C-V characteristics of an n-channel MOS capacitor

$$C = \frac{Q}{V}$$

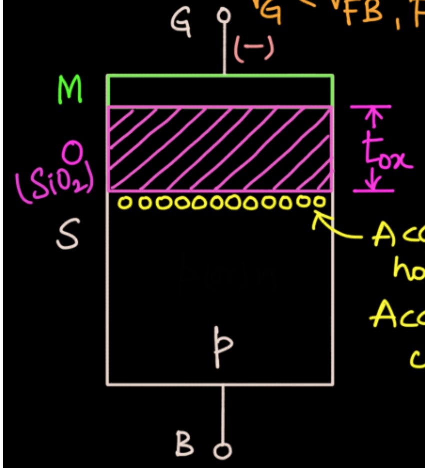
$Q = Q_s$ → Semiconductor (or) Surface charge

$$C = \frac{dQ}{dV}$$

$$C = \frac{dQ_s}{dV}$$

Accumulation Mode

$V_G < V_{FB}$, For ideal MOS capacitor, $V_{FB} = 0$



Accumulated holes (or) Accumulation charge

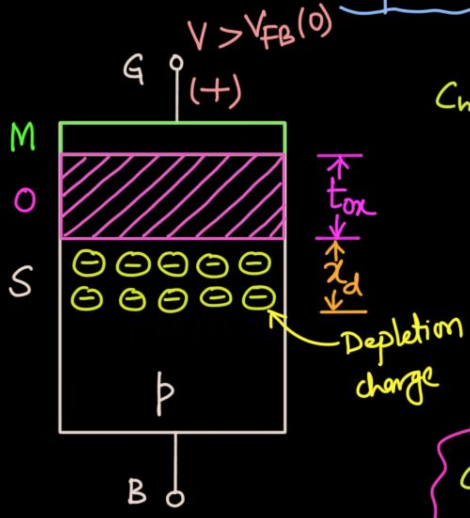
$$\boxed{C = C_{ox}}$$

$$\boxed{C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}}$$ – oxide capacitance per unit area (F/cm^2)

⇓

"t_{ox} can be calculated"

Depletion Mode



$$\overset{C_{min}}{C} = \frac{\overset{C_{max}}{C_{ox}} C_d}{C_{ox} + C_d}$$

$C_d = \frac{\epsilon_s}{x_d}$ — Depletion capacitance

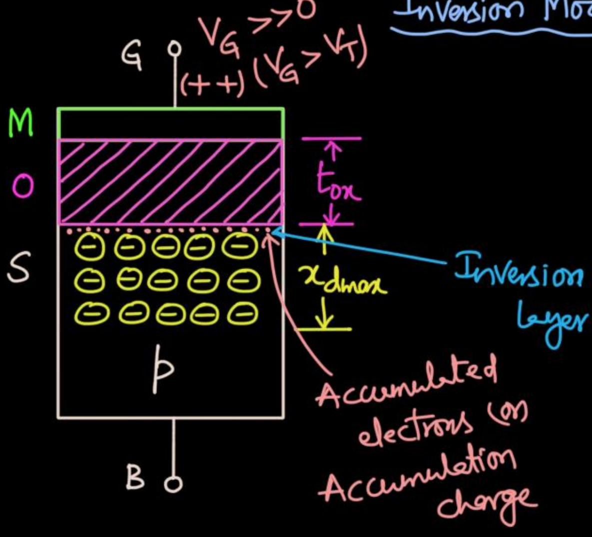
$V_G \uparrow \rightarrow x_d \uparrow \rightarrow C_d \downarrow \rightarrow C \downarrow$

If $x_d = x_{dmax} \Rightarrow C_{dmin} \Rightarrow C_{min}$

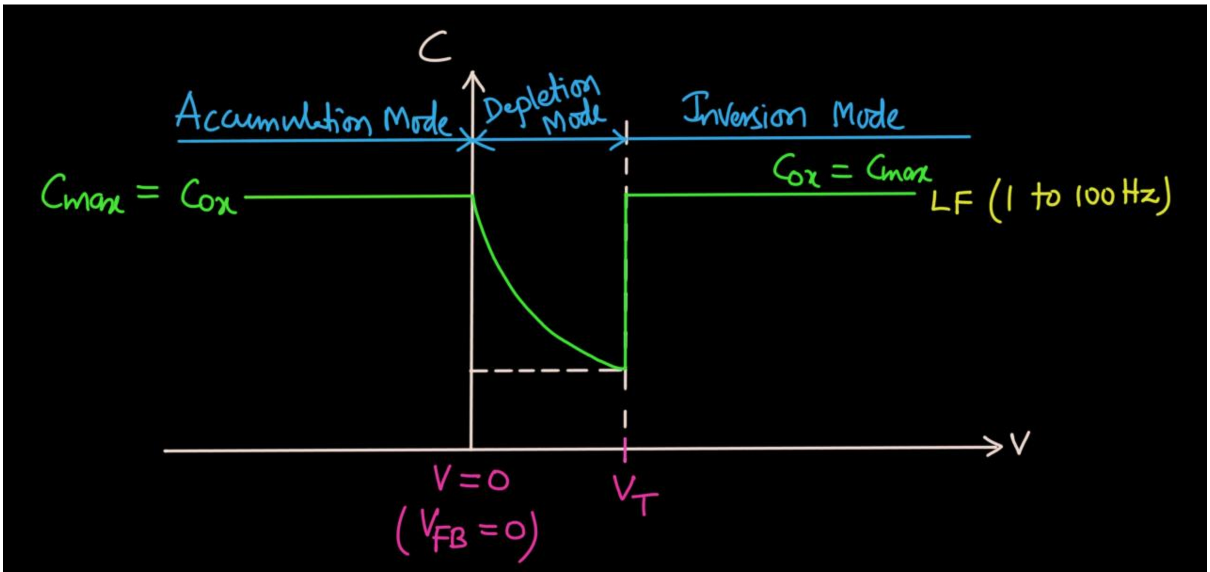
"x_{dmax} can be obtained"

$$\boxed{C_d = \frac{C_{max} C_{min}}{C_{max} - C_{min}}}$$

Inversion Mode



$$\boxed{C = C_{ox} = C_{max}}$$



* Accumulation Mode

Majority carriers ⇒ respond in the time scale of

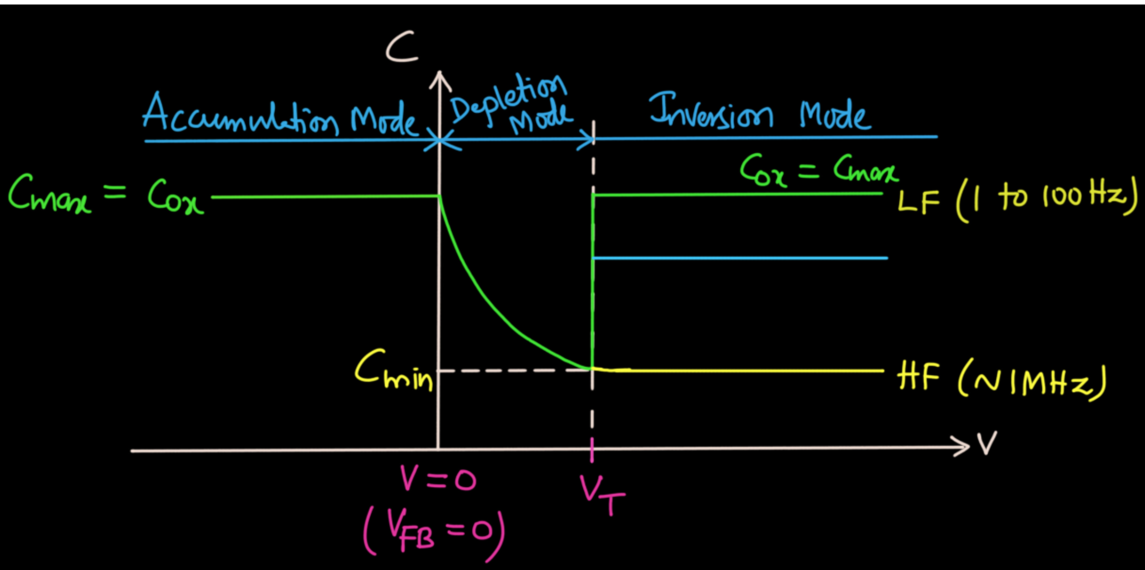
$$\tau = \frac{\epsilon}{\sigma}$$ – Dielectric time constant

$$\boxed{\tau \simeq 10^{-13}\,s}$$

* Inversion Mode

Minority Carriers ⇒ respond in the time scale of generation and recombination.

Usually this will be in the order of hundreds of microseconds.



Non-Ideal MOS Capacitor

Ideal MOS capacitor

* $\Phi_{ms} = 0$

* $Q_{it} = 0$

$Q_{ot} = 0$

$Q_f = 0$

$Q_m = 0$

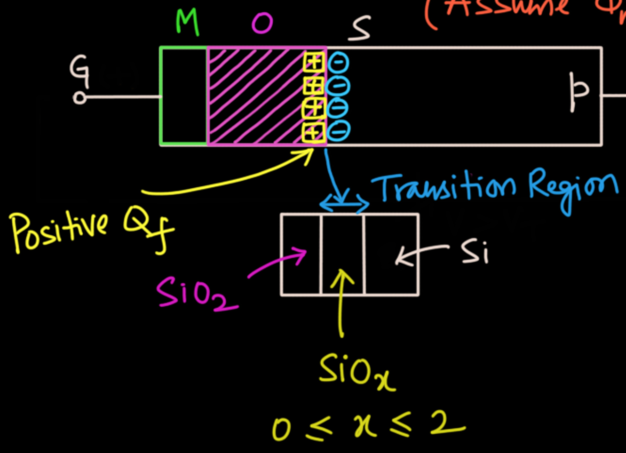
Non-Ideal MOS capacitor

→ The effect of work function difference between M and S ($\Phi_{ms} \neq 0$)

→ The effect of fixed charge (Q_f)

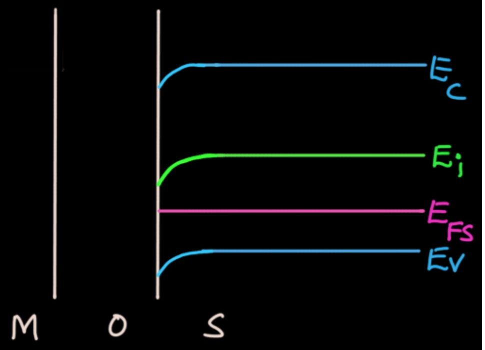
Effect of fixed charge (Q_f)

(Assume $\Phi_{ms} = 0$)

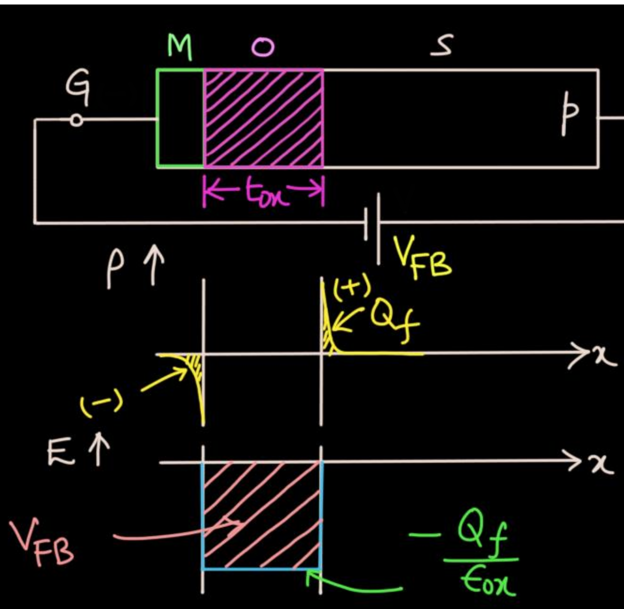


p ↓

E_i bend or tilt downward



* The voltage that is required to make the energy levels flat is referred as Flat-Band Voltage (V_{FB}).



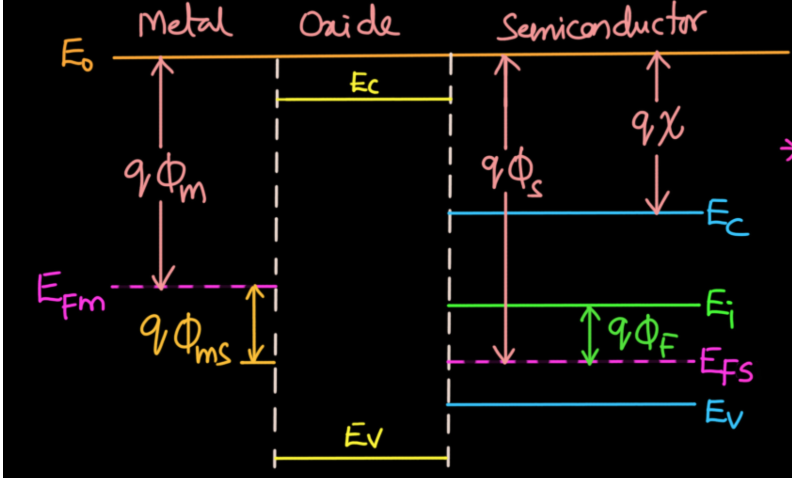
$$V_{FB} = \frac{-Q_f}{\epsilon_{ox}} t_{ox}$$

$$\boxed{V_{FB} = -\frac{Q_f}{C_{ox}}}$$

→ The effect of Q_f is, V_{FB} is shifted to negative value.

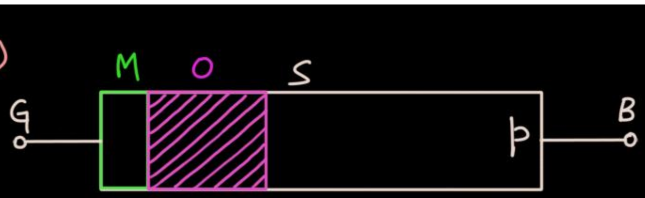
Effect of work function difference between M and S

(Assume $Q_f = 0$)



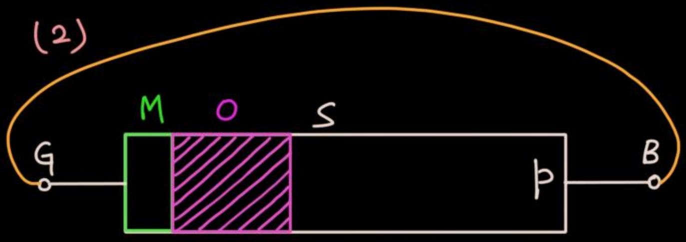
* Energy band diagram when all the components of MOS capacitor are isolated. (Metal, oxide and Semiconductor)

(1)



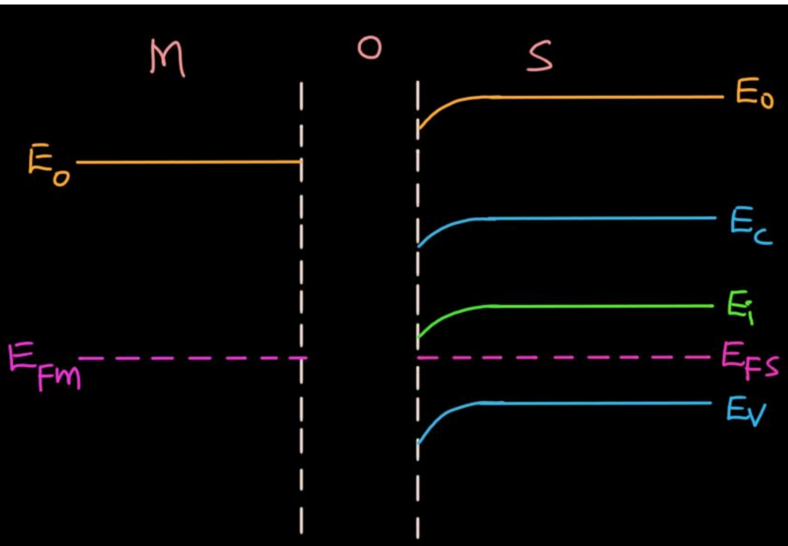
→ Here still the system is not formed, because in a system various components should interact with each other. But oxide layer is insulating M and S.

(2)



* System

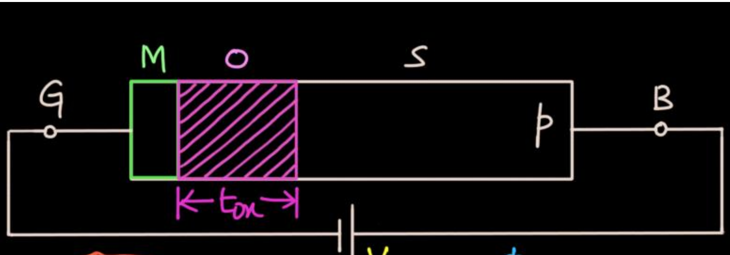
→ If the system is formed, there is a transfer of electron from M to S, because E_{Fm} is above E_{FS} but not due to higher electron concentration in M than S.



At the interface or surface, $p \downarrow$

$\Downarrow$

E_i bend or tilt downward

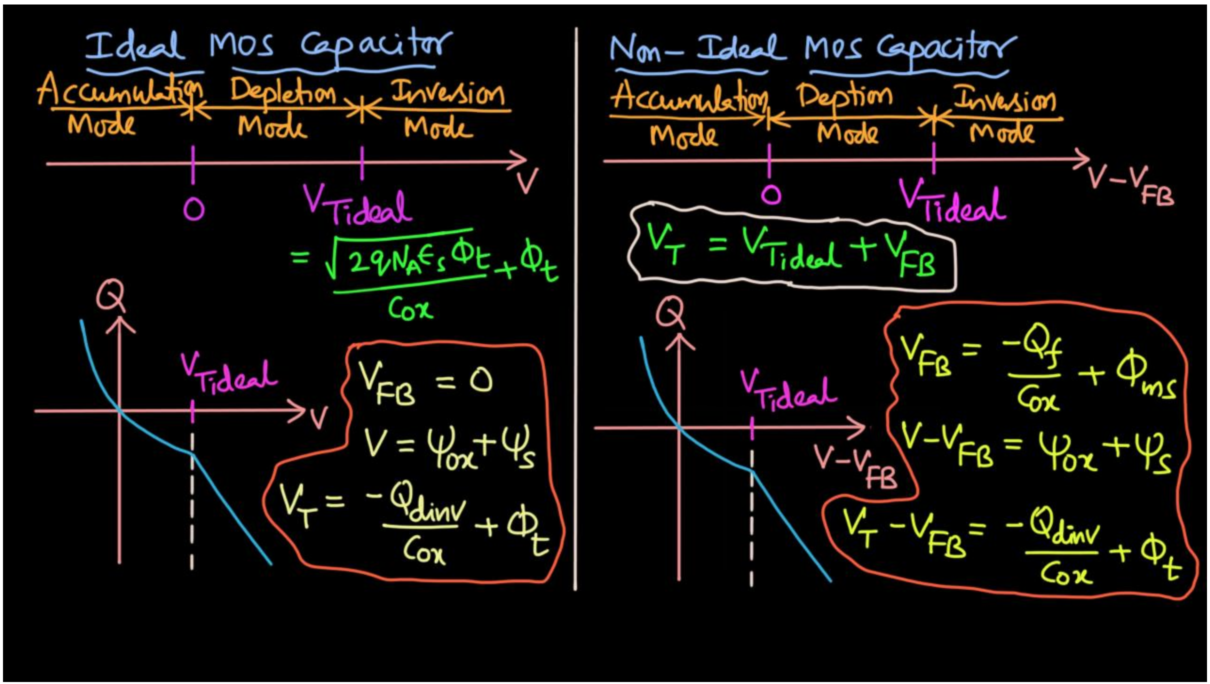


$V_{FB} = \phi_{ms}$

$$\boxed{V_{FB} = \phi_{ms} = \phi_m - \phi_s}$$

→ In presence of Q_f and ϕ_{ms}

$$\boxed{V_{FB} = -\frac{Q_f}{C_{ox}} + \phi_{ms}} \; *$$



Ideal MOS Capacitor

Accumulation Mode | Depletion Mode | Inversion Mode

V: 0, V_{Tideal}

$$V_{Tideal} = \frac{\sqrt{2qN_A\epsilon_s\Phi_t}}{C_{ox}} + \Phi_t$$

$$V_{FB} = 0$$

$$V = \psi_{ox} + \psi_s$$

$$V_T = \frac{-Q_{dinv}}{C_{ox}} + \Phi_t$$

Non-Ideal MOS Capacitor

Accumulation Mode | Deption Mode | Inversion Mode

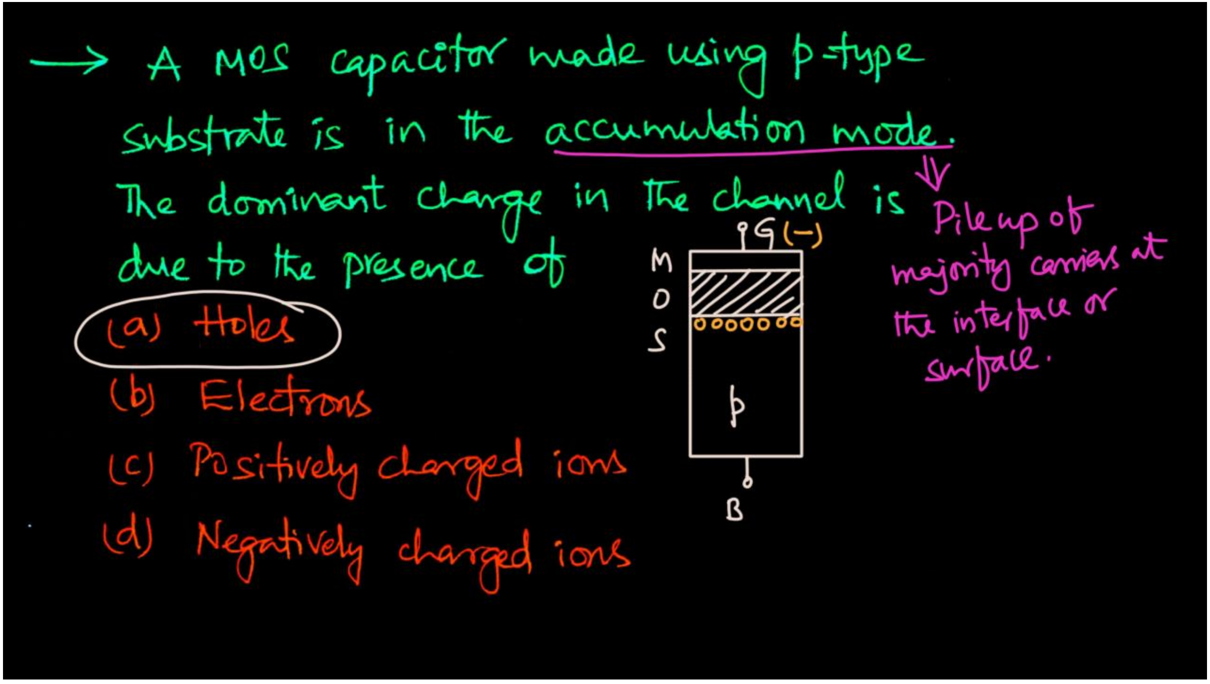
$V - V_{FB}$: 0, V_{Tideal}

$$\boxed{V_T = V_{Tideal} + V_{FB}}$$

$$V_{FB} = \frac{-Q_f}{C_{ox}} + \Phi_{ms}$$

$$V - V_{FB} = \psi_{ox} + \psi_s$$

$$V_T - V_{FB} = \frac{-Q_{dinv}}{C_{ox}} + \Phi_t$$



→ A MOS capacitor made using p-type substrate is in the accumulation mode. The dominant charge in the channel is due to the presence of

⇓ Pile up of majority carriers at the interface or surface.

[✓ (a) Holes]

(b) Electrons

(c) Positively charged ions

(d) Negatively charged ions

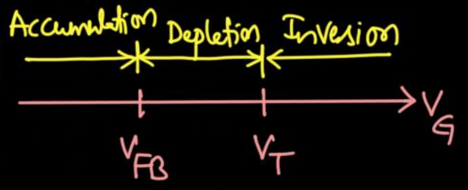
→ Consider a MOS junction with p-type substrate. If the applied gate voltage $V_G < V_{FB}$ (Flat band voltage), then the MOS capacitor is in

[✓ (a) Accumulation mode]

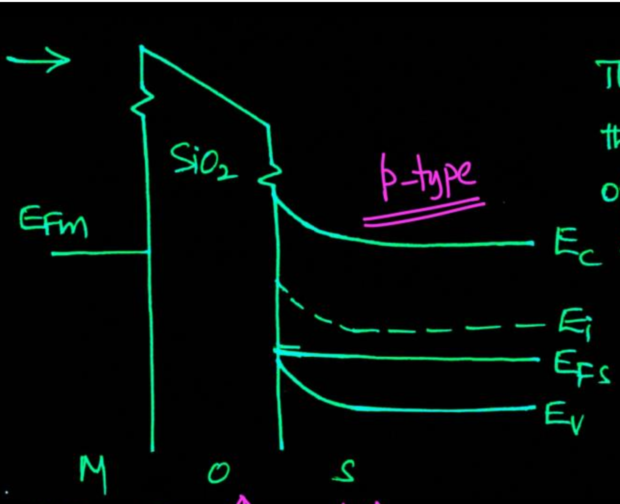
(b) Depletion mode

(c) Inversion mode

(d) Flat-band condition



→



$(E_i - E_{FS})\uparrow \Rightarrow p\uparrow$

The figure shows the band diagram of a MOS. The surface region of this MOS is in

$p = n_i e^{(E_i - E_{FS})/kT}$

(a) Inversion

[✓ (b) Accumulation]

(c) Depletion

(d) Flat band

→ An ideal MOS diode is fabricated on a p-type silicon substrate having a doping concentration of $N_A = 1\times10^{16}/cm^3$. If the gate oxide thickness (t_{ox}) is 1000 Å, calculate the maximum depletion width x_{dmax} and the threshold voltage V_T. $[(\epsilon_r)_{Si} = 11.9$ and $(\epsilon_r)_{SiO_2} = 3.9]$

$$|Q_d| = qN_A x_d$$

$$x_d = \frac{|Q_d|}{qN_A}$$

$$x_{dmax} = \frac{|Q_{dinv}|}{qN_A}$$

$$= \frac{\sqrt{2qN_A\epsilon_s\phi_t}}{qN_A}$$

$$\boxed{x_{dmax} = \sqrt{\frac{2\epsilon_s\phi_t}{qN_A}}}$$

$$\phi_t = 2V_t\ln\left(\frac{N_A}{n_i}\right) = 2\phi_F$$

$$\boxed{x_{dmax} = \sqrt{\frac{4\epsilon_s\phi_F}{qN_A}}}$$

$$\Phi_t = 2V_t \ln\left(\frac{N_A}{n_i}\right) = 2\times 0.026\times \ln\left(\frac{10^{16}}{1.5\times 10^{10}}\right) \simeq 0.7V$$

$$x_{dmax} = \sqrt{\frac{2\times 11.9\times 8.854\times 10^{-14}\times 0.7}{1.6\times 10^{-19}\times 10^{16}}} = \underline{0.304\,\mu m}$$

$$\boxed{1\mathring{A} = 10^{-10}m = 10^{-8}cm}$$

$$V_T = \frac{\sqrt{2qN_A\epsilon_s\Phi_t}}{C_{ox}} + \Phi_t\ , \quad C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.9\times 8.854\times 10^{-14}}{1000\times 10^{-8}}$$

$$= 34.5\ nF/cm^2$$

$$V_T = \frac{\sqrt{2\times 1.6\times 10^{-19}\times 10^{16}\times 11.9\times 8.854\times 10^{-14}\times 0.7}}{34.5\times 10^{-9}} + 0.7$$

$$\underline{V_T = 2.11V}$$

→ In the above problem if the gate oxide area is $\underline{1\times 10^{-4}}$ cm² then what is the maximum capacitance and minimum capacitance?

$$C'_{max} = C_{ox} \Rightarrow C_{max} = C_{ox}A$$

$$= 34.5nF/cm^2\times 10^{-4}\,cm^2$$

$$= \underline{3.45pF}$$

$$\boxed{\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_d}}$$

$$\frac{1}{C'_{min}} = \frac{1}{C_{ox}} + \frac{1}{C'_{dmin}}$$

$$C'_{dmin} = \frac{\epsilon_s}{x_{dmax}} = 35\,nF/cm^2$$

$$C'_{min} = 17.4\,nF/cm^2$$

$$C_{min} = C'_{min} \times A = \underline{\underline{1.74\,pF}}$$

$$\left.\begin{array}{l} C'_{max} \\ C'_{min} \\ C_{ox} \\ C'_{dmin} \end{array}\right\} \rightarrow F/cm^2$$

$$\left.\begin{array}{l} C_{max} \\ C_{min} \end{array}\right\} - F$$

→ An ideal MOS capacitor has boron doping concentration of $10^{15}/cm^3$ in the substrate. When a gate voltage is applied, a depletion region of width $0.5\,\mu m$ is formed with a surface potential of $0.2V$. Given that $\epsilon_0 = 8.854 \times 10^{-14}$ F/cm and the relative permitivities of silicon and silicon diode are 12 and 4 respectively, the peak electric field in the oxide region is ____

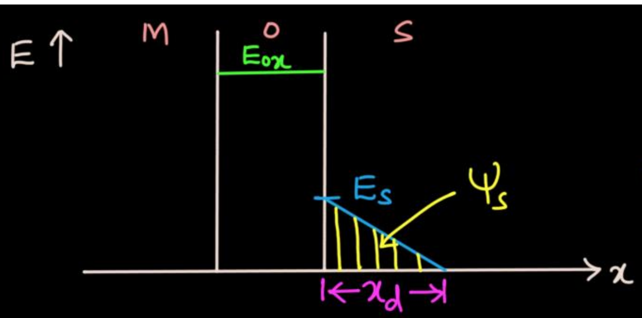
p-type

$N_A = 10^{15}/cm^3$

$x_d = 0.5\,\mu m$

$\psi_s = 0.2V$

$E_{ox} = ?$



$$\psi_s = \frac{1}{2} x_d E_s$$

$$\Rightarrow E_s = \frac{2\psi_s}{x_d}$$

$$= \frac{2\times 0.2}{0.5\mu}$$

$$= 0.8 \text{ V}/\mu m$$

$$\epsilon_{ox} E_{ox} = \epsilon_s E_s$$

$$E_{ox} = \left(\frac{\epsilon_s}{\epsilon_{ox}}\right) E_s$$

$$= \frac{12}{4} \times 0.8 \text{ V}/\mu m = \underline{\underline{2.4 \text{ V}/\mu m}}$$

→ A voltage V_G is applied across a MOS capacitor with metal gate and p-type silicon substrate at $T = 300K$. The "inversion carrier density" for $V_G = 0.8V$ is $2\times 10^{11}/cm^2$. For $V_G = 1.3V$, the inversion carrier density is $4\times 10^{11}/cm^2$. What is the value of the inversion carrier density for $V_G = 1.8V$?

The inversion charge is given by

$$(C/cm^2) \rightarrow Q_i = -C_{ox}(V_G - V_T)$$

$$Q_i \propto (V_G - V_T)$$

$$Q_i = q n_i' \Rightarrow n_i' \propto (V_G - V_T)$$

Inversion carrier density

$$\frac{n_{i1}'}{n_{i2}'} = \frac{(V_{G1} - V_T)}{(V_{G2} - V_T)}$$

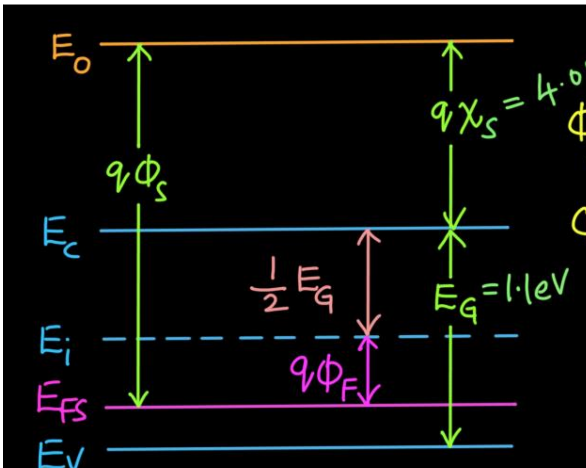
$$\frac{2\times10^{11}}{4\times10^{11}} = \frac{0.8 - V_T}{1.3 - V_T} \Rightarrow V_T = 0.3V$$

$$\frac{n_{i3}'}{2\times10^{11}} = \frac{1.8-0.3}{0.8-0.3}$$

$$\Rightarrow n_{i3}' = 6\times10^{11}/cm^2$$

→ An ideal MOS diode is fabricated on a p-type silicon substrate having a doping concentration of $N_A = 1\times10^{16}/cm^3$. what is the workfunction ϕ_s of the silicon substrate at room temperature? (For Si $E_G = 1.1eV$, $\chi = 4.05V$, $n_i = 1.5\times10^{10}/cm^3$ and $V_T = 26mV$ at 300K)

$\phi_s = ?$



$$\phi_s = \chi_s + \frac{E_c - E_{FS}}{q}$$

$$\phi_s = \chi_s + \frac{E_G}{2q} + \phi_F$$

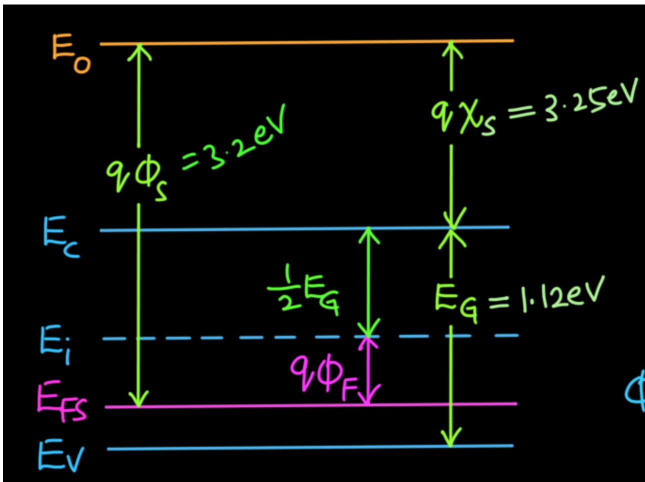
$$\phi_F = V_t \ln\left(\frac{N_A}{n_i}\right) = 0.35V$$

$$\phi_s = 4.05 + 0.55 + 0.35$$

$$\underline{\underline{\phi_s = 4.95V}}$$

→ Determine the metal-semiconductor work function difference $\underline{\underline{\phi_{ms}}} \overset{?}{=}$ for a given MOS system. Let $\underline{\phi_m = 3.2V}$ and electron affinity of semiconductor is $\chi_s = \underline{3.25V}$, energy gap of semiconductor is $E_G = \underline{1.12eV}$ and $\underline{N_A = 10^{15}/cm^3}$. Given $V_t = 25.9\ mV$

$\phi_{ms} = ?$



$$\phi_s = \chi_s + \frac{E_G}{2q} + \phi_F$$

$$\phi_F = V_t \ln\left(\frac{N_A}{n_i}\right) = 0.288V$$

$$\phi_{ms} = \phi_m - \phi_s$$

$$= 3.2 - (3.25 + 0.56 + 0.288)$$

$$= \underline{-0.898V}$$

→ Consider a MOS capacitor with p-type silicon substrate doped to $N_A = 10^{16}/cm^3$, a silicon dioxide insulator with a thickness of $t_{ox} = 200\,Å$ and a n^+ polysilicon gate. Assume $\phi_{ms} = -1.1V$ and the fixed charge $Q_f = 5\times10^{10}$ electronic charges per cm^2. (Given that ϵ_r of SiO_2 is 3.9). Calculate the Flat-band voltage.

$$V_{FB} = \phi_{ms} - \frac{Q_f}{C_{ox}}$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.9 \times 8.854 \times 10^{-14}}{200 \times 10^{-8}} = 1.726 \times 10^{-7}\ F/cm^2$$

$$V_{FB} = -1.1 - \frac{1.6 \times 10^{-19} \times 5 \times 10^{10}}{1.726 \times 10^{-7}}$$

$$= \underline{\underline{-1.15V}}$$

→ A MOS capacitor is fabricated on $\underline{\text{p-type}}$ silicon where the metal work function is $\overset{q\phi_m}{\underline{4.1eV}}$ and electron affinity of Si is $\overset{q\chi_s}{\underline{4.0eV}}$. $E_c - E_F = \underline{0.9eV}$. Oxide $\underline{\epsilon_r = 3.9}$ and $\underline{t_{ox} = 0.1\mu m}$. If the measured flat band voltage of this capacitor is $\overset{V_{FB}}{\underline{-1V}}$, then the magnitude of the fixed charge at $Q_f = ?$ the oxide-semiconductor interface is —

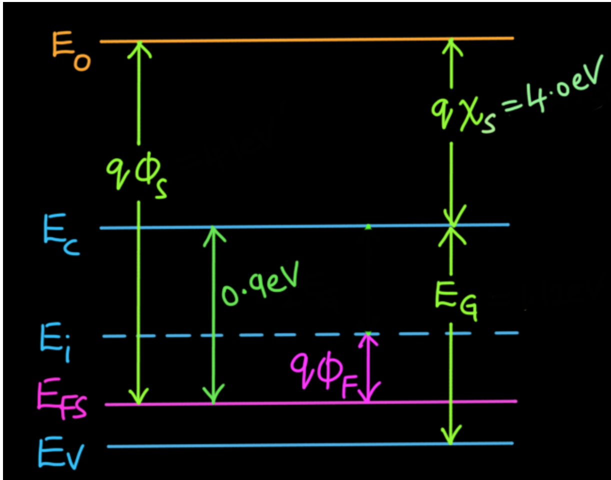
$$V_{FB} = \Phi_{ms} - \frac{Q_f}{C_{ox}}$$

$$Q_f = C_{ox}(\Phi_{ms} - V_{FB})$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.9 \times 8.854 \times 10^{-14}}{0.1 \times 10^{-4}} = 34.5\ nF/cm^2$$

$$\Phi_{ms} = \Phi_m - \Phi_s = 4.1 - 4.9 = -0.8V$$

$$Q_f = 34.5 \times 10^{-9}(-0.8 - (-1)) = \underline{\underline{6.9\ nc/cm^2}}$$



$$\Phi_s = \chi_s + \frac{(E_c - E_{FS})}{q}$$

$$= 4 + 0.9 = 4.9$$

→ Consider a p-type silicon substrate at $T = 300K$ doped to $N_A = 10^{15}/cm^3$. Let $t_{ox} = 12nm$ and the fixed charge at the interface is 10^{10} electronic charges per cm^2. Assume the oxide is silicon dioxide with $\epsilon_r = 3.9$, $V_t = 25.9mV$, $\phi_{ms} = -0.88V$, $n_i = 1.5\times10^{10}/cm^3$ and ϵ_r of silicon is 11.7.

$x_{dmax} = ?$

(1) Calculate the maximum depletion region width in the semiconductor.

(2) Find the threshold voltage. $V_T = ?$

$$x_{dmax} = \sqrt{\frac{4\epsilon_s\phi_F}{qN_A}}$$

$$\phi_F = V_t \ln\left(\frac{N_A}{n_i}\right) = 0.288V$$

$$x_{dmax} = \sqrt{\frac{4\times11.7\times8.854\times10^{-14}\times0.288}{1.6\times10^{-19}\times10^{15}}}$$

$$= \underline{\underline{0.864\,\mu m}}$$

$$\boxed{V_T = \frac{-Q_{dinv}}{C_{ox}} + 2\phi_F + V_{FB}}$$

$$Q_{dinv} = -qN_A x_{dmax} = -13.8\ nc/cm^2$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = 0.288\ \mu F/cm^2$$

$$V_{FB} = \phi_{ms} - \frac{Q_f}{C_{ox}} = -0.88 - \frac{1.6\times10^{-19}\times10^{10}}{0.288\times10^{-6}} = -0.885V$$

$$V_T = \frac{-(-13.8)}{0.288\times10^{-6}} + 2\times0.288 + (-0.885) = \underline{\underline{-0.261V}}$$

Pinch-off Voltage(V_p)