HOMOGENEOUS SYSTEMS: A system of m linear equations in n unknowns

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = 0$$

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = 0$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = 0$$

tomogeneous sys are solution always consistent.

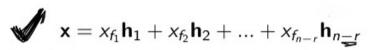
in which the right-hand side consists entirely of 0s is said to be a homogeneous system.

Let $A_{m \times n}$ be the coefficient matrix for a homogeneous system of m linear equations in n unknowns, and suppose rank(A)=r.

$$V_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
 $V_2 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$



- The unknowns that correspond to the positions of the basic columns (i.e., the pivotal positions) are called the basic variables, and the unknowns corresponding to the positions of the nonbasic columns are called the free variables.
- ► There are exactly r basic variables and n-r free variables.
- ➤ To describe all solutions, reduce A to a row echelon form using Gaussian elimination, and then use back substitution to solve for t basic variables in terms of the free variables. This produces the general solution that has the form



where x_{f_1} , x_{f_2} , . . . , $x_{f_{n-r}}$ are the free variables and \mathbf{h}_1 , \mathbf{h}_2 , ..., \mathbf{h}_{n-r} are n \times 1 columns.

- ▶ These n-r solutions are said to linearly independent.
- A homogeneous system possesses a unique solution (the trivial solution) if and only if rank (A) = n i.e., if and only if there are no free variables

$$X = K_1 h_1 + K_2 h_2$$

$$\begin{pmatrix}
V_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} & V_2 = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \\
V_2 = 2 V_1$$

Homogeneous System

unique solution

$$\chi_1 = \chi_2 = - = \chi_1 = 0$$
 Trivial ACE
 $\chi_1 = \chi_2 = - = \chi_1 = 0$ Solution Zero
 $\chi_1 = \chi_2 = - = \chi_1 = 0$ Solution

Infinitely many solution

Non trivial solution Non zero



NON-HOMOGENEOUS SYSTEMS

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1$$

 $a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2$
 \vdots
 $a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m$

a system of m linear equations in n unknowns is said to be non homogeneous whenever $b_i \neq 0$ for at least one i. Unlike homogeneous systems, a non homogeneous system may be inconsistent.



To describe the set of all possible solutions of a consistent nonhomogeneous system, construct a general solution by exactly the same method used for homogeneous systems as follows.

- Use Gaussian elimination to reduce the associated augmented matrix [A|b] to a row echelon form [E|c].
- Identify the basic variables and the free variables.
- Apply back substitution to $[\mathbf{E}|\mathbf{c}]$ and solve for the basic variables in terms of the free variables.
- Write the result in the form

free variables.

If the form

$$\mathbf{x} = \mathbf{p} + x_{f_1} \mathbf{h}_1 + x_{f_2} \mathbf{h}_2 + \dots + x_{f_{n-r}} \mathbf{h}_n - \mathbf{r}^{x,5}$$

If the form

$$\mathbf{x} = \mathbf{p} + x_{f_1} \mathbf{h}_1 + x_{f_2} \mathbf{h}_2 + \dots + x_{f_{n-r}} \mathbf{h}_n - \mathbf{r}^{x,5}$$

where x_{f_1} , x_{f_2} , . . . , $x_{f_{n-r}}$ are the free variables and $\mathbf{p}, \mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_{n-r}$ are $n \times 1$ columns. This is the general solution of the non-homogeneous system.



The general solution of the non-homogeneous system is given by a particular solution plus the general solution of the associated homogeneous system.



No solution $R(A) \neq R(A|b)$

Non Homogeneous

System

Unique solution

R(A) = R(A/b) = 0

Infinitely many

Solutions

R(A) = R(A|b) < 0

Consider the linear equations:

$$x - 2y + z = 3,$$

 $2x + \alpha z = -2,$
 $-2x + 2y + \alpha z = 1.$

In order to have unique solution to this linear system of equations the value of α should not be equal to

(a)
$$\frac{-2}{3}$$

(b)
$$\frac{2}{3}$$

(c)
$$\frac{4}{3}$$

(d)
$$\frac{-4}{3}$$

unique solution For $R(A) = R(A|b) = \frac{3}{2}$

mca

12. If
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 4 & 3 & 10 \end{bmatrix}$$
, then which of the

following is *not* true?

- (a) Rank of A = 2 True
- (b) The system AX = O has infinitely many solutions, where $X = [x \ y \ z]^T$ True O O O
- (c) The system AX = B has a unique solution False
- A^{-1} does not exist (d)

$$R_{2} \rightarrow R_{2} - 2R_{1}, R_{3} \rightarrow R_{3} - HR_{1}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & -5 & -2 \\ 0 & -5 & -2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$R_{3} \rightarrow R_{3} - R_{2}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & -5 & -2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$R(A) = 2$$

$$(X = 0) \quad R(A) < 0$$

13. Consider the system
$$\begin{cases} kx + y + z = 1 \\ x + ky + z = 1 \end{cases}$$
$$x + y + kz = 1$$

If the system has a unique solution then which of the following is true?

(a)
$$k = 1$$
 and $k = -2$

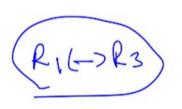
(b)
$$k \neq 1$$
 and $k \neq -2$

(c)
$$k = 1$$
 and $k \neq -2$

(d)
$$k \neq 1$$
 and $k = -2$

$$\begin{bmatrix} A[b] = \begin{bmatrix} K & I & I & I \\ I & K & I & I \\ I & I & K & I \end{bmatrix}$$

and c are eliminated For K=1 RIAD = 1



Let AX = B be a system of three equations in three variables x, y and z. The augmented matrix of the system is given by

$$[A \mid B] = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}.$$

Which of the following is *not* true?

- True (a) Rank of $[A \mid B] = 2$
- (b) The system AX = B has infinitely many True solutions
- (d) Rank of A = 2 True

$$\begin{bmatrix} 2 & 1 & 0 & 1 \\ 6 & 3 & 4 & 7 \\ 4 & 2 & 1 & 3 \end{bmatrix}$$

$$\begin{pmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

solutions
$$\Upsilon U \in C$$

(c) The system $AX = O$ has unique solution $FaU \in C$
(d) Rank of $A = 2$ $\Upsilon U \in C$

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R3-> R3-2R

15. The following system of equations

$$x + y + z = 3,$$

$$x + 2y + 3z = 4,$$

x + 4y + kz = 6

has infinitely many solutions when

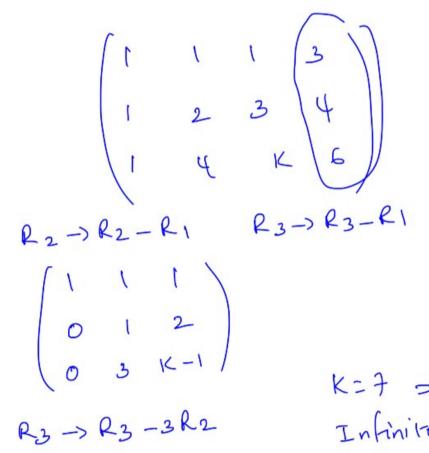
(a)
$$k \neq 0$$

(b)
$$k = 0$$

(c)
$$k = 7$$

(d)
$$k \neq 7$$

GATE 2015



(=7 =) R(A) =2 Tolinitely_ many (olutions

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & K-7 \end{pmatrix}$$



16. Consider the matrix $A = \begin{bmatrix} k & k & k \\ 0 & k-1 & k-1 \\ 0 & 0 & k^2-1 \end{bmatrix}$.

If the system AX = O has only one independent solution then k =____.

(a)
$$0, -1$$

$$(b) -1, 1$$

(c)
$$0, 1$$

(d)
$$0, 1, -1$$

Given
$$N-X=1$$

$$3-X=1$$

$$8=2$$

verification
$$K=1$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$R(A)=1$$

eliminate options with K=1
b, c, d are eliminated

Ans (a)



18. Consider the following system of equations:

$$3x + 2y = 1$$
, $4x + 7z = 1$,
 $x + y + z = 3$, $x - 2y + 7z = 0$

Which of the following is true?

- (a) The system has no solution
- (b) The system has infinitely many solutions
- (c) The system has unique solution
- (d) The rank of the augmented matrix of the system is 2

GATE 2014

$$\begin{pmatrix} 1 & 1 & 1 & 3 \\ 4 & 0 & 7 & 1 \\ 3 & 2 & 0 & 1 \\ 1 & -2 & 7 & 0 \end{pmatrix} \begin{array}{c} R_2 - 7 R_2 - 4R_1 \\ R_3 - 7 R_2 - 4R_1 \\ R_3 - 7 R_2 - 4R_1 \\ R_4 - 7 R_4 - R_1 \\ R_5 - R_5 - R_5 - R_5 \\ R_5 - R_5 \\ R_5 - R_5 \\ R_5 - R_5 -$$

$$\begin{pmatrix}
1 & 1 & 1 & 3 \\
0 & -H & 3 & -11 \\
0 & -1 & -3 & -8 \\
0 & -3 & 6 & -3
\end{pmatrix}$$

R21-> R3



$$\begin{pmatrix}
1 & 1 & 1 & 3 \\
0 & -1 & -3 - 8 \\
0 & -4 & 3 - 11 \\
0 & -3 & 6 & -3
\end{pmatrix}$$



227. If the following system has non – trivial solution

$$px +qy + rz = 0$$

$$qx + ry + pz = 0$$

$$rx + py +qz = 0$$

Then which one of the following Options is

(a)
$$p - q + r = 0$$
 or $p = q = -r$

(b)
$$p + q - r = 0$$
 or $p = -q = r$

(c)
$$p + q + r = 0$$
 or $p = q = r$

(d)
$$p - q + r = 0$$
 or $p = -q = -r$

$$\begin{array}{ll}
\text{If} & P = Q = X \\
A = \begin{pmatrix} P & P & P \\
P & P &$$

$$\begin{pmatrix} P & P & -P \\ P & -P & P \\ -P & P & P \end{pmatrix}$$

$$\begin{array}{cccc}
 & P & -P \\
 & 0 & -2p & 2p \\
 & 0 & 2p & 0
\end{array}$$

$$\begin{array}{cccc} R_3 \rightarrow R_3 + R_2 \\ \hline P & P & -P \\ \hline O & -2p & 2p \\ \hline O & O & 2p \end{array}$$

leads to infinitely many solutions Nontrivial solution P+9+8:0

- Given B is linear

 combination of Columns of A

 ACE

 The Ax=B is consistent

 Let us solve this after covering

 Linearly independent and dependent

 vector Concept.
- 17. Let AX = B be a system of three equations in three variables x, y and z. If A has three linearly independent columns and B is a linear combination of the columns of A, then which of the following is true?
 - (a) The system has unique solution
 - (b) The system has infinitely many solutions
 - (c) The system has no solution
 - (d) The system AX = O has non-zero solution



268. Let c1,..., cn be scalars, not all zero, such that $\sum c_i a_i = 0$ where a_i are column vectors

in Rⁿ. Consider the set of linear equations Ax = b

Where
$$A = [a_1, ..., a_n]$$
 and $b = \sum_{i=1}^n a_i$.

The set of equations has

- (a) a unique solution at $x = J_n$ where J_n denotes a n-dimensional vector of all 1
- (b) no solution
- (c) infinitely many solutions
- (d) finitely many solutions

b = £ai = a, +azt - + an
i=1
b is linear combination of columns of A => Ax=6 is (onsistent eliminate (b) and (d) Let us solve this after covering Linearly independent and dependent vector Concept.

Find the values of k, for which the following system of linear equations has an infinite number of solutions?

$$x_1-x_2 + 2x_3 = 7$$
, $x_1 + x_2 - x_3 = 1$, $-x_1 + kx_2 + 3x_3 = 0$

- (a) -7/3 & 4/3 (b) -7/3 & -4/3

- (c) 7 & 4 (d) -7 & -4

Question not properly framed

