→ Diffusion Approximation

* Consider transport equation for h^+'s

$$J_p = pq\mu_p E - qD_p \frac{\partial p}{\partial x}$$

If $\left| qD_p \frac{\partial p}{\partial x} \right| >> \left| pq\mu_p E \right|$

$$\boxed{J_p \simeq -qD_p \frac{\partial p}{\partial x}}$$

Diffusion approximation for h^+'s in an n-type semiconductor

* Consider transport equation for $\bar{e}$'s

$$J_n = nq\mu_n E + qD_n \frac{\partial n}{\partial x}$$

If $\left| qD_n \frac{\partial n}{\partial x} \right| >> \left| nq\mu_n E \right|$

$$\boxed{J_n \simeq qD_n \frac{\partial n}{\partial x}}$$

Diffusion approximation for $\bar{e}$'s in p-type semiconductor

→ The drift component depends on carrier concentration and applied electric field.

→ But the diffusion component depends only on concentration gradient.

→ There is a possibility that the carrier concentration is small, but concentration gradient is large, where the diffusion approximation is valid.

> The diffusion approximation is applicable for minority carriers.

The consequences of diffusion approximation

→ Consider continuity equation for holes

$$\frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + G' - \frac{\delta p}{\tau}$$

$$J_p = pq\mu_p E - qD_p\frac{\partial p}{\partial x}$$

Under diffusion approximation $\Rightarrow J_p \simeq -qD_p\frac{\partial p}{\partial x}$

$$\frac{\partial \delta p}{\partial t} = D_p\frac{\partial^2 p}{\partial x^2} + G' - \frac{\delta p}{\tau}$$

Under steady state condition $\rightarrow \frac{\partial}{\partial t} = 0$

$$0 = D_p\frac{\partial^2 p}{\partial x^2} + G' - \frac{\delta p}{\tau}$$

$p = p_0 + \delta p$, p_0 is uniform $\Rightarrow \frac{\partial p}{\partial x} = \frac{\partial \delta p}{\partial x}$

$$0 = D_p\frac{\partial^2(\delta p)}{\partial x^2} + G' - \frac{\delta p}{\tau}$$

$$\frac{\partial^2(\delta p)}{\partial x^2} = \frac{1}{D_p}\left(\frac{\delta p}{\tau} - G'\right) = \frac{\delta p - G'\tau}{D_p\tau}$$

→ If G' is uniform (G' is not varying with distance or uniform volume generation)

$$\frac{\partial^2(\delta p - G'\tau)}{\partial x^2} = \frac{(\delta p - G'\tau)}{D_p \tau}$$

τ – Average life time of excess carriers

(or)

Average life time of minority carriers

$$\frac{\partial^2(\delta p - G'\tau_p)}{\partial x^2} = \frac{(\delta p - G'\tau_p)}{D_p \tau_p}$$

$$L_p^2 = D_p \tau_p$$

$$\Rightarrow \boxed{L_p = \sqrt{D_p \tau_p}}$$ Diffusion length of the holes

$$\boxed{L_n = \sqrt{D_n \tau_n}}$$ Diffusion length of the electrons

$$\boxed{L = \sqrt{D \tau}}$$

$$p = p_0 + \delta p = p_0 + \Delta p \, e^{-x/L_p}$$

$$\Delta p = \delta p\big|_{x=0}$$

$$n = n_0 + \delta n = n_0 + \Delta n \, e^{-x/L_n}$$

$$\Delta n = \delta n\big|_{x=0}$$

→ Under low level injection assumption, the injected minority carrier current for an extrinsic semiconductor is essentially the

(a) diffusion current
(b) drift current
(c) recombination current
(d) induced current

→ Electron mobility and life time in a semiconductor at room temperature are respectively $\underbrace{0.36\ m^2/V\text{-}s}_{\mu_n}$ and $\underbrace{340\ \mu s}_{\tau_n}$. The diffusion length is ____

$$L_n = \sqrt{D_n \tau_n} \qquad \frac{D_n}{\mu_n} = V_T$$

$$D_n = \mu_n V_T$$

$$L_n = \underline{\underline{1.78\ mm}}$$

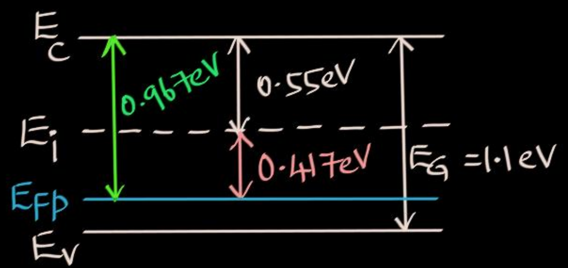
→ In a very long p-type Si bar with cross-sectional area $\overset{A}{0.5\ cm^2}$ and $N_A \overset{p_0}{=} 10^{17}/cm^3$, we inject holes such that the steady state excess hole concentration is $\Delta p = 5 \times 10^{16}/cm^3$ at $x = 0$.

(1) What is the steady state separation between E_{fp} and E_c at $x = 1000\ \text{Å}$?

(2) What is the hole current there?

(3) How much is the excess stored hole charge?

Assume that $\mu_p = 500\ cm^2/V\text{-}s$ and $\tau_p = 10^{-10}\ s$, $E_G = 1.1\ eV$ and $n_i = 1.5 \times 10^{10}/cm^3$.



$L_p = \sqrt{D_p \tau_p}$

$D_p = \mu_p V_T = 13 \ cm^2/s$

$L_p = 3.6 \times 10^{-5} \ cm$

$1000 \ \text{Å} = 1000 \times 10^{-8} cm = 10^{-5} cm$

(1) $E_i - E_{Fp} = kT \ln\left(\frac{p}{n_i}\right)$

$p = p_o + \Delta p \, e^{-x/L_p}$

$p = 10^{17} + 5\times10^{16} \, e^{-10^{-5}/3.6\times10^{-5}}$

$= 1.377 \times 10^{17}/cm^3$

$E_i - E_{Fp} = 0.026 \times \ln\left(\frac{1.377\times10^{17}}{1.5\times10^{10}}\right)$

$= 0.417 eV$

$\underline{\underline{E_c - E_{Fp} = 0.967 eV}}$

(2) $I_p = ?$ $\quad x = 1000 \ \text{Å}$

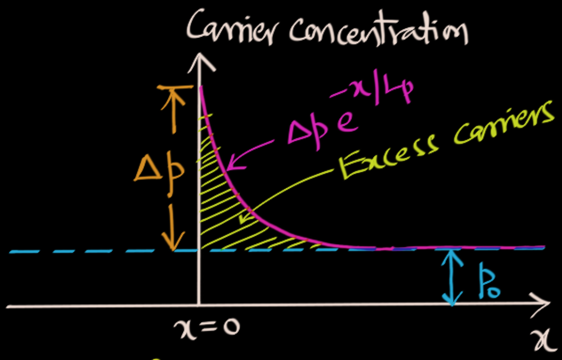
$J_p = -q D_p \frac{\partial p}{\partial x} \Rightarrow I_p = -A q D_p \frac{\partial p}{\partial x}$

$I_p = -A q D_p \frac{\partial}{\partial x}\left(p_o + \Delta p \, e^{-x/L_p}\right) = \frac{A q D_p}{L_p} \Delta p \, e^{-x/L_p}$

$= \frac{0.5 \times 1.6\times10^{-19} \times 13}{3.6\times10^{-5}} \times 5\times10^{16} \, e^{-10^{-5}/3.6\times10^{-5}}$

$= 1.09 \times 10^{3} A$

(3)



$$Q_p = \int_0^{\infty} Aq\,\Delta p\, e^{-x/L_p}\,dx$$

$$= Aq\,\Delta p \int_0^{\infty} e^{-x/L_p}\,dx$$

$$Q_p = Aq\,\Delta p\, e^{-x/L_p}\Big|_0^{\infty}(-L_p)$$

$$\boxed{Q_p = Aq\,\Delta p\, L_p}$$

$$Q_p = 0.5\times 1.6\times 10^{-19} \times 5\times 10^{16}\times 3.6\times 10^{-5}$$

$$Q_p = 144\ \text{nC}$$

Space charge Neutrality

It means that "$\rho = 0$"

Consider Gauss law $\Rightarrow \dfrac{\partial E}{\partial x} = \dfrac{\rho}{\epsilon}$

$$\boxed{\frac{\partial E}{\partial x} = 0}$$

E is constant w.r.t x, where $\rho = 0$

* Quasi-neutrality $\rightarrow$ $\rho \simeq 0$, $\delta p \simeq \delta n$

$$\boxed{\delta n, \delta p >> |\delta n - \delta p| \Rightarrow \rho \simeq 0}$$

* Depletion approximation

→ Consider space charge

$$\rho = q\,(p + N_D^+ - n - N_A^-)$$
$$= q\,(p - n + N_D^+ - N_A^-)$$

If $|p-n| << |N_D^+ - N_A^-|$

$$\boxed{\rho \simeq q\,(N_D^+ - N_A^-)}$$ Depletion approximation

* Debye Length (L_D)

→ Consider an n-type semiconductor

$$\rho = q\,(p - n + N_D^+ - N_A^-)$$

Under depletion approximation

$$\rho \simeq q\,(N_D^+ - \cancel{N_A^-})$$

$$\rho \simeq q N_D^+$$

Consider the Gauss law

$$\frac{\partial E}{\partial x} = \frac{\rho}{\epsilon}$$

$$\frac{\partial E}{\partial x} = \frac{q N_D^+}{\epsilon}$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{q N_D^+}{\epsilon} \qquad \because E = -\frac{\partial \psi}{\partial x}$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\left(\frac{q N_D^+}{\epsilon V_T}\right) V_T$$

$$\frac{1}{L_D^2} = \frac{q N_D^+}{\epsilon V_T}$$

$$\boxed{L_D = \sqrt{\frac{\epsilon V_T}{q N_D^+}}}$$ Debye length

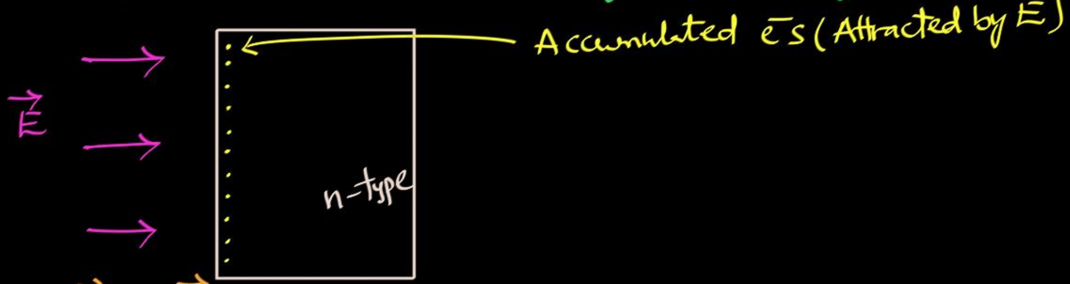
L_D depends on doping

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{V_T}{L_D^2} \Rightarrow$$ The Debye length describes the variation of the potential with distance in space charge region (or) depletion region

* L_n and L_p describes variation of carrier concentration with distance.

Charge sheet Approximation

* This approximation is usually used to describe free carriers near the surface (or) interface.



$\vec{F} = q\vec{E}$

* The carrier concentration is very high close to the surface and decreases very radiply when moving away from the surface within a small distance. So it can be considered as sheet of charge.

* This approximation is used in MOS capacitors

Gradual channel Approximation

* It is used to convert a 2D situation into 1D situation approximately.
* This approximation is used in MOSFETs

Consider Gauss law in 2D

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = \frac{\rho}{\epsilon}$$

If $\frac{\partial E_x}{\partial x} >> \frac{\partial E_y}{\partial y}$ then

$$\frac{\partial E_x}{\partial x} \simeq \frac{\rho}{\epsilon} \quad - 1D$$

Procedure for Device Analysis – Summary

ACE

Step(1) Same set of approximations may not hold over the entire device volume. Hence, partition the device into different regions.

Step(2) Analyse each region using a suitable set of approximations and boundary conditions to obtain n, p, J_n, J_p and E in that region. Approximations for different regions may be different.

ACE

Step(3) Combine the information regarding n, p, J_n, J_p and E obtained in different regions, ensuring "continuity of these parameters" across boundaries separating the regions to obtain the complete picture.

Basic Device Building Blocks

(1) Metal – Semiconductor Interface

Metal	Semiconductor

→ Rectifying contacts (Schottky barrier) – Allows current only in one direction

→ Ohmic contacts (Non-rectifying contacts) – Allows current in both the directions with negligible voltage drop.

(2) pn Junction

p-type Semiconductor	n-type Semiconductor

* BJT → pnp, npn
* Thyristor → pnpn

(3) Heterojunction

Ex:

GaAs	AlAs
Semiconductor-A	Semiconductor-B

These are key components in high-speed and optoelectronic devices

(4) Metal – Oxide – Semiconductor structure

Metal	Oxide	Semiconductor

* MOS Capacitor/ Junction/ Diode
* MOSFET

pn Junction

→ Equilibrium (zero bias or open circuit)
→ Ideal V-I characteristics
→ Variation of V-I characteristics with temperature
→ Diode resistance
→ Diode equivalent circuits
→ Breakdown mechanisms
→ Junction capacitance
→ Small signal characteristics
→ Switching times of the diode

Equilibrium Analysis

Qualitative

Quantitative

$\boxed{n,\ p,\ J_n,\ J_p \text{ and } E}$

* Width of the depletion region
* Built-in potential (or) contact potential
* Maximum electric field
* Charge stored in the depletion region

Junction

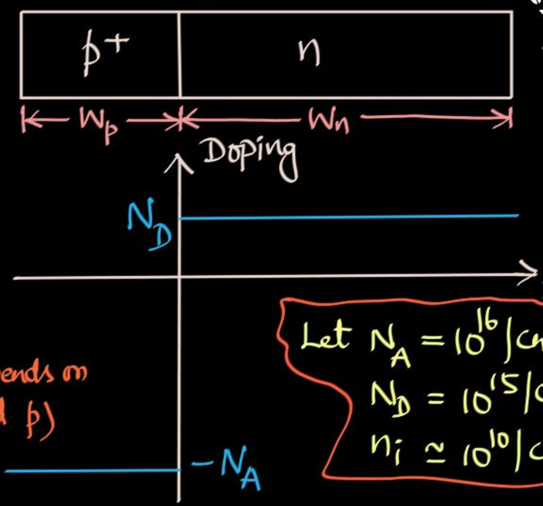
Abrupt (or) Step graded junction (Non-linear)

Linearly graded junction (Linear)

Equilibrium (zero bias or open circuit) – Qualitative Analysis

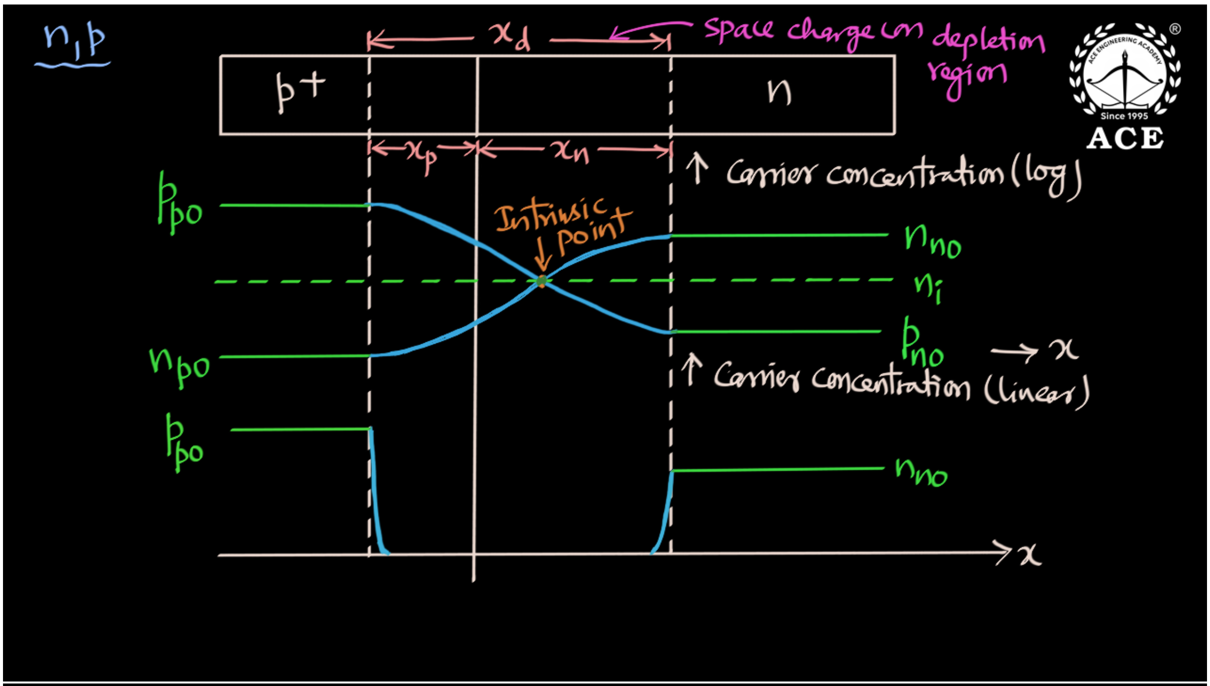
* Abrupt and uniform junction

$$\boxed{\begin{matrix} L_n << W_p \\ L_p << W_n \end{matrix}}$$

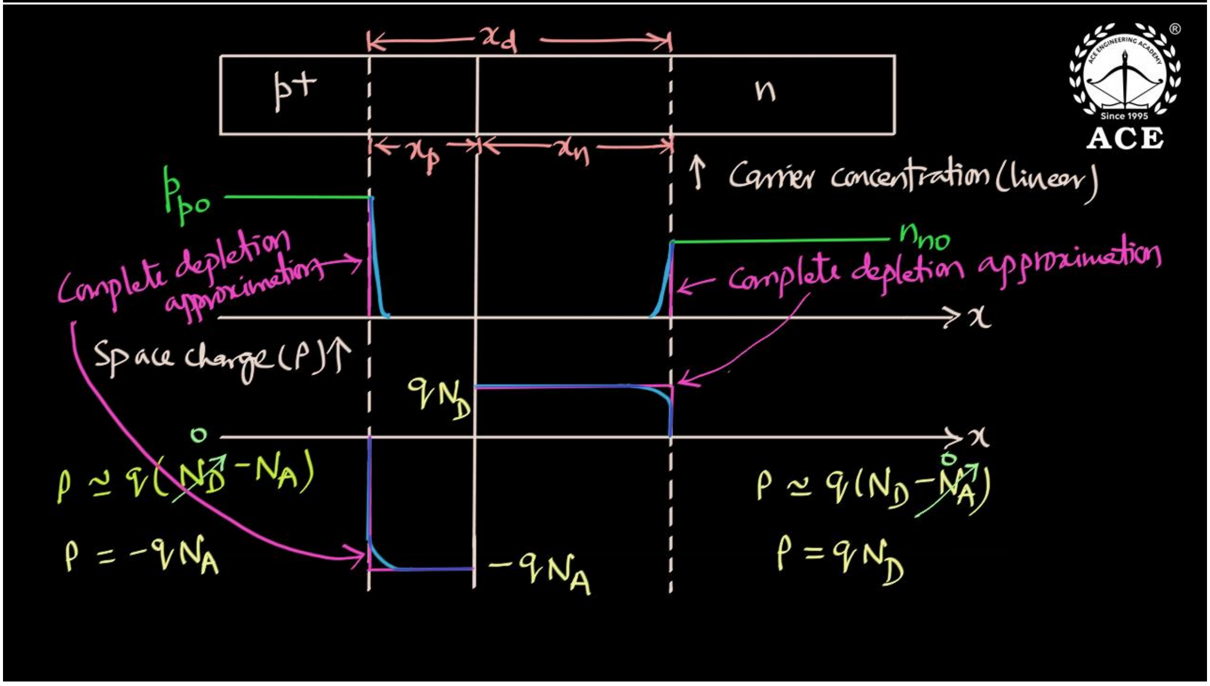


→ n, p, J_n, J_p and E (depends on n and p)

Let $N_A = 10^{16}/cm^3$
$N_D = 10^{15}/cm^3$
$n_i \simeq 10^{10}/cm^3$



n, p
p+
n
x_d
Space charge (or) depletion region
x_p
x_n
↑ Carrier concentration (log)
p_{po}
Intrinsic point
n_{no}
n_i
n_{po}
p_{no}
→ x
↑ Carrier concentration (linear)
p_{po}
n_{no}
x



x_d
p+
n
x_p
x_n
↑ Carrier concentration (linear)
p_{po}
n_{no}
Complete depletion approximation
complete depletion approximation
x
Space charge (ρ)↑
qN_D
0
x
$\rho \simeq q(\cancel{N_D}^{0} - N_A)$
$\rho = -qN_A$
$-qN_A$
$\rho \simeq q(N_D - \cancel{N_A}^{0})$
$\rho = qN_D$

→ Consider space charge $\rho = q(p - n + N_D^+ - N_A^-)$

Let all the impurities are ionized

$$\rho = q(p - n + N_D - N_A)$$

Apply complete depletion approximation

$\boxed{p - n = 0}$

$$|p-n| << |N_D - N_A| \Rightarrow \boxed{\rho \simeq q(N_D - N_A)}$$

Depletion approximation