

Linear Algebra

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Reference Book: Matrix Analysis and applied Linear

Algebra by Carl D Meyer

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Contents

- 1. System of linear equations
- 2. Matrix Algebra
- 3. Eigen values and Eigen vectors

Since 1995

Imagine there are three triends. They go to a market

Friend 4 3apples + 2bananas + 1orange = 39 -
$$\bigcirc$$

Friend 2 2 apples
$$+$$
 3bananas $+$ 1 orange $=$ 26 \bigcirc

Friends
$$1$$
 apples $+ 2$ bananas $+ 3$ orange $= 25 - 3$

How to find cost of each fruit??

Elimination of variables



$$2x + y + z = 1$$

$$6x + 2y + z = -1$$

$$-2x + 2y + z = 7$$

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 6 & 2 & 1 & 1 \\ -2 & 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & 1 & 1 \\ 3 & 2 & 1 & 3 \end{bmatrix}$$

$$\begin{cases} 2 & 1 & 1 & 1 \\ 6 & 2 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 3 & 2 & 3 & 2 & 8 \\ 0 & 3 & 3 & 8 \\ 0 & 3 & 2 & 8 \\ 0 & 3 & 2 & 8 \\ 0 & 3 & 2 & 8 \\ 0 & 3 & 2 & 8 \\ 0 & 3 & 2 & 8 \\ 0 & 3 & 2 & 8 \\ 0 & 3 & 2 & 8 \\ 0 & 3 & 2$$



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix}
2 & 1 & 1 \\
0 & -1 & -2 & -4 \\
0 & 3 & 2 & 8
\end{pmatrix}$$

$$\begin{pmatrix}
2 & 1 & 1 \\
0 & -1 & -2 & -4 \\
0 & 0 & -4 & -4
\end{pmatrix}$$

$$-\frac{1}{3} - 28 = -4$$

$$-\frac{1}{3} - 28 = -4$$

$$\frac{1}{3} = 2$$

$$2x + 3 + 3 = 1$$

$$2x + 2 + 1 = 1$$

$$x = -1$$

Bosic Columns.

Columns not Containing prot elements are called

Non Bosic Columns.

Rank(A) =3

Rank (A/b) = 3

Solve the following system of linear equations

$$x_{1} + 2x_{2} + x_{3} + 3x_{4} + 3x_{5} = 5$$

$$2x_{1} + 4x_{2} + 4x_{4} + 4x_{5} = 6$$

$$x_{1} + 2x_{2} + 3x_{3} + 5x_{4} + 5x_{5} = 9$$

$$2x_{1} + 4x_{2} + 4x_{4} + 7x_{5} = 9$$

$$(A|b) = \begin{cases} 1 & 2 & 1 & 3 & 3 & 5 \\ 2 & 1 & 0 & 1 & 4 & 4 \\ 1 & 2 & 3 & 5 & 5 & 9 \\ 2 & 1 & 0 & 1 & 4 & 4 & 9 \end{cases}$$

Solve the following system of linear equations
$$x_1 + 2x_2 + x_3 + 3x_4 + 3x_5 = 5$$

$$2x_1 + 4x_2 + 4x_4 + 4x_5 = 6$$

$$x_1 + 2x_2 + 3x_3 + 5x_4 + 5x_5 = 9$$

$$2x_1 + 4x_2 + 4x_4 + 7x_5 = 9$$

$$2x_1 + 4x_2 + 4x_4 + 7x_5 = 9$$

$$2x_1 + 4x_2 + 4x_4 + 7x_5 = 9$$

$$2x_1 + 4x_2 + 4x_4 + 7x_5 = 9$$

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$$2x_1 + 4x_2 + 4x_4 + 7x_5 = 9$$

$$2x$$

$$\begin{pmatrix}
1 & 2 & 1 & 3 & 3 & 5 \\
0 & 0 & -2 & -2 & -2 & -4 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3 & 3
\end{pmatrix}$$

R3 (-> R4

$$-2 2 3 - 2 2 4 - 2 2 5 = -4$$

$$-2 2 3 - 2 2 4 - 2 = -4$$

$$-2 2 3 - 2 2 4 - 2 = -4$$

X3+ X4 = 1

Non basic Columns of A are 2 and4

=> 22 and 24 are called free variables.

Let $\chi_H = 1 \Rightarrow \chi_{3+1} = 1$ =) $\chi_3 = 0$

21 + 212 + 23 + 314 + 315 = 521 + 212 + 0 + 3 + 3 = 5



$$\chi_{1} + 2\chi_{2} = -1$$

set $\chi_{2} = 0 = \chi_{1} = -1$

$$\begin{pmatrix} 21 \\ 22 \\ 23 \\ 24 \\ 25 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\lambda_{S} = 1$$
 $\lambda_{S} = 1$
 λ_{S



1	2	1		3	1
0	0	- 2	-2		-4
0	0	0	0	3	3
0	0	0	0	0	0



Solve the following system of linear equations

$$x_1 + 2x_2 + x_3 + 3x_4 + 3x_5 = 0$$

 $2x_1 + 4x_2 + 4x_4 + 4x_5 = 0$

$$x_1 + 2x_2 + 3x_3 + 5x_4 + 5x_5 = 0$$

$$2x_1 + 4x_2 + 4x_4 + 7x_5 = 0$$



$$Rank(A) = 3 R(A|b) = 3$$

$$\frac{\text{Rankia}}{=} = \delta = 3$$

Number of variables = n = 5

Rank (A) = 3 Rank (A(b) = 3 Rank (A) =
$$8 = 3$$

$$(1 = 3)$$

 $(2 + 1) = 3$ $(2 + 1) = 4$

$$N=5$$

Rank (A) = 3, R(A|b) = 4 Rank (A) $+$ Rank (A|b)

 $OX_1 + OX_2 + OX_3 + OX_4 + OX_5 \neq 1$
 NO Solution

 NO Solution

 $Column$

00000 2



m linear equations in n unknowns is given by

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1$$
 $a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2$
 \vdots
 $a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m$
 \vdots

The above equations in matrix form can be written as $\mathbf{A}\mathbf{x} = \mathbf{b}$.

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{mxn} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \times = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

$$A_{mxn}$$



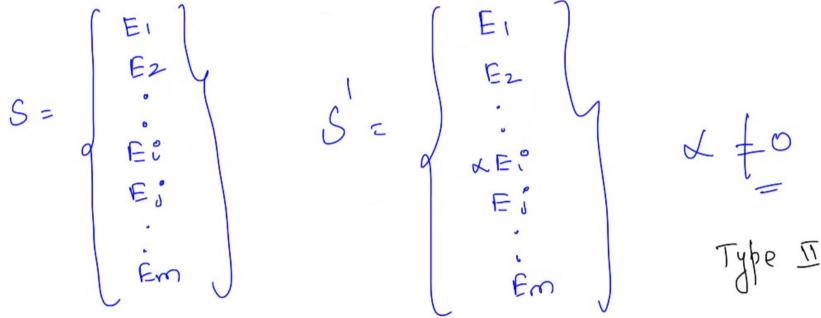
Let
$$S = \begin{cases} E_1 \\ E_2 \\ \vdots \\ E_m \end{cases}$$
 be the linear system with m equations and n unknowns.

For a linear system S, each of the following three elementary operations results in an equivalent system S'.

(1) Interchange the i^{th} and j^{th} equations. That is, if

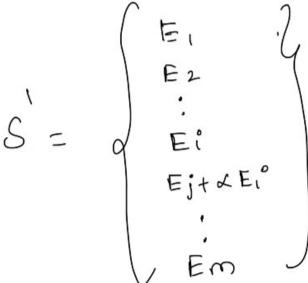


(2) Replace the i^{th} equation by a nonzero multiple of itself. That is





(3) Replace the j^{th} equation by a combination of itself plus a multiple of the i^{th} equation. That is,





A typical structure for a matrix in row echelon form is illustrated below with the pivots circled.

г								٦
	(*)	*	*	*	*	*	*	*
	0	0	*	*	*	*	*	*
	0	0	0	*	*	*	*	*
	0	0	0	0	0	0	(*)	*
	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0

What is a Consistent System?

A System which has atleast one solution
is called Consistent system

Inconsistent System. A System with no solution in called Inconsistent System.



Consistency

Each of the following is equivalent to saying that [A|b] is consistent.

✓ ► In row reducing [A|b], a row of the following form never appears:

$$(00..0|\alpha)$$
 where $x \neq 0$
 $A|b|$. $Ox_1 + Ox_2 + Ox_n \neq x$

- **b** is a nonbasic column in [A|b].
- ightharpoonup rank[$\mathbf{A}|\mathbf{b}$] = rank (\mathbf{A}).
- **b** is a combination of the columns in **A**.



b is a combination of the columns in **A**.

$$2x + y + z = 1$$

$$6x + 2y + z = -1$$

$$-2x + 2y + z = 7$$

$$-1\begin{pmatrix} 2\\6\\-2 \end{pmatrix} + 2\begin{pmatrix} 1\\2\\2\\2 \end{pmatrix} + 1\begin{pmatrix} 1\\1\\1 \end{pmatrix} = \begin{pmatrix} 1\\-1\\2\\2 \end{pmatrix}$$