step to be selected from k available predictions. Extending the classical potential-based analysis, the framework achieves an $O(\log k)$-competitive ratio against the best dynamic solution. This competitive ratio depends on the number of experts rather than problem-specific factors, allowing scalability for large problem instances where traditional bounds may fail. Another strength is the robustification technique, which balances performance between predictions and worst-case online guarantees. Kevi and Nguyen (2023) critically evaluate limitations of the earlier proposed benchmark (Anand et al., 2022a) by identifying flawed assumptions such as the unrealistic requirement for tight expert solutions, which lead to unbounded performance guarantees under certain conditions. Instead, the authors propose a new benchmark, called LIN-COMB, which models linear combinations of expert solutions. Instead of applying potential-based methods, their novel algorithm uses a primal-dual framework and achieves a competitive ratio of $O(\log k)$. Moreover, the authors extend similar ideas to general optimization problems beyond 0-1, such as ski rental and facility location. Recently, Antoniadis et al. (2023) tackled the challenge of combining multiple predictors in Metrical Task Systems (MTS) problems. The paper demonstrates the hardness of the problem by showing no algorithm can perform better than an $O(k^2)$-competitive ratio against the best dynamic combination without problem structure assumptions. However, there exists a $(1+\epsilon)^2$-competitive randomized algorithm leveraging the structure of prediction errors for any $\epsilon > 0$. The paper also extends its analysis to a bandit setting, where there are multiple predictors, but the algorithm only selects one and interacts with it, gaining information only about the chosen predictor.

These approaches rely on optimization techniques rather than problem-specific structures, which allow them to solve for a wider range of problems. Despite introducing stronger benchmarks, these algorithms can achieve competitive performance, which demonstrates their potential real-world applications. However, we can further explore open research questions, such as expanding these frameworks to address packing problems and non-linear objectives. Given the same online problem, for instance ski rental, another research direction is to empirically evaluate the general and problem-specific approaches on the same benchmark to highlight their difference as well as their relative computational complexity.

5 Grey-box predictors

While the majority of OLA algorithms assume ML predictions came from **black-box** machine learning models, recent advancements emphasize **grey-box** models to further improve algorithmic performance. Black-box models make no assumptions about the training procedure or the generation of predictions. Although this simplifies the integration of ML predictions, it limits the algorithm's ability to fully exploit consistency when the predictions are correct. Additionally, because predictions are considered untrusted, this assumption requires OLA algorithms to provide worst-case guarantees. However, in practice, we usually have partial knowledge about the potential algorithms used to train these models. Therefore, OLA algorithms can exploit this knowledge to enhance performance. This survey focuses on grey-box predictors, which follow two main strategies: *learning predictions from samples or distributional advice* and *assuming predictions to be correct with a certain probability.*

5.1 Learning predictions from samples/distributions

Recent studies have shifted to relax this assumption by allowing the model to learn to adapt during the optimization process. The key difference between common learning settings and this learning process is that we are not interested in optimizing the traditional loss functions but rather the performance of online algorithms. In other words, ML models do not necessarily make perfect predictions, but they do not adversely degrade the performance of the online algorithms.

Anand et al. (2022c) shift the focus from merely integrating ML predictions to customizing ML models for optimization objectives, using the rent-or-buy problem as a case study. Unlike traditional statistical approaches, the paper proposes to optimize the modified ML loss functions to include competitive ratios. The new loss functions align predictions with online optimization goals rather than solely maximizing prediction accuracy, which enables the algorithm to focus on the online optimization objective, leading to better performance. A key contribution is a new *learning-to-rent* algorithm, which frames the problems as a classification task by introducing an auxiliary binary variable that captures the optimal online algorithm policy. While the algorithm achieves near-optimal competitive ratios, it comes at the cost of exponential sample complexity in high-dimensional spaces. To maintain the performance, the authors propose using margin-based techniques to reduce the complexity. The paper also extends the model to handle noisy data and provides robustness guarantees even in adversarial settings. By contrast, Anand et al. (2022b) introduce a regression-based approach to learning ML models. Previous research (Anand et al., 2022c) used simple classification methods that did not suit many real-value problems. This paper proposes the ONLINESEARCH framework, a regression-based approach applicable to various problems such as ski rental, scheduling, and bin packing. The main contribution is the design of a new loss function that aligns with the competitive ratio, surpassing the limitations of traditional regression tools. Building on this, the author proposes the PREDICT-AND-DOUBLE algorithm, achieving a tight trade-off between robustness and consistency with reasonable sample complexity. Finally, their analysis extends to agnostic settings, demonstrating the algorithm's effectiveness even when true predictors lie outside the assumed function class. Khodak et al. (2022) present a general framework for learning predictions in online and adversarial settings, which they apply to classic problems, such as bipartite matching, ski rental, and job scheduling. Previous methods (Anand et al., 2022b,c) are very specific and do not always take into account the fact that input instances may not follow a fixed distribution in online learning settings. In contrast, this paper offers a general-purpose solution framework for learning predictions. This paper makes two main contributions. First, it derives a measurable and optimizable upper bound on the algorithm's cost based on prediction quality. Second, it applies online learning methods to refine the predictions while providing guarantees on performance and complexity. Unlike previous approaches that improve predictions within an algorithm, this framework emphasizes learning predictions as a standalone process, broadening its applicability and enhancing its potential impact.

There has been significant progress on other problems, such as ski rental, prophet inequalities (Diakonikolas et al., 2021), caching, scheduling (Elias et al., 2024), matching, and load balancing (Lavastida et al., 2021). In conclusion, these studies demonstrate how to integrate the learning process of ML models to create online algorithms that are more efficient. Despite these developments, several questions remain unanswered. One of the future projects could involve identifying hard-instance problems where no plausible prediction can improve the performance of existing algorithms. Moreover, we need a general framework to convert competitive ratios into loss functions. Another intriguing direction is to minimize the number of samples to train an ML predictor for online problems.

5.2 Assuming to be correct within ϵ-accurate

In contrast to previous approaches, these methods are based on specific assumptions about the prediction quantity. Instead of assuming black-box models, this approach specifies more information on prediction quality, which allows online algorithms to better exploit it and improve performance.

Gupta et al. (2022) introduce ϵ-accurate predictions, a model where predictions are correct with probability ϵ but otherwise arbitrarily inaccurate. Previous works primarily rely on total prediction error to determine the competitive performance of algorithms, a method that cannot handle high prediction noise. By parameterizing the accuracy of prediction with a confidence parameter ϵ, this paper provides robust solutions. One of the most important contributions of this paper is the TwoStrikes algorithm for caching, which gets an $O(\log(1/\epsilon))$-competitive ratio by using a probabilistic eviction strategy along with randomized fallback strategies. The authors create the Set-Hedge and Cover-Hedge algorithms for general covering problems. These algorithms use submodular properties and probabilistic selection to get a competitive ratio of $O(1/\epsilon)$. Sun et al. (2023) generalize the prediction paradigm in previous work (Gupta et al., 2022) by introducing uncertainty-quantified predictions. While earlier approaches treat predictions as single-point estimates, this method leverages conformal inference techniques to generate probabilistic prediction intervals. The paper also proposes a new benchmark, called the distributionally-robust competitive ratio (DRCR), which aims to bridge the gap between online learning and average-case scenarios. It then introduces an optimization-based method to integrate uncertainty-quantified predictions into online algorithms. The authors apply the new algorithm to solve problems such as ski rental and online search, aiming to achieve optimal DCRC. Further, the authors adapt online learning to utilize uncertainty-quantified predictions in scenarios with multiple problem instances, achieving sublinear regret.

Overall, these algorithms propose simple alternatives to the explicit learning mechanisms of previous approaches by incorporating stronger assumptions about prediction quality. One appealing area of research is applying uncertainty-quantified predictions to problems like covering and packing, and another is testing how well the predictions work with real-world datasets.

6 Optimal consistency-robustness trade-off

Typically, analysis of OLA algorithms considers the trade-off between robustness and consistency. This trade-off is inherent, meaning that improving robustness often comes with the cost of not fully exploiting correct predictions and vice versa. However, existing research does not thoroughly explore whether these trade-offs are necessary or optimal. In particular, most OLA algorithms exhibit **non-optimal** trade-offs between robustness and consistency, which can affect their smoothness and cause the sensitivity to small changes in prediction errors. Consequently, our survey focuses on approaches achieving this **optimal** trade-off in OLA algorithms.

The study (Wei and Zhang, 2020) aims to balance the robustness and consistency of OLA algorithms in two well-known problems: ski rental and non-clairvoyant job scheduling. The authors extend prior work by providing the tight lower bounds on these trade-offs, offering a deeper understanding of their necessity and optimality. The paper also underscores the challenge of enhancing performance under perfect predictions at the expense of compromising robustness. While confirming the optimality of the previously defined trade-off for the ski rental problem, they establish a new tight bound for non-clairvoyant job scheduling. By rigorously proving these results, the paper emphasizes that the trade-offs are intrinsic to designing OLA algorithms. Building on the previous work, Christianson et al. (2023) extend the above idea to the Metrical Task Systems (MTS) problem and propose DART, a novel algorithm that achieves optimal tradeoffs between robustness and consistency. DART uses an adaptive weighting scheme to keep the reliance on learned advice (ADV) and a baseline robust algorithm (ROB) in balance over time. It also uses optimal transportation techniques to make sure that switching between strategies goes smoothly. It achieves nearly optimal performance and polynomial robustness improvements for structured problems like k-server. This work not only resolves issues with previous methods that relied on bounded metric spaces or demonstrated additive penalties when given incorrect advice, but it also establishes a theoretically tight tradeoff between robustness and consistency. A new paper by Angelopoulos et al. (2024) looks at the brittleness of Pareto-optimal algorithms that have already been found. It focuses on how easily prediction errors can hurt the performance of these algorithms. To address this issue, the authors propose a novel framework that incorporates user-specified profiles to create algorithms that ensure smooth performance degradation as a function of prediction error. Using the one-way trading problem as an example, they demonstrate how these profiles allow algorithms to adapt dynamically, overcoming the limitations of static threshold-based methods that are often impractical in real-world scenarios. Traditional Pareto-optimal approaches focus solely on trade-offs between consistency and robustness but fail to handle errors smoothly, leading to dramatic performance drops even under slight inaccuracies. Finally, experimental evaluations using synthetic and real-world data validated the proposed methods, demonstrating significant improvements in robustness and adaptability.

Overall, OLA algorithms with optimal trade-offs can fully characterize the relationship between robustness and consistency. However, there are usually two ways to characterize this relationship, using either prediction errors or confidence parameters in prediction quality. Therefore, it would be intriguing to explore the relationship between these two optimal trade-offs.

7 Systematic algorithmic design

While the majority of OLA algorithms (Gollapudi and Panigrahi, 2019; Wang et al., 2020) have been developed to solve specific problems, called an **ad-hoc** perspective, recent progress has shown a more **systematic** algorithmic framework capable of solving a wider range of problems. Since ad-hoc algorithms often rely heavily on the unique problem structures, they are difficult to generalize to new problem settings. In contrast, systematic frameworks, such as the *online primal-dual* approach, provide a unified perspective of various algorithms. This unification reveals connections between algorithms for solving different problems while removing the dependency on specific structural assumptions. Given the generality of this framework, we will focus on recent progress in designing **systematic** approaches, particularly those derived from the *online primal-dual* framework.

Étienne Bamas et al. (2020) introduce a novel extension of the primal-dual method for designing online algorithms that incorporate machine learning predictions to enhance performance on online covering problems. Previous online algorithms are problem-specific and rely on worst-case guarantees, resulting in overly cautious approaches for real-world applications. To address this issue, the authors introduce a general framework, called Primal-Dual Learning-Augmented (PDLA), which is an extension of previous primal-dual techniques by incorporating ML predictions. Specifically, this new framework uses predictions and the confidence parameter λ to modify the previous rate of updates for primal variables. In other words, these modifications allow the primal variables to converge faster toward optimal solutions. The paper presents experimental results to demonstrate the practicality of the PDLA framework in the TCP acknowledgement problem. Following up on their previous work, Grigorescu et al. (2022) generalized the PDLA framework to online linear and semidefinite programming problems. In particular, the new framework relaxes the assumption of integral advice to fractional advice, enabling greater flexibility. It also replaces the discrete version of primal-dual updates with the continuous one. Finally, the authors use guess-and-double schemes to allow a potentially large number of arriving constraints. Grigorescu et al. (2024) recently expanded the framework beyond linear objectives to solve online concave packing and convex covering problems. Their results show that the framework is better at adapting to prediction accuracy while still being robust when predictions are wrong. For the online concave packing problem, the authors proposed a general algorithm, called Switching, that adaptively balances between the predictive advice and the previously classical online algorithms using the confidence parameter. Meanwhile, since Switching strategy is not efficient to solve convex covering problems, the PDLA framework is extended further with l_q-norms to address these problems. To prove their concepts, the authors demonstrate the proposed frameworks in areas such as resource management, network optimization, and inventory-constrained problems.

The new framework will offer a unified framework for the majority of previously specific solutions. However, not all online learning problems, such as prophetic inequalities and random ordering problems, can be classified as covering problems. Therefore, designing OLA algorithms using primal-dual methods is one of the future research directions.

8 Conclusion

Motivated by the limitations of problem-centric approaches in the previous survey Mitzenmacher and Vassilvitskii (2020), we propose a new taxonomy for OLA algorithms based on their settings and algorithmic designs. The survey highlights its practicality as a bridge between traditional online algorithms and ML predictions. These algorithms address the limitations of purely online approaches by integrating predictive components. By categorizing across the above five dimensions, we emphasize not only the practical relevance but also unified design paradigms that span multiple problems. In each dimension, we also critically evaluate several outstanding OLA algorithms and propose some research problems. Another promising direction, such as designing principal approaches to handle multiple and restricted predictors with refinement trade-offs, can lead to a broader range of applications of OLA algorithms.

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